ERNST MACH'S CONTRIBUTION TO THE PHILOSOPHY OF SCIENCE IN LIGHT OF MARY B. HESSE'S POSTEMPIRICISM

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Ernst Mach's definition of the relationship between thoughts and facts is well known, but the question of how Mach conceived of their actual relationship has received much less attention. This paper aims to address this gap in light of Mary B. Hesse's view of a postempiricist approach to natural science. As this paper will show, this view is characterized by a constructivist conception of the relationship between theory and facts that seems to be consistent with Mach's observations on scientific knowledge. The paper first explores Hesse's account of postempiricism and her project of a new epistemology. It then considers Ernst Mach's conception of facts as the middle term of a triad of concepts that includes thoughts and elements as extreme terms. Finally, the paper will offer concluding remarks on Mach's contribution to the debate on scientific realism and his attempt to redefine the notions of correspondence and objectivity in science.

1. Introduction

It is difficult to overstate Ernst Mach's role in the history of the philosophy of science. His influence on his contemporaries has been significant, both on the plane of pure research and on that of epistemological reflections on scientific knowledge (cf. Einstein 1992; Frank 1949, chaps. 2–3). Furthermore, the fact that several members of the Vienna Circle were influenced by him—although

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I would like to thank the anonymous reviewers who commented on my paper. Their help has been especially important to strengthen my argument. Also, from their remarks I realized how crucial it is to deal with a plurality of scholarly opinions in order to carry out a well-informed interpretative work on such a rich, complex, and multifaceted issue as Mach's epistemology.

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sometimes misleadingly—contributed to his positive reception in the early twentieth-century debate. At the same time, however, the received view seems to be largely limited to this perspective, and it is only recently that a discussion of Mach as an actual "phenomenalist" has been initiated.[1] A key problem in this regard is that general agreement regarding the established interpretation of Mach's views has meant that certain issues related to his view of science have not been thoroughly studied. This general agreement, in turn, is likely due to the fact that the relative narrowness of Mach scholarship to this point has made the development of a proper critical debate on the more subtle aspects of his work impossible. Times have changed, however, and the current renewed interest in Ernst Mach means that we can count on a much richer account of the scientist and philosopher (for he in fact developed philosophically significant reflections), which will help us to shed new light on his fundamental conceptions.[2]

Within this framework, the present paper aims to deal with an often (albeit uncritically) referenced issue in Mach's view of scientific knowledge (i.e., the relationship between thought and facts). As I will show, the issue is not as trivial as it may appear, and the resulting inquiry allows us to outline an original view on one of the fundamental subjects in the philosophy of science. In other words, I will defend the idea that Mach's position on how and in what sense it is possible for science to *explain facts* is a viable alternative to traditional metaphysical commitments that are still debated in the current literature on scientific realism versus antirealism.

In order to achieve my aim, I will take an unconventional path. Mach's writings will be addressed directly, of course, but the general viewpoint of the research is inspired by the work of Mary B. Hesse—more precisely, by her conception of a postempiricist approach to natural science. As is well known, Hesse dealt with the value of theoretical explanation in science as a "metaphoric redescription of the domain of the explanandum" (1980, 111). Most importantly for our purposes here, she reflected on the way in which what she calls the "standard empiricist account" (vii) has been undermined by focusing on how the relationship between theory and facts is described in the relevant philosophical literature. According to Hesse, during the first half of the twentieth century, the image of natural science as a realm of objective and testable experience—of "exact, formalizable, and literal language," of facts that are supposed to be independent of theoretical explanation—was ultimately abandoned. In its place arose a

1. On this see, e.g., Banks (2013).

2. This interest was stimulated by the centenary of Mach's death in 2016. On that occasion, a large conference was organized in Vienna, the proceedings of which have now been published (Stadler 2019), thus providing us with an important contribution to the secondary literature on Mach.

postempiricist view, according to which "the language of natural science is irreducibly metaphorical and inexact," "data is not detachable from theory," and "facts themselves have to be reconstructed in the light of interpretation" (170, 172–73). Furthermore, that view entails a new epistemology based on the idea that a direct and unambiguous representation of the world is an unattainable ideal, but also admits that "a moderate realism [of pragmatic success] for science can be maintained" (1994, 447, 453). As will be shown in what follows, Hesse tries to defend a "'consensus' theory, as distinct from the traditional 'coherence' and 'correspondence' theories'" (1980, 145). That theory is not limited to mere agreement within the language community but rather takes into account "the way we learn or evolve to use our observational vocabulary" (146), thus allowing one to argue that "meaning is not given independently of observation constraint and purely by theoretical context" (153).

It is my opinion that certain aspects of this new conception of science can be found in Mach, especially with regard to his conception of the relationship between theory and facts and the value of actual scientific "knowledge." I do not maintain that Mach was a postempiricist, of course, for it would be impossible to deny the empiricist side of his epistemology. On the other hand, though, I believe that it is possible to approach his conception from a broader viewpoint and to show that it somehow inspired (or indirectly contributed to) the kind of discussion of the empiricist commitment that took place in twentieth-century philosophy of science.[3]

I will therefore attempt to read Mach through Mary Hesse, with the secondary aim of offering a new account of Mach in the wider framework of the history of ideas. This will be accomplished in three steps. In section 2, I will deal with Hesse and her outline of a postempiricist account of the relationship between facts and theory in the natural sciences. Section 3 will be devoted to Ernst Mach, with a thorough exploration of his way of dealing with facts as a middle term in a triad of concepts that includes thoughts and elements as extreme terms. Finally, in section 4, I will present my conclusions on Mach's contribution to the debate on scientific realism, with reflections on how he can help us to redefine the notions of correspondence and objectivity in science.

3. To be completely clear, I would like to note that Hesse's postempiricist conception is not anti-empiricist at all. In exploring how authors such as Wittgenstein, Quine, Black, Kuhn, and Feyerabend contributed to undermining the standard empiricist account, Hesse did not maintain that contemporary philosophy of science resulted in a complete rejection of empiricist views of all kinds. On the contrary, she tried to assess the actual function and value of empiricism (e.g., in light of recent studies on scientific language) and suggested that a more flexible view of scientific knowledge (including the fundamental concepts of *truth*, *objectivity*, and *correspondence*) can be developed.

2. A Wider Epistemological Framework

2.1. Postempiricism

Mary B. Hesse provided us with an original and interesting view of the scope and value of scientific knowledge. Inspired by the way in which the premises of what she calls "the standard empiricist account"—including "the assumptions of *naïve realism*, a *universal scientific language*, and of the *correspondence theory of truth*" (Hesse 1980, vii)—have been undermined by several authors from the philosophical tradition (e.g., Duhem, Quine, Kuhn, Feyerabend), Hesse developed an alternative view of theoretical explanation focused on the metaphorical value of scientific language (e.g., 1980, 111ff.). Furthermore, Hesse maintains that we ought not to overlook the modeling role played by the observer in ordinary scientific inquiry; consequently, she argues that the very relationship between our theories and the objects they describe is much more complex than one might think, and the confidence that Hesse believed standard empiricists gave to the descriptive and explanatory power of the language of science is untenable. More precisely, she calls for a reconsideration of the "picture of science and the world," according to which "there is an external world which can in principle be exhaustively described in language" and "science is an ideally linguistic system in which true propositions are in one-to-one relation to facts" (1980, vii). Hesse develops this apparent radical form of anti-empiricism in an original way, however. In fact, she does not maintain that strong antirealism must be defended; rather, she tries to stress the actual value and function of language in science and focuses on how it provides us with access to the natural world that, while not literally accurate, still depends on an independent "world structure" (1994, 447). Thus, we might say that an unconventional (for Hesse's time) form of empiricism is preserved, and it is precisely this view that I believe can be profitably compared to Mach's.

Hesse's view on this issue is first expressed in her early writings and then developed throughout her career. For example, she maintains that the "dictionary theory" (according to which "there is some set of [phenomenal] statements whose truth or falsity is known directly by observation") and then, contrasted with this, another set of "theoretical statements" the meaning of which is "dependent upon a dictionary which translates some of them into phenomenal statements" (Hesse 1958, 13–14) simply cannot accommodate the problems raised (e.g., by quantum theory). The dictionary theory, Hesse continues, has been adopted "firstly, to contrast the clarity and certainty of empirical tests with the tentative nature of theories, and secondly to show nevertheless how theories could be unambiguously tested and hence given meaning by experiment, and so distinguished from speculative metaphysics for which no such tests were

available" (14). For Hesse, however, "if phenomenal statements are to be tests of theories, then their meaning cannot be entirely independent of that of the theories, and consequently the function of the dictionary has been [traditionally] misconceived" (14).

The actual problem that Hesse has in mind is the classic riddle about the boundaries of observability. What can be properly defined as "observable" and "unobservable" in science? How are these two realms related? What is the actual aim of scientific theories, or the proper meaning of scientific "explanation"? Hesse tries to answer these questions by arguing that "a confusion has been perpetuated by some writers in physics about the notion of 'unobservable entities'" (1958, 21), for the very use "by physicists of the words 'observable,' 'factual,' 'objective,' 'physically significant,' [has been contrasted] to such phrases as 'mental conceptions,' 'fictitious entities,' and 'mathematical equipment'" (22). This view is the product of a classic conception of science that would seem to place undue confidence in sense perception, thus radically separating phenomenal data and theoretical elaborations. For Hesse, the difficulties that follow from this presupposition have been tentatively solved by philosophers and scientists "of an empiricist persuasion" through instrumentalism—that is, by interpreting "scientific theories which go beyond immediate experience" as mere "tools for correlating and predicting the result of possible experiments, but not as descriptions of physical reality" (22). That may have worked in the age of Newtonian science (Berkeley and Kant) and in nineteenth-century physics (Mach, Pearson, Duhem), "but in twentieth-century physics, the word 'observable' has become a technical term of quantum theory," and the misleading equation between unobservable entities and "theoretical (as opposed to phenomenal) concepts" is currently untenable (22). In plain language, this means that it is currently quite difficult not to consider the actual explanatory function of theoretical models that describe events that are not directly observable by us. Antirealist views such as Van Fraassen's can still be defended, of course, but the contraposition between purely theoretical versus phenomenal statements corresponding to indirect versus direct or literal representations of natural events simply does not stand, as recent attempts to preserve attenuated forms of realism in science based on either fictionalism or perspectivism show (on this, cf. Suárez [2008] and Massimi [2017]).

Hesse's view on this issue is more oriented toward stressing the theoretical component of any observation statement than toward the empirical relevance of theories. By this I mean that Hesse argues that the phenomenalist contraposition between mental conceptions and facts is untenable, insofar as in twentieth-century physics "all observation for the purpose of testing a theory involves some interpretation, and the interpretive report of an experiment may be given in theoretical terms" (1958, 23). Accordingly, a redefinition of words such as "fact,"

"observable," "actual," "real," and "objective" is required in light of a new conception that accepts that "the distinction between theoretical and phenomenal statements . . . does not call for translation but for interpretation of observations at different interpretive levels involving more or less reference to theoretical concepts" (21). That is to say, a dictionary theory presupposing a direct and univocal relationship between theory and facts is no longer satisfactory. Instead, one should conceive of observations as theory laden, as the product of a kind of theoretical activity that does not necessary build facts but rather participates in the process of scientific discovery.

The new epistemology that Hesse subsequently elaborates is an attempt to develop these early considerations. She in fact focuses on such concepts as *hypothesis, analogy, model*, and *metaphor*, thus stressing the constructive and interpretive role of the scientist, who attempts to make sense of world events and organize them in a coherent system of efficient predictions, thereby preserving an objective report of the facts explored.[4]

Further examining the aspects of Hesse's work in detail would lead me far beyond the scope of the present paper. What I would like to stress before turning to Ernst Mach, however, is that the constructivist view that Hesse elaborates in her later writings constitutes the basis of a critical elaboration of the empiricist conception of scientific knowledge. We find interesting observations on this issue by Hesse and Arbib (1986), especially regarding the relationship between theories and facts. In that text, a captivating image of science is outlined—one that is explicitly contrasted with the traditional conception, which believes (although moderately) in the explicatory power of what can be viewed as the most complex and highly developed expression of human knowledge. For Hesse, science is "constructive of new facts," with "no fixed body of facts passively awaiting explanations" (Hesse and Arbib 1986, 8), and the development of successful theories determines the constant extension of "the range of phenomena that exist to be described and explained" (8).This means that further work on the system of theories adopted is constantly needed, and the value of that system can be assessed in light of its explanatorily power with regard to the new facts encountered.[5] Most importantly, this view of science implies that "the facts themselves are *theory laden*. There is no representation of facts without the observation language, and no observation language is just 'given' as theory-free" (8). Hence, the very object of scientific explanation is not as literarily definable as it may seem. Even within a

4. Cf. Hesse (1953). For more on this, it is worth reading the special issue of the journal *Philosophical Inquiries*, vol. 3, no. 1 (2015) dedicated to Mary Hesse. Among the contributions, see especially Bartha (2015) and Nersessian (2015).

5. That is, it must explain these new facts in an efficient way and help to predict future occurrences.

relatively stable range of inquiry, the constructive character of our observation determines that the facts explored by science cannot be described as directly as the ordinary correspondence theory supposes.[6]

Hesse further remarks: "The attempt to represent the world in knowledge as a neutral independent object is not like a mirror image; rather, it is a projection on the world of a mental model whose framework is given by schemas of kinesthetic activity and by the categories of language" (Hesse and Arbib 1986, 159). Nevertheless, she continues: "This projection of a constructed world-model is not arbitrary, nor does it mean that the *objectified* nature is unconnected with *objective* reality" (159). This picture presupposes essential interaction between the knowing subject and the world, whose actual existence is never questioned. In fact, it is on this interaction that Hesse develops her own demarcation criterion by arguing that a key difference between science and other kinds of world interpretation is that the constructed models provided by scientific theory are "constrained by feedback loops involving experimentation in the natural world" (159). This means that the constructive activity of science is constantly tested against nature and that the actual agreement between theory and facts is a matter of the feedback the tested hypothesis receives from the latter. It is worth noting that this picture allows us to preserve some sort of objectivity in science, which Hesse attempts to reconceive and redefine on a new basis.

Allow me to sum up what has been argued thus far. For Hesse:

1. There is an external world, "independent of human beings and their knowledge" (Hesse and Arbib 1986, 159).
2. Any observational statement is a construction of the image of the world that we know, explore, and try to explain through scientific theory.
3. The natural world is constructed "in a complex feedback process in which theoretical models and sensory input are assimilated and accommodated in a self-modifying sequence of prediction and test" (176).
4. Therefore, facts are theory laden insofar as they cannot be explained independently of the inquiry of which they are a part, while at the same time scientific objectivity is preserved by 3, for our construction is constantly tested against the external world.

6. Hesse is not willing to give up on objectivity; she rather tries to preserve it based on a "consensus theory," as I will shortly show.

5. Consequently, the theory of knowledge as construction determines a *new epistemology*, which "combines coherence and correspondence criteria of truth and dissolves the barriers between 'objective' science and nonscience." Just as there is "a continuum between literal and metaphorical meanings, so we do not posit a sharp dichotomy between the natural sciences on the one hand and the social or literary hermeneutical sciences on the other" (171).

This last point requires further exploration in light of Hesse's papers "In Defence of Objectivity" (1974) and "Truth and the Growth of Scientific Knowledge" (1976), both of which are included in her 1980 collection of essays. As shown above, this book explores how "the turn from logical models to historical models [in empiricist philosophy of science] . . . has undermined several of the premises upon which the standard empiricist account depended. The most important of these are the assumptions of *naïve realism*, of a *universal scientific language*, and of the *correspondence view of truth*" (Hesse 1980, vii). According to Hesse, traditional empiricism, with its firm dependence on the objectivity of facts, is challenged by the new trend in the philosophy of science (Kuhn, Feyerabend, Toulmin, Quine), but this does not imply that empiricism must be abandoned. On the contrary, it seems possible to develop a different view, one that maintains certain basic assumptions of empiricism while showing that a more flexible view of the relationship between theory and facts can be beneficial, even in the natural sciences. Hesse's "critique of the empiricist presuppositions" (xiv) is therefore aimed at an attempt to steer a course between scientific realism and instrumentalism, a goal that, for her, "require[s] integration of natural science into a wider epistemological framework embracing the philosophy of social science, hermeneutics, and the sociology of knowledge" (xiv).

The notion of "hermeneutics" is crucial to understanding what Hesse thinks natural science is or will become, according to an ideal of the integration of methodologies from other fields. The view expressed in her 1974 paper is that the sort of hermeneutic approach that one encounters in the social sciences can be found in the natural sciences as well; that is, a "concern for knowledge as interpretation, sometimes explicitly distinguished from what is taken to be the direct, literal, uninterpreted modes of description proper to the natural sciences" (Hesse 1980, 168).[7] In her study, Hesse argues that the development

7. Hesse's paper aims to discuss Habermas's view "of the similarities and differences between empirical and hermeneutic method" presented in his *Knowledge and Human Interests* (1972). It is within this context that Hesse's remarks must be read.

of the history and philosophy of sciences of her time gradually led to dissatisfaction with logical empiricist accounts of the structure of science and with an overly sharp and largely unviable "demarcation" principle. As a result, "the imperialism previously claimed for natural science in the empiricist tradition has now turned in some quarters into its opposite, namely an assimilation of natural science itself to something approaching the hermeneutic critique" (169). On this basis, Hesse provides an updated account of the natural sciences, with the declared aim of showing that "almost every point [traditionally] made about the human sciences has recently been made about the natural sciences" (171), whereas the empiricist view of the natural sciences that one encounters in the works of the critics of a scientific world description, who hastily reduce empiricism to a firm belief in given experience as the sole basis of scientific knowledge and in a theory-independent and stable language available to describe this given, has nowadays been almost universally discredited (cf. 172). For Hesse, "the work of Wittgenstein, Quine, Kuhn, Feyerabend, and others has . . . made it increasingly apparent that the descriptive language of observables is 'theory-laden'" (172); consequently, a *postempiricist* account of natural science can be outlined as follows:

1. In natural science data is not detachable from theory, for what count as data are determined in the light of some theoretical interpretation, and the facts themselves have to be reconstructed in the light of interpretation.
2. In natural science theories are not models externally compared to nature in a hypothetico-deductive schema, they are the way the facts themselves are seen.
3. In natural science the lawlike relations asserted of experience are internal, because what count as facts are constituted by what the theory says about their interrelations with one another.
4. The language of natural science is irreducibly metaphorical and inexact, and formalizable only at the cost of distortion of the historical dynamics of scientific development and of the imaginative constructions in terms of which nature is interpreted by science.
5. Meanings in natural science are determined by theory; they are understood by theoretical coherence rather than by correspondence with facts. (Hesse 1980, 172–73)

According to these five points, it is the very contraposition of facts and theory or interpretation that is problematized, the crucial point being that the object of scientific explanation is *not* a datum with which the observer must comply

and that must be justified *passively*. On the contrary, insofar as access to the facts available to scientific explanation depends on theoretical interpretation, it is possible to argue that an active and even constructive role pertains to theory—which is still based on an "objective reality" (Hesse and Arbib 1986, 159). This leads Hesse to a new conception of scientific meaning based on a coherence theory of knowledge: instead of searching for objectivity in the domain of facts, as the traditional correspondence theory recommends, we might explore the realm of theories. But this is only the first step for Hesse, for both solutions would seem to be unsatisfactory. Neither instrumentalism nor strong forms of scientific realism accommodate the full complexity of the theorizing process, which, she argues, involves a "circular interpretation, reinterpretation, and self-correction of data in terms of theory, theory in terms of data" (Hesse 1980, 173). As anticipated, her new epistemology aims to combine coherence and correspondence criteria of truth, which might be accomplished through the "consensus theory" she presents in her 1976 paper (1980, chap. 6).

2.2. A Moderate Realism of Pragmatic Success

Hesse's 1976 paper explores the issue of truth and meaning as approached by the philosophy of language from the late 1960s. Hesse refers to Kuhn, Tarski, and Putnam in particular, with the aim of arguing that it is possible to outline a criterion of truth that allows for the translation of theoretical sentences in different language communities and, furthermore, justifies "the historical fact that true sentences have accumulated" independently of theoretical revolutions (Hesse 1980, 159). Within this framework, Hesse outlines her consensus theory of truth, which depends not "on the assumption of any privilege for the truth of the theoretical framework of our science, but rather on the property of defining our science in terms of a certain category of observation sentences and a particular inferential method" (159). The consensus theory is a theory of reference that is "distinct from the traditional 'coherence' and 'correspondence theories,'" based on the minimal agreement that "for a given language community, 'true' observation sentences and 'correct' application of general observation terms are at least those that are reinforced as such by the consensus of the community" (145).

Hesse's original observation is that "more than mere consensus is implied by inclusion of the word 'observation' in its specification. It does not follow in such a theory that everything that is agreed is true, nor that truth is wholly dependent on the language community. Such objections rest on a misunderstanding, for the mechanics of language learning and reinforcement of 'correctness' themselves depend on external reference of language. It is not that anything

goes if agreed by the language community, but that the language community does or does not agree according to external constraints" (Hesse 1980, 145). Thus, a sentence like "That tree is green" is true *not* because of a universally recognized extralinguistic "greenness," but *also not* because "there is a capricious and arbitrary agreement that the sentence should be true" (146). Hesse's conclusion—which I cannot thoroughly discuss here—is that consensus is rather "the way we have learned, or evolved, to use our observational vocabulary" (146).

The philosophical issue Hesse aims to deal with in light of her consensus theory is that of the descriptive function of theories. Starting from the idea that "every scientific system implies a conceptual classification of the world into an ontology of fundamental entities and properties," and given that "these ontologies are most subject to radical change throughout the history of science" (Hesse 1980, 147), she elaborates a conception of the growth of knowledge not as "a convergence of ontologies approximating better and better to a [literal or direct] description of the true essence of the world," but rather as "an instrumental growth, as pragmatic as our desire to have true, controllable predictions" (159). Quite significantly for a comparison with Mach's epistemology, Hesse defines this "pragmatic . . . accumulation of true observation sentences . . . in the sense that we have better learned to find our way about in the natural environment, and have a greater degree of predictive control over it" (158). This is unsatisfying for Hesse, however, for it leaves unexplained how we can assess our theoretical sentences as true or objective, especially if they are not directly constrained by the external world. It is on this issue that the consensus theory can make the difference. Combined with a probability model (interpreted in an epistemological sense as a "rational or warranted degree of belief in the relevant scientific community that the sentence in question is true" (148), that theory allows us to maintain that the truth value of a theoretical sentence implies that "there are entities and properties in the world as [it] describe[s] them" (153). Therefore, Hesse argues that "meaning is not . . . given independently of observation constraint and purely by theoretical context" (153); it is not a matter of instrumentalism, but is rather determined by the interplay between theoretical and observational activity in a constructive sense that is fundamentally focused on the plane of actual scientific knowledge (or description, or explanation).

We thus reach the core of the *new epistemology* that Hesse outlines in 1994, which is based on the idea that "there is no ideal, univocal language for science which can directly and unambiguously represent the world" (453), but also that a "moderate realism of pragmatic success" can be maintained (447, 456). That epistemology stresses the metaphorical value of scientific language but does not aim to leave behind the world's "true state." On the contrary, Hesse argues that "acceptable theories do not need to be put forward as true in order to be pragmatically

useful. But they do tell a story about the world which captures some of its structure, and in that sense relates to its true state" (447). In agreement with those authors who undermined the fundamental presuppositions of standard empiricism, Hesse is not interested in denying the very existence of an "underlying structure of the natural universe" (446) that science aims to discover in a set of true propositions constituting theories. Rather, she problematizes the idea that this structure "can ever be accurately or completely *known* or represented in language," for although theories "may be better or worse representations of the natural world, none can ever be known to be definitively *true*" (446).

Hesse's view is both multifaceted and controversial. It is not my intention to discuss her position here, but rather to show whether and to what extent certain of her assumptions can profitably be compared with Mach's. In particular, the attention she pays to the relationship between theories and facts as the domain of actual scientific knowledge, and her defense of a moderate realism of pragmatic success, are especially promising. I will therefore focus on these, starting in the following section.

3. Thoughts, Facts, Elements

What has been argued thus far has helped me to outline the picture in light of which I believe Ernst Mach's contribution to the philosophy of science might profitably be read. It may be worth remarking once more that it is not my intention to maintain that Mach was a postempiricist tout court (and even less an anti-empiricist, which is far from what Hesse had in mind). I would instead like to argue that he contributed to the development of that perspective—that his view of the role and value of the scientific world description, and his critical remarks on the metaphysical commitment of the scientists who were active at his time, played an important role in the problematization of the empiricist view, which remains the framework of Mach's inquiry. What I therefore aim to show in the following is that the epistemological view we can infer from Mach's writings seems to be consistent with certain presuppositions of the postempiricist approach to science expressed by Mary Hesse and with the resulting view of scientific knowledge. Importantly, this implies the redefinition of such notions as "correspondence" and "objectivity," which Mach's account of theory as mental-economical reconstruction of facts in thought allows us to reconceive on a moderate-realist basis. Finally, Mach's postempiricist account is helpful in relation to two main goals. First, it allows us to evaluate Mach's pivotal role in the history and philosophy of science from an original perspective, one that offers an alternative to the received view and to the traditional continental versus analytic divide. Second, it sheds light on certain aspects of Mach's conception

of scientific knowledge that remain unclear and largely neglected in the literature. If properly explored, these aspects may help to reveal interesting features of Mach's epistemology that could be relevant to the ongoing debate on scientific realism and the actual value of scientific conceptualization.

The main issue to be addressed is the relationship between thoughts (ideas or theories) and facts. As I have tried to show, this issue plays a central role in Hesse's elaboration of the postempiricist view to the extent that she focuses on the role that interpretations play in the redefinition of the domain of facts that science attempts to explain (Hesse 1980, 111; Hesse and Arbib 1986, 8). This relationship is equally central to Mach's work, although this has largely been ignored in the secondary literature. Mach deals with it several times, focusing on how concepts arise and on their function in both science and ordinary thought; precisely how concepts represent facts, however, remains an unexplored background question.[8] What seems to be widely accepted is that, like Hesse, Mach reflected on the relationship between theoretical and observation statements in a way that calls for a problematization of that relationship—as if he were not completely satisfied with the ordinary conception of truth that defends the idea of a literal representation of facts through scientific theories (cf. sec. 2.2). On the other hand, Mach's empiricism—that is, the idea that facts are the roots of scientific conceptualization, the latter consisting in an activity of reconstructing facts in thought oriented toward "as faithful a picture as possible of nature herself" (Mach 1976, 81)—can hardly be bypassed. That is why Paul Feyerabend's (1984, 8–9) claim that for Mach "adapting ideas to facts . . . is a dialectical process that transforms both ingredients," albeit intriguing, is highly controversial, for it seems to imply a modification of the facts themselves. As John Preston (2020) convincingly shows, Feyerabend goes too far in arguing that for Mach scientists are engaged in "*positive* and *constructive* work" that allows them to actually "rebuild facts" (Feyerabend 1984, 8; cf. Mach 1976, 234), for he

8. The issue of what a fact is, for Mach, is also scarcely considered in the relevant scholarship. It has been indirectly addressed by John Blackmore (1972, 32) and Gerald Holton (1988, 247), but thus far no thorough study has been devoted to it, the only exception being a paper that was published as I was working on the revision of the present contribution (De Waal and Ten Hagen 2020). In that paper, a thorough exposition of the use of the concept of a "fact" in modern (philosophy of) science is provided, and De Waal and Ten Hagen focus on the case of Mach and Einstein. As I do, they (a) argue that "Mach's understanding of a 'fact' seems to have been mistaken as self-evident"; (b) make clear that "facts" and "elements" or "sensations" are not synonymous in Mach; (c) maintain that "Mach made a clear distinction between fact and theory, but he also acknowledged that the two crucially depended on one another" (i.e., facts alone have little scientific value in Mach); and (d) remark that "Mach's reputation as a positivist only interested in facts is an oversimplification" of his view (cf. De Waal and Ten Hagen 2020; the open access edition does not provide page or section numbers).

mistakes the reconstruction of facts in thoughts (i.e., a purely mental-economical elaboration), which Mach discusses several times, with an actual building activity.[9]

In what follows, by taking a different path than Feyerabend, I will try to explore the apparent tension (*not* a contradiction, for they in fact coexist in Mach) between the ideal of mirroring facts in thought and the aim that Mach (1895, 186; 1986, 361) ascribes to science—that is, to provide us orientation through theories (i.e., indirect descriptions; 1976, 2–3, 354; 1986, 365). This, I think, allows us to properly understand how Mach conceived of actual scientific "knowledge"—that is, the conceptual elaboration of facts that implies both simplification and amplification of the explored area of facts and can never provide us with literal correspondence between theory and facts (Mach 1976, 98, 355). As a result, I will outline a moderate constructivist view—a view that does not pretend to grasp the ontological level of our world representation but that is limited to the inaccurate, albeit operationally efficient, mental reconstruction of facts of which we are capable (e.g., Mach 1976, 355).

3.1. Connections of Elements

In Mach, the notion of "facts" appears as the middle term in a triad of concepts: *thoughts-facts-elements*.[10] In order to understand what constitutes the actual object of scientific inquiry on Mach's view, we must therefore examine the role that facts play in this triad—that is to say, how they are related to the other two concepts, starting from what is supposed to be the realm of natural world itself: the *elements*.

The issue of the elements is perhaps the most explored topic of Mach's thought—and for good reason, given its importance to his work and the interesting features that pertain to it. A key reference in this regard is the work of

9. Preston (2020) especially remarks that Feyerabend "attributes to Mach various views characteristic of the mid-twentieth century historical turn in the philosophy of science (more specifically, of *his own* such turn). Notably, Feyerabend credits Mach with the beliefs that science involves transforming facts, and that all concepts are theoretical concepts." As Preston observes elsewhere in his paper, the latter claim "is Feyerabend's extreme version of the claim that scientific observation is theory-laden."

10. It may be worth clarifying that the thoughts involved in this triad are the result of the mental ("*gedänklich*," i.e., intellectual or cognitive) activity that elaborates concepts (scientific concepts in particular). That is, it is not the psychical ("*psychisch*," sometimes misleadingly translated as "mental"), which appears in Mach's reflections on the elements, of which I am willing to speak. In the following pages, I will focus on the theoretical ideas to which Mach refers in the several papers and book chapters where he deals with the transformation of scientific thoughts or ideas and their adaptation to facts (e.g., 1986, chap. 25; 1976, chap. 10). These ideas are a product of the mental-economical ("*denkökonomisch*") activity of conceptual completion of facts in thought, which, according to Mach, especially characterizes the scientific world description.

Erik C. Banks (see esp. Banks 2003, 2004, 2014), who defends a direct realist view of Mach. For Banks, "Mach was convinced that the most realistic portrait of the world was in terms of these individually unique, *irrepeatable* states or events, not substances, objects or laws" (2004, 25). A thorough exploration of Mach's *neutral monism*—the view that the world consists of a flux of states that are neither physical nor psychical in themselves but rather subjects of interpretation that depend on the observer who deals with them (cf. Mach 1959, chap. 1)—leads Banks to argue that Mach defended "a kind of ontological, i.e. meta-scientific, observation about the manner in which things exist" (Banks 2004, 40). That is to say, not only was Mach concerned with how science is structured and the kind of methodology it employs, but he also aimed to provide a realist world description (an "empiricist realist" description, in fact), his efforts being directed at providing "entrée for a new metaphysics" (Banks 2004, 46).[11]

11. In presenting his case for Mach's neutral monism, Banks seems to be unaware of the fact that an alternative conception of that issue can be defended within the positivist tradition. In fact, neutral monism can be seen as an expression of the so-called *Immanenzphilosophie*, a perspective advocated, e.g., by Wilhelm Schuppe and that played an important role in the German and Austrian philosophical debate in the late nineteenth and early twentieth centuries (cf. Acham 1976; Sass 1976. On Schuppe's antitranscendentalist philosophical conception, see also Hiebert 1976, xxiii). As well as Banks's view, which is supported by Mach's writings and has been recognized both by his contemporaries (e.g., Paul Carus, Hans Kleinpeter, William James) and by a great variety of philosophers throughout the past century (cf. Banks 2018, 77), the interpretation of Mach as a typical representative of *Immanenzphilosophie* and *Immanenzpositivismus* has been defended over the years. (David Romand has recently defended this interpretation in his interesting attempt to "provide a new perspective in the history of positivism" based on a comparison between Mach's concept of "sensation" and Heinrich Gomperz's notion of "feeling." Quite significantly, Romand [2019, 106] also remarks that "the paradigm of *Immanenzpositivismus* should not be confounded with the research program elaborated by Mach, which represents only a particular tendency of the positivist school.") According to the upholders of the immanentist interpretation of Mach's epistemological concerns, the most important feature of his positivist epistemology resides in the idea that knowledge makes sense only insofar as its constituents are given to individual consciousness. The foundations of knowledge rest basically on "sensations," while the ultimate ontological level should be what Richard Avenarius (another key figure of the immanentist movement) called "pure experience." That level is, per se, neither physical nor psychical, neither external nor internal, neither objective nor subjective (cf. Romand 2019, 94). Therefore, the "immanentist stance" might be the very gist of Mach's neutral monism. What is especially interesting is that, at the beginning of the twentieth century, Mach and the other immanentist philosophers were commonly regarded as the supporters of an idealist epistemology (cf. Sass 1976, 237). In light of this, there might be space for an idealist interpretation of Mach's neutral monism, for, as a matter of fact, the immanentist stance implies that in the final analysis, every kind of cognizance and every form of reality has to do with the consciously given, and is therefore of an "experiential" nature. In this respect, immanentist positivists admit the primacy of the mental over the material and acknowledge the epistemologically and ontologically founding nature of individual consciousness (I owe these remarks to the third anonymous reviewer, to whom I am thankful). I am not willing to deal with an issue that would lead me far beyond the aim and scope of the present paper. I only would like to remark that, in fact, Mach (1959, 46) expressed his sympathy

Albeit fascinating and grounded in good textual evidence, this view is permeated by at least two problems. The first is the function versus element contraposition outlined by Banks, which does not seem to account for the tentative approach to the latter that can be found in Mach (the elements are the "ultimate component parts [of things], which *hitherto* we have been unable to subdivide any further" [Mach 1959, 6; emphasis added]). Banks in fact distinguishes between "first-order" theories of real events and "causal-functional relations grounded in the real causal behaviour of the elements and their qualities as they exert forces on each other," on the one hand, and a "second-order sorting of these patterns, [which] is necessary for the purposes of economy or codification of many phenomena under few laws and principles," on the other (2014, 51). The former "are not subject to thought economy at all"; they "are really there," and the development of physiological psychology would allow us to grasp them (2004, 44). Thus, Banks takes the two realms to be continuous, but at the same time a clear separation between them can be accomplished at the theoretical level. Furthermore, Banks argues that, especially in the last stage of his work, Mach allowed for the possibility of a direct representation of "real" events. As I argued in another paper (Gori 2018), given Mach's particular care not to replicate the sort of metaphysical speculation he finds expressed in the work of most of the scientists working at his time, which he so strongly criticizes throughout his career, it seems to me that Banks tried to go too far in ascribing that sort of realism

for "the representatives of a philosophy of immanence" (esp. Schuppe), but also his aversion to any attempt to accuse him of "idealism, Berkeleianism" and other "-isms" (1959, 48, 361). He also claims to have been an upholder of the Kantian critical idealistic standpoint in his early years, and that this standpoint also influenced his later work (357, 367). Finally, he declares he is thankful to Avenarius for having dealt with the same tendencies toward idealism and pan-psychism that he had to work through, and for "having developed the same conception of the relation between the physical and psychical on an entirely realistic or . . . materialistic foundation" (362). These observations leave us with difficult interpretative work to do, all the more so because Mach's famous rejection of both realism and idealism as an dispensable starting commitment of any research (56) seems to be as hasty as inconsistent with what we actually read in some of his works. But if we start from a contextualization of his view, things become clearer. That is, if we consider that Mach's criticism was oriented toward the *materialistic* realism, on one side, and toward the *solipsistic* tendencies following from idealism, on the other side (Goeres 2004, 45, 54), there seems to be space for a more nuanced reading of the basic tenets of his epistemology. This reading, far from being aimed at pronouncing the final word on such a multifaceted and subtle issue, might allow us to defend moderate idealist or moderate realist interpretations of Mach's epistemology, leaving the discussion open to a variety of scholarly opinions. As for the immanentist positivist versus empiricist realist interpretation of neutral monism, as said, I think it deserves to be thoroughly explored in another paper. Given its relevance in the current debate, I decided to start from Banks's view, which I do not endorse uncritically. On the contrary, it is precisely in light of Mach's phenomenalist stance (i.e., the role he attributes to sensations as the foundation of actual meaningful knowledge), and his reiterated attempt not to defend metaphysical commitments of any sort, that I think it is possible to discuss Banks's interpretation.

to Mach. On the contrary, I am inclined to think that a *moderate realist* view might work better in his case—that is, a view that accepts the existence of a world of elements, of whose functional relationship we can only be conscious of *as* sensations, but which rejects the possibility of gaining actual *direct* knowledge of that world itself. This view seems to be more coherent with Mach's reiterated interest in the phenomenal side (the plane of experienced facts, as I will show below) as the only realm of which one can meaningfully speak.[12] The second problem I find in Banks's remarkable study is that he leaves facts out of the picture. Banks deals with the way in which scientific knowledge elaborates an economical strategy for taking care of the flux of (what we might call) "elementary states" and organizing them in a fruitful system of concepts for the purposes of orientation.[13] Science is not directly concerned with elements, however, and Mach seems to be quite clear on that. Science aims to explain facts, not elements, and it is to facts that thoughts must be adapted (cf. sec. 3.2). Here we touch on the problem at issue: what are facts, and what is their relation to elements on the one hand and to theory on the other?

A clue in this regard is provided by Mach himself in an 1892 paper published in *The Monist*. In this paper, he observes that "every physical notion is nothing more than a definite connection of the sensory *elements* which I denote by A B C . . . , and every physical fact rests therefore on such a connection. These *elements*—elements in the sense that no further resolution has for the present been effected of them—are the most ultimate building stones of the physical world that we have as yet been able to seize" (Mach 1892, 205). What can be inferred from this excerpt is that, for Mach, (1) both physical *facts* and physical *notions*

12. That is why I am also inclined to use the expression "epistemological agnosticism" to define Mach's view, for I believe that, in the end, he was not interested in metaphysical investigations that try to grasp things that cannot be meaningfully expressed (Gori 2018, 167ff.). As observed by Wolters (2008, xiv–xv), Mach insists on the fact that all knowledge comes in the form of a conscious content (*Bewusstseinsinhalt*) and discards as useless the metaphysical question regarding any nonsensible reality lying beyond or behind the experienced world. But at the same time, it is important to deal carefully with the value Mach devoted to the sensations and their relationship with the elements. For example, Ralf Goeres (2004, 50) defends that we must be aware that, according to Mach's phenomenalist neutral stance, "it is misleading to conceive of elements as 'conscious contents' [*Bewusstsinsinhalt*]," for they are a far more fundamental entity. They are *Urphänomene* (archetypal phenomena), in fact, which can be *perceived as* sensations, depending on the way we look at them (50). Actually, given the three classes of elements that Mach famously describes ("world elements," A B C; "body elements," K L M; "intrapsychical" elements, α β γ), only the functional relationship between the first and the second classes can be *seen as* a compound of "sensations" (1959, 16). Therefore, argues Wolters (2008, xvii), "sensations are a special type of elements" (on this, see also Mach 1976, 356; Goeres 2004, 52). Yet, the role of sensations in Mach's epistemology cannot be understated, for he repeatedly argues that the world we can know and meaningfully describe actually consists in our sensations (1959, 12).

13. Cf. Mach (1959, 37; 1976, 2–3, 354). On this and for further remarks on Banks's interpretation, see Gori (2018, 2019).

rest on a *connection of elements*; and (2) the *elements* themselves are only a temporary resting point of our research, the further development of which may reveal quite a different picture. The second point is something that Mach repeatedly stresses, most likely because he seems to have no intention of offering the final word on an issue that is metaphysical at its very core. (As said, this, in my view, at least undermines any attempt to portray Mach as a direct realist.) I am more interested in 1, however, for it allows me to introduce the other part of the triadic relationship between thoughts, facts, and elements.

If I am not mistaken, Mach affirms that the facts that science deals with are not the elements themselves but a product of the *relationship between groups of elements*.[14] More precisely, they can be seen as clusters or relatively stable connections of "world elements" (to use Banks's expression) that come to us, such as "the green of a tree" or "refracted light" (Mach 1892, 202). It is this kind of natural event that scientists describe and try to explain by "mentally *supplying*" them with economical symbols (200).[15] Here is where the relationship between facts and thoughts plays out. Physical facts rest on connections of elements; physical notions (even the least-interpreted physical notions we can imagine) are expressions of the same connection. But this expression is the result of a categorization and simplification activity that "puts in the place of a fact something *different*, something more simple, which is qualified to represent it in some *certain* aspect, but for the very reason that it is different does *not* represent it in other aspects. . . . Our theories are abstractions, which . . . neglect almost necessarily, or even disguise, what is important for other cases" (201). This does not mean that theories create facts, of course. At the same time, however, Mach seems to make room for the idea that it is impossible for us to directly mirror in thought the observed phenomena, in fact, and that we instead ordinarily work with "models" that only "imitate in form" actually occurring events (202). Anticipating an observation that he would go on to elaborate further in other works, Mach argues that "we have to distinguish sharply between our theoretical conceptions of phenomena and that which we observe. The former must be regarded merely as auxiliary instruments that have been created for a *definite* purpose and which possess permanent value only with respect to that purpose" (202). Thus, the question at stake seems to be this: as scientists, we live in a world of theoretical elaborations that reproduce natural events in the form of mental-economical

14. On this, see also Wolters (2008, xvii) and Goeres (2004, 52). The sort of "production" I refer to needs not to be interpreted as a secondary event that actually follows the "flow" of the elements. Affirming this would of course be inconsistent with the way Mach describes our relationship with the natural world (that, for him, "exists once only"; Mach 1895, 199).

15. On facts as "complexes of elements, as well as the relations holding among them," see Brecevic (2021), De Waal and Ten Hagen (2020), and Banks (2003, 43–44).

symbols (may we call them "metaphors," as Hesse does?).[16] Our theoretical statements are rooted in observational statements, however; they somehow and to some extent correspond to phenomena and to what Mach (1976, 356) calls "sensible reality." How is this so? What kind of access to facts does scientific knowledge provide us with, throughout theory? Also, given that, for Mach, meaningful knowledge only pertains to that sensible reality, what is the actual value of the scientific world representation?

3.2. Physiological Reactions and Intellectual Conceptualization

Throughout his writings, Mach repeatedly stresses that science is a process of *adapting thoughts to facts and thoughts to each other* (cf. 1895, chap. 12; 1976, chap. 10; 1986, chap. 25) and devotes highly interesting sections of his later works (1976, 1986) to explaining how this adaptation must be interpreted. As is well known, Mach maintains that "knowledge is a product of organic nature, and . . . if Darwin reasoned rightly, the general imprint of evolution and transformation must be noticeable in ideas also" (1895, 217–18). Given that science is simply a highly elaborated version of human knowledge on Mach's view, this can also be applied to theories and laws of nature. In fact, Mach argues that "ideas, and especially scientific ideas, are transformed and adapted in the same manner as that which Darwin supposed to be the case for organisms" (1986, 350), for "thoughts are organic processes. . . . Thus, our whole scientific life appears to us as one side merely of our organic development" (358). The way in which Mach deals with this issue allows us to argue that he adhered to a strong form of *evolutionary epistemology* that maintains that it is possible to "account for the characteristics of cognitive mechanisms in animals and humans by a straightforward extension of the biological theory of evolution to those aspects or traits of animals which are the biological substrates of cognitive activity," and that the same can be done "for the evolution of ideas, scientific theories and culture in general by using models and metaphors drawn from evolutionary biology" (Bradie 1986, 401–3).[17]

16. An interesting treatment of the issue of scientific models in light of Hesse's conception of metaphors is provided in Bailer Jones (2009).

17. In fact, Mach believes that the process through which ideas adapt to facts (and to each other) is the same as that through which a species adapts itself to its environment, including the mechanism of *exaptation* (i.e., the idea that "an organ originally constructed for one purpose . . . may be converted into one for a wholly different purpose" [Darwin 2008, 143; cf. Gould and Vrba 1982]) of members of a single organism (Mach 1895, 63, 228–29). On Mach's commitment to an evolutionary epistemology or, less radically, to a biological conception of knowledge, cf. Čapek (1968), Haller (1988), and Pojman (2011). For a contextualization of this conception in Mach's intellectual framework, see also von Mises (1992, 251ff.), Wolters (1985, xiii), and Goeres (2004, 46ff.).

If this is the case, it is possible to infer something interesting about the relationship between thoughts and facts, in Mach. On the one hand, there are those thoughts, ideas, theories, and so forth that must be coherent among themselves. Their survival within this system is determined by their ability to agree with one another and to contribute to mutual support. On the other hand, there is the environment to which thoughts must adapt (i.e., the domain of facts to be explained or described). If the analogy with natural selection must be understood in a nonmetaphorical sense, it might be said that a mutual adaptation is involved, for the Darwinian model implies that the environment constantly changes in compliance with the biological modifications of the species it hosts. This would suggest that Mach held a strong constructivist view, according to which thoughts determine a modification of facts themselves. However, no evidence for this can be found in Mach. On the contrary, he repeatedly maintains that ideas are rooted in facts and that it is thoughts that adapt to facts, not the reverse. At the same time, Mach conceives of theoretical activity as determining an extension of the domain of facts to be explained—not of facts themselves but of the "constantly widening sphere of action" of ideas, whose transformation "appears as a part of the general evolution of life" (1895, 233).

Just how this type of modification of the environment should be understood can be clarified on the basis of Mach's biological conception of knowledge. He argues, "scientific thought arises out of popular thought, and so completes the continuous series of biological development that begins with the first simple manifestation of life" (Mach 1976, 1). At the lower level of the relationship with the external world, we find pure physiological reflexes, which are subsequently perfected through (*a*) the development of more elaborate sense organs, (*b*) the rise of imagination and memory (of two different kinds: individual and communicated), and (*c*) the activity of thinking (1). The emergence of scientific thought is therefore first and foremost a matter of increasing complexity on the model of *trial and error*, to use the famous expression adopted by Karl Popper: at the final stages of the series (ordinary and scientific knowledge), we find an attempt to mentally supply incomplete observational findings. Accordingly, we can say that the concepts we elaborate "consist in consciousness tied to a word of the reactions to be expected from the class of objects or facts denoted. . . . The object corresponds to the concept, if it yields the expected reaction when tested in the way intended" (Mach 1976, 97; cf. 93). Although concepts are "rooted in facts" (99) and always have a "factual correlative" (102), Mach argues that the agreement between our mental symbols and the results of observation should be evaluated *pragmatically*, as a matter of the response received by our attempts at explanation. In fact, Mach defines scientific knowledge as "a mental experience directly or indirectly beneficial to us" that "flows from the same mental

source" as error, and argues that "only success can tell the one from the other" (84).[18] What is relevant, in order to evaluate our judgements, is therefore only whether they "stand up" (84) and allow us to achieve the expected result.

Of course, this is not a matter of mere instrumentalism for Mach. He indeed assumes that the "potential knowledge" that concepts contain (Mach 1976, 83) can be properly assessed only on the basis of how well they help us to "cope with the wealth of experience" (81). In principle, "the course of representations should adapt itself as closely as possible to physical and psychical experience"; it "is to be as faithful a picture as possible of the course of nature herself" (81–82).[19] What Mach never gives up is the notion that our engagement with the world is grounded on the physiological plane. Accordingly, he argues that "while concepts do indeed not exist as physical 'things,' our reactions to objects [i.e., facts] of the same class of concept are psycho-physically similar, and those to objects of different classes dissimilar" (Mach 1976, 99). This means that the concepts elaborated by science do correspond to something independent of theory. It is thus possible to outline a principle of objective evaluation of knowledge based on how efficiently theories "keep pace with facts" (we might say "save the phenomena"; cf. 1895, 227) and make successful predictions. The question is, should we believe that our theories will ultimately faithfully represent the realm of facts (at some point in the future), that the process of adapting our thoughts to them will ever determine a direct or literal "representation of facts in thought" (1976, 2)? It seems to me that Mach has not (always) been overly confident in this regard.

The question at stake here is how we should interpret the relationship between theoretical and observational statements in Mach, and what we can say about his view of correspondence and objectivity in science in light of that. What I would like to defend is the claim that Mach situates the scientific world representation in the *relationship* between thoughts and facts, for it is only at the meeting point of these two realms that we produce actual scientific "knowledge." But

18. The way in which Mach conceives of these concepts is well expressed by William James in an annotation to his personal copy of *Knowledge and Error*: "Error in Mach's eyes has the exclusively practical meaning of a concept that leads to disappointment in expectation. *Täuschung* [deception] leads to *Enttäuschung* [disappointment], *Wahrheit* [truth] to *Bestätigung* [confirmation]" (Hiebert 1976, xxvi). On the pragmatist character of Mach's epistemology, see, e.g., Gori (2018).

19. I have slightly modified the English translation of this passage, using "representation" instead of "idea" for the German *Vorstellung*, as well as "psychical" instead of "mental" for the original *psychisch*. The latter substitution in particular is intended to stress the reference to the psycho-physical parallelism that is so relevant in Mach and that, as the quoted excerpt also suggests, is to be distinguished from the mental (i.e., intellectual [*gedänklich*]) activity of conceptual completion of observed facts that takes place at another level of world description.

by this I do not mean that Mach maintains a strong constructivism according to which facts are a product of theoretical activity. Rather, my view is that Mach takes care of the role played by the mental activity of the conceptual completion of facts *in thought* in order to make sense of scientific world representation (which is made of abstract models and symbols) and of laws of nature to which nothing literally corresponds in the world of our experience (cf. 1895, 193, 201). It is this world representation that is constructed, and the relationship between this construction and the facts it aims to reproduce in thought is worthy of consideration.

From what has been argued above, the picture that can be drawn is the following:

1. There is a natural world that "exists once only" (Mach 1895, 199); this is the realm of what Mach calls the *elements*, consisting in a flux of events, states, or whatever we can call a process of pure development. Since we can interact with that realm only through our bodies, we ordinarily mistakenly view it as divided into two separate realms: the *physical* (i.e., "the totality of what is immediately given in space for all") and the *psychical* (i.e., "the totality of what is given immediately only to one" [1976, 5]).
2. We naturally perceive these elements combined in relatively stable groups, to which we attribute relevance insofar as they help us to discern a pattern and, consequently, to organize our immediate experience; at this level, Mach talks of "observed facts, phenomena in the outer or inner world" (1986, 363; cf. Mach 1959, 10).
3. Finally, there is the system of thoughts, theoretical representations, theories, and so forth, which results from the economic activity of the "mental completion of a fact from partial data" (Mach 1976, 1; cf. 1895, chap. 9). On this, an apparent tension between two views of scientific knowledge seems to be at work. On the one hand, we find in Mach the idea that the human mind "attempts to mirror in itself the rich life of the world" (1895, 187; cf. 1986, 361), which is consistent with the ideal of mental adaptation to facts as an attempt to reproduce observed phenomena as adequately as possible, which has been explored above. On the other hand, it seems uncontroversial that Mach conceives of science as a tool for orientating ourselves in the natural world (1959, 37; 1976, 2, 98, 354). For him, the elaboration of conceptual symbols allows us to spare mental effort and reproduce in thought a large amount of facts (1895, 191ff.), but this activity implies an intervention into the received data and,

> consequently, determines a contraposition between theoretical and observational statements (cf. 1986, 365).[20]

It is at this point that constructivist language comes into play. For example, Mach defines scientific explanation as the attempt to "comprehend without effort" a province of fact in order to "no longer feel lost in this province," arguing that this goal is achieved once we "mentally construct the whole province" (1895, 194). Furthermore, in the quite interesting (albeit largely neglected) chapter titled "Comparison as a Scientific Principle," Mach maintains that "description" in science "is a construction of facts in thought" (1986, 370) and that "the strict definition of a concept and, in case it is familiar, even the name of the concept is a stimulus to a precisely determined though often complicated, testing, comparing or constructing *activity* whose result, in most cases perceptible by the senses, is a term in the extension of the concept" (369). In agreement with the pragmatic conception of knowledge introduced above, Mach finally argues that "the concept is not a finished idea, but [a] body of directions for testing some actually existing idea with respect to certain properties, or of constructing some idea from given properties. The definition of the concept, or the name of the concept, releases a definite activity, a definite reaction, which has a definite result" (381). This construction activity takes place in thought, of course. As shown above, concepts are rooted in facts, for Mach, and our mental activity cannot modify the latter on the ontological plane. It is only in this sense that his further statement that "conceptions are instructions for building, and facts are the result of building" (Mach 1986, 370), must be understood, I think. But we must consider the fact that, for Mach, scientific knowledge actually takes place at the mental-economical level, and for that reason it is crucial, on his view, to focus on that construction activity. By this I mean that it is thought that provides us with access to facts. Therefore, what is in fact relevant to scientists is the class of facts that the theoretical activity allows them to deal with.[21]

On this, it may be worth considering a couple of passages in which Mach focuses on the relationship between theoretical and observational statements. He asks: "What is a theoretical idea? Why does it seem to us to stand higher than the mere adherence to a fact or an observation?," arguing that "the idea can, and is intended to, present more than we see at the moment in the new

20. Mach observes that "a 'theory' or a 'theoretical idea,' which is the starting point of a theory, falls into the category of indirect descriptions," that is, "a description in which we make use of one already given elsewhere" (1986, 365).

21. On this, cf. Mach (1976, 102): "Thought does not occupy itself with things as they are in themselves, but with our concept of them; we know things only through their relations with other things."

fact; it can widen and enrich this fact with new features for which we are stimulated to seek—and which are often found" (Mach 1986, 365). Thus, a theoretical idea determines "an extension of knowledge which gives a quantitative advantage over mere observation" (365). Accordingly, elsewhere he observes that concepts, whose purpose is "to allow us to find our way in the bewildering tangle of facts . . . can represent and symbolize in thought large areas of fact. . . . By bridging it under a concept, we simplify a fact, by leaving out of account those factual features that are irrelevant to our purpose, while at the same time amplifying the fact, by including all the characteristics of the class" (1976, 98).[22] These excerpts outline a conception that can be compared with Hesse's constructivism, which affirms that theoretical explanation is a "metaphoric redescription of the domain of the explanandum" (1980, 111), and that the development of successful theories determines the constant extension of "the range of phenomena that exist to be described and explained" (Hesse and Arbib 1986, 8). Neither Mach nor Hesse has in mind the construction of facts themselves. Rather, they stress the way in which our theoretical activity continuously determines a modification of how the phenomena to be explained are represented by us, both extending their range and focusing on different or new features that pertain to them. But this activity is only the first part of a process of elaboration of hypotheses that must be successfully tested against sensible reality in order to become "knowledge." The factual correlative of concepts can never be neglected, and it is of course not subject to any modification by the concepts themselves.

To conclude this section, let me briefly return to the issue of the adaptation of thought to facts. The apparent tension between an ideal of intellectual mirroring and the merely economical aim of science seems to disappear in what Mach states about scientific progress. Indeed, he observes that "experience never ceases, and science, accordingly, stands midway in the evolutionary process" of adaptation that determines ever "richer" (i.e., more complete and therefore adequate) conceptions of facts (Mach 1895, 227). Similarly, he remarks that "more accurate quantitative enquiry aims at determining facts as completely as possible," and that "the progressive refinement of the laws of nature and the increasing restriction of expectations corresponds to a more precise adaptation of thought to fact" (1976, 355). Nevertheless, he also admits that "it is not possible to achieve perfect adaptation to every individual and incalculable future fact" (355).[23] Therefore, the idea of a literal reproduction of facts in

22. Cf. Mach (1959, 325): "Facts . . . are extended and enriched, and ultimately again simplified, by conceptual handling."

23. Similarly, Mach argues that "it remains an open question how far nature corresponds to . . . our formal need of a very simple, palpable, substantial conception of the processes in our environment . . . or how far we can satisfy it" (1894, 54).

thought can only be maintained as a regulative ideal, while a moderate realist conception of science seems better able to adhere to the actual features of scientific knowledge explored by Mach.

4. Conclusion: On Correspondence and Objectivity in an Extra-Empirical Sense

In light of what has been argued thus far, it is now possible to say something on the sort of constructivism that can be found in Mach and, especially, on how he tried to reconceive ordinary correspondence and objectivity in science. As noted above, I think that his views can be profitably compared with Hesse's (1994) attempt to develop a new epistemology based on a moderate realism of pragmatic success and, from a broader perspective, it can be related to the path leading to the postempiricist conception that she outlines. With this, it is not my intention to maintain that Mach was himself a postempiricist or a forerunner of Hesse's view, of course. My aim is rather to stress that Mach, the empiricist, reflected on the relationship between theoretical and observational statements in an original way, focusing on the tension between the ideal of a direct mirroring or literal representation of the natural world through scientific conceptualization and the actual value and function of theoretical ideas. Furthermore, he explored how these ideas allow us to deal with facts and argued that the scientific world description is the result of an activity that involves *both* the theoretical reconstruction of a class of facts *and* the reactions of the factual correlatives of the concepts denoting that class.

It seems to me that all of these elements are fundamental to a first reconsideration of standard empiricism and an attempt to outline a theory of knowledge that "combines coherence and correspondence criteria of truth" (Hesse 1986, 171).[24] In fact, Mach undermines the conception of a literal correspondence between facts and theories when admitting that the latter only provide us with an "indirect description" of the natural world and, consequently, that their complete adherence to facts is an unachievable goal (1976, 355; 1986, 365). At the same time, however, scientific knowledge is not a matter of mere coherence at all for Mach. Concepts are rooted in facts, and thoughts are involved in a process of adaptation to them that determines both further elaboration of the system of theories and the extension of the domain of facts to be explained. Furthermore, the agreement between a concept and the denoted object is conceived of by Mach

24. It is worth noting that the concept of truth is largely absent in Mach's *Sketches on the Psychology of Enquiry* (1976), where he instead speaks of "knowledge" and "error" from a functional-pragmatic viewpoint, as shown above.

as the result of a pragmatic activity of testing facts through theory, for he argues that "the object corresponds to the concept, if it yields the expected reaction when tested in the way intended" (1976, 97).

Thus, the alternative to the ordinary correspondence and coherence theories elaborated by Hesse seems to be a viable option for Mach as well, or at least we can say that some aspects of her view seem to be consistent with Mach's pragmatist epistemology. Hesse's consensus theory of scientific knowledge is of course grounded in a basis that differs markedly from Mach's, and their semantic frameworks are not the same. Nevertheless, something of value can be said, I think, starting from the relationship between theoretical and observational statements explored by both authors. As shown above, Hesse rejects the idea of a direct correspondence between theory and facts and argues for the dependence on "theoretical interpretation" of "what count as data" (1980, 172). Yet she also maintains that agreement between theories is not a satisfactory principle for the evaluation of truthful knowledge, for it leads to mere instrumentalism with no apparent correlate in the natural world. As she argues, "meaning is not given independently of observation constraint and purely by theoretical context" (1980, 153). Thus, the viable alternative must consider both the function of theoretical statements and their relation to observation. The consensus theory outlined by Hesse in fact combines the two alternative views in a moderate realist conception that stresses the pragmatic value of scientific knowledge, in a sense that can also be found in Mach. Although the kind of constructivism we find in his writings is not as strong as Hesse's (it must nevertheless be said that Hesse, too, primarily focuses on the *domain* of the explanandum as the actual subject of theoretical interpretation, i.e., hermeneutical reconstruction), Mach likewise elaborates a view of scientific knowledge focused on the interplay of theories and facts. As shown above, this interplay does not imply that theories *rebuild* facts in the sense of an antirealist constructivism. At the same time, it can be argued that the extension of the class of facts to be explained is not fixed for Mach, but rather changes depending on the theory.[25]

The plane on which this constructivism must be assessed is that of the *meaningfulness* of conceptual language. On this, too, a certain consistency between Hesse and Mach can be observed. For Hesse (1958, 14), "the meaning [of phenomena statements] cannot be entirely independent of that of the theories," insofar as these statements "are to be tests of theories." Similarly, Mach defines scientific knowledge as an agreement between theoretical ideas and facts that must be judged pragmatically, based on the success in gaining the expected result from

25. Incidentally, Mach affirms that "facts are always somewhat arbitrarily and forcibly defined with a view to the momentary intellectual aim" (1976, 3).

the testing activity. This does not entail giving up on the *ideal* of a literal correspondence between thought and sensible reality—at least not for Mach. Rather, it follows from an attempt to show how science in fact works, with the aim of stressing both the power and the limits of that theoretical enterprise. The mental-economical systematization of the natural world is indeed quite a powerful tool, but one must be aware of the fact that it is a purely theoretical elaboration with no ontological or metaphysical value. On the other hand, this is the only access we have to sensible reality, of which we cannot say anything independently of theoretical statements (with Hesse [1980, 172], it can be affirmed that "theories . . . are the way the facts themselves are seen"), and it is therefore important to focus on the factual roots of these statements and how they are related to theory. This is the only way to keep intact a form of objectivity of the scientific world representation. On the one hand, instrumentalism can indeed lead to skeptical forms of relativism—the sort of relativism that claims that there are really no truths, no objective facts, and no universal validity claims (cf. Bernstein 2010, 109)—which can be detrimental to scientific inquiry. On the other hand, in light of what I have tried to show thus far, it seems impossible to defend any direct realism that argues for the literal meaning of theoretical statements, according to Mach. Nevertheless, a criterion for objectivity in scientific knowledge, and the correspondence between theoretical and observational statements, can still be found if one looks at the dependence of theory on its factual correlative.

Thus, the moderate realist conception of the theoretical reconstruction of facts in thought outlined above makes it possible to reconceive two fundamental notions in the philosophy of science. This conception is empiricist insofar as it maintains that a meaningful world description is based on facts to which thought must adapt; at the same time, it takes the first step toward critical (in Mach's sense) reflection on the ideal of a direct representation of facts in thought by focusing on the actual value and function of the theoretical statements from which that world description arises.[26]

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26. Mach's observations on our scientific engagement with facts involve aspects that should be further explored in order to complete the picture I have tried to outline here, e.g. Mach's view of analogy, abstraction, and, most importantly, poetic imagination in science (on this, cf. Haller [1982] and, for a recent discussion, Brecevic [2021]). Given the space that such a study would require, I will leave it for a future paper.

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