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TRUTH, OMNISCIENCE, AND THE KNOWER*

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In 'A Paradox Regained'[1] David Kaplan and Richard Montague offer a purified form of the paradox of the surprise examination that they call the paradox of the Knower. In later work, Montague uses a form of the paradox against syntactical treatments of modality.[2]

The full impact of the Knower, however, has not yet been realized — or so I will argue. For what the Knower offers is a surprisingly powerful argument against the coherence of a broad range of common notions if taken in full generality. Most importantly for my purposes here, it offers an intriguing argument against any notion of all truth or of omniscience.[3]

In the first three sections that follow I want to outline the paradox and sketch what I take to be its general impact, with truth and omniscience foremost in mind. But of course a number of responses are possible to the Knower as to other paradoxes. In Sections IV, V, VI, and VII I consider possible ways out.

The argument at issue is complicated enough that one must be wary of dogmatic and precipitate conclusions; here as with other paradoxes one may quite justifiably wonder whether some new response, or some further variation on an old response, might yet save us from at least initially counterintuitive conclusions.

Far too often, however, it is asked what has gone wrong with paradox rather than what paradox may have to teach us. What the Knower may genuinely have to teach us, I think, is that there really can be no coherent notion of all truth or of omniscience.

I. THE PARADOX OF THE KNOWER

The paradox of the Knower is a somewhat complex but solid argument

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for the surprising conclusion that an apparently innocuous list of statements is in fact inconsistent.

We start by borrowing the syntax of any system adequate for arithmetic, for example system Q of Robinson arithmetic. To the alphabet of this system we add a single symbol 'Δ'. We broaden the grammar so as to include '$\Delta(x)$' as a well-formed formula (where numbers will fill in for 'x'), and specify that '$\Delta(x)$' will form compound wffs in the standard ways.

So far, then, we have simply broadened slightly our acceptably grammatical class of wffs. All standard symbols will appear in their standard interpretations, and 'Δ' can for the present be thought of as some sort of knowledge predicate. More about 'Δ' in a moment.

The language of our list is borrowed from any system adequate for arithmetic. The first set of statements of our list is borrowed from the same source, and consists simply of the axioms of such a system.[4] All of these we take to be true, of course, both because they seem transparently true on the standard interpretation intended and because we take *arithmetic* to be true.

Notice now two key facts.

The first is this. All of the strings of the language we are using, including those in which 'Δ' appears, can be recoverably encoded as numbers. This is guaranteed simply by the fact that the strings of any such language can be so encoded. Let us then assume a particular Gödel numbering of this sort and refer for our purposes to the number which corresponds to string 'A' as '$\overline{A}$'. '$\Delta(\overline{A})$', then, will be the application of 'Δ' to a number '$\overline{A}$' (in place of 'x'), encoding a formula 'A'.

The second key fact is this. We have so far implicitly outlined a *system* Q', consisting of the syntax of Q broadened to include 'Δ' and the axioms of Q borrowed for our list. As in the case of Q — and of any system adequate for arithmetic — the derivability relation for Q' is itself definable in the system. For our purposes we'll symbolize this as '$I(\overline{A}, \overline{B})$'.

We now have notation in hand to complete our list. 'Δ', we've proposed, is to be read as a knowledge predicate of some kind. But we've been anything but specific about that, and will in fact entertain a broad range of interpretations in what follows. For now, let us use '$\Delta(x)$' to mean 'the formula with Gödel number x is known to be true,' and add to our list the claims indicated by the following schemata:

$$\Delta(\overline{A}) \supset A$$

$$\Delta(\overline{\Delta(\overline{A}) \supset A})$$

$$I(\overline{A}, \overline{B}) \supset . \Delta(\overline{A}) \supset \Delta(\overline{B})$$

On the interpretation outlined these will read, in effect,

1. If something is known to be so, it is so.
2. (1) is known to be so.
3. If B is derivable from A in Q', then if A is known to be so, B is known to be so.[5]

We can express the paradox of the Knower as follows. The list we've constructed, composed merely of axioms borrowed from Q and the claims indicated in our three schemata above, is provably inconsistent. If the initial and axiomatic items of our list are true, our auxiliary claims concerning knowledge logically cannot be true.

This will hold, moreover, for whatever interpretation we choose to give for 'Δ'. For no notion of knowledge — or anything else, for that matter — can claims represented by our three schemata be true.

II. THE ARGUMENT

The technical argument for our paradoxical conclusion builds on the Diagonal Lemma, provable for Q':

> *For any formula $B(y)$ of Q', containing just the variable y free, there is a sentence G of Q' such that the following is demonstrable as a theorem*:
>
> $G \equiv B(\overline{G})$.[6]

Let us use '$\Delta(\text{neg}(y))$' in place by $B(y)$, where 'neg' is a recursive function representable in Q' which gives us the Gödel number of the negation of the formula with Gödel number y. Then by the Diagonal Lemma there will in particular be a sentence S of Q' such that

$$\vdash S \equiv \Delta(\text{neg}(\overline{S}))$$

or simply

$$\vdash S \equiv \Delta(\overline{\sim S}).^7$$

Note that for '$\vdash$' or 'provable in Q'' here we might also substitute 'derivable from the initial entries of our list,' since these are the axioms of Q'.

Now let us take that instance of our first schema above in which '$\sim S$' appears in place of 'A'. We will call this instance (i):

(i) $\Delta(\overline{\sim S}) \supset \sim S.$

(i) with '$\vdash S \equiv \Delta(\overline{\sim S})$' gives us

$$S \supset \sim S,$$

so

$$\text{(i)} \vdash S \supset \sim S.$$

But then of course

$$\text{(i)} \vdash \sim S$$

— i.e., '$\sim S$' is derivable from (i), an instance of our first schema.

Derivability of this sort, we've noted, is definable in Q'. Thus the fact that the demonstration holds will itself be demonstrable:

$$\vdash I((\bar{\text{i}}), (\overline{\sim S})).$$

Consider now our third schema above with '$(\bar{\text{i}})$' and '$\overline{\sim S}$' in place of '$\bar{A}$' and '$\bar{B}$'. This gives us

(iii) $\Delta((\bar{\text{i}})) \supset \Delta(\overline{\sim S}).$

Our second schema, of course, gives us '$\Delta((\bar{\text{i}}))$'. So:

(ii) $\Delta((\bar{\text{i}})).$

By *Modus Ponens*,

$$\text{(ii), (iii)} \vdash \Delta(\overline{\sim S}).$$

Using our derivable '$S \equiv \Delta(\overline{\sim S})$,' then,

$$\text{(ii), (iii)} \vdash S.$$

We now have, from steps above, '(ii), (iii) $\vdash S$' and '(i) $\vdash \sim S$'. Thus:

$$\text{(i), (ii), (iii)} \vdash S \cdot \sim S.$$

But of course (i), (ii), and (iii) are merely particular instances of our three schemata.

Given what can be derived from the initial axiomatic entries of our list, then, our three auxiliary schemata lead to contradiction. Our list as a whole, however plausible its entries individually — and whatever interpretation we choose for 'Δ' — is inconsistent.

III. THE POWER OF THE PARADOX

What the argument above really shows is this: that for no 'Δ' can all three of our schemata be consistently maintained:

1. $\Delta(\overline{A}) \supset A$
2. $\Delta(\overline{\Delta(\overline{A}) \supset A})$
3. $I(\overline{A}, \overline{B}) \supset . \Delta(\overline{A}) \supset \Delta(\overline{B})$.

Or at least, for no 'Δ' can these three be maintained consistently with the axioms of any system adequate for arithmetic. Arithmetic, surely, is not to be abandoned.

This is, I think, a very powerful result, precisely because there are many common and important notions for which we would intuitively insist on all the schemata above. What the argument shows is that we can't.

In what follows I want to outline implications of the Knower — or at least apparent implications — for notions of knowledge and truth. But this is merely a first sketch. Whether the apparent implications of the Knower are genuine or merely apparent is a matter that requires careful consideration of possible replies in subsequent sections.

With only the interpretation sketched for 'Δ' so far in mind, however, the argument above might not appear to have a particularly startling result. For let us take 'Δ' as 'is known,' and take this as 'is known by a particular person' — Patrick Grim, for example.

Here schema (1) — '$\Delta(\overline{A}) \supset A$' — will apparently hold simply because knowledge, in order to be knowledge, must be veridical. (2)

might hold if Grim knows this. But (3),

$$I(\bar{A}, \bar{B}) \supset. \Delta(\bar{A}) \supset \Delta(\bar{B}),$$

seems quite *unlikely* to hold for Grim or any comparable agent; Grim surely does *not* know every logical consequence of the things he knows. If (3) is false on the face of it, an argument that (1) through (3) cannot all be true may not seem particularly unsettling.

Within a system of epistemic logic such as that offered by Hintikka in *Knowledge and Belief*, of course, something like (3) will hold.[8] For given any '$p \supset q$' valid in ordinary propositional logic, one is committed in such a system to

$$Kap \supset Kaq$$

with 'Kap' read as 'a knows that p.'[9]

Precisely this principle, however, has been widely thought to be a particularly implausible feature of such systems. Here again, if (3) is implausible on its own, the fact that (1) through (3) cannot be true together might seem a less than startling result.

Even here, however, the argument above should give us pause. For although Grim does not know all consequences of what he knows, this would intuitively seem to be a contingent limitation; it just so happens that he is not an ideal knower. Hintikka's original response to something like (3) in his account is similarly to stress its idealistic intent:

> Our results are not directly applicable to what is true or false in the actual world of ours. They tell us something definite only in a world in which everybody follows the consequences of what he knows as far as they lead him. (*Knowledge and Belief*, p. 36)

What the argument above shows, however, is not merely that our schemata are not in fact true together. What it shows is that they logically cannot be true together.

That (1) through (3) do not hold where 'Δ' is read as 'is known by Grim,' then, is apparently not merely a contingent limitation of Grim. That the knowers that we know do not satisfy (1) through (3) is apparently not to be shrugged off by appeal to ideal knowers. For what the argument seems to show is that such ideal knowers are as logically impossible as married bachelors or circular squares — the ideal itself appears to be incoherent.

We can perhaps make the point clearer, and the impact of the paradox more forceful, if we interpret 'Δ' not as 'is known by Grim' or the like but as 'is in principle knowable.' Now all of our schemata would seem to hold, regardless of our personal or present limitations, given an optimistic enough view of the extendibility of knowledge in principle. If 'knowable in principle' is taken to include 'knowable on the basis of (unlimited) demonstration within Q',' in fact, the truth of our troublesome (3) would seem to be guaranteed.

But our argument holds regardless of what interpretation we choose for 'Δ'. So not even for knowability in principle can our schemata all hold, it appears, no matter how far we choose to stretch 'in principle.' Alternatively put: a knowledge in principle of which (1) through (3) would hold appears to be logically impossible.

Let us read 'Δ', finally, as 'is known by an omniscient God.' Surely now all of our schemata must be true. For they read, in effect:

4. If something is known by an omniscient God, it is so.

5. (4) is known by an omniscient God.

6. If B is derivable from A, and A is known by an omniscient God, then B is known by an omniscient God.

But the force of the argument is that these logically cannot all be true. Genuine omniscience would demand no less, and thus it appears that there logically cannot be any omniscient being.

Up to this point we have emphasized the implications of the paradox for notions of knowledge. But the argument at issue will hold for any interpretation of 'Δ', epistemic or not, and thus the paradox of the Knower will apply to other common notions as well.

One of these is truth — not surprisingly, perhaps, in virtue of certain points of contact between Montague's original development of the Knower and Tarski's theorem.

If 'Δ' is read simply as 'is true,' all of our schemata would seem to hold without a hitch:

7. If something is true, it is so.

8. (7) is true.

9. If B follows from A, and A is true, B is true as well.

But for 'Δ' as truth no more than for 'Δ' as knowledge will these be consistent with axioms for arithmetic. Even a notion of truth for which (1) through (3) would hold, it appears, must be incoherent.[10]

But does the argument above genuinely have these philosophical implications for knowledge and truth, or are they merely apparent? Formal arguments and their philosophical implications are subtle things, and here we must carefully consider possible replies.

IV. WAYS OUT

What the argument of Section II shows is that for no 'Δ' can all three of our schemata be consistently maintained. This much, moreover, is a solidly formal result. We may be able to philosophize around it, but we won't be able to philosophize it away.

As noted in the preceding section, however, (1) through (3) do seem to hold intuitively for common notions of truth and of knowledge — at least for knowability in principle and for omniscience. Yet any notion of truth or knowledge for which these do hold must be abandoned as incoherent.

What are our possible ways out?

The basic strategy of any escape here is to fight initial intuitions: to deny, despite appearances, that truth or knowledge in the sense at issue is something for which (1) through (3) will hold. These are *not*, on any such strategy, to be properly represented by 'Δ' in the schemata above.

In succeeding sections I want to consider several variations on this central strategy.

Note that in our schemata above 'Δ' appears as a predicate — a single predicate, and a predicate which applies to '$\overline{A}$' and the like as names of sentences.

One way out is to insist that although truth and knowledge do apply as predicates, they apply as predicates not of sentences but of something else — propositions, perhaps.

Another possible escape is to insist that these are not to be construed as predicates at all, but as operators or in the manner of redundancy theories of truth.

A third way out, the most traditional and in some ways the simplest, is to concede truth and knowledge as predicates, and as predicates of sentences, but to deny that these can be represented by a *single* predicate such as 'Δ'. This is the strategy of the hierarchical reply.

Each of these strategies of escape, I think, has something to be said for it, and each will be considered in a following section.[11] But none, I want to suggest, effectively obviates the force of the Knower with respect to a notion of all truth or of omniscience.

V. PROPOSITIONS AND THE STRENGTHENED KNOWER

In the schemata of the Knower, 'Δ' appears as a predicate of sentences. One way out is to insist that knowledge and truth, so tempting and yet so disastrous as interpretations for 'Δ', are properties of something else instead. Here the traditional candidate is propositions.

The obscurity of propositions is of course legion: it is far from clear what such things would *be*, if things they be at all. Here, however, I want to press a tidier objection against any propositional reply to the Knower.

It won't work. For just as propositional replies to the Liar standardly fall afoul of the Strengthened Liar,[12] so any propositional reply to the Knower will fall afoul of the Strengthened Knower.[13]

Here we might fruitfully distinguish a negative from a positive aspect of the propositional reply.

The negative aspect is simply the charge that knowledge and truth cannot properly be represented by the 'Δ' of our three schemata. But that negative aspect alone, of course, fails to distinguish a propositional response from any other; it is simply the crucial strategy of any response to the Knower.

What genuinely distinguishes the propositional response is its positive aspect. For the propositionalist appears to have an alternative way of representing truth and knowledge in mind. The positive propositional claim is that these apply not to sentences, as does 'Δ', but to some sort of propositions somehow expressed by sentences.

It is the positive aspect of the propositional reply that falls afoul of the Strengthened Knower.

For suppose that we add to our language those symbols necessary to properly represent knowledge or truth in line with the propositional proposal. Here we will need not only 'Δ', interpreted now as the real truth or knowledge predicate, but a mechanism for applying it strictly to propositions.

Let us thus introduce

$$E(\overline{A}, p)$$

as an expression relation between sentences and propositions of precisely the type that the propositionalist proposes, where 'p' is a new type of variable for propositions. '$E(\overline{A}, p)$', then, applies just in case the sentence with Gödel number '$\overline{A}$' expresses proposition 'p'.

Now, it appears, we will have the linguistic resources necessary for expressing genuine truth and knowledge predicates as the propositionalist conceives them. Note also that we need go no further in specifying what propositions *are*.[14]

Within such a language, however, our paradox reappears. For let us define '$\mathbf{\Delta}(\overline{A})$' as:

$$\mathbf{\Delta}(\overline{A}) =_{\mathrm{df}} \exists p\, (E(\overline{A}, p) \cdot \Delta p),$$

in which 'Δ' applies — as the propositionalist demands — to propositions alone.

With regard to '$\mathbf{\Delta}$', analogues of our previous schemata will now be propositionally unobjectionable. For

$$\mathbf{\Delta}(\overline{A}) \supset A$$

$$\mathbf{\Delta}(\overline{\mathbf{\Delta}(\overline{A}) \supset A})$$

$$I(\overline{A}, \overline{B}) \supset. \mathbf{\Delta}(\overline{A}) \supset \mathbf{\Delta}(\overline{B})$$

will now read, in effect:

10. If the sentence numbered $\overline{A}$ expresses a Δ proposition (a true proposition, or one known by God, let us say), then A.

11. (10) expresses a Δ proposition.

12. If the sentence numbered $\overline{A}$ expresses a Δ proposition and the sentence numbered $\overline{B}$ follows from that numbered $\overline{A}$, then the sentence numbered $\overline{B}$ also expresses a Δ proposition.[15]

In our original system, '$S \equiv \Delta(\overline{\sim S})$' was demonstrable on the basis of axioms borrowed from Q. Within the present system Q' — consisting of our expanded language and the same axioms — there will similarly be a sentence U such that

13. $U \equiv \mathbb{\Delta}(\overline{\sim U})$

is demonstrable.

But now we have all the pieces needed — the demonstrable (13) plus our schemata for '$\mathbb{\Delta}$' — to repeat the argument precisely as before.

Our schemata even for '$\mathbb{\Delta}$', then, will be inconsistent with any axioms for arithmetic, and this despite the fact that our knowledge or truth predicate 'Δ' is now reserved exclusively for propositions. The propositional reply apparently fails.[16]

What might a propositionalist say here?

He might insist that something has gone wrong in extending our language to include the expression relation '$E(\overline{A}, p)$'. Perhaps just as it was claimed that '$\Delta(\overline{A})$' did not capture the genuine knowledge or truth predicate, it might be claimed that '$E(\overline{A}, p)$' somehow fails to capture the genuine expression relation.

But the problem is that given any such general relation proposed it appears that the argument can be repeated. If the notion of a proposition as something expressed by a sentence can itself be expressed at all, it appears — and that they are expressed by sentences is one of the few relatively clear things that *can* be said about propositions — the propositional proposal will fall afoul of the Strengthened Knower.[17] Hence the justice of C. Anthony Anderson's comment with regard a propositional treatment of the Knower:

> . . . if we take the suggestion as something that can itself be expressed, the difficulty reappears. (*op. cit.*, p. 347)

There is also, perhaps, a general lesson here. Propositional approaches have often been faulted for incompleteness: for failing to make explicit what the expression relation between sentences and propositions is supposed to be, in a way that would tell us what sentences will fail to express propositions and why.[18] But the work above suggests that this is perhaps more than a contingent limitation. For given any expressible relation proposed between sentences and propositions, we can construct — in terms of that relation itself — a

Strengthened paradox which the propositionalist seems powerless to handle.

There is, however, one further way that the propositionalist might make objection to the Strengthened Knower stick. For he might insist that the expression relation between sentences and propositions cannot properly (or coherently) be represented by a *single* relation '$E(\overline{A}, p)$'.

Expression, we might propose, must instead be represented using a hierarchy of expression relations, with familiar restrictions. An expression relation '$E_n(\overline{A}, p)$' of this hierarchy, we might insist, can apply grammatically only to sentences involving expression relations with lower subscripts. Such a move would, indeed, stop the Strengthened Knower. We could not for example then use our second schema,

$$\Delta(\overline{\Delta(\overline{A}) \supset A}),$$

in which the stacked 'Δ's are defined in terms of expression relations, in the manner required for paradox.[19]

There is also much to be said for such a proposal in other regards. If a wide range of terms seems to call for similar hierarchical treatment, for example — including perhaps truth, necessity, knowledge, belief, and other propositional attitudes[20] — it would be convenient to localize centrally the source of that hierarchy. Given something like a propositional account of these, the expression relation would seem a primary candidate.

A propositionalist approach *can* escape the Strengthened Knower, then, by auxiliary recourse to hierarchy. But propositional treatment modified by hierarchy will also inherit any difficulties of a purely hierarchical response.

VI. REDUNDANCY THEORIES AND OPERATORS

In Section V we considered treating truth and knowledge, unlike the 'Δ' of our schemata, as predicates of something other than sentences. With negative results.

Another way out is to insist that these are not legitimately predicates at all; they are, perhaps, to be construed as sentence-forming operators instead. This is essentially the route of redundancy theories of truth, exemplified classically by brief comments in Ramsey and developed elegantly in Dorothy L. Grover's prosentential theory.[21] Let me start by concentrating on redundancy theories, broadening my comments at a later stage to include operator responses to the Knower in general.

Redundancy theories face at least initial intuitive implausibilities; truth is normally and syntactically treated in English as a straightforward predicate. Nonetheless the guiding idea of redundancy theories is that 'is true' can be eliminated from all contexts without semantic loss. 'It is true that *p*,' Ramsey argues, means nothing more than '*p*'.

As a treatment of cases in which '*p*' is displayed, as it were, this seems a promising start. For if truth is not a predicate at all, we can conveniently avoid philosophical thickets as to what it is a predicate *of* (sentences or propositions) and what the conditions of its application are (correspondence, coherence, etc.).

But what of ordinary usages of 'true' in which '*p*' is not displayed, such as (14)?:

14. All that Joe says is true.

Here omniscience and a notion of all truths are our immediate targets, it will be remembered, and if (14) could not be expressed within the confines of a redundancy theory it's clear that these couldn't either.

As Ramsey notes and Grover develops in detail, however, the effect of a truth predicate in such cases can be achieved using propositional quantification. (14), for example, becomes:

15. (p) (Joe says that $p \supset p$).

Difficulties *may* remain. As Herbert Heidelberger notes,

> It is not clear whether Ramsey intended the last occurrence of '*p*' to fall within or outside the scope of the universal quantifier; either way, however, the paraphrase is unsuccessful. If '*p*' falls within the scope of the quantifier, then it is an isolated variable to which no predicate is adjoined.[22]

The predicate we are tempted to add, of course, is 'is true', and if that were required the redundancy theory would fail to eliminate truth as a predicate.

Here, however, I want once again to press a tidier objection: that once redundancy theories are equipped to simulate a truth predicate by means of propositional quantification, paradox threatens again in the form of both the Liar and the Strengthened Knower.

Consider first the Liar, and let us introduce an operator '§' which forms a term from a sentence. '§' might be read, for example, as representing 'the statement that . . . .'

Let 'c' abbreviate '§16', where (16) is

16. $(p)(c = \S p \,.\!\supset\, {\sim} p)$.

Now empirically,

17. $c = \S(p)(c = \S p \,.\!\supset\, {\sim} p)$,

and this is all we need to derive a contradiction in the standard manner of the Liar.[23]

Here we can also go one step further, by treating 'c' as an abbreviation for a sentence rather than as a term for a sentence. Let 'c' abbreviate sentence (18):

18. $(p)((c \equiv p) \supset {\sim} p)$.

In virtue of our abbreviation,

19. $c \equiv (p)((c \equiv p) \supset {\sim} p)$,

and we can obtain a Liar-like contradiction as before.

Elimination of a truth predicate alone, then, in the manner of redundancy theories, fails to avoid a quantificational Liar. It is similarly insufficient — as is mere recourse to operators — to avoid a form of the Strengthened Knower.

The Strengthened Knower of the preceding section, it will be remembered, employs

$$\Delta(\overline{A}) =_{\mathrm{df}} \exists p\, (E(\overline{A}, p) \cdot \Delta p).$$

Redundancy theories of truth seem explicitly to demand propositional quantification, as will operator views in general if adequate to express

20. All that God knows is true

or the like.

But here let us simply substitute '$N(\overline{A}, p)$' in place of '$E(\overline{A}, p)$', using '$N(\overline{A}, p)$' to indicate that '$\overline{A}$' forms a singular term for sentence

'p'. One way of quite naturally reading '$N(\bar{A}, p)$', for example, would be '$\bar{A}$ is the Gödel number of formula p.'

Now let:

$$\mathbf{\Delta}(\bar{A}) =_{\mathrm{df}} \exists p\,(N(\bar{A}, p) \cdot \Delta p).$$

Just as 'Δ' applied only to propositions in the earlier form of the Strengthened Knower, 'Δ' can here be interpreted purely as an operator on p.

Alternatively, we might introduce a term-forming operator '§' as above, using

$$\mathbf{\Delta}(\bar{A}) =_{\mathrm{df}} \exists p\,((\S p = \bar{A}) \cdot \Delta p).^{24}$$

In either case, though 'Δ' can now appear purely as an operator, the paradox of the Knower will reappear for '$\mathbf{\Delta}$'. Note also that the paradox will hold for various interpretations for 'Δ'. In particular, our schemata for '$\mathbf{\Delta}$' will seem obvious not only for 'Δ' as an operator 'is true,' as in redundancy theories, but for 'Δ' as 'is knowable in principle' (optimistically construed) or 'is known by an omniscient God.' Appeal to operators alone, then, will apparently be insufficient against the Knower.[25]

There is an attractive way out here, however, for redundancy theories and operator approaches in general: a resort to hierarchy.

Both the Liar and the Knower as they appear above can be blocked effectively if term-forming operators or relations are made subject to a hierarchy, or if propositional quantification itself is made subject to such a hierarchy. The first proposal is that of Gilbert Harman;[26] that if 'p' is an expression of a language L, '§p' or the like can appear only in a metalanguage of L. The second proposal is that which Grover favors in 'Propositional Quantification and Quotation Contexts';[27] that in a substitutional interpretation of quantification we allow as substitutions for 'p' only formulae with fewer quantifiers than that in which 'p' occurs.

Resort to hierarchy of either sort will stop paradox precisely as before. With an eye to the Knower in particular, hierarchy of either sort will prevent us from using for example our second schema,

$$\mathbf{\Delta}(\overline{\mathbf{\Delta}(\bar{A}) \supset A}),$$

with stacked 'Δ's defined in terms of propositional quantification and a term operator or relation, in the manner required for paradox.[28]

In either case, however, it appears that neither abandonment of a truth predicate nor an insistence that certain notions be treated as sentence-forming operators will alone suffice to dispose of the Knower. Any redundancy or operator theory modified by an auxiliary recourse to hierarchy, on the other hand, will predictably inherit the difficulties of a purely hierarchical response.

VII. HIERARCHIES

The remaining hope for a response to the Knower, it seems, is recourse to hierarchy, either in a pure form or as an attractive auxiliary to appeals to propositions or operators which prove insufficient without it.

What I will argue here, however, is the following: that whatever the virtues of a hierarchical response in general, it prohibits both any global notion of truth suitable for speaking of *all* truth and any coherent notion of omniscience. If hierarchy is required to escape the contradictions of the Knower, then, abandonment of any notion of all truth and of omniscience seems a necessary cost of that escape.

The hierarchical route regarding paradox is by far the best travelled. Here I will concentrate on three I take to be sterling representatives of such an approach: Tarski, Kripke, and Burge.

A. *Tarski*

The 'Δ' of our schemata, we've noted, is not only a predicate, and a predicate of sentences, but a single and univocal predicate. The strategy of hierarchy in general is to deny that knowledge, truth, or the like can properly be represented by any *one* such 'Δ'.

What Tarski proposes is a hierarchy of languages L_0, L_1, L_2, . . . , each language of which contains the truth predicate (and here we might alternatively propose a knowledge predicate) for the language below it in the hierarchy.[29] In place of a single predicate 'true,' then, we have an ascending series of predicates 'true-in-L_0,' 'true-in-L_1,' 'true-in-L_2,' . . . , which we might alternatively envisage as an ascending series of subscripted predicates 'true$_0$,' 'true$_1$,' 'true$_2$,' . . . .

The applications of a Tarskian hierarchy to the Knower should be clear. For if 'true,' say, appears only with a subscript, and if a truth predicate 'true_n' is allowed to apply only to sentences involving a truth predicate of subscript $< n$, our second schema

$$\Delta(\overline{\Delta(\overline{A}) \supset A}),$$

for example, will be unable to appear grammatically in the manner required for paradox.[30]

A Tarskian hierarchy, however, does not come without significant cost. Such a hierarchy is technically limited to finite levels, those levels are fixed intrinsically and in advance rather than floating on wayward empirical facts in the way paradox often does, and it is unable to deal intuitively with cases in which Nixon and Dean call each other liars.[31] Here, however, I want to press two simpler objections.

The first — and this is a difficulty which will plague hierarchical approaches throughout — is that appeal to hierarchy inevitably appears *ad hoc*. Familiar notions of neither truth nor knowledge seem to come with anything like subscripts attached, and thus at best hierarchical replies have the air of clever technical impositions rather than fully satisfying philosophical solutions.

Secondly, and for our purposes most importantly, a Tarskian hierarchy effectively prohibits any global notion of truth or knowledge. Once 'true' is replaced by an infinitely fragmented series of truth predicates 'true_0,' 'true_1,' 'true_2,' . . . , for example, we have no way grammatically even to *state*

21. Every proposition is either true or false

or other basic logical laws. For if, as intended, (21) is about propositions of *all* levels, neither its truth predicate nor any truth predicate applied to it can have *any* level. With a similarly hierarchical treatment in terms of knowledge levels, neither the knowledge predicate of any true (22),

22. God knows all truths,

nor of the claim that God knows (22), could be assigned any level.

This feature of Tarskian hierarchies, moreover, is far from accidental. For the very purpose of a hierarchical strategy is precisely to prohibit any global form of such predicates. Were we to keep Tarskian levels but reintroduce a global predicate, paradox would return. The Global Liar, for example, could take the form

23. (23) is not globally true.

If (23) is globally true, it is true at all levels that there is some level on which (23) is *not* true. But if there is any level on which (23) is not true, it appears, it is simply true.

Any global knowledge or truth predicate would moreover fill the schematic role outlined for 'Δ', and thus saddle us again with the paradox of the Knower.

Short of restoring paradox, then, a Tarskian hierarchy allows us no global notion of truth suitable for speaking of all truth, and no global notion of knowledge of the sort required for any coherent notion of omniscience.

This too, I will argue, is a characteristic of hierarchical treatments throughout.

B. *Kripke*

One standard approach to the Liar is the invocation of a third truth value or the option of none — 'neither true nor false.'[32] Another is appeal to a Tarskian hierarchy. What Kripke does is to combine these in a technically sophisticated and ingenious way.[33]

Truth for Kripke, unlike for Tarski, will be only partially defined. Those sentences which are not assigned truth values are termed *ungrounded*, and it is this crucial notion of groundedness that is technically specified in terms of a Tarskian hierarchy of languages.

"Suppose," Kripke says, "we are explaining the word 'true' to someone who does not yet understand it" (p. 701). We might do so by means of the following principle:

> One may assert that a sentence is true just when one is entitled to assert that sentence, and may assert that it is not true just when one is entitled to deny it.

Our learner, we assume, starts off entitled to assert 'Snow is white' and the like and thus, by the principle above, ''Snow is white' is true.'

Repeated applications of the principle allow iterations of 'is true,' and using existential generalization and other statements we can envisage him eventually capable of handling, say, 'Some sentence printed in the *New York Times* is true.'

In this manner, the subject will eventually be able to attribute truth to more and more sentences involving the notion of truth itself. There is no reason to suppose that *all* statements involving 'true' will become decided in this way, but most will. Indeed, our suggestion is that the 'grounded' sentences can be characterized as those which eventually get a truth value in the process (p. 107).

Somewhat more technically, though I simplify, the approach is as follows. We introduce a hierarchy of languages beginning with L_0, L_1, L_2, . . . and '$T(x)$' is interpreted within any language $L_{\alpha+1}$ as the truth predicate for sentences of L_α, much as before. At the lowest level L_0, then, '$T(x)$' is completely undefined. At L_1 it is assigned to wffs which do not themselves contain '$T(x)$,' and so on, precisely in line with the intuitive sketch above. At each step wffs previously assigned truth values retain them,[34] but definite truth values are also assigned to new wffs for which '$T(x)$' was previously undefined. In that sense '$T(x)$' becomes more defined as the process continues.

A very pretty technical aspect of the Kripkean approach is that after sufficiently many applications — indeed transfinitely many — the process saturates at a fixed point; the set of true and false sentences is the same as at the preceding level.[35] A wff will be *grounded*, then, just in case it is assigned a truth value at the smallest fixed point. And among those sentences which will not be grounded will be:

24. (24) is false

and

25. (25) is true.

A Kripkean hierarchy of this sort has a number of advantages over a Tarskian. It is specified beyond finite levels, it can intuitively handle Nixon-Dean cases alluded to above, and it is consistent with allowing the level of sentences to float on empirical facts.

It does not, however, escape the particular difficulties of hierarchical accounts here at issue.

Kripke's answer to the Liar is essentially that of any three-valued approach: (24) is neither true nor false but ungrounded.

But what then of a Strengthened Liar,

26. (26) is false or ungrounded

or simply

27. (27) is not true?[36]

Surprisingly, Kripke does not directly address the issue of the Strengthened Liar. But his response would evidently be that 'true' as it appears in (27), as well as 'ungrounded' as in (26), must be treated as predicates of a further metalanguage:

> . . . Liar sentences are *not true* in the object language, in the sense that the inductive process never makes them true; but we are precluded from saying this in the object language by our interpretation of negation and the truth predicate. . . . The necessity to ascend to a metalanguage may be one of the weaknesses of the present theory (p. 714).

If the predicates of (26) and (27) are consigned to a metalanguage, of course, the Strengthened Liar has no grammatical place within Kripke's original hierarchy of languages for precisely the same reason that the standard Liar has no place within Tarski's hierarchy.

We now have two senses of 'true', however. Let us write the 'true' of (27), exiled to the metalanguage, as 'true$_m$', using a similar subscript to mark 'ungrounded' as a purely metalinguistic predicate.

What now of

28. (28) is not true$_m$?

This clearly cannot be a sentence of the original hierarchy, since it contains an explicitly metalinguistic predicate. But if taken as a sentence of the metalanguage, is (28) true or untrue in the metalinguistic sense?

Paradox will return for any true-like construal of 'true$_m$'. For if (28) is true$_m$, it appears, it is not true$_m$. If it is not true$_m$, it must be true$_m$.[37]

One option here is to reject (28) as a sentence of the metalanguage at all, on the grounds that a sentence of linguistic level m cannot be assigned a truth predicate of level m. This is, of course, the Tarskian strategy with regard to the Liar. Statements of our metalanguage can still be said to be true or false, but only in terms of a meta-metalinguistic predicate 'true$_{m'}$'.

Another option is to accept (28) as a sentence of the metalanguage, but as an ungrounded sentence. Groundedness and ungroundedness within the metalanguage might in fact follow the same pattern of ascension to a fixed point as in Kripke's original hierarchy. But of course (28) could not be said to be 'ungrounded$_m$' or 'not true$_m$'. Short of paradox we would have to say perhaps that (28) is 'ungrounded$_{m'}$' or 'not true$_{m'}$', using the distinct groundedness and truth predicates of a further meta-metalanguage.

Either option clearly leads to an ascending series of metalanguages with distinct truth predicates, on precisely the Tarskian model. Kripke's comment that "the ghost of the Tarski hierarchy is still with us" (p. 714) thus seems a serious understatement.

If something so like a Tarskian hierarchy is inevitable on a Kripkean approach, of course, the difficulties noted above with regard to Tarski will return. Truth once again, and perhaps despite initial appearances, will be infinitely fragmented into levels. Any statement regarding *all* truths will have no metalanguage to call home, and any Kripkean hierarchy for knowledge[38] will have no place for a true (22):

22. God knows all truths.

A Kripkean hierarchy, then, offers no more hope for a notion of all truths or for omniscience than does the Tarskian hierarchy which, in the end, it so closely resembles.[39]

C. *Burge*

The final hierarchical variation that I want to consider is an intriguing indexical-schematic account put forward by Tyler Burge.[40]

In Burge as in Tarski truth appears only with implicit subscripts, and paradox is avoided by similar restrictions on the application of subscripted truth predicates. Burge's approach, however, is to take truth as an indexical notion. 'True' is to remain constant in meaning; subscripts are to indicate its shifting extensions, fixed pragmatically by context.[41]

What is of most interest for our purposes here, however, is the following. Burge effectively admits that a pure indexical account affords us no way of making sense of a global notion of truth, suitable for "All statements are either true or not" or "God is omniscient."[42]

For precisely that reason, however, he ultimately insists on a dual, indexical-*schematic* account. Most uses of the predicate 'true' in natural language are indexical, Burge proposes, but some are schematic.[43] Global applications of 'true' are to be represented as schematic generalizations:

> ... The subscripts marking contexts of use stand open, ready to be filled in as the occasions arise. ... The first sentence above, for example, ["All sentences are true or not"] should be formalized: $(s)(Tr_i(s) \vee \sim Tr_i(s))$. When we judge the schematic statement itself to be true, we make an equally schematic statement with the context of our evaluation schematically fixed at $\ulcorner \text{true}_{i+1} \urcorner$.[44]

Here a number of objections might be raised. What Burge offers us in place of a statement concerning all statements, for example, is a bare schema — not a statement at all.[45] But here as in previous cases I want to press a tidier objection: that Burge's schemata, *if* adequately global, do not suffice to avoid paradox.

Consider again Burge's rendering of a global principle of bivalence:

29. $(s)(Tr_i(s) \vee \sim Tr_i(s))$.

(29), Burge insists, is not — short of paradox — to be understood in terms of quantification:

> For quantification into the argument place will provide an open sentence just as subject to paradox as the "naive" truth-predicate formalization. ... For example, one might suggest a sentence like (a), '(a) is not true at any level.'[46]

Quantification aside, however, can we take (29) or something like it as genuinely global, and thus as providing a principle of bivalence which governs for example its own truth as well? Apparently not. For, if genuinely global, (29) will take itself as a substituend —

$$Tr_i(29) \vee \sim Tr_i(29) —$$

and here we have implicitly stacked 'Tr_i's, in apparent violation of the standard hierarchical stricture that a truth predicate be applicable only to formulae involving truth predicates of lower subscripts.

A new schema, taking perhaps schemata as substituends, *will* give us a principle of bivalence adequate at least for (29):

30. $(s)(Tr_{i+1}(s) \vee \sim Tr_{i+1}(s))$.

But (30) will again not be genuinely global, since it will still be unable to take itself as a subtituend.

If we attempt to make room for genuinely global claims by waiving hierarchical strictures against stacked predicates of the same level, moreover — at least in the special case of 'Tr_i', say — we get paradox again.

For consider

31. $\sim Tr_i(31)$,

which we might term the Schematic Liar.

(31) is of course a schema. As such it is a 'schematized direction for making statements,' in Burge's terms, and in effect licenses the following list:

$$\sim Tr_1(31)$$
$$\sim Tr_2(31)$$
$$\sim Tr_3(31)$$
$$\vdots$$

But what truth value can we assign to (31) itself?

To claim that (31) is true, following Burge's outline above, would be to claim that Tr_{i+1} (31). But this is a further schema which licenses the following:

$$Tr_2(31)$$
$$Tr_3(31)$$
$$Tr_4(31)$$
$$\vdots$$

To claim that (31) is not true, on the other hand, would be to claim that $\sim Tr_{i+1}(31)$. What this licenses is:

$$\sim Tr_2(31)$$
$$\sim Tr_3(31)$$
$$\sim Tr_4(31)$$
$$\vdots$$

The first of these — the 'true' option — licenses a list which contradicts that of (31) itself at an infinite number of points. By claiming both (31) and its truth, it appears, we would be committed to:

$$Tr_2(31) \cdot {\sim} Tr_2(31)$$
$$Tr_3(31) \cdot {\sim} Tr_3(31)$$
$$Tr_4(31) \cdot {\sim} Tr_4(31)$$
$$\vdots$$

The 'false' option, on the other hand, gives a subset of what (31) itself gives us. Not good. Given standard double negation, moreover, we would be unable to maintain the falsity of (31) together with (31)'s negation without a similarly infinite list of contradictions. For '$\sim\ \sim Tr_i(31)$' — (31)'s negation — gives us

$$Tr_1(31)$$
$$Tr_2(31)$$
$$Tr_3(31)$$
$$\vdots$$

whereas '$\sim Tr_{i+1}(31)$' — the statement of (31)'s falsity — gives us

$${\sim} Tr_2(31)$$
$${\sim} Tr_3(31)$$
$${\sim} Tr_4(31)$$
$$\vdots$$

Let me also offer another form of the paradox, using the talk of schemata and their instances that Burge's account would seem to demand.

Consider:

32. Every instance of schema (33) is true.

33. $\sim Tr_i(32)$.

Now does the schema

$$Tr_i(32)$$

have a true instance or not? If it *does* have a true instance, (32) is false. If it has *no* true instance, on the other hand, it appears that (32) is simply true.[48]

If 'Δ_i' is to be iterable in the manner required for a genuinely global principle on the model of (29), moreover, we can form a Schematic Knower.

Let us add 'Δ_i' to our language as a symbol required to express genuinely global claims regarding truth or knowledge. Our system will now have the slight peculiarity that some formulae — those containing 'Δ_i' — will be read as schemata.

But for some S,

$$S \equiv \Delta_i(\overline{\sim S})$$

will be demonstrable in the system as before. For 'Δ_i' as a representation of truth or knowledge it will also seem obvious that:

34. $\Delta_i(\overline{A}) \supset A$

35. $\Delta_i((\overline{34}))$

36. $I(\overline{A}, \overline{B}) \supset . \Delta_i(\overline{A}) \supset \Delta_i(\overline{B})$,

giving us a contradiction precisely as before.

Burge's account, then, if capable of handling genuinely global claims regarding truth or knowledge, would be insufficient to avoid precisely the type of paradox he anticipates with respect to quantification.

Despite recourse to schemata, we are left with all the difficulties of a simple hierarchy. Short of paradox, there is again no level of statement or schemata on which one can genuinely speak of all truths. A hierarchy for knowledge on the pattern Burge suggests can similarly be expected to leave no place for a true (22):

22. God knows all truths.

Let me emphasize this last point.

With a hierarchy of divine knowledge predicates K_1, K_2, K_3, . . . , including perhaps a schematic K_i, K_{i+1}, K_{i+2}, . . . , each claim c that 'God knows . . .' will use a knowledge predicate of a certain level. But if any such claim is true, that c is *known* will be expressible only by means of a higher knowledge predicate. Given hierarchical restrictions crucial to avoid paradox, c cannot be claimed to be known within the scope of c itself. No such knowledge predicate, then, will be adequate to express omniscience.

With our indices read as indicating times or persons, of course, this tells us that within a hierarchical account all truths cannot be said to known at any time or by any person. Will it help for theological purposes to construe indices as 'orders of knowledge' of some more occult sort? No order of God's knowledge exhausts omniscience, we might propose, but His series of orders does.[49]

But even this will not do. For the proposal is in effect that we take 'is known within the series' as a new global knowledge predicate. Using 'K_Ω' to represent 'is known within the series', we get a form of the Liar with

37. $\sim K_\Omega(37)$.

If (37) is known within the series, it is true, and thus it is not so known. But if it is not known within the series, it is true, and thus some truth is omitted from K_Ω.

With 'K_Ω' as a global knowledge predicate we will also get the Knower in full force.

Our choice once again, then, is between the contradictions of our paradoxes and abandonment of a genuinely global notion. In the case of knowledge that global notion is omniscience.

In preceding sections we found appeal to propositions and operators alone apparently insufficient against the Knower, and auxiliary recourse to hierarchy to be an attractive way out. The general lesson of this section is that the cost of hierarchy, if adequate to handle paradox in other regards, is an abandonment of a notion of all truth or of omniscience.

VIII. CONCLUSION

Let us sum up.

The paradox of the Knower poses a direct and formal challenge to the coherence of common notions of knowledge and truth. We've considered a number of ways one might try to meet that challenge: propositional views of truth and knowledge, redundancy or operator views, and appeal to hierarchy of various sorts. Mere appeal to propositions or operators, however, seems to be inadequate to the task of the Knower, at least if unsupplemented by an auxiliary recourse to hierarchy. But the cost of hierarchy appears to be an abandonment of any notion of *all* truth or of omniscience. What the contradictions of the Knower seem to demand, then, is at least an abandonment of these.

As noted in the introduction, the argument is complicated enough that one must be wary of dogmatic and precipitate conclusions. One may legitimately wonder whether some new response, or some variation on an old one, will yet offer a way out.

Far too often, however, it is asked what has gone wrong with paradox rather than what paradox may have to teach us. What the Knower may have to teach us, I think, is that there really can be no coherent notion of all truth and really can be no coherent notion of omniscience.[50] In its own way that conclusion is perhaps as humbling as is any traditional notion of God.

NOTES

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[1] *Notre Dame Journal of Formal Logic,* **I** (1960), 79—90. See also C. Anthony Anderson, 'The Paradox of the Knower,' *Journal of Philosophy,* **80** (1983), 338—355, and Tyler Burge, 'Epistemic Paradox,' *Journal of Philosophy,* **81** (1984), 5—29.

[2] 'Syntactical Treatments of Modality, with Corollaries on Reflexion Principles and Finite Axiomatizability,' *Acta Philosophica Fennica,* **16** (1963), 153—167. A form of Montague's argument also appears against syntactical treatments of indirect discourse in Richard H. Thomason, 'Indirect Discourse Is Not Quotational,' *Monist,* **60** (1977), 340—354. These applications of the Knower have been contested, in each case with a hierarchical treatment in mind, in Bryan Skyrms, 'An Immaculate Conception of Modality,' *Journal of Philosophy,* **75** (1978), 368—387, and Tyler Burge, 'Epistemic Paradox,' *op. cit.* Hierarchical responses to the Knower with an eye to knowledge and truth are considered in Section VII.

[3] Implications for necessity, though perhaps tempting throughout, I will leave to another paper.

[4] See for example George S. Boolos and Richard C. Jeffrey, *Computability and Logic, Second Edition* (New York: Cambridge University Press, 1980), p. 158.

[5] For the moment we will act as if such claims in their full generality are added to our list. In fact, however, it is only particular instances of these that are required for the argument below.

[6] See for example Boolos and Jeffrey, *op. cit.*, p. 173.

[7] Another way of getting the same result is to prove a variation of the Diagonal Lemma which gives us

$$\vdash_{Q'} G \equiv B(\overline{\sim G}).$$

One form of such a proof complicates Boolos and Jeffrey's for the Diagonal Lemma by building on a notion of *double-neg diagonalization* (where the double-neg diagonalization of A is the expression '$\sim \sim \exists x (x = \overline{A} \cdot A)$') and by adding a few well-placed negations.

On the interpretation sketched so far, of course, '$S \equiv \Delta(\overline{\sim S})$' says in effect 'my negation is known' in the same (rough) sense that the standard Gödel sentence says 'I am not a theorem.'

[8] Jaakko Hintikka, *Knowledge and Belief* (Ithaca: Cornell University Press, 1967). My comments here are directed at 'something like' Hintikka's system, however, rather than Hintikka's work *per se*. For one thing, Hintikka's 'K' appears as an operator rather than a predicate of sentences, Recourse to operators is further discussed in Sections IV and V.

[9] This is Hintikka's original gloss of '*Kap*' in *Knowledge and Belief, op. cit.*, p. 29. Later in the same work he proposes a reinterpretation; that '*Kap*' should perhaps be read not as '*a* knows that *p*' but 'it follows from what *a* knows that *p*.'

Hintikka has since changed his tune. He now emphasizes that difficulty for the principle at issue arises only if we take every epistemically possible world to be logically possible. See Kriste Taylor, 'Worlds in Collision,' *Philosophia*, **13** (1983), 289—297, and Jaakko Hintikka, 'Impossible Possible Worlds Vindicated,' *Journal of Philosophical Logic*, **4** (1975), 478—484.

[10] Here a contrast with Tarski's theorem is perhaps also in order, however. What Tarski shows in 'The Concept of Truth in Formalized Languages,' in J. H. Woodger, trans., *Logic, Semantics, Metamathematics* (Oxford: Clarendon, 1956, pp. 152—278), is in effect that the schema

$$\Delta(A) \equiv A$$

cannot consistently be added to the axioms of Q'. As Montague notes in 'Syntactical Treatments of Modality, with Corollaries on Reflexion Principles and Finite Axiomatizability,' *op. cit.*, however, the schemata of the Knower seem intuitively much weaker than this, and in that regard the argument of the Knower is the more powerful result.

[11] I have not here reserved a separate section for possible three-valued and 'gapped' responses to the Knower. See, however, note 16 below and the discussion of Kripke's three-valued hierarchy in Section VII.

[12] On the Liar and propositions in a theological context, see my 'Some Neglected Problems of Omniscience,' *American Philosophical Quarterly*, **20** (1983), 265—276.

[13] The notion of a Strengthened Knower dervies from C. Anthony Anderson, 'The Paradox of the Knower,' *op. cit.* I have also benefited from personal correspondence with Anderson.

[14] We might, for example, go on to treat propositions as sets of possible worlds or equivalence classes of sentences under a synonymy relation (see Anderson, *op cit.*, pp. 346—347). But our symbolic treatment here demands no such commitment.

[15] Here as before, of course, it is only particular instances of these schemata that are actually required for the paradoxical argument.

[16] In the present paper I have not reserved a section for three-valued and 'gapped' responses to the Knower. These also fall afoul of a Strengthened Knower, however,

much as three-valued approaches to the Liar fall afoul of a Strengthened Liar (see Nicholas Rescher, *Many-valued Logic* (New York: McGraw-Hill, 1969), esp. pp. 87—90).

For let us suppose the three-valued or 'gapped' objector to claim that knowledge or truth applies only to sentences with (classical) truth values, and that some of the sentences to which 'Δ' applies in our argument (such as perhaps the self-referential '$\sim S$' or '$S \equiv \Delta(\overline{\sim S})$') either have some third value or lack a truth value entirely.

In order to incorporate this proposal into our language and to express the real knowledge or truth predicate, let us use p as a variable reserved exclusively for sentences with classical truth values and use '$\exists p\,(N(\overline{A}, p))$' to indicate that '$\overline{A}$' numbers a sentence of the proper sort.

We can now introduce

$$\dot{\Delta}(A) =_{\mathrm{df}} \exists p\,(N(\overline{A}, p) \cdot \Delta p),$$

in which 'Δ' is reserved exclusively for the right kind of sentences.

Paradox reappears. For analogues of our schemata with '$\dot{\Delta}$' will stand, and there will be a sentence U such that

$$U \equiv \dot{\Delta}(\overline{\sim U})$$

will be demonstrable as before. Note that if '$\sim U$' has a non-classical truth value, or no truth value at all, '$\Delta(\overline{\sim U})$' is simply false. The argument will go through as before.

[17] It would be a mistake here, by the way, to think that by introducing '$E(\overline{A}, p)$' or the like we have built in an assumption that all sentences express propositions or that all propositions are expressable.

In fact, no such general assumption is at stake. In the argument of the Knower at issue, use of '$\dot{\Delta}$' as defined commits us to claiming only that one proposition is known (or whatever) or is expressed by a sentence: an instance of our second schema

$$\dot{\Delta}(\overline{\dot{\Delta}(\overline{A}) \supset A})$$

employed in the argument tells us that an instance of our first schema expresses a proposition and that that proposition is known. This clearly seems innocuous, and all other elements of the argument are hypothetical in the relevant respects.

[18] See, for example, Susan Haack, *Philosophy of Logics* (New York: Cambridge University Press, 1978), p. 140.

[19] A stacked '$\dot{\Delta}$' is also buried in '$U \equiv \dot{\Delta}(\overline{\sim U})$,' since '$\dot{\Delta}$' will appear in '$U$' itself. C. Anthony Anderson (*op. cit.*, pp. 350—351) also gives reason to suppose that the derivability relation 'I' would have to be hierarchically treated.

[20] See for example Sarah Stebbins, 'Necessity and Natural Language,' *Philosophical Studies,* **37** (1980), 1—12, and Tyler Burge, 'Epistemic Paradox,' *op. cit.*

[21] F. P. Ramsey, 'Facts and Propositions,' *Proceedings of the Aristotelian Society,* Supplement 7, 1927, pp. 142—143. For Grover's work see especially Dorothy L. Grover, Joseph L. Camp, Jr., and Nuel D. Belnap, Jr., 'A Prosentential Theory of Truth,' *Philosophical Studies,* **27** (1975), 73—125.

As Sarah Stebbins' work in 'Necessity and Natural Language,' *op. cit.*, indicates, redundancy theories are in a sense simply special cases of an operator approach, in which 'is true' is treated as a peculiarly vacuous sentence-forming operator.

Operator treatments of necessity are of course familiar from the work of Montague and Kripke, and of epistemic notions from Hintikka, *op. cit.* An operator theory of belief developed with the general motivation of redundancy theories appears in A. N. Prior, *Objects of Thought*, ed. by P. T. Geach and A. J. P. Kenny (Oxford: Clarendon, 1971), esp. pp. 19—20 and 131—143.

[22] The Indispensability of Truth,' *American Philosophical Quarterly,* **5** (1968), 212—217. See also however Grover's comments in 'Propositional Quantifiers,' *Journal of*

Philosophical Logic, **I** (1972), 111—136. Further discussion appears in Susan Haack, *Philosophy of Logics, op. cit.,* pp. 130—134.

[23] At a number of points here I follow closely Susan Haack's discussion in *Philosophy of Logics, op. cit.,* pp. 149 ff.

Another form of the Liar, using propositional quantification and some empirical assumptions but without a truth predicate, can be formulated along lines suggested by John L. Pollock, abstract of 'The Liar Strikes Back,' *Journal of Philosophy,* **74** (1971), 604—606.

Consider S:

$$\exists p\,(\text{It has been asserted that } p \text{ in the inner sanctum} \cdot \sim p),$$

where the inner sanctum is a special room in which only the Pope is allowed, and only when he speaks *ex cathedra.*

I manage to sneak in, however, and assert that S, which is now true if and only if it is false.

Note also that the paradox would hold were nothing else asserted in the inner sanctum or, in a variation on the paradox, were nothing else asserted on certain lines of a certain page.

[24] It would be a mistake here, by the way, to think that by introducing '$N(\overline{A}, p)$' or the like we are building in an assumption that all p are somehow 'termable.'

In fact, no such general assumption is at stake. In the argument of the Knower at issue, use of '$\mathbb{\Delta}$' as defined commits us to claiming only that one p is known (or whatever) or 'termed': an instance of our second schema

$$\mathbb{\Delta}(\overline{\mathbb{\Delta}(\overline{A}) \supset A})$$

employed in the argument tells us that an instance of our first schema, for which there is a term, is known. This clearly seems innocuous, and all other elements of the argument are hypothetical in the relevant respects.

[25] It might appear that one course for redundancy and operator theories here would be to explicitly prohibit term relations and term-forming operators such as '$N(\overline{A}, p)$' and '§'. In this regard it must be conceded that simple systems which do not have the resources to handle these, and in which 'Δ' appears as an operator with analogues of the schemata above, are provably consistent. (I am indebted to Evan W. Conyers for an elegant syntactical proof.) It might be argued that in using propositional quantification and a term relation or term-forming operator we are — in spirit, at least — violating Tarski's prohibition against quotation functions, legislated precisely with such truth — predicateless paradoxes in mind (Tarski, *op. cit.,* p. 162).

There are important limits to consistency results for operator systems, however; for some operators within some systems the Diagonal Lemma and paradox of the Knower hold much as for predicates. I leave technical presentation of this result to another paper.

Effective prohibition of term relations, term-forming operators and the like would at any rate have to be a recourse of heroic extremes. As Susan Haack notes in 'Mentioning Expressions,' *Logique et Analyse,* **17** (1974), 277—294, quantificational forms of the Liar will arise with *any* means of denoting expressions, and we can expect much of the same to hold for the Knower. A Tarskian treatment of quotation is moreover quite counter-intuitive (see G. E. M. Anscombe, 'Analysis Puzzle 10,' *Analysis,* **17** (1957), 49—52, and Susan Haack, *ibid.*), and quantification into quotational contexts seems both formally and informally to be of quite general and significant value (see especially Nuel D. Belnap, Jr., and Dorothy L. Grover, 'Quantifying In and Out of Quotes,' in Hugues Leblanc, ed., *Truth, Syntax,* and *Modality* (London: North-Holland, 1973), pp. 17—47).

The case against a Tarskian strategy of prohibition is clinched, I think, by Donald

Davidson's 'Quotation,' *Theory and Decision,* **11** (1979), 27—40. As Davidson notes, any adequate theory of quotation — necessary for an adequate theory of truth — must do justice to the fact that "one can form the name of an arbitrary expression by enclosing it in quotation marks." (pp. 34—45)

> ... if you want to refer to an expression, you may do it by putting quotation marks around a token of the expression you want to mention. (p. 37)

This, at least with substitutional quantification of the sort familiar in operator and redundancy theories, seems to be all that the argument above demands.

In the end, I think, simple prohibition of quotation functions and the like would be comparable in extremity to simple prohibition of, say, a notion of truth, especially since a similar hierarchical treatment is adequate to hedge each against paradox.

[26] 'Substitutional Quantification and Quotation,' *Noûs,* **5** (1971), 213—214.

[27] In Hugues Leblanc, ed., *op. cit.,* pp. 101—110. Later in her work, however, Grover takes a different tack. In '"This is False" on the Prosententialist Theory,' *Analysis,* **36** (1975), 80—83, 'Inheritors and Paradox,' *Journal of Philosophy,* **74** (1977), 590—604, 'Truth,' *Philosophia,* **10** (1981), 225—252, and 'Truth: Do We Need It?,' *Philosophical Studies,* **40** (1981), 69—103, she attempts to deal with paradox in terms of 'inheritors,' much on the pattern of both Kripke's approach (considered in Section VII) and propositional approaches (considered in Section V). As these affinities indicate, however, this later approach has nothing essentially to do with a prosententialist theory. In the attempt to escape the Strengthened Liar, moreover, she seems forced to drastic measures (which even so may not prove effective): to insist that Liar-like sentences lack content, for example, but to prohibit concluding that they are not true (see especially 'Inheritors and Paradox,' *op. cit.,* and 'Truth: Do We Need It?' *op. cit.*). In her later work, it might be noted, Grover also more or less concedes a role for 'true' as a predicate.

[28] See also note 19 above.

In the Liar paradox, either form of hierarchy would similarly prohibit '*c*' as defined as a substitution in (16) itself. Grover's proposal seems adequate to handle the Liar as it appears in (18) and (19), however, in a way that Harman's does not — unless, of course, we insist that the *abbreviations* crucial there appear only in metalanguage as well.

[29] Tarski, *op. cit.*

[30] See also note 19 above.

[31] Each of these charges is made effectively in Saul Kripke, abstract of 'Outline of a Theory of Truth,' *Journal of Philosophy,* **72** (1975), 690—715. See also Susan Haack, *Philosophy of Logics, op. cit.,* pp. 143—145.

[32] On three-valued and 'gapped' approaches to the Knower see note 16 above.

[33] Kripke, *ibid.* It should perhaps be noted, however, that Kripke does not commit himself to any particular three-valued approach, merely noting strong and weak Kleene systems and von Fraassen's supervaluations as options.

[34] In 'Notes on Naive Semantics,' *Journal of Philosophical Logic,* **11** (1982), 61—102, Hans G. Herzberger considers somewhat similar but non-monotonic progressions.

[35] Kripke, *ibid.,* pp. 701—705. This result is anticipated in Robert L. Martin and Peter W. Woodruff, 'On Representing "True-in-*L*" in *L*,' *Philosophia,* **5** (1975), 217—221.

[36] We might here have used a Strengthened Knower instead, of perhaps the form outlined in note 16 above.

[37] The difficulty remains whether we conceive of our metalanguage as bivalent, as Kripke suggests (p. 714) or as 'gapped' in the sense of the original hierarchy. In a bivalent metalanguage (28) will function as a form of the Liar. In a 'gapped' metalanguage it will function as a form of the Strengthened Liar.

[38] A possibility which Kripke entertains on p. 714.

[39] For reasons of conciseness, I have not discussed here a further variation in the hierarchical tradition proposed by Anil Gupta in 'Truth and Paradox,' *Journal of Philosophical Logic,* **11** (1982), 1—60. Closely related but independent work appears in Hans G. Herzberger, 'Notes on Naive Semantics,' *op. cit.*

Important details aside, Gupta offers a (bivalent) hierarchy in terms of progressive applications of a rule of revision. Truth values for Liar-like sentences remain unstable throughout.

For Gupta, however, 'is not stably true' is in much the position of Kripke's 'is false or ungrounded,' and applears to demand a similar treatment in terms of ascending metalanguages or something equally troublesome. Though I cannot here give his account the attention it deserves, I would argue that Gupta offers no more hope than does Kripke for a notion of *all* truth or of omniscience.

[40] 'Semantical Paradox,' *Journal of Philosophy,* **76** (1979), 169—198; 'The Liar Paradox: Tangles and Chains,' *Philosophical Studies,* **41** (1982), 353—366; and 'Epistemic Paradox,' *op. cit.* A similar account is employed in C. Anthony Anderson, 'The Paradox of the Knower,' *op. cit.*

[41] Here Gupta objects that Burge's pragmatic rules

> . . . like all pragmatic rules . . . are sloppily stated and do not constitute a theory of levels . . . If we follow the Tarskian route then the theory of levels constitutes the heart of the theory of truth. It does not belong on the garbage dump of informal pragmatics (*op. cit.*, p. 28).

This is not, however, the route that my criticism will take.

[42] 'Semantical Paradox,' *op. cit.*, p. 191.

[43] This is perhaps clearest in 'The Liar Paradox: Tangles and Chains,' *op. cit.*

[44] 'Semantical Paradox,' *op. cit.*, p. 192.

Note by the way that the schematic portion of Burge's account is logically distinct from the indexical portion. We might thus consider Burge's schematic suggestion a general strategy for hierarchical accounts; we could, for example, envisage it grafted onto a purely Tarskian hierarchy.

[45] As noted below, Burge explicitly prohibits a quantificational reading of schemata, which *would* give us genuine statements. For this would also give us paradox once again.

[46] 'Semantical Paradox,' *op. cit.*, p. 192.

Schemata are of course often interpreted as involving implicit quantification in a metalanguage. See for example Baruch Brody's definition in 'Logical Terms, Glossary of,' *The Encyclopaedia of Philosophy*, and Christopher Kirwan's discussion in *Logic and Argument* (New York: New York University Press, 1978), esp. pp. 20—21.

[47] One might attempt to avoid paradox here by insisting that the truth or falsity of (31) be represented by '$Tr_{i+1}(\sim Tr_i(31))$' and '$\sim Tr_{i+1}(\sim Tr_i(31))$', blocking substitution of '(31)' for '$\sim Tr_i(31)$.' One might even resist (31) itself on the grounds that formulae themselves, rather than their names, must follow 'Tr_i'.

This, however, would be effectively to propose that 'Tr_{i+1}' or 'Tr_i' be treated on the model of an operator, violating Burge's emphatic insistence that truth be treated as a predicate. The difficulties of operator views, of course, have been detailed in Section VII.

[48] A variation is this. Consider:

32′. Some instance of (33′) is not true.

33′. $Tr\,(32')$.

Are all instances of (33′) true or not? If every instance of (33′) *is* true, (32′) is false. But if some instance of (33′) is false, it appears, (32′) is simply true.

I am grateful to David Boyer for planting the seed of this paradox.

[49] This might not even be theologically satisfactory, of course — it appears for example to conflict with a traditional notion of God's simplicity.

[50] This conclusion is, moreover, buttressed by work in other quarters. See my 'Some Neglected Problems of Omniscience,' *op. cit.*, 'Against Omniscience: The Case from Essential Indexicals,' *Noûs,* **19** (1985), 151—180, 'There Is No Set Of All Truths,' *Analysis,* **44** (1984), 206—208, and 'Logic and Limits of Knowledge and Truth,' forthcoming in *Noûs.*

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