Worlds by supervenience: some further problems

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Allen Hazen has recently proposed a new approach to possible worlds, designed explicitly to overcome the Cantorian difficulties I've emphasized elsewhere for possible worlds construed as maximal consistent sets of propositions.[1] In what follows I want to emphasize some of the distinctive features of Hazenworlds, some of their weaknesses, and some further Cantorian problems for worlds against which they seem powerless.

1. Hazen concedes that worlds cannot be construed as, or as corresponding to, maximal sets of propositions: there can be no such maximal sets, and thus can be no such possible worlds. For slightly more complex reasons, Hazen also concedes that worlds cannot be construed as, or as corresponding to, maximal sets of contingent propositions. But perhaps worlds *can* be thought of as corresponding to distinguishing *non*-maximal sets of contingent propositions. Perhaps, for every world, there is a set of contingent propositions on which all other propositions true at that world are supervenient.

Hazen's approach is in some ways reminiscent of atomism – he uses Democritean worlds to introduce Hazenworlds – but it is important to note that his are not the worlds of classical *logical* atomism. Those were worlds in which all propositions, contingent or necessary, were logical complexes built from some set of atomic propositions. But logical atomism falls victim to a Cantorian modus tollens. Logical complexes built from a starter set of atomic propositions can give us at most a set of propositions true at a world. There is no set of all propositions true at any world, and thus for no world will logical atomism be adequate.

Hazenworlds promise a narrow escape from such an argument by relying on a notion of *supervenience* rather than logical complex. The difficulty for logical atomism is that 'is a logical complex of' preserves set-sizedness: logical complexes constructed from a starter set of atomic propositions must also form a set. Hazen's proposal is that supervenience need not be like that: those propositions supervenient on a starter set need not form a set.

There is a weakness in the motivating argument for Hazenworlds, however, and also a basic weakness in what we finally get.

Hazen builds a case for the plausibility of his worlds by starting with

[1] See Hazen 1996 on Grim 1984, 1986, and 1991. See also Plantinga and Grim 1993.

Analysis 57.2, April 1997, pp. 146–151.

Democritean worlds, consisting of a continuum of space-time points each of which is either simply full F or empty E. If we assume space can be filled only by chunks with continuous surfaces, a denumerable set of contingent truths for such a simple world will be specifiable in a finite language. More complex worlds can be envisaged with more than continuum-many points and with some corresponding large set of independent properties. But 'under a weak hypothesis', Hazen says, each world we imagine in such a progression will be 'fully described' by a set of atomic sentences in some formal language, where 'languages' are extended to include the possibility of non-denumerable vocabularies, sentences of infinite length, and the like. Hazen's crucial 'weak hypothesis' is that each world contain a mere set of 'physical' objects, characterized by a mere set of independent properties.

Despite Hazen's characterization, however, there is no need to restrict the 'weak hypothesis' to 'physical' objects: what is required is simply that each world contain a mere set of contingently existent particulars characterized by a mere set of properties. Whether the particulars are 'physical' or not has nothing to do with the basic problems of cardinality: a set of spiritual entities, say, with sets of beliefs or states of consciousness irreducible to physical states, would satisfy the necessary restrictions just as well.

The 'weak hypothesis' is in fact the weak point in the proposal. What it demands is that all worlds be restricted to mere sets of independent things with mere sets of independent properties. One might think that hypothesis was true if one thought that there was a limit to conceivable totalities, and that that limit was given by principles very much like the axioms of set theory.[2] But Hazen doesn't think that the limits of sets are the limits of conceivable totalities: on his approach we are supposed to accept a larger-than-set number of propositions true at our world, a larger-than-set number of contingent propositions true at our world, and evidently (as outlined below) a larger-than-set number of worlds. Given *that* acceptance of non-set totalities, why should one balk at a similarly non-set totality of independent things or independent properties in a possible world? If the numbers are the same, it can't be a matter of numbers.

A world populated by a complete academy of experts, each an expert on a particular ordinal, would constitute this kind of world. A world populated by an academy of experts, each an expert on a particular possible

2 This is of course something like the position argued in Grim 1991 and Plantinga and Grim 1993. For rhetorical purposes I speak here of 'non-set totalities' where I in fact think there can be no totalities at all. A complex issue related to this other work, but which I haven't pressed here, concerns universal quantification in the closure propositions required for Hazenworlds.

world, or a set of possible worlds, would apparently constitute such a world. Beliefs themselves might plausibly count as contingent entities in the sense required, and thus a world in which each true proposition is believed would be such a world. No such world, it appears, could qualify as a Hazenworld. But then without further reason to believe there really can be no such worlds there seems little reason to believe Hazen's as an outline adequate for worlds in general: there seems little reason to believe that worlds in general must be Hazen.

There are also some weaknesses in what we'd ultimately get from Hazenworlds. The promise of an outline of possible worlds in terms of maximal sets of propositions was that our modal notions could then be understood derivatively. 'Truth in a possible world' would simply amount to inclusion in such a maximal set. Contingency would amount to inclusion in some such set, necessity to inclusion in all. Despite the fact that Hazen speaks of worlds 'defined as' the atomic sets of contingent propositions at issue, 'determined by', 'fully specified' by, or 'fully described' by those sets, however, it's not the case that something is true at a possible world, in the familiar sense, just in case it is a member of the definitional set. Contingency doesn't amount to inclusion in one such set, and necessity doesn't amount to inclusion in all. Unlike standard possible worlds, Hazen's sets are in fact quite carefully designed to be incomplete: were they to include all the relevant truths they would once again fall victim to Cantorian argument. Thus the most Hazen's sets can do is *index* possible worlds, themselves construed in their entirety as including all that supervenes on those basic sets. In that richer sense possible worlds can still not correspond to complete sets or complete novels.

A major weakness in Hazenworlds is of course that they are defined in terms of an essential notion of 'supervenience', for which neither Hazen nor anyone else seems to be able to offer a satisfactory outline. There are a few things it is clear that 'supervenience' cannot mean for Hazen's purposes, however. '*y* supervenes on propositions of set X' *cannot* mean, for any formal system, that members of X *entail y*, even if we broaden 'formal system' like 'language' so as to allow non-denumerably many axioms, proofs of infinite length, and the like. A basic set of contingent propositions in any formal system in even this extended sense will generate a set of entailments. Hazen has to maintain that there is no set of all propositions supervenient on his basic sets, however, and thus has to maintain that supervenience cannot be explicated in terms of entailment.

Indeed 'supervenience' cannot even be a *relation* in the familiar sense, modellable by a set of ordered pairs. A set of ordered pairs with elements of the basic set X on one side would represent a relation only between X and a set of propositions true at a world. There is no set of all propositions true at a world, and thus no such relation could do what supervenience is supposed to.

Perhaps these points merely reinforce an impression clear from the beginning, however, that supervenience must be a quite thoroughly modal notion, begging for explanation in terms of the familiar mechanism of possible worlds. If that is required, of course, Hazenworlds will clearly not have taken us very far.

2. Hazenworlds were constructed in order to avoid one set of Cantorian difficulties: the difficulties of construing worlds as complete sets of propositions. But there are other and perhaps deeper Cantorian difficulties facing possible world semantics as well. Against these Hazenworlds seem as helpless as more standard alternatives.

The deeper difficulties at issue concern not the sets that have been proposed *as* possible worlds, but the basic notion of sets *of* possible worlds. Even those not committed to the former seem bound to the latter: the whole promise of possible world semantics, after all, has been the promise of a set-theoretical semantics for modal logic written in terms of sets of possible worlds.

Can there be a set of all worlds? Here I want to strengthen an argument, credited to David Kaplan and Christopher Peacocke,[3] that there cannot. The original Kaplan-Peacocke argument runs as follows:

> Suppose for a proof by contradiction that there is a set of all worlds, and that its cardinality is κ.
>
> To each subset S of the set of worlds will correspond a distinct proposition. (A proposition true in precisely the worlds of S, perhaps, or a proposition of the form '$P \in S$' for a chosen proposition P.)
>
> For each such proposition, it is possible that David Kaplan is contemplating that proposition and no other on April 1st, 1997, at 3:00 EST.
>
> There are at least as many worlds as these distinct Kaplan-contemplation possibilities.
>
> There are 2^κ such propositions, 2^κ distinct Kaplan-contemplation possibilities, and thus at least 2^κ possible worlds. $2^\kappa > \kappa$, and we have derived a contradiction.

In this form the argument is not quite conclusive. The obvious response is David Lewis's, denying the third step: propositions regarding sets of worlds, however they might be constructed, will outstrip possibilities of Kaplan contemplation. 'It is absolutely impossible that anyone should

[3] Davies 1991: 262, and Lewis 1986: 104.

think a thought with content given by one of these ineligible sets of worlds.' (Lewis 1986: 105)

The argument can be strengthened, however, so as to close the Lewis gap. Consider the following one-place propositional function, with a variable for ordinals at *o*:

T: On April 1st, 1999, at 3:56 p.m. EST, Grim writes a phrase true of ordinal *o* and no other.

For each ordinal, it appears, T will give us a distinct proposition. But if each such proposition is contingent, we will have as many distinct possible worlds as ordinals. There are too many ordinals for any set, and thus could be no set of all worlds.

Here the Lewis gambit is also clear, however: maintain that T does *not* gives us a contingent proposition for each ordinal. For some ordinals T will give us a necessarily false proposition instead: 'it is absolutely impossible that Grim should write a phrase true of precisely one of these ineligible ordinals.'

At this stage in the strengthened argument, however, our problems just get worse. If there are ordinals for which the instantiation of T is necessarily false, there must be a *first* ordinal for which the instantiation of T is necessarily false. But we can certainly bring it about as a contingent matter of fact that on April 1st, 1999, at 3:56 p.m. EST, Grim writes the following phrase:

'The first ordinal for which the instantiation of T is necessarily false.'

On the defence outlined, a mere set of possible worlds entails that there is a first ordinal for which the instantiation of T is necessarily false. But in that case what Grim is contingently going to write will be a phrase *true* of that unique ordinal, and thus the instantiation of T for that ordinal will be contingently true rather than necessarily false.

Contradiction. There can be no first ordinal for which the instantiation of T is necessarily false, and thus contrary to the Lewis gambit there can be no ordinal at all for which the instantiation is necessarily false. Given that Grim is not a necessary being, each instance of T must be contingent. But there are then at least as many distinct propositions and hence as many possible worlds as there are ordinals. There can then be no set of them all.

3. The supervenient atomism of Hazenworlds is an important addition to the literature on possible worlds, and certainly worthy of further work. What I've tried to show is that there are nonetheless important gaps in the argument that possible worlds are generally Hazen, that there are major philosophical deficiencies in what they yet have to offer, and that

Cantorian problems remain for possible world semantics against which Hazenworlds seem as powerless as their alternatives. [4]

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[4] I am grateful to Allen Hazen for fruitful discussion.