The buried quantifier: an account of vagueness and the sorites

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Contrary to the great bulk of philosophical work on vagueness, the core of vagueness is not to be found in vague monadic predicates such as 'bald', 'tall', or 'old'. The true source of vagueness – at least vagueness of the type that typically appears in the sorites – lies beneath these, in a mechanism using a buried quantifier operative over the comparatives 'balder', 'taller' and 'older'.

Or so I propose. Here the quantifier account is presented in its simplest form, with the limited claim that it offers a paradigmatic treatment for paradigmatic vague predicates in the sorites. Questions remain as to whether the account or something like it can be extended to all sorites-vulnerable predicates, and qualifications and concessions in this regard are offered in §9. What the approach promises, however, even in this limited form, is deeper understanding of vagueness through a deeper understanding of non-comparative adjectives derived from comparatives, a central explanation for a range of otherwise puzzling and disparate phenomena, and a new resolution for the sorites.

1. The comparative base

The sorites is standardly run on monadic predicates 'bald', 'tall', and 'old' for which there are clear comparatives or graded adjectives: 'balder', 'taller', and 'older'. Each of these comparatives suggests a calibration scale: 'taller' in terms of inches of height, 'older' in terms of years of age, and 'balder' (with some obvious forcing) in terms of numbers of hairs. In these cases, it appears, any two candidates will be equally bald or one will be balder than the other: $\forall x \forall y(xRy \vee yRx \vee xR{=}y)$. But the sorites can also be run on multi-dimensional comparatives such as 'nice' or 'intelligent', where there is both no clear calibration scale and where it may remain indeterminate whether two candidates are equally nice or one is nicer than the other. Even in these cases, however, we can expect the following to hold:

irreflexivity: $\sim xRx$ No x is balder or nicer than itself, for example.
asymmetry: $xRy \rightarrow \sim yRx$ If x is balder than y, y is not balder than x.
transitivity: $xRy \;\&\; yRz \rightarrow xRz$ If x is nicer than y, and y is nicer than z, then x is nicer than z.

ANALYSIS 65.2, April 2005, pp. 000–000. © Patrick Grim

One point of interest about the partial orderings established by comparatives of this sort is that they satisfy an elementary real algebra in the sense of Tarski (1930, published 1948). A real algebra for comparatives, unlike the number theory required for fuzzy logic, for example, can be shown to be consistent, negation-complete, and decidable (Hunter 1971).

2. *The buried quantifier*

Philosophers have been obsessed with just two quantifiers – 'all' and 'some'. Linguists recognize far more, including 'a few', 'several', 'many,' 'lots of' and 'almost all'. These are quantifiers one and all, though they are of course vague quantifiers. Their logic is not nearly so tidy as 'all' and 'some', though some of that contrast is illusion: philosophers have regimented even their quantifiers away from common use. Despite logical regimentation and despite the injection of that regimentation into standardized tests like the GRE and LSAT, for example, 'some' does not mean 'at least one'. In the linguistic literature, 'some Fs' is recognized as entailing 'a few Fs' (Channell 1994: 97).

None of the vague quantifiers 'a few', 'several', 'many', or 'almost all' are reducible to expressions in terms of $\forall x$ or $\exists x$, and none are specifiable in terms of a precise number or percentage. These are inherently vague quantifiers.

It is another vague quantifier, I want to propose, that lies buried beneath vague monadic predicates. I will use 'Zx' to represent a quantifier that might be expressed as:

For the great bulk of *x*'s …
For the overwhelming majority of *x*'s …
It is quite generally true for *x*'s that …
For a large percentage of *x*'s …
With relatively few exceptions among *x*'s …

It might be better for my purposes were I able to introduce Zx ostensively, as it might be introduced in teaching a language. With piles of beans, grapefruits, or gerbils I am confident I would be able to introduce you to the quantifier Zx in a short period of time.

3. *The core account*

For at least paradigm cases, on this proposal, vague monadic predicates can be defined derivatively. Someone is 'bald' just in case they are balder than the great bulk of the comparison class. They are 'tall' if they are taller than the overwhelming majority of the population. Someone is 'old' if it is quite generally true that they are older than the others.

Using 'Bx' for 'x is bald' and 'xBy' for 'x is balder than y', we can outline our monadic predicate as follows:

$$Bx \equiv Zy(xBy)$$

The quantifier Zx is indefinite in two very intuitive ways, as will be expressions built from it. There is, first of all, no answer to the question 'How many (or what proportion) constitute 'the great bulk of'?' In this regard Zx is like the quantifiers 'many', 'lots of', 'almost all', 'a few', and 'a significant number of', none of which correlate with any definite number or proportion of the population. Like these, Zx is *essentially* imprecise.

Zx is also indefinite as to the comparison class. Balder than the great bulk of whom? Taller than the overwhelming majority of what group? These issues are specified by context, and can only be so specified. Because 'bald' is defined in terms of Zx over a comparison class, it follows immediately that 'x is bald' will be true only with reference to a particular comparison class. This is not a weakness of the theory, but a strength; this is precisely how we use 'bald', 'tall', and 'nice'. The hirsute man at the bald men's convention may be the bald man at the hirsute men's convention. The old kid in kindergarten may be all of 6½. The young guy in the nursing home is 59.

The comparison classes at issue need not be set by companion nouns: when speaking of a well-educated trapeze artist it may be clear in context that the comparison class extends beyond the class of trapeze artists. Nor need the comparison class be a class of actually existing individuals: a comparison class may remain contextually salient despite the demise of a large number of its members, for example. Were we to kill off all but the tall men, the class of recently existing men might remain as the comparison class. Context can also indicate comparison classes of normal, ideal, or merely possible individuals. Much of the resistance to accounts grounded in comparatives stems from Kamp's classic (1975), but much of that opposition is founded on a short-sighted limitation to comparison classes of actual individuals or set by companion nouns.

4. *The quantifier Zx*

The key to understanding vagueness, on this account, lies in understanding the buried quantifier. The familiarity of $\forall x$ and $\exists x$ are often obstacles to understanding Zx, however, and it is clear that the logic appropriate to $\forall x$ and $\exists x$ will fail for Zx.

Consider a comparison class of men numbering in the millions, the relation 'x is balder than y' mapped onto that class, and a notion of 'x is bald' defined as 'balder than the overwhelming bulk of' those men. Consider a 'forced march', in which we line the men up in terms of relative baldness.

As we step down the line of candidates from the most bald to the least, will there be a particular point at which we pass from an individual who is 'balder than the overwhelming bulk of' the members of the group to a next individual who is not 'balder than the overwhelming bulk'? Will there be a transition step such that

$\exists x \exists z(Zy(xBy)$ & z is next in line behind x & $\sim Zy(zBy))$?

Clearly not. The explanation for the lack of a transition step lies precisely in the nature of 'the overwhelming bulk of' quantifier. If our men are lined up in terms of numbers of hairs, we will correspondingly deny a transition step phrased in terms of our derivative 'is bald':

$\sim\exists x \exists z(Bx$ & z has one more hair than x & $\sim Bz)$.

There is no step at which we go from 'bald' to 'not bald', precisely because there is no step at which we go from 'balder than the great bulk of' to 'not balder than the great bulk of'.

If we maintain that there is no transition step, are we not forced to the classically equivalent

$\forall x \forall z(Bx$ & z has one more hair than $x \rightarrow Bz)$?

No. This too we can deny. Quantifier Negation holds only on the assumption of excluded middle, which here comes down to the assumption that $\forall x(Bx \vee \sim Bx)$. But if this abbreviates $\forall x(Zy(xBy) \vee \sim Zy(xBy))$, it is clear from the nature of our quantifier Zy that excluded middle simply will not hold. 'Balder than the great bulk of' gradually loses applicability as we walk down the line, and thus there will be cases in which we won't want to maintain either that x is balder than the great bulk of the comparison class or that x is not balder than the great bulk of the comparison class. Excluded middle fails for $Zy(xBy)$ and thus for Bx, and Quantifier Negation fails with it.[1]

The most natural logic for $Zy(xBy)$ will be more than bivalent: There will be cases where Bx is true, where Bx is false, but also cases where Bx is neither true nor false. Were our logic merely trivalent, however, we would expect a law of excluded third to hold: $\forall x(Bx \vee \sim Bx \vee \sim(Bx \vee \sim Bx))$. But a trivalent logic also seems false to the gradual inapplicability of $Zy(xBy)$ across a spectrum of cases. There are clear cases where $Zy(xBy)$, clear cases where $\sim Zy(xBy)$, and clear cases where neither holds. But there also appear to be cases where none of *these* three options holds. To the extent that this reasoning can be extended, the

[1] Although not detailed here, Zx will also block arguments in Read and Wright (1985).

natural logic for $Zy(xBy)$ will have *no* number of values n: it will not only be non-bivalent but non-n-valent. If so, we can expect a logic for Bx to lack not only a principle of excluded middle but of excluded n-dle: There will be no set $\{s_1, s_2, \ldots, s_n\}$ of relevant expressions written in terms of $Zy(xBy)$ such that $\forall x(s_1 \vee s_2 \vee \ldots \vee s_n)$. All of this, I would argue, is as it should be: these inherent characteristics of Zy are precisely those that appear in the literature under the category of 'higher-order vagueness'.

5. *Alternative accounts for Zx phenomena*

What *does* happen with 'balder than the great bulk of' as we walk down the line of increasingly hairy men? The quantification $Zy(xBy)$ progressively and gradually loses its applicability. But what does that loss of applicability amount to? Here it is interesting that there are alternative possible accounts. It is also interesting that they parallel a range of competing contemporary approaches to vagueness.

One might claim that $Zy(xBy)$ loses its applicability epistemically: we become less certain that we know whether a candidate is balder than the great bulk of the others or not. Developed in certain ways, such a claim would parallel contemporary epistemic approaches to vagueness.

One might claim that $Zy(xBy)$ loses its applicability metaphysically or semantically: It is simply less true that $Zy(xBy)$ for an x further down the line. That claim, developed in terms of degrees of truth, generates a second contemporary approach.

Although I don't believe the current account is tied to any of these alternatives, I tend to favour the answer that $Zy(xBy)$ loses its applicability as a matter of slipping appropriateness of use. In one broad sense of the term, $Zy(xBy)$ loses its applicability as a matter of pragmatics (Levinson 1983). Conventions for the use of 'balder than the great bulk of' or $Zy(xBy)$ support fully its application in some cases. As we move across a continuum of possible cases, however, support for appropriateness of application tapers off.

6. *The sorites*

The crucial test for any theory of vagueness is the sorites. In imagination we line up men with increasing numbers of hairs, each perhaps with one more hair than his predecessor.

A standard formulation for the sorites uses the induction principle

$$\forall x \forall z(Bx \ \&\ z \text{ has one more hair than } x \rightarrow Bz).$$

If baldness is definable in terms of a vague quantifier over the comparison adjective 'balder', this amounts to:

$$\forall x \forall z(Zy(xBy) \ \&\ z \text{ has one more hair than } x \rightarrow Zy(zBy)).$$

Such a principle will clearly be false. As we walk down our line of men, it will eventually be clear that we no longer have a 'great bulk of' the population for a candidate to be balder than. The induction principle fails.

If we deny the induction principle, however, are we not forced to admit that there is a crucial transition step:

$\exists x \exists z(Bx$ & z has one more hair than x & $\sim Bz)$?

No, for this amounts to

$\exists x \exists z(\mathrm{Z}y(xBy)$ & z has one more hair than x & $\sim \mathrm{Z}y(xBy))$,

which we also deny. The character of Zx makes it clear why predicates incorporating such a quantifier will not obey excluded middle (or even perhaps an excluded *n*-dle). Only with a principle of excluded middle would an inference from the negation of the induction principle to an existential claim of a crucial transition step hold. Our understanding of 'bald' as incorporating the buried Zx quantifier allows us to see how both the existential and universal quantifications will fail.

7. *The plausibility of the induction principle*

There is one further desideratum that any account of vagueness targeted for the sorites should satisfy. The notion of a crucial transition step – the notion that

$\exists x \exists z(Bx$ & z has one more hair than x & $\sim Bz)$ –

is inherently implausible. The fact that the Zx account denies such a claim is therefore a point in its favour. The counter-intuitive claim that there *is* a crucial transition step is one that epistemic accounts, for example, struggle to make palatable.

The Zx account also denies the induction principle, however:

$\forall x \forall z(Bx$ & z has one more hair than $x \rightarrow Bz)$.

Unlike the transition step, this generalization is taken to be eminently plausible. If it is false, why are we so tempted to think it is true (Graff 2000)?

One of the appealing claims of a fuzzy logic is that the induction principle, though not fully true, is very close to being true. The Zx approach suggests a similar tack. Although the induction principle above is false, and must be so, it has a close relative that may well be true:

$\mathrm{Z}x \forall z(Bx$ & z has one more hair than $x \rightarrow Bz)$.

All that has changed is the first quantifier. Unlike the induction principle, this is not a universal quantification over x at all. It is instead a vague quantification: For the great bulk of cases, the overwhelming majority of cases, if x is bald and z has one more hair then z is bald as well. Because it holds 'for the great bulk of cases', it may be quite generally usable as a rule of thumb for individual applications. Because it falls short of a full universal quantification, however, it doesn't saddle us with the sorites.

8. The virtues of Zx

It is obvious that 'bald', 'old' and 'tall' are conceptually related to the comparatives 'balder', 'older', and 'taller. The core of the *Zx* account is an analysis of the former in terms of the latter using a buried vague quantifier. A first virtue is that such an account seems to run in the right direction, deriving 'bald' from 'balder', rather than the other way around. Indeed, were 'bald' the monadic predicate that it is often taken to be, obedient to the law of excluded middle, it is unclear how any account *could* derive 'balder' from 'bald'. As Hans Kamp notes, 'It is quite obvious that if adjectives were ordinary predicates no such transformation could exist. How could we possibly define the relation *x is bigger than y* in terms of nothing more than the extension of the alleged predicate *big*?' (Kamp 1975: 127).

Here I have tried to outline a paradigmatic account for paradigmatically sorites-vulnerable predicates such as 'bald', 'old', and 'taller'. Though rarely remarked, it is a remarkable fact that the paradigmatically sorites-vulnerable predicates *are* those related to comparatives. The *Zx* account offers a clear explanation for that fact.

Zx offers a unified account of vagueness across radical differences in predicate character: the source of vagueness for terms as different as 'old', 'tall', 'bald', and even 'nice' can be traced to a buried quantifier they share in common.

As noted, a buried quantifier 'For the great bulk of …' or 'For the vast majority of …' brings with it quite naturally two familiar aspects of vagueness: the lack of sharp transition steps and relativity to a contextually specified comparison class. Matters of degree and contextual sensitivity have sometimes been treated as separate aspects of vagueness, leaving it a mystery why they should be found together and in precisely sorites-vulnerable terms. The *Zx* account offers a deeper explanation for their union.

The present account has some of the virtues of a fuzzy logic treatment – it allows, for example, a similar explanation for the plausibility of the induction principle. But it also avoids many of the vices of a fuzzy logic. Though comparatives bring with them a smooth continuum of 'balder than' in the *Zx* account, they don't bring artificial numerical values along that continuum. By avoiding precision, *Zx* avoids false precision. It is also often said that fuzzy logic fails to do justice to higher-order vagueness. By bringing in 'higher-order vagueness' at the bottom, as an inherent characteristic of Zx, the account avoids both that charge and the need to try to introduce higher-order vagueness by complex means from above (Smith 2004).

9. *Objections*

It might be objected that the Zx account succeeds not by analysing vagueness but by smuggling it in via the buried quantifier. How does this take us further than simply saying that 'bald' is vague?

Any successful account of vagueness will have to incorporate vagueness in one way or another; at the core of the Supervaluational approach, for example, lies the vagueness of 'acceptable precisifications'. Any hope for a fully precise account of vagueness is doomed. The closest we might get in that direction is something like fuzzy logic, with of course an accompanying charge of false precision. What Zx introduces is the possibility of understanding qualitative vagueness – the vagueness of 'bald' – in terms of an underlying quantitative vagueness – the vagueness of 'the great bulk of …'. It also offers a unified account of different vague predicates in terms of a vague quantifier they share, it ties the vagueness of 'bald', 'tall', and 'old' to their comparatives 'balder', 'taller', and 'older', and it offers an account of why the induction principle must fail but why something close may well be true.

Here I have offered a simple Zx account, with the limited claim that it offers a paradigmatic treatment for paradigmatic vague predicates in the sorites. Serious questions remain whether such an account can be scaled up: whether the Zx account can be extended, beyond the paradigmatic cases of sorites-vulnerable predicates emphasized here, to handle them all. It is heartening that some of the initial problem cases do not appear to pose serious difficulties. As Stanley has emphasized (2003), the sorites can be run on verbs such as 'shout' and nouns such as 'heap' as well as adjectives such as 'bald'. But to shout is to vocalize loudly, and 'louder' or 'more loudly' offer a clear comparative on which Zx can operate. 'Heap' is particularly interesting. There is no comparative 'heaper'. But a heap of sand, for example, is a pile of many grains. Here the buried vague quantifier is 'many', and it's not even buried very deep.

There are also cases that seem to run contrary to a Zx account. In 50 years, perhaps, with improvements in medicine and nutrition, everybody will be tall. Here it seems plausible that the comparison class for 'tall' is not the future class but people as we know them now. There are more difficult cases, however, in which categories or divisions are set up. Kamp notes that most English cars are small, a fact that seems to turn on standard size categories for cars rather on the sizes of the vast majority of cars

(Kamp 1975: 126). Graf (2002) constructs a case in which a teacher divides a class between an A team of taller children and a B team of shorter children, after which the A children are naturally spoken of as the tall kids and the B children as the short kids – despite the fact that the division might have been made at a different point and despite the relative numbers on the two teams. In these and other cases of salient divisions, I think, the Zx account will fail: 'small' or 'tall' will properly be analysed not as 'smaller than the great bulk of …' or 'taller than the vast majority of …' but as something more like:

'among those that are taller than the others'.

In these cases it is salient divisions that mark the separation between 'those that are taller' and the 'others'. To the extent that salient divisions are in place, however, a primary requirement for the sorites will disappear: salient divisions *will* mark particular points at which someone is no longer to be classified as one of the tall kids, for example. Where a Zx account fails because of salient divisions, the smooth transitions required for the sorites will fail as well.

The concession here is that the Zx account may in the end prove insufficiently general as a full account of vague positives in terms of comparatives. 'Balder than the vast majority of …' may be something like a default reading for 'among those balder than the others …,' capable of being overridden in the presence of other contextual markers. Even short of full generality, however, a Zx account may offer a better understanding of paradigmatically vague terms, including the great bulk of those that fall victim to the sorites.

10. Conclusion

At the core of the sorites, on the current proposal, are two related mistakes. The first mistake is treating familiar predicates which are actually vague quantifications over comparative adjectives as if they were something quite different – monadic predicates which obey the excluded middle. The second and related mistake is to think that the induction principle

$\forall x \forall z(Bx \;\&\; z \text{ has one more hair than } x \rightarrow Bz)$

will hold, whereas in fact what holds is only its close but vague relative

$Zx \forall z(Bx \;\&\; z \text{ has one more hair than } x \rightarrow Bz)$.

A better understanding of vagueness in general will come from a richer consideration of relations between families of terms – comparatives, superlatives, hedges and derived monadic predicates. It will also come with a study of quantification that broadens consideration from the few easily regimented cases to the vast range of vague quantifiers.[2]

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[2] Thanks to Jason Stanley, Graham Priest, Roy Sorensen, Samuel C. Wheeler III, JC Beall and Delia Graff for comments on an earlier draft.