What Do Infinite Sets Look Like? It Depends on the Perspective of the Observer

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Abstract

Consider an infinite set of discrete, finite-sized solid balls (i.e., elements) extending in all directions forever. Here, infinite set is not meant so much in the abstract, mathematical sense but in more of a physical sense where the balls have physical size and physical location-type relationships with their neighbors. In this sense, the set is used as an analogy for our possibly infinite physical universe. Two observers are viewing this set. One observer is internal to the set and is of the same finite size scale as the ball elements. Another observer is external to the set and is infinite in size relative to the balls and observer within the set. This observer is of the same size scale as the set as a whole. What do these sets look like to the two observers? The finite-sized (relative to the balls inside the set) observer within the set would view the set as a space composed of discrete, finite-sized objects. The external infinite-sized (relative to the inside the set) observer would view the very same set as a continuous space and would see no distinct elements within the set. How does this relate to us? First, the differing views of set N as being composed of a discrete versus continuous space may be related to the differing views of spacetime as discrete and continuous. Second, the perspective of the observer would have an impact on the assignment of a cardinality to an infinite set.

Introduction

Despite a large number of papers having been written on the infinite and infinite sets, very little or no attention has been paid to how observers might view an infinite set. That is, what does an infinite set look like? While this may at first sound unimportant and trivial, mathematicians and physicists think about the infinite all the time, and we may live in an infinite universe. Thus, how mathematicians, physicists and philosophers think about and visualize infinite sets would seem to have some impact on their theorizing about the infinite and the universe. This short and simple paper is an attempt to describe how two observers, one inside and one outside, an infinite set would view that set and what relationship this might have to their thinking on the infinite. Section 1 describes how an infinite set of finite-sized balls would appear to an internal observer of the same size scale as the balls. Section 2 describes how an external observer of an infinite size relative to the balls would view that same set. Section 3 briefly explores how these differing views might affect the observer's theorizing about the universe.

The Appearance of An Infinite Set to An Internal Observer

Consider a set, N, defined as containing an infinite number of discrete, finite-sized elements such as balls. Observer Internal-O is located inside this set and of the same finite size scale as the balls. For instance, if a ball is one foot in diameter, observer Internal-O may be 6 feet tall. The balls extend outward in infinite numbers relative to any location and orientation of Internal-O. That is, wherever Internal-O is in the set and in whichever direction Internal-O is "looking", the elements of the set extend without bounds the same potentially infinite distance in all directions relative to Internal-O. Note that set N is not a strictly abstract, mathematical-type infinite set because it contains multiple copies of identical elements (the balls) and the spatial, location-like relationships of the balls matters. That is, their adjacency. Identical elements are weeded out in mathematical sets and the location-like relationships are of no importance. So, set N is more of a "physical" analog of an infinite set. Given this, what does set N look like from internal observer Internal-O 's perspective? Given the above, it seems reasonable to conclude that:

- Internal-O sees the contents of the set as a discrete space made of finite-sized balls.

- Internal-O sees its size relative to the entire set as approaching, but never quite reaching, zero. It never reaches zero because Internal-O exists, and so its size can't be zero. Also, no matter how far Internal-O looks, it can never see an actually infinite endpoint of the set. From Internal-O's viewpoint, the set is always potentially infinite and, thus, its size gets smaller and smaller relative to the whole set but never quite reaches zero. If Internal-O were ever able to see the whole set in its entirety, then Internal-O's size would finally reach zero relative to that whole. Luckily, for Internal-O, that can't happen!

- No matter how far Internal-O travels within its reference frame (i.e., inside set N), it will never reach the edge, boundary or "exit door" of the set due the unending nature of infinity.

- By definition of this set, relative to any location of observer Internal-O, the other elements progress radially away from it the same potentially infinite distance in all directions, and, thus, Internal-O would view the set as a potentially infinite sphere. It would be an odd sphere because the observer can be at any point inside the sphere. Of course, elements within the sphere can never view the edge, so that, for them, the shape of the overall set only approaches that of an infinite sphere.

- Internal-O cannot "step outside" N and hope to be able to see the set as a whole. This is because Internal-O is of a different size scale than the set as a whole. So, even if Internal-O were able to step outside N, it would still be infinitesimally small relative to the set as a whole and would, therefore, still see set N as a potentially infinite-sized object. Thus, Internal-O is physically trapped in its reference frame, or "dimension", inside the set.

In sum, relative to finite-sized internal observer Internal-O, set N would appear as a potentially infinite, spherical space composed of finite-sized, discrete elements.

The Appearance of An Infinite Set to An External Observer

Next, how would the same set N appear to a hypothetical, external observer? Consider the same set N, defined above as having an infinite number of discrete, finite-sized ball elements extending forever relative to any location and orientation of an observer, Internal-O, within the set. However, now assume that there is a second observer, External-O, outside this set and that External-O's size relative to Internal-O and the other ball elements is actually infinite. That is, External-O is of the same size scale as the entire set N, which is actually infinite relative to Internal-O. Given this, it seems reasonable to conclude that:

- External-O views the entire set N itself as of finite size, which means that External-O could see set N in its entirety.

- If External-O's size relative to Internal-O is actually infinite, then Internal-O's size relative to External-O is infinitesimally small. Because Internal-O is of the same size scale as all the other elements inside the set, these elements are also all infinitesimally small relative to External-O. This means that the boundaries that define Internal-O and the other ball elements and that separate them from each other are also infinitesimally small relative to External-O. The individual elements still exist, by definition of set N, but boundaries become indiscernible to External-O. They disappear from view and the balls merge into a continuous space from External-O's perspective. That is, External-O would observe the inside of set N as a continuous space, as opposed to Internal-O's view of it as a space filled with discrete elements.

- External-O can see the whole amount of set N and, thus, can see the edge or boundary of N, which means that set N, in its entirety, is seen as an existent, finite-sized, discrete object by External-O.

- External-O cannot "step inside" N and hope to be able to see its elements as discrete. This is because External-O is of a different size scale than the elements inside the set. External-O's scale is the same as that of the entire set, which is actually infinite relative to Internal-O. So, even if External-O tried to step inside N, it would still be infinitely big relative to the elements and would, therefore, still just see a continuous space. Thus, External-O is also physical trapped in its reference frame, or "dimension", just as Internal-O is trapped in its reference frame inside the set.

Thus, relative to infinite-sized, external observer External-O, set N would appear as a finite-sized existent object with an internal continuous, smooth, infinitely divisible space. Note that these arguments don't prove the existence of an infinite external observer; they just suggest how this observer, if it existed, would view the set.

Implications of Set N For Our Perceptions of the Universe

The main takeaway of the differing ways internal and external observers view set N seems to be that the appearance of a physically-existent thing as being composed of either discrete elements or a continuous space depends on the perspective, or reference frame, of the observer relative to that thing. How does that relate to us and our perception of the universe? While I'm neither a mathematician nor a physicist, my layperson's view suggests the following possibilities.

First, consider if the thing we're observing is our universe. There is a debate in physics about whether or not spacetime is discrete or continuous (Dowker, 2014; Wallden, 2010; Kempf, 2010). While it's not known if the universe is infinite in size like set N (Lehners and Quintin, 2024), the obvious analogy between the differing views of set N and the debate about spacetime suggests that those physicists who view spacetime as discrete are thinking about the universe from the perspective of the finite-sized observer inside the universe and those who view spacetime as continuous are thinking about the universe from the perspective of the infinitely big observer outside the universe. Perhaps, taking the observer/physicist's viewpoint into account may offer some insights into the debate.

Second, the different appearances of set N to internal and external observers suggest that it is important to use an internally consistent perspective throughout any mathematical description of reality. For instance, if a theory describes spacetime as discrete, indicating that the scientist's perspective is similar to that of internal observer Internal-O, then it should use the same perspective in its calculations, such as in its calculations of the probability of an event occurring at a specific location in space. That is, assuming a continuous, real number-like distribution of probabilities for location while also assuming a discrete spacetime would mean that the theory is switching back and forth in its perspective of reality. This would be inconsistent

Third, if set N is our universe, the differing views of this set as continuous or discrete have some rough analogy with McTaggart's differing views of time as a changing flow (A-series) or a set of static events (B-series) (Freeman, 2010).

Finally, and this point is more tenuous since it will tie set N back to more traditional, mathematical sets, one implication of set N is that the cardinality we assign to an infinite set depends on the perspective, or reference frame, of the observer (e.g., the mind of the mathematician) relative to the set. For example, within infinite set N, observer Internal-O would assign the set's cardinality as equal to that of the set of integers. However, outside set N, observer External-O would assign it a cardinality equal to that of the real numbers. A second implication is that the perception of the integers as being a potentially infinite set of finite, discrete elements (e.g., 1, 2, 3....) and the real numbers within an integral range (e.g., from 0 to 1, from 1 to 2, etc.) as being a continuum will vary depending on the perspective of the observer. For instance, if an observer could decrease his or her size scale to that of the real numbers, they might appear as finite-sized, bounded and discrete elements instead of their usual external observer-based description as being infinitesimally small. Additionally, a hypothetical external observer of infinite size would view the set of integers as a continuous, infinitely divisible space similar to how we observe the real numbers.

In sum, being aware of one's perspective view when thinking about an infinite set and maintaining consistency in that view seem to be of value in theorizing about the universe.

Conclusions

This short, simple paper describes what a "physical" infinite set of finite-sized balls would look like to both internal and external observers. An internal observer will view such a set as a potentially infinite, spherical space composed of finite-sized, discrete elements. An external observer whose size is actually infinite relative to a ball element in the set will view the set as a finite-sized unit whole object containing a continuous, smooth, infinitely divisible space. These differing views of the same set have implications for theorizing about mathematics and physics and suggest that switching back and forth between the internal/discrete and external/continuous views can lead to inconsistencies. Overall, the perspective of the observer (i.e., the scientist's mind) should be taken into account when using infinities in physics and mathematics.

References

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