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ORIGINAL RESEARCH

Lagrangian possibilities

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Abstract
Natural modalities are often analysed from an abstract point of view where they are associated with putative laws of nature. However, the way possibilities are represented in physics is more complex. Lagrangian mechanics, for instance, involves two different layers of modalities: kinematical and dynamical possibilities. This paper examines the status of these two layers, both in the classical and quantum case. The quantum case is particularly problematic: we identify four possible interpretive options. The upshot is that a close inspection of the way possibilities are represented in physics could lead to new ways of thinking about natural modalities.

Keywords Lagrangian mechanics · Modalities · Laws of nature · Feynman path integral · Kinematic possibilities

1 Introduction

Natural possibilities can be understood as the subset of metaphysical possibilities that are compatible with the laws of nature. This understanding is associated with different conceptions of laws of nature: (a) they constrain what is naturally possible or not in our universe (or derive from such constraints), in an objective, mind-independent sense of "possible" that cannot be analysed in terms of what is actual (Armstrong, 1983; Dretske, 1977; Bird, 2007), or (b) they supervene on non-modal natural facts in such a non arbitrary way that they license talking in modal terms (Lewis, 1973, Chap. 3).[1] It is generally thought that science, and physics in particular, is in the business of uncovering these laws.

[1] From these seminal works many research avenues were explored, but these more recent propositions do not impact our argument.

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All this could let us think that examining the structure of models and theories in physics could give us a good grasp of what natural possibilities are, metaphysically speaking. When the theories of physics are considered, the idea generally put forth is that each model of the theory represents a "possible world according to the theory", and that therefore, natural modalities can be simply analysed in terms of what is "true in all models", at the level of theories, that is (See for example (Ruetsche, 2013, Chap. 1; van Fraassen, 1980, p. 47). However, this widespread idea is prima facie disconnected from actual science. A model of a pendulum is not a representation of a world in any relevant sense, for instance. In practice, different models rather correspond to different *applications* of the theory. Furthermore, probabilistic models represent various alternative possibilities rather than the extensional content of a single possible world.[2]

One can always postulate that the standard possible-world picture could in principle accommodate all the messiness of scientific practice, but this is a mere postulate, and we believe that focusing on this practice directly rather than on idealistic reconstructions can lead to interesting insights that are easily overlooked by the traditional approach. In particular, we believe that looking at the modal structure of the models used in scientific applications can provide us a more accurate understanding of the ontological commitments associated with modal talk in physics. That is, it can be instructive to start by looking for modalities *within models* in order to extract the ontological commitments associated with their use. The main objective of this paper is to show by example that adopting this bottom-up approach leads to interesting insights with regards to the metaphysics of natural possibilities and enriches the debate.

These analyses will be carried out in the case of classical and quantum Lagrangian formalism (Feynman functional integral). The reasons for this choice are (1) that Lagrangian formalism allows us to explore in a rigorous way a vast class of physical models, from Newtonian mechanics to quantum field theories. (2) Lagrangian formalism focuses on the dynamics, the core of physics. (3) Lagrangian formalism allows for a smooth transition from classical mechanics to quantum physics (relativistic or not). (4) Lagrangian formalism remains relatively neutral in the metaphysical debate about the status of space and time (Belot, 2005). For these reasons, the Lagrangian formalism looks like a promising framework to understand the modal aspects of physical models. Having said that, it could be interesting to carry out the same analyses in other general formalisms, which we leave for future research.

Model construction in Lagrangian mechanics involves two modal structures: the set of kinematical possibilities and the set of dynamical possibilities (not our choice of terminology). This already brings up an apparent conflict with traditional understandings of natural modalities, which generally assume only one layer of natural possibilities. The paper will focus on the different ways of interpreting these two layers.

The upshot of our analysis is the following. While the classical case could in principle be amenable to traditional understandings of natural modalities (but already opens more options), the quantum case raises an issue for the metaphysics of modal-

[2] Similar remarks were made by Kripke (1980, preface), with his contention that possible worlds should be construed as "miniworlds" attached to particular objects, by Cartwright's and her notion of a nomological machine, and by the research on modelling activities that her work initiated within the so-called "practice turn" in philosophy of science (see for example the collective monograph (Morgan & Morrison, 1999).

ities, because incompatible classical and quantum "possibilities" coexist in the same modelling practice. This demonstrates that examining the ontological commitments associated with modelling practice, in particular when it comes to modalities, can substantially enrich the metaphysical debate. In Sect. 2, we present the classical case and examine the various interpretive options. In Sect. 3, we examine the quantum case, and show that it leads to a puzzle concerning the status of kinematical possibilities. In Sect. 4, we express this puzzle in the form of a quadrilemma and examine the four possible solutions. We conclude with a few general comments in Sect. 5.

2 Classical kinematical possibilities

2.1 The Lagrangian formalism

Let us first give an informal presentation of classical Lagrangian mechanics.[3] Consider a classical system.[4] A possible history of this system is called in the physics literature a *kinematical possibility*. It is the primitive notion of this formalism.[5] Together these histories form a set K. Note that K is defined independently of the actual dynamical laws acting on the system. In fact, among the elements of K only a subset D, called *dynamical possibilities*, are compatible with the actual dynamics at play on the system. The role of the Lagrangian formalism is precisely to provide an efficient way to identify this subset. Characterizing the sense of "possibility" that is at stake in this modelling practice will be the object of this section.

In practice, one builds K from sets of independent variables (typically time) and dependent variables of the system (its degrees of freedom, e.g. positions). Dependent variables are usually defined using a configuration space: a space of parameters (called generalized coordinates) that are supposed to represent the possible states of the system under known constraints, in one sense of possible. We understand the word "constraint" here as meaning any limitation of the possible histories of a physical system that are "a priori" in the sense of being part of the conditions of application or relevance of the model, not its conditions of empirical accuracy: if these constraints did not obtain, the physical system would be faulty (not the one intended by the application) rather than the model. We could assume, for example, that the model represents a gas trapped in a box, and therefore limit K to the histories confined in the box. This is to be contrasted with the natural constraints on possibilities *given these limitations* that a wrong model can potentially misrepresent (the histories of the gas that are naturally possible, given that it is trapped in the box–however, we will see later that this distinction between a

[3] For an introduction to the Lagrangian formalism, see (Scheck, 2010). For a mathematical introduction, see (Choquet-Bruhat et al., 1982).

[4] Note that defining precisely a system is not trivial. It can be understood as (a) a collection of entities forming a whole, (b) a connected portion of spacetime, (c) an entity for which what happens on its frontiers, understood in a broad sense, is known, or (d) the subject of dynamics. In this paper, we take systems to be entities at the intersection of c and d, which seem to be the most appropriate definitions for Lagrangian mechanics, in particular in the quantum context.

[5] This assertion could surprise the reader that only learned about the Lagrangian formalism in physics textbooks. We provide reasons to defend it below.

priori and a posteriori constraints is not always obvious). The set K encodes these "a priori" limitations or conditions of relevance for a model, but not only them: it also encodes general constraints that are imposed by the Lagrangian framework, such as the continuity and differentiability of the histories, as well as general theoretical posits (e.g. that a gas is composed of particles).

Once we are given the relevant sets of independent and dependent variables, we can define the set of kinematical possibilities K as the set of continuous and differentiable mappings from the values of independent variables to values of dependent variables (histories). As mentioned above, specifying the values of dependent variables amounts to specifying a *configuration* for the system, a point in configuration space, so a kinematical possibility can also be represented as a (typically time-like) evolution in the configuration space of the system. However, the Lagrangian formalism is relatively neutral with regards to metaphysical conceptions of change and time. For example, a history can typically be re-parametrised in order to make time a dependent variable (Rovelli et al., 2015, Chap. 2).[6] This confirms that the notion of "history" of a system is primitive in this modelling practice. The status of time, change or what is a system is not fixed by the Lagrangian formalism, but histories are assumed. Our ontological analysis is relative to this basic choice of taking histories as a starting point of the modelling.

The Lagrangian procedure is the following:

- Encode all dynamical properties in the action functional S generically defined as a differentiable functional mapping that assigns to each kinematical history a real number, $S : K \to \mathcal{R}$.[7]
- A dynamical history is a stationary point of S. The set of all stationary points forms D.

This procedure is sometimes called a principle of least action because in many cases the elements of D are not only stationary points but also minima of the action functional.[8]

Once we have D, we can deduce from initial conditions (the beginning of a history) and potential final conditions the history that will actually occur (if it exists according to the laws of nature): it is the element of D compatible with these conditions. We assume, here, that the initial and potential final conditions are not built into the model, but are invoked only in a specific application. In this context, it is natural to think that the set D supports counterfactuals of the form "if the initial conditions at time t_0 had been c_0, then the final conditions at time t_1 would have been c_1". This is because D encodes the action of dynamical laws, and the material conditional associated with this counterfactual is true of all dynamical possibilities (see (Willamson, 2016)) for arguments to that effect). Such subjunctive conditionals are usually interpreted in terms of objective possibilities. Following this reasoning, perhaps the set of dynamical

[6] Note that we will put aside the problem of the correspondence between histories and measured physical evolution. In this paper, we concentrate on natural possibilities from a theoretical point of view. A complete picture should include the notion of experimental or empirical possibilities. For an introduction to the problem of the correspondence between theoretical and empirical states, see (Redhead, 2003).

[7] Note that S depends on the choice of variables that parametrise kinematical possibilities.

[8] For a discussion of this point, see (Arnold, 1989) part 2.

possibilities is simply the set of *natural* possibilities. But it should be noted that it only constitutes a subset of the larger set of kinematical possibilities.

Kinematical possibilities are proper objects used in the construction of models. They cannot be identified with what is merely conceivable, or with epistemic possibilities (what is compatible with what we know about the system), since they must respect continuity and differentiability constraints, whereas arguably, a discontinuous trajectory is conceivable, and compatible with our empirical knowledge. Kinematical possibilities are assigned properties, the actions. Furthermore, the space of kinematical possibilities is somehow structured: selecting stationary actions implies a notion of distance between possibilities (the action of a history is stationary *relative to nearby histories*; this notion of distance can be analysed in terms of Lewis's possible worlds semantics (Butterfield, 2002)).

This prompts the question: what exactly is the metaphysical status of kinematical possibilities, since they apparently form a larger class than the natural possibilities but a smaller one than the conceptual possibilities? Or, to say it differently, what kind of ontological commitment comes with the use of kinematical possibilities in our representations of the world?

Not everyone agrees that isolated possibilities are ontologically committing, in the sense that believing that something is objectively possible instead of remaining agnostic would make a practical difference. However, the idea that relations of necessity are committing (because they constrain inferences, support counterfactual reasoning, etc.) is less controversial, and can be agreed on even by an agnostic with regards to the existence of objective possibilities (Divers, 2004).[9] The sense in which scientists would be committed to a set of kinematical or dynamical possibilities can, in any case, be interpreted as a commitment to a statement of necessity associated with the delimitation of this set. That is, when defining a relevant set of possibilities, scientists assume that *necessarily*, the history of the system lies within it, and this assumption constrains their inferences about the system (this is true at least in the classical case– the quantum case will be discussed later). From this perspective, the question that we want to ask in this paper is: in what sense of "necessarily"?

In the rest of this section, we provide a short analysis of the options at our disposal in the classical case.

2.2 Interpreting modalities in classical Lagrangian mechanics

When asked for the metaphysical status of an entity that figures in our representations, there is always the option of taking an instrumentalist stance. In the case of kinematical possibilities, this would mean interpreting them as mere calculation tools with no counterpart in reality.

However, most contemporary philosophers are not satisfied with instrumentalism in general. Note that this dissatisfaction is not restricted to scientific realists. It can be expressed in the form of Quine's indispensability thesis, according to which we should be ontologically committed to whatever plays an indispensable role in our best science. The indispensability thesis likely applies to the set of kinematical possibilities

[9] We thank an anonymous reviewer for drawing our attention to this aspect.

(and associated constraints of necessity) in the context of Lagrangian mechanics. The indispensability of the Lagrangian formalism itself could be debated of course. However, the centrality of this formalism in relativistic physics, especially in general relativity, let us think that an indispensability argument could be produced in the context of classical (non-quantum) physics. We do not think that this thesis necessarily implies a strong realist understanding of ontological commitments in terms of outright belief, transcendental reference and correspondence truth. However, let us leave this discussion aside for the moment and examine other options. It will be raised again in the quantum case.

We have already rejected in the previous section an epistemic reading of kinematical possibilities.[10] If we also reject instrumentalism and understand kinematical possibilities as a species of objective possibilities, how shall we interpret them?

We know that in this classical context, for any admissible dynamics, the set D is strictly included in K. Metaphysicians often assume that natural possibilities are a subset of metaphysical possibilities. A first option (A) is therefore to interpret K as the set of metaphysical possibilities for a system, and D as its natural possibilities. This is the option defended in Hirèche et al. (2021). In so far as it is distinguished from conceptual necessity, metaphysical necessity is generally understood as referring to what must be true in virtue of the fundamental nature, essence or identity of things (Fine, 2002; Hale, 2012) (the most unrestricted form of mind-independent necessity). Here, the idea could be that having continuous and differentiable histories is part of the metaphysical nature of physical systems, or at least of classical physical systems.

If kinematical possibilities did not respect continuity and differentiability constraints, it would be impossible to assign an action functional to them using the usual techniques. Another way to say this is that kinematical possibilities must be compatible with the existence of laws of nature (of a classical kind), *whatever these laws are*. If, say, the law of gravitation had been different, then the action functional, hence the set D of a given system, would have been different, but K would be the same, since it does not depend on the action. In this restrictive sense, we can say that kinematical possibilities are the different histories of the same system governed by different classical physical laws. This reading corresponds to the ordinary way physicists understand the variational principle[11] of the Lagrangian formalism: conserve the identity of the system under study and allow the dynamical laws to vary.

Many philosophers assume that the laws of nature are metaphysically contingent, so that it makes sense to ask "what would the world be like if the laws were different?"

[10] Note that epistemic constraints are obviously involved in scientific applications from the first stages of model building, when it comes to evaluating whether the system is relevant for the application. We shall implicitly assume, from now on, that the possibilities considered in our analysis are relative to there being a system of the relevant type (the one that the model aims at representing, including all "a priori" constraints: see previous remarks). This includes kinematical possibilities. The point of the previous section is that kinematical possibilities do not merely correspond to all the logical or conceptual possibilities compatible with these relevance conditions, because they incorporate continuity and differentiability constraints which are neither empirical nor a priori. What is at stake is understanding the commitments associated with these constraints, and the associated objective necessity (assuming Lagrangian models are accurate).

[11] Variational principles are formal methods used in mathematics and physics where we optimise a function in order to get a solution. For example, in the Lagrangian context, we vary the laws (look at different S) in order to explore different dynamics.

(for example (Roberts, 2008)). We can see that interpreting kinematical possibilities as metaphysical possibilities is congruent with this assumption. A difficulty of this reading is that it requires that laws of a certain (classical physics) kind be always applicable, so that metaphysical possibilities must accord with a classical physics ontology. In light of more recent theories, this could seem implausible. If we knew that the universe were classical, we could postulate that it is part of the nature of physical entities to have continuous and differentiable histories, but quantum mechanics seems to invalidate this idea.

Another option (B) is to assume that kinematical possibilities constitute an intermediate level between metaphysical and natural possibilities. This would be the set of metaphysical possibilities for a specific system compatible with "meta-laws" or higher-level laws, associated with the general framework of our physical theories (its spacetime symmetries, etc.), but independent from more specific dynamical laws, such as the Newtonian law of gravitation. Similar ideas have been entertained by Lange (2009).[12] In our context, applying Lange's account, both K and D would correspond to sub-nomically stable sets of truths (stable under counterfactual suppositions compatible with all members of the set), with the set of kinematical laws strictly contained in the set of dynamical laws.

The difficulty with this option is firstly practical, but has philosophical consequences. If we define kinematical possibilities as metaphysical possibilities of a certain type, in order to make the Lagrangian formalism able to accommodate non-classical models (assuming its generality beyond classical mechanics), we would need to build alternative sets of possible histories that are compatible with different "meta-laws" and at the same time support the variational principle at play in the Lagrangian formalism. However, we do not know how to build such sets, which casts doubt on their conceivability. As we will see soon, in the quantum case, we still rely on the same classical K, which apparently contradicts this option. This does not strictly exclude it, but places the burden of proof on its defender to convince us that alternative "meta-laws" to the classical ones are metaphysically possible.

None of options (A) and (B) are available for dispositional essentialists, who assume that the laws of nature are empirical regularities that result from the action of the essential dispositions of physical entities (Bird, 2007). According to dispositional essentialism, laws of nature are metaphysically necessary conditional on the instantiation of properties. For a given system that instantiates a set of properties, there is only one layer of objective modalities; natural and metaphysical possibilities are the same. In this nomological conception, the same system could not have been governed by different laws and the status of kinematical possibilities remains unexplained.

In order to defend the next option, a possible confusion should be lifted. A naive interpretation of the Lagrangian formalism would claim that all a priori dynamical limitations are encoded in the set of histories K and all nomological information in the action functional S. This is not always the case. Let us imagine a situation A where a gas is trapped in a closed box, and a situation B where the lid of the box is now open and the gas can diffuse freely. There are two ways of modelling this difference. In a

[12] An anonymous reviewer mentions a difference, in Lange, between a first-order law that would be high on the hierarchy and a meta-law. What we have in mind is presumably closer to the former.

first model, dynamical laws are the same in the two situations ($S_A = S_B$), but the set of kinematical possibilities is different ($K_A \neq K_B$) because we passed from a closed system to an open one. This is the default interpretation. In a second model, the set of possible histories is unchanged ($K_A = K_B$), but the dynamics changed ($S_A \neq S_B$) because new regions of K became dynamically accessible in situation B. When we model a system, nomological and contingent information can be found in every aspect of the formalism. In consequence, we should be careful not to read too rapidly the metaphysics from the physics: the formalism is open to interpretation concerning what exactly corresponds in it to contingent or necessary aspects.

This last case (the second way) supports a final option (C), which consists in assuming that K corresponds to natural possibilities and D to sub-natural possibilities. What we call sub-natural possibilities are a species of *relative modalities*: they correspond to what is dynamically possible *given local environmental constraints* (see (Ruyant, 2021, Chap. 5)) for an account of situated possibilities). In other words, as far as K is concerned, dynamical laws are fixed, but the environment of the system is allowed to vary, yielding different possible D. The set K does not represent the histories of a system subjected to different possible laws, but, under fixed laws, the histories of a system subjected to different possible environmental constraints (Note that this is also compatible with Lange's account of a hierarchy of laws, assuming that D still corresponds to a sub-nomically stable set of truths). This interpretation is a consequence of conventionalism (Menaem, 2006). All things equal, the same inverse r^2 attraction force behaviour measured between two particles could be the result of an inverse r^2 attraction force acting between two particles in an inert vacuum (the environment as far as the model is concerned) or the result of an r^3 attraction force acting between the two particles plus a specific force produced by vacuum acting on particles.

This option makes sense in so far as the systems modelled in Lagrangian mechanics never correspond to the universe as a whole (otherwise the required distinction between system and environment would break down). They more generally correspond to subsystems of the universe subject to particular constraints, due to the way they interact with their environment (and to the laws of nature of course). The action, from which dynamical possibilities are selected, integrates in part these environmental constraints, for example in the form of an external potential. An interesting conjecture is that any given kinematical possibility would be a dynamical possibility *had environmental constraints been different* (they could have a stationary action with at least one possible classical S) with the same laws of nature. If this were so, then kinematical possibilities would just be the set of possibilities that are compatible with actual laws of nature, and dynamical possibilities woud be the subset of them compatible with specific constraints.

Under this conjecture, being compatible with specific laws of nature, such as Newton's law of gravitation (assuming possible variations in the environment) or being compatible with higher-level laws, that is, with the fundamental principles of classical mechanics, would not make any real difference, because changes in the environment could "mimic" any change in the low-level laws. This could have interesting implications with regards to the metaphysics of laws. One could, for example, combine a dispositionalist account of bounded systems with a best system analysis of laws at the level of the universe: they would correspond to regularities in instantiated dispositions (this conception of laws has been suggested recently in Kimpton-Nye (2017) and Demarest (2017)).

As we can see, there is room for various options regarding the metaphysical status of kinematical and dynamical possibilities in the classical case. Excluding instrumentalism, the first of these options (A) is conservative with regards to the traditional understanding of natural possibilities being a subset of metaphysical possibilities, but it is somehow implausible, because it imposes that metaphysical possibilities should only be compatible with classical dynamical laws. The second option (B) is somehow revisionary, because it assumes a layer of modalities in between metaphysical and natural modalities corresponding to what is possible according to laws of a certain type (compatible with higher-level laws). This layer would play an important role in science. This option avoids the difficulty of the first one, but it is practically implausible: we do not know how to build a Lagrangian formalism that would be general enough to be compatible with several "meta-laws", which casts doubts on its viability. The third option (C) is more compatible with traditional modal metaphysics, but it could imply novel conceptions of laws that have only been considered recently.

It is not clear whether one option stands out as the right one, but this is quite routine in metaphysical discussions, so the case of classical Lagrangian mechanics should not cause much trouble to the metaphysician. Nevertheless, adopting a bottom-up approach has proved fruitful: different options regarding the status of objective modalities and their costs have been clarified. As we will now see, the quantum case raises new interesting issues.

3 Quantum kinematical possibilities

3.1 The Feynman functional integral formulation

The functional integral (or path integral) formulation of quantum mechanics was developed by Richard Feynman in an attempt to provide a Lagrangian formulation of quantum mechanics in place of the Hamiltonian one that was used by physicists at the time. In the spirit of Feynman (1948), we consider Feynman's formalism to be a genuine formulation of quantum mechanics, and not a mere computational tool. Belief in the autonomy of this formulation is reinforced by the fact that when it was developed, having such a formulation seemed essential to quantize general relativity. Moreover, it has been used successfully in the context of perturbative and topological quantum field theory, for which we do not always have available alternative formulations.

The formalism can be presented informally by adapting our recipe from the previous section (for a technical but nevertheless introductory presentation, see (MacKenzie, 2000)).

According to this formulation, the transition amplitude $\langle out|in\rangle$ is the basic way to represent the quantum process between in and out.[13] They are analogous to the dynamical possibilities in the classical case because they are what the formulation

[13] Transition amplitudes are a generalization of the notion of Feynman propagator: the probability amplitude that a particle is at a certain spacetime location if it was at another before.

produces as physical results. Each transition amplitude is a complex number. The absolute square value of this number gives us the probability for the system if it was in to become out. How are the transition amplitudes computed from their associated classical counterparts? We have to follow these steps:

- Consider the set of kinematical possibilities of the classical system for which we want a quantum description.
- Consider the action functional for this classical system.
- Assign to each kinematical possibility a *phase*, which is proportional to the action associated to this possibility.
- To obtain the transition amplitude between in and out, integrate the phases of all kinematical possibilities that start at in and finish at out.[14] Note that this requires knowing the measure of the space K, that is, its local density.

The relationship between the classical and quantum cases is the following. When the phases of histories are summed to compute an amplitude, they can interfere either constructively or destructively (either they add up or they cancel each other out). In general, the closer a history is to a dynamical (stationary) history of the corresponding classical model, the more it interferes constructively with others. In a sense, the quantum model *explains* why classical histories have a stationary action: this is because they are the locus of constructive interferences. Nevertheless, it should not be forgotten that all possible histories between in and out contribute to the amplitude $\langle out|in\rangle$.

3.2 Interpreting modalities in Feynman functional integral formulation

As in the classical case, it is natural to assume that the kind of representation provided by Lagrangian mechanics supports counterfactuals of the kind "if the system had been initially in conditions in, then it would have been in conditions out with probability p". Since we can compute, using the transition amplitude $\langle out|in\rangle$, the corresponding probability, we can associate to this amplitude a natural possibility, conditional on the fact that this probability is not 0. But as in the classical case, transition amplitudes are not the only modal structure: we also have kinematical possibilities, which respect various criteria (continuity and differentiability), and which are assigned a (physical) quantity, the action phase. Furthermore, the space K is also structured: integrating action phases over histories implies that there is a well defined *density* of histories in configuration space. So, again, the modal structure of K is not trivial, and it deserves a proper metaphysical interpretation.

However, there is an important difference. In the classical case, dynamical possibilities are a subset of kinematical possibilities, the ones that are "selected" by the action functional. This is not so in the quantum case. Whereas kinematical possibilities are continuous histories, the transition amplitudes (the analogues of classical dynamical possibilities) are discontinuous transitions between two configurations. This leads to a puzzle that will occur however we choose to interpret these modalities.

[14] Note that we use the same notation "in" and "out" for quantum transition amplitudes and classical kinematical histories even if these notions are not exactly the same. The difference will not have any effect on our arguments.

Assume, for example, that transition amplitudes represent natural possibilities, and that kinematical possibilities are the set of metaphysical possibilities of the system. Then what is naturally possible for the system is not a subset of what is metaphysically possible, and none of the metaphysical possibilities of the system is eventually realised. This does not seem to make much sense. The same goes if we associate K with an intermediate level between natural and metaphysical modalities, or if we consider transition amplitudes to be sub-natural possibilities. In all cases, kinematical possibilities are, in a sense, *impossible*: they cannot be realised. This is actually a well-known result that follows directly from Kochen–Specker theorem: in standard quantum mechanics, one cannot attribute a definite value to all the quantities for the system that parametrise the set K.[15] Nevertheless, we need to consider this set of "impossible histories" in order to find the dynamical possibilities of the system.

This shows how our bottom-up approach, which consists in analysing modalities *within* models, is informative for the metaphysics of modalities: it raises puzzles regarding the status of objective modalities that would go unnoticed if we adopted the traditional top-down approach.

4 A quadrilemma for kinematical possibilities

4.1 The quadrilemma

The last section ended with a puzzle for the interpretation of kinematical possibilities in Feynman functional integral formulation: although apparently indispensable, these possibilities do not represent natural possibilities.

Let us first formulate more clearly our puzzle. We will use Quine's indispensability thesis for this purpose (the thesis is discussed below).

QUINE: We should be ontologically committed to the entities that play an indispensable role in our best scientific theories or models.

INDISP: The set of kinematical possibilities plays an indispensable role in the Feynman functional integral formulation. This theoretical framework is the structure in which we built among the best models we have of fundamental interactions, solid state physics, etc.

NECESS: Being committed to a set of possibilities for a given system means accepting a corresponding necessity claim: that it is necessary, for the system, to lie in these possibilities.

IMPOSS: It is impossible, for any given system, to have a history corresponding to a kinematical possibility, given that these are incompatible with quantum dynamical possibilities.

If we accept the four premises, we reach a contradiction. QUINE and INDISP entail that we should be committed to the set of kinematical possibilities. This and NECESS entail that we should assume that the dynamical history of the system is necessarily

[15] See (Skow, 2010; Darby, 2010) for an interpretation in terms of "deep" metaphysical indeterminacy: it makes explicit that the corresponding possible worlds are excluded by quantum mechanics.

within the kinematical possibilities. This directly contradicts IMPOSS. One premise must be rejected.

Let us examine them in turn: why they seem prima facie plausible, how they can be rejected, and what this would imply.

4.2 First option: instrumentalism

The first premise of the quadrilemma, QUINE, states that we should be ontologically committed to the entities that are indispensable to our best scientific theories.

This thesis, due to Quine (1948; 1981, Chap. 1), is part of a naturalistic worldview, according to which science is continuous with metaphysics, and according to which science is our best source of knowledge and our best guide in ontological inquiry (see (Guay & Pradeu, 2020) for a taxonomy of such metaphysical projects). The indispensability thesis has been challenged in the context of the philosophy of mathematics. However, some of these challenges do not apply here, since we are not concerned with the existence of abstract entities, such as real numbers, but with the existence of concrete (albeit modal) entities attached to a particular physical system that is being represented. Indeed, Quine's thesis is often accepted in the context of empirical sciences, where entities are assumed to exist when they have explanatory power or are invariant under relevant symmetries (this is a typical abductive argument for realism).

So, what does it mean to be indispensable? Quine understands it in terms of quantification.[16] If the canonical formulation of a theory quantifies over an entity, either existentially or universally, then we should commit to this entity.

Of course, it is always possible, *in principle*, to reformulate a theory so that it only quantifies over a subset of its vocabulary (Craig, 1956). If the only aim of theories were to make empirical predictions concerning, say detection of particles at various positions, then no theoretical variable would strictly be indispensable except for position. However, given that such drastic reformulations are not used by scientists (they are not canonical), it is plausible that making predictions about particle positions is not the only aim of science, and a reformulation of our theories in terms of positions only, even if achievable, would probably fare *worse* than the original theory with respect to other important aims, such as understanding the world, associated with different criteria, such as simplicity and unification. We can presume that the entities referred to in canonical formulations (charge, mass, spin, etc.) are indispensable for these broader aims (confirmation holism, according to which no hypothesis is ever confirmed in isolation, can be used to support this rationale). The naturalist stance assumes that the aims of philosophy are congruent with the aims of science, which implies an ontological commitment towards these entities. This is how the indispensability thesis is generally understood (see (Colyvan, 2019)).

Talking about quantification over entities using variables assumes that theories have a propositional form. It is more common, nowadays, to assume that theories are families of models. However, it is not difficult to transpose Quine's thesis to

[16] There are propositions to go beyond simple quantification, for example (Braillard et al., 2011) but for our argument, quantification will be sufficient.

model-based conceptions of theories: we should be committed to the entities that are represented in our best models, or we should assume that the variables used in canonical model descriptions *refer*.[17] This is the case of the set K in Feynman functional integral models.

This thesis, and notably the reference to non-empirical criteria for being a good theory, seems to imply a realist stance. Assuming that the aim of science is to describe reality, Quine's thesis could be understood as a simple abductive argument from explanatory power to existence of the kind typically entertained by realists. However, we do not think that this is required for our quadrilemma. Quine himself did not seem to understand ontological commitment in a strong realist fashion (Price, 2009).

An ontological commitment can be understood in a fallibilist framework: we assume that our commitments could be defeated in the future. It can be understood in terms of acceptance rather than belief, following van Fraassen (1980)'s distinction. It can also be interpreted in terms of pragmatist notions of truth, objectivity and reference, associated with ideal assertability and norms of inquiry. Quine's indispensability thesis is actually quite natural (perhaps even tautological) in a pragmatist framework. But however we express our ontological commitments, we face a puzzle: accepting all the premises of the quadrilemma forces us to be at the same time committed and uncommitted to the set of kinematical possibilities.

In sum, our premise QUINE can and should be interpreted as a weak indispensability thesis. Rejecting QUINE for *all* theoretical entities amounts to adopting a radical form of instrumentalism, wherein indispensability implies no form of commitment at all, not even acceptance. Scientific discourse should not be taken at face value. It should not be considered truth-apt, whatever the notion of truth one adopts. This kind of position is scarcely ever adopted today.

A rejection of QUINE for *some* theoretical entities could be supported by a recent fictionalist trend in the debates on scientific representation (for example (Frigg, 2010)). Idealisations are ubiquitous in science: physicists often describe frictionless planes, infinitely extended gases, point particles, massless and inelastic strings. They usually present their models as if they were talking about a real object, even though no real object can have such unrealistic features. This has motivated the view that scientific models in general are fictions, or "props in a game of make believe". These fictions can be compared to real objects, but there is no reason to be committed to the full content of our models. Note, however, that contrary to frictionless planes and the like, kinematical possibilities are not specific to particular models. They must be used in *all* models of classical or quantum Lagrangian mechanics. They cannot be "de-idealised" by adding more parameters to the model, because they are full part of the way models are constructed. So, it is not appropriate to view them as idealisations. It might be possible to deny QUINE by being selective in our commitments for other reasons, but the existence of idealisations is not one of them.

Furthermore, contrary to abstract mathematical entities which are used across all sciences, kinematical possibilities are specific to one type of theory. So they are also immune to Sober (1993)'s argument against mathematical realism. Sober argues that

[17] Note that we should exclude from this commitment what is explicitly conventional. For example, it is not significant to thermodynamics whether a temperature is expressed in Celsius or Fahrenheit degrees.

empirical confrontation is contrastive, and that therefore the existence of mathematical entities is not really confirmed together with our theories, because the same entities figure in all theories we can conceive and contrast. This is not the case with kinematical possibilities.

From these considerations, it seems that the interpretation of kinematical possibilities as being at the same time indispensable, but mere tools in order to produce predictions is a no go.

4.3 Second option: eliminativism

Even accepting QUINE as a general thesis, it is possible to interpret the space of kinematical possibilities as a mere calculation tool on the ground that it would not be indispensable to our best science after all. That is, we can reject premise INDISP.

Kinematical possibilities *are* indispensable, in Quine's sense, to the Feynman functional integral formulation because this formulation quantifies over this set. So much cannot be denied. What could be questioned is whether this formulation is one of our best scientific theories.[18] Maybe there exist other formulations of quantum mechanics that dispense with kinematical trajectories. Maybe these other formulations are *better*, and our ontological commitments should be based on them.

The assumption that Feynman's formalism is one of our best scientific theories is based on the observation of scientific practice: this formulation is widely used in contemporary physics with great success. It is also based on the virtues of Lagrangian mechanics mentioned in the introduction and in the previous section: it allows for a smooth transition between classical, quantum and relativistic contexts. This is especially true in the context of quantum gravity research where the Lagrangian framework plays a central role (Oriti, 2009). However, this inference is defeasible. Scientific practice might be rooted in traditions and habits, and there might exist better formulations that do not refer to a structured set K.

Note that "better" should not be understood as "providing better ontological commitments", or "being more compatible with traditional metaphysics", because this would divert Quine's thesis from its original purpose: taking science as it is as a reliable guide in ontological inquiry. Scientists, not philosophers, should judge which theory is better. So, for example, the fact that our puzzle about kinematical possibilities arises should not be a reason to dismiss the Lagrangian formulation altogether.

In any case, even if it could be shown that other formulations fare better than the Feynman functional integral formulation by scientists' own criteria, it is hard to eliminate entirely classical concepts from the interpretation of quantum mechanics, and we suspect that a similar puzzle would arise in other formulations as well, such as the Hilbert space formulation (although we leave this kind of analysis for future research).

[18] We assume that different formulations of quantum mechanics are different theories here: this is required for an application of Quine's indispensability thesis, otherwise ontological commitments would be ambiguous (even taking theories to be families of models instead of sets of propositions: distinct formulations come with different models). Problems of theory identification will not be addressed for lack of space, but we do not think that this affects our conclusions.

In sum, eliminativism could be a viable option, but in the absence of a positive proposal, it is reasonable to assume that sets of kinematical possibilities are indispensable parts of our best scientific theories.

4.4 Third option: reductionism

If we accept QUINE and INDISP, we are led to the conclusion that we should be committed to the existence of a set of kinematical possibilities for any given system. However, this tells us nothing about the nature of this set, nor about the nature of our commitment: why assume that kinematical histories exist *qua possibilities*, and what does a commitment to a set of possibilities amount to?

Let us address the second question first. As already mentioned, not everyone agrees that possibility claims are committing. However, the case is less contentious for necessity claims, because they have a clear role in inferences, such as being usable when reasoning about mere hypotheses (Divers, 2004). In a scientific context, the content of theories is typically thought to have a force of necessity, which indeed allows theories to be used in hypothetical reasoning, or when planning possible future actions. So, in science too, our commitments seem to take the form of accepting necessity claims, and it is not clear how accepting a mere possibility claim instead of remaining agnostic would affect practical inferences. This seems true in particular when the set of kinematical possibilities is introduced in the Lagrangian formalism: the point is not to refer to a disparate collection of possibilities which would each be relevant individually for whatever reason. The point is, apparently, to define an *exhaustive set* that is relevant as a whole. If the set was truncated, the calculation of transition amplitudes would lead to different results.

A natural interpretation is that introducing the set K amounts to introducing a specific constraint on the way the system could be. The introduction of K would come with a commitment to a claim of the form: necessarily, the history of the system is such that ϕ (for example, its trajectory is differentiable and continuous), where ϕ characterises K. This is what NECESS asserts.

This interpretation works well in the classical case, which gives us a prima facie reason to adopt it in the quantum case too. But rejecting it, and claiming instead that a commitment to K is only a commitment to disparate possibilities without any corresponding necessity, is an available option in order to solve our quadrilemma. Kinematical possibilities could be, for example, metaphysical possibilities that are naturally impossible. However, this option requires explaining (contra Divers) in what sense mere possibilities would be individually relevant, and more precisely why a very particular set of possibilities, *none of which are actual*, should play a particular role in our inferences, including when it comes to determining the actual history of a system.

Another option in order to reject NECESS is to deny that kinematical possibilities are possibilities at all, which leads us back to the first question above. The main reason to accept that we are talking about possibilities is that this is the way physicists talk, and we should take scientific discourse seriously. The canonical way of presenting Feynman's formalism is as involving a sum over elements of K, construed as a space of *possibilities*. This kind of discourse is ubiquitous (see as an example the textbook

(Zee, 2010, Chap. 1). Another reason in favour of this interpretation is continuity with classical mechanics. The path functional formulation is a transposition of classical Lagrangian mechanics to quantum mechanics. Classical Lagrangian mechanics is still used in physics today. It is still a good scientific framework. So, it seems reasonable to assume a continuity of reference between the two theories. And as we have seen, the kinematical possibilities of classical Lagrangian mechanics are naturally interpreted as objective possibilities.

One could remain unmoved by these two reasons. Perhaps talk of possibilities is just an entrenched way of speaking that should be abandoned in light of recent theories. Kinematical possibilities could exist in another sense: as real, actual, unobservable entities that contribute to our observations.

This kind of view seems compatible with the Many-Worlds Interpretation of quantum mechanics. This interpretation considers that what is commonly referred to as possibilities actually corresponds to the branches of a multiverse. In a Many-Worlds theory, kinematical possibilities are interpreted as actual paths in the branching structure of the universe (each branch of the structure is coarse-grained, and so, is constituted of a superposition of kinematical possibilities (Wallace, 2014, Chap. 3)). This non-modal interpretation would likely extend to transition amplitudes, which would occur *within* branches instead of being genuine possibilities.

On one possible reading, this family of approaches has no deep implications for the metaphysics of modalities: kinematical possibilities are not modal entities, so their nature does not affect the way we should think about natural possibilities. In the context of the Many-Worlds theory, this would correspond to what Wilson (2013) calls *collectivism about Everettian branches* (a thesis he rejects): the alternative branches of the universe, as well as the kinematical possibilities that compose them, all exist inside the actual world. However, it remains to be explained why physicists call them possibilities. In the case of the Many-Worlds theory, this is the problem of the source of probabilities in a deterministic multiverse (Wallace, 2014, pt. 2). It seems, at least, that quantum mechanical models have a structure that is very similar to a modal structure (or as Wilson (p. 713) puts it, they include "structures that we naturally want to think of as representing entities that are alternatives to one another").

On another reading, there *are* deep implications. What we usually call possibilities are actually something else (actual entities that contribute to our observations, or branches of a multiverse). This reinterpretation could be extended to modal talk in general, and not only in the context of physics. This could imply a revisionary stance with respect to traditional modal metaphysics, which could take the form of a reinterpretation of the options detailed in Sect. 2. This corresponds to the approach adopted by Wilson and his Quantum Modal Realism: he identifies Everettian branches with concrete Lewisian possible worlds (see (Harding, 2021) for implications on modal metaphysics).

Even in a single universe theory, one could in principle propose an ontology where kinematical possibilities are not actually possibilities, but rather processes interfering with each other that together produce the quantum phenomenon. In this case, transition amplitudes could retain their modal status. An option that could be interpreted along this line is Conroy (2012). Conroy champions a variant of collectivism, wherein all facts are construed as relative, rendering each Everettian branch either a factual or counterfactual description of the actual world. Within such a framework, Everettian branches are accorded a modal status. However, the status of kinematical possibilities (which are descriptively finer than coarse-grained branches) is not directly addressed since the elements of K are not described in terms of relations. In (Conroy, 2012), certain discussions about classical objects suggest that kinematic possibilities might be conceptualized as constituting possible branches. From the perspective of any given branch, these possibilities are actual entities that participate to the constitution of the branch itself. Consequently, this interpretation would also challenge the validity of NECESS, and give us only one layer of modalities within models: the dynamical possibilities.[19]

4.5 Fourth option: realism

We have seen so far three possible solutions to our quadrilemma: being an instrumentalist about kinematical possibilities, reformulating the theory to dispense with them or rejecting the idea that they commit us to a necessity claim. The fourth and final option is to accept that the set of kinematical possibilities corresponds to a constraint of necessity on any given system, and therefore to reject IMPOSS: it must be possible, for any given system, to have a history corresponding to one of the kinematical possibilities. This is where an examination of the structure of scientific models, our bottom-up approach, can really enrich the metaphysics of modalities.

IMPOSS can be justified by the fact that quantum mechanics, in its traditional formulation, precludes the instantiation of classical histories. In so far as the set of kinematical possibilities is associated with a necessity claim, assuming that what is actual (discrete transitions) is not part of it violates a theorem of modal logic: if p is necessary, then p is the case ($\Box p \rightarrow p$).[20]

Another way of expressing the problem is to remark that kinematical and dynamical possibilities constitute disjoint sets, so they cannot both be necessary. In order to solve the problem, we need to bring them closer together until one set becomes a subset of the other.

As far as we can see, there are two possible ways of doing this. The first one consists in considering quantum dynamical possibilities to be continuous histories instead of transition amplitudes, so as to force them to be contained in the set K. The second one consists in turning kinematical possibilities into transition-like entities, and enlarging the set K until it contains the dynamical possibilities (transition amplitudes).

The idea behind the first strategy is to consider transition amplitudes to be partial descriptions of underlying classical histories, which would be the real dynamical possibilities. If this were a viable option, we would have a well defined set of dynamical possibilities D contained in K. The various interpretive options detailed in Sect. 2 could be recovered (and the first option, consisting in identifying kinematical and

[19] This viewpoint is similar to, yet distinctly differs from, the position presented in Ardourel and Guay (2018).

[20] This is not a theorem of *deontic* modal logic: what is mandatory is not always respected. However, we cannot think of any sensible interpretation along this line (metaphysical possibilities are supposed to be alethic).

metaphysical possibilities, would not be implausible after all: the ontology of the universe would be classical in its properties, even if not in its dynamics). These options could even be simplified in case no probability amplitude weight is strictly equal to zero, because then the sets K and D would be identical.

Unfortunately, this is not a viable solution unless we leave the Feynman functional integral formulation behind. This is because probabilities in Feynman's formulation are not additive.

Consider two points in configuration space in and out, and a set of mutually exclusive intermediate conditions between the two that cover the space of kinematical possibilities. The probability of a transition between in and out is generally *not* the sum of the probabilities of all the combinations of transitions that go from in to out with such an intermediary step. Thinking so would neglect interference effects between all these histories. However, if a transition were merely a partial description of an underlying classical history, probabilities would be additive in this sense: the probability of a transition would be the probability that one of the dynamical histories realising this transition occurs, which is just the sum of the probabilities of these histories, and this would also correspond to the sum of the probabilities of transitions with an intermediary step. Since this is not the case, reinterpreting transition amplitudes as partial descriptions of kinematical possibilities is not an available move in Lagrangian quantum mechanics.

Bohmian mechanics assumes that particles follow classical trajectories in spacetime, which implies that physical systems also follow classical trajectories in configuration space. Since Bohmian mechanics is widely considered a consistent theory that is empirically equivalent to standard quantum mechanics, one could wonder how it avoids the problem of additivity just mentioned, and why this move is unavailable in our context. From the perspective of Bohmian mechanics, considering a transition amplitude in the Feynman functional integral formulation implicitly assumes that the corresponding configurations are measured by the environment, and considering the probability for a different transition assumes a different way of measuring the system. The strategy adopted by Bohmians in order to handle measurement situations is to incorporate measuring instruments or relevant parts of the environment into the model, so as to reduce any measurement of a physical quantity to a position measurement (Daumer et al., 1996). In this context, from the Bohmian perspective, the measuring instruments affect the dynamics of the system (the pilot wave) in a way that is not captured by the Lagrangian. From our perspective, this merely shows that Bohmian mechanics is *not* the same theory as the Feynman functional integral, since it requires redefining the boundaries of systems of interest and their dynamics (note, in this respect, that the calculation tools of the Feynman functional integral can be used in the context of Bohmian mechanics, but the paths considered in the calculation are distinct from the possible Bohmian trajectories (Tumulka, 2005)).

The second strategy is to bring kinematical possibilities closer to dynamical possibilities, and to enlarge the former until it contains the latter. This can be done by first reconceptualising classical histories as particular limiting cases of *composite transitions*, namely infinite successions of infinitesimal transitions. Then we can enlarge the space K so that it contains all possible transitions in configuration space (in one sense of possible), composite or not, including classical histories as a special case.

What we call a composite transition here corresponds to what is called a *history* in the consistent histories formulation (which is close in spirit to the functional integral (Griffiths, 2003)).[21] In this context, we can take quantum mechanics to inform us about salient relations between these possible histories, from which the probability weights of various dynamical histories in particular contexts can be inferred. The two sets K and D need not be identical: restricting the range of relevant dynamical possibilities by taking into account the context (specifying relevant ins and outs, as is normally done with the Feynman functional integral) could be required in order to avoid inconsistencies with standard probability calculus.[22] These restrictions must be applied to D, because a more complete set K is required to calculate the probabilities. With this approach, the special case of classical histories (now re-conceived of as infinite sequences of transitions) will not figure in D due to these restrictions.

In this context, all interpretive options detailed in Sect. 2 are a priori available. The new set K can be identified with metaphysical, intermediate or natural possibilities, and D with natural or sub-natural (context relative) possibilities. Note that identifying K with metaphysical possibilities does not mean restricting oneself to a classical ontology any more, so considering an intermediate level does not seem required. Identifying D with sub-natural possibilities looks particularly promising, given the contextual restrictions on this set just mentioned. However, if adopted, this reading will move us away from the usual interpretation of Feynman's formulation.

This is merely a sketch at this point. What remains to be done under this strategy is providing a metaphysical interpretation of these "relations between possibilities" (the phases, their association with probability weights, and the structure of possibility space involved in this association). Doing so would certainly have deep implications for the metaphysics of modalities and laws of nature, because it would force us to rethink the structure of natural possibilities in a way that is not necessarily allowed in a "flat" possible-world semantics. This is why looking at modalities *within models* can potentially advance the metaphysics of science, which is the main message of this article.

5 Concluding remarks

Lagrangian mechanics comes equipped with two layers of modal structures: kinematical possibilities and dynamical possibilities. We have examined, in this article, various options for interpreting them, both in the classical and quantum cases.

Although the classical case is more easily amenable to a traditional analysis in terms of natural and metaphysical possibilities, it also offers more revisionary options and potentially leads to a re-conceptualisation of laws of nature. The option most congruent with traditional metaphysics appears to be the less plausible one, since it implies a commitment to classical ontology. This already shows that the debate on laws

[21] However, consistent histories can be defined from any kind of observable, whereas configuration space is normally defined on the basis of positions.

[22] This is what motivates the introduction of "frameworks" in Griffiths (2003)'s consistent histories approach. Note that this is not the same as postulating an effect of the environment on the dynamics on top of the Lagrangian.

of nature could benefit from a closer examination of the modal structure of scientific models.

The quantum case is more interesting, because it breaks the homogeneity between kinematical and dynamical possibilities: the former are classical histories, while the latter are transition amplitudes. The question of how to interpret them remains largely open. However, we have provided a landscape of possible solutions (instrumentalism, eliminativism, reductionism and realism about kinematical possibilities) in the form of a quadrilemma that could be helpful for future inquiry on the subject. In the end, equivalents to the options we found in the classical case could be recovered, but new possibilities were opened up as well.

The lesson we draw from this analysis is that examining more closely the structure of theoretical models in order to extract ontological commitments proves fruitful for metaphysics. This attitude can substantially enrich the metaphysical debate, not by allowing one to decide between traditional positions, but rather by opening up new options, and pointing to potential irrelevancies in the old construals. Arguably, this has been the import of the theory of relativity for the metaphysics of space and time. This could be the import of quantum mechanics for the metaphysics of modalities.[23]

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[23] A similar insight is provided by Saunders (1995), who extends a relational analysis of tense to modalities. By centring our discussion on the Lagrangian formalism, we have sidestepped the challenging question of the relation between time and modality. Readers seeking insight into this particular aspect are encouraged to refer to Saunders (1995, 1996, 1998). We thank an anonymous reviewer for drawing our attention to these works.

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