THE KALAM COSMOLOGICAL ARGUMENT: THE QUESTION OF THE METAPHYSICAL POSSIBILITY OF AN INFINITE SET OF REAL ENTITIES

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Abstract: This paper examines the Kalam Cosmological Argument, as expounded by William Lane Craig, insofar as it pertains to the premise that it is metaphysically impossible for an infinite set of real entities to exist. Craig contends that this premise is justified because the application of the Cantorian theory to the real world generates counterintuitive absurdities. This paper shows that Craig's contention fails because it is possible to apply Cantorian theory to the real world without thereby generating counter-intuitive absurdities, provided one avoids positing that an infinite set of real entities is technically a set within the meaning of such theory. Accordingly, this paper proposes an *alternative version* of the application of Cantorian theory to the real world thereby replacing the *standard version* of such application so thoroughly criticized by Craig.

The Kalam Cosmological Argument (KCA) purports to establish that God exists based upon the alleged metaphysical impossibility of an infinite regress of past events. According to KCA, given that an infinite temporal regress is metaphysically impossible and that everything that begins to exist has a cause of its existence, further analysis discloses that such cause is a personal creator who changelessly and independently willed the the beginning of the universe.[1]

Professor William Lane Craig justly deserves much credit for having presented, with great analytical and polemical skill, the KCA in its most persuasive and challenging form in contemporary times.[2] Craig's version of the KCA relies upon two separate philosophical arguments to establish the premise that a beginningless temporal series is metaphysically impossible. The first is that: a) an actual infinite cannot exist in the real world; and b) an infinite temporal series is such an actual infinite.[3] The second is that a temporal series cannot be an actual infinite, assuming than an actual infinite can exist in the real world, because: a) a temporal series is a collection formed by successive addition; and b) a collection formed by successive addition cannot be an actual infinite.[4] When Craig denies that an actual infinite can exist in the real world, he is denying that there can be infinitely many natural or supernatural entities of any kind. Craig denies that abstract entities (e.g., numbers, universals) exist in the real world; but such entities may properly be said to exist, in a rather pickwickian sense, in the mathematical realm.[5] In any event, I shall use the term *real entities* to refer exclusively to natural or supernatural substances (or continuants), and properties and events pertaining to them, as distinguished from abstract entities, whatever their true ontological status.[6] I shall also use the term *real infinite* to refer to an infinite set consisting of *real entities*. Whatever the true ontological status of abstract entities, Craig is quite emphatic that he does not deny the logical possibility of an actual infinite, as distinguished from its metaphysical possibility. According to Craig, the existence of a real infinite set is an example of what is logically but not metaphysically possible.[7]

This article focuses upon Craig's first philosophical argument. I do so because the second assumes, as I have noted, that a real infinite set can exist. Therefore, the issues and how to resolve them are quite different, however related they are. Nevertheless, discussion of the first argument actually facilitates discussion of the second since Craig holds that the problem of the metaphysical possibility of a real infinite set is exacerbated in the case of a beginningless temporal series, assuming the same is an actual infinite set.[8] Additionally, if a real infinite is metaphysically possible, it then follows that God is without power to create a world with infinitely many entities or a superworld of infinitely many worlds each with finitely many entities.

Essentially, Craig's first argument is that real infinites are metaphysically impossible because the Cantorian theory of transfinite numbers,[9] which may be perfectly consistent in the mathematical realm, must therefore be limited to that domain because it cannot successfully be applied to a real world without generating counterintuitive absurdities.[10] Craig does not claim, as his selection of purported counterintuitive absurdities abundantly discloses, that the existence of real infinite sets are metaphysically impossible simply because they may be factually impossible upon some other, nonmathematical, ground.[11]

This paper, as far as I am aware, is unusually (if not uniquely) different from others critical of the KCA in that I agree with Craig that counterintuitive absurdities are indeed generated by the application of Cantorian theory to the real world according to (what I call) the *standard version* (hereafter *SV*) of such application. But I hasten to add, not everything Craig claims to be a counterintuitive absurdity is actually one.[12] I chiefly differ from him in that I believe Cantorian theory may be applied to the real world without generating counterintuitive absurdities, provided that SV (i.e., the *standard version* of the application of Cantorian theory to the real world) is abandoned.

Before proceeding with a brief review of relevant Cantorian theory,[13] let us first consider finite sets. The number (0 or a positive integer) that constitutes the size (or power) of a finite set is called its cardinal number.[14] To state what is the cardinal number of a finite set is to answer the question: how many members of the set are there? Two finite sets A and B are said to be equipollent when their respective members correspond one-to-one to each other; that is, their members can be so related such that to every member of A there corresponds one and only one member of B, and conversely.

Here I digress for a moment to note that various terms are commonly used in the literature to refer to sets the members of which are respectively in one-to-one correspondence (or bijection), e.g, "equivalent," "equinumerous," "equipotent," "equipollent." I shall use "equipollent" (except when quoting others) as the most theoretically neutral expression, for my purposes, because: a) "equivalence" is often used to refer to a relation that by definition is reflexive, symmetrical, and transitive; and b) "equinumerous" may suggest that the cardinality of two sets not in one-to-one correspondence is necessarily not identical. I prefer to use "equipollent" although "equipotent" has the same literal meaning. Accordingly, this paper accepts the definition: "Two sets A and B are said to be [*equipollent*] (in symbols, $A \sim B$) if and only if there exists a one-to-one correspondence between them" (i.e., such one-to-one correspondence is a pairing of the members of either set with those of the other set).[15]

Two finite sets have the same cardinality if and only if they are equipollent. Thus, two non-equipollent finite sets do not have the same cardinality. If, for example, there is a two-to-one correspondence between members of A and of B, then the cardinality of A is twice that of B, and conversely. Equipollence between two sets is the necessary and sufficient condition for both having the same cardinality (i.e., numerical equivalence) (symbolically, $A \sim B \;.=.\; [A] = [B]$). The relation of equipollence is reflexive (i.e., each set is equivalent to itself), symmetric (e.g., if $A \sim B$, then $B \sim A$), and transitive (e.g., if $A \sim C$ and $B \sim$ C, then $A \sim B$). The cardinality of a non-empty finite set is determined by counting, that is the process of pairing members of a finite set with the progressive sequence of natural numbers (but starting with 1) until the set A is *exhausted*; that is, there is no remaining member of A to be paired with the next natural number. The highest-paired natural number is the cardinal number of the finite set.

Let us next consider mathematical infinites according to relevant Cantorian theory. A natural number cannot be the cardinal number of the set of all natural numbers (i.e., $\{1, 2, 3, 4, \ldots .\}$)[16] since there is no highest natural number.[17] The size or magnitude of the set of all natural numbers (N) must be a transfinite cardinal number, which is termed $\aleph_0$ (aleph zero) because it is the smallest transfinite number.[17] A mathematical infinite with $\aleph_0$ as its cardinal number is said to be denumerable (or denumerably infinite) because it is equipollent to the set of all natural numbers.[18] $\aleph_0$ (aleph zero) is also the cardinal number of all rational numbers (i.e., numbers expressible as a fraction with two integers as numerator and denominator, respectively), and of all algebraic numbers. There are cardinal transfinite numbers greater than $\aleph_0$, such as the cardinal number of the set of all geometrical points (on a line, in a square, or in a cube, for example), and that of the set of all geometrical curves, the latter being a cardinal number greater than the former.

However, I should like to here emphasize that this paper does not concern itself with issues pertaining to transfinite numbers greater than $\aleph_o$. Unless otherwise indicated, my discussion of mathematical infinites is limited to the set of natural numbers and other integers. I do so because, first, we have enough problems just talking about denumerable infinites. Second, Craig himself asserts that it is metaphysically impossible for non-denumerable infinites to exist in the real world even were it the case that denumerable infinites exist.[19] Thus, for purposes of convenience only, my paper assumes arguendo that non-denumerable infinites cannot exist in the real world.

According to Cantorian theory, mathematical infinites are like finite sets in that two sets have the same cardinality if and only if they are equipollent. Equipollence is an equivalence relation, and hence it is transitive. Therefore, two mathematical infinites, each of which is equipollent to another mathematical infinite, are necessarily equipollent to each other. However, unlike finite sets, any mathematical infinite is necessarily equipollent to any of its infinite proper subsets.[20] Indeed, as Craig reminds us, modern set theory defines an infinite set as being equipollent to one of its proper subsets.[21]

Members may be removed from a denumerably mathematical infinite (provided an infinite set remains) or added to it without changing its size (i.e., $\aleph_o$) or affecting its denumerability. Additionally, the cardinal number of the union of two or more denumerably infinite sets of numbers is $\aleph_o$, and so is that of the union of a denumerably infinite set and any finite number.

A commonly given way of illustrating the equipollences that obtain among infinite mathematical sets is to juxtapose symbols representing the set of all natural numbers with that of some other set or sets. Thus, for example, we have:

N {1, 2, 3, 4, 5, . . . [n] . . . } the set of all natural numbers
E {2, 4, 6, 8, 10, . . . [2n] . . . } the set of all even natural numbers
Z {–1, –2, –3, –4, –5, . . . [–n] . . . } the set of all negative integers

N and Z are two disjoint sets, but a function-equation ($z = -n$) is a rule that orders every member of N into a one-to-one correspondence with a member of Z.[22] E is a proper subset of N, but a function-equation ($e = 2n$) is a rule that orders every member of N into a one-to-one correspondence with an even number. The cardinal number of the finite subset of consecutively ordered numbers {1, 2, 3, 4, 5, 6} in N—that is, 6—is the same as that of its complementary subset {2, 4, 6, 8, 10, 12} in E; but larger than the cardinal number of its own proper subset of even numbers, {2, 4, 6}—that is, 3. Nevertheless, as we see, each member of every finite subset of consecutive members of N has its own corresponding member in E. That E has the same cardinality as N is an example illustrative of the rule, so disturbing to many, that with respect to actual mathematical infinites, the whole is *not* greater than any of its parts.[23]

So long as one is concerned with only finite sets, the same "arithmetic" practically applies to both abstract and real entities. However, things are not quite so simple when we turn our attention to infinite sets of real entities. For, according to Craig, Cantorian theory itself is only within the domain of pure mathematics and, as such, does not apply to the real world. Thus, he writes: "Cantor's system and set theory are concerned exclusively with the mathematical world, whereas our argument concerns the real world."[24] Craig further explains: "Cantor's definition of a set made it clear that he was theorizing about the abstract realm and not the real world for, it will be remembered, he held that the members of a set were objects of our intuition or of our thought."[25] It is thus necessary to devise appropriate bridging (or correspondence) rules in order to apply such theory to the real world. What one must bear in mind is that the bridging rule, whereby a *pure* mathematical theory of transfinite numbers is rendered applicable to a real world, is not itself part of that theory whether as a theorem or otherwise. It should also be borne in mind that Craig himself holds that the term "set" in Cantorian theory is used to pertain only to abstract entities. What Craig has done, and this appears to be commonly (if not universally) assumed in the literature, is to uncritically posit that real sets (whether finite or infinite) are to be deemed as being also among those entities denoted by the term "set" as used in Cantorian theory as a term of art.[26] The significance in SV (the *standard version* of the application of Cantorian theory to the real world) of the bridging rule, which may be thought of as being in the nature of a metaphysical axiom, chiefly lies in that the relation of equipollence of sets is transitive. Accordingly, given the bridging rule posited in SV, if two real infinites A and B are each equipollent to N (the set of all natural numbers), then they are necessarily equipollent to each other, and such equipollence is the necessary and sufficient condition for both having one and the same cardinal number. So, for example, if there are infinitely many humans, each with exactly two hands, then the set of humans and that of their hands are equipollent. According to SV, a one-to-one correspondence between the set of infinitely many humans and that of pairs of human hands entails a one-to-one correspondence between infinitely many humans and their infinitely many hands. This, surely, is not simply a strange result. It is, as Craig would maintain, a counterintuitive absurdity.[27]

However, Craig has failed to show, and indeed has not even attempted to show, that Cantorian theory cannot successfully apply to the real world if so much of *applied* Cantorian theory that generates the counterintuitive absurdities of which he justly complains is eliminated. More precisely, what is to be eliminated is the bridging rule itself whereby a real infinite is posited to be a *set* within the meaning of modern set theory for the purpose of applying such theory to the real world. Such positing requires the application to real infinites of the transitivity rule to every mixed situation involving an illation of an equipollence between two real infinites given their respective equipollences with a denumerable mathematical infinite.[28]

The proposition that a real infinite set is necessarily equipollent to another real infinite set is not at all evident to me; anymore than the proposition that equipollence is not only sufficient but also necessary in order to have numerical equivalence. We should here bear in mind Georg Cantor's admonition:

> All so-called proofs of the impossibility of actually infinite numbers are, as may be shown in every particular case and also on general grounds, false in that they begin by attributing to the numbers in question all the properties of finite numbers, whereas the infinite numbers, if they are to be thinkable in any form, must constitute quite a new kind of number as opposed to the finite numbers, and the nature of this kind of number is dependent on the nature of things and is an object of investigation, but not of our arbitrariness or our prejudice.[29]

The same things can be more or less as well said *mutatis mutandis* concerning real infinites with respect to mathematical infinites. Thus, we should also be prepared to consider the possibility that the mathematical properties of real infinites may radically differ in some respects from those of infinite sets of abstract entities.

In place of SV, this paper proposes an *alternative version* (hereinafter referred to as AV) of the application of Cantorian theory to the real world. AV includes four principal propositions. The first (AV1) is that every real infinite and *N* (the set of all natural numbers) are equipollent because the members of the former correspond one-to-one with the members of the latter. Instead of a bridging rule that *intrinsically* entails the equipollence of any real infinite with *N* because "set," as standardly used in Cantorian theory, is erroneously deemed by Craig to encompass real infinites, we have a bridging rule that *extrinsically* matches every member of any real infinite with one and only one member of *N*, and conversely. The second (AV2) is that the cardinal number of *N* is the cardinal number of every real infinite because each such infinite is equipollent with *N*. The third (AV3) is that equipollence between two real infinites is a sufficient but not necessary condition for such two sets (as commonly understood) to have the same cardinality. The fourth (AV4) is that no real infinite is equipollent with any of its infinite proper subsets, although both have the same cardinality, i.e., $\aleph_0$.

I provisionally consider (subject to further analysis) each of these four propositions (AV1–4) to be an axiom (or, if you prefer, a postulate) in AV. These proposed axioms cannot be proved to be inconsistent or invalid in a non–question-begging way (e.g., by stipulating that SV is true). I believe that AV has a higher epistemic status that those of SV, taken as a whole, for the reasons I give in this paper. Accordingly, I regard the axioms of AV as metaphysically necessary truths.[30]

The proposition (AV1) that any infinite set of real entities is equipollent to *N* is minimally necessary in order to apply Cantorian theory of transfinite numbers to the real world. Were a set with infinitely many real entities to exist, its cardinal number could not possibly be a natural number. Hence, its cardinal number must be $\aleph_0$ since $\aleph_0$ (according to Cantorian theory) is the smallest transfinite number.[31]

Hence, it follows (AV2) that the cardinality of the set of all natural numbers is the common cardinality of all real infinites. Given AV1 (and so also in SV), the equipollence between infinitely many pairs of human hands (where each human has exactly two hands) and *N* coheres with the equipollence between *N* and the set of infinitely many human hands (taken individually: [{infinitely many pairs of human hands} ~ *N*] and [*N* ~ {infinitely many human hands individually}]. A real infinite is equipollent not only to *N*, but also to any other denumerable mathematical infinite, for example, {infinitely many human hands} ~ *N* ~ *E*.[32]

Such equipollences necessarily obtain by virtue of the mathematical properties of purely mathematical infinites as intuitively discerned, as further embodied in naïve or logicist set theory (as adequately amended by some theory of types), or by virtue of some particular axiomatic set theory (such as that of Zermelo-Fraenkel, for example). Because such equipollences are mathematically determined, the one-to-one correspondence between any two mathematical infinites does not depend upon a factually contingent or definitional matter pertaining to a real world consisting of natural or supernatural substances and events pertaining to them.[33]

I propose that the relation of equipollence of each of two real infinite with *N* is not transitive with respect to another real infinite. Unlike mathematical infinites, whether or not two real infinites are equipollent to each other depends upon the existence of a factually contingent or definitional matter pertaining to real entities. For example, let us again suppose that there is an infinite set of humans, and that each such human has two and only two hands. Contrary to SV, there cannot possibly be a one-to-one correspondence between the infinite set of humans and the infinite set of their hands (individually taken) since it has already been given that each human has two and only two hands which, I might add, are his or her very own hands. In yet other cases whether or not two real infinites are equipollent to each other depends upon a definitional fact pertaining to the real world. For example, if an inch is defined as being a twelfth part of a foot, then there cannot be a one-to-one correspondence between an infinite set of feet and an infinite set of inches.[34] Given the foregoing, I maintain (AV3) that equipollence between two real infinites is a sufficient but not necessary condition for both having the same cardinality.[35] Accordingly, the equipollence of each of two real infinites with the set of all natural numbers does not entail the equipollence of the two real infinites.[36] Nevertheless, non-equipollent real infinites have the same size in the sense that every real infinite is equipollent to *N* and thus has its cardinality; i.e., $\aleph_0$. Therefore, one cannot properly say that the size of one real infinite can possibly differ from that of another.

It is not at all evident that a real infinite set must be equipollent to a proper infinite subset, although both sets are equipollent to *N*.[37] Thus, for example, let us take again the case of an infinite set of humans (each with exactly two hands) and the complementary infinite set of their hands. There is a one-to-two correspondence between sets of all humans and all human hands. However, there is a one-to-one correspondence between the sets of left hands and of right hands, and the same correspondences, respectively, between the sets of all humans with that of their right hands, or that of their left hands. Accordingly, there cannot be a one-to-one correspondence between human hands and members of either of its subsets; i.e., those of right and left hands. Were there to be such a correspondence, there would then also have to be a one-to-one correspondence between humans and all their hands—a proposition contrary to our original hypothesis. Hence, I propose (AV4) that it is impossible for a real infinite and any of its infinite proper subsets to be equipollent, although both sets are numerically equivalent in that they have the same cardinality.

Some readers may well experience difficulty in accepting the notion that one may not infer the equipollence of two real infinites because each is equipollent to N. First, recall that there is an equivocation with respect to the term "set." A real infinite is not to be deemed a *set* as standardly understood in Cantorian set theory as a technical term of art. Accordingly, whether or not an equipollence between two real infinites obtains depends upon contingent or definitional matters of fact rather than upon the mathematical properties of abstract sets as determined by the axioms of the theory in question. Moreover, equipollences between real infinites and mathematical infinites are similar in some respects to, but also different in others from, equipollences between real infinites. For example, the infinite set of all human hands is equipollent to N, and so respectively are the sets of all human left hands, and of human right hands, and of all humans. Furthermore, for example, the infinite set of all pairs of all human hands (pairing right and left hands) is equipollent to N. In short, the equipollence of the infinite set of all human hands to N and the equipollence of the infinite set of all pairs of all human hands to N mutually entail each other. On the other hand, the equipollence between the real infinite of all humans and that of all pairs of human hands, given that each human has exactly two hands, does not entail an equipollence between the former set and the set of all human hands. Given the foregoing, if a real infinite, A, is not equipollent to another, B, in that there is a two-to-one correspondence between the members of A and B, then there is an equipollence between A^{2*} (the infinite of pairs of members of A) and B. The equipollence between A^{2*} and N thus entails an equipollence between A and N. Somewhat similarly, if two real infinites A and B are equipollent, but then n additional members are added to A, then B is no longer equipollent to A^{+n} (the original members of A plus the n additional members). But both B and A^{+n} are each equipollent to N, and the equipollence of $A+n$ and N entails the equipollence of A and N.

Ultimately, the approach to be taken in determining what version of the application of Cantorian theory to the real world is to be adopted depends upon first hypothetically assuming that there are real infinites, such as those of humans and their hands, and then immediately discerning that such real infinites cannot possibly be equipollent. Nevertheless, the version proposed in this paper is not calculated to drive pure mathematicians "out of the paradise which Cantor has created for [them],"[38] since propositions AV1, AV2, AV3, and AV4 do not pertain to the domain of mathematical objects.

Before we proceed to consider some ostensibly compelling counterintuitive absurdities commonly cited by Craig and others, and which are generated by the application of Cantorian theory to the real world according to the *standard version*, I should first like to address Craig's general objections to a beginningless temporal series based upon the alleged metaphysical impossibility of an infinite set of real entities. These general objections appear to pertain to any version of the application of Cantorian theory to the real world.

One such general objection is that a new member cannot be added to a real infinite because its members, prior to any addition, already correspond one-to-one to *N*. Craig argues that *N* has been already completely exhausted or used up, in the sense of thereby precluding the addition of another member.[39] Craig rightly contends that it is not sufficient to answer that that the members of the augmented set can be reassigned *numerals* in order to take the addition of new members into account. Nevertheless, Craig is mistaken in his basic view. The mere fact that one infinite set of real entities corresponds one-to-one with members of *N* does not entail that there cannot be other infinite sets of real entities whose members simultaneously correspond one-to-one with the members of *N*. In the case of finite real sets, we would not say that the presence of seven cups in a cupboard exhausts or uses up that subset of natural numbers which has the cardinal number 7, thereby precluding the concurrent existence of other real finite sets, each with seven members. Similarly, the existence of one real infinite set, itself necessarily *exhausting* (in one sense) the infinite set of natural numbers, does not preclude the concurrent existence of other real infinite sets which are similarly equipollent to *N*. Even more to the point, every infinite subset of a real infinite (such as, say, those of an infinite series of years ending in 2000 with each subset ending respectively in a different, earlier year) is equipollent to *N* and, therefore, to any of its infinite subsets. Hence, I do not think that there is any merit to the contention that there cannot be an addition to an existing infinite set of real entities because *N* is said to have already been exhausted or used up. Similarly, a removal of members from a real infinite does not entail that there is no longer a one-to-one correspondence between the members of the reduced but yet infinite real set and *N*. Craig describes his argument on the matter as "one of the most tentative I presented."[40] He does so, I think, for good reason.

Craig also generally objects to the metaphysical possibility of real infinites upon the general ground that it is absurd to maintain that the cardinal number of members of one real infinite and the number of a union of two or more other such infinites must be the same, or that the cardinal number of a real infinite remains the same even though a proper subset of infinitely many members is removed.[41] This general objection appears to rest upon his rejection of the applicability of the principle of correspondence, which "asserts that if [and only if] a one-to-one correspondence between the elements of two sets can be established, the sets are [numerically] equivalent [i.e., they have the same cardinality]," to infinite sets.[42] First, whatever appears to be absurd obtains by the proposition that common cardinality of two or more real infinites necessitates that they are equipollent. Given this assumption, there is absolutely no sense in which, for example, it can be properly said that there are many more human hands than humans, given that the two sets have the same cardinality, notwithstanding that each of the infinitely many humans has exactly two hands. Second, the common cardinality of all real infinites obtains because the cardinal number of any real infinite cannot be a natural number, and so it must be the smallest transfinite number; i.e., $\aleph_0$. To say that there are as many members in one real infinite as there are in another real infinite, in the sense that both are numerically equivalent, simply means that they have one and the same cardinality. Any real infinite, like a mathematical infinite, retains its cardinality although members are added to or removed from it, provided there is always a surd infinite. Hence, the numerical equivalence per se of two real infinites does not entail the equipollence of the two sets.

Yet another general objection, which Craig advances, is what he characterizes as his "strongest arguments in favor of the impossibility of the existence of an actual infinite, those based on inverse operations performed with transfinite numbers."[43] His position is that "contradictions [are] entailed by inverse arithmetic operations performed with transfinite numbers, operations which are conventionally prohibited in transfinite arithmetic in order to preserve logical consistency."[44] Craig also indicates similar inconsistencies with respect to attempts to divide transfinite cardinals numbers.[45] Thus, as Craig puts it, "the extension of computational operations beyond the realm of finite cardinals is possible only for the direct operations—addition and multiplication—not for their inverses."[46]

Although according to Cantorian theory inverse computational operations are not possible for transfinite cardinal numbers, it does not follow that non-computational operations of addition or removal of members are impossible with respect to infinite sets of entities, whether abstract or real. Indeed, such operations are clearly admitted in Cantorian theory.[47] There is a manifestly clear difference between the *removal* of an infinite subset from an infinite set of real or abstract entities and the attempted *subtraction* of one transfinite cardinal number from another. Hence, it appears that Craig's third general objection to the metaphysical possibility of an actual infinite set of real entities also fails because the mathematical impossibility of inverse operations performed with transfinite numbers does not preclude inverse operations of addition or removal of members with respect to real infinite sets.[48]

We turn now to alleged counterintuitive absurdities, other than those more general objections noted above, which appear to specifically pertain to the SV.[49] A favorite way in which Craig seeks to show that Cantorian theory, as applied to the real world according to SV, entails counterintuitive absurdities pertains to his hypothetical library of infinitely many books.[50] Let us suppose that there is an infinite set of spaces, of equal size and dimensions, in a column starting from a fixed position in an absolute, Euclidian space and extending in a straight line in a given direction. Suppose that each space is fully occupied by one and only one book. Let us further suppose that every book has either a red or black cover, and that for every black-covered book there is a red-covered book, and conversely. Let us assume arguendo that two infinite sets of real entities, each equipollent to *N*, are necessarily equipollent to each other. As Craig rightly asserts, counterintuitive absurdities are indeed entailed given this assumption. Thus, for example, it follows that there is a one-to-one correspondence between the members of the set of all the books in the column and each of its subsets of red or black-covered books—although there is also a one-to-one correspondence between the two subsets of red- and black-covered books. Moreover, if all the black-covered books are removed, it still remains the case that there continues to be a one-to-one correspondence between the remaining books and all the spaces.

However, these counterintuitive absurdities are not entailed by AV (the theory of this paper). By virtue of this theory, there is a one-to-one correspondence between the red- and black-covered books, and there is not a one-to-one correspondence between the set of all books and either of its two subsets in question. There is a one-to-one correspondence between the spaces described in the foregoing paragraph and such books as occupy them—if all spaces are occupied. If all the red-covered books are removed, there consequently is a two-to-one correspondence between the spaces and the remaining books. All other things remaining equal, only so many books can be added to the library as would fill the empty spaces. If only one book is removed from the otherwise complete library, a new book can only be inserted at the *near end* or at some space at a finite distance from it. It is nonsense to speak of a new book being inserted at the *far end*, because there is no such end. The only way to add members to the set of spaces in the column is to add spaces by expanding the column beyond the near end, that is, away from the far end. Prior to any addition, there would have been a one-to-one correspondence between the entire set of books and *N*. If there is an addition of one or more books, then there will be *de novo* another one-to-one correspondence between the augmented set of books and *N*.

Similar considerations apply *mutatis mutandis* to that (hypothetical) infinite set of years ending in the year 1998 and that set ending in the year 2001. The former is a proper subset of the latter. However, both sets have one and the same cardinal number ($\aleph_0$), because the members of each set correspond one-to-one with *N*. Fully applying Cantorian theory, according to the SV, the set of years ending in 1988 and the set of years ending in 2001 necessarily correspond one-to-one to each other.[51] Craig would say, and rightly so, that this is absurd. But the anomaly is discharged if we eliminate the assumption that a given real infinite set must be equipollent to at least one of its infinite proper subsets.

Another example of a counterintuitive absurdity is the paradox of an immortal Tristram Shandy, who writes his autobiography so slowly that it takes him a whole year to record the events of a single day.[52] Much has been written about whether it is theoretically possible for him to finish or bring up to date his autobiography giving an account of each day because, according to the SV, there is a one-to-one correspondence between the infinite set of years and the infinite set of days. Given this assumption, any proposed solution of this paradox is doomed to failure. But if this assumption were rejected, then both infinite sets of days and of years cannot be equipollent to each other since there is in fact a 365-to-one correspondence between days and years.[53] This is not simply the case of asserting that the ratio of 365 days to one year holds with respect to only a finite period of time, no matter how large. Rather it is the 365-to-one correspondence that holds throughout the infinite period of time as such. Accordingly, the factual situation in the Tristam Shandy story is utterly impossible given that there cannot be a one-to-one correspondence between infinitely many years and infinitely many days. At the end of any given year, Tristam Shandy will at best have written about only one of the 365 days of the same year—thereby always keeping hopelessly behind with his autobiography.[54]

Richard Sorabji, in his *Time, Creation, and the Continuum*, has asked us to consider the following:

> Suppose we imagine the column of past years stretching away from our left eye infinitely far into the distance, and parallel to it, stretching away from our right eye, the column of past days, also receding infinitely far. The two columns should be aligned at the near end, starting at the present, and the members of the two columns should be matched against each other one to one. I can now explain the sense in which the column of past days is not larger than the column of past years: it will not *stick out beyond the far end of* the other column, since neither column has a far end.[55]

Alas! Sorabji patently errs in asserting, "the two columns [of past years and of past days] should be matched against each other one to one." This assertion is inconsistent with his opinion that "the sense in which one infinity is greater than the other will be better brought out by saying, however large a *finite* period we take, the ratio of days . . . remains [365]: 1."[56] Craig, in his review of Sorabji's book, comments in a most interesting fashion as follows:

> [I]f we divide the columns into hand-long segments and mark one column as the years and the other as the days, then one column is as long as the other and yet for every hand-length segment in the column of years, 365 segments of equal length are found in the column of days![57]

This is surely astonishing, although Craig is quite right in pointing out the absurd consequence of applying SV. Contrary to Sorabji and Craig, the first 365 segments in the column of years should collectively stand for only one year, but each of the first 365 segments in the column of days should stand for one and only one day, and so on. Alternatively, contrary to Sorabji and Craig, there would be one infinitely extended column of segments with each segment representing a day and another parallel infinite column of segments with each segment representing $1/365^{th}$ part of a year. Therefore, the only way another segment could be added to both columns would be for the unfortunate fellow with the beams in his eyes to step back a hand—all other things remaining equal.

Yet one more example of an alleged counterintuitive absurdity pertains "to the argument of al-Ghãzãli concerning the concentric spheres which revolved such that the innermost sphere completed one rotation in a year while the outermost sphere required thousands of years to complete a single rotation."[58] Craig comments: "According to Cantor, if his system were descriptive of reality, the number of revolutions would be equal, for they could be placed in a one-to-one correspondence." And this, Craig rightly observes "is simply unbelievable."[59] But the absurdity obtains by virtue of the application of the theorem in the SV of *applied* set theory that two real infinites are necessarily equipollent to each other.[60]

Another of Craig's favorites is Hilbert's Hotel.[61] The reader is asked to conceive of a hotel with a finite number of rooms, with each room occupied by one and only one guest. The hotel manager apologizes to anyone arriving and requesting a room: "Sorry—all the rooms are full." The reader is then asked to imagine a hotel with an infinite set of rooms at ground level and above. We have to assume, I suppose, that members of this set correspond one-to-one with the members of an infinite set of spaces in some spatial manifold. Let us assume that one and only one occupant occupies each room. However, according to Craig, Cantorian theory as applied to this situation entails that a prospective new guest could be accommodated without any existing occupant being required to vacate the hotel. Thus, for example, all current occupants could be simultaneously directed to simultaneously move into the next numbered room in order to make room number 1 available for the new guest. But Craig is quite right in rejecting this as an absurdity: "For," he comments, "if the hotel has an actually infinite collection of determinate rooms and *all* the rooms are full, then there is no more room."[62] Since all the infinitely many rooms are occupied, the only way to accommodate new guests would be to construct new rooms, perhaps by adding them below ground level, instead of engaging in quasi-magical operations suggested by the adherents of the SV.

Sorabji also refers to Hilbert's Hotel. The reason why, according to Sorabji, it can be fully occupied and yet accommodate infinitely many new guests is: "There is a temptation to think that some unfortunate resident at the far end of the hotel will drop off into space. But there *is* no far end. It is like the column of whole numbers which we considered before: the line of residents will not *stick out beyond the far end of* the line of rooms."[63] However, Craig comments:

> Now Sorabji is certainly correct that Hilbert's Hotel illustrates an explicable truth about the nature of the actual infinite. If an actual number of things could exist, a Hilbert's Hotel would be possible. But Sorabji seems to fail to understand the heart of the paradox: I, for one, experience no temptation to think of people dropping off the end of the hotel, for there is none, but I do have difficulty in believing that a hotel in which all the rooms are occupied can accommodate more guests. Of course, the line of guests will not stick out beyond the line of rooms, but if *all* of those infinite rooms already have guests in them, then there is no room for more guests. [64]

This passage is quite remarkable because I am unaware (subject to correction) of any other place where Craig appears to acknowledge that an infinite set of real entities is not per se absurd, but is rendered absurd because the application of Cantorian theory as applied to the real world according to SV entails the conclusion that the set of all fully occupied rooms may be increased without changing the hotel in some way.

Indeed, there is virtually no discussion of philosophical theology or philosophy of religion in this paper. Nevertheless, our discussion is very relevant to the question as to whether God exists, or more precisely whether one form of the Kalam Cosmological Argument is valid. It is also most relevant as to whether God is eternal or, instead, temporally everlasting, and as to whether he can create a beginningless temporal world, or a world with real infinites, or a superworld consisting of infinitely many different worlds none of which is spatially related to another. But I confess that by now the metaphysical possibility of a real infinite has become a matter of great interest for me for its own sake.[65]

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Notes

1. William Lane Craig, *The Kalam Cosmological Argument* (New York: Barnes & Noble, 1979), 63–64, 149–153. Of course, if the beginning of the universe is preceded by other events, then the entire temporal regress regress is nevertheless finite assuming the metaphysical impossibility of an infinite temporal regress.

2. Professor Craig's other writings on KCA include: *Apologetics: An Introduction* (Chicago: Moody Press, 1984), 73–93; "Time and Infinity," *International Philosophical Quarterly* 31 (1991): 387–401; "Reply to Smith: On the Finitude of the Past," *International Philosophical Quarterly* 33 (1993): 225–231; "Feature Book Review: *Time, Creation, and the Continuum—Richard Sorabji*," *International Philosophical Quarterly* 25 (1985): 319–326; "A Swift and Simple Refutation of the *Kalam* Cosmological Argument," *Religious Studies* 35 (1999), 57; his contributions in William Lane Craig and Quentin Smith, *Theism, Atheism, and Big Bang Cosmology* (Oxford: Clarendon Press, 1993). Craig "find[s] [the Kalam Cosmological Argument] for a temporal first cause of the universe to be the most plausible argument for God's existence" (*Apologetics*, 73). See also *The Kalam Cosmological Argument*, at 63, where he describes the Kalam Cosmological Argument as the "most likely . . . sound and persuasive proof for the existence of God."

3. Ibid., at 69. This paper assumes arguendo that an infinite temporal regress is an actual infinite, which can exist in the real world, albeit it is instantiated successively.

4. Ibid., 103. A potential infinite differs from an actual infinite in that the former is a finite set that can be indefinitely increased but that always remains finite. Craig states, "[m]odern set theory, as a legacy of Cantor, is thus exclusively concerned with the actual as opposed to the potential infinite" (67).

5. Ibid., at 69-72, 87–92.

6. I think that abstract entities should not be said to exist in the real world. *What really exists* should be limited to only natural or supernatural entities. What I have in mind is that universals and other abstract entities (but not their instantiations) can properly be said to *objectively subsist* if they are indeed mind-independent and thus await human discovery rather than invention. To be sure, to say that abstract entities objectively subsist appears to be very close to what Platonic realists claim: that abstract entities exist in a real domain, albeit radically different from the domain of,

say, "concrete" entities in a natural or supernatural world. In any event, contrary to Craig (see *The Kalam Cosmological Argument*, 87-92; "Swift and Simple Refutation," 60–61, 69–70), I maintain that the existence per Platonic realism (assuming its truth arguendo) of infinite sets of abstract entities, such as numbers of various kinds, does not entail those counterintuitive absurdities that he cites as reasons for rejecting the metaphysical possibility of real infinites. In any event, some persons who believe that actual infinites are metaphysically impossible in any domain are nevertheless Platonic realists (e.g., J. P. Moreland, *Scaling the Secular City: A Defense of Christianity* [Grand Rapids, Mich.: Baker Book House, 1987], 25). However, we need not further discuss the matter since Craig acknowledges that "the actual infinite may be a fruitful and consistent concept in the mathematical realm" (*The Kalam Cosmological Argument*, 69).

7. Ibid., 69-71; Craig, "Graham Oppy on the Kālam Cosmological Argument," *Sophia* 32 (1993): 1–3.

8. Craig, *The Kalam Cosmological Argument*, 97.

9. I use "Cantorian theory" in a broad sense to include naïve set theory or any axiomatic set theory (such as that of Zermelo-Fraenkel set theory) in pure mathematics that accepts the notion of an actual mathematical infinite and includes the propositions that one-to-one correspondence between sets is the necessary and sufficient condition of the sets having the same cardinality, and that every such infinite set has at least one proper subset with which it is in one-to-one correspondence.

10. Ibid., 69, 92.

11. Indeed, virtually all instances of counterintuitive absurdities cited by Craig and other writers are, or appear to be, instances of what are factually impossible for other, non-mathematical reasons in any real world of spatially and causally related objects (e.g., Hilbert's hotel, a library with infinitely many books). However, to suppose that there are infinitely many worlds of spatially and causally related objects, none of which are spatially related to another world, seems *prima facie* coherent and presents most clearly the problem of the metaphysical possibility of real infinites based upon mathematical considerations. For the sake of convenience, I shall use "real world" to also apply to the superworld (or "many worlds"), if any, of worlds of spatially and causally related objects, none of which are spatially related to another world.

12. A proposition may initially be counterintuitive without being absurd, or its counterintuitiveness may be overridden by countervailing considerations.

13. For expositions of Cantorian theory, see: Georg Cantor, *Contributions to the Founding of the Theory of Transfinite Numbers*, trans. Philip E. B. Jourdain (New York: Dover Publications, 1955); Joseph Breuer, *Introduction to the Theory of Sets*, trans. Howard F. Fehr (Englewood Cliffs, N.J.: Prentice-Hall, 1958); Patrick Suppes, *Axiomatic Set Theory* (New York: Dover Publications Inc., 1972); B. Rotman and G.T. Kneebone, *The Theory of Sets and Transfinite Numbers* (New York: American Elsevier Publishing Company, 1968); Peter Suber, "A Crash Course in the Mathematics of Infinite Sets," *St. John's Review* XVIV, 2 (1998): 35–59 (available at **www.earlham.edu/~ peters/wrting/infinity.htm**).

14. The cardinal number of an empty set is 0.

15. Rotman and Kneebone, *The Theory of Sets and Transfinite Numbers*, 36; "equipotent" in original. To be sure, Cantor and others understood that equipollent sets have the same cardinal number, but to conclude this is to take another step beyond the definition of equipollence just given in the accompanying text. At times I shall use "numerically equivalent" (in symbols, $[A] = [B]$) to refer to sets which have the same cardinality.

16. The term "set of all natural numbers" is used in this paper to refer to all positive whole numbers. Although for some purposes, not relevant to this paper, it is necessary to define that set so as to include 0.

17. Ibid., 105: "The next smallest cardinal number after the natural numbers is $\aleph_o$, the cardinal number of any denumerably infinite set, and in particular of the set

N of all natural numbers [defined by the authors to include 0]. The least ordinal number that can belong to a denumerable set is ω, the sequent in *W* [the class of all ordinal numbers] of all the natural numbers, and it therefore follows . . . that $\aleph_0$ is the ordinal number ω itself." (cf. ibid., 36–37, 99; Suppes, *Axiomatic Set Theory*, 156, 225.)

18. The set of all natural numbers is also a denumerable (or denumerable infinite) set.

19. Craig, *The Kalam Cosmological Argument*, 86–87.

20. *B* is a proper subset of set *A* if *A* contains at least one member more than *B*. The determination that *B* is a proper subset of *A* is made by the process of "cancelling" the common members and seeing whether there is a remainder.

21. Ibid., 67, 73.

22. There are many other actual mathematical infinites as to which it is intuitively very plausible that a one-to-one correspondence obtains by virtue of a function-equation that orders such a correspondence. In other cases, the correspondence is purportedly established if there is a rule-governed sequence for each set that runs throughout the members without omission or repetition. Cantor explicitly states "that two aggregates M and N are 'equivalent,' . . . if it is possible to put them, by some law, in such a relation to one another that to every element of each one of them corresponds one and only one element of the other" (Cantor, *Contributions*, 86–87). In other words, when *M* and *N* are equipollent "there is a law of co-ordination by means of which *M* and *N* are uniquely and reciprocally referred to one another" (ibid., 88).

23. Of course, *N* is *larger* (in one sense) than *E*, its proper subset, in that *N* has members (i.e., odd numbers) remaining after the "cancellation" of the common members of *N* and *E* (i.e., even numbers). On the other hand, that *N* and *E* are equipollent is not determined by the application of the matching process used in ascertaining whether two real sets are equipollent—a process that presupposes the "cancellation" of common members. Hence, there is no contradiction in asserting that *N* is larger (in one sense) than *E*, but that nevertheless the two sets are equipollent and also numerically equivalent.

24. Craig, *The Kalam Cosmological Argument*, 69.

25. Ibid., 70. Other writers agree with Craig. Rotman and Kneebone observe: "[In Zermelo-Fraenkel axiomatic set theory] [b]oth the notion of set and the notion of membership are taken as primitive (i.e., unanalysed and undefined) and no properties are attributed to sets beyond those conferred on them by the stated axioms of set theory" (*The Theory of Sets and Transfinite Numbers*, 57). Furthermore, in the Zermelo-Fraenkel theory "a certain domain of entities is postulated as the universe of discourse, and these entities are referred to as sets" (ibid.). "[T]he 'universe' of sets to which the Zermelo-Fraenkel theory refers is in no way intended as an abstract model of an existing Universe, but serves merely as the postulated universe of discourse for a certain kind of abstract inquiry" (ibid., 61). These authors refer to "set theory [as] exclusively . . . a branch of *pure* mathematics . . ." (ibid.). As another text tersely puts it: "In this book, we want to develop the theory of sets as a foundation for other mathematical disciplines. Therefore, we are not concerned with sets of people or molecules, but only with sets of *mathematical* objects, such as numbers, points of space, functions, or sets" (Karel Hrbacek and Thomas Jech, *Introduction to Set Theory*, 2nd ed. [New York: Marcel Dekker, 1984], 2). See also, for example, Stewart Shapiro, *Thinking About Mathematics* (Oxford: Oxford University Press, 2000), 223: "As branches of pure mathematics, modern set theories do not concern sets of physical objects. The set-theoretic hierarchy is thoroughly abstract, consisting of the empty set, the powerset of the empty set, and so on."

26. That Craig assumes SV is the only otherwise plausible version of the application of Cantorian theory to the real world is certain. See, for example, his "A Swift and Simply Refutation," 64, where he peremptorily rejects the idea that an infinite of days and one of years cannot be put into a one-to-one correspondence, remark-

ing: "This is obviously false, since both have the cardinality of $\aleph_0$ . . . . [T]he days and years cannot fail to correspond." See also *The Kalam Cosmological Argument*, 98.

27. Let us suppose that each human has a negative integer for his name, without omission or repetition, and that each human's hands are designated respectively by placing the letter *A* for the left hand and the letter *B* for the right hand after the negative integer which serves as his name. One commentator on an earlier version of this paper, who suggested the foregoing scenario, contended that a one-to-one correspondence could be set up between infinitely many humans and their hands. But, alas, the result would be that hands –1*A* and –2*A* are respectively paired with humans –1 and –3, and hands –1*B* and –2*B* would be respectively paired with humans –2 and –4, and so forth. These results are patently inconsistent with the supposition that every human has exactly two hands, such that the sequence should be human –1 is matched with both his hands –1*A*, and –1*B*, and so forth.

28. For example, ({infinitely many humans} ~ {all natural numbers}) and ({infinitely many hands} ~ {all natural numbers}); therefore ({infinitely many humans} ~ {infinitely many hands}).

29. Quoted in introduction by Philip E. B. Jourdain in Cantor, *Contributions*, 74.

30. I owe the substance of this paragraph to the suggestion of the editor of *Philo*.

31. It might be suggested that if a real infinite is not equipollent to *N* then it must be equipollent to an infinite proper subset of all natural numbers. But according to Cantorian theory, *N* and any of its infinite proper subsets are equipollent, and therefore any real infinite must be equipollent to *N*.

32. Although any real infinite is equipollent to any denumerable mathematical set, it does not follow that a particular real infinite can have any conceivable order type. For example, the order type of an infinite temporal series, each member of which is a finite distance from the present, has the order type ω^*, which is the order type of negative numbers (i.e., {. . . –3, –2, –1}). It cannot have order type $\omega^*+\omega^*$. The order type of natural numbers in its ordinary progression is ω. For an interesting discussion of this, and related matters, see Craig, "Time and Infinity," 387–391; Quentin Smith, "Reply to Craig: The Possible Infinitude of the Past," *International Philosophical Quarterly* 33, 1, (1993): 109–119; Craig, "Reply to Smith: On the Finitude of the Past," 226–229. Craig commendably agrees with Smith that that there cannot be "a series of past events order $\omega^* + \omega^*$, where temporal distance is correlated with ordinal numbering" (ibid., 229).

33. Moreover, in determining whether there is a one-to-one correspondence between the two sets of real entities, the matching process presupposes the "cancellation" of members common to both sets, if any. This is quite different from the matching process as applied to sets of mathematical entities; a process that does as such not presuppose the "cancellation" of common members, if any—as in the case or *N* and *E*.

34. In a written but private commentary on an earlier draft of this paper, its writer rightly points out that some correspondences involving sets of abstract entities are conventional in nature, rather than being given by some function-relation. The writer is correct: some correspondences are conventional or arbitrary. But surely, epistemologically speaking, the equipollence between (for example) the set of all positive whole numbers and the set of all even numbers "given in terms of the 'multiplication by 2' function" (to use his words) is very useful (if not necessary) in order to see why it is at least plausible that an infinite mathematical set and one of its proper subsets are equipollent. The writer asserts, "correspondences in the case of sets whose members are concrete entities are conventional in nature." But he overlooks what I had stated in the draft of this paper, which he had reviewed, and which is included in this paper: that equipollences between real infinites depend upon factually contingent or definitional matters. Some empirically contingent equipollences may well obtain because of a law of nature (e.g., that each normal human naturally has two and only two hands), rather than being a grand cosmic coincidence.

35. The writer of the private, written commentary (referred to above) argues that two real infinites must be equipollent to each other given that each is equipollent to the set of all natural numbers based upon "idea of a relative product of two relations . . . [as] given by Russell and Whitehead in section 34 of *Principia Mathematica*: "The relative product of two relations R and S is the relation which holds between x and z when there is an intermediate term y such that x has the relation R to y and y has the relation S to z. . . . The relative product of R and S is denoted "R/S". . ."" (Alfred North Whitehead and Bertrand Russell, *Principia Mathematica* [Cambridge: Cambridge University Press, 1910], 256, 300). The writer asserts that given any two relations *R* and *S* there must exist the relative product *R/S* of these relations. Since his argument is too long to briefly summarize, I cannot do it full justice. Nevertheless, it appears to me that some relations *R* and *S* do not have relative products. For example, what is the *R/S* where *A* is the father of *B* and *B* is a friend of *C*? Or, what is the *R/S* where *A* is a partner of *B* and *B* is a partner of *C*? Second, determination of what is the relative-product of two relations depends upon the theories of the particular nature of the relations, and whether these relations entail and/or are entailed by the candidate *R/S*. For example, the conclusion that two real infinites, each equipollent to *N*, are necessarily equipollent to each other depends upon what version of the application of Cantorian theory to the real world is adopted. Third, the writer's argument starts with the premise "that there are an infinite number of humans . . . [and] that each individual has exactly two shoes," but nevertheless he concludes that there is "a one-to-one correspondence between the set of shoes and the set of humans." The writer's conclusion is therefore inconsistent with the assumption that every human has exactly two shoes.

36. On the other and, two real infinites, each equipollent to another real infinite, are equipollent to each other. Moreover, a real infinite *A* is equipollent to mathematical set *O* if *A* is equipollent to mathematical set *M* and *O* and *M* are equipollent. Of course, the relation of having the same cardinality is transitive such that if $[A] = [B]$ and $[B] = [C]$ then $[A] = [C]$ whether or not *A*, *B*, or *C* are real or mathematical infinites.

37. Since the real infinite and its infinite proper subset are equipollent to *N*, they then have the same size (i.e., cardinality). They can be said to be numerically equivalent in the sense that the cardinal number of the members of former is the same as that of the members of the latter set. It is in this sense that it can be said that Euclid's maxim, that the whole cannot be greater than any of its parts, does not apply to real infinites.

38. David Hilbert, "On the Infinite," *Philosophy of Mathematics*, ed. with introduction by Paul Benacerraf and Hilary Putnam (Englewood Cliffs, N.J.: Prentice-Hall, 1964), 141, quoted in Craig, *The Kalam Cosmological Argument*, 67.

39. See ibid., 83–84; "Time and Infinity," 393–394.

40. Ibid., 394.

41. Craig, *The Kalam Cosmological Argument*, 83, where he expresses his disbelief "that the number of [infinitely many] red books in the library is the same as the number of red books *plus* the number of [infinitely many] black books," and page 84, where he denies the possibility of the number of an infinite set of real entities remaining the same after the removal of a proper subset.

42. Ibid., 94–95. Craig is here a bit inaccurate, and this is surely not his meaning. The principle essentially asserts that numerical equivalence (i.e., having the same cardinality) obtains if and only if two infinites are equipollent (i.e., are in one-to-one correspondence). See "A Swift and Simple Refutation," 63, 64; and "Time and Infinity," 396.

43. "Reply to Smith," 230.

44. William Lane Craig, "Graham Oppy on the Kalam Cosmological Argument," *Sophia* (1993), 1, 3.

45. Craig, *The Kalam Cosmological Argument*, 81.

46. Ibid., 82.

47. The very table provided on page 81 by Craig, purportedly inserted to demonstrate his point, shows the possibility of removal from infinites, as distinguished from subtraction from transfinite numbers. See also: Cantor, *Contributions*, 104–105; J. Breuer, *The Theory of Sets*, 32 ("If a finite number of elements is added to or subtracted from a denumerable set, the new set is denumerable") and 33 ("If a denumerable set of elements is subtracted (removed or cancelled) from an infinite set, then if the resulting complementary set is still infinite, it has the same cardinal number as the original set"); Rotman and Kneebone, *The Theory of Sets and Transfinite Numbers*, 43 (inviting the reader to "[s]how that the cardinal number of an infinite set is not affected by the removal of a denumerable subset, provided that the set which remains is infinite").

48. Craig, *The Kalam Cosmological Argument,* 86, complains: "While we may correct the mathematician who attempts reverse operations with transfinite numbers, we cannot in the real world prevent people from checking out books they please from our library [containing infinitely many books]." To which I can only reply: right.

49. I review these specific alleged counterintuitive absurdities because Craig has complained that his critics fail to sufficiently discuss them. See, for example, Craig, "A Swift and Simple Refutation," 59: "Worse . . . Taylor simply breaks off his discussion at this point, ignoring all the even more counter-intuitive absurdities entailed by the existence of an actual infinite, such as those illustrated by Hilbert's Hotel. . . ." See also similar remarks in "Time and Infinity," 394.

50. *The Kalam Cosmological Argument*, 82–84, 86–87. To be sure, Craig also generally objects to the idea that any two or more real infinites can have the same cardinality, but the counterintuitive absurdity considered in this paragraph involves the interrelationship between books and spaces on library shelves.

51. Steven T. Davis, in his *God, Reason & Theistic Proofs* (Grand Rapids, Mich.: Wm. B. Eerdmans Publishing Company, 1997), 153, although acknowledging that an infinite temporal set is an actual infinity, contends that "such a series is [not] the sort of actual infinite that Craig's paradoxes rule out" because these paradoxes apply only to actual infinites whose members exist simultaneously. Davis, as we see, is incorrect in concluding that all of "Craig's paradoxes [are] rule[d] out" in the case of an infinite series of successive events. See Craig, "A Swift and Simple Refutation," 62.

52. *The Kalam Cosmological Argument*, 97–99; "Time and Infinity," supra, 396–399; "A Swift and Simply Refutation," 64.

53. The ratio of about 365 days to one year, which for convenience I reduce to an exact 365-to-one ratio, is a contingent fact if a day is defined in terms of the earth's rotation on its axis and a year is defined in terms of the earth's revolution around the Sun. Similarly, for the sake of convenience, our discussion assumes that the ratio is invariable.

54. I do not touch on everything relating to Tristram Shandy because it appears more to involve Craig's second philosophical argument against the metaphysical possibility of an infinite temporal regress.

55. Richard Sorabji, *Time, Creation and the Continuum: Theories in Antiquity and the Early Middle Ages* (Ithaca, N.Y.: Cornell University Press, 1983), supra. 217. Sorabji cogently discusses the view of John Murdoch (14th century) that there is a "sense in which one (denumerable) infinite set might be called greater than another, and in a sense in which it might not. It might be called greater in the sense of containing all of the members of the other and some members besides (*preter*, elsewhere *praeter*)" (ibid.). He notes that "[t]he mediaeval discussions explain nicely the sense in which the infinite set of past years can be thought of as having grown larger by next year: next year's collection will contain the same members, and one more besides (*praeter*)" (ibid., 218). See Sorabji's discussion of various paradoxes pertaining to "actual and traversed" real infinite sets, 219–24. However, he failed to apply consistently the insight that two real infinities need not be equipollent to each other.

56. Ibid., 218. We slightly paraphrase Sorabji, who is writing of a somewhat different matter.

57. Craig, *"Time, Creation and the Continuum—Richard Sorabji,"* 322–3.

58. Craig, *The Kalam Cosmological Argument*, 98.

59. Ibid.

60. That two real infinites are necessarily equipollent to each other is a theorem in SV given the axiom in that theory (i.e., the bridging rule) that real sets are to be deemed as being among those entities denoted by the term "set," as used in Cantorian theory as a term of art, and the theorem in Cantorian set theory that the relation of equipollence of sets is transitive.

61. Ibid., 84–85.

62. Ibid., 85.

63. Sorabji, *Time, Creation and the Continuum*, 223.

64. Craig, *"Time, Creation and the Continuum"—Richard Sorabji,"* 323.

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