truthmakers for 1st order sentences - a proposal

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Abstract

The purpose of this paper is to communicate - as a proposal - a general method of assigning a 'truthmaker' to any 1st order sentence in each of its models. The respective construct is derived from the standard model theoretic (recursive) satisfaction definition for 1st order languages and is a conservative extension thereof.

The heuristics of the proposal (which has been somewhat idiosyncratic from the current point of view) and some more technical detail of the construction may be found in my article on part I of Spinoza's 'ethica, ordine geometrico demonstrata'[1], which is the context within which I elaborated the assignment. But this context need not be repeated here, the presentation of the truthmaker assignment will be comprehensible to anybody with solid basic knowledge in 1st order model theory, anything used is standard and no advanced techniques are required.

Contents

[1]'a formal analogy to elements of "de deo" '[7]

1 introductory remarks

It has become popular, to frame discussions on truth (at least in the context of a correspondence theory of truth) in terms of 'truthbearer' (e.g. 'sentence', 'proposition', 'judgement', 'belief', ... or the like), 'truthmaker' ('facts', 'events', ... or other) and the relation between truthmaker and truthbearer, called sometimes the 'truthmaking relation'.[2]

From the abundant literature in the field I mention, for sake of shortness, in this context only these two: first, a Tarski-related discussion by Stephen Yablo in chapter 4 'A Semantic Conception of Truthmaking' of his book 'aboutness' [13] , and secondly Kit Fine's 'Truthmaker Semantics' [6], which contains both, discussion and a general survey of research on 'semantic truthmakers'.

When the discussion turns on truthmakers for quantified sentences, Kit Fine (op.cit., chapter 1 §7 Quantifiers) reflects unwelcome restrictions of considered truthmaker assignments[3], and Stephen Yablo, looking for a Tarski style general solution, raises an objection to Donald Davidsons 'True to the Facts' plea for Tarski semantics as the better correspondence theory of truth:

"4.2 RECURSIVE TRUTHMAKERS Tarski gave us the semantic conception of truth. Might there be room in his system also for truthmakers? Davidson considers this question in his 1969 article "True to the Facts" (Davidson [1969]). He believes there is not only room for truthmakers in Tarski, they are in some sense already there. He defines truth, recall, in terms of satisfaction. Sentences are true because of what they are true of : certain sequences of objects. Sequences are what facts become in Tarski's system. Satisfaction is all that remains of correspondence. Davidson's idea here is puzzling, for truths are satisfied by all sequences. ..."

and concerning Davidson's supposed way out

" ... The problem is not entirely solved, however, for distinct truths may agree too in their derivational history. Take for instance two universal generalizations, $\forall \mathbf{x}\ \mathbf{Fx}$ and $\forall \mathbf{x}\ \mathbf{Gx}$, understood both to be true, and where the predicates are atomic. **Fx** and **Gx** are both satisfied by all sequences, and there is no more to the story than that. This is the wrong result; true generalizations are not all true for the same reason. ..."[13] p.44 f.

The passage of Davidson's article, which evocates Yablo's "... is not entirely solved ..." criticism, reads in full

"Since different assignments of entities to variables satisfy different open sentences and since closed sentences are constructed from open, truth is reached, in the semantic approach, by different routes for different sentences. All true sentences end up in the same place, but there are different stories about how they got there; a semantic theory of truth tells the story for a particular sentence by running through the steps of the recursive account of satisfaction appropriate to the sentence."([5], p.759)

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[2]a concise survey of this terminology is contained e.g. in the article 'correspondence theory' of the Stanford Encyclopedia of Philosophy, §2. Truthbearers, Truthmakers, Truth[4]

[3]e.g. "One problem with these clauses is that they presuppose a fixed domain of individuals.For suppose that the actual individuals are $a_1, a_2, ...$ and that a is a merely possible individual (distinct from each of $a_1, a_2, ...$). Then in a possible world in which a exists, the truth of the instances $\phi(a_1), \phi(a_2), ...$ is not sufficient to guarantee the truth of $\forall x\ \phi x$ and hence the fusion of verifiers for $\phi(a_1), \phi(a_2), ...$ need not be a verifier for $\forall x\ \phi x$ (this is a familiar problem, going back to the early days of logical atomism)."[6], p. 12

In what follows, it will appear, that the fact that " ... truths are satisfied by all sequences ... " is not a problem at all in assigning a truthmaker to 1st order sentences via the Tarski-type recursive satisfaction definition, though Yablo is of course right in this insistence, that *in general* the truthmakers for two different true sentences should differ[4]. But Davidson's statement cited above, as far as it goes, seems to me perfectly alright, and Yablo's general request and Davidsons statement will be seen to be compatible. Details will become obvious, as we proceed.

2 truthmaker assignment

2.1 preliminaries

In this communication discussion is restricted to 1st order sentences, i.e., closed formulae in 1st order language, as truth bearers, and the assignment of truthmakers to them. The nature of the respective truthmaking relation will be clear from this assignment. The proposed method of assignment uses and is restricted to the set theoretic semantics of 1st order languages (theory of models) in the tradition originating with Alfred Tarski(1935)[11],(1954)[12].

By this methodology to any 1st order sentence (closed formula) of a 1st order language **L** [5], in any relational structure $\mathcal{RS}$ for **L**, a truth value ('true' or 'false') is assigned.This assignment of a truth value is supplied by the corresponding recursive definition of satisfaction for any formula **A** of **L** in any relational structure $\mathcal{RS}$ for **L**.[6]

2.2 truthmaker assignment step by step

The assignment of a truthmaker for each sentence (closed formula) **A** of a 1st order language **L** in a relational structure $\mathcal{RS}$ for **L**, in which **A** is true, runs thus:

(I) we use the fact from 1st order model theory, that for each formula (including closed formulae) **A** of **L** there exists a logically equivalent prenex normal form **pnf** (**A**) = **QB** , where **Q** is some finite sequence of universal and existential quantifiers , and **B** is a quantifier free formula; **B** is called the matrix of prenex normal form **QB** .

[4]whereas in the special case mentioned [$\forall$**x Fx** and $\forall$**x Gx**] , in relational structures, in which both sentences are true, **Fx** and **Gx** need have the same extension and thus presumably $\forall$**x Fx** and $\forall$**x Gx** will have the same truthmaker in the framework of any extensional semantics

[5]1st order languages **L**, as we presuppose here, may or may not, in addition to n-ary predicate letters, contain n-ary function symbols and/or identity.

[6]For technical details especially concerning the recursive satisfation definition I refer in the main to Kreisel-Krivine(1967)[9] chapter 2 'Predicate Calculus', for the use of the term 'relational structure' instead of the Kreisel-Krivine term 'realization' see e.g. Bell & Slomson(1974)[1]. The reason for my preference of Kreisel-Krivine is, that in this text the recursive definition of satisfaction is remarkably concise formulated; but in principle any other serious introductory textbook of model theory will do as well as a reference.

(II) the set of all formulae of **L** valid in $\mathcal{RS}$, including the set of all closed formulae(sentences) of **L** true in $\mathcal{RS}$, is called the theory of $\mathcal{RS}$, in symbols $\mathcal{TH}(\mathcal{RS})$ As we proceed, we define 'truthmaker' for all sentences $\mathbf{A} \in \mathcal{TH}(\mathcal{RS})$

(III) The recursive satisfaction definition assigns to any formula **A** of **L** in a relational structure $\mathcal{RS}$ for **L** a value $\overline{\mathbf{A}}$, viz. a set of variable assignments (a variable assignment being a map, assigning to each individual variable of **L** an element of the universe of discourse **U** of the relational structure $\mathcal{RS}$). And thus is assigned the value $\overline{\mathbf{B}}$ to the matrix **B** of $\mathbf{pnf}(\mathbf{A}) = \mathbf{QB}$.

Now we restrict (without loss of generality) the value $\overline{\mathbf{B}}$ of **B** in $\mathcal{RS}$, by restricting any map in $\overline{\mathbf{B}}$ to the individual variables occurring (free) in **B**, and get the (finite or infinite) set $/\overline{\mathbf{B}}/$ as a truthmaker for **QB**. We may view $/\overline{\mathbf{B}}/$ as (finite or infinite) set of finite sequences of objects from **U**, the domain ('universe of discourse') of $\mathcal{RS}$ (, each object in each sequence 'indexed' by the variable it is assigned to). In what follows, it is of utmost importance, to keep in mind, that this truthmaker assignment is relative to the relational structure $\mathcal{RS}$.

As, due to logical equivalence transformations, the prenex normal form of a given formula need not be unique, and as not every sentence $\mathbf{A} \in \mathcal{TH}(\mathcal{RS})$ is in prenex normal form, the truthmaker definition extends the truthmaker assignment for sentences $\mathbf{A} \in \mathcal{TH}(\mathcal{RS})$ in prenex normal form to all sentences $\mathbf{A} \in \mathcal{TH}(\mathcal{RS})$ by

(IV) If $/\overline{\mathbf{B}}/$ is a truthmaker for 1st order sentence **A** in the relational structure $\mathcal{RS}$, $/\overline{\mathbf{B}}/$ is also a truthmaker in $\mathcal{RS}$ for any 1st order sentence $\mathbf{A}'$ with $\Vdash \mathbf{A} \leftrightarrow \mathbf{A}'$

The **Definition** is summarized by:

(V) A truthmaker $/\overline{\mathbf{B}}/$ for 1st order sentence **A** in the relational structure $\mathcal{RS}$ is either constructed according to (I)-(III) or inherited via logical equivalence by (IV)

Corollary

In each relational structure $\mathcal{RS}$ the set of all truthmakers of a sentence (closed formula) **A** true in $\mathcal{RS}$ is the smallest set, that contains the truthmaker of some prenex normal form $\mathbf{pnf}(\mathbf{A}) = \mathbf{QB}$ and in addition the truthmakers of all sentences $\mathbf{A}' \in \mathcal{TH}(\mathcal{RS})$ with $\Vdash \mathbf{A} \leftrightarrow \mathbf{A}'$.

'having the same set of truthmakers with respect to relational structure $\mathcal{RS}$' is of course an equivalence relation, and thus splits the set of all sentences $\mathbf{A} \in \mathcal{TH}(\mathcal{RS})$ into equivalence classes, but the respective equivalence relation is n o t simply induced by logical equivalence. Again the dependence of the truthmakers on $\mathcal{RS}$ plays a crucial role: Simple example is Yablos $\bigwedge_{\mathbf{x}} \mathbf{Fx}$. In relational structures, in which $\bigwedge_{\mathbf{x}} \mathbf{Fx}$ is true, $\bigvee_{\mathbf{x}} \mathbf{Fx}$ is also true and has the same truthmaker(s).

3 truthmaking relation

The dependency of such truthmakers on the respective relational structure was already tacitly used in my note on Yablo's supposed counterexample to Davidsons statement. Really one get's into a logical mine field, if the dependence of the truthmaker assignment on the respective relational structure is ignored or violated; let's have a look on the following 'trap':

3.1 on whether a truthmaker for a sentence A can be said in some sense to logically imply A

Let $\mathbf{A} = \mathbf{QB} \wedge \mathbf{A} \in \mathcal{TH}(\mathcal{RS})$ with truthmaker $/\overline{\mathbf{B}}/$:

We adopt proper names for all the objects from the universe of discourse contained in $/\overline{\mathbf{B}}/$, and from the matrix formula **B** by substituting proper names corresponding to truthmaker sequences $\{\phi_{\mathbf{i}}\}$ for the variables in **B** we get a finite or infinite set $\{\mathbf{B_{sub}}\phi_{\mathbf{i}}\}$ of quantifierfree sentences true in the relational structure. Now the trap is the conjecture, that this set of 'matrix sentences' does logically imply our 1st order sentence **A**, in symbols

$$(\_?\_) \quad \{\mathbf{B_{sub}}\phi_{\mathbf{i}}\} \Vdash \mathbf{A}$$

This were of course a complete misunderstanding of the situation. The truth of the sentences in $\{\mathbf{B_{sub}}\phi_{\mathbf{i}}\}$ relates to the current relational structure $\mathcal{RS}$, and the truth of the sentences in $\{\mathbf{B_{sub}}\phi_{\mathbf{i}}\}$ necessitates the truth of **A** in the general case only with respect to $\mathcal{RS}$. All this is far from logical implication, as logical implication is the situation that *in any* relational structure, in which all the premises are true, the conclusion is also true.

And there is another argument to the effect, that the conjecture $(\_?\_)$ is not valid, which, while it does not cover the general case, is still instructive in another way. Consider the following counterexample:

Suppose, our relational structure were the set $\mathbb{N}$ of natural numbers, with the successor-function $\mathbf{s} : \mathbb{N} \mapsto \mathbb{N}$, and **A** be the sentence stating that 0 is not successor of any number and that the successor function is injective (1-1), this sentence already in prenex normal form. The set of all true 'matrix sentences' $\{\mathbf{B_{sub}}\phi_{\mathbf{i}}\}$ in this intended model is of course infinite.

Now, by indirect proof: suppose

$$\{[\neg \mathbf{s}(\mathbf{x}) = \mathbf{0} \wedge [\mathbf{s}(\mathbf{x}) = \mathbf{s}(\mathbf{y}) \to \mathbf{x} = \mathbf{y}]]_{\mathbf{sub}}\phi_{\mathbf{i}}\} \Vdash \bigwedge_{\mathbf{x}} \bigwedge_{\mathbf{y}} [\neg \mathbf{s}(\mathbf{x}) = \mathbf{0} \wedge [\mathbf{s}(\mathbf{x}) = \mathbf{s}(\mathbf{y}) \to \mathbf{x} = \mathbf{y}]]$$

then, by compactness,

$\bigwedge_{\mathbf{x}} \bigwedge_{\mathbf{y}} [\neg \mathbf{s}(\mathbf{x}) = \mathbf{0} \wedge [\mathbf{s}(\mathbf{x}) = \mathbf{s}(\mathbf{y}) \to \mathbf{x} = \mathbf{y}]]$
need follow already from a finite subset of $\{[\neg \mathbf{s}(\mathbf{x}) = \mathbf{0} \wedge [\mathbf{s}(\mathbf{x}) = \mathbf{s}(\mathbf{y}) \to \mathbf{x} = \mathbf{y}]]_{\mathbf{sub}}\phi_{\mathbf{i}}\}$,

which is obviously false, because any such finite subset has a model in a finite domain, viz. here in an initial segment of $\mathbb{N}$, whereas **A** clearly has not.[7]

3.2 truthmaking relation in detail

We conclude from this concerning the

(VI) truthmaking relation:
the truth of **A** is necessitated by its truthmaker $/\overline{\mathbf{B}}/$, but this necessitating relation is given by the recursive satisfation definition working on the current relational structure, and not by logical implication from a set of matrix-instances.[8]

At this stage one might perhaps be inclined to say something like
"ok, the truthmaker is the relational structure, the truthmaking relation between the sentences of the 1st order language and the relational structure given by the recursive satisfaction definition, so what? I knew that already ... "

The reply is of course, that the assignment (with respect to the relational structure) of a uniquely determined set of truthmakers to any set of logically equivalent sentences true in this relational structure, is far more specific.

As well much more specific is the truthmaking relation:

For sentences **A** already in prenex normal form $\mathbf{A} = \mathbf{QB}$ its simply the working of the satisfaction definition on the quantifier prefix **Q**, taking as a starting point the value $\overline{\mathbf{B}}$ of the matrix **B** [9], [10].

In case of sentences **A** not in prenex normal form, the truthmaking relation is given by (IV), i.e. inherited from the above case.

In section 4 'first conclusions', I'll try to show, that this is not only a formal exercise, but gives philosophically relevant structural information on "the concept of truth in the formalized languages".

3.3 truthmaking selection apology

Another objection might be raised with respect to my selecting the matrix of prenex normal form as the anchor point of the truthmaker definition. Ok, in some sense, for every term frequently used but without sharp contours of usage,

[7] and generally, in any case, where the considered sentence **A** is satisfiable only in an infinite domain, this reasoning by compactness applies

[8] both counter arguments stated above do not only serve as refutations of (_?_), but relate - in some way and to some extent - as well to Kit Fine's dealing with quantification within the 'state space' semantics sketched in [6], Part I, §§ 3-4 and 7.

[9] using e.g. projection along $\mathbf{x_i}$ for any existential quantifier $\bigvee_{\mathbf{x_i}}$, and complement for any negation sign - for details see e.g. Kreisel-Krivine(1967) [9],chapter 2, p.17

[10] Thus, the set of matrix sentences $\{\mathbf{B_{sub}}\phi_\mathbf{i}\}$ may be generated from the truthmaker $\overline{\mathbf{B}}$ and the matrix formula **B**, but the working input in 'truthmaking' is not this set of matrix sentences, but the set of variable assignments satifying the matrix in the current relational structure.

the selection of a definiens has a shadow of arbitraryness. In this case, in order to define truthmakers for 1st order sentences, this arbitrariness to me seems little if not marginal. The selection of the matrix of the prenex normal form has two indisputable advantages. First, that it serves our ubiquitous intuitions in the use of matrix sentences in order to cope with quantification in defining truthmakers, and let's us understand, why and in what respect, these intuitions fail. Secondly, by the metatheorem on the existence of a logically equivalent prenex normal form for any formula $\mathbf{A} \in \mathbf{L}$, we have the means to spread the truthmakers derived from sentences in prenex normal form within the same relational structure $\mathcal{RS}$ to any sentence $\mathcal{RS} \Vdash \mathbf{A}$. Thus, if someone still insisted, that this selection is arbitrary, I would concede, that it is, but, in my view, much less so than anything else, I met in the field.

4 first conclusions

4.1 concerning the correspondence theory of truth: Occam's razor cuts Plato's beard ?

In an obvious sense, the proposed truthmaker construction is in accordance with Tarski type recursive satisfaction definition, first, in that it renounces the introduction of additional abstract entities duplicating the linguistic entities(e.g. propositions assigned to sentences, properties or the like assigned to unary predicate letters, ...), and secondly, in that it does not support 'falsemakers' (unwilling to serve Plato's beard[11]).

Concerning the first point, not so much can be said in favour of 'intensions' and the like in the current context. The model theoretic recursive satisfaction definition proceeds with reference to extensions (set theoretic entities, maybe including 'atoms'[12]) only and does so successfully. The whole spectrum of discussion concerning say 'sense' as opposed to 'reference', intension, propositions, properties, tropes, ... etc. is seemingly bypassed here without any loss with respect to truth value distribution or to the proposed truthmaker definition.

Concerning the second point: An adequacy condition on any theory of truth is, that besides providing an account of 'truth', to provide also an account of 'falsehood'. And the discussion concerning Plato's beard concerns the question of whether the provided account of falsehood will be in some sense symmetric or asymmetric to the given account of truth.[13]

Seemingly innocent but nevertheless a trap concerning 'the riddle of nonbeing' is:

[11] for Quine's 'On what there is' epitheton "Plato's beard" ([10], pp.1-2) see e.g. Sophie-Grace Chappell, Plato on Knowledge in the Theaetetus, 7.1 The Puzzle of Misidentification: 187e5–188c8 [3]

[12] 'atoms' in set theory are members of sets, which are not itself sets; see e.g. Ketland[8]

[13] in the beginnings of the 20th century, Bertrand Russell and G.E. Moore switched a number of times between different symmetric and asymmetric variants

... the fact, that exists, if p is true = the fact, that not exists, if p is false ...

[the problem here, of course, is the right hand side of the equation].

Whatever in this respect, it is clear, that the truthmaker proposal, as here presented, does not supply a 'falsemaker' for false sentences in a symmetric manner. This might perhaps be misunderstood. Of course, a sentence **A** false in the relational structure $\mathcal{RS}$, is as well assigned the value of the matrix of any of its prenex normal forms, and such a value may be considered as a falsemaker. The asymmetry then lies in this: consider the respective matrix sentences, generated from this kind of 'falsemaker'. What they are about, is, how facts are in this relational structure $\mathcal{RS}$, and these matrix sentences do not refer to something would-be, if **A** were true.

This asymmetry is of course grounded in the fact, that the (recursive) generation process for the value of a formula **A** of **L** in $\mathcal{RS}$ starts from the values of those atomic formulae, which are part of **A**, each value of each atomic formula corresponding to a set of 'positive' sentences true in $\mathcal{RS}$. The recursive satisfaction definition assigns to 1st order sentences the truth value 'true' or 'false', according to the facts in the relational structure, but while it does not supply a symmetric falsemaker for a 1st order sentence **A** false in this relational structure, it does of course supply a truthmaker for $\neg$**A** in any relational structure, in which $\neg$**A** is true.

4.2 concerning Quine's "standard" for judging ontological commitment of a theory

Lastly, let's consider Willard van Orman Quine's often cited "To be ..., is ... to be ...the value of a variable" [14], more explicitly "...: a theory is committed to those and only those entities, to which the bound variables of the theory must be capable of referring in order that the affirmations made in the theory be true" [15]

A modification of Quine's standard might be suggested by the above truthmaker definition, tentatively: the ontology, which a say finitely axiomatized 1st order theory **T** is committed to, with respect to its intended model $\mathcal{RS}_{\mathbf{T_I}}$ is given not by the (complete) domain of the intended model but by the truthmakers of its theorems (special sets of sequences of objects over that domain). Thus the suggestion is, to take as ontologically relevant for e.g. a theory in physics neither say real numbers in isolation nor fictive physical objects (e.g. 'masspoints'), but only complex arrangements of them as the "objects, the theory 'deals with', or is about' ". But this is a great topic of its own, it is reflected in a suitable frame in Jeffrey Ketland's paper 'Foundations of Applied Mathematics I' under the topic of 'mixed mathematical objects' [16].

[14] 'On what there is'[10], cf. pp. 13-18

[15] ibid., while Quine immediately relativizes the importance of this "semantical formula" with respect to conventionalist or phenomenalist preferences of theory selection, he keeps the "formula" still running as valid standard.

[16] [8], sections 1.2-1.3, pp.3-7

5 'true to the facts' is true to the facts

In my introductory remarks I stated, that the passage of Davidson's 'True to the facts', quoted by Stephen Yablo, up to my understanding, seems perfectly alright. This passage is taken from a context, in which Davidson compares Tarki's account of truth of sentences (via the recursive satisfaction definition) with versions of the 'correspondence theory of truth', which rely on the relation of true sentences to the facts, they express.

The summary of this comparison in Davidson's article follows shortly after the already cited passage. And now, having stated my truthmaker proposal, and argued to some extent for it, I would like to conclude this paper by quoting Davidson again with this summary:

"Seen in retrospect, the failure of correspondence theories of truth based on the notion of fact traces back to a common source: the desire to include in the entity to which a true sentence corresponds not only the objects the sentence is "about" (another idea full of trouble) but also whatever it is the sentence says about them. One well-explored consequence is that it becomes difficult to describe the fact that verifies a sentence except by using that sentence itself. The other consequence is that the relation of correspondence (or "picturing") seems to have direct application to only the simplest sentences ('Dolores loves Dagmar'). This prompts fact-theorists to try to explain the truth of all sentences in terms of the truth of the simplest and hence in particular to interpret quantification as mere shorthand for conjunctions or alternations (perhaps infinite in length) of the simplest sentences. The irony is that, insofar as we can see quantification in this light, there is no real need for anything like correspondence. It is only when we are forced to take generality as an essential addition to the conceptual resources of predication and the compounding of sentences, and not reducible to them, that we appreciate the uses of a sophisticated correspondence theory"[5], p.579

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