Axiomathes
https://doi.org/10.1007/s10516-020-09479-7

ORIGINAL PAPER

Quantum Epistemology from Mimicry and Ambiguity

Han Geurdes[1]

Received: 18 September 2019 / Accepted: 28 March 2020

Abstract
In the paper we look into the epistemology of quantum theory. The starting point is the previously established mathematical ambiguity. The perspective of our study is the way that Schrödinger described Einstein's idea of physics epistemology. Namely, physical theory is a map with flags. Each flag must, according to Einstein in Schrödinger's representation, correspond to a physical reality and vice versa. With the ambiguity transformed to quantum-like operators we are able to mimic quantum theory. Therefore we have created little flags. The question is raised whether nature itself is ambiguous. The created flags point at ambiguous nature. Or, nature is not ambiguous and the ambiguity can be repaired in mathematics.

Keywords Quantum mimicry · Mathematical ambiguity · Semantics of physical concepts (flags on the map)

1 Introduction

In the paper we will explore the structure of quantum theory to gain insight into the foundation of this science. Because physics is an old and established natural science, we claim that what we say here will affect chemistry, biology and fields of (social) science (Haven and Khrennikov 2013) where quantum physics is applied.

Let us start with some very basic mathematics. Nobody can doubt the arithmetic fact that, with the real number system at hand, $1 + 1 = (1/2) + (3/2)$. Furthermore nobody can doubt the arithmetic fact that $3 \times (1/2) = (1/2) \times 3$. It also is an elementary arithmetic fact that therefore $y^{1+1} = y^{1/2}y^{3/2}$ and that $y^{3/2} = \{\{y\}^3\}^{1/2}$ and/or $y^{3/2} = \{\{y\}^{1/2}\}^3$ because $3 \times (1/2) = (1/2) \times 3$.

To proceed to quantum mechanics. In that theory use is made of complex number wave functions that finally lead to real numbers in Born's probability interpretation.

✉ Han Geurdes
han.geurdes@gmail.com

[1] GDS Applied Mathematics, BVC vd Lijnstraat 164, 2593 NN Den Haag, the Netherlands

Published online: 17 April 2020

Therefore, we can make no objections to computations where complex numbers occur as an intermediate result, as long as the final result is in the real numbers.

As a response to the discussion, described in e.g. Howard (1985), revolving around Einstein's criticism on the completeness of quantum mechanics (Einstein et al. 1935) Bell wrote an important paper (Bell 1964). In short, Bell supposed that a distinction can be made between entanglement caused by classical hidden variables λ and the quantum mechanical description of entanglement. His basic correlation formula embraces all possible hidden variables models. In very general terms we have:

$$E(a,b) = \int_{\lambda \in \Lambda} \rho(\lambda) A(a,\lambda) B(b,\lambda) d\lambda \tag{1}$$

In this formula, Λ is the set of values for the λ. The a and b represent the unit length parameter vectors to measure ± 1 spin. The $\rho(\lambda)$ represents the distribution of the hidden variables λ. The latter is positive definite and normalized, $\int_{\lambda \in \Lambda} \rho(\lambda) = 1$. The $A(a,\lambda)$ and $B(b,\lambda)$ represent the measurements of the spin on two distant spin measuring instruments A and B. Bell, following EPR (Einstein et al. 1935) postulated local hidden variables and assumed that λ influences the outcome of measurement. Ideally, $A(a,\lambda) = \pm 1$ and $B(b,\lambda) = \pm 1$.

1.1 The Anti-axiom Ambiguity

In recent work we argued that Bell's formula is the source of concrete mathematical incompleteness. This work is supported by a complete counter model that can be found in Geurdes et al. (2017). The incompleteness of Bell's formula is demonstrated by connecting each possible valid physics model under the umbrella of Bell (1) to the *anti-axiom*; see e.g. (Yessenin-Volpin and Hennix 2001, p. 10). Below a short version of the connection of the *anti-axiom* to each Bell model, is presented.

Suppose we define a probability density for a single real variable $x \in \mathbb{R}$.

$$\rho_X(x) = \begin{cases} -x, & x \in [-1,0] \\ +x, & x \in [0,1] \\ 0, & otherwise \end{cases} \tag{2}$$

We have, $\rho_x(x) \geq 0$ for all $x \in \mathbb{R}$ and $\int \rho_x(x)dx = 1$. Then let us also define a kind of spin function: $\text{sign}(x) = 1$ when $x \geq 0$ and $\text{sign}(x) = -1$ when $x < 0$. We use, $\theta(x) = 1$ for $x \geq 0$ and $\theta(x) = 0$ for $x < 0$ and, $\text{sign}(x) = 2\theta(x) - 1$ (Lighthill 1966; Farasat 1996).

Please do subsequently note that Bell's formula can always be transformed with $\rho(\lambda, x) \to \rho(\lambda)\rho_X(x)$ and e.g. A with $A(a,\lambda,x) \to A(a,\lambda)\text{sign}(x)$. The $\rho_X(x)$ is defined in (2). There is no physical reason that may disallow it to happen. The reason is that there is no explicit physics theory behind the Bell formula. It can therefore not be excluded from experiment that the evaluation of each Bell formula is connected to the evaluation of the integral.

$$E = \int_{-1}^{1} |x| \text{sign}(x) dx \tag{3}$$

In Geurdes and Nagata (2019a, b) it was demonstrated that E is ambiguous. Key element in the argument is $|x|\text{sign}(x) = x\,\text{sign}(x) \times \text{sign}(x)$ in (3). In order to process the $\text{sign}(x) \times \text{sign}(x)$ we note that e.g.

$$\text{sign}(x) \times \text{sign}(x) = \{\text{sign}(x)\}^{1/2} \times \{\text{sign}(x)\}^{3/2} \tag{4}$$

is a possibility with complex numbers as intermediate result. There can be absolutely no objections against the use of complex numbers as intermediate result. As we already stated: $1 + 1 = 2 = \frac{1}{2} + \frac{3}{2}$.

In the evaluation of $\{\text{sign}(x)\}^{3/2}$ we may employ two principles to compute E. Let us study the following form

$$F(x) = \{\text{sign}(x)\}^{3/2} \tag{5}$$

Then, we can write down two principles

Principle 1 The evaluation of $\{\text{sign}(x)\}^{3/2}$ of the $F(x)$ of (5) is based on first the power 3 then the power 1/2.

Principle 2 The evaluation of $\{\text{sign}(x)\}^{3/2}$ of the $F(x)$ of (5) is based on first the power 1/2 then the power 3.

Note, furthermore, that $\text{sign}(x)$ can also be written as: $\exp[3i\pi(1 - \theta(x))]$. From the previous we can derive two conflicting forms for the E in (3). According to principle 1

$$\begin{aligned} E_1 &= \int_{-1}^{0} x\sqrt{\exp[3i\pi(1-\theta(x))]} \times \{\text{sign}(x)\}^{1/2} dx \\ &\quad + \int_{0}^{1} x\sqrt{\exp[3i\pi(1-\theta(x))]} \times \{\text{sign}(x)\}^{1/2} dx \\ &= \int_{-1}^{0} x(i \times i) dx + \int_{0}^{1} x dx = 1 \end{aligned} \tag{6}$$

and according to principle 2

$$\begin{aligned} E_2 &= \int_{-1}^{0} x\left(\exp\left[\frac{i\pi}{2}(1-\theta(x))\right]\right)^3 \times \{\text{sign}(x)\}^{1/2} dx \\ &\quad + \int_{0}^{1} x\left(\exp\left[\frac{i\pi}{2}(1-\theta(x))\right]\right)^3 \times \{\text{sign}(x)\}^{1/2} dx \\ &= \int_{-1}^{0} x((-i) \times i) dx + \int_{0}^{1} x dx = 0 \end{aligned} \tag{7}$$

Therefore, the ambiguity

$$\left[\{\text{sign}(x)\}^3\right]^{1/2} \not\equiv \left[\{\text{sign}(x)\}^{1/2}\right]^3 \tag{8}$$

cannot be pushed aside in favour of a preferred particular view concerning Bell type experiments.

Note that the *anti-axiom* shows from $(1/2) \times 3 = 3 \times (1/2)$. This implies ambiguity in any physics experiment that follows Bell's formula and inequalities derived thereof. We also note that in defending Bell it is meaningless to fall back on the computer challenge to reproduce the quantum correlation with a computer simulation of the Bell experiment. The physics connection of the anti-axiom to the formula destroys its meaning.

1.2 Epistemology Quantum Theory

In a letter from Schrödinger to Pauli, Schrödinger gives a description of Einstein's view of the epistemology of a physics theory. We quote from Howard (1990): "He (Einstein) has ...a map with little flags. *To every real thing there must correspond on the map a little flag, and vice versa*". Let us employ this nice picture to find out where in quantum theory is the place of the ambiguity. The question we raise here is related to Wigner's question why mathematics is so effective in describing nature (Wigner 1959).

In order to perform our study, we try to mimic quantum behaviour with the use of ambiguity based operators. Let us define for $y \in \mathbb{R}$ the operators $\widehat{P_{1/2}}$ and $\widehat{P_3}$

$$\begin{aligned} \widehat{P_{1/2}}y &= \{y\}^{1/2} \\ \widehat{P_3}y &= \{y\}^3 \end{aligned} \tag{9}$$

Note,

$$\widehat{P_{1/2}}(\widehat{P_3}(-1)) \not\equiv \widehat{P_3}(\widehat{P_{1/2}}(-1)). \tag{10}$$

The order of the operations is depicted with the brackets.

2 Quantum Mimicry

We ask the question whether it is possible that epistemological strange characteristics of quantum theory actually arise from ambiguity in mathematics. In order to study the role of the ambiguity in quantum theory, we aim to mimic some aspects of it basing ourselves on the ambiguity. Let us start in the first place to define a wave function for a time-of-observation interval $0 \leq t \leq T$ and a space variable $x \in \mathbb{R}$, with $-1 \leq x \leq 1$.

$$\psi(x,t) = e^{it}(\theta(x) - 1)C_{<} + e^{-it}\theta(x)C_{>} \tag{11}$$

For the ease of the presentation we repeat the definition already used above. The θ is defined by a Heaviside function

$$\theta(x) = \begin{cases} 1, x \geq 0 \\ 0, x < 0 \end{cases} \tag{12}$$

2.1 Normalization

The normalization reflects itself in the condition on the constants $C_<$ and $C_>$. We have,

$$|\psi(x,t)|^2 = \psi^*(x,t)\psi(x,t) = |\psi_<|^2 + |\psi_>|^2 \tag{13}$$

With $\psi_< = e^{it}C_<(\theta(x) - 1)$ and $\psi_> = e^{-it}C_> \theta(x)$. Equation (13) holds because from (12) it follows, $\theta(x)(\theta(x) - 1) = 0$. From, (11) we may deduce that $(x<0)$

$$|\psi_<|^2 = (\theta(x) - 1)^2 |C_<|^2 \tag{14}$$

and so,

$$\int_{-1}^{1} |\psi_<|^2 dx = |C_<|^2 \int_{-1}^{0} dx = |C_<|^2 \tag{15}$$

Furthermore, $(x \geq 0)$

$$|\psi_>|^2 = \{\theta(x)\}^2 |C_>|^2 = |C_>|^2 \tag{16}$$

Therefore,

$$\int_{-1}^{1} |\psi_>|^2 dx = |C_>|^2 \int_{0}^{1} dx = |C_>|^2 \tag{17}$$

and this together with (13) leads us to

$$\int_{-1}^{1} |\psi(x,t)|^2 dx = |C_<|^2 + |C_>|^2 = 1 \tag{18}$$

We can conclude that $|C_<|^2 + |C_>|^2 = 1$ is the condition alluded to previously.

2.2 Hamiltonian

In the second place, let us define an operator $\hat{H}(x,t)$.

$$\hat{H}(x,t) = ie^{it}C_<(\theta(x) - 1)\widehat{P_{1/2}}(\widehat{e^{-it}/C_<}) + \theta(x) \tag{19}$$

With $\hat{H} = \hat{H}_< + \hat{H}_>$. In the previous formula especially the wide hat symbols need some attention. The sequence

$$\widehat{P_{1/2}}(\widehat{e^{-it}/C_<})f(x,t)$$

means: First multiply *f*(*x*, *t*) with $e^{-it}/C_<$. Second, take the square root of that result. In symbols:

$$\widehat{P_{1/2}}(e^{-\widehat{it/C}_<})f(x,t) = \{(e^{-it}/C_<)f(x,t)\}^{1/2} = \sqrt{(e^{-it}/C_<)f(x,t)}$$

In a different form with $\widehat{P_3}$ this recipe gives

$$\widehat{P_3}(e^{-\widehat{it/C}_<})f(x,t) = \{(e^{-it}/C_<)f(x,t)\}^3$$

Let us subsequently suppose that $x<0$. Then the outcome of $\hat{H}(x,t)\psi(x,t)$ is

$$\hat{H}(x,t)\psi(x,t) = ie^{it}C_<(\theta(x)-1)\widehat{P_{1/2}}(e^{-\widehat{it/C}_<})e^{it}C_<(\theta(x)-1) \tag{20}$$

Hence, in $x<0$,

$$\hat{H}(x,t)\psi(x,t) = ie^{it}C_< \times (-1)\sqrt{-1} = -\psi(x,t) \tag{21}$$

because for $x<0$, we have $\theta(x)-1=-1$ and $\psi(x,t)=e^{it}(-1)C_< \equiv \psi_<$. Furthermore, for $x<0$ we also have

$$i\frac{\partial}{\partial t}\psi(x,t) = -\psi(x,t) \tag{22}$$

For $x\geq 0$ it subsequently is easy to acknowledge that $H(x,t)\psi(x,t)=\psi(x,t)$. Observe that $\psi_> = e^{-it}\theta(x)C_>$ which is $\psi_> = e^{-it}C_>$ for $x\geq 0$. So, $H(x,t)\psi_> = \psi_>$. Moreover, $i\frac{\partial}{\partial t}\psi_> = \psi_>$. Hence, for $T\geq t\geq 0$ and $-1\leq x\leq 1$ we may write the Schrödinger equation

$$\hat{H}(x,t)\psi(x,t) = i\frac{\partial}{\partial t}\psi(x,t) \tag{23}$$

and $\hat{H}(x,t)$ such as defined in (20). A kind of Schrödinger equation is derived from an ambiguity producing operator $\widehat{P_{1/2}}$. Below, a similar form is obtained for $\hat{D}$ in $x<0$.

2.3 Hermiticity

Hermiticity of an operator is an indication for a possible physical existence of the process measured with the operator. In the sense of Einstein; the flag stands where a physics reality is. We refer to (Merzbacher 1970, p. 155). Therefore, it is required to have

$$\int_{-1}^{1}\psi^*(x,t)\hat{H}(x,t)\psi(x,t)dx = \left\{\int_{-1}^{1}\psi^*(x,t)\hat{H}(x,t)\psi(x,t)dx\right\}^* \tag{24}$$

For the clarity of presentation let us write $\hat{H}_<(x,t)$ for $\hat{H}(x,t)$ and $\psi_<(x,t)$ for $\psi(x,t)$ when $x<0$ and $\hat{H}_>(x,t)$ for $\hat{H}(x,t)$ and $\psi_>(x,t)$ for $\psi(x,t)$ when $x\geq 0$. Hence,

$$\left\{\int_{-1}^{1}\psi^{*}(x,t)\hat{H}(x,t)\psi(x,t)dx\right\}^{*}=\left\{\int_{-1}^{0}\psi^{*}_{<}(x,t)\hat{H}_{<}(x,t)\psi_{<}(x,t)dx\right\}^{*} + \left\{\int_{0}^{1}\psi^{*}_{>}(x,t)\hat{H}_{>}(x,t)\psi_{>}(x,t)dx\right\}^{*} \tag{25}$$

Looking at the first term on the right hand of (25) we have for $x<0$

$$\left\{\int_{-1}^{0}\psi^{*}_{<}(x,t)\hat{H}_{<}(x,t)\psi_{<}(x,t)dx\right\}^{*} = \int_{-1}^{0}\left(\hat{H}_{<}(x,t)\psi_{<}(x,t)\right)^{*}\psi_{<}(x,t)dx \tag{26}$$

Because,

$$-\psi^{*}_{<}(x,t) = \left(\hat{H}_{<}(x,t)\psi_{<}(x,t)\right)^{*}=\hat{H}^{*}_{<}(x,t)\psi^{*}_{<}(x,t)$$

and so,

$$\left\{\int_{-1}^{0}\psi^{*}_{<}(x,t)\hat{H}_{<}(x,t)\psi_{<}(x,t)dx\right\}^{*} = -\int_{-1}^{0}|\psi_{<}(x,t)|^{2} = \int_{-1}^{0}\psi^{*}_{<}(x,t)\hat{H}_{<}(x,t)\psi_{<}(x,t)dx \tag{27}$$

Hence in $-1\leq x\leq 0$ the operator is Hermitian.

Looking at the second term on the right hand of (25) we see for $x\geq 0$

$$\left\{\int_{0}^{1}\psi^{*}_{>}(x,t)\hat{H}_{>}(x,t)\psi_{>}(x,t)dx\right\}^{*} = \int_{0}^{1}\left(\hat{H}_{>}(x,t)\psi_{>}(x,t)\right)^{*}\psi_{>}(x,t)dx \tag{28}$$

Because,

$$\psi^{*}_{>}(x,t) = \left(\hat{H}_{>}(x,t)\psi_{>}(x,t)\right)^{*}=\hat{H}^{*}_{>}(x,t)\psi^{*}_{>}(x,t)$$

it follows

$$\left\{\int_{0}^{1}\psi^{*}_{>}(x,t)\hat{H}_{>}(x,t)\psi_{>}(x,t)dx\right\}^{*} = \int_{0}^{1}|\psi_{>}(x,t)|^{2}dx = \int_{0}^{1}\psi^{*}_{>}(x,t)\hat{H}_{>}(x,t)\psi_{>}(x,t)dx \tag{29}$$

From the previous Eqs. (29) and (27) we can conclude that $\hat{H}(x,t)$ is Hermitian. This implies that a physical reality can exist behind the operator.

2.4 Commutation

2.4.1 Principle forms $\hat{H}_<$ and $\hat{D}_<$

Let us define an operator $\hat{D}_<$ for $x<0$, based on the cubic, as

$$\hat{D}_< = e^{it}(-1)C_<\widehat{P_3}(\widehat{e^{-it}/C_<}) \tag{30}$$

Note, for $x<0$, we have $\psi_<(x,t) = e^{it}C_<(\theta(x)-1)$. Hence,

$$\hat{D}_<\psi_<(x,t) = e^{it}C_<(-1)\{-1\}^3 = e^{it}C_< = -\psi_<(x,t) \tag{31}$$

Let us recall that $x<0$ implies from (19) that

$$\hat{H}_< = ie^{it}C_<(-1)\widehat{P_{1/2}}(\widehat{e^{-it}/C_<}) \tag{32}$$

Because of (10) we may suspect that $\left[\hat{H}_<,\hat{D}_<\right] \not\equiv 0$. We also note that $\hat{D}_<$ is Hermitian. With $\psi_<$ and observing (31), an equivalent of Eq. (27)

$$\begin{aligned}\left\{\int_{-1}^{0}\psi^*_<(x,t)\hat{D}_<(x,t)\psi_<(x,t)dx\right\}^* &= -\int_{-1}^{0}|\psi_<(x,t)|^2 \\ &= \int_{-1}^{0}\psi^*_<(x,t)\hat{D}_<(x,t)\psi_<(x,t)dx\end{aligned} \tag{33}$$

is valid for $\hat{D}_<$.

2.4.2 Operator Definition

Because the operators $\widehat{P_{1/2}}$ and $\widehat{P_3}$ are nonlinear, the introduction of a summation is a tricky enterprise. Let us introduce the operator $\sum$. We only will need sum of two terms. So, $\sum(a,b) = a+b$. Subsequently it is noted that $\widehat{P_3}\sum(a,b) \not\equiv \sum\widehat{P_3}(a,b)$, i.e. $(a+b)^3 \not\equiv a^3+b^3$. The operators are evaluated from right to left. So in e.g. $\sum\widehat{P_3}(a,b)$ first we have $\widehat{P_3}(a,b) = (a^3,b^3)$. Secondly, we have $\sum(a^3,b^3) = a^3+b^3$. Let us subsequently look at the following definition

$$\hat{\mathcal{H}}_<(\gamma) = \sum(\hat{H}_<,\gamma\hat{D}_<) = \hat{H}_< + \gamma\hat{D}_< \tag{34}$$

If this operator works on $\psi_<$ we see

$$\begin{aligned}\sum(\hat{H}_<,\gamma\hat{D}_<)\psi_< &= \sum(\hat{H}_<\psi_<,\gamma\hat{D}_<\psi_<) \\ \sum(-\psi_<,-\gamma\psi_<)) &= \psi_<\sum(-1,-\gamma) = -\psi_<(1+\gamma)\end{aligned} \tag{35}$$

The last step in the previous equation is vital. $\hat{D}_<$ and $\sum$ don't commute.

Let us look at the commutator of the Hamiltonian of (34) and $\hat{D}_<$ i.e.

$$\left[\hat{\mathcal{H}}_<(\gamma),\hat{D}_<\right]=\hat{\mathcal{H}}_<(\gamma)\hat{D}_<-\hat{D}_<\hat{\mathcal{H}}_<(\gamma) \tag{36}$$

We then have for the first term

$$\begin{aligned}\hat{\mathcal{H}}_<(\gamma)\hat{D}_<\psi_< &= \hat{\mathcal{H}}_<(\gamma)(-\psi_<)=\sum(\hat{H}_<,\gamma\hat{D}_<)(-\psi_<)\\ &=\sum\left(\hat{H}_<(-\psi_<),\gamma\hat{D}_<(-\psi_<)\right)=\sum(i\psi_<,\gamma\psi_<)=\psi_<(i+\gamma)\end{aligned} \tag{37}$$

The term $\hat{H}_<(-\psi_<)$ in the previous equation can explicitly be evaluated with,

$$\begin{aligned}\hat{H}_<(-\psi_<) &= ie^{it}C_<(\theta(x)-1)\widehat{P_{1/2}}(\widehat{e^{-it}/C_<})e^{it}C_<\\ &= ie^{it}C_<(\theta(x)-1)\widehat{P_{1/2}}1=i\psi_<\end{aligned} \tag{38}$$

In addition, the term $\gamma\hat{D}_<(-\psi_<)$ in (37) is explicitly

$$\begin{aligned}\gamma\hat{D}_<(-\psi_<) =&\gamma e^{it}(-1)C_<\widehat{P_3}(\widehat{e^{-it}/C_<})e^{it}C_<\\ =&\gamma e^{it}(-1)C_<\widehat{P_3}1=e^{it}(-1)C_<1^3=\gamma\psi_<\end{aligned} \tag{39}$$

The second term of (36) is $\hat{D}_<\hat{\mathcal{H}}_<(\gamma)$. Looking at (35) this term results in

$$\hat{D}_<\hat{\mathcal{H}}_<(\gamma)\psi_<=\hat{D}_<(-(1+\gamma)\psi_<)=(1+\gamma)^3\psi_< \tag{40}$$

Here,

$$\begin{aligned}\hat{D}_<(-(1+\gamma)\psi_<) =&e^{it}(-1)C_<\widehat{P_3}(\widehat{e^{-it}/C_<})e^{it}C_<(1+\gamma)\\ =&e^{it}(-1)C_<\widehat{P_3}(1+\gamma)=(1+\gamma)^3\psi_<\end{aligned} \tag{41}$$

Note again, $\hat{D}_<$ and $\sum$ do not commute. I.e. $\hat{D}_<(-(1+\gamma)\psi_<)\not\equiv \hat{D}_<(-\psi_<)+\hat{D}_<(-\gamma\psi_<)$.

Hence, the commutator of (35) is

$$\left[\hat{\mathcal{H}}_<(\gamma),\hat{D}_<\right]\psi_<=(i+\gamma)\psi_<-(1+\gamma)^3\psi_< \tag{42}$$

This implies $\left[\hat{\mathcal{H}}_<(\gamma),\hat{D}_<\right]\psi_<=i\psi_<$ when $\gamma\approx-2.3247$ with $|\epsilon|\approx 1.823\times10^{-7}$. With $\epsilon=\gamma-(1+\gamma)^3$. In (35) the Hamiltonian equation is an eigenvalue equation

$$\hat{\mathcal{H}}_<(\gamma)\psi_<=-(1+\gamma)\psi_< \tag{43}$$

which is a Hermitian operator for eigenfunction $\psi_<$.

2.4.3 Uncertainty

Now from the commutation $\left[\hat{\mathcal{H}}_<(\gamma),\hat{D}_<\right]\psi_<=i\psi_<$ it is possible to derive an uncertainty relation noticing that $\psi_<$ is not normalized but has

$$\int_{-1}^{1} dx|\psi_<|^2 = |C_<|^2 \tag{44}$$

and $|C_<|^2$ finite, positive and real. If, similar to the quantum theory we are trying to mimic, we write $\Delta\mathcal{H}_<$ for the uncertainty in $\hat{\mathcal{H}}_<(\gamma)$ and $\Delta D_<$ for uncertainty in $\hat{D}_<$, we have similar to quantum mechanics (Merzbacher 1970, p. 158 vv),

$$\Delta\mathcal{H}_< \Delta D_< \geq \frac{1}{2}|C_<| \tag{45}$$

The $|C_<|$ from the previous equation arises from (15) and (44). The physical measurements behind $\Delta\mathcal{H}_<$ and $\Delta D_<$ are unknown. But that does not at all mean that those flags on the map refer to nothing.

3 Conclusion and Discussion

The previous sections show that certain aspects of quantum theory can be construed from the mathematical ambiguity. This may shine a different light on what is generally called quantum weirdness. If we look at what Schrödinger wrote about flags on a map representing for Einstein a 1-1 relation between theory and physical entities, it can be observed that the *mathematical ambiguity* in the order of how $\widehat{P}_{1/2}$ and $\widehat{P}_3$ are operated on (-1) i.e.

$$\widehat{P_{1/2}}(\widehat{P_3}(-1)) \not\equiv \widehat{P_3}(\widehat{P_{1/2}}(-1)) \tag{46}$$

hides behind the uncertainty relation in (45). Note, $\{\{-1\}^3\}^{1/2} = i$ and $\{\{-1\}^{1/2}\}^3 = -i$. The uncertainty for position and momentum measurement (Merzbacher 1970) holds physical reality. We may, hence, wonder if the flags construed inside the quantum formalism we mimic here, represent something in the physical reality. Let us please observe that the connection of the concepts with the to-be-explained phenomena in the foundation of science is not established fact. In terms of the map-and-flags. Nobody knows if flags and reality are connected or perhaps even that flags create reality or are introducing a reality on an other map.

The finding of the anti-axiom connection to each Bell experiment may appear as a negative result. Nevertheless, to the author, the question raised here is interesting. Is the ambiguity an ambiguity of nature or of our language to describe nature? The former concurs with Wigner's idea that mathematics effectively describes physical nature. If the ambiguity in mathematics reflects an ambiguity of nature, then perhaps there are ways to find this in experiment. The concept that the language is the limiting factor concurs with Wittgenstein's view of philosophy (Wittgenstein 1958, lemma 119). There is actually no reason whatsoever why Wittgenstein's lemma 119 might not be true for a theory of natural sciences as well. Another point we can raise is the following. If successful application of mathematics in science is an important trigger for the objectivity of mathematical knowledge Molinini (2019), then what is the ambiguity and its quantum theoretical application telling us about mathematical knowledge?

References

Bell JS (1964) On the Einstein Podolsky Rosen paradox. Physics 1:195

Einstein A, Podolsky B, Rosen N (1935) Can quantum-mechanical description of physical reality be considered complete? Phys Rev 47:777

Farasat F (1996) Introduction to generalized functions with applications in aerodynamics and aeroaucoustics. NASA

Geurdes H, Nagata K (2019a) A question about the consistency of Bell's correlation formula. arXiv:1907.00621v1

Geurdes H, Nagata K (2019b) Bell's correlation formula and anomalous spin. arXiv:1907.0448v1

Geurdes H, Nagata K, Nakamura T, Farouk A (2017) A note on the possibility of incomplete theory. arxiv:1704.00005

Haven E, Khrennikov A (2013) Quantum social science. Cambridge University Press, Cambridge

Howard D (1985) Einstein on locality and separability. Stud Hist Philos Sci 16:171–201

Howard D (1990) Nicht sein kann was nicht sein darf or the prehistory of EPR 1909–1935. In: Miller A (ed) Sixty-two years of uncertainty. Plenum, New York

Lighthill MJ (1966) Einfürung in die Theorie der Fourieranalysis und der verallgemeinerten Funktionen: chap 2, Die theorie der verallgemeinerte Funktionen. Hochschultaschenbücher, Mannheim

Merzbacher E (1970) Quantum mechanics. Wiley, New York

Molinini D (2019) The weak objectivity of mathematics and its reasonable effectiveness in science. Axiomathes. https://doi.org/10.1007/s10516-019-09449-8

Wigner E (1959) The unreasonable effectiveness of mathematics in the natural sciences. Commun Pure Appl Math 23:1–4

Wittgenstein L (1958) Philosophical investigations. Basil Blackwell, Oxford

Yessenin-Volpin AS, Hennix C (2001) Beware of the Gödel-Wette paradox. arXiv:0110094v2