MEINONG ON MAGNITUDES AND MEASUREMENT

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"The work of Herr Meinong on Weber's Law, is one from which I have learnt so much, and with which I so largely agree ..."

Bertrand Russell,
The Principles of Mathematics

Abstract

The paper comprises a presentation and defence of Meinong's discussion on magnitudes and measurement found in his *Über die Bedeutung des Weber'schen Gesetzes*. The first and longer part of the presentation examines Meinong's analysis of magnitudes. According to Meinong, we must distinguish between divisible magnitudes and indivisible ones. He argues that relations of distance, or dissimilarity, are indivisible magnitudes that coincide with divisible magnitudes called stretches. The second part of the presentation is concerned with Meinong's account of measurement as a comparison of parts. Meinong holds that measuring is comparing parts and, thus, only divisible magnitudes are directly measurable. When indivisible magnitudes like distances are indirectly measured, they are measured by means of divisible magnitudes like stretches. Meinong's account allows us to reject important objections against measurement of similarity and to reconsider the logical form of the sentences involving comparative similarity.

Few works of Meinong seem to be as neglected as his work on Weber's Law. It remains inaccessible to non-German speakers and except the noticeable book of E. Tegtmeier, *Komparative Begriffe*[1], no

[1] E. Tegtmeier (1981).

important work in philosophy discusses it. This lack of interest is all the more astounding when we consider that it is the only work of Meinong ever substantially endorsed by Russell. Part III and half of Part IV of *The Principles* are explicitly grounded on Meinong's work. If *The Principles* is an important work in the history of Analytic Philosophy, which of course it is, Meinong's work on Weber's Law is a non-negligible part of this history.

This paper is about Meinong's 'foundation of measurement' in his *Über die Bedeutung des Weber'schen Gesetzes.* Meinong's work on Weber's Law, as Russell calls it, is chiefly concerned with the analysis of the basic notions at work in the 'fundamental law' of psychophysics; an analysis that leads Meinong from the concept of magnitude (Grösse) to psychical measurement. Meinong's aim is to show that most of the mistakes and confusions that beset his contemporaries in their writings on psychical measurement are rooted in confusions about measurement. He achieves this aim by means of an impressive and meticulous study of the basic notions involved in our practice of measurement.

This paper is primarily about this account that first aroused Russell's admiration and that I shall call "the foundation of measurement". Therefore, the focus will be on sections one to four of Meinong's work; the fifth section, which is about measurement of psychical magnitudes, will not be discussed here. The aim of this study is to show the scientific interest of Meinong's analysis and foundation of measurement. To this end, I will first present and clarify Meinong's account of magnitudes; then Russell's reception of this account will be presented; finally, I will apply Meinong's discussion to the special topic of logic for comparative similarity.

1. The Foundation of Measurement

In more recent times[2], the work of a foundation of measurement has consisted in answering the question: how do we represent quantities numerically? Numerical representation of quantities is only one feature of a foundation of measurement according to Meinong; and on this particular topic the questions he is interested in are rather (1) 'what is it

[2] Especially the famous and wonderful book of David H. Krantz, & Duncan R. Luce & Patrick Suppes & Amos Tversky (1971). The phrase "Foundation of Measurement" is the title of this book.

that we represent numerically?' and (2) 'what do we really do when we represent numerically?'

A foundation of measurement according to Meinong is not a model-representation of magnitudes but rather a rigorous analysis of the concepts that ground our practice of measurement. These grounding concepts are primarily the concepts of *magnitude* (Grösse) and *comparison* (Vergleichung). Magnitudes ground measurement in the sense that what we intend to measure when we measure are magnitudes. A good understanding of measurement requires a good understanding of what we intend to measure. Comparison equally grounds measurement simply because, following Meinong, measurement is a kind of comparison, a comparison of parts.

Therefore, the foundation of measurement is achieved in the four steps that constitute sections one to four of Meinong' discussion: the first section[3] introduces a comprehensive definition of a magnitude and a distinction between the main kinds of magnitudes; the second section[4], accounts for comparison, in particular for comparison of magnitudes; the third section[5], discusses the relation between comparison of parts (Teilvergleichung) and measurement, and a distinction between general kinds of measurement; the fourth section[6], offers an account of the particular measurement of dissimilarities of magnitudes.

The foundation of measurement is introduced in order to advance what Russell calls the single thesis of Meinong's work, that is:

> The true import of Weber's Law is that equal dissimilarities (Verschiedenheiten) in the stimuli correspond to equal dissimilarities in the corresponding sensations; while the dissimilarity of two measurable quantities [Grössen] of the same kind may be regarded as measured by the difference of the logarithms of these quantities. [7]

This thesis, in itself, is not what interests me here. My purpose is to examine the account of magnitudes and measurement that grounds it. Nevertheless, this thesis will constitute the starting point.

[3] « Vom Grössengedanken und dessen Anwendungsgebiet »
[4] « Über Vergleichung, insbesondere Grössenvergleichung »
[5] « Über Teilvergleichung und Messung »
[6] « Über Messung von Grössenverschiedenheiten »
[7] Russell (1899) p. 251.

1.1. Meinong on Magnitudes

1.1.1. Defining Magnitudes.

First, in his review, Russell's translation of the German "Grösse" is the English "quantity". This translation, which already appears in his report of the main thesis of Meinong, will be rectified in Russell (1903). In the review, Russell never uses "magnitude". Considering that *Grössen* of a same kind could be dissimilar, i.e. they could be distinguishable by means of asymmetrical relations like greater and less, the concept of *Grösse* clearly corresponds to what Russell (1903) calls magnitudes[8]. I will use, then, the translation of (1903). [9] The point is purely terminological. What is a magnitude according to Meinong?

At the beginning of the first section on "the notion of magnitude and its area of application"[10], Meinong proposes to characterise the notion of magnitude. He aims to offer a non-circular characterisation. Meinong's starting point[11] is clearly Kant's definition of an intensive magnitude as that which:

> Nur als Einheit apprehendiert wird, und in welcher die Vielheit nur durch Annährung zur Negation = 0 vorgestellt werden kann[12].
> (is only apprehended as a unity, and in which the multiplicity can only be represented by approaching the negation = 0.)

What Meinong retains from Kant in his characterisation of magnitudes is this conception of zero as the negation, or contradictory opposite of a magnitude and this idea of a magnitude approaching (Annährung) zero.

He keeps the idea without keeping the relation of approaching. For this relation of approaching presupposes that, for a given magnitude A_n, there is another magnitude A_m such that A_m is closer to zero than A_n; i.e. such that A_m is less than A_n. Relations of order like less and greater, according to Meinong, presuppose the concept of magnitude. Therefore,

[8] Russell (1903) chapter XIX, especially 151, p. 159.
[9] See below, his translation of Meinong's characterisation of magnitudes.
[10] Our Translation
[11] See Meinong (1896) Erster Abschnitt §1 'Das Limitieren gegen die Null pp.218-219, footnote 7.
[12] see I. Kant (1974) p. 208.

keeping the relation of approaching would commit him to a vicious circle.

After some approximations, the following characterisation of a magnitude is adopted:

> Grösse ist oder hat, was zwischen sich und sein kontradiktorisches Gegenteil Glieder zu interpolieren gestattet.[13]
> (Magnitude is or is had by that which allows the interpolation of terms between itself and its contradictory opposite.)[14]

To avoid circularity, this interpolation of terms should not be thought of as saying that the interpolated term approaches zero but rather as saying that the interpolated term "falls in the same direction as non-*x* [*i.e.* zero]."[15] The concept of direction, says Meinong, does not presuppose the idea of magnitude.

This brief characterisation of magnitudes leads Meinong to introduce an important distinction concerning magnitudes, the distinction between divisible and indivisible magnitudes.

1.1.2. Divisible and Indivisible Magnitudes

The originality and theoretic value of Meinong's work appear with this distinction between divisible (teilbare) and indivisible (unteilbare) magnitudes. Divisible magnitudes are those that can be partitioned into other magnitudes of the same nature. Integers, for instance, are such divisible magnitudes. Other magnitudes, psychical magnitudes for instance, are not divisible as such. Meinong's examples of psychical magnitudes are sound and heat. He claims:

> Es hätte keinen Sinn, von einem lauten Geräusch zu sagen, es enthalte ein leises von übrigens genau der nämlichen Qualität als Teil in sich.[16]
> (It would be nonsensical to say of a loud sound that it has as a part a low sound of exactly the same kind.)

[13] Meinong (1896) Erster Abschnitt §1, p. 219
[14] Russell's translation ; cf. Russell (1903), note on chapter XIX pp. 168-169.
[15] „fällt in die nämliche Richtung wie non-*x*" Meinong (1896) p. 220.
[16] Meinong (1896), pp. 232-233.

Even if indivisible, sounds are nevertheless magnitudes, given that we agree that between any loud sound and silence, sounds that are less loud can be interpolated. The same holds for pleasure and heat: between an intense heat and the absence of heat, distinct heats can be interpolated that fall in the same direction as the zero heat.

The important contribution of Meinong is to have shown that some relations are indivisible magnitudes, and in particular that a certain kind of relation is. This kind of relation is the relation of dissimilarity (Verschiedenheit)[17] or distance (Distanz). For Meinong, the terms of dissimilarity and distance refer to the same kind of magnitude, or, more precisely, distance is a kind of dissimilarity. Meinong asks:

> Ist die "Distanz", welche ich zwischen die Zirkelspitzen nehmen und übertragen kann, zunächst und in erster Linie [69] wirklich eine Verschiedenheit und nicht vielmehr eine Strecke?[18]
> (Is distance, which I can take and take back between the extremities of the compasses, first and really a dissimilarity and not rather a stretch?)

We will see in the following pages that, according to Meinong, distances are not stretches and that, therefore, they are dissimilarities. Distances are magnitudes, since between any distance distinct from zero, and the zero distance, other distances, shorter than the former one, can be interpolated. However, distances are indivisible magnitudes

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[17] The translation of the German Verschiedenheit by the English dissimilarity was introduced in Russell (1899). The translation is not perfect. German vocabulary has another word corresponding to the English dissimilarity: Unähnlichkeit, which is clearly the contrary for the German Ähnlichkeit that is the correct translation for the English similarity. A simple example shows that the translation of Verschiedenheit by dissimilarity is not perfectly correct. In German, we could say that two twins are perfectly similar (ähnlich), but nevertheless verschieden. Thanks to Barbara Berger who gave me this example. In English, it is impossible for things that are exactly similar to be dissimilar. In the example of the twins, the German verschieden is closer to the English diverse. That is true, but this is not the meaning of the word 'Verschiedenheit' that Meinong is considering. We will see that, according to him, there are degrees of Verschiedenheit: two things could be more or less verschieden; but it is clearly false to claim that two things are more or less diverse. Diversity is supposed to be a sharp relation. However, things could be more or less dissimilar. The linguistic postulate of this paper is that the word 'Verschiedenheit' is ambiguous in ordinary German. The disambiguated meaning that interests Meinong corresponds to the English 'dissimilar'.

[18] Meinong (1896), Dritter Abschnitt, § 15 p. 278.

because they are relations and relations are not divisible. Meinong offers no real argument for the indivisibility of relations, just a *prima facie* conviction that things could not be otherwise:

> Vielmehr scheinen Relationen als solche einfach sein zu müssen. […] die Unteilbarkeit der Distanz verrät sich ohne weiteres von selbst.[19]
> (It seems rather that relations as such must be simple. […] The indivisibility of distance betrays itself without any ado.)

Let me propose an argument for the indivisibility of relations that are magnitudes. To say that a magnitude of a kind is divisible entails that the relations of greater and less between magnitudes of this kind depend on the number of parts these magnitudes have. For instance, the divisibility of integers entails that 6 is greater than 5 because of the fact that 6 is divisible in a greater numbers of unit parts than 5 is; take 1 as the unit part, 6 is divisible in 6 parts when 5 is divisible in 5 parts.

Consider relations that are magnitudes now. If relations were divisible, then the relations of greater and less between distinct relations of the same kind depend on the number of parts these relations have. Parts of a magnitude of a certain kind are magnitudes of this kind too. Therefore, a relation of a certain kind that is a magnitude, if divisible, must have for parts relations of the same kind. Consider, for instance, two shades of red, red_1 and red_{12}. First, it seems undeniable that two shades of red are less dissimilar to each other than either is to a shade of blue. This relation of order entails that dissimilarity between shades of colour is a kind of magnitude. Then, if the distance between red_1 and red_{12} were a divisible magnitude, the dissimilarity between red_1 and red_{12} would be partitioned in the following way:

$$Dis(red_1,red_{12}) = dis(red_1,red_2) + dis(red_2,red_3)+ dis(red_3,red_4),\ldots, + dis (red_{11},red_{12}).$$

Of course, it is nonsensical to claim that the dissimilarity between red_2 and red_3 is a part of the dissimilarity between red_1 and red_{12} simply because the relations do not stand between the same entities. A part of an entity is supposed to be *in* this entity. No meaning of the little word '*in*' - which does not overlap the meaning of 'between'[20] - allows us to

[19] ibid.,§3 p. 234.
[20] 'between' does not express parrthood relations, but order relations.

claim that the dissimilarity between red_2 and red_3 is in the dissimilarity between red_1 and red_{12}. Therefore, relations of dissimilarity or distance are indivisible magnitudes.

1.1.3. Distance and Stretch.

The indivisibility of relations and especially of distances leads Meinong to introduce the important distinction between distances and *stretches* (Strecke). Russell (1903) maintains this distinction and uses it. Meinong claims that:

> […] der Gedanke an die Verschiedenheit zweier Punkte im Raume etwas anderes ist, als der Gedanke an die zwischenliegende Strecke.[21]
> (The idea of the dissimilarity between two points in space is something distinct from the idea of the stretch that lies between them.)

To any distance corresponds some stretch that is conjoined with it. However, distances are relations, stretches are not. That distance, or dissimilarity, is a relation is undeniable. A distance is necessarily a distance between something and something else; a distance cannot obtain without the existence of distant things. Contrary to distances, stretches can obtain without distant things:

> die Strecke zwischen zwei Raum- oder Zeitpunkten besteht, mag sie übrigens existieren oder nicht.[22]
> (The stretch between two points of space or time obtains, do them, by the way, exist or not.)

Meinong adds about spatial and temporal stretches:

> Räumliche und zeitliche Strecken bieten die geläufigsten und zugleich durchaus einwurfsfreie Beispiele: jeder Raum "besteht" aus Räumen, jede Zeit aus Zeiten. (…) Jede Strecke hat Strecken zu Bestandstücken, und diese wieder Strecken usf. ins Unendliche;[23]

[21]Ibid. § 3 p. 234
[22] Ibid. Vierter Abschnitt § 17 p. 288
[23] Ibid. §3 p. 232.

> (Spatial and temporal stretches offer the most common and at the same time completely uncontroversial examples: each space "consists" of spaces, each time of times. (...) Each stretch has stretches for constituents, and these stretches again and so on to infinity;)

The quotation gives us some information about the nature of stretches and their characteristic difference from relations of distance. Stretches, Meinong says, have stretches for components, so they are divisible magnitudes. Since relations are indivisible magnitudes, stretches are not relations. What are they? From Meinong's quotation, it is possible to infer that stretches of space and of time, are regions of space and time. The distance between two points in space is the relation between these points, but the stretch that lies between them is the region of space that separates the two points. Such regions of space or time are divisible into sub regions and so on. Regions of space and time are not abstract entities like relations, they are concrete and particular entities. Therefore, stretches are particulars.

Even if distance and stretch are different kinds of entity, and in particular different kinds of magnitude, it is also true that to any distance corresponds some particular stretch:

> Und zwar ist nicht nur jeder Streckengrösse eine Distanzgrösse, sondern auch jeder Distanzgrösse eine Streckengrösse zugeordnet.[24]
> (That is to say, not only is each magnitude of stretch a magnitude of distance, but each magnitude of distance is also conjoined with a magnitude of stretch.)

This point requires no demonstration. Consider any two points in an order, to their distance, there corresponds a proper part of this order.

The distinction between distance and stretch allows Meinong to introduce another important distinction for the analysis of Weber's Law, the distinction between dissimilarity and difference.

1.2. Dissimilarity and difference

[24] Ibid. § 15 p. 278.

At the heart of the fourth section of Meinong's discussion on Weber's Law[25] we find the distinction between dissimilarity and difference (Unterschied). To exemplify the distinction, Meinong says:

> In diesem Sinne ist etwa der Unterschied zwischen zwei Linien wieder eine Linie, indes die Verschiedenheit zwischen zwei Linien so gut wie sonst irgendeine Verschiedenheit eine Relation nichts weniger als eine Strecke ist.[26]
> (In this sense, the difference between two lines is a line, but the dissimilarity between two lines, like any other dissimilarity, is a relation and nothing like a stretch.)

Consider a line with three points A, B, and C. The numerical difference between the two lines AB and AC, is the line BC. The dissimilarity between AB and AC is equivalent to the dissimilarity between B and C, but these dissimilarities are relations not lines. Stretches are differences. This reveals an important distinction between dissimilarity and difference, namely, that the former is a type of indivisible magnitude while the second is a type of divisible one:

> Das ergibt sich einfach daraus, dass Verschiedenheit ihrem Wesen nach mit Teilung und Teilbarkeit nichts zu tun hat, die Differenz[27] aber, wie wir sahen, erst auf der Teilvergleichung hervorgeht;[28]
> (It follows simply from this that the nature of dissimilarity has nothing to do with partitioning and divisibility; difference, however, as we saw, first involves comparison of parts.)

Meinong offers another example of the distinction between dissimilarity and difference:

> 1 ist von 2, man kann dies auch ganz wohl von den Zahlengrössen aussagen, erheblich verschiedener als 6 von 7; dennoch ist der

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[25] Ibid. Vierter Abschnitt 'Über Messung Grössenverschiedenheit'.
[26] Ibid. § 21 p. 304
[27] In Meinong's text, the words 'Unterschied' and 'Differenz' are clearly synonyms: "Differenzen oder Unterschiede aber können überhaupt nur zwischen Grössen vorkommen […]" § 21 p. 303.
[28] Ibid. § 20 p. 300

Unterschied oder die Differenz in beiden Fällen von gleicher Grösse.[29]
(1 is from 2 - one can also very well state that they are magnitudes of number - much more dissimilar than 6 is from 7; nevertheless, the difference is in both cases of the same magnitude.)

The dissimilarity between 1 and 2 is greater than the dissimilarity between 6 and 7 simply because 2 is the double of 1 while the dissimilarity between 6 and 7 is obviously less. Their difference, however, is equal; it is of 1.

Like the difference between 1 and 2 and 6 and 7, the difference between 0 and 1 is of 1. What about the dissimilarity between 0 and 1? Meinong claims about this relation of dissimilarity that:

Die Verschiedenheit zwischen 1 and 0 ist grösser, als irgendeine Verschiedenheit zwischen endlichen Grössen, oder auch: sie ist grösser, als irgendeine endlich grösse Verschiedenheit, sie ist unendlich gross;[30]
(The dissimilarity between 1 and 0 is greater than any dissimilarity between finite magnitudes, in other words: it is greater than any great finite dissimilarity, it is infinitely great;)

Russell agrees with Meinong on this distinction between the infinite dissimilarity between 1 and 0 and the finite difference between them. One of the main interests that Russell finds in Meinong's discussion is the great similarity between Meinong's conditions for having a function for measuring dissimilarity and the conditions for functions of distance in non-Euclidean Geometry. He emphasizes that the infinite dissimilarity between a finite magnitude like 1 and 0 is one of these conditions:

In finding a function for measuring dissimilarity, certain requirements are laid down. (I) The dissimilarity must vanish when the quantities [Grössen] are equal; (2) It must be infinite when one quantity is finite and the other is zero or infinite; (3) The dissimilarity between *A* and *B plus* that between *B* and *C* must be equal to that between *A* and *C*. These conditions are essentially similar to those which, in non-Euclidean Geometry, regulate the

[29] Ibid. §19 p. 295.
[30] Ibid. § 18 p. 293.

expression of distance in terms of coordinates, and Herr Meinong might have simplified a needlessly complicated piece of mathematics by reference to this analogous case.[31]

The distinction between dissimilarity and difference is of central importance to the analysis of Weber's Law. Weber's Law is about the relative difference between magnitudes of stimuli and corresponding magnitudes of sensations. Magnitudes of sensations, however, are indivisible magnitudes. Therefore, difference of sensations (Unterschiedsempfindlichkeit) is first a kind of dissimilarity rather than a kind of difference:

> […] Ausdrücke wie "Unterschiedsschwelle", "Unterschiedsempfindlichkeit", bei denen es sich zweifellos nicht um Unterschied im eben angegeben Sinne, sondern um Verschiedenheit handelt, […].[32]
> ([…] phrases like "thresholds of discrimination"[33], "difference of sensations", which undoubtedly do not refer to difference in the given meaning, but rather to dissimilarity, […].)

Not only does difference of sensations refer to a relation of dissimilarity, relative difference is, according to Meinong, a relation of dissimilarity. See below section 1.4.3., for Meinong's notation of relative difference.

1.3. Meinong on Comparison

1.3.1 Meinong's definition of Comparison

In the light of the preceding results, I return now to Meinong's main thesis about Weber's Law in order to understand his account of comparison. The second part of the main thesis talks of dissimilarities that are measured. Measurement, as it will appear later, is a special kind of comparison. The foundation of measurement, then, should first

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[31] Russell (1899) p. 254.
[32] Meinong (1896) § 21 p. 305.
[33] See below, section 1.3.4., for a discussion of these thresholds and an explanation of this translation.

explain how it is possible to compare magnitudes that are, as we saw, indivisible.

Meinong characterises the action of comparing by its aim: an action of comparing has as its aim a judgement about likeness (Gleichheit) and dissimilarity (Verschiedenheit):

> Das Vergleichen ist ein Tun, das Ziel aber, auf das es gerichtet und durch das es völlig natürlich und ausreichend bestimmt wird, ist ein Urteil über Gleichheit und Verschiedenheit[34].
> (Comparing is an action, the end it is directed towards is a judgement about likeness and dissimilarity; this end determines this action in a natural and complete way.)

The notions of comparison and dissimilarity are thus tightly related according to Meinong. The end of an action of comparison is a judgement about likeness or dissimilarity. Judgements about likeness and dissimilarity are also called judgements of comparison by Meinong. Judging that A and B are alike or dissimilar is such an action of comparing. This tightly relation entails that the compared entities are alike or dissimilar. What are these compared entities?

1.3.2. The compared entities

1.3.2.1. The grounds of comparison

Meinong's definition of an action of comparing is sufficiently broad to allow that not only magnitudes could be compared. Meinong will focus on comparing magnitudes in order to discuss Weber's Law but he recognises that other entities could be compared:

> Denn zwischen zwei gegebenen Grössen gibt es, wie auch zwischen zwei sonstigen Vergleichungsfundamenten, nur eine Verschiedenheit.[35]
> (For, between two given magnitudes, even between two other grounds of comparison, only one dissimilarity obtains.)

[34] Meinong (1896) Zweiter Abschnitt, § 4 p. 236.
[35] Ibid. Dritter Abschnitt, § 12 p. 268.

Meinong calls the terms of a comparison, its grounds. Meinong's student Konrad Zindler defines them as such:

> Fundamente einer Relation heissen die Dinge, die in Relation stehen, […], bei Vergleichungsrelationen die Dinge (Vorstellungen etc.), die miteinander verglichen werden.[36]
> (The things that are called grounds of a relation are the things that stand in a relation, […]; concerning relations of comparison, the grounds are the things (presentations, etc.) that are compared to each other.)

According to the previous quotation from Meinong, entities that are not magnitudes could also be such grounds of comparison, and could also be the grounds of a dissimilarity. Therefore, it would be wrong to restrict Meinong's notion of dissimilarity (Verschiedenheit) to a relation between magnitudes. The relation is a very broad one indeed.

Nevertheless, comparison between magnitudes has a special feature, or a special end, that is not shared by other comparisons:

> […], dass, wenn man "Grössen vergleicht", man sein Absehen normalerweise nicht einfach auf das Urteil "verschieden" gerichtet hat, sondern auf ein Glied der Disjunktion "gleich gross, grösser oder weniger".[37]
> ([…] that, if one compares magnitudes, her focus is normally not simply directed on the judgement "dissimilar"; her focus is rather directed toward one member of the disjunction "equal, greater, or less".)

Meinong adds:

> Grössen vergleichen sich im allgemeinen nicht anders als andere Objekte; dagegen fällt in Betreff der Ergebnisse [(110)] der Grössenvergleichung eine zunächst terminologische Eigentümlichkeit ins Auge. Wer die Grössen A und B miteinander vergleicht, wird, wenn er nicht Gleichheit gefunden hat, das Resultat doch nicht leicht in der Form ausdrücken: "A ist von B verschieden;

[36] Konrad Zindler (1889), p. 5, footnote 1.
[37] Meinong (1896), Zweiter Abschnitt, § 7, p. 246.

er wird vielmehr normalerweise etwa sagen: “A ist grosser” oder “B ist kleiner”.[38]
(Comparing magnitudes is, in general, not distinct as comparing other objects; but, concerning the results of a comparison of magnitudes, we first remark a terminological peculiarity. Who compares between magnitudes A and B will not, if she has not found likeness, express the result simply in the following form: “A is dissimilar from B”; she will rather say something like “A is greater” or “B is less”.)

Judgements of comparison of magnitudes involve more than these simple judgements: “A and B are alike”, or “A and B are dissimilar”. Such judgements involve that the compared magnitudes enter into some ordering. When magnitudes are compared, order relations such as “greater than” or “less than” are introduced. These order relations will allow measurement.

1.3.2.2. The nature of magnitudes

If it is obvious that the grounds of a comparison between magnitudes are magnitudes, the real nature of these magnitudes is far from clear for the moment. Let us examine if, for Meinong, magnitudes constitute a single kind of entity, and what should be this kind of entity. Consider the following examples:

Sieht man in den Strassen der Stadt etwa Gasflammen, elektrisches Glühlicht und Petroleumflammen ausreichend nahe nebeneinander, so kann man sie “unmittelbar vergleichen”; nicht so die Länge des Rheins mit der der Donau.[39]
(If one sees in the streets flames of gas, the light of an electric light bulb, and flames of paraffin that are sufficiently near to each other, she can “immediately compare” them; this is not the case when comparing the length of the Rhine with the length of the Danube.)

The light of an electric light bulb, the length of the Rhine, etc. are clear examples of a particular type of entity: individual properties (Eigenschaften). An individual property is a property had by an

[38] ibid, § 7 p. 245.
[39] Ibid. § 5 p. 237.

individual and only by it. These examples are also clear cases of magnitudes. Thus, if an act of comparison has for its end a judgement about likeness and dissimilarity, relations of dissimilarity could have for grounds such magnitudes as individual properties. Some magnitudes are individual properties, but is every magnitude such an individual property?

E. Tegtmeier answers affirmatively:

> Ich habe Größen definiert als Eigenschaften aus einem linear geordneten Eigenschaftsbereich.[40]
> (I defined magnitudes as individual properties from a linear ordered domain of individual properties.)

This, however, seems to be a counterintuitive restriction of the domain of magnitudes and something far from Meinong's advise. As I emphasised in section 1.2., integers like 1, 0, 6 etc., are, according to Meinong, dissimilar (verschieden). Their dissimilarity, moreover, is not reducible to their difference. Integers are magnitudes, they enter into the relevant order relations of greater and less. Are integers, and *a fortiori* numbers, individual properties? Of course not. It would thus be a mistake to reduce magnitudes to individual properties and to think that, according to Meinong, there exists a unified ontological kind of magnitude. Magnitudes may be individual properties, but they may be something else; the important thing being that they enter into the relevant kind of order.[41, 42]

1.3.2.3. A note on dissimilarity, distance, and direction

[40] Tegtmeier (1981) p. 43

[41] According to Höfler and Meinong (1890) p.54, individual properties have a peculiar place in the domain of grounds of comparison:

> Alle Vergleichungsrelationen pflegen sowohl von Dingen, wie von Vorgängen und Eigenschaften ausgesagt zu werden; doch ist leicht zu erkennen, dass unmittelbar immer nur letztere […] verglichen werden können.
> (Every relations of comparison would express comparing between things as well as between processes and individual properties; but it is easily recognisable that only the latter could always be immediately compared.)

If Meinong maintains this, and is right, this gives to magnitudes that are individual properties a privileged place in our epistemic processes.

[42] See also Meinong (1900) pp. 460-464 about actions of comparing of simples for more informations on the comparison of individual properties.

As it was shown in section 1.1.1., Meinong uses the notion of direction (Richtung) to define the concept of magnitude. He first rejected a definition in terms of "approaching" because it presupposes order relations like greater than and less than that lead to a vicious circle. The reason why there is a vicious circle is obvious now: such order relations are peculiar to dissimilarity between magnitudes.

It could be claimed, as Höfler does, that the relation of dissimilarity presupposes the relation of direction too:

> Abstand ist die umkehrbare Komponente -, Richtung ist die nicht umkehrbare Komponente der Verschiedenheitsrelation zweier Orte.[43]
> (Distance is the symmetrical component and direction the asymmetrical component of the relation of dissimilarity between two locations.)

According to Höfler, thus, a relation of dissimilarity has two components, distance, which is symmetrical, and direction, which is asymmetrical. Distance is, therefore, more a component of a relation of dissimilarity than a kind of dissimilarity. This might be accepted from a psychological point of view, which is, after all, Höfler's own; but he also recognises that:

> Schliesslich sind, was wir "Komponente" oder "Seiten" nannten, doch auch wieder species desselben genus "Ortsverschiedenheit", […][44]
> (After all, what we called "components" or "sides" are, of course, also species of the same genus "dissimilarity between locations", […])

This interpretation is closer to Meinong's account. Distance is *a* species of the kind of dissimilarity relations, direction being another species. Meinong defines magnitudes with one of these species, the asymmetrical one, and then focuses on the other species: distance. Distance is clearly the species of dissimilarity that interests Meinong in his discussion of Weber's Law.

What is important here is that distance, and thus the dissimilarity we are dealing with, is a symmetrical relation. Russell, in 1903, will define

[43] Höfler (1896) p. 226.
[44] Ibid. p. 228

distance as an asymmetrical relation.[45] This point will be considered when Russell's reception of Meinong's account will be discussed.

1.3.3. Typical and atypical relations

If, as Meinong says, the action of comparing magnitudes is directed toward one member of the disjunction "equal, greater, or less", then comparing magnitudes entails an ordering between magnitudes. This involves a restriction of judgements of comparison to magnitudes of the same kind.

Comparisons of magnitudes of distinct kinds, which Meinong calls after von Kries "atypical relations" (atypische Beziehungen)[46], should be ruled out because of the incommensurability of magnitudes of distinct kinds. In order to make judgements of comparison involving expressions like "greater than, less …", the possibility of a continuous line between the compared magnitudes and the zero magnitude is necessary. However, magnitudes of different kinds do not converge toward the same zero:

> Es ist ferner unmittelbar ersichtlich, dass die Wege, auf [35] denen Grössen verschiedener Klassen sich der Null nähern oder von ihr entfernen können, keineswegs zusammenfallen.[47]
> (It is further immediately evident that the lines, on which magnitudes of distinct classes can draw near to and move away from zero, do not converge at all.)

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[45] Also in Höfler & Meinong (1890) p. 53:

> Die umkehrbaren Relationen sind von den nicht umkehrbaren meist leicht schon dadurch zu unterscheiden, dass die Sprache bei ersteren für beide Glieder der Relation (Freund – Freund) oder für die Relation (Gleich – Gleich) Ein und dasselbe Wort gebraucht;
> (The symmetrical relations could most easily be distinguished from the asymmetrical relations; for the first relations [the symmetrical ones], the language needs for both terms of the relation (friend – friend) or the relation (alike – alike) only one and the same word.)

As it was showed when Meinong's definition of the action of comparing was given, the relation of likeness is the contradictory of the relation of dissimilarity. Therefore, if the first is a symmetrical relation, so is the second relation.

[46] Meinong (1896) § 8 'Von Kries über "atypische Beziehungen"'.

[47] Ibid. § 7 p. 245.

Therefore, atypical comparisons, i.e. comparisons between magnitudes of distinct kinds or classes, should be avoided. According to Meinong, however, there are some contexts in which atypical relations seem admissible:

> Man wird sicher geneigt sein, Farben- und Tonhöhenverschiedenheiten für a priori "unvergleichbar" zu halten, und doch urteilt man mit vollster Evidenz, dass die Verschiedenheit zwischen zwei Farben oder die zwischen zwei Tönen kleiner ist als die zwischen Ton und Farbe.[48]
> (One will surely be inclined to regard dissimilarities of colour and pitch as *a priori* "incomparable"; nevertheless, one judges with certainty that two colours, or two pitches, are less dissimilar two each other than a pitch is to a colour.)

Colours and pitches seem *a priori* incomparable (Unvergleichbar[49]) for the lines of colour, on the one hand, and of pitches, on the other hand, do not converge toward the same zero. Nevertheless, Meinong knows that ordinary language allows atypical comparison between such magnitudes; and we are clearly justified to compare them as such. Avoiding atypical comparisons is not Meinong's aim, but he assumes that the magnitudes compared in an atypical relation must nevertheless be sufficiently close to each other:

> Während ferner nichts im allgemeinsten unvergleichbar heissen kann, ist die Grössenvergleichung, die Beurteilung auf Grösser [45] und Kleiner, an die Bedingung geknüpft, dass die auf ihre Grösse zu vergleichenden Objekte ihrer Qualität nach einander ausreichend nahe stehen.[50]
> (Furthermore, while nothing can be called incomparable in general, comparison of magnitudes, i.e. judgements of greater and less, are attached to the condition that the quality of the objects, i.e. the magnitudes which are to be compared, be sufficiently close to one another.)

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[48] Ibid, § 8 p. 254.
[49] Höfler und Meinong (1890) p. 55-56, use the German word 'disparat' as synonym to 'unvergleichbar'.
[50] Meinong (1896) § 8 p. 255.

The German ‘Qualität’ in this quotation can be understood as the *kind* to which a compared magnitude belongs. Meinong’s thought is that, in cases of atypical relation, compared magnitudes must belong to kinds of magnitudes that are, in nature, sufficiently similar to support a comparison. This, thus, entitles the distinction between two kinds of dissimilarity: an objectual dissimilarity, which is the dissimilarity between the magnitudes or things that are compared; and a generic dissimilarity, which is the dissimilarity between the kinds to which the compared magnitudes or things belong. This distinction, which does not belong to Meinong’s vocabulary, will not be further discussed in this paper.[51, 52]

1.3.4. The thresholds of discriminability

One of Meinong’s wonderful development concerns the notions of “Unterschiedsschwelle” and “Ebenmerklichkeit”, which can be translated respectively by the phrases “the threshold of discrimination” and “equal discriminability”. The threshold of discrimination and cases of equal discriminability - that I will generically call the thresholds of discriminability[53] - are some of the important results of Fechner experiments[54]. Meinong characterises these thresholds of discriminability in the following way:

> Es gibt Gebiete, auf denen sich Gleichheit streng genommen niemals mit Sicherheit erkennen lässt;[55]
> (There are some contexts in which likeness, in the strict sense, can never be recognised with certainty;)

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[51] This idea of a generic dissimilarity is close to one of Meinong’s student, Ernst Mally, who talks about *similarity* (Ähnlichkeit) *of types* in Mally (1922) p. 100:

> Die ähnlichkeit dieser Typen besteht darin, dass die Bestimmung des Rot und des Gelb etwas gemeinsan haben.
> (The similarity between the types of red and yellow is explained by the fact that the determination of red and of yellow have something in common.)

Unfortunately, Mally does not develop the idea.

[52] See also Meinong (1900) pp. 457-458 for a brief discussion of atypical Relations.

[53] ‘Thresholds of discriminability’ is the name used in English psychological literature to refer to these phenomenons. See, for instance, Hardin (1988) p. 214.

[54] G. T. Fechner, (1888)

[55] Meinong (1896) §9 p. 256.

Let me specify further the distinction between the two kinds of threshold. On the one hand, the result about the threshold of discrimination is that dissimilarities in the stimuli are discriminable only between an upper and a lower limit of light that are characterised as the thresholds of discrimination. Under a lower limit of light, dissimilarities cannot be discriminated; the same holds of an upper limit of light. On the other hand, cases of equal discriminability occur when the dissimilarity between two things is too small to be discriminated as such. In such cases, we perceive likeness where there is dissimilarity. [56]

The ideas of 'threshold' and of 'equal discriminability' entail that:

> Was verschieden erscheint, ist auch verschieden; was hingegen verschieden ist, erscheint als verschieden nur bis zu einer Grenze, jenseits welcher der Schein der Gleichheit eintritt.[57]
> (What appears dissimilar is dissimilar; what, however, is dissimilar, appears dissimilar only until a certain limit, beyond which the appearance of likeness arises.)

What appears dissimilar is always dissimilar, but what is dissimilar cannot always appear as such. Not every dissimilarity can thus be discriminated. It also entails that, on the one hand, claims of dissimilarity, and thus negation of likeness, when based on perception, are infallible. On the other hand, claims of likeness and negation of dissimilarity can turn out false if some threshold of discrimination does not allow us to perceive dissimilarities.[58]

This discussion concerning the thresholds of discrimination is external to Meinong's account of magnitude and measurement. It is worth emphasising however that, according to Meinong, dissimilarity and likeness are not cognitive relations; for, if they were, appearance of dissimilarity would be dissimilarity and appearance of likeness would be likeness.

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[56] Meinong (1900) p. 486 concerns a case of equal discriminability.

[57] Meinong (1896) p. 256.

[58] In fact, the idea of fixed, sharp thresholds of discriminability has been abandoned in psychophysics for many years. See, for instance, Schrödinger (1926) and Hardin (1988) p. 215. Cases of thresholds of discriminability are typical cases of decision making under conditions of uncertainty. However, this does not affect the conclusions Meinong draws on these thresholds.

1.4. Meinong on Measurement

1.4.1. Definitions of Measurement

Measuring is, according to Meinong, an operation derived from comparison. It is an indirect method for comparing magnitudes. He says:

> Die Messoperationen sind Verfahrungsweisen, eventuell auch ohne ausdrückliche Vergleichung Gleichheiten mit grösserer Zuverlässigkeit festzustellen, als der Unvollkommenheit unserer Vergleichungsfähigkeit nach durch direktes Vergleichen ohne solche Hilfsmittel zu erzielen wäre.[59]
> (Operations of measuring are, possibly without explicit action of comparing, methods for stating likenesses with more reliability than that which could be achieved by our imperfect capacity for direct comparison without such aid.)

This characterisation of actions of measuring provides, - as did the one Meinong offered for actions of comparing -, the end of measurement. It could be said that measuring is an action, the end of which is a more precise method for stating likeness and dissimilarity than that delivered by our direct capacity for comparing. If this characterisation gives the end of measuring, it does not capture what is, according to Meinong, the real nature of measurement:

> Alle Messen ist seiner Natur nach Teilvergleichung, aber es gehört mit zu dieser Natur, nicht nur Teilvergleichung zu sein. [60]
> (Every measure is, by nature, a comparison of parts; but it does not belong to its nature to be solely a comparison of parts.)

How should we understand this apparently contradictory account?

1.4.2. Substitutive Measurement

Meinong recognises that numerical measurement proper depends on divisibility. Numbers, without which there could be no such

[59] Meinong (1896) § 13 pp. 272-273.
[60] Ibid § 13 p. 271.

measurement, are divisible magnitudes. Therefore, to measure numerically some magnitudes, these magnitudes must be divisible as numbers are:

> Ist alle Messung […] Teilvergleichung, so können [68] selbstverständlich nur solche Grössen messbar sein, die in gleichbenannte Teile zerlegbar sind, also die bereits oben im besonderen so genannten teilbaren Grössen.[61]
> (Since every measurement is a comparison of parts, it is obvious that only magnitudes that are divisible in parts bearing the same name are susceptible of measure; these magnitudes are the 'divisible magnitudes' already mentioned.)

This justifies the first part of Meinong's characterisation of measurement and reinforces the impression that the characterisation is contradictory.

Some magnitudes, as we saw, are indivisible. For such magnitudes, comparison of parts is not directly possible, given that they have no parts. In such cases, how could we measure these magnitudes if measuring is a comparison of parts?

The type of indivisible magnitude in which Meinong is interested is the dissimilarity relations type. As we saw, to any distance, there corresponds a *stretch* that is a divisible magnitude. Then, following Meinong, when we measure distances or dissimilarities, what we really do is measuring their corresponding stretches:

> Es liegt unter solchen Umständen nahe genug […] von Messung der Distanzen zu reden, wo man zunächst nur von Messung der zugeordneten Strecken reden dürfte.[62]
> (In such circumstances, it seems very likely sufficient to speak of measure of distances where one may first just speak of measure of the conjoined stretches.)

When we are measuring distances by means of their corresponding stretches, says Meinong, we are measuring distances in an indirect (mittelbare) way; that is to say that a case of measurement of indivisible magnitudes by means of divisible magnitudes like stretches is one of *substitutive* (surrogative) measurement. It is called 'substitutive

[61] Ibid. § 15 p. 277.
[62] Ibid. § 15 p. 278.

measurement' because we measure some magnitude by means of another substitute magnitude, namely, a stretch:

> Vielmehr wird hier als Messung des A etwas bezeichnet, was eigentlich nur Messung eines B ist. Bei Messung der Distanz wird eigentlich nicht diese gemessen, sondern die zugeordnete Strecke […]; ich stelle daher Messungen dieser Art als surrogative Messungen den früher betrachteten als eigentlichen Messungen gegenüber.[63]
> (What, in such a case, stands for a measure of A is rather properly just a measure of some B. When measuring distance, it is rather the conjoined stretch that is properly measured and not the distance […]; Measurements of this kind, being substitutive measurement, are contrasted with the above-mentioned proper measurement.)

Substitutive measurement shows that "it does not only belong to the nature of measurement to be a comparison of parts" and it teaches us how the dissimilarities that are involved in Weber's Law are measured. They are measured by means of their corresponding stretches. Therefore, having identified that relative difference between stimuli (*i.e.* the logarithm of their ratios), on the one hand, and between sensations, on the other hand, are magnitudes of dissimilarity and not of difference, we can measure these dissimilarities by means of their conjoined stretches.

1.4.3. Operations by means of substitutive measurement

Just as an illustration, let me mention some of the operations that are, according to Meinong, obtainable by means of substitutive measurement.

The dissimilarity between two magnitudes a and b is expressed by Meinong using the following formula: ${}_aV_b$.

The geometrical difference between a and b is one of the following ratios:

${}_aV_b = C\ a/b$ or ${}_aV_b = C\ b/a$[64]

[63] Ibid. § 15 pp. 281-282.
[64] Ibid, § 22 p. 306.

where C is:

> wo C eine […] durch geeignete Wahl der Einheit eventuell auch zu beseitigende proportionalitätskonstante bedeutet.[65]
> (where C is a proportionality constant that could be removed by means of a suitable choice of unity.)

The crux of Weber's Law is precisely that the proportion between intensity of stimuli and intensity of sensations is not one of geometrical difference but one of relative difference. Relative difference between *a* and *b* is one of the following ratios:

$$ {}_aV_b = C\,(b\text{-}a)/a \quad \text{or} \quad {}_aV_b = C\,(b\text{-}a)/b $$ [66]

To conclude on these operations, note that, as there is substitutive measurement, there is substitutive addition too:

> Kann man also Distanzen surrogativ messen, so wird man sie auch […] surrogativ addieren können.[67]
> (As one can measure distances by means of substitutes, one could add them by means of substitutes.)

The operation of addition between dissimilarities is formalised as follows:

$$ {}_xV_z = {}_xV_y + {}_yV_z $$

The conjoined addition between stretches is expressed symbolically:

$$ \overline{xz} = \overline{xy} + \overline{yz} $$

This ends the presentation of Meinong's account of measurement. Russell's reception of Meinong's work should now be discussed.

[65] Ibid, §18 p. 292.
[66] Ibid, § 23 p. 308.
[67] Ibid § 24 p. 314.

2. Russell and Meinong's work on Weber's Law

2.1. Russell's review

As a transition between the presentation of Meinong's work and that of its application to contemporary issues in philosophy, let me take a brief look at the kind of reception Russell offered to Meinong's work, first in his critical review and then in *The Principles.*

Let me first examine the critical side of the critical review. Russell makes only one objection to Meinong. The objection is that dissimilarities are magnitudes, in the proper meaning of the word, only if we are talking about dissimilarity between magnitudes of the same kind. There is another type of dissimilarity, however, namely diversity of content, which is not strictly a magnitude:

> Had he applied his doctrine to the relations of other pairs of terms than quantities of the same kind, it would, I think, led him into serious troubles. If the relations in question are reducible to identity and diversity of content, they cease to be properly quantities.[68]

What Russell calls here diversity of content is, for instance, the dissimilarity between a colour and a pitch. A pitch is diverse in content with respect to a colour because they are object of distinct *quality* (Qualität) as Meinong says[69] or are magnitudes of distinct kinds.

If Russell's objection does raise trouble, the discussion of the so-called atypical relations[70] shows that Meinong was aware of it. Atypical relations are comparisons of magnitudes of distinct types. Such comparisons lead to judgements of dissimilarity between magnitudes of distinct kinds. Meinong was aware that in such cases, he could not apply his notion of dissimilarity. For the magnitudes that are dissimilar, according to his notion of dissimilarity, should converge toward the same zero. The critical side of Russell's review, thus, only reiterates the limitations of Meinong's theory that Meinong himself already noticed.

The positive side of Russell's critical review is full of compliments for Meinong's work:

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68 Russell (1899) p. 252

69 See the discussion of atypical relations section 1.3.3.

70 See above, section 1.3.3

> The thesis [of the work] is at once simple and ingenious, the argument at once lucid and subtle. [71]

Russell's interests in Meinong's discussion are the following: (i) the distinction between dissimilarity and stretch that Russell himself accepts; (ii) the theory of substitutive measurement; (iii) the similarity between conditions on functions for measuring dissimilarity and conditions on functions of distance in non-Euclidean geometries mentioned in section 1.2. ; (iv) the measure of psychical magnitudes that constitutes the fifth section of Meinong's work that was not discussed in this presentation.

If Russell is interested in those topics, he offers no indication as to how he intends to use Meinong's contribution in the area. Russell (1903) does not consider measurement of psychical magnitudes; hence, point (iv) will be left aside. Points (i)-(iii), however, will be of great importance in *The Principles*. Russell's use of Meinong's contribution on these points should be considered.

2.2. Meinong's contribution to *The Principles*

At the beginning of the third part of the book, which is devoted to quantities, Russell offers a clear and ambitious thesis. The thesis of this part of the book is that modern mathematics since Descartes had, as a postulate, that "numbers and quantity were *the* objects of mathematical investigation, and that the two were so similar as not to require separation"[72]. Weierstrass, Dedekind, and Cantor, however, have shown that some numbers, irrational numbers, "must be defined without reference to quantity"[73].

Given this thesis, Russell's aim is to distinguish conceptually and logically the concept of number from the concept of quantity. He will show that, on the one hand, the notion of quantity is not a necessary condition for numbers (what is proved by the example of irrational numbers), and that, on the other hand, numbers are not necessary for having quantity. This means that "some quantity could not be

[71] Russell (1899) p. 251.
[72] Russell (1903) p. 157.
[73] Ibid, p. 157.

[numerically] measured, and some things which are not quantities (for example anharmonic ratios projectively defined) can be measured."[74]

The first step of the argument according to which some quantities cannot be measured is directly borrowed from Meinong's discussion.

A quantity is, following Russell (1903), something that has a magnitude. A weight for instance is a quantity that has a magnitude of 20 grams.[75] Russell first introduces the two traditional features of magnitudes:

> The usual meaning [of magnitude] appears to imply (1) a capacity for the relations of greater and less, (2) divisibility.[76]

Russell shows then that there are entities that have the capacity for the relations of greater and less but are not divisible. In so doing, Russell only reproduces Meinong's argument: some relations, for instance similarity, enter into relations of greater and less but are not divisible;[77] psychical magnitudes have the same features as these relations.[78]

Russell distinguishes between the relations of greater and less, which do not presuppose divisibility, and the relation of parthood, which does, arguing that, because of this distinction, Euclid's Axiom according to which the whole is greater than the parts "is not a mere tautology".[79] On this point Russell explicitly refers to Meinong.

When considering *The range of Quantity* in Chapter XX, Russell introduces the properties of order relations like greater and less. He defines the relations into which the magnitudes of a same quantity enter as relations of *distance*.[80] If a magnitude A is greater than B, then A is

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[74] Ibid, p. 158.

[75] It is remarkable that, even if the German ' Grösse' is best translated by 'magnitude', Meinong identifies what Russell will distinguish as quantity, on the one hand, and magnitude, on the other hand. Remember Meinong's characterisation of a magnitude: "Magnitude is or is had by that which allows the interpolation of terms between itself and its contradictory opposite". According to Russell, what allows etc. is a quantity, what is had by a quantity, is a magnitude. Following this distinction, individual properties (Eigenschaften) are not magnitudes at all, but quantities.

[76] Russell (1903), p. 159.

[77] Ibid, 153 (_) pp. 159-160.

[78] Ibid, 154 p. 160.

[79] Ibid, 153 (_) p. 160.

[80] Ibid, 160 p. 171.

at some distance from B. As already mentioned, Russell defines distance as an asymmetric relation:

> I shall mean by a kind of distance a set of quantitative asymmetrical relations of which one and only one holds between any pair of terms in a given class;[81]

The explanation for this is simply that he believes that it is easier to identify relations of distance with relations of greater and less which are obviously asymmetrical.

Russell shows then that distances are indivisible magnitudes[82]. Measurement of magnitudes is "a one-one correspondence between magnitudes of a kind and all or some of the numbers".[83] Measurement requires divisibility because:

> Measurement demands that […] there should be an intrinsic meaning to the proposition "this magnitude is double of that." […] Now so long as quantities are regarded as inherently divisible there is a perfectly obvious meaning to such a proposition: a magnitude A is double of B when it is the magnitude of two quantities together, each of these having the magnitude B.[84]

Measurement of divisible magnitudes suppose then a relation of part-whole, since, in Russell's example, A is the sum of two magnitudes B; A has for parts two quantities of magnitude B.

Distances and other indivisible magnitudes, thus, could not be measured as divisible magnitudes are. This leads Russell to introduce Meinong's distinction between distances and stretches in order to measure distances:

> On the straight line, if, as is usually supposed, there is such a relation as distance, we have two philosophically distinct but practically conjoined magnitudes, namely the distance, and the divisibility of the stretch.[85]

[81] Ibid, p. 180
[82] Ibid. p. 173
[83] Ibid. p. 177
[84] Ibid. p. 178
[85] Ibid. p. 182

How does Russell use these results of Meinong's discussion? The distinction between distance and stretch is used by Russell to define continuity, order, and finally, the distinction allows Russell to introduce the distinction between angles, on the one hand, and areas and volumes, on the other hand; a distinction of the greatest importance for Russell's account of geometry.

Russell, nevertheless, disagrees with Meinong on two points that will be briefly presented. The first disagreement concerns Meinong's characterisation of magnitudes in terms of limitation toward zero[86]. According to Russell, it is wrong or at least ambiguous to say that zero is the contradictory opposite of any magnitude of its kind. This is hardly deniable given the Kantian origin of this characterisation. Nevertheless, as Russell assumes[87], if a better characterisation of magnitude as the one he himself proposes in *The Principles* is given, then the rest of Meinong's discussion and results is not affected.

The second objection concerns the relation between distances and stretches. Meinong often suggests that distance, or dissimilarity, is a more important kind of magnitude than the kind to which stretches belong. Russell, however, doubts that distances really exist:

> On the whole, then, it seems doubtful whether distances in general exist; and if they do, their existence seems unimportant and a source of very great complications.[88]

The question of the existence of distances is left open in the present paper. It is clear that rejecting the existence of distances is a mere counterintuitive position, and thus, I am inclined to think that Russell is wrong. Nevertheless, this does not affect the truth of Meinong's thesis that if there are indivisible magnitudes like dissimilarities and distances, these magnitudes are measured by means of some substitutes, which are stretches. This thesis will be useful in the domain of a logic for comparative similarity that will be presented now.

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[86] Ibid. pp. 168-169, and pp. 184-187.
[87] Ibid. 177 pp. 186-187.
[88] ibid. p. 255.

3. Meinong and a Logic for Comparative Similarity

In the last decades, two distinguished contemporary philosophers, namely David Lewis[89] and Timothy Williamson [90], have each offered a system of logic for comparative similarity. Their systems are distinguishable from standard accounts of the logic of comparative terms. Standard accounts[91] of the logic for comparative terms account for comparatives by means of order relations between magnitudes. For instance

$\exists x \exists x'$ (Tall (Alfred, x) & Tall (Paul, x') & $x > x'$)

says that Alfred is taller than Paul is. At first sight, we might express comparative similarity in a similar way:

$\exists x \exists x'$ (D(Alfred, Paul, x) & D(Alfred, Sam, x') & $x > x'$)

(Read: the dissimilarity between Alfred and Paul is greater than the dissimilarity between Alfred and Sam.)

This last notation is called here the *standard* notation for comparative similarity because the notation is standard for any kind of comparative; is the same for any comparative.

However, the logics for comparative similarity of Lewis and Williamson treat the comparative as a predicate between things, not between magnitudes. For instance, the primitive in Lewis' logic is: $j \leq_i k$, which means that j is at least as similar to i as k is. The primitive in Williamson's logic is the expression T(w,x,y,z), which means that w is at least as similar to x as y is to z. Notice also that in their systems dissimilarity is primitive on similarity.

In this section, the standard notation will be defended. There are three distinct reasons why the standard notation must be preferred over a notation in the Lewisian or Williamsonian way.

3.1. Expressive Power

[89] See D. Lewis (1973) pp. 48 ff.
[90] See T. Williamson (1988).
[91] For instance, A. Morton (1984) and B. Katz (1995).

The first reason is that the expressive power of the standard notation is greater than the expressive power of the notations of Lewis and Williamson. Briefly, we can compare similarity with dissimilarity and express inferences of the following kinds:

$\exists x \exists x'$ (D(Alfred, Paul, x) & S(Alfred, Paul, x') & $x > x'$)

$\exists x \exists x'$ (D(Alfred, Paul, x) & D(Alfred, Sam, x') & $x < x'$)

then: $\exists x \exists x'$ (S(Alfred, Paul, x) & D(Alfred, Sam, x') & $x < x'$)

With this notation, dissimilarity and difference can also be compared. Remember that, following Meinong, the dissimilarity between 1 and 0 is infinite when the difference between them is finite. Therefore, the following sentence is meaningful:

$\exists x \exists x'$ (D(1, 0, x) & dif(1, 0, x') & $x > x'$)

Such comparisons and inferences are expressible in neither Lewis' nor in Williamson's notation because their special notation for similarity prevents us from comparing anything with (dis)similarity. The standard notation has thus a greater expressive power.

3.2. The grounds of comparative similarity

What are supposed to be the grounds for a relation of comparative similarity? At first sight, comparative similarity is supposed to compare similarities or dissimilarities. When one says that he is more similar to his mother than he is to his father, the terms of the comparison are the similarity between the speaker and his mother and the similarity between the speaker and his father. "The dissimilarity between red and orange is less than the dissimilarity between red and blue" compares the dissimilarity between red and orange and the dissimilarity between red and blue.

Therefore, the relation expressing comparative similarity in a notation for comparative similarity should be, it seems, a relation between such similarities.

Neither the notation of Lewis, nor that of Williamson, nor the one I will favour construe comparative similarity as a relation between similarities. Nevertheless, this similarity between our two philosophers' account and my favoured notation is a similarity in dissimilarity. It will be argued, following Meinong, that the terms of the relation of comparative similarity are stretches and not directly (dis)similarities. However, as it was shown, the terms of the relation are, according to Lewis and Williamson, the things that are said to be similar, not their similarities.

Thus, while Lewis and Williamson offer to compare things, I will offer to compare magnitudes that are necessarily conjoined with (dis)similaries. It is worth emphasising that, as Meinong, Lewis and Williamson agree that comparison introduces an order relation, in particular an order relation that entails concepts of "greater, less, at least as, etc.". Some order relations can put things, ordinary things, in order; for instance, genealogic order of precedence. Things are not the same, however, for order relations entailing concepts of "greater, less, etc." as the order relations that comparatives introduce. As far as the latter concepts are concerned, following Meinong[92], we deal with magnitudes.

E. Tegtmeier[93] already defends, following Meinong and against Carnap and Hempel, that:

> Komparative Begriffe sind Grö_envergleichsbegriffe, Begriffe, mit denen man Grö_en vergleicht.
> (Comparative concepts are concepts of compared magnitudes; with which one compares magnitudes.)

New arguments will not be offered here for this account of comparatives as instances of comparison between magnitudes. The presentation of Meinong's discussion constitutes sufficient support for this account. Since the comparison in question introduces order relations of "greater than, etc.", and since such order relations are peculiar to comparing magnitudes, the favoured notation for comparative similarity will, then, account for comparative similarity as comparison between similarities; which are magnitudes.

3.3. Measurement and quantification

[92] See section 1.3.2. on grounds of comparison
[93] E. Tegtmeier (1981) pp. 42 ff.

Intuitions clearly suggest that comparatives compare magnitudes. A comparison is something of the form "*x* is more something than *y* is", and the terms of the "more" in this sentence are the distinct values of the "something" that *x* and *y* share. My opinion is that Lewis and Williamson share this intuition on comparatives. So why do they prefer quantification and predication over things rather than the more intuitive quantification and predication over magnitudes concerning comparative similarity?

They reject the standard notation in terms of order relations between magnitudes for comparative similarity because they think it involves quantification over, and ontological commitment to, degrees of dissimilarity. Such quantification over degrees of dissimilarity implies a measurement of dissimilarity. They are very suspicious about measures of similarity, and thus they reject quantification over degrees of dissimilarity.

It can be shown that their qualms are unwarranted, and this can be done by means of Meinong's contribution to Weber's Law. I shall first address their worries individually, and then I shall give a general objection against their reservations based on Meinong's distinction between dissimilarity and stretch.

3.3.1. The assumption of symmetry

The first motivation is concerned with the fact that an important constraint on order could be invalid in an order of degrees of (dis)similarity. The constraint is: if *x* is at least as similar to *y* as *z* is and if *z* is at least as similar to *x* as *y* is, then *x* is at least as similar to *z* as *y* is. The constraint is not always satisfied because, says Lewis, similarity and dissimilarity are sometimes symmetrical, sometimes asymmetrical relations. While it cannot be demonstrated here, the point can be illustrated thus: while most people will assent to the judgement that "Korea is similar to China", few will assent to the judgement that "China is similar to Korea".

Lewis' explanation of such cases of asymmetrical similarity is that similarity is a vague relation which is context-dependent in the sense that the truth of "*a* is (dis)similar to *b*" depends on our interests, our culture, our point of view on the things under consideration.

Therefore, Lewis offers a purely cognitive account of similarity and dissimilarity, one which grounds his objection against a possible measure of objective degrees of similarity. Fechner's experiments and Meinong's discussion on the thresholds of discriminability show that some dissimilarities cannot be discriminated in particular circumstances. If one accepts the vagueness of (dis)similarity thesis, that is, the thesis according to which (dis)similarities depend on our point of view, then one accepts that every dissimilarity could be discriminated. A dissimilarity that could not be discriminated by a subject could not depend, in any sense of the phrase, on a point of view.

Therefore, if Meinong's discussion on equal discriminability is correct, which it undoubtedly is, (dis)similarity is not a cognitive relation. Discrimination, or awareness of dissimilarity, in contrast, is a cognitive relation. It really seems that Lewis confuses dissimilarity and discrimination[94]. Given that dissimilarity is not a cognitive relation, it is always symmetrical and the constraint on order mentioned above is always satisfied. Thus, the first motivation does not provide a conclusive objection against order-relations between degrees of similarity, and thus against measurement of, or quantification over, degrees of similarity.

3.3.2. The cardinality of similarity-order

The second motivation, mainly developed by Williamson (1988), concerns the cardinality of the order of degrees of similarity; more precisely, it concerns the possibility of mapping the order of degrees of similarity onto the order of real numbers. According to Lewis and Williamson, to measure degrees of similarity is to give a real-valued measure of similarity. This means that the order of degrees of similarity must be order-isomorphic to the order of real numbers. Moreover, to have such a measure of degrees of similarity, the domain of degrees of similarity cannot exceed the domain of real numbers.

Now it is possible to have a domain of degrees of similarity exceeding the domain of real numbers. Williamson offers the following illustration: someone might claim that, for any infinite cardinals c, c', c'', if $c < c' < c''$, then c' is more similar to c'' than c is. The claim entails that there are at least as many degrees of similarity as cardinals greater than c. However, since there are more cardinals greater than c

[94] T. Williamson, who gives few importance to this motivation against measure of similarity, could not be accused to get them mixed up, at least when similarity is understood as identity of content. See T. Williamson (1989) pp.10-23.

than there are real numbers, there are more degrees of similarity than there are real numbers.

When considering Meinong's discussion, but also Russell's considerations on distance in non-Euclidean geometry, it seems clearly true that there are more degrees of (dis)similarity than there are real numbers. Some dissimilarities, for instance, one between a finite and an infinite magnitude, are infinite; Williamson is right, but what is exactly the problem? Nothing rules out the case of an order of degrees of similarity that is order-isomorphic and even to the order of infinite cardinals.

3.3.3. Quantification over Stretches

Although I have addressed the particular concerns of Williamson and Lewis, it will be useful to have a more general response to worries concerning (1) measurement of (dis)similarity and, more particularly, (2) quantification over degrees of similarity based on Meinong's account of the substitutive measurement of dissimilarity.

Let me first introduce a distinction between magnitudes and degrees of (dis)similarity. Following Lewis and Williamson, quantification over degrees of dissimilarity implies measurement of dissimilarities. However, as Meinong claims, measurement of dissimilarities is a comparison between magnitudes of dissimilarity; it implies such magnitudes. Both claims appear consistent when degrees of dissimilarity are defined as numerical representations of magnitudes of dissimilarity. If degrees are such numerical representations, it is true that it implies measurement; and if Lewis and Williamson are suspicious about measurement of dissimilarity, they must suspect quantification over degrees.

Nevertheless, the standard notation for comparative involves quantification over magnitudes, not over their numerical representation. Magnitudes ground measurement, and not *vice versa*.

Suppose Lewis and Williamson reject this distinction between magnitudes and degrees, Meinong's theory of substitutive measurement could still be helpful. If Meinong is right, then what we are really measuring when we measure dissimilarity is a substitute of it; namely a stretch. Any objection against measurement of dissimilarity misses the target where measurement of stretches is concerned. Stretches are not vague, context dependent, and there is no objection against infinite stretches. If what we are really, directly, measuring when we measure (dis)similarities are stretches, then, what we are really, directly, quantifying over when quantifying over degrees of (dis)similarity are stretches.

In short, following Meinong, nothing forbids us from having a standard notation for a logic for comparative similarity; nothing prevents us from quantifying indirectly over degrees of similarity and nothing prevents us from having an indirect measurement of them.

A standard and developed notation for comparative similarity that assumes Meinong' theory of substitutive measurement must then have the following form:

$\exists x \exists x' \exists y \exists y'$(D(Alfred, Paul, x) & D(Alfred, Sam, x') & ($x \approx y$) & ($x' \approx y'$)& $y > y'$)[95, 96].

Where x and x' are magnitudes of dissimilarity, and y and y' are their coinciding stretches, and where the sign ≈ must be read 'is conjoined with'.

If I am on the right track and Russell is correct, then Meinong's contribution to Weber's Law is a wonderful piece of analysis. The manner in which indivisible magnitudes and their measurement is treated is very insightful and difficult to argue against. Moreover, it leads to fruitful and innovative application in some areas of contemporary analytic philosophy. The hope in this study was to rehabilitate one of the most neglected and maybe one of the most important works of Meinong. We could wish for a more widespread recognition of its value.[97]

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[95] The developed notation is longer than the notations of Lewis and Williamson, but we can abbreviate it as Katz (1995) does for "taller" in the following way: D(Alfred, Paul) > D(Alfred, Sam).

[96] The notation quantifies over magnitudes of dissimilarity, which are relations. Does this quantification commit us to relations? I do not think so. This quantification is non-nominal, and thus, non-committal. See A. Rayo & S. Yablo (2001).

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References:

Fechner Gustav Theodor (1888), Über die psychophysischen Massprinzipien und das Webersche Gesetz, *Wundts Philosophische* Studien [vol. 4], 1888, p. 192.

Hardin Larry C. (1988), Phenomenal Colors and Sorites, *Noûs* [vol. 22], 1988, pp. 213-234.

Höfler Alois (1896), Zur Analyse der Vorstellungen von Abstand und Richtung, *Zeitschrift für Psychologie* [vol. 10], 1896, pp. 223-234.

Höfler Alois & Meinong Alexius (1890), *Logik*, Prag, Wien, Leipzig 1890.

Kant Immanuel (1974), *Kritik der reinen Vernunft*, Frankfurt am Main 1974.

Katz Bernard (1995), Making Comparisons, *Mind* [vol.104], 1995, pp. 369-392.

Krantz David H. & Luce Duncan R. & Suppes Patrick & Tversky Amos (1971), *Foundations of Measurement*, New York, London 1971.

Lewis David K. (1973), *Counterfactuals*, Oxford 1973.

Mally Ernst (1922), *Studien zur Theorie der Möglichkeit und Ähnlichkeit: Allgemeine Theorie der Verwandtschaft gegenständlicher Bestimmungen*, Vienna 1922.

Meinong Alexius (1896), *Über die Bedeutung des Weber'schen Gesetzes*, reprinted in *Meinong's Gesammelte Abhandlungen*, Vol. 2, Leipzig 1913.

Meinong Alexius (1900), Abstrahieren und Vergleichen, reprinted in *Meinong's Gesammelte Abhandlungen*, Vol. 2, Leipzig 1913.

Morton Adam (1984), Comparatives and Degrees, *Analysis* [vol.44], 1984, pp. 16-20.

Rayo Agustin & Yablo Stephen (2001), Nominalism Through De-nominalisation, *Noûs* [vol.35:1], 2001, pp. 74-92.

Russell Bertrand (1899), Review of Meinong's *Ueber die Bedeutung des Weber'schen Gesetzes*, *Mind* [n.s.8], 1899, pp. 251-256.

Russell Bertrand (1903), *The Principles of Mathematics*, 2nd ed., London 1937.

Schrödinger Erwin (1926), *Thresholds of Color Differences*, in MacAdam D. L. (ed.), Sources of Color Science, Cambridge 1970, pp. 183-193.

Tegtmeier Erwin (1981), *Komparative Begriffe: eine Kritik der Lehre von Carnap und Hempel*, Berlin 1981.

Williamson Timothy (1988), First-Order logic for Comparative Similarity, *Notre Dame Journal of Formal Logic* [vol.29], 1988, pp. 457-481.

Williamson Timothy (1989), *Identity and Discrimination*, Oxford 1989.

Zindler Konrad (1889), *Beiträge zur der mathematischen Erkenntnis*, Wien 1889.