Being There and Getting There:

A View on the Nature and Application of Models

by

William M. Goodman, Ph.D.

**ABSTRACT**

This paper updates a previously-presented model of models, which can be used to clarify discussion and analysis in a variety of disputes and debates, since many such discussions hinge on displaying or implying models about how things are related. Knowing about models does not itself supply any new information about our world, but it might help us to recognize when and how information is being conveyed on these matters, or where possibly it is being obscured. If a claim P is expressed in the context of a certain model, a disagreement about P may actually not be about whether the underlying model is accurate; it could reflect misunderstandings about the model’s assumptions or conventions. Also, in practice, constructed models are rarely fully elaborated for every detail, so misunderstandings can arise from apparent “gaps” in the model. In that case, developing and considering complementary versions of models can sometimes clarify ambiguities. Following the main analysis are several extended Case examples to illustrate the wide applicability of the models approach, for clarifying, if not necessarily “resolving”, numerous debates and issues. For example, a famous “counter example” by Nelson Goodman to a logical model proposed by Carnap is shown to be not as deal-breaking as presumed historically, if only plausible complementary models had been explored. And models sometimes presented as competing in Artificial Intelligence research— “Declarative” versus “Procedural”—can instead be viewed as good examples of “complementary” models. Also reconsidered in this light are some well-known, competing positions, by Russell, Strawson, and Kripke, in the literature regarding names, pointing, definite descriptions and “using” sentences, and so on. All these concepts are locatable within the presently proposed models analysis. Questions such as where “truth” resides (whether in sentences or in uses of sentences) are not settled in this paper; yet, practical questions about how testable claims about the world can be expressed by models are clarified.

**Keywords: Models. Modeling. Declarative versus Procedural Models. Carnap. Hofstadter. Russell. Strawson. Kripke.**

**INTRODUCTION**

This paper presents a general view of the nature of models and modeling. The proposed model of models is relatively simple, yet it provides a broadly applicable framework for understanding, and sometimes helping to resolve, numerous problems and disputes arising in philosophy, artificial intelligence (AI) research, and other areas.

AI researchers, for instance, have debated whether human knowledge is best represented “declaratively" or "procedurally”. In the former, knowledge is modeled as effectively a large body of stored *facts*. The role of intelligence is manipulating those facts in some general ways (similar to how formal logics “manipulate” a large body of theorems using a small number of rules). A "procedural" position, on the other hand, holds that much knowledge is not a collection of distinct memory contents, but rather a body of procedures which the mind executes (or is able to execute). For instance, my “knowing how to multiply 28 x 24" lies in my *performing* this multiplication, not in recalling contents from a large “times table” in memory. Terry Winograd convincingly argues, against these two extremes, for balanced roles for *both* procedural and declarative models of knowledge: To know a fact, there must be processes in place to learn and use it; yet, procedures require data on which to operate.

This balanced declarative/procedural distinction, however, is not limited to modeling knowledge, but is one of many useful observations that can be made about models—of *any* objects—in general. That specific pairing is an example of what can be called “complementary” versions of models. Since few, if any, constructed models are fully elaborated for every detail, misunderstandings can arise about a model’s interpretation or application. So, developing and considering complementary versions of models in tandem can often help clarify ambiguities.

This paper presents the basic elements for a proposed analysis of models, in an outline form. It consists of six “claims” and four “definitions”. There follow some extended Case examples where the analysis could be applied (and/or has been applied by the author). With that goal, I wade into some famous historical debates—such as Nelson Goodman’s “counter example” to Carnap about a knowledge-construction model, or Russell-Strawson-Kripke on naming and definite descriptions. These cases show how the proposed model of models might shed light on those discussions, if not necessarily settle them.

**ON MODELS**

Definition 1:

A "model" is a (usually) complex symbol whose central effect is to specify or illustrate the relations which hold between a number of actual or possible states, entities, persons, or conditions.

According to this definition, what a model "specifies or illustrates” are the *relations* among things, rather than those things as such. A New York State road map, for instance, shows that the Hudson River stands to Buffalo in the geographical relationship "east", yet the map has nothing to say about Buffalo "in itself". The map portrays the geographical relations among things, such as streets, bridges, rivers, and buildings.

Other examples of models can be found in "schematic diagrams" for electronics hardware or “flowcharts" for computer software. Each of these illustrates the relations (in terms of signal or program flow) between different segments of a product or program. Similarly, a blueprint is a model which may either illustrate the layout of an already-constructed building or else specify in advance what that layout should be after its construction (where "layout" signifies the spatial relationships among the indicated doors, walls, windows, and so on).

In principle, almost any type of symbolic indicating device could be called a "model", since each could be described as "illustrating the relations which hold between a number of actual or possible states (or) entities". Even a simple fuel gauge, for instance, shows the relation which currently holds between the volume of gasoline in a fuel tank and the volume of that tank itself. But clearly devices of this sort (or, more accurately, the symbolic, displayed outputs from such devices) are the weakest of all possible models. For these symbols show the relation between only two or three things or states (such as "the fuel’s volume vs. the tank's volume", or, in the case of a household thermometer, "today’s temperature here vs. the freezing and boiling temperatures of water at sea level"). Against this, a weather map is a model which can show the relations of temperatures across a whole continent.

Claim 1:

"To model”, or to engage in "modeling", is to establish a relation over three distinct arguments:

1) The OBJECT which is modeled;

2) The MODEL proper; and

3) The CONTEXT-DOMAIN of the model.

*To say that this relation has been established is to affirm that (3) (the context-domain) has transformed (2) (the model, proper), into an apparent* claim *about (1) (the object).*

Claim 1 presents the heart of this account. “To model” an object requires more than simply having, or creating, an entity which is intended to satisfy Definition 1. Modeling requires that there be an "object" being modeled. And there must be a "context-domain" in which the modeling attempt occurs. In this section, I attempt to support the above claims, and elaborate on the meaning of my terms.

The OBJECT:

In conventional terms, an accurate model elucidates facts and relationships in the world. If someone has “knowledge” about an object, then a corresponding model attempts to represent that object in the way that accords with what one knows about it. This is not intended as a deep argument about ontology or epistemology. Claim 1 does not require specific opinions on whether a model can be fully accurate in representing knowledge of something real, or on how (or if) its accuracy could ever be fully known or confirmed. Yet, models are used conventionally *as if* they point to objects of potential knowledge, in the above sense.

For example, suppose I claim I know the musical range of a harmonica. One way to express this knowledge would be to employ musical staff notation, with the harmonica-playable notes depicted on the appropriate lines of the staff. The object of my purported knowledge is the range of notes actually playable on a harmonica, and this is also the object of my corresponding model. Note that this "object" is not a simple unity, but a relation among playable sounds or notes on the instrument.

Not all models point to known or knowable objects in such a straightforward way. I am thinking, for instance, of the map/model for "The Island of Atlantis" which R.G. Bury includes his translation of Plato’s dialogue, *Critias*.1 I doubt that either Plato or Bury wished to convey as knowledge or facts that Atlantis actually existed, or had the described or depicted appearance.

There are countless reasons that people offer descriptions—or models—of objects. If Socrates says “The plain of Atlantis is surrounded by a canal," he *may* be intending to express genuine knowledge. On the other hand, he may be lying, or making a wild guess, or writing in the context of fiction. In such cases, no knowledge is really being expressed (at least not literally); though the speaker may hope that the listener or reader will spend some time thinking, speculating, or pretending that it is.

The case is analogous with regard to models. The "default assumption" of presenting a model, such as Bury’s map of Atlantis, is that it is making a straightforward claim about an object. But actually, one could be using the model for purposes of fiction, speculation, or falsification. In such cases, the model still has an *apparent* object (from the viewpoint of its structure *as a model*), but does so in the same qualified sense that incomplete, feigned, illusory expressions of knowledge have an object. That is, one can present a model *just to make you think* (or hope, or wonder if) it has a real object, as depicted--just as one can make propositional statements to make you think, hope, or wonder if some real knowledge is being expressed. Either way, the object of modeling is describable as that about which the model either does, or tries to, or pretends to, inform us.

The MODEL, proper:

The model proper is the collection of symbols or procedures intended to convey the model’s apparent claim about the object. For example, a blueprint might serve as a model to represent architectural features of the Kremlin. Alternatively, one could construct in plaster a “scale model” of the architectural features. In either case, the model’s features symbolically portray some salient relationships within the object itself.

The CONTEXT-DOMAIN:

 In practice, a model is always employed in a specific context. I might print out a musical score using a printer that has only blue ink—but this has no relevance for understanding the model of the music. But if the score’s modeller and reader do not share common assumptions about musical staff notation in general, the model cannot do its job. The set of potential contexts (of shared assumptions, available implementation options, etc.) in which a model could be used intelligibly is the model’s *context-domain*. (This concept is similar to what in AI is called the “frame” that must be specified when employing representations of knowledge.) A model’s context-domain is essentially a *family of symbols or prior-accepted models* within which the information contained in the given model can be interpreted.

In the example of modeling the Kremlin's architecture, use of a blueprint presupposes a prior familiarity with symbols commonly found a blueprint, and an acceptance of their common interpretation. If Samantha interprets a symbol on the model in an idiosyncratic fashion, then the model proper might still be generated, but the drawing might not serve for her as a useful, or even an accurate, model of the intended object. Even building a Scale Model still requires a shared symbol-domain to be effective: How a viewer interprets the model depends on whether he or she accepts to ignore the overall physical size of the model and the materials of the model’s construction, and focus only on its features’ relative sizes and positions and shapes.

In short, for a model to be truly communicative of the relations implicit in the object, it must rely upon a Context-Domain of symbols which is not presently under scrutiny or dispute by those who are trying to work with and learn by the model.

Claim 2: Models can be 'used' or 'displayed'.

 The use/display distinction for models is roughly analogous to the use/mention distinction of linguistic analysis. Figure 1, below, "displays" (i.e., brings to the reader’s explicit attention) a model for a mythical village, Modelville. Dialogue 1, on the other hand, relates a conversation in which the same model is “used" or implied, but is not explicitly displayed.

In general, to use a model is to make directly some claim which could also be made by displaying that model in full. The model-user (as opposed to a model-displayer) avoids the requirement of producing the full model, and pointing to specific features of interest, as it were. Nonetheless, he or she is still *presupposing* features of the (potentially) displayable model, and of its underlying context-domain.



 **Figure 1:** The Model of Modelville (Displayed)

 Algernon: How do I get from City Hall to Whynot Hall?

 Bertha: Take Northmount Street south to Whyheck Street; then turn left and proceed forward. Stop at the bend in the road.

 **Dialogue 1:** A Use of the Model of Modelville

To say that Bertha, who is speaking in quite conventionally in Dialogue 1, is "modeling" may seem odd. This is because she does not explicitly display the model that she is using. Nonetheless, her reply is dependent upon the model; and the model is implicit in her reply. This should become clearer in the following section.

Claim 3: There are three ways to dispute a model.

There are three ways that Algernon could object to Bertha's reply in Dialogue 1. This is instructive because, I would argue, these three "ways" are, in fact, the three types of objections which can be raised against *any* model—whether used or displayed. If Algernon takes one of these tacks, he is not disputing Bertha's words, per se, but rather disputing the model she is using.

🡺 Objection 1: One can deny that the presumed context-domain (or relevant part of it) used for constructing the model is applicable. This approach effectively halts fruitful discussion based of the model. For example:

Algernon: "But Northmount Street has been entirely under water since Hurricane Irma!”

Clearly the context-domain for Figure 1’s model includes assuming that all roads depicted there can actually be travelled on. If this assumption is not tenable (since Algernon is not travelling by boat), then the model does not convey useful information for him.

 🡺 Objection 2: One can accept the assumptions of the context- domain, but argue technical points about the vocabulary for referring to it. For instance:

Algernon: "Whyheck Street! I thought the only 'Whyheck’ in Modelville is called 'Whyheck Crescent'?"

Here the dispute is strictly about nomenclature. Once Algernon and Bertha can agree on what to *call* the road which Bertha clearly has in mind then, after this, the nomenclature is basically irrelevant; and common discourse can begin.

 🡺Objection 3: One can accept the context-domain, but reject the factual accuracy of the model.

Algernon: "Are sure you are not giving me the directions to the Book Store?!"

In this third case, the context-domain is totally agreed upon by both parties—so that, finally, substantive discussion of the *relations within the object* can begin. At this level, the question of what sequence of directions actually leads to Whynot Hall is *empirical*; one can examine the object itself to decide the question. In other words, the issue can be decided by traveling from City Hall to Whynot Hall, in the manner suggested, and observing whether Bertha‘s instructions actually work.

Observe that while Algernon raised his three objections against Bertha's verbal answers (which *used* the model of Figure 1), these same three objections could be raised if Bertha had instead *displayed* the model on which her claims were based. That is, barring merely grammatical criticisms and the like, objections to her answers must really address the model itself and its context-domain, regardless of whether the model was used or displayed.

Claim 4: No Model is Absolute.

We have seen that a model can inform about its object only once its context-domain is settled, that is, once the domain is effectively agreed on by all who will be using the model in common. Otherwise, one can easily become embroiled in circular or pointless debates on the models, themselves, and veer off from discussing the objects of interest.

But the context-domains that need to be assumed, are themselves, we have noted, families of symbols (and of other models). Hence, every model inevitably depends on *other* models, and no "rock-bottom model” can he uncovered. No model can depict direct “knowledge" of its object, unfiltered by assumptions in its context domain.

That last point need not entail extreme skepticism. Countless travellers might use Figure 1’s model successfully to get from one place on the map to another. What more could one ask of the model? Claim 4 merely reminds us that notions such as "North", "roads”, "traversable distances", and so on, which models presuppose, are provided to the models from outside, as it were. No model on its own is self-sufficient, but requires instead a pool of symbols and background on which its users can draw.

Claim 5: Models range in type from Mundane Models to Creative Models

The criteria for a "mundane" model are: (1) The object of the model can be observed independently from that model, itself; and (2) the context-domain of the model is so universally accepted as to render the model virtually transparent to its object. A "creative" model has the opposite properties.

While no model is purely "mundane" or purely "creative" by these criteria, it is useful to distinguish these two opposing tendencies in models. The map in Figure 1 is paradigmatically mundane. If you do not believe the map, its object (the relations among streets and buildings in Modelville) can be independently verified by personally touring the town. This fulfils condition 1 for mundane models. Furthermore, the assumptions and techniques of road maps are, by now, so thoroughly ingrained that few users of such maps would dispute them. The result is that (in satisfaction of condition 2) we willingly "see" the streets themselves through our representation of them in the model.

On the other hand, a possible example of "creative" modeling is the use of Tarot cards to purportedly determine features of a subject's "past lives". It lacks mundane property (1) since what would count as independent means for checking its claims? And it lacks property (2) because the model itself is controversial, so attention is drawn from the intended object of the model to the veracity of its own domain of assumptions.

Most models lie between those extremes. Consider the molecular model for water. While means of testing its claims by experiment are available, they are certainly not available to the average citizen. Nor is everyone familiar enough with the notations of chemists to “see” the intended relationships through the model's symbols. These concepts are relatable to Richard Rorty's terminology: When people engage in what Rorty calls “normal discourse”,2 they tend to view themselves as using "mundane" models common to their shared area of inquiry; whereas when discourse has turned “abnormal” (i.e., when not everyone accepts the same, shared, background models), new models introduced may be viewed as being more “creative” models.

Claim 6: The context-domains for models generally contain a hierarchy of interacting levels (or sub-domains). These levels range from the most “global" to the most “model-specific".

Consider the domain for Figure 1. At the most global levels are found notions such as “buildings” and traversability of distances. These concepts clearly stand behind a great many models—not all of which are maps. On the other hand, there are conventions for street-name-representation, compass-direction-indicating, and so on which apply *only* to maps—though, of course, to many varieties of these. Some options that are implicit in the more global domains may not be explicitly used by a particular individual model at all, and this can be a common source of ambiguity. For instance, is Figure 1 intended to be drawn "to scale" or not?

At the most model-specific level, Figure 1 relies on our willingness (in this particular case) to interpret the star symbols as representing buildings, and blue, parallel line segments as representing roads, and so on. Often a model will include a "key" to alert potential users of its own more specific conventions; though of course, the conventions for specifying the keys are themselves more “global”.

Definition 2:

Two models are "in the same domain" if all but the more model-specific levels of their respective domains are mutually shared.

Models in the same domain use generally the same conventions. In other words, they are all fairly standard roadmaps or standard blueprints or standard flowcharts, etc. Where such "standards" can vary, models are in the same domain if they all share the same general conventions.

(Definition 2’s precise application can get “fuzzy”, given the hierarchy of “sub-domains” for models, per Claim 6. The issue is similar, again, to Rorty’s discussion of “normal discourse”: If people agree in finding their models’ domains “the same enough” to continue discourse and focus on the objects of the models, then Definition 2 can be applied. If one person is using “Google Maps” and another is using a paper map, but they are not bothered by their specific, different conventions, then both maps’ domains are the same enough for practical purposes.)

Definition 3:

Two models are "equivalent" if they are in the same domain *and* they represent the same object.

According to this definition, any model would be equivalent to Figure 1 which (1) shares the more global domains associated with road maps and also (2) represents those features of Modelville which are portrayed in Figure 1. The scale, colour of paper, drawing style—all being quite model specific—need not be duplicated.

Definition 4:

 Two models are "complementary" if they represent the same object but share only the more global levels of the context-domain.

An example pair of complementary models might be a Mercator-projection map of the earth vs. a Mollweide projection map. These are not equivalent models, because although their objects are the same, their mapping conventions differ substantially, i.e., at a rather global Level. Nonetheless, there are important, higher-level, global concepts which they both share. For instance, the context-domains for both projections include the concepts of geographical features such as oceans and mountains, and of national boundaries, and so on.

An important, special category of complementary models is the pairing of “declarative” versus “procedural” models that was mentioned in this paper’s Introduction. These are discussed at length in the next section.

Complementary “Declarative” and “Procedural” Models

In many cases (and possibly for all cases): For a "Declarative" type model there is (often, or always) a "Procedural" type model which is its complement--and vice-versa.

Below the most global context domains shared by models, there are two distinct modeling approaches that can often be used to model the same objects: A *procedural* model includes conventions for representing the object by a sequence of inputs, transformations, and outputs. A *declarative* model includes conventions for representing the object by a set of declarations, illustrations, or other static depictions. A model’s being “procedural” versus "declarative" does not, in itself, determine or limit the possible objects of the model. In fact, for many objects (maybe all objects), a complementary pair of models—one procedural, the other declarative—could in principle be constructed. Applying Definition 4, if there is a common object being modeled by different model-specific approaches (in this case, based on the procedural/declarative distinction), the results would be complementary models.

The model of Modelville which is displayed in Figure 1 is an example of a declarative model. It lays out what is "there" in the object. When Bertha used the model (in Dialogue 1) to explain to Algernon how to get from City Hall to Whynot Hall, she could be taken as constructing a procedural *complement* to Figure 1 (but focused on “how to get there"). Technically, the complement would not be complete unless she said how to get from *anywhere* on that map to *anywhere else*; but, obviously, Algernon only cares about the parts relevant to his itinerary. The two model versions share many common global assumptions, such as about distances being traversable, and about the distinction between streets and their environs, and so on. And both versions have the same object (namely, Modelville’s street and key buildings’ relationships). Hence, they are complementary models.

Although the example just given suggests an analog between the use/display distinction (of what is done with models) and the procedural/declarative distinction (of complementary model types), these two dichotomies should not be confused. Models of *either* type can be used or displayed. For example, if an article publishes a sequence of computer program code that is part of a statistical software package, the published code-sequence is a procedural model that is being displayed. But if one actually *runs* the computer code to generate a statistic, that would be *using* the model. (We have seen already that a declarative model can be either used or displayed, as well.) Nonetheless, part of the appeal of a procedural model is that if it is presented in tandem with its declarative complement, it can provide a kind of guide to how to *use* the declarative version. (For example: “If you’re standing at the location where I’m pointing on the map, notice it’s telling you to go south on this street and turn left, if you want to get to the museum.”)

An interesting case of the declarative/procedural complementation is the relation between the “schematic” model vs. the "building instructions" model for something that has to be constructed, like a building or a radio. In this context the declarative "schematic” (or “blueprint”, etc.) model appears to *describe the result* (output) of correctly *using* the procedural instructions. Because models presented in practice are never totally compete, it’s often helpful to have both types in view together. For example, a schematic may statically depict two parts of an electronic device placed near each other; but for structural stability, or to avoid “shorting”, etc. it might be that to actually get the parts into that position, without damage, a certain sequence would have to be followed. In principle, the schematic version could have some notation to show the two parts’ vulnerability during construction, but it might not happen to include this; therefore, an extra, procedural guideline would be a helpful supplement. The Case examples section below includes an illustration of where considering the procedural complement to a declarative model could potentially resolve a dispute about a models’ object.

\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*

It is hoped the above sketch of an analysis of models can be used to clarify discussion and analysis over a broad spectrum of inquiry and philosophy. Understanding models does not itself supply any new information about our world, but it might help us to recognize when and how information is being conveyed on these matters, or where possibly it is being obscured. For instance, if a claim P is expressed in the context of a certain model, a disagreement about P may actually not be about whether the underlying model is accurate; it could reflect misunderstandings about the model’s assumptions or conventions.

In the remainder of this paper, several extended Case examples are presented. These illustrate potential applications of this models analysis to some topics of traditional interest to philosophers, in particular. Not all the above claims about models are specifically utilized for each discussion area. Nonetheless, I believe that what follows will be suggestive of the wide potential applicability of this analysis.

**CASE STUDIES**

**Case 1: Carnap’s Construction of Similarity Circles in his *Aufbau***

This section discusses a hypothetical step proposed by Rudolf Carnap in his *Aufbau* (The Logical Structure of the World), in his modeling of how human minds could construct, logically, the complex world of our conceptual experience, with the only inputs being raw sense impressions (prior to processing, labeling or interpreting). Carnap’s preferred model type is declarative: His proposed “constructions” are statements about the relations among objects, expressed in a formal, logical notation (much as a physicist expresses objects’ relations mathematically). Carnap *supplements* his declarative model with corresponding steps that he calls “fictitious operations”, suggesting that these are only for illustration, and secondary in importance. Yet arguably, these “fictitious operations” are complementary, procedural models that are *co-informative* to the declarative versions.

In either version, Carnap’s models are a good example of “creative” models. It is difficult to conceive of any independent method for observing the object of the model. The objects are not simply classes of grouped experiences, but relations which are alleged to hold between such classes and other (supposedly more "basic") classes of objects.

Key parts of the context-domain for his models, *which Carnap expects his readers to recognise and accept as prerequisites to understanding them*, are logical rules and methods that would be familiar to his contemporaries, based on Russell and Whitehead’s *Principia Mathematica.* Alternative (complementary) models could also be constructed using more current logical languages. (A possible reframing of this sort is illustrated in this Author’s paper, referenced below.3)

Reflection on a model’s context domain and possible alternatives can sometimes resolve misunderstandings. For example, Carnap introduced in his model a new relation-symbol called “Abstr’”, which does not appear in the *Principia*; and not all readers accepted the new usage as fitting closely-enough into the otherwise agreed on context-domain for the work. Readers expected (and some found lacking) an explicit, convincing justification for the innovation. It can be argued, however, that Carnap’s intended, logical point in using the “Abstr’” symbol *could* have been re-expressed alternatively, using some (now more current, and uncontested) logical formulations, and this would have diffused the purported dispute. Similarly, a timely switch from a declarative model to a procedural complement can sometimes sidestep apparent disagreements.

Consider one of the central declarative constructions in the *Aufbau*: namely, a concept he called “Similcirc” (for “similarity circles”). It has a key role in explaining how people’s undifferentiated raw sense-impressions can become grouped into what become perceived as our experienced world of objects, with various shared (i.e., “similar”) properties. The declarative Similcirc model represents the central relation between constructed classes, by means of distinct, interrelated propositions, expressed as logical derivations

But as noted above, Carnap also sketches procedural complements to the declarative statements in his main model. These ‘fictitious operations’ purportedly describe sequences of *operations one could perform* on raw perceptual inputs, or on lower-level constructed objects as inputs, to generate a new level of constructed objects as output—in this case, the Similcircs. This alternative approach is like using a sequence of procedural route instructions, for driving from location A to B, as opposed to viewing a map (declarative model) that visually depicts points A and B. Both the model versions, declarative and procedural, represent *the same object*—in Carnap’s case, the relations between certain classes of experience. And both versions of Similcirc model share many global assumptions, such as concepts and notations of logical “sets” (taken from the *Principia*), and various constructional definitions and notions introduced earlier in the *Aufbau* itself.

If both of these model types could be fully elaborated, in theory they could be redundant; i.e., both would convey the same information about the object. But in practice, models leave out details, whether intentionally for simplifying, or due to oversight. If complementary models are considered in tandem, one can sometimes glean important new information. For example, a driver using a map might think she can take a certain shortcut; but the map might fail to convey critical information about one-way road restrictions. If she consults Google Map’s Directions sequence—and it factors in one-way road restrictions—she could possibly correct the misimpression about the shortcut.

Precisely this type of situation occurs when comparing Carnap’s two model versions for Similcirc: the declarative logical derivation, and his procedural, fictitious operations. As described in this author’s paper *Structures and Procedures: Carnap’s Construction in the Aufbau*3*,* a “counter-example” to Carnap’s logical Similcirc construction was famously put forward by philosopher Nelson Goodman, which many took (and still take) to have effectively refuted Carnap’s whole project. But in fact, a replicable computer sequence can demonstrate that Carnap’s fictitious operation (procedural model) for Similcirc’s *does do* exactly what he claims, for constructing Similcirc’s—using only prior-generated objects. It can be shown that a simple extension to Carnap’s logical model would have cleared up declaratively, as well, an ambiguity that Nelson’s counter-example played on; but no one had realized this, and the ambiguity does not impact the procedural alternative model.

**Case 2: Theorems for successful additions of two numbers in Hofstadter’s p-q system.**

 In his book *Goedel, Escher, Bach*, Hofstadter presents a simplified logic called the "p-q" system, for modeling the addition of natural numbers. The system is designed such that each of its “theorems” is a model of the successful addition of two natural numbers. For example, the theorem **--p---q-----** can be interpreted as modeling **2 + 3 = 5**. The theorem **----p---q-------** represents **4 + 3 = 7**, and so on. Since every valid addition of this sort is representable by a p-q theorem, the p-q system as a whole might be taken as a model for the addition of natural numbers in general.

In the terms of this paper, what would be the object of the p-q model? Perhaps it could be described as "the relation which always holds between the addends and the sum of any natural number addition." Hofstadter's p-q system is suggestive of a declarative model for that object. It says, in effect: Each theorem *states* one particular relation of the indicated sort, while all theorems collectively state all the forms the addition-relation can take.

The p-q system is "suggestive of" a declarative model, because this is a case where a fully elaborated declarative model is not possible. There are infinite possible theorems; so obviously, not all can be displayed explicitly. Instead, the presented system contains "rules", which can be used to *generate* individual theorems (and theoretically, all the infinite possible theorems). That sounds interestingly akin to a *procedural* model. Many logical models are really a blend of declarative/procedural models in this sense that they include rules for generating theorems that *could* be shown declaratively, though most are not actually shown explicitly.

A strictly procedural complement to the p-q model could be implemented by a computer program, which the author has done using code written in the Logo language (similar to Lisp). The model consists of an executable procedure called “**Th? *X***”, where "X" can be replaced by any string of symbols. Running **Th? *X*** leads to an output which asserts or denies the p-q theoremhood of *X* (which is to say, it assesses the correctness of interpreting X as a model of correctly adding two natural numbers). The **Th? X** model is an alternative representation for the object "the relation between addends and the sum in natural number addition"; but instead of doing this by displaying, as theorems, particular instances of valid addition, the performance of the procedure is itself the model of the "performance" of successful addition: If the automated p-q check can he performed, then the model is affirming that the corresponding addition can be performed validly, as well.

**Case 3: Application to issues of reference and definite description.**

 Debates about the proper way to analyse “denoting”, reference, and definite description have continued apace since Bertrand Russell’s classic paper on denoting appeared over a century ago. Two alternative accounts have become equally iconic: Strawson’s (1950) and Kripke’s (1972). Detailing the long evolution of these three positions and debates is beyond this paper’s scope. Yet, reframing the core positions in terms of the analysis of models can provide some useful insights.

***Russell's Theory of Descriptions***

Russell’s classic theory of descriptions is introduced in his paper "On Denoting".4 In brief, he rejects the idea that descriptive phrases such as "the present king at France" have meaning in isolation. Rather, he claims, if such phrases are combined in sentences that are meaningful as a whole, then an analysis is possible for determining the net truth or falsity of the overall, constructed sentences. In this way, paradoxes arising from references to non-existent objects can be avoided.

The core of Russell’s analysis can be summarized as follows: To say in a sentence S that “The x is A” amounts, logically, to a three-part assertion that "Something is x, *and* not more than one thing is x, *and* that x is A," Therefore, if it turns out that the definite description "the x" does not refer to anything, then Russell’s analysis neatly renders the whole sentence S “false” (rather than meaningless), because the existential conjunct included in the sentence’s logical interpretation is false. S is true, on the other hand, only when all three conditions enumerated above are satisfied.

A typical objection to Russell is that his analysis for definite descriptions is largely counterintuitive. After all, the sentences “This man here is bald” and “The Canadian Prime Minister of 2017 is bald” appear to have essentially the same form. Yet the first sentence can be straightforwardly analyzed as the attribution of the predicate to the subject; while we are asked to treat differently the sentence which has the definite description “The Canadian Prime Minister of 2017” as the subject. The *grammatical* subject in the second case, we are told, is not identical with a simple logical subject for analysis. Russell’s method ensures that when the denoting phrase (the grammatical subject) does not refer, the complete sentence is rendered false; however, it may seem more natural to say: “Such a sentence is neither true nor false; it simply makes no sense.”

By employing this paper’s models analysis, one could handle definite descriptions in a way which is both (1) compatible with the strength’s of Russell’s view, yet (2) able to avoid the counterintuitiveness of that view. In what follows, refer back to the model in Figure 1, and consider the term “node” to represent a distinct, individual feature or intersection of features on the model.

Suppose someone, A, says S: "The intersection of Northmount and Anne is south of City Hall." Taken out of context, the sentence is not even interpretable. What is “Anne"? Which "City Hall"? A statement like the above—call it a “claim statement”—requires a context for being understood because it is an instance of *using* a model that is not being explicitly displayed.

I would argue that, in general, claim-making sentences can be understood as, in effect, using a model that may or may not be explicitly displayed. To utter a sentence "in context" is to speak under circumstances such that the model being used, and hence the context-domain for that model as well, are clearly understood by both the speaker and hearers of the claim.

Yet, we saw that the actual object of a model is not things, per se, but relations among things. What we think of as "things" are generally represented in the model as nodes whose interrelations the model displays. Therefore, when a claim-statement appears to make assertions about specific things or persons, it is in fact using a model which, as a whole, shows the *relations* between particular nodes.

Thus, returning to A's assertion that S: A is making a claim using a displayed or implied model like Figure 1 (or equivalent). What claim is A making? It is the claim that precisely that node which can be identified by the phrase "the intersection of Northmount and Anne” stands in the relation south to the node identifiable as "City Hail". The purpose of the definite description in the claim is straightforward: it serves to help us pinpoint the node whose relations we are being asked to consider. Once we know which node is meant, we can then either learn from, or question the factual veracity of, the claim which is made.

I believe this analysis accords with what is the most important insight in Russell's approach. The very purpose of the definite description is to locate on the model that node whose relations are being detailed. Clearly, such "locating" of a node can be successful only when (1) there is indeed a node on the model which corresponds to the descriptive phrase, and (2) there is no more than one node which corresponds to it. These two conditions are essentially the first two conditions included in Russell's three-part analysis (i.e., there should exist one, and only one, x which satisfies the definite description). RusselI's third requirement for the truth of a claim sentence is retained in this proposed analysis: namely, that whatever is claimed about the pinpointed node should be true. In the present account, that would mean that the model's display of identified node's relationships should be as independently verifiable as possible.

However, note that specifically using a definite description to pinpoint a specific node on the model is just one possible strategy for identifying that individual node. Labeling a feature such as "City Hall" on the map, itself, would also work. As well, the ostensive expression "this intersection here” (while physically pointing to it on the map) could be used to specify a unique node; or even in some contexts a "you are here” arrow could be included on the model.

ln short, Russell’s definite descriptions are one way—but not the only way—to make a claim that is, in effect, relying on (“using”) an underlying model. Every claim-statement that has a singular subject presupposes a model in which there is a distinct, corresponding node being referred to. There is a variety of techniques (including using a definite description) for identifying the node of interest.

A corollary of Russell’s position, following from the logical form of his analysis, is that if a sentence uses a definite description which fails to refer (i.e., if there fails to be exactly one x which is identified), then that sentence is false. It *is* possible, if one wishes, to accommodate Russell’s logical position in the terms of this paper’s account; but in my view, it is equally plausible to reject that logic. More specifically, I suggest that Russell’s position actually adds an extra, tentative claim to this paper’s models analysis—and that extra claim is optional to the present analysis, rather than required:

Claim R

A claim-statement, S, which is a use of model M is "true" if and only if there is at least one submodel M\* of M such that:

1) M\* is itself a true model of its (more restricted) object, and

2) M\* could itself be used for making the claim S;

*Otherwise S is "false".*

This claim could be appended to this paper’s account, but does not have to be. Definite descriptions are not really a problem having to do with models themselves. A model simply portrays whatever is its object—and either does so truly or falsely (i.e., such that its depicted relations either would, or would not, be verified by independent testing, if such is possible5). Yet Russell‘s research goal here is discovering a rule for *determining* *the truth or falsity of individual claim-statements* (rather than of the models they use), *when such statements include definite descriptions*. If one accepts Claim R, one can follow a models approach for sentence analysis, and still produce the same true/false evaluations for individual sentences as Russell’s account would give.

What Claim R says, in effect, is that a sentence is true if and only if it is the use of a true model (or at least of a true sub-model). To be "a use" of such a model means that its descriptions (if any) cannot fail to denote specific nodes on the model (compare Russell’s existential content conditions); while to be a "true" sentence means that the model being used must itself be true (or "accurate”) at least insofar as it portrays the relations among nodes that are specified.

A submodel M\* of model M can be defined as a separate model which (1) shares the original context-domain of M, but (2) includes a more restricted object (i.e., leaves out some nodes in M, and their relations, but adds none). For statement S to be true requires only that some "submodel" that includes all the nodes referenced in S would be accurate. (If *other* parts of the full, original model happen to be mistaken, this would not be relevant to the evaluation under consideration.)

If Claim R is accepted, a claim-sentence would be "true" only if Russell's threefold conditions, listed earlier, are satisfied, and otherwise it would be false. For example, “The intersection of Darn and Belowmount is south of Anne” would be false, since it is certainly not a use of the true model in Figure 1 (or of any subpart of it), which clearly shows there is a different relationship between the identified nodes. “The intersection of Whyheck and Anne is east of Northmount” would also be false, because there is no portion of the map on which the referenced node—necessary to make the claim at all—can be found. Neither of the above two claim sentences satisfies all the truth conditions in Claim R.

Nonetheless, Claim R seems to be “optional” so far as analyzing models is concerned. Various possible schemes for relating the "truth" of models to the "truth" of statements could be elaborated. Russell suggests one possible approach, which Claim R formalizes. But does Claim R really need its “Otherwise…” clause? A statement could be evaluated as "true" if it uses a submodel that is accurate—but be left unclassified, if the “Otherwise” is omitted, for cases where no node in the model is even referred to in the claim. Whether or not the “correct” version of a Claim R should include the “Otherwise” goes beyond simply applying a models analysis.

***Strawson‘s Rebuttal to Russell***

Strawson’s two main counter claims to Russell are that (1) definite descriptions do have *meaning* whether or not they successfully refer; and (2) the truth or falsity of sentences is dependant on the *use* of those sentences, and cannot be assessed, once-and-for-all, by formal or logical criteria, alone. According to Strawson’s essay "On Referring",6 if a speaker P uses a sentence of the form “the x is A", this certainly does *imply* a belief on P’s part that there is an x, and only one x, and that x is A. Yet, Strawson argues, it is not the case that such an existence claim is included in the logical structure of P’s remark. Instead, the definite article is acting "as a *signal* that a unique reference is being made—a signal, not a disguised assertion."7

Much of Strawson’s argument is based on a view of meaning, according to which to "give the meaning of an expression…is to give *general directions* for its use to refer to or mention particular objects or persons.”8 Phrases like "the x" are meaningful precisely because they create in us our expectation that some unique object is being referred to. It is a contingent question, however, for particular uses of these expressions, whether the called-for references can he made successfully.

At one point, Strawson distinguishes between a sentence, its use, and its utterance.9 For instance, the sentence "This man is bald" cannot he true or false in itself; as its truth or falsity depends on the use which is made of it when it is uttered on a specific occasion. All who uttered it, say, with reference to Don Rickles when he appeared on the “Tonight Show” spoke truly, though all who meant to refer to Justin Trudeau when he was elected Canadian Prime Minister spoke falsely.

Sentences involving definite descriptions follow this same general analysis: "The intersection of Darn and Belowmont is north of Anne" is neither true nor false, simply as a sentence. If used with reference to Modelville (assuming Figure 1 is accurate), it is used truly. If used with reference to Someotherville, it may be used falsely. But if used with reference to Toronto, Ontario, it fails to make any claim at all—for there is no such intersection in that city.

What Strawson calls "using" a sentence to make a claim roughly corresponds to what this paper calls *using a model* to state a claim. Both usages suggest that a sentence, considered as such and stated out of context, is not informative. For a claim to be made by a sentence, the sentence must he uttered under appropriate circumstances, and in line with certain presuppositions about the sentence's meaning. In this paper’s terminology: The sentence makes a claim only if it uses an underlying model, and thus draws on that model’s context-domain. (If the model itself was simply displayed, statement's claims about the object would be represented directly.)

That claims “use models” is a more general observation than Strawson’s focus on using “sentences”, in particular. An accountant could express the finances of a company by listing key profit and loss variables in a table format. The table is a “statement” that is communicable to trained auditors, etc., but does not stand alone; it depends on (i.e., uses) an underlying accounting model for the company’s finances. In that sense, a “sentence” is not the *only* vehicle that could be used to possibly make a claim based on an underlying model; but Strawson’s key point is correct that a sentence *is* a common vehicle for making a claim, and as such, cannot be assessed out of context.

Strawson’s approach to "meaning" is also consistent with this paper’s analysis, and relates to the idea of a context-domain for a model. If a sentence expresses a model, then shared conventions of “meaning” guide what one looks for in the model to correspond to what is said. How does one know, for instance, what to look for in Figure 1 if a claim is made about “the intersection of Bach and Northmount”? Or to realize that a claim about “the intersection of Anne and Whyheck” makes no sense?

Strawson and Russell agree that there is a problem when sentences contain unsuccessful references. Their substantive disagreement is whether Claim S (below, implied by Strawson) is more justifiable than Claim R (above, implied by Russell) *as an analysis of assigning “truth”.* The truth or falsity of sentences (or of “uses” of sentences, for Strawson) is derivative upon the accuracy of underlying models; but Strawson opts to not to assign any truth values at all to sentences as such, but only to their use—and only if that use is successful (i.e., if its references refer). Both claims, R and S, are compatible with this paper’s models analysis, and the relative advantages of R versus S are left for the reader’s consideration.

Claim S

A use of claim-statement, S, is “true” if and only if S is a possible use of model M, and there is at least one submodel M\* of M such that:

1) M\* is itself a true model of its (more restricted) object, and

2) M\* could itself be used for making the claim S.

*If S is not a possible use of model M, then no truth value is assigned to S.*

***Kripke and Names***

Although the definite description is an important form of reference, not all reference is accomplished by means of such descriptions. The use of names (i.e., "proper names") is another viable method for identifying specific individuals when making claims. This final case example briefly explores Kripke’s views on naming.

In the Russellian view, a name is a sort of shorthand for a definite description. Thus, for instance (to consider the simplest version of the theory), "Aristotle" might simply *mean* “The author of the Metaphysics". It this were so, we could analyze a sentence like "Aristotle taught Alexander” in exactly the manner we were taught by Russell (or Strawson)—that is, treat the sentence as equivalent to "The author of the Metaphysics taught Alexander."

But this approach quickly needs qualification. The claim "Aristotle is *not* in fact the author of the Metaphysics” seems to be a perfectly acceptable, contingent proposition. But, if "Aristotle" is no more than a disguised reference to "the author of the Metaphysics," how can we even discuss the above proposition? It would be logically inconsistent to affirm that "The author of the Metaphysics is *not* the author of the Metaphysics."

A popular approach to sidestep the above dilemma has been to adopt a "cluster-of-descriptions" theory to fix the reference of a name. To borrow from Kripke’s summary of the gambit:

(1) To every name or designating expression 'X', there corresponds a cluster of properties, namely the family of those properties such that A [the one who refers to ‘X’] believes [that 'X’ has those properties]."

(2) One of those properties, or some conjointly, are believed by A to pick out some individual uniquely.

(3) If most, or a weighted most, of the properties are [in fact] satisfied by one unique object y then y is the referent of [the name or expression] ’X'.

(4) If the [above process does not identify] any unique object y, ‘X’ does not refer.10

By this approach, “Aristotle” would not correspond simply to "the author of the Metaphysics", but rather to a whole "cluster" of properties, such as {"the author of the Metaphysics”, "the author of the Ethics", "the student of Plato“, "the teacher of Alexander", …..}. Provided there is indeed some person who satisfies a "weighted most" of these properties, then "Aristotle" refers to precisely that person. Any specific property in the list can be denied of Aristotle, so long as the "weighted most" of properties continues to refer to that individual.

As Kripke delights in illustrating, however, such a theory is easily refuted. On the one hand, it is surely logically possible (though unlikely) that the real Aristotle did or was *none* of the things we have historically attributed to him. (Perhaps the real Aristotle was Meno’s slave boy, referred to in Plato’s dialogue.) Though it is logically possible for this to be the case, it would be impossible to even suggest such a possibility according to the cluster-of-properties account. Making that suggestion would be like saying: "The man who possessed most of the properties j, k, l, …, etc. possessed *none* of properties j, k, l, …, etc.”

Moreover, if there was *somebody* who actually satisfied the "weighted most" of what is attributed to Aristotle, then the cluster theory requires that *that specific person* must be the referent of "Aristotle". But that requirement would make it *logically inconsistent* to speculate that the cluster may happen empirically to refer to someone *other* than Aristotle—maybe to the brother of Aristotle. Saying that suggesting this would be inconsistent (rather than just factually unlikely) runs counter to our intuitions, which tell us that when we refer to "Aristotle", *we mean Aristotle*—regardless of whatever historical relations, if any, he might have to whomever, if anyone, happens to possess most of the properties which are usually attributed to Aristotle himself.

So how are we to get around these difficulties? What Kripke suggests in his *Naming and Necessity* is that naming--or, more specifically, reference by naming--is an essentially social phenomenon. At some paint, a person (or building, or whatever) is "baptized" (in a "generic" sense of "baptized") with a certain name. "Here", says Kripke, "the object may be named by ostension [i.e., pointing], or the reference of the name may be [initially] fixed by a description." From that point on, unless the chain is broken (which does sometimes happen), "the name is 'passed from link to link', [and each new] receiver of the name must… intend when he learns it to use it with the same reference as the [person] from whom he heard it."11

Hence, we suppose that a particular ancient Greek was "baptized" with the name "Aristotle", in Kripke’s sense. This was the name he learned to use when referring to himself, and the name by which others came to refer to him. When we now speak of Aristotle, we mean to refer to just that man who was so baptized, and whose name has come down to us "from link to link". Whether any other things we attribute to this Aristotle are true is a thoroughly empirical question; and there is no inconsistency if we question them.

That account seems plausible and consistent with this paper’s models analysis. However, I suggest that the approach introduced in this paper can capture the “picture" of how naming works that Kripke is trying to express, without needing to find some “theory” for the process (which Kripke has said he found elusive).12  That is, there is no need for any extra theory beyond what has already been provided.

As the reader will recall, this paper’s models analysis addresses, basically, the *relations* among nodes or features in the model, and not the "essence" of the nodes as such. If a name appears on a model, then it is merely another feature—another node. It is unneeded, and really adds nothing extra, to inquire into the "nature" of the name itself. Instead, the model portrays the relation between that name and other features on the model.

Nonetheless, Kripke’s "baptism" concept does provide a useful terminology for referring to the (name🡨🡪named-object) relationship. Figure 1, for instance, portrays a building with the label (name) "Whynot Hall". Employing Kripke's terminology, we might say about this model: "Figure 1 is making a claim that the building which stands in such-and-such a relation to other features of Modelville stands also in the relation 'baptized’ to the name “Whynot Hall”. That implied claim could turn out to be false. It may be that the building which stands in the indicated relations to the rest of Modelville was actually baptized “White Hall”; but the map-maker misread its name. Or, perhaps there was *some* building which stands in the baptismal relation to the name "Whynot Hall", but such a building is not factually located on Whyheck Street, as the map suggests.

In short, a model's use of a name label is one of many conventions for claiming that a certain relation holds—in this case, a “baptismal” relation between some identifiable node and a name written somewhere on the model. If you are not disposed to question this relationship in a particular instance, you can simply use the name as a device to refer to its associated node. Yet the question whether this implied relationship between the node and the depicted name can be empirically verified is no more, or less, problematic than any other question about the veracity of relationships which the model shows. One will think a model "true" if one accepts that all the relations it depicts (including baptismal ones) are accurate.

To close this section of the paper, I present a simple model, followed by four (of many) possible criticisms that could be made of it.



Figure 2: A Brief Historical Model

*Four Possible Criticisms of the Model:*

 1: “I disagree that it was Aristotle’s parents who named Aristotle ‘Aristotle’. I think he was named by his godparents.”

2: “Though I agree that who we think of as Aristotle was named by his parents, I disagree that the name his parents actually gave him was “Aristotle”. Rather, as a youth, he was (re)baptised as “Aristotle” by a rich benefactor.

 3: “No, there was never any person at all assigned the name "Aristotle"! (An old copyist made an error on a manuscript, which others kept repeating.) Therefore, it is wrong of Figure 2—or any similar model—to show a node for a person having the name-label "Aristotle".

4: “I concur with Figure 2’s claims regarding the relations among Aristotle, his name, and his parents who named him. However, the model is mistaken because, in my view, the *Metaphysics* is a forgery; and so, the model is incorrect to display Aristotle as standing in an authorship relation to that node."

This example illustrates that names can play various possible roles in models. Figure 2 is a model that depicts a single node labelled "Aristotle". All the speakers acknowledge that, in part, this model displays a claim that there is some person who (1) is related to the name "Aristotle" by something like Kripke’s “baptism", and also (2) stands in other relations to other nodes as depicted in the model.

 Objector 1 accepts most of the model’s claims; yet he disputes that those standing in the "baptizing" relation to Aristotle can be accurately labeled "Aristotle’s parents". The node for those who named Aristotle should be labeled “Aristotle’s godparents”.

Objector 2 accepts that Aristotle's parents did historically play a baptizing role in the life of the person who later became known as Aristotle…. However, he argues that the model is incomplete, insofar as the baptizing of Aristotle *as “Aristotle”* did not occur until afterwards, by a different person.

Objector 4, one the other hand, accepts the whole depicted history of Aristotle's baptism as such; and moves on (like most model users) to concern himself with the accuracy of depicted relations among that node (properly so labeled), and *other* features on the model of interest.

Only Objector 3 goes so far as to argue that no node at all should be labelled "Aristotle", interpreting that to mean that no person ever got connected to that name by baptism. Yet that too is just another type of factual claim, like the other three, that can be offered against the model’s implied account of history. In that sense, there is nothing inherently different about this third objection.

In light of the above case examples, I submit that my models analysis can have a wide applicability, for clarifying, if not “resolving”, a variety of debates and issues that turn on claims being made via models. Nelson’s Goodman’s “counter example” to Carnap, for example, was shown to be not as deal-braking as presumed, if only plausible complementary models had been explored. The “declarative” versus “procedural” approaches of AI models can be viewed as such complementary models. And some well-known, competing positions, by Russell, Strawson, and Kripke, in the literature regarding names, pointing, definite descriptions and “using” sentences, etc., are shown to all be locatable within the presently proposed models analysis. Leaving aside questions such as where “truth” resides inherently (e.g., whether in sentences or in uses of sentences), models are shown to be practical for expressing claims about the world in a way that can potentially be verified or falsified, making them useful for scientific inquiry and for critical evaluation of claims.

 *Notes*:

This paper, completed in 2017, updates and expands on the contents the following paper originally presented by the author:



1. Plato. *Plato: Timaeus, Critias, Cleitophon, Menexenus, Epistles,* (London: William Heinemann Ltd., 1929), frontispiece.

2. Rorty, R., *Philosophy and the Mirror of Nature,* (Princeton, NJ: Princeton University Press, 1979), p. 320.

3. Goodman, W.M., Structures and Procedures: Carnap’s Construction in the *Aufbau*. *Philosophy Research Archives*, 11(March, 1985), 551-578. *Available online at:*  <http://philpapers.org/rec/GOOSAP-2>

4. Russell, B., On Denoting. *Mind,* 14 (1905), 473-493.

5. Of course, for more “creative" models, such verification is inherently difficult to obtain.

6. Strawson, P.F., On Referring. *Mind,* 59 (1950), 320-44.

7. Strawson, p. 331.

8. Strawson, p. 327.

9. Strawson, p. 325.

10. Kripke, S.A., *Naming and Necessity,* (Oxford: Basil Blackwell, 1972), p. 71.

11. Kripke, p. 96.

12. Kripke, p. 87.