REVIEW OF *PLATO'S GHOST* BY JEREMY GRAY

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This book is both a narrative of late nineteenth-century and early twentieth-century mathematics, and a historical analysis of what it is for mathematics to be "modern". This is an admirably ambitious project and to make it more manageable, Gray focuses on the development of geometry, analysis, and mathematical logic in this era. In addition he discusses the history and philosophy of mathematics of the era, which developed hand-in-hand with mathematics, often by authors with substantial mathematical training and accomplishments of their own. It is an era whose investigation promises fruitful insight, and Gray handles it with aplomb.

Gray argues that in this era mathematics underwent a transformation that parallels other contemporaneous "modernist" transformations in art, literature and music. By a "modernism", Gray offers as a definition

> an autonomous body of ideas, having little or no outward reference, placing considerable emphasis on formal aspects of the work and maintaining a complicated—indeed, anxious—rather than a naïve relationship with the day-to-day world, which is the de facto view of a coherent group of people, such as a professional or discipline-based group that has a high sense of the seriousness and value of what it is trying to achieve. (p. 1)

There is much to wonder at in this definition, but having done so elsewhere (cf. Arana [2008]), I will simply grant that Gray's definition at least gestures toward characteristic aspects of this period. Anxiety over the multiplicity of geometries suddenly on offer, contrasted with past confidence in Euclidean geometry; awareness of the seriousness of choosing geometric relativism (or

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"tolerance") over Euclidean absolutism; movement toward formal systems and away from "intuition": these are essential to understanding the period in question and Gray's definition has room for each. While one could with good cause ask for a definition that provides a conceptual analysis of modernity in mathematics, or makes more explicit how the parts of its definiens relate to the particulars of the practice of this era, this isn't Gray's purpose in giving such a definition. Instead, its purpose is to provide a framework for weaving the events and understandings of this period into a coherent narrative.

Gray's narrative begins by chronicling the period before modernism (from roughly the turn of the nineteenth century through the 1870s), to the arrival of modernism (the 1880s to the mid 1890s), to high modernism (the late 1890s to the 1910s). There are then a couple of interlude chapters on the interface of modernist mathematics with physics, measurement theory, linguistics, and psychology, as well as on the popularization and history of mathematics in the high modernist period. The narrative then concludes with a discussion of the "foundational crisis" in mathematics after the First World War, focusing on the work of Hilbert, Brouwer, Zermelo, and Gödel.

A brief survey of the central narrative concerning geometry will give a glimpse of what Gray means by modernist mathematics. Gauss, Bolyai and Lobachevsky proved many theorems of what came to be called hyperbolic geometry. Their work planted the seeds of modernist geometry by spurring anxiety concerning the truth of the parallel postulate. Kant had claimed that its truth is grounded in the structure of rational cognition, but since Kant's claim was in part a product of ignorance of plausible alternatives to the parallel postulate, the development of non-Euclidean geometries gave reasons to demur at Kant's position. Modernism in geometry was born with the results of Beltrami and Klein that came to be understood as having shown the equiconsistency of hyperbolic geometry and Euclidean geometry. Both were thus plausible; which was true? Helmholtz suggested an empirical approach to this question by means of experiments, while Poincaré maintained that axioms were mere conventions suited to our investigative ends, and could be freely chosen or rejected. High modernism arrived with the widespread recognition that the axioms of geometry need not be taken to express propositions, but rather only the "scaffolding" of propositions, as Hilbert put it in a letter to Frege. The non-logical terms of geometric expressions admit multiple interpretations, and the content of these expressions is taken simply to be their inferential role in an axiomatic system. Modernist geometry thus offered radically new views of meaning and truth, which would have dramatic consequences for the development of modernist logic and algebra as well. These views give substance to the book's title, which refers cleverly both to the evident discrediting of Platonism by these new views of truth, and to the refrain of Yeats' poem "What Then?", the book's epigraph.

One particularly important component of Gray's narrative is its emphasis on actors who have received less attention in contemporary Anglo-American philosophy of mathematics, in particular philosophers and philosophically-reflective mathematicians who undertook foundational investigations. Among these philosophers are Fries, Natorp, and Cassirer; among these mathematicians are Study, du Bois-Reymond, and Veronese. As far as I know there is no other single source surveying such work. Gray's volume promises to enrich and even open up philosophical projects by bringing these underappreciated actors to attention, and could even be of use to historians of late nineteenth-century philosophy whose concerns extend beyond mathematics.

While developments in geometry, analysis, and logic are detailed, much less attention is given to algebra. Gray's choice is understandable: the book's heft is already considerable, and there is a clear sense in which the three areas that are covered in detail intertwine. In light of the consistency of non-Euclidean geometries relative to Euclidean geometry and the consistency of Euclidean geometry relative to the theory of the reals, consistency concerns led directly to investigations of the foundations of analysis; and investigations of the foundations of analysis led directly to the set-theoretic investigations covered here. Indeed, mathematicians' awareness of these interconnections—that the foundations of mathematics demand the attention of practicing mathematicians, shaping their choices of problems and methods—is a key instance of what Gray calls "anxiety", his term for the period's concerns with consistency and error borrowed from other literature on modernism (one thinks for instance of Auden's poem "The Age of Anxiety" from 1947). Gray sheds much light on these interconnections, and they provide the structure keeping the narrative together. Still, algebra is a critical part of this story also, and I wish the book had delved more deeply into its development in this period. Surely it is as much a modernism as the other areas covered here, as van der Waerden's choice of title *Modern Algebra* for his epochal textbook makes quite explicit. Algebraic geometry especially seems paradigmatic of Gray's take on modernist mathematics, from worries concerning the rigor of the Italian school, to the embrace of Noether and van der Waerden's structuralist ideas incorporating the new views of meaning offered by modernist geometry. A treatment of it here would have been welcome, though as Gray notes it would have expanded the scope of the text considerably and an author has to draw the line somewhere.

A word is in order concerning Gray's treatment of mathematical logic. Given the orientation of much present work in Anglo-American philosophy of mathematics, I suspect most readers of this journal will find Gray's history of logic familiar and will find other parts of the book more enlightening. However, I think the logic narrative will be welcome to philosophers of mathematics of other traditions who want to engage the Anglo-American philosophical world but whose training and practice have placed less importance on mathematical logic. Here I am thinking particularly of French philosophers of mathematics since the Second World War. Gray's treatment promises to be a valuable service to such philosophers.

In closing I commend Gray for writing an extraordinarily detailed and fascinating history of modernist mathematics, whose philosophical fruits remain ripe for the picking. The sections on geometry shine with clarity and convey the drama of modernism in a compelling and page-turning way. The treatments of lesser-studied actors are fascinating and promise to be of much use in incorporating their work into ongoing scholarship. The book could be fruitfully used as a supplement to a variety of courses in philosophy, including philosophy of mathematics and logic, history of analytic philosophy, and philosophy of science. It is a monument of scholarship and will reward careful study.

References

Andrew Arana. Review of *The Architecture of Modern Mathematics*, by José Ferreirós and Jeremy Gray. *Mathematical Intelligencer*, 30(4), 2008. 1

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