

a formal analogy to elements of 'de deo'

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Abstract

We aim to compile some means for a rational reconstruction of a named part of the start-over of Baruch (Benedictus) de Spinoza's metaphysics in 'de deo' (which is 'pars prima' of the 'ethica, ordine geometrico demonstrata' [3]) in terms of 1st order model theory. In so far, as our approach will be judged successful, it may, besides providing some help in understanding Spinoza, also contribute to the discussion of some or other philosophical evergreen, e.g. 'ontological commitment'. For this text we assume the reader familiar with 'de deo' as well as with some basic concepts and results of 1st order model theory. Before we start reconstruction, we will first revisit shortly the concept of 'attributum' (definitio IV) in its setting in 'de deo', next scan for formalizable aspects of 'in suo genere finita' ('de deo', definitio II), subsequently list the model theoretic constructs we will make use of. Then we begin reconstruction by stating "coordinative definitions" for the notions of 'attribute (of a substance)', 'modus (as conceived by an attribute)' and 'substance (as conceived by an attribute)', reasoning shortly for each of them. The "coordinative definitions", we will arrive at, must not be understood as literal translations of Spinoza's concepts - of course, there can't be such a thing as a literal translation - they are meant as formal analoga of these concepts, mapping some logical structure. But even with this caveat they may seem strange to the reader at this stage of discussion. Additional justification for them then should be found in our endeavour, to map some argumentation of Spinoza's proofs of some of his propositions from this starting point. ¹

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¹Our interest in Spinoza is systematic, not historical, aiming for rational reconstruction, although we'll try hard to respect historical scrutiny as far, as this approach allows for.

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1 introduction

The part of the text of 'de deo', to which we confine our endeavour for reconstruction, is rather small, starting from definitio II [3](1,45)² up to propositio XIII,[3](12,56). This is, because we view 'ethices pars prima de deo' as built/composed from a merge of two basic lines of argument:

the first, which we would like to call the "**logical line of argument in 'de deo'**", starting from definitiones II, III, IV and V, this "line" is, what we try to picture here

the complementary line of argument in 'de deo', starting from definitiones I, VI, VII and VIII, which we nickname "**the ontological line of argument in 'de deo'**"

Rather obviously, in this second line of argument Spinoza uses assumptions and presuppositions, as implied in the 'ens perfectissimum' therefore 'ens necessarium'-argument (Anselm of Canterbury, Descartes, ... for a vivid historical account see Harry A. Wolfson, 'The Philosophy of Spinoza', [7] Chapt. VI, § 1 The ontological proof, pp.158 ff.). Now this 'ontological' part might in principle be capable of being pictured in model theory as well - and to this end, a suggestion might be, to take a start with Kurt Gödel's discussion of 'ontological proof' in terms of modal logic³ - but this would be a task of it's own and is completely bypassed here.

The only thing from that second ontological line of argument, we will make use of here for our restricted purpose, is correlating the here elaborated logical line of argument to the subsistence-inherence-schema as expressed in Axioma I of 'de deo' : 'omnia, quae sunt, vel in se vel in alio sunt' ([3](2,46), which, from our point of view, is a lot weaker than the entire ontological argument, and which seems indispensable for every account of Spinoza's reasoning. This correlation will be given in the form of a 'meaning postulate', relating the ontological line main concept 'causa sui' from Definitio I of 'de deo' to the logical line concepts 'attribute' and 'mode, as conceived by an attribute' (sections 4.4.2 and 6.1).

2 heuristic notes

2.1 concerning 'attributum' (definitio IV)

Before stepping into technicals, we have to consider the 'de deo'- concept of 'attribute' (definitio IV), the combination of which with the substance-inherence-

²the pagination (**x**,**y**) in this edition [3] of Spinozas 'Ethica,...' is , according to the editor Konrad Blumenstock, [3] p. 565, to be taken as :

x is the pagination of 'Opera Posthuma', Amsterdam 1677,

y is the pagination of 'Spinoza : Opera' Vol. II, ed. Carl Gebhardt, Heidelberg 1925

³Kurt Gödel, Collected Works, Volume III [19], Ontological Proof(1970*), pp.403-404, with a highly adjuvant introduction to (1970*) by Robert M. Adams (l.c. pp.388-402), see also Appendix B, pp.429-437

scheme is, what is special and characteristically new in Spinoza's treatment of substance as opposed to medieval tradition.⁴

Harold F. Hallett, 'Benedict de Spinoza, The Elements of his Philosophy' ([9] p.16) points out "that the term [*'attribute'*] is not to be taken in the vulgar sense of a characteristic or quality related to substance as, e.g. sobriety is related to Peter, or redness to a rose: 'By attribute I understand that, which the intellect perceives of substance as constituting its essence' ... ", citing thus from definitio IV of 'de deo'. Wolfgang Cramer, 'Spinozas Philosophie des Absoluten', in his discussion of definitio IV ([10] pp. 30 ff.) emphasizes the difference between propria of substance and attributes of substance⁵. These propria according to W. Cramer relate to definitio I and are stated in propositiones belonging to what we have nicknamed the 'ontological line of argument'. Six propria of (the one substance) God are summed up by Spinoza in the ending passage of 'de deo': "Appendix: His Dei naturam, ejusque proprietates explicui, ... , ... (1) That He necessarily exists; (2) that He is one; ... ", (for a complete English transcription with comments relating to propositiones I-XIII see Wolfson [7] pp.112ff.) .

For reasons which hopefully will get clearer, as we proceed, we will choose as target for the coordinative definition of 'attribute' in the theory of models a special case of what is called a 'formalized theory', viz. the closure under logical implication of a set of formulae in 1st order language. But, as we learn from propositiones I and II in 'ethices pars secunda, de natura et origine mentis' (henceforth 'ethices ... pars II') [3] (42-43, 86-87), that 'res cogitans' and 'res extensa' are attributes of God in the sense of 'de deo' Definitio IV, we have to take care, that such 'formalized theory', in order to count as a representative for 'attribute' in the sense of 'de deo', should be understood as a large range comprehensive, else universal, theory of its subject; in terms of theories of our time e.g. a (hopefully somewhen) unified theory of physics, or still likewise ambitious a unified theory of cognitive sciences, and from our contemporary point of view still better a union of both - the latter (imhop) would do less harm to Spinoza's theory of substance, as one might be tempted to think with respect to the 'ethices' Cartesian dichotomic heritage.

2.2 concerning 'in suo genere finita' (definitio II)

Next we revisit Spinoza's finite/infinite-distinction, which has rather nothing to do with infinite cardinal numbers, but stems from old Greek philosophical tradition and may have reached Spinoza's thinking directly or indirectly via what we may nickname the Anaximander-Proklos-Connection: "Ich meine aber die Grenze und das Unbegrenzte ($\tau\omicron\ \pi\epsilon\rho\alpha\varsigma\ \kappa\alpha\iota\ \tau\omicron\ \alpha\pi\epsilon\iota\rho\omicron\nu$). Denn von diesen

⁴On medieval vs. Spinoza's treatment of concepts 'substance', 'modes', ... in this respect see Wolfson "When we come, however, to Spinoza's formal definition of that thing which is in itself, labelled by the good old name 'substance' and compare it with the mediaeval definition, we find that while in part they read alike, Spinoza's definition contains a new additional element. The mediaeval definition simply reads, as has been said, that substance is that which is in itself, i.e., not in a subject ... But Spinoza adds to 'that which is in itself' the statement "and is conceived through itself" (Def.III). Again the mediaeval definition of accident is that which is in another thing ... Here again, using the term 'mode' (*modus*) which he identifies with the affections (*affectiones*) of substance, Spinoza first defines it like the traditional accident as 'that which is in another thing', but then adds the clause 'through which also it is conceived' (Def.V) ..." [7] pp.63-64 and Chap. III ff. passim

⁵see also H. H. Joachim, 'A Study of the Ethics of Spinoza' [6] p.41f. note 2

beiden ersten Prinzipien nach der unergründbaren und allen unfäßbaren Wirkursache des Einen gewann alles andere Bestand und auch das mathematische Sein, ...”⁶

While definitio II restricts to ‘in suo genere finita’, the use of ‘necessario infinita’ in the proof of propositio VIII shows the connection to definitio II . Thus we might need look for something common to both, when adopting a formal picture of definitio II (please note that throughout this article we treat ‘infinite’ as synonymous with ‘not finite’)

The concept of ‘infinite’ is discussed besides others by Harold H. Joachim, ([6], Book I Chapter 1§2, pp. 27-35 , see also Book I, Chapter 4) and by Hallett ([9], Chapter III pp. 31 ff.) ‘infinity of natura naturata’. For our endeavour, to find formal correspondences to Spinoza’s concepts we draw from these discussions that we should preserve the following structure:

‘infinite’ as applying to ‘substance’ viz. ‘necessarily infinite’ qua ”natura naturans”

(‘de deo’, definitio III , see e.g. Joachim [6], p. 65 note 1, referring to ‘de deo’, propositio XXIX scholium [3] (27, 71))

‘infinite’ as applying to ‘(infinite)modes’ viz. ‘infinite’ qua ”natura naturata”
(‘substance in it’s affections’)

Joachim writes: ”The infinity of modes – when they are infinite – is the infinity of the cause on which they depend, and that cause is (not themselves, but) Substance.” ([6] p.30)

‘finite’ as applying to ‘finite modes’ viz. ‘finite’ qua ”natura naturata” (‘substance in it’s affections’)

And this last is rather opaquely commented in the above cited literature with an emphasis on the durational aspect of finite modes, this opaqueness but inherited from Spinozas own treatment of the topic of ‘modes’⁷

2.2.1 the ‘limits’-relation

So let’s take a fresh start to the topic of ‘finite mode’ from definitio II, confining ourselves to the ‘logical aspects’ and let’s - for the time being - abstract from any ‘ontological aspects’ of definitiones II and V.

‘Ea res dicitur in suo genere finita, quae alia ejusdem naturae terminari potest’ where we translate ‘terminari potest’ as ‘can be limited by ’.

Now, in order to gain some formal stuff, we start with a few observations on properties of relation ‘y limits x’ , used in definitio II, which we tentatively formalize as

⁶Proklus Diadochus, Euklid-Kommentar’ [1], Vorrede p.165

⁷see e.g. ‘de deo’ propositio XV scholium [3] (14, 58) ‘unde ejus [sic: materiae] partes modaliter tantum distinguuntur, non autem realiter’ (parts of matter can be distinguished only modaliter, not realiter) together with propositiones XXII, XIII [3] (22 – 23, 66 – 67) on the necessary existence of finite and infinite modi

$$\mathbf{x} \in \mathbf{finite} \Leftrightarrow \forall \mathbf{y} (\mathbf{y} \neq \mathbf{x} \wedge \text{'x, y are of the same kind'} \wedge \langle \mathbf{y}, \mathbf{x} \rangle \in \overline{\mathbf{limits}}) \text{ }^8$$

where we use \Leftrightarrow as the sign for 'equivalent by definition' and use $\overline{\mathbf{limits}}$ for the extension of the 'limits'-Relation. We argue that from the proof of propositio VIII [3]⟨4, 48⟩ it is obvious, that definitio II is *not* meant as a partial definition (defined only for objects of the same kind), but as an explicit definition, and this is reflected by the logical form of the above formalization.

properties of the 'limits' – relation

First of all, 'limits' is **irreflexive**, thus has a domain \mathbf{D} with $\mathbf{card}(\mathbf{D}) = \mathbf{0} \vee \mathbf{card}(\mathbf{D}) \geq \mathbf{2}$

Next, 'limits' should be understood to be **symmetric**.

We hold this classification 'symmetric' valid, because requested for a consistent interpretation of 'de deo'⁹, though it seems to conflict with Spinozas example in definitio II 'Ex. gr. corpus dicitur finitum, quia aliud semper majus concipimus', insinuating that the picture of a 'Russian nesting doll' , else of concentric spheres or cubes and the like would give decisive information on the logical structure of 'y limits x'. We argue, that Spinozas choice of this example for res extensa in definitio II is simply influenced by a felt need to escape examples, which support Descartes' thesis of 'divisibility of substance as res extensa' (which is battled against by Spinoza e.g. in 'de deo', propositiones XII, XIII). But of course he can't escape accounting for single physical objects and he does account for them in 'ethices ... pars II', see there definitio I ('per corpus intelligo modum ...' [3]⟨40, 84⟩ and the various Axiomata and Lemmata following pars II, propositio XIII [3]⟨53 – 58, 97 – 102⟩ . And again, in his handling of physical bodies here, Spinoza avoids talking about the space inhabited by an 'corpus', but steps fast to the dynamic embedding of the corpora by 'motion and rest' (e.g. Lemma I [3] ⟨53, 97⟩) , which picture is likewise drawn from the physics of Descartes Principia Pars II, but this part of Descartes' presentation in principle adopted and emphasized by Spinoza.

Next, we note the 'limits'-relation to be not transitive, not (weakly) antisymmetric, and not (weakly) dichotomic, thus deny more typical ordering properties (there has been a temptation to construe the 'limits'-relation as a weak partial ordering, with 'infinite modes' as maximal elements of chains of 'finite modes', and a construct like this might fit very well into the ontological line of argument, which is but bypassed here).

Thus, concerning the logical line for definitio II, we sum up, what has been said so far and try the following :

Be \mathbf{M}_f the set of 'finite modes' , and again $\overline{\mathbf{limits}}$ the extension of the 'limits'-relation, and \mathbf{limits} the set of pairs $\{\mathbf{x}, \mathbf{y}\}$ with $\langle \mathbf{x}, \mathbf{y} \rangle \in \overline{\mathbf{limits}}$,

⁸we use the same signs for logical connectives in our text and in the later defined 1st order language \mathbf{L} , which may be viewed as the 'object language', while our discussion may be said to be in the respective 'metalanguage', because, in the rare cases we happen to cite formulae or sets of formulae of \mathbf{L} , it's clear from the context, that the signs for the logical connectives in these cases belong to \mathbf{L}

⁹see note on 'undirected' below

then

$$\mathbf{G}_m = \langle \mathbf{M}_f, \underline{\text{limits}} \rangle$$

shows the properties of a simple graph (undirected¹⁰, no loops, no multi-edges)

(see e.g. Reinhard Diestel, Graph theory [20], Chapter I, or simply en.wikipedia.org on 'graph' and 'neighbourhood')

Thus we have found a scarce, but nevertheless instructive 'logical line' formal characteristics for 'in suo genere finita', and hence too for

'finite' as applying to 'finite modes'.

It's a first characteristics only; at this stage we must leave open questions like whether or not the graph of our 'limits'-relation is connected , because we hold, that every attribute of substance implements the 'limits'-relation in it's own way:

e.g. René Descartes in his description of the laws governing 'res extensa' (in Part II, § XVI ff., [2] pp.49 ff.) asserts that any part of space is (necessarily) occupied by some part of extended matter ('no vacuum'). If the 'res extensa'-attribute were thus construed, the graph of it's associated 'limits'-relation of course would be connected.

2.2.2 'infinite'

Now what about

'infinite' as applying to 'substance' ?

The answer is explicitly given by Spinoza in his proof of 'de deo' propositio VIII:

substance is necessarily infinite, because there is 'in suo genere' necessarily only one substance (as shown by preceding propositiones) In our terms, a simple graph $G_m = \langle M_f, \underline{\text{limits}} \rangle$ (see above) does not allow for $\text{card}(D) = 1$, which accounts for '... infinite', the 'necessarily ...' qualification due to the ontological line of argument.

And last not least, what about

'infinite' as applying to '(infinite)modes'

From the above cited discussions of this question by Joachim (emphasis on "infinity inherited" see the passage quoted above [6] p.30) and Hallett (emphasis on "natura naturata") we draw the following sketch: The existence, interdependence, else interaction of the finite modes is governed by a framework of universal laws. We referred already to the description of 'res extensa' by Descartes in 'Principia Philosophiae', Part II, here we find with 3-dimensionality of space, inertia, actio=reactio, etc. a framework of 'res extensa' , given by these universal principles. Such framework (the German word 'Trägersystem' might be somewhat more suggestive here) for the finite modes as covered by an attribute

¹⁰if we would have determined the 'limits' relation to be asymmetric, of course, the graph would be directed, and if determined neither symmetric nor asymmetric, the result still worse would be completely counterintuitive

of substance (formally rather a set of universally quantified sentences), describes the infinite mode(s) of substance as apprehended by the attribute. And again, this framework , viewed as the logical product (conjunction) of these universal principles, describes the unique infinite modus exhibited by the attribute in question, and because of being one in suo genere, infinite.

Joachim's accounting "infinity of infinite mode inherited from substance as it's infinite cause", is of course valid, because e.g. the infinite mode 'motion and rest' ('ethics ... pars II', Axioma I and Lemmata I,III [3](53 – 54, 97 – 98))¹¹ implements together with the 'corpora' (the 'finite modes' in 'res extensa', 'ethics ... pars II', definitio I [3](40, 84))) the 'res extensa'-attribute of substance¹², but this account is formulated elliptic - in not mentioning explicitly 'de deo', definitio II (introducing implicitly 'infinite' as 'not finite')¹³.

In Spinoza's correspondence with (Schaller on behalf of) Tschirnhaus, there is a passage which gives the following account of the infinite modes¹³:

LETTER LXV.

G. H. SCHALLER TO SPINOZA.

... Fourthly, I should like to have examples of those things which are immediately produced by God, and those which are produced through the means of some infinite modification. Thought and extension seem to be of the former kind; understanding in thought and motion in extension seem to be of the latter.

...

G. H. SCHALLER.

Amsterdam, 25 July, 1675.¹⁴

LETTER LXVI

Spinoza to . . .

[Spinoza answers by reference to the first three books of the 'Ethics.']

...

Lastly, the examples you ask for of the first kind are, in thought, absolutely infinite understanding; in extension, motion and rest; an example of the second kind is the sum of the whole extended universe (*facies totius universi*) which, though it varies in infinite modes, yet remains always the same. Cf, Ethics II. note to Lemma vii. before Prop. Xiv.

...

The Hague, 29 July, 1675.

Thus, Spinoza counts in each attribute one infinite mode: '... in cogitatione intellectus absolute infinitus, in extensionem autem motus et quies ...'¹⁵

¹¹see Joachim on 'Modal System of Extension' [6],pp. 82-88, see also Hallett on 'Infinite and Eternal Modes' [9] pp. 31f.

¹²for the infinite mode of the 'res cogitans'-attribute of substance 'intellectus infinitus' see also 'de deo' propositiones XXX-XXXI [3] (27 – 28, 71 – 72)

¹³cited from R.H.M. Elwes English translation in 'Philosophy of Benedict de Spinoza'[4] p. 403 and p. 406

¹⁴English translation from Elwes, 'Philosophy of Benedict de Spinoza' [4]), from which we cite here of p.403 and p.406 respectively

¹⁵cited from Ginsberg, 'Der Briefwechsel des Spinoza im Urtexte' [5] p.182 [- in other editions these two letters are counted not as LXV and LXVI, but as LXIII and LXIV, and 'Schaller' is written 'Schuller'] .

2.3 concluding remark on this section

The above notes and conclusions should be understood as heuristic ones, thus only tentative, never apodictic, but as a trial to explore the scenario in 'de deo', in which the real definitions (!) of 'substance', 'attribute', and 'mode' happen to reside.

3 modeltheoretic concepts and results

In establishing our coordinative definitions we make free use of some well known basics of 1st order model theory without explicitly citing anything separately. Generally, for the concept 'satisfiability' we appeal to Georg Kreisel and Jean-Louis Krivine (model theory), [17], chapters 0 and 2, see especially the recursive definition of 'value of a formula', which is the only thing we do cite here in some detail. Instead of the Kreisel-Krivine term 'realization' we use the meanwhile more common term 'relational structure', and we refer also to the model theoretic concept 'elementary equivalence' (see for many John L. Bell and Alan B. Slomson [18]).

3.1 1st order language \mathbf{L}

3.1.1 notes on the syntax of \mathbf{L}

We define a 1st order language \mathbf{L} in the usual way, build from an infinite set of variables $\mathbf{V}_{\mathbf{L}}$, a sequence of sets \mathbf{f}_i^n of n -ary function symbols, and a sequence of sets \mathbf{R}_i^n of n -ary relation symbols, unary logical connectives \neg , and for any $\mathbf{x} \in \mathbf{V}_{\mathbf{L}}$ the existential quantifier $\forall_{\mathbf{x}}$, and only one binary connective \vee . For any $\mathbf{x} \in \mathbf{V}_{\mathbf{L}}$ universal quantifiers $\wedge_{\mathbf{x}}$ and more binary connectives $\wedge, \rightarrow, \leftrightarrow, \dots$ when needed, may be defined in the usual way. Further, the set of terms of \mathbf{L} , $\mathbf{T}_{\mathbf{L}}$ build (as usual) as the functional closure¹⁶ from the Variables in $\mathbf{V}_{\mathbf{L}}$ as 0-ary symbols and n -ary functional symbols, the atomic formulae of \mathbf{L} are of the form $\mathbf{R}_i^n \mathbf{t}_1, \dots, \mathbf{t}_n$, where \mathbf{R}_i^n is an n -ary relation symbol and $\mathbf{t}_1, \dots, \mathbf{t}_n$ are terms of \mathbf{L} . Again, the formulae of \mathbf{L} are build in the usual way as the functional closure from the set of atomic formulae and the logical connectives \neg , existential quantifiers $\forall_{\mathbf{x}}$, and \vee . Be \mathcal{F} the set of all formulae \mathbf{A} of \mathbf{L} . For each formula $\mathbf{A} \in \mathcal{F}$ there exists a unique (finite, and possibly empty) set of variables occurring free in \mathbf{A} ¹⁷

3.1.2 notes on the (model theoretic) semantics for \mathbf{L}

A relational structure¹⁸ \mathcal{RS} for \mathbf{L} is defined to consist of

(1) a non-empty set \mathbf{U} , the domain of the structure (often nicknamed 'the universe of discourse' of the structure)

(2) for each $\mathbf{n} \geq 0$ a mapping $\phi_{\mathbf{n}}$ of the set of n -ary function symbols \mathbf{f}_i^n into the set of functions defined on \mathbf{U}^n and taking values in \mathbf{U} . For the value of $\phi_{\mathbf{n}}(\mathbf{f}_i^n)$ we write \mathbf{f}_i^n . This mapping induces a mapping from $\mathbf{T}_{\mathbf{L}}$ (the set of terms

¹⁶for the term 'functional closure' see Kreisel-Krivine [17], chapter 0 page 1

¹⁷for a definition see e.g. Kreisel-Krivine [17] p.18

¹⁸'realization' in Kreisel-Krivine [17], the following definitions and theorems with slight (mostly typographical) modifications taken from [17] chapter 2: Predicate Calculus, pp.16-20

with exactly n distinct variables) into the set of functions defined on \mathbf{U}^n (let Φ the set of all such mappings ϕ_n)

(3) For each $\mathbf{n} \geq \mathbf{0}$ a mapping $\psi_{\mathbf{n}}$ of the set of n -ary relation symbols $\mathbf{R}^{\mathbf{n}}$ into $\mathcal{P}(\mathbf{U}^n)$ the power set of \mathbf{U}^n For the value of $\psi_{\mathbf{n}}(\mathbf{R}_i^{\mathbf{n}})$ we write $\overline{\mathbf{R}}_i^{\mathbf{n}}$ (let Ψ the set of all such ψ_n)

Thus \mathcal{RS} has the form : $\mathcal{RS} = \langle \mathbf{U}, \Phi, \Psi \rangle$

Given \mathcal{RS} , the set of all mappings ρ from the set of Variables $\mathbf{V}_{\mathbf{L}}$ into the 'universe of Discourse' \mathbf{U} is given too

$$\mathbf{U}^{\mathbf{V}_{\mathbf{L}}} = \{ \rho \mid \rho : \mathbf{V}_{\mathbf{L}} \mapsto \mathbf{U} \}$$

satisfiability, value of a formula

Now we are ready to replay the recursive definition of **the value $\overline{\mathbf{A}}$ of \mathbf{A} in \mathcal{RS}** (which is central to our approach) using the concept of satisfiability 'a variable assignment $\rho \in \mathbf{U}^{\mathbf{V}_{\mathbf{L}}}$ satisfies formula \mathbf{A} in \mathcal{RS} '

Accordingly we intend to define the value $\overline{\mathbf{A}}$ of \mathbf{A} in \mathcal{RS} as the subset of variable assignments, which satisfy \mathbf{A} , implying $\overline{\mathbf{A}} \subseteq \mathbf{U}^{\mathbf{V}_{\mathbf{L}}}$

To specify this subset $\overline{\mathbf{A}}$ for every formula, we have to recur to the definition of these formulae (see above 'notes on the syntax ...), and proceed:

(S - 1) be \mathbf{A} of the form $\mathbf{R}_i^{\mathbf{n}} \mathbf{t}_1, \dots, \mathbf{t}_n$ (\mathbf{A} atomic)

the value $\overline{\mathbf{A}}$ of \mathbf{A} is given by $\overline{\mathbf{A}} = \{ \rho \in \mathbf{U}^{\mathbf{V}_{\mathbf{L}}} \mid \langle \rho(\mathbf{t}_1), \dots, \rho(\mathbf{t}_n) \rangle \in \overline{\mathbf{R}}_i^{\mathbf{n}} \}$

(S - 2) be \mathbf{A} of the form $\neg \mathbf{B}$

the value $\overline{\mathbf{A}}$ of \mathbf{A} is given by $\overline{\neg \mathbf{B}} = \mathbf{U}^{\mathbf{V}_{\mathbf{L}}} - \overline{\mathbf{B}}$
the complement of the value of \mathbf{B}

(S - 3) be \mathbf{A} of the form $\mathbf{B} \vee \mathbf{C}$

the value $\overline{\mathbf{A}}$ of \mathbf{A} is given by $\overline{\mathbf{B} \vee \mathbf{C}} = \overline{\mathbf{B}} \cup \overline{\mathbf{C}}$
the union of the values of \mathbf{B} and \mathbf{C}

(S - 4) be \mathbf{A} of the form $\forall_{\mathbf{x}} \mathbf{B}$

the value $\overline{\mathbf{A}}$ of \mathbf{A} is given by $\overline{\forall_{\mathbf{x}} \mathbf{B}} = \{ \rho \in \mathbf{U}^{\mathbf{V}_{\mathbf{L}}} \mid \rho \in \overline{\mathbf{B}} \vee \forall_{\sigma} (\sigma \in \overline{\mathbf{B}} \wedge \rho =_{\mathbf{x}} \sigma) \}$
viz. the set of all variable assignments ρ of $\overline{\mathbf{B}}$ plus all variable assignments σ , which differ only in the value for variable \mathbf{x} from some $\rho \in \overline{\mathbf{B}}$

3.2 some more modeltheoretic concepts and results for \mathbf{L}

We need still (only a few) more basic concepts and results of 1st order model theory. These are

3.2.1 value of a formula continued, truth and falsehood as values of closed formulae (sentences) of \mathbf{L}

We cite without proof from Kreisel-Krivine [17] p.18 the following basic metatheorem for \mathbf{L} :

Let \mathbf{A} be a formula whose free variables are $\mathbf{x}_1, \dots, \mathbf{x}_n$. Then $\overline{\mathbf{A}}$ depends only on the variables $\mathbf{x}_1, \dots, \mathbf{x}_n$

An immediate consequence is: If \mathbf{A} is closed (does not contain any free occurrences of variables), then $\overline{\mathbf{A}} = \mathbf{U}^{\mathbf{V}_L}$ or $\overline{\mathbf{A}} = \emptyset$. \mathbf{A} is then said to be 'true' in \mathcal{RS} or 'false' in \mathcal{RS} respectively.

If $\overline{\mathbf{A}} = \mathbf{U}^{\mathbf{V}_L}$, we say also, \mathbf{A} is valid in \mathcal{RS} and write $\mathcal{RS} \Vdash \mathbf{A}$

3.2.2 prenex normal form

First we cite (from Kreisel-Krivine [17] p.19) the **definition of prenex normal form**

A formula is said to be in prenex normal form or to be a prenex formula if it is of the form \mathbf{QA} , where \mathbf{Q} is a finite sequence of symbols \neg and $\forall_{\mathbf{x}_i}, \mathbf{x}_i \in \mathbf{V}_L$, and \mathbf{A} is a quantifier free formula.

Now we cite again(from Kreisel-Krivine [17] p.20) without proof another important metatheorem for \mathbf{L} (**existence of prenex normal form**):

If \mathbf{V}_L is infinite, then for each formula \mathbf{A} there is a prenex formula \mathbf{A}' which is (logically) equivalent to \mathbf{A} , i.e. $\mathbf{A} \leftrightarrow \mathbf{A}'$ is a theorem of \mathbf{L} , or $\overline{\mathbf{A}} = \overline{\mathbf{A}'}$ (in any relational structure \mathcal{RS} for \mathbf{L})

3.2.3 theory of a relational structure

Next we cite the concept **theory of relational structure \mathcal{RS}**

$$\mathcal{TH}(\mathcal{RS}) = \{ \mathbf{A} \mid \mathbf{A} \in \mathcal{F} \wedge \overline{\mathbf{A}} = \mathbf{U}^{\mathbf{V}_L} \}$$

where \mathbf{U} again is the domain of \mathcal{RS} .

$\mathcal{TH}(\mathcal{RS})$ then is the set of all formulae of \mathbf{L} valid in \mathcal{RS} ¹⁹. Obviously $\mathcal{TH}(\mathcal{RS})$ is deductively closed, consistent and complete.

3.2.4 elementarily equivalent structures

Two relational structures $\mathcal{RS}_1, \mathcal{RS}_2$ for \mathbf{L} are said to be **elementarily equivalent** if they share the same theory²⁰, i.e., if the same set of formulae of \mathbf{L} is valid in both .

In this case we write $\mathcal{RS}_1 \equiv \mathcal{RS}_2$

¹⁹see e.g Bell and Slomson [18] p. 140

²⁰see e.g Bell and Slomson [18] p. 74

This completes our review of basic modeltheoretic concepts and results, we will make use of.

4 coordinative definitions

By a **coordinative definition** for a concept of 'de deo' in 1st order theory of models we understand a mapping, which assigns some target concept defined in the theory of models to the definition of the source-concept in 'de deo'.

4.1 coordinative definition for 'attribute'

'IV. Per attributum intelligo id,quod intellectus de substantia percipit, tanquam ejusdem essentiam constituens' ('de deo', definitio IV [3] (1,45))

Thus 'attributes' describe the essence of substance (i.e. of reality), as known to/perceived by the intellect.

And, important too, 'attributes' are expressed by the 'modi' :

'the particular things are nothing else but affections of the attributes of God , i.e. modi, by which the attributes of God are expressed in a certain and definite way' (our translation of 'de deo' propositio XXV, corollarium [3] (24,68))

We amend then, what has been said above in the heuristic section 'concerning attributum', to arrive at the following picture:

Be $\mathcal{T}_{\mathcal{A}\mathcal{T}\mathcal{T}}$ a comprehensive theory of it's (large range) subject²¹, (preferably axiomatized), in 1st order language \mathbf{L} , and be $\mathcal{TH}_{\mathcal{A}\mathcal{T}\mathcal{T}_{\mathcal{I}}}$ the theory of it's intended model $\mathcal{RS}_{\mathcal{A}\mathcal{T}\mathcal{T}_{\mathcal{I}}}$

we propose to picture 'Attribute intended by theory $\mathcal{T}_{\mathcal{A}\mathcal{T}\mathcal{T}}$ ' as

$$\boxed{\mathcal{TH}_{\mathcal{A}\mathcal{T}\mathcal{T}_{\mathcal{I}}} = \{ \mathbf{A} \mid \mathbf{A} \in \mathcal{F} \wedge \mathcal{RS}_{\mathcal{A}\mathcal{T}\mathcal{T}_{\mathcal{I}}} \Vdash \mathbf{A} \}}$$

Obviously $\mathcal{TH}_{\mathcal{A}\mathcal{T}\mathcal{T}_{\mathcal{I}}}$ is an extension of $\mathcal{T}_{\mathcal{A}\mathcal{T}\mathcal{T}}$

We try to show, that $\mathcal{TH}_{\mathcal{A}\mathcal{T}\mathcal{T}_{\mathcal{I}}}$ is a good choice , first, by showing the correlation with our picturing of 'modi'

4.2 coordinative definition for 'mode'

'V. Per modum intelligo substantiae affectiones,sive id, quod in alio est, per quod etiam concipitur' ('de deo', definitio V [3] (1,45))

'mode' is conceived through 'quod intellectus de substantia percipit', i.e., through an 'attribute', which description is enriched by

'de deo' propositio XXV, corollarium 'res particulares nihil sunt, nisi Dei attributorum affectiones, sive modi, quibus Dei attributa certo, et determinato modo exprimatur. ...'[3](24,68)

²¹for 'res extensa' we might imagine as an example a theory of 'cartesian mechanics', or for sake of argument some up to day encoding of classical mechanics

and we propose that 'attribute' in our approach should be pictured as a (formalized) theory.

Now, if we have a theory, a legitimate question is - what is the theory about ? And if we state, that 'attribute' in the sense of 'de deo' should be pictured by some (axiomatized) 1st-order theory, and we learn (besides others) from the cited corollary, that 'attribute' is about 'modi', we should try to show the pictures of modi related to a the kind of theory, which is selected to serve picturing 'attribute'.

4.2.1 Quine's "standard" for judging ontological commitment of a theory

Willard van Orman Quine's often cited "To be ..., is ... to be ...the value of a variable" ²², more explicitly "...: a theory is committed to those and only those entities, to which the bound variables of the theory must be capable of referring in order that the affirmations made in the theory be true" ²³ We hold, that following Quine's standard in principle but with some qualification, will give us an acceptable picture, in what sense $\mathcal{TH}_{ATT_{\mathcal{I}}}$ is "about" it's modes, which constitute the ontology, to which we hold our attribute ATT to be committed.

4.2.2 representing modes as definite expression of the corresponding attribute

In order to do this, we use some formalism, which has already been introduced above.

(a) $\mathcal{TH}_{ATT_{\mathcal{I}}}$, being the theory of a relational structure, is deductively closed, consistent and complete.

(b) By the above cited theorem on prenex normal forms,

$\bigwedge_{\mathbf{A} \in \mathcal{F}} [\mathcal{TH}_{ATT_{\mathcal{I}}} \Vdash \mathbf{A} \rightarrow \bigvee_{\mathbf{A}', \mathbf{B} \in \mathcal{F}} (\mathbf{A}' = \mathbf{QB} \wedge \Vdash \mathbf{A}' \leftrightarrow \mathbf{A})]$
 where \mathbf{Q} is a finite sequence of symbols \neg and $\bigvee_{\mathbf{x}_i}, \mathbf{x}_i \in \mathbf{V}_L$, and \mathbf{B} is a quantifier free formula.

(c) The (quantifier-free) matrix \mathbf{B} of the prenex normal form \mathbf{QB} contains finitely many, say $\mathbf{n} \geq 1$, different variables $\mathbf{x}_1, \dots, \mathbf{x}_n$. All occurrences of variables in \mathbf{B} are free, thus $\overline{\mathbf{B}}$, the value of \mathbf{B} , does depend only on the variables $\mathbf{x}_1, \dots, \mathbf{x}_n$

$\overline{\mathbf{B}}$ may be finite or infinite. The mappings $\rho \in \mathbf{U}^{\mathbf{V}_L}$ may be viewed as infinite sequences of objects of \mathbf{U} , each object indexed by the variable, to which it is assigned. As $\overline{\mathbf{B}} \subseteq \mathbf{U}^{\mathbf{V}_L}$ depends only on the \mathbf{n} distinct variables $\mathbf{x}_1, \dots, \mathbf{x}_n$ occurring in \mathbf{B} , $\overline{\mathbf{B}}$ may (without loss of generality) be restricted to these variables only, getting

²²On what there is'[16], cf. pp. 13-18

²³ibid., while Quine immediately relativizes the importance of this "semantical formula" with respect to conventionalist or phenomenalist preferences of theory selection, he keeps the "formula" still running as valid standard.

$$\overline{\mathbf{B}}/ = \{ \rho \in \overline{\mathbf{B}} \mid \forall \mathbf{a}_1, \dots, \mathbf{a}_n [\langle \mathbf{a}_1, \dots, \mathbf{a}_n \rangle \in \mathbf{U}^{\{\mathbf{x}_1, \dots, \mathbf{x}_n\}} \wedge \rho(\mathbf{x}_i) = \mathbf{a}_i] \}$$

where $\mathbf{x}_1, \dots, \mathbf{x}_n$ are the \mathbf{n} variables free in \mathbf{B}

Accordingly we define the set of **modes conceived by attribute** $\mathcal{TH}_{\mathcal{AT}\mathcal{T}\mathcal{I}}$

$$\boxed{\text{Modes}_{\mathcal{TH}_{\mathcal{AT}\mathcal{T}\mathcal{I}}} = \{ \overline{\mathbf{B}}/ \mid \forall \mathbf{A}, \mathbf{B} \in \mathcal{F} [\mathbf{A} = \mathbf{QB} \wedge \mathcal{TH}_{\mathcal{AT}\mathcal{T}\mathcal{I}} \Vdash \mathbf{A}] \}}$$

where \mathbf{Q} is a finite sequence of symbols \neg and $\forall_{\mathbf{x}_i}$, $\mathbf{x}_i \in \mathbf{V}_L$, and \mathbf{B} is a quantifier free formula.

4.2.3 slight difference to Quine's suggestion on ontological commitment of a theory

We argue, that some more clarification is gained for the concept of 'ontological commitment' by taking thus the value of the matrix of the prenex normalform (for any theorem of the theory)

first: this determination evades the seeming necessity, to count any object in the universe of Discourse \mathbf{U} in itself as ontological relevant, but only special configurations of them,

secondly: these 'ontological relevant configurations of objects from the universe of discourse' are singled out by the theorems of the theory itself.

4.2.4 now what about 'infinite modes' in the sense of de deo by our way of picturing 'mode'

We recall: the **mode** assigned to $\mathcal{TH}_{\mathcal{AT}\mathcal{T}\mathcal{I}} \Vdash \mathbf{A} = \mathbf{QB}$ is given by $\overline{\mathbf{B}}/$, which may be a finite or infinite set of finite sequences of objects of \mathbf{U} . But this $\overline{\mathbf{B}}/$ being a finite or infinite set, does of course not settle the question, whether the mode, characterised by $\overline{\mathbf{B}}/$, is an infinite mode in the sense of 'de deo'. In the heuristic section above, we saw the unique 'infinite mode' given with an 'attribute' as a framework for the finite modes, formally represented by the logical conjunction of the universal principles stated by the attribute in question. Suppose, such a conjunction of universal sentences of our attribute were given as $\mathcal{TH}_{\mathcal{AT}\mathcal{T}\mathcal{I}} \Vdash \mathbf{A}_1 \wedge \dots \wedge \mathbf{A}_n$. Then again we take a prenex normal form $\mathbf{A}' = \mathbf{QB}$ with $\Vdash \mathbf{QB} \leftrightarrow \mathbf{A}_1 \wedge \dots \wedge \mathbf{A}_n$, and the infinite mode of the attribute is characterised by $\overline{\mathbf{B}}/$, viz. by the restricted value of the matrix of the prenex normal form of the conjunction of these universal principles. Our above remarks on the general definition of 'mode' apply, but in this case we would expect $\overline{\mathbf{B}}/$ to be an infinite set anyway).

4.3 coordinative definition for 'substance'

We mentioned already the scholium to propositio XXIX ('de deo' [3] (27, 71)), where Spinoza explains/recalls, what he understands by 'natura naturans' and 'natura naturata'. For 'natura naturans' he refers via 'de deo' propositio XIV, corollarium I ([3] (12, 56)) to 'de deo' definitio VI ('Deus'), and via 'de deo' propositio XVII, corollarium II ([3] (17, 61)) to 'de deo' definitio I ('causa sui');

hence, Spinoza's discussion of 'natura naturans' lays it's emphasis on the ontological line of argument, and we can't deal with 'substance' viewed as 'natura naturans' within our logical line-approach ²⁴, but only with 'substance' viewed as 'natura naturata', which Spinoza explains in propositio XXIX scholium:

'Per naturatam autem intelligo id omne, quod ex necessitate Dei naturae, sive uniuscuiusque Dei attributorum sequitur, hoc est, omnes Dei attributorum modos, quatenus considerantur, ut res, quae in Deo sunt et quae sine Deo nec esse, nec concipi possunt'

Thus, we are entitled to choose as a coordinative definition for 'substance' qua 'natura naturata' the totality of all modes conceived by any of God's attributes.

So far we dealt within our reconstructive approach only with one attribute at a time, without making any special assumptions on the contents of the attribute, except for illustrating by examples. Accordingly, we state our coordinative definition for 'substance' qua 'natura naturata' again with respect to our one (further unspecified) attribute $\mathcal{TH}_{\mathcal{AT}\mathcal{T}_{\mathcal{I}}}$ as the totality of modes conceived by this attribute.

But now, we have a choice :

if we pictured the 'inesse'-relation ('de deo', Axioma I, [3] (2, 46)) between a mode conceived by the attribute and the totality of these modes ('natura naturata') as the relation of a set (substance) to it's elements (modes), we would fail to picture the information, that modes are 'affectiones' of substance - by which we understand: 'local concretions' of substance. In addition, modes may 'overlap'²⁵, which does decrease the plausibility of the 'set of modes'-construction again.

Therefore we suggest, to use the obvious alternative, viz., 'substance' qua 'natura naturata' be the union of the set of modes, which is again a subset of the set of all finite sequences of objects of the universe of discourse \mathbf{U} singled out by our 'attribute' $\mathcal{TH}_{\mathcal{AT}\mathcal{T}_{\mathcal{I}}}$, like the modes themselves were thus singled out.

More formally

$$\text{Substance}[\mathcal{TH}_{\mathcal{AT}\mathcal{T}_{\mathcal{I}}}] \Leftrightarrow \bigcup_{\overline{\mathbf{B}}} [\bigvee_{\mathbf{A}, \mathbf{Q}, \mathbf{B}} (\mathcal{TH}_{\mathcal{AT}\mathcal{T}_{\mathcal{I}}} \Vdash \mathbf{A} \wedge \mathbf{A} = \mathbf{QB})]$$

where again A is a theorem of $\mathcal{TH}_{\mathcal{AT}\mathcal{T}_{\mathcal{I}}}$ in prenex normal form, with prefix Q and quantifierfree matrix B .

4.4 concluding remarks on this section

4.4.1 note on 'inesse' pictured by the subset-relation for substance qua 'natura naturata'

By our selection of coordinative definitions for 'substance' (qua 'natura naturata') and 'mode' trivially the following hold:

²⁴but see our "Concluding remarks on logical line of argument" in the end of this paper

²⁵technically speaking, our 'mode'-pictures $\overline{\mathbf{B}_i}$, $\overline{\mathbf{B}_{i+1}}$, ... need not be pairwise disjoint

$$\bigwedge_{/\overline{\mathbf{B}}/\in\text{Modes}_{\mathcal{TH}_{\mathcal{AT}\mathcal{T}_{\mathcal{I}}}}} \text{Substance}[\mathcal{TH}_{\mathcal{AT}\mathcal{T}_{\mathcal{I}}}] \cup / \overline{\mathbf{B}}/ = \text{Substance}[\mathcal{TH}_{\mathcal{AT}\mathcal{T}_{\mathcal{I}}}]$$

$$\bigwedge_{/\overline{\mathbf{B}}/\in\text{Modes}_{\mathcal{TH}_{\mathcal{AT}\mathcal{T}_{\mathcal{I}}}}} \text{Substance}[\mathcal{TH}_{\mathcal{AT}\mathcal{T}_{\mathcal{I}}}] \cap / \overline{\mathbf{B}}/ = / \overline{\mathbf{B}}/$$

This serves us as a further hint for a 'grosso modo' formal adequacy of our coordinating definitions.

4.4.2 note on 'inesse' for substance qua 'natura naturans'

Things are different concerning 'inesse' for substance qua 'natura naturans', which concept is complementary to 'inesse' for substance qua 'natura naturata' just considered.

While 'modes' and 'natura naturata' concern "substance in it's affections being conceived by some attribute", 'natura naturans' is substance viewed as the (immanent) cause of all this²⁶, which turns the 'x inest y'-relation into the 'y is the (immanent) cause of x'-relation with the notable special case 'x is the (immanent) cause of x' (causa sui) . This complementary concept of 'inesse' obviously belongs to the ontological line of argument (by 'de deo', definitio I), but we have a possibility to refer to it using not an explicit but a contextual definition relating this concept to our coordinating definitions, viz.

$$\mathbf{x} \text{ causa } \mathbf{x} \leftrightarrow \bigvee_{\mathcal{L}, \mathcal{RS}_{\mathcal{AT}\mathcal{T}_{\mathcal{I}}[\mathcal{L}]}} \bigwedge_{/\overline{\mathbf{B}}/} [/\overline{\mathbf{B}}/ \in \text{Modes}(\mathcal{TH}_{\mathcal{AT}\mathcal{T}_{\mathcal{I}}[\mathcal{L}]}) \rightarrow \mathbf{x} \text{ causa } / \overline{\mathbf{B}}/]$$

which we spell out as follows:

\mathbf{x} is causa sui if and only if
there exist attributes $\mathcal{TH}_{\mathcal{AT}\mathcal{T}_{\mathcal{I}}[\mathcal{L}]}$, formulated each as a theory of some intended model $\mathcal{RS}_{\mathcal{AT}\mathcal{T}_{\mathcal{I}}[\mathcal{L}]}$, such that for any mode $/\overline{\mathbf{B}}/$ determined by $\mathcal{TH}_{\mathcal{AT}\mathcal{T}_{\mathcal{I}}[\mathcal{L}]}$
 \mathbf{x} is the cause of mode $/\overline{\mathbf{B}}/$, or in other words, $/\overline{\mathbf{B}}/ \text{ inest}_{ontological} \mathbf{x}$ "

Please note,

(a) that the formula should not be taken as an explicit definition for 'x is the cause of x' ('causa sui'), because the definiens then would contain 'x is the cause of y'

(b) but that the formula may be taken as well as a contextual definition for 'x is the cause of y' (the ontological line connotation of 'y inest x'), and as well as a meaning postulate for 'x is the cause of x' (relating to 'de deo' definitio I [3](1, 45)).

4.4.3 where theory of models comes in ...

The concept of 'mode' appears to be the link between the concepts of 'attribute' and 'substance' qua 'natura naturata', and this mediating role is reflected by our coordinating definitions for 'de deo' definitiones III-'substantia', IV-'attributum' and V-'modus'.

²⁶'de deo' propositio XVIII 'Deus est omnium rerum causa immanens; non vero transiens'[3](19, 63)

And it is for the coordinative definition for 'modus', that we essentially make use of facts of the theory of models, first for each formula \mathbf{A} of \mathbf{L} the existence of a logically equivalent prenex normal form $\mathbf{A}' = \mathbf{QB}$, secondly, for each formula \mathbf{B} of \mathbf{L} and relational structure \mathcal{RS} for \mathbf{L} the existence of the value of that formula $\overline{\mathbf{B}}$ in \mathcal{RS} and it's dependency only on the variables free in \mathbf{B} , which allows us to strip $\overline{\mathbf{B}}$ down to $\overline{\mathbf{B}}/$.

4.4.4 'mode' in it's role as "truth-maker", correlation to theories of 'truth'

The following remarks, again, are meant heuristically and hence tentative only, but the idea seems to suggest itself and therefore shall be presented here:

We view the mode $\overline{\mathbf{B}}/$ of some closed formula in prenex normal form $\mathcal{TH}_{ATT_{\mathcal{I}}} \Vdash \mathbf{QB}$ as the "truth-maker" of \mathbf{QB} in $\mathcal{RS}_{ATT_{\mathcal{I}}}$, because (by construction), the value of matrix \mathbf{B} is, what makes the closed prenex formula \mathbf{QB} true in $\mathcal{RS}_{ATT_{\mathcal{I}}}$ where ' \mathbf{QB} is true in $\mathcal{RS}_{ATT_{\mathcal{I}}}$ ' means

$$\overline{\mathbf{QB}} = \mathbf{U}^{\mathbf{V}_L}$$

(a) this is of course in accordance with the modeltheoretic definition of truth, which, by meeting Tarski's adequacy condition (' A is true iff A)²⁷, conforms to the correspondence theory of truth.

(b) In the sense explained, the 'mode' ("truth-maker") of a closed prenex formula \mathbf{A} , is dependent on $\mathcal{TH}_{ATT_{\mathcal{I}}}$. But on whether our scenario may be counted as a case for the coherence theory of truth as well as for the correspondence theory of truth, obviously depends on whether we judge the functions and relations referred to by $\mathcal{RS}_{ATT_{\mathcal{I}}} = \langle \mathbf{U}, \Phi, \Psi \rangle$ as dependent on theory ATT or some other theory, or as theory-independently given.

(c) thus we might arrive at considering these theories of truth, which are occasionally supposed to contradict each other, to show simply two faces of the same coin.

5 picturing 'de deo' proofs of 'de deo' propositions

5.1 preliminary note on picturing proofs

Our endeavour now is trying to picture some parts of Spinoza's proving propositions in 'de deo' with reference to our 'coordinative definitions'²⁸

²⁷for a less laconic and more precise formulation see 'the concept of truth in the formalized languages'[14] pp. (45 - 46, 305 - 306) (original pagination), reprinted in Karel Berka, Lothar Kreiser 'Logik Texte' [15] pp. 447-559 [there pp. 481-482]

²⁸there is an attitude of negatively commenting on 'de deo' propositions to the effect, that they were vacuous else unnecessary in order to prove e.g. the uniqueness of substance ('de deo' propositio XIV [3](12,56)), because the conclusion viz. 'de deo' propositio XIV were already immediately obvious from 'de deo' definitiones, e.g. I, III, IV. This attitude is described by Konrad Cramer in his inaugural lecture (Heidelberg 1975) "Es ist nämlich ein Standardeinwand gegen das gesamte Verfahren des Spinoza, daß die berufene Einzigkeit der Substanz und ihre Identifikation mit Gott schon aus den *Definitionen*, die Spinoza seinen Beweisen vorangestellt hat, ohne alle weiteren Vermittlungen direkt folgen. ..." [11], p. 2,

This is not easily done, and the result may be felt to suffer of more or less important shortcomings. Shortcomings here may arise from different causes:

first, that our coordinative definitions are in part or completely beyond the point, not providing a workable formal analogy to 'de deo' (which we hopefully tried to avoid in what has been done so far in this text)

secondly, that a formal analogy is provided to some extent, but is not sufficient in detail, to master the task, we try to ...

third, that the 'de deo'-proof, we try, relies too much on ontological line connotations of the concepts involved in the proof, and thus our logical line account of the proof fails to grasp one or other central point.

The judgment concerning all this must faithfully be left at the disposal of the reader.

In what follows, we select some of 'de deo' propositiones I-XIII in order to exemplify our way of picturing 'de deo' proofs, and omitting explicitly those propositions which rather obviously belong to the ontological line of argument. We take Spinoza's 'ordine geometrico' procedere seriously by holding, that for 'de deo' definitiones and axiomata there is nothing to prove.

5.2 Propositio I. 'de deo' [3] (3, 47)

Propositio I. Substantia prior est natura sui affectionibus.

Demonstratio. Patet ex definitione 3. et 5.

'de deo' definitiones III ('substance') and V ('modus') [3] (1, 45) consist (III) of a pair of clauses 'in se esse' and 'per se ... concipitur', and (V) of a pair of clauses 'in alio esse' and 'per quod ... concipitur'.

Thus, to picture propositio I, we have to picture both the 'inesse'-clauses with respect to our coordinative definitions, as well as the 'per ... concipitur' clauses. As we already mentioned, there is the ontological connotation of 'substance' as 'natura naturans' and thus an ontological line component of 'inesse' between modes and natura naturans, related to 'de deo' definitio I ('causa sui') and nominally defined by $y \text{ inest}_{ontological} x \Leftrightarrow x \text{ causa } y$, of the latter 'x causa y' we tried a contextual definition, viz

and in what follows K. Cramer argues under various aspects against the 'Standardeinwand' . Likewise observed was the situation already by Joachim, op.cit. in his preface "Barren abstractions, tortured into the form of 'geometrical demonstration' by a pedantic logic, appear to constitute the larger portion of it [sic:The Ethics]: and he remainder has been taken for poetry pure and simple. It has seemed easy to annihilate the first with a few catchwords of criticism, dismissing the second as the dreams of a mystic. I have assumed that the 'poetry' and 'imagination' which breath through its pages are - as in a great thinker they must be - in the service of a mind, which is pedantic only in its endeavour to think clearly and reason logically ..." [6], p. iii . In this question on a moderately contrary position we find Wolfson, op.cit. "If, as we have been trying to show, there is no logical connection between the substance of Spinoza's philosophy and the form, in which it is written, his choice of the Euclidian geometrical form is to be explained on other grounds. Primarily, we say, the reason for its choice was pedagogical, ..." [7]"The geometrical method', p. 55 . But there are more scholars in the study of the ethics of Spinoza, which at least in the main take Spinoza's argumentation 'ordine geometrico' seriously, we mention W. Cramer op.cit in his section 4. 'Die Propositionen und ihre Beweise', especially 'a) Der Beweis der Einzigkeit der Substanz' [10] pp. 58 ff., and Stuart Hampshire in his 'Spinoza'-Book in the section 'Outline of Metaphysics' [8], pp. 30 ff.

$$\mathbf{x\ causa\ x} \leftrightarrow \forall_{\mathbf{L}, \mathcal{R}_{\mathcal{S}_{\mathcal{A}\mathcal{T}\mathcal{T}_{\mathcal{I}}[\mathbf{L}]}} \wedge_{/\overline{\mathbf{B}}/} [/\overline{\mathbf{B}}/ \in \mathbf{Modes}(\mathcal{T}\mathcal{H}_{\mathcal{A}\mathcal{T}\mathcal{T}_{\mathcal{I}}[\mathbf{L}])} \rightarrow \mathbf{x\ causa\ }/\overline{\mathbf{B}}/]$$

And on behalf of the logical line of argument, we pictured the logical line component of 'inesse' between modes and natura naturata' by

$$\wedge_{/\overline{\mathbf{B}}/ \in \mathbf{Modes}_{\mathcal{T}\mathcal{H}_{\mathcal{A}\mathcal{T}\mathcal{T}_{\mathcal{I}}}} \mathbf{Substance}[\mathcal{T}\mathcal{H}_{\mathcal{A}\mathcal{T}\mathcal{T}_{\mathcal{I}}}] \cup / \overline{\mathbf{B}}/ = \mathbf{Substance}[\mathcal{T}\mathcal{H}_{\mathcal{A}\mathcal{T}\mathcal{T}_{\mathcal{I}}}]$$

and concerning the 'per se/per quod ...concipitur'-clauses of 'de deo'-definitiones III ('substance') and V('mode'), we claim, that they are directly pictured by the model theoretic relations between our three coordinative definitions for **Substance**, **Modes** and **Attribute**.

In the heuristic section we already pointed to 'de deo' propositio XXIX, Scholium [3](27, 71), where Spinoza explains 'natura naturans' as the free cause ('causa libera'), which, by it's attributes with necessity determines it's 'natura naturata'. We should keep in mind, that nothing said or pictured so far, does imply, that 'natura naturata' and 'natura naturans' were two different things, just in the opposite: we get the partial description of a system of substance, attributes and modes, (picturing Spinoza's system hopefully), in which

the role of substance as 'natura naturans', joining 'de deo' definitiones I('causa sui'), III('substance'), IV('attribute'),

and the role of substance as 'natura naturata', joining 'de deo' definitiones III('substance'), IV('attribute'), V('modus'),

supply only different perspectives into this system.

In the end of our introductory section we stated, that while confining our endeavour rather exclusively to the logical line of argument in 'de deo', we need but one digression to the ontological line, viz. 'inesse' as used in 'de deo' axioma I 'omnia, quae sunt, vel in se, vel in alio sunt'[3](2, 46), and Spinoza's laconic demonstratio of 'de deo' propositio I now is the first case of this. We will refer to *inesse_{ontological}* a second time when dealing with 'de deo' propositio IV, see below.

5.3 Propositio II. 'de deo' [3] (3, 47)

Propositio II. Duae substantiae, diversa attributa habentes, nihil inter se commune habent.

Demonstratio. Patet etiam ex Defin. 3 . Unaquaeque enim in se debet esse, et per se concipi, sive conceptus unius conceptus alterius non involvit.

... because each be in se and be conceived per se ... is of course derived from that they are 'duae substantiae', the difficulty is with the informal clauses 'nihil inter se commune habere' and 'conceptus alterius non involvere'.

We try the following:

Be $\mathcal{T}\mathcal{H}_{\mathcal{A}\mathcal{T}\mathcal{T}_{\mathcal{I}}[1]}$ and $\mathcal{T}\mathcal{H}_{\mathcal{A}\mathcal{T}\mathcal{T}_{\mathcal{I}}[2]}$ two different attributes of two different substances

$$\mathbf{Substance}_1[\mathcal{T}\mathcal{H}_{\mathcal{A}\mathcal{T}\mathcal{T}_{\mathcal{I}}[1]}] \cong \cup_{/\overline{\mathbf{B}}/} [\forall_{\mathbf{A}, \mathbf{Q}, \mathbf{B}} (\mathcal{T}\mathcal{H}_{\mathcal{A}\mathcal{T}\mathcal{T}_{\mathcal{I}}[1]} \Vdash \mathbf{A} \wedge \mathbf{A} = \mathbf{QB})]$$

and

$$\mathbf{Substance}_2[\mathcal{TH}_{AT\mathcal{T}_I[2]}] \cong \cup_{\overline{B}} [\vee_{A, Q, B} (\mathcal{TH}_{AT\mathcal{T}_I[2]} \models \mathbf{A} \wedge \mathbf{A} = \mathbf{QB})]$$

We argue, that in order to 'have something in common', $\mathbf{Substance}_1[\mathcal{TH}_{AT\mathcal{T}_I[1]}]$ and $\mathbf{Substance}_2[\mathcal{TH}_{AT\mathcal{T}_I[2]}]$ need share the same language and thus share the same set of relational structures.

But, of course, as both (model theoretic pictures of) attributes being theories of relational structures, they are complete theories (as already mentioned above). Now, if both theories have all their closed theorems in common, all their models are elementarily equivalent, which amounts to they are not two but one attribute. Else there is a closed formula \mathbf{A} such that $\mathcal{TH}_{AT\mathcal{T}_I[1]} \models \mathbf{A}$ and $\mathcal{TH}_{AT\mathcal{T}_I[2]} \models \neg \mathbf{A}$, hence, in this case these theories ('attributes') contradict each other.

Thus, if we allow counting 'mutual contradiction' of two attributes as implying, that the substances, whose essence is expressed by these attributes, do have nothing in common, we are done, and proof picturing for 'de deo' propositio II is complete.

5.4 Propositio III. 'de deo' [3] (3, 47)

Propositio III. Quae res nihil commune inter se habent, earum una alterius causa esse non potest.

Demonstratio. ...

Spinoza's proof refers directly to 'de deo' axiomata V and IV, none of our pictured concepts 'attribute', 'mode', 'substance' is essentially involved, hence for Spinoza's proof of propositio III nothing for us to picture.

5.5 Propositio IV. 'de deo' [3] (3 - 4, 47 - 48)

Propositio IV. Duae aut plures res distinctae, vel inter se distinguuntur ex diversitate attributorum substantiarum, vel ex diversitate earundum affectionum.

Demonstratio. Omnia, quae sunt, vel in se, vel in alio sunt (per Axiom. 1.) hoc est (per Defin. 3. et 5.), extra intellectum nihil datur praeter substantias earumque affectiones. Nihil ergo extra intellectum datur, per quod plures res distingui inter se possunt praeter substantias, sive quod — idem est (per Defin. 4.) earem attributa, earumque affectiones. Q.E.D.

There is an investigation by Robert Schnepf on the occurrences, uses and (maybe context-dependent) meanings of Spinoza's terminus 'res' in the text of 'de deo'. Concerning propositio IV we read in "§ 3: Die Bedeutung des Begriffs 'res' und ihre Variation relativ auf die Begriffe 'Substanz', 'Attribut' und 'Modus' bis Lehrsatz 15" ([13], pp. 146-152) "... "An insgesamt sieben Stellen wird die Rolle des Ausdrucks 'res' zumindest implizit durch sein gemeinsames Auftreten mit den Ausdrücken 'Substanz', 'Attribut' und 'Modus' bestimmt: nämlich in den Lehrsätzen 3, 4, der Anmerkung zu Lehrsatz 8, in Lehrsatz 9, der Anmerkung zu Lehrsatz 11, im 2. Folgesatz zu Lehrsatz 14 sowie in Lehrsatz 15. ... " (ibid., p. 147)

While, we agree with this, 'res' in propositio III should be taken to refer implicitly to 'substance', the proof of propositio III is, from our point of view, complete by it's reference to axiomata V and IV.

Quite another case is propositio IV. Axioma I and definitiones III, IV, and V are used essentially. The '... sive quod idem est ...' clause states, that every substance is exhaustively and completely characterized by it's attributes and it's affections, and that there exists nothing else but substances, each with it's attributes and affections. We have two remarks on this:

(a) 'res' qua 'substance' in propositio IV comes as a package of 'substance', 'attributes', and 'modes'²⁹

(b) we hold, that 'substantias, sive quod earem attributas' does refer to the concept of substance qua 'natura naturans/causa sui', and citing 'de deo' axioma I in the proof refers to **inesse_{ontological}**, which we discussed in the concluding remarks of the previous section, and we cite the contextual definition of 'x causa y', there given :

$$\mathbf{x\ causa\ x} \leftrightarrow \forall_{L, \mathcal{R}S_{\mathcal{A}T\mathcal{T}_I[L]}} \wedge_{\overline{B}} [/\overline{B}/ \in \mathbf{Modes}(\mathcal{TH}_{\mathcal{A}T\mathcal{T}_I[L]}) \rightarrow \mathbf{x\ causa\ } \overline{B}/]$$

And we repeat, that this formula, beyond it's role of being a contextual definition for 'x causa y', plays a second role as a meaning postulate for 'x causa x'. From the latter we take, that for any 'causa sui' **x** there exists the non-empty set of it's attributes, and from our coordinative definition of mode we take, that under each attribute the set of all modes conceived by this attribute is determined also, and again from our meaning postulate for 'causa sui' we take, that any 'causa sui' is uniquely determined by these sets.

Now we apply 'de deo' axioma I, and 'de deo' definitio III (in it's double role 'substance' qua 'natura naturata' and 'substance' qua 'causa sui') and 'de deo' definitio V ('mode'), and thus infer

omnia, quae sunt, vel in se, vel in alio sunt

⊨ **omnia quae sunt, vel causae sui, vel modi sunt**

This, although we presented the argument in reverse order, completes the proof, because, two or more causae sui can only differ, in what characterizes them uniquely, viz. differ in it's set of attributes and/or in it's sets of correlated modes.

5.6 Propositio V. 'de deo' [3] ⟨4, 48⟩

Propositio V. In rerum natura non possunt dari duae, aut plures substantiae ejusdem naturae, sive attributi.

(Spinoza's demonstratio refers to propositiones I and IV, [Definitio III and axioma VI] and might be reconstructed accordingly)

²⁹"Substanz, Attribut und Modus sind in diesem Lehrsatz als Komponenten einer in sich strukturierten *res* zu lesen" Schnepf [13], p. 148, cf. too "It seems clear, that *in some sense* Attributes and Substances are the same thing ..." Joachim, 'Substance and Attribute' [6], p.18

Demonstratio. Si darentur plures distinctae, deberent inter se distingui, vel ex diversitate attributorum, vel ex diversitate affectionum (per Prop. praed.)

Allright, starting from already proven propositio IV.

Si tantum ex diversitate attributorum, concedetur ergo, non dari, nisi ejusdem attributi.

The first alternative of proposition IV is exactly, what proposition V asserts

At si ex diversitate affectionum, cum substantia sit prior natura affectionibus, (per Prop. 1) depositis ergo affectionibus, et in se considerata, hoc est (per Defin 3. et Axiom. 6.) vere considerata, non poterit concipi ab alia distingui, hoc est (per Prop. praed.), non poterunt dari plures, sed tantum una. Q.E.D.

Our 'coordinative definition' for **Modes** shows, that the set of modes is **uniquely** determined by its corresponding attribute $\mathcal{RS}_{AT\mathcal{T}_I}$ due to the modeltheoretic definition of 'value of a formula of **L**' in a relational structure \mathcal{RS} for **L**'³⁰, which implies, that for each relational structure \mathcal{RS} 'attributes' (qua theories which share \mathcal{RS} as a model) are different, if **Modes** are different. This completes the proof.³¹

5.7 remarks on next 'de deo' propositiones VI-VII [3] ⟨4, 48⟩

Propositio VI: Spinoza's demonstratio refers to propositiones II and III, and might be reconstructed accordingly

Propositio VII: Spinoza's demonstratio refers to propositio VI (see above) and definitio I (ontological line), essentially referring on the ontological proof, which is suggested valid by definitio I. Thus, propositio outside our scope.

5.8 Propositio VIII. 'de deo' [3] ⟨4 – 5, 48 – 49⟩

Propositio VIII: Omnis substantia est necessario infinita

Demonstratio: Substantia unius attributi non, nisi unica, existit, (per Prop. 5) et ad ipsius naturam pertinet existere. (per Prop. 7) Erit ergo de ipsius natura, vel finita vel infinita existere. At non finita. Nam (per Defin.2.) deberet terminari ab alia ejusdem naturae, quae etiam necessario deberet existere; (per Prop. 7.) adeoque darentur duae substantiae ejusdem attributi, quod est absurdum (per Prop. 5.) Existit ergo infinita. Q.E.D.

Spinoza's demonstratio refers to 'de deo' definitio II, propositio V (logical line) and proposition VII (ontological line). We hold, that reference to propositio

³⁰see above our section 'notes on the (model theoretic) semantics for **L**'

³¹the reference to 'de deo' axioma 6 (Spinoza's referring to correspondence theory of truth) is here implicitly contained in using logical inference (based on model theoretic truth definition) from our coordinative definition of **Modes**.

VII is only for the qualification 'necessario', leaving to picture concerning the logical line 'Omnis substantia est infinita'

A short way is provided then thus: we already pictured proving 'de deo' propositio V above, 'there can't be to or more substances of the same attribute' which we take to imply 'there can't be two or more substances in suo genere', and now we pick up from our heuristic section, when considering the 'limits'-relation (attaining to 'de deo' definitio II), that if something it is the only object 'in sui genere', it is **not finite** \Leftrightarrow **infinite**

5.9 Propositio IX. 'de deo' [3] (7, 51)

Propositio IX: Quo plus realitatis, aut esse unaquaeque res habet, eo plura attributa ipsi competunt.

Demonstratio: Patet ex Defin. 4.

In our opinion, Spinoza's "obvious by definitio IV" does only make some sense, if the grammatical subject 'res' again is meant as 'substance' qua 'causa sui' (see our above remarks on propositio IV). The argument from 'degrees of existence/reality' is common with the ontological line of argument³². Thus, for the logical line of argument, nothing to picture.

5.10 remarks on next 'de deo' propositiones X-XIII [3] (7-12, 51 - 56)

Propositio X: Spinoza's demonstratio refers to definitiones III and IV.

Nothing to reconstruct.

Propositio XI: Spinoza's demonstratio refers to ontological line definitio VI (implicitly), axioma VII, propositio VII

Thus, propositio outside our scope.

Propositio XII, XIII concern (in-)divisibility of 'substance' (as we noted already: battle against Descartes Principia Philosophiae).

But in spite of this historical context, there is, except implicitly by Spinoza's handling of the both proofs, in our opinion no comprehensible meaning for phrases like 'divisibility of substance', if the term 'substance' is to be understood in accord with 'de deo' definitio III or definitio I - - - although there is a lot of argumentation concerning 'divisibility' and 'res extensa' in the 'scholium' to 'de deo' propositio XV, including the already in our heuristics section cited passage stating that 'partes modaliter ... distinguuntur, non autem realiter'.

Anyway, let's have a short look on

Propositio XII: Spinoza's demonstratio starts with a dichotomy 'partes enim, ..., vel naturam substantiae retinebunt, vel non. ...' [3] (11, 55) For proving the first case, he refers to the already mentioned (more or less) logical line 'de deo'

³²Wolfson comments with regard to 'de deo' propositio IX "Spinoza further uses *realitas* as the equivalent of *perfectio* [7], p.141

propositions (IV, V, VI, VIII, X), in proving the alternative, he refers to ontological line 'de deo' propositio VII

Propositio XIII: Spinoza's demonstratio starts again with a dichotomy 'partes enim, ..., vel naturam substantiae absolute infinitae retinebunt, vel non. ...' [3](11, 55) For proving the first case, he refers to the logical line 'de deo' propositio V, in proving the alternative, he refers to ontological line 'de deo' propositio XI

6 concluding remarks on logical line of argument

6.1 concerning Deus

We stop considering 'de deo' proofs at this point, and we had already argued for this by distinguishing a logical line and a ontological line of argument in 'de deo'. Of course, 'de deo' propositiones XIV and XV are a highlight of 'de deo', but definitely belong to the ontological line via reference to besides others 'de deo' definitio VI ('Deus').

To exemplify, what might be involved by this, we try picturing 'de deo' definitio VI by recurring once more to our 'causa sui' meaning postulate

$$\begin{aligned}
 & \mathbf{x} = \mathbf{Deus} \leftrightarrow \\
 & [\forall_{\mathbf{L}, \mathcal{R}_{S_{AT\mathcal{T}_I[\mathbf{L}]}}} \wedge_{/\overline{\mathbf{B}}/} \{ [/\overline{\mathbf{B}}/ \in \mathbf{Modes}(\mathcal{TH}_{AT\mathcal{T}_I[\mathbf{L}]}) \rightarrow \mathbf{x} \text{ causa } / \overline{\mathbf{B}}/] \} \\
 & \quad \wedge \\
 & \neg \forall_{\mathbf{L}, \mathcal{R}_{S_{AT\mathcal{T}_I[\mathbf{L}]}}} \vee_{/\overline{\mathbf{B}}/} \{ [/\overline{\mathbf{B}}/ \in \mathbf{Modes}(\mathcal{TH}_{AT\mathcal{T}_I[\mathbf{L}]}) \wedge \neg \mathbf{x} \text{ causa } / \overline{\mathbf{B}}/] \}]
 \end{aligned}$$

The very impact of this formalization (as with the source, viz. 'de deo' definitio VI) is, that we have to deal with a totality of 'attributes', and by this of course, with Spinoza's use(s) of ontological proof.

6.2 different appreciation of 'de deo'

While, to be sure, the ontological line of argument leads to 'de deo' propositiones (XIV,... ff.), which presumably had considerable impact on the development of 'german idealism' of the nineteenth century, we hold, that perhaps the most important accomplishment of Spinoza's metaphysics in 'de deo' is , to combine and reconcile a thoroughgoing, uncompromising ancient-rooted account of metaphysics (subsistence-inherence) with an at the time new, extremely rational, deductive style of thinking, which in 'de deo' leads to the 'natura naturata as conceived by an attribute' - construction.

6.3 liability for plurality of 'attributes'

Obviously, as 'modi' and 'natura naturata' as a whole are construed according to the respective attribute, the question arises, of whether and how 'modi' of different attributes, e.g. 'res cogitans' and 'res extensa', are correlated. Spinoza deals with this extensively in 'ethics pars secunda, de natura et origine mentis'³³, e.g. 'propositio VII, scholium : '... Sic etiam modus extensionis et idea illius modi una, eademque est res, sed duobis modis expressa ...'[3]⟨46, 90⟩. That this construction might lead into some conceptual trouble or, at least, perplexities especially in view of 'Deum ... substantiam constantem infinitis attributis ...' ('de deo', definitio VI et passim), is already part of Spinoza's correspondence, especially the epistolae from and to Tschirnhaus in 1675³⁴

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³³see e.g. Joachim, 'II . The Human Mind, §2 The Human Mind as the Idea of the Body' [6], p. 125 ; W.Cramer, 'II . Mens humanum und corpus humanum'[10], pp. 85 ff., "Jedem endlichen Modus in einem Attribute korrespondiert ein endlicher Modus in einem anderen Attribute", ibid. p90

³⁴see e.g. Joachim [6] pp. 70 note 2, 72 note 2 and p.136 - for a more recent account of the debate see Errol A. Harris 'Infinity of Attributes and Idea Ideae'[12]

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