Classical Probability, Shakespearean Sonnets, and Multiverse Hypotheses

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Abstract

We evaluate classical probability in relation to the random generation of a Shakespearean sonnet by a typing monkey and the random generation of universes in a World Ensemble based on various multiverse models involving eternal inflation. We calculate that it would take a monkey roughly 10^{942} years to type a Shakespearean sonnet, which pushes the scenario into a World Ensemble. The evaluation of a World Ensemble based on various models of eternal inflation suggests that there is no middle ground between eternal Poincare-Zermelo recurrence and a 0 probability in regards to the natural generation of the initial conditions of the universe.

1. Introduction

We evaluate classical probability in relation to the random generation of a Shakespearean sonnet and the random generation of universes in a World Ensemble based on various multiverse models involving eternal inflation. According to tradition, Thomas Henry Huxley explained the evolutionary importance of accidents and random events with the scenario of a monkey, whom we will call “M,” who would randomly type a Shakespearean sonnet if given enough time. Below is a hypothetical calculation for this creative event.

2. Shakespearean Sonnet by Monkey

All sonnets have 14 lines, and each line averages 8 words, and the standard word has 5 letter-spaces. The average sonnet, consequently, has 560 letter-spaces, and the typical typewriter has about 50 keys, and M has a keyboard with 50 unbiased keys while we ignore capital letters and other shift-key options for the purpose of mathematical simplicity. Assuming classical probability and the above variables, the probability of 560 consecutive random strikes on a keyboard resulting in 1 of the 154 Shakespearean sonnets = $154/50^{560} \approx 10^{-949}$. For the purpose of a binomial experiment, $\pi = 10^{-949}$, where π is the proportion of success for typing a Shakespearean sonnet with a trial consisting of 560 consecutive random strikes on a keyboard. If M types 50 words averaging 5 letter-spaces per minute for 40 hours per week and 50 weeks per year, then M types 3×10^{7} letter-spaces per year. The trials can overlap without significantly skewing the probabilities based on binomial theory because the probability that M would type 2 or more Shakespearean sonnets within 560 trials is approximately 10^{-1893} so each strike of the keyboard effectively begins an independent trial of 560 random strikes, except for the last 559 strikes. If M has 14 billion years (BY), M types roughly 10^{17} letter-spaces and $n_e \approx 10^{17}$ with $P_{(\geq 1)} \approx 10^{-932}$, where n_e is the effective number of independent trials and $P_{(\geq 1)}$ is the probability of 1 or more successes. Unfortunately, 14 BY are improbably enough time. Perhaps a supervisor could push M to type 70 words per minute for 100 hours per week for a billion BY (10^{18} years), then $n_e \approx 10^{26}$ with $P_{(\geq 1)} \approx 10^{-923}$, yet a billion BY are improbably enough time. Fortunately, 10^{941} years makes $n_e \approx 10^{949}$ with $P_{(\geq 1)} \approx .63$ and 10^{942} years makes $n_e \approx 10^{950}$ with $P_{(\geq 1)} \approx .99995$, but

M would have to be older than the possible sum of the years of all solar systems in the universe and the signal-to-noise ratio would make it nearly impossible to find a sonnet among all of the typing. Moreover, M lacks enough matter to type 10^{950} letter-spaces. For example, if M types 10^{100} letter-spaces on every neutron and proton in the universe, M would fall short of typing 10^{950} letter-spaces by 770 orders of magnitude.

The above literary analogy depends upon a limited number of possibilities for each trial. For example, M has only 50 keys to strike. If on the other hand, M scribbles in sand, then generating a Shakespearean sonnet with 10^{942} years of random sand scribbles would be an improbability instead of the near determinism of the typing scenario. Moreover, the typing scenario requires googols and googols of universes with monkeys and intelligently designed typewriters or word processors that produce the English alphabet.

3. Eternal Inflation in a Multiverse

Physicists propose various models of World Ensembles to explain the probabilities related to the apparently delicately tuned features of the observed universe[1] such as various models of eternal inflation.[2] And now we see that a World Ensemble is also needed to explain the scenario of a monkey typing a Shakespearean sonnet. Here we will use classical probability to examine some models of eternal inflation that form a multiverse. We will examine probabilities related to the generation of initial conditions similar to the initial conditions of the observed universe, which we will call *i,* where *i* is the value for the same initial conditions of the observed universe, give or take an order of magnitude of 1 for the values of all physical constants and mass and initial entropy (an arbitrary decision).

Since we do not know the values of *i* and π_i, we will consider two hypothetical values for π_i, where π_i is the proportion for a universe birth generating the value *i.* In the first case we will use the proportion from Penrose,[3] where $\pi_i = 1$ in $10^{(10^{123})}$. (Penrose calculated the probability for generating a universe similar to the observed universe with the following considerations: the universe has 10^{80} baryons with maximum entropy in a big crunch of 10^{43} photons per baryon while the entropy of the early universe was less than 10^{8} photons per baryon. And in this case, the universe is roughly 14 billion years old while there are roughly 10^{11} galaxies while each galaxy averages 10^{11} stars.) In the second case, we will assume that $\pi_i = 1$ in ∞. And we consider the second case because if the proportion for the generation of the required value for at least one physical constant would equal 1 in ∞, then we need $\pi_i = 1$ in ∞.

3.1 Eternal Inflation without a Beginning

Here we will look at a hypothetical model of inflation without a beginning by Aguirre and Gratton, where the values for the physical constants of the universe are from eternal past and there already has been an infinite number of universe births.[4] And since the physical constants of the observed universe are from eternal past, we only need to calculate for the generation of the mass and initial entropy. So $\pi_i = 1$ in $10^{(10^{123})}$. And since there already has been an infinite number of universe births, then $n_i = \infty$, where n_i is the number of trials of universe births. Likewise, in this model, there would be infinite number of universes with the value *i.* So there would be an infinite number of universes similar to the observed universe, which implies eternal Poincare-Zermelo recurrence.

Now we will look at a hypothetical model of inflation without a beginning where $\pi_i = 1$ in ∞ and there already has been an infinite number of universe births so $n_i = \infty$. Unfortunately, calculations with $\pi_i = 1$ in ∞ and $n_i = \infty$ will lead to arguments about various answers, but we attempt to outline various potential answers. First, we need to review some of the related difficulties. For example, i is an interval with a finite range while there are an unlimited number of intervals within the range of i. We will illustrate this by looking at the hypothetical interval of Gravitation Length (L_{pl}) that is required for a universe similar to the observed universe as defined by Carroll,[5] where the required value for L_{pl} is in the order of 10^{-32}. In this case in the context of classical probability, L_{pl} is 1 of an infinite number of successive intervals that has a range of 9 x 10^{-32}. On the other hand, it would make no difference if $L_{pl} \sim 10^{-100}$ or 10^{100} because in each of the cases the range of L_{pl} could be divided into an unlimited number of proportionately smaller intervals while each respective interval would be 1 in an infinite number of successive intervals. (Here is a brief review of an infinite regression: there are an infinite number of fractions from 10 to 1, and from 1 to .1, and from .1 to .01, and ad infinitum.)

Here we attempt the challenge to calculate for the mean of the distribution (μ_i) and the standard deviation (σ_i) when $\pi_i = 1$ in ∞ and $n_i = \infty$, where we calculate μ_i and σ_i as follows:

(1) $$\mu_i = n_i\pi_i$$

(2) $$\sigma_i = \sqrt{\pi_i(n_i - \mu_i)^2}$$

Each of these equations could arguably have three answers while we may never gain a unanimous agreement on the correct answer. In the case of $\mu_i = n_i\pi_i$, then ∞ x $1/\infty = 0$; or ∞ x $1/\infty = 1$; or ∞ x $1/\infty = \infty$. If the various calculations of ∞ x $1/\infty$ for the mean are consistently used for the respective calculations of the standard deviation, then the mean will equal the standard deviation in the context of these calculations. Likewise, when $\mu_i = 0$ then σ_i is irrelevant, and when $\mu_i = 1$ then $\sigma_i = 1$, and when $\mu_i = \infty$ then $\sigma_i = \infty$.

In the method where $\mu_i = 0$, then $P_{(i\geq 1)} = 0$, where $P_{(i\geq 1)}$ is the probability of generating at least 1 universe with i. And in the method where $\mu_i = \infty$ with $\sigma_i = \infty$, then there would be an infinite number of universes with the value i, which implies eternal Poincare-Zermelo recurrence. And in the method where $\mu_i = 1$ with $\sigma_i = 1$, then $P_{(i\geq 1)} \approx .63$ as suggested according the calculation of $P_{(\geq 1)}$ for the Shakespearean sonnet or $P_{(i\geq 1)} \approx .68$ as suggested by the normal distribution. And since all intervals are subject to an unlimited regression of proportionately smaller intervals, then there is an unlimited number of intervals within the range of i while each of these proportionately smaller intervals would also have a .63 probability of being generated at least once. So according to the unlimited regression of proportionately smaller intervals, $\mu = 1$ with $\sigma = 1$ regresses to an infinite number of universes similar to the observed universe, which implies eternal Poincare-Zermelo recurrence.

The calculation for the unlimited regression of proportionately smaller intervals, however, depends upon an imaginary smallest interval because there is no real smallest interval. For example, we define the smallest interval as in the order of i_i, where the value for $i_i = 10^{-\infty}$. Likewise, the size of the range for $\sim i_i$ is r_i, where $r_i = (10 \text{ x } i_i) - i_i = 0$. So the value of r_i is comparable to the substance of a geometric point. For example, we will never identify the smallest real fraction.

Since r_i is comparable to the substance of a geometric point, then an infinite regression of r_i is also comparable to the substance of a geometric point. And this suggests that ∞ x $1/\infty = 0$. And this also suggests that an infinite set of universes with every potential value for i would be an imaginary set of universes.

3.2 Eternal Inflation with a Beginning

On the other hand, several models of eternal inflation have an ultimate beginning. And in the case of eternal inflation with an ultimate beginning, there would always be a finite number of universe births though the eternal inflation never ends.

When eternal inflation with a beginning faces π_i = 1 in $10^{(10\text{^}123)}$, then $10^{(10\text{^}123)}$ universe births or $n = 10^{(10\text{^}123)}$ would result in $P_{(i \geq 1)}$ = .63 while $n = 10 \times 10^{(10\text{^}123)}$ would result in $P_{(i \geq 1)}$ = .99995. In this case, eternal inflation with a beginning would eternally generate universes similar to the observed universe, which implies eternal Poincare-Zermelo recurrence.

When eternal inflation with a beginning faces π_i = 1 (or any finite number > 0) in ∞, then n will always be a finite number so $P_{(i \geq 1)}$ = 0. In this case, eternal inflation with a beginning does not help cosmogony to explain the apparently delicately tuned features of the observed universe.

4. Discussion

This examination of eternal inflation assumes classical probability and suggests that there is no middle ground between eternal Poincare-Zermelo recurrence and a 0 probability for the natural generation of a universe similar to our universe. We also clarify that a hypothesis of eternal inflation may not follow classical probability were all potential intervals have the same probability. And some of the cases with an infinite number of universes that do not follow classical probability could totally miss the value for *i.* We will illustrate this by describing sets with an infinite number of values that exclude other sets with an infinite number of values. For example, the infinite number of fractions between 2 and ∞ has nothing to do with the infinite number of fractions between 0 and 1 while there are no more fractions between 2 and ∞ than between 0 and 1. Likewise, there are an infinite number of scenarios where an infinite number of values for *i* would not include all potential values for *i.*

In the case of events with a 0 proportion, we can appreciate the irony that events with a 0 proportion occur every Planck time. And here is a hypothetical example of an event with a 0 proportion. In this event, there is no wind while we randomly toss a tablespoon of dry grains of sand on a smooth and level meter-square tabletop. (The quantity of sand and size of the tabletop are an arbitrary decision). So in this event, there would be an infinite number of potential spatial arrangements for the end result of each sand toss. And all potential outcomes have a 0 probability while the sand toss results in an outcome with a 0 probability.

Now that we examined some probability related to various World Ensemble models, we will look back to the scenario of the Shakespearean sonnet by monkey. The 10^{942} years of random typing makes the scenario meaningless to the observed universe. It can only make sense in an unlimited multiverse or other type of unlimited World Ensemble with eternal Poincare-Zermelo recurrence for generating universes with monkeys and intelligently designed typewriters or word processors that produce the English alphabet. However, at this point in the history of scientific thought, we do not know if there is a World Ensemble. And we are uncertain about the probabilities related to three critical factors: 1) assuming a World Ensemble, we do not know the probabilities related to the ultimate origin of the World Ensemble; 2) assuming the origin of a World Ensemble, we do not know the probabilities related to the origin of the initial conditions of the observed universe; 3) assuming the initial conditions in the observed universe, we do not know the probabilities related to the origin of intelligently designed typewriters or word processors that produce the English alphabet.

References

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[2]For examples, see J. Garriga and A. Vilenkin, "A Prescription for Probabilities in Eternal Inflation," *Physical Review D* 61, 023507 (2000), URL = <http://arxiv.org/abs/gr-qc/0102090>; A. Aguirre and S. Gratton, "Inflation without a Beginning: A Null Boundary Proposal," *Physical Review D* 67, 083515, (2003), URL = <http://arxiv.org/abs/gr-qc/0301042>; L. Susskind, "The Anthropic Landscape of String Theory," *High Energy Physics – Theory* (Feb, 2003), URL = <http://arxiv.org/abs/hep-th/0302219>; S.M. Carroll. "Is Our Universe Natural," *High Energy Physics – Theory* (Dec, 2005), URL = <http://arxiv.org/abs/hep-th/0512148>.
[3]R. Penrose, "Time-Asymmetry and Quantum Gravity," in *Quantum Gravity 2: A Second Oxford Symposium* eds. C.J. Isham, R. Penrose, and D.W. Sciama (Oxford: Clarendon Press, 1981), 245-72.
[4]A. Aguirre and S. Gratton (2003).
[5]S.M. Carroll (2005).