ON THE LOGICAL STRUCTURE OF REALITY AND CONCEPTUAL RELATIVISM

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Part 1. The Relativity of Objects

1. Conceptual Schemes and Analyticity

Conceptual schemes have had a variety of notions. An early and prototypical proposal being the transcendental idealism of Kant wherein fixed rules for mental activity synthesize a manifold of empirical data into intelligible experience. Another being the paradigms of Kuhn. Another being the simple intuition involving differing points of view. Wang (2009) explains that while widespread confusion lingers over the notion of a conceptual scheme, a particular version dominates the discourse, namely the Quinean linguistic model which was ironically popularized by Davidson's critique (1973). We will take this *linguistic model* (or the Quinean model) for our conceptual schemes so that languages and schemes are approximately correspondent. As such, our starting position is the same which Davidson attacks in his critique of conceptual relativism, which associates having a language to a conceptual scheme, and that, if conceptual schemes differ, so do languages.

Start then with a linguistically equipped observer recording into language the world as it appears to them. Assume this observer to be reliable and a conceptually matured. As facts appear, our observer documents each as a proposition. The Sultans of Mughal India would commission historians to chronicle the proceedings of court life and all factual details of administering the empire. One of the most thorough and detailed of these chronicles is the three volume Akbarnama (the official chronicle of the reign of Emperor Akbar) written by the courtier Abul Fazl. So detailed, in fact, as to include minutia of the Sultan's daily diet. I imagine an observer like Abul Fazl. A trusted and reliable historian documenting every fact using the resources of his language.

Assume further that Abul Fazl is permitted a certain omniscience, granted the ability of bringing his chronicle everywhere and into every corner of the Indian subcontinent, such that all the facts comprising a state of his world fills a volume. This volume includes all facts expressible or reachable through language, as "Akbar is mortal," "Bodies are extended," "The cat is on the mat," and so on.

Suppose further that this language is formalizable into a first-order logic (even if that mathematical machinery is unknown to Abul Fazl), such that concepts like "cat" or "mortal" correspond to a unary predicate symbols, on-ness to a binary relation (we also permit of relations and functions of arbitrary order as well as constants), so that all the elements of the scheme are collected into a signature $\mathcal{L}$.[1]

[1]It is not here implied that the entirety of Abul Fazl's natural language is reducible to a first-order system. There are several dimensions of language (giving orders, telling jokes, religious

The state of India, all objects and their arrangements as facts, is thereby an $\mathcal{L}$-structure. For Quine (1951) the conceptual scheme itself ought to be subject to revision according to the demands of empirical observation. Concepts might be introduced to service novel patterns within experience, and others discarded, as well their meanings rearranged. However, we will suppose the scheme immutable (if only to hold it still and explore its consequences). Whichever novelties, and however the "flux of experience" refluxes, the subsequent state of things decomposes into propositions with respect to this fixed scheme. Today's facts will be an $\mathcal{L}$-structure. Tomorrow – whichever tomorrow – will be an $\mathcal{L}$-structure. Had a past contingency been otherwise, the alternative now and its objects are an $\mathcal{L}$-structure.

Suppose we list these worlds as $\mathcal{L}$-structures $\mathcal{U}_1, \mathcal{U}_2, \mathcal{U}_3, \ldots$ each with their respective domain of existing objects. Were all these viewable at once, a cross-examination would disclose propositions true of all worlds. Propositions always confirmed from wherever the linguistically equipped observer chooses to observe from. This set of necessary propositions would be non-empty, since they would at least include what Kant describes as *containments* such as "bodies are extended," in symbols $\forall x\,(b\,(x) \rightarrow e\,(x))$.

Propositions of this form are true by a relation of meanings (are *analytic*) and are law-like from within the scheme. Whatever possible world becomes actual "bodies are extended," since this is true by the conventions of the language and derived from meanings alone.

As philosophy stands there is a great number of contentions and conflations among the categories of analytic/synthetic, necessary/contingent and a priori/a posteriori, and to progress it will take some effort to untangle these.

Quine (1951) undermines the distinction between *analytic* and *synthetic* by exposing the entirety of an empirical theory (as well its innermost logical structure and arrangements of meanings) to revision. Any statement can be "held true come what may" should the system be adjusted to ensure its necessity. Even the laws of logic (like that of excluded middle) are revisable as the evidence requires. Here Quine is correct. Both descriptively, in that, were the meaning of a *line* to change, their intersections are no longer known come what may; and prescriptively, since our models should be adaptable, and our lines ought curve with the shape of experience so to speak.

However we have assumed the scheme immutable (if only to fix its consequences), and thereby the category of analytic truths is valid, since the positions of meanings cannot change. The example provided "bodies are extended" is serviceable. If the meaning "body" and the meaning "extended" are constant, the containment "bodies are extended" comes what may. Or it might be said that the *analyticity* of a proposition is respective to the choice of scheme itself, any scheme *possesses* analytic propositions, and swapping schemes also changes-out the analytic propositions.

Any analytic proposition is represented formally as a sentence (ie, a formula where every variable is bounded). Simplest among these are single variable Kantian containments, as "all bodies are extended" symbolized by $\forall x\,(b\,(x) \rightarrow e\,(x))$. But more complicated propositions involving functions, higher-order relations, etc. would also be found. Perhaps the proposition "if a is heavier than b and b is heavier

experience, etc.) which could not be represented in that reduction. Rather I propose that terms used for description can be so reduced. Those elements used in the recording of fact. In terms of later Wittgenstein: The special langauge game of factual reporting.

than c, then a is heavier than c" or in symbols,

$$\forall xyz\,(H\,(x,y) \wedge H\,(y,z) \to H\,(x,z))$$

which is so by the meaning of the heavier than relation.

Collect all analytic propositions into a set of sentences $\mathcal{A}$ (alternatively, collect a set of sentences $\mathcal{A}$ serving as axioms for analyticity: If s is analytic, then $\mathcal{A} \vdash s$). Since these are necessary, any world models $\mathcal{A}$. Similarly necessary propositions must also be sentences. Collect these propositions into a set of sentences $\mathcal{Q}$. Will analytic and necessary propositions coincide? Imagine Abul Fazl working with unlimited time and writing unlimited chronicles. At the end of his modal travels he finally visits the last possible world, and after committing every fact therein to a volume, verifies at last that s was indeed necessary. Does it follow that s might have been derived *a priori* from the start? That it might have been deduced by simply unpacking the definitions of the terms and this modal journey was wasteful?

An example from Quine (1951) might be useful: The terms "creature with a heart" and "creature with a kidney" likely overlap in extension but differ in meaning. Perhaps (because all creatures with hearts need kidneys to clean the same blood the heart pumps) $s = \forall x\,(k\,(x) \leftrightarrow h\,(x))$ is true in all worlds. But might we *deduce so* by meanings alone? Or following Kant, and making analytic depend upon the principle of contradiction: Does $\neg s$ contradict $\mathcal{A}$? Would the existence of a creature with a heart but not with kidneys violate the conventions of the language? Intuitively not, and to know s with absolute certainty would require a modal journey to the last possible world. Such a proposition is an *a posteriori necessary* truth. The existence of such propositions was elaborated by Kripke (1980). He provides the example "the morning star is the evening star." Both morning star and the evening star identify the same object Venus, but this equivalence cannot be deduced a priori through and appeal to the meanings of "morning star" and "evening star." The proposition required observation; yet venus is venus, and in all worlds the morning star is the evening star; therefore the proposition is necessary.

Supposing Abul Fazl were permitted to observe every possible world forming a large family of $\mathcal{L}$-structures $\{\mathcal{U}_\alpha : \alpha \in I\}$. Each would be a model $\mathcal{U}_\alpha \models \mathcal{A}$. By completeness, if every model of $\mathcal{A}$ models s then $\mathcal{A} \vdash s$, which builds an intuitive connection between necessary truths (true in every model) and a priori/analytic truths (deductively true by the conventions of the language). Of course we cannot assume that possible worlds exhaust models of $\mathcal{A}$. By the upper Skolem theore: If $\mathcal{A}$ has an infinite model then $\mathcal{A}$ has a model at every cardinality (much too large for possible worlds). Not to mention the models of $\mathcal{A}$ that are semantically incommensurate to human experiences (modeling the conventions of the language, but with alien objects). Even so, this gives some mathematical intuition that analytic/a priori propositions and necessary propositions are connected. It cannot be assumed that every model of $\mathcal{A}$ is a possible world, for the moment I will assume so to exploit completeness (this is a technical error, but one that will be rectified later).

There are two cases. The simplest occurs when necessary truths are analytic truths. In this case,

- A sentence s is analytic when $\mathcal{A} \vdash s$.
- Then s is a priori as it has a deduction from $\mathcal{A}$.
- If $\mathcal{A} \vdash s$ then for all models $\mathcal{U} \models A$ we have $\mathcal{U} \models s$, therefore s is necessary.

- Analytic propositions are by definition necessary (are a subset of necessary propositions).
- A synthetic proposition s is by definition one which does not follow from $\mathcal{A}$. Or alternatively $\mathcal{A}$ and $\neg s$ are consistent. So there exists models of $\mathcal{A} \cup \{\neg s\}$. Therefore synthetic propositions are contingent.
- Synthetic propositions are a posteriori (are true of particular worlds $\mathcal{U} \models s$). A posteriori propositions are synthetic, since if an a posteriori proposition is not synthetic it is analytic, then $\mathcal{A} \vdash s$ and s is a priori.

The more complicated case occurs when there exists a necessary proposition s that is not a deductive consequence of $\mathcal{A}$. In this case there exists an $s \in \mathcal{Q}$ so that $\mathcal{A} \nvdash s$. Because $\mathcal{Q}$ are all necessary truths, for any possible world $\mathcal{U}$ it it must be that $\mathcal{U} \models \mathcal{Q}$.

- The *modal scope* of $\mathcal{A}$ is larger than physical possibility. Since there exist models of $\mathcal{A}$ that are not models of $\mathcal{Q}$. If s is a posteriori and necessary then $\mathcal{A} \nvdash s$. Since $\neg s$ is consistent with $\mathcal{A}$ and there is a model $\mathcal{U} \models \mathcal{A}$ where $\mathcal{U} \models \neg s$. But this could not be a possible world.
- Models of $\mathcal{Q}$ are models of $\mathcal{A}$. Therefore analytic propositions (s where $\mathcal{A} \vdash s$) are still necessary.
- Since the a posteriori necessary propositions are *discoveries* that could not be arrived at a priori, the a priori category remains the conclusions of $\mathcal{A}$.
- Further, synthetic propositions (s that is not an consequence of $\mathcal{A}$) are still a posteriori (since s is not deducible by $\mathcal{A}$, and must be discovered through observation). As before, synthetic propositions are a posteriori and a posteriori propositions are synthetic.

Thus there is only one distinction between these cases: The existence of synthetic (equivalently, a posteriori) necessary propositions. In the simpler case: A proposition is either a contingent empirical truth or is a logical consequence of the language. In the complexified case, a proposition is classified into one of the following: Analytic truths (which are a priori truths, and form a subset of necessary truths), synthetic truths (a posteriori and contingent, the three are equivalent), and finally necessary but a posteriori (equivalently, necessary but synthetic).

2. Conceptual Schemes and Objects

The purpose of classifying analyticity and the above complications is to ground an ontology. There exists an intuitive connection between objects and properties. Common-sensically, a property describes some attribute or quality inhering in the object. A given property P is "true of" the object a and is written $P(a)$. But the object might equally be attributed to the property: The form $P(a)$ might be rethought as $a(P)$. After all, listing the properties of an object is also to know it, and an exhaustive list is to know it exhaustively. In other words, a might be reconstructed by $a(P)$ across P.

Our chronicler Abul Fazl searches out every fact that can be found and records each. These labors produce an $\mathcal{L}$-structure $\mathcal{U}$ such that each fact has been represented as an $\mathcal{L}$-formula $\psi(a_1, \ldots, a_n)$ where $\mathcal{U} \models \psi(a_1, \ldots, a_n)$. So if the "cat is on the mat" then $\mathcal{U} \models \mathrm{on}(\mathrm{cat}, \mathrm{mat})$, and if not then $\mathcal{U} \not\models \mathrm{on}(\mathrm{cat}, \mathrm{mat})$. All this documentation must therefore posit the objects $a_1, \ldots, a_n$ that arrange within those same formulas. Formally, this is the domain of interpretation which, in our context, is an ontology of existing things. When tomorrows facts are recorded, those formulas will arrange from tomorrows existents. And the day after. So that each $\mathcal{U}_1,\mathcal{U}_2,\mathcal{U}_3,\ldots$ comes with an ontology of existing objects. While an overlap between these domains is expected (the same cat is alive tomorrow), it is not evident from here that there would be a global inter-relation, and each domain could be something of an ontological island.

We will construct a more global ontology. As before let $\mathcal{A}$ be analytic sentences capturing the conventions of the language. These propositions verified everywhere in observation provided observation is conducted through the scheme. In the language of model theory the $\mathcal{L}$-structures $\mathcal{U}_\alpha$ are *models* of the theory $\mathcal{A}$, or $\mathcal{A} \models \mathcal{U}_\alpha$ for $\alpha \in I$.

Construct the 1-*types* of the theory $\mathcal{A}$ according to the following procedure:

(1) Define an equivalence relation on unary formulas $\psi(x) \sim \varphi(x)$ when $\mathcal{A} \vdash \forall x\,(\psi(x) \leftrightarrow \varphi(x))$.
(2) Order the equivalence classes by setting $[\psi] \leq [\varphi]$ when $\mathcal{A} \vdash \forall x\,(\psi(x) \rightarrow \varphi(x))$. This partial order defines a Boolean algebra (the Lindenbaum-Tarski algebra). Denote this algebra as $B(\mathcal{A})$.
(3) Construct the Stone space $S(\mathcal{A}) = \mathrm{St}(B(\mathcal{A}))$. This is the space of 1-types.

Let $\mathcal{U}$ be a model of $\mathcal{A}$. Given $u \in \mathcal{U}$, the type of that object $\mathrm{tp}(u)$ is the function defined on equivalence classes of unary formulas by,

$$\mathrm{tp}(u)([\psi]) = \begin{cases} T & \text{when } \mathcal{U} \models \psi(u) \\ F & \text{otherwise} \end{cases}$$

This is a homomorphism of $B(\mathcal{A})$ into $\{T, F\}$. Notice that if $\mathcal{A} \vdash \forall x\,(\psi(x) \leftrightarrow \varphi(x))$ then $\mathcal{U} \models \psi(u)$ if and only if $\mathcal{U} \models \varphi(u)$, so the function is well defined on equivalence classes. While if $[\psi] \leq [\varphi]$, then $\mathcal{A} \vdash \forall x\,(\psi(x) \rightarrow \varphi(x))$, and because $\mathcal{U} \models \mathcal{A}$ we know $\mathcal{U} \models \forall x\,(\psi(x) \rightarrow \varphi(x))$ and therefore $\mathrm{tp}(u)([\psi]) \leq \mathrm{tp}(u)([\varphi])$. Thus $\mathrm{tp}(u)$ is an order-preserving map from $B(\mathcal{A})$ to the two element Boolean algebra. Which is to say that any object $u \in \mathcal{U}$ specifies a type $\mathrm{tp}(u)$. Or the map $u \rightarrow \mathrm{tp}(u)$ sends objects to respective types.

Suppose the model $\mathcal{U}$ satisfies a second-order logical property corresponding to Leibniz's law (that is, the identity of indiscernibles), formulated as: If for all ψ, $\mathcal{U} \models \varphi(u)$ if and only if $\mathcal{U} \models \varphi(v)$, then $u = v$. In other words, given $u, v \in \mathcal{U}$ where $u \neq v$, there exists a single-variable formula distinguishing them. Under this law $\mathrm{tp}(u) = \mathrm{tp}(v)$ implies $u = v$. Such that the map $\mathcal{U} \rightarrow S(\mathcal{A})$ given by $u \rightarrow \mathrm{tp}(u)$ is injective. In other words, every domain $\mathcal{U}$ satisfying Liebnez's law identifies a *subspace* of $S(\mathcal{A})$. If Leibniz's law fails then $u \rightarrow \mathrm{tp}(u)$ is no longer injective and maps together indiscernible objects (with respect to unary formulas).

This will need some exploration and cleaning up. For now the intuition is: An object $u \in \mathcal{U}$ *is* (or is *up to classification*) its truth-values when paired with unary formulas ψ. Consider that all simple concepts (as "animate," "inanimate," "organic," "synthetic," "hard," "triangular," ...) have been represented as predicate symbols $p(x)$, then knowing $\mathrm{tp}(u) : B(\mathcal{A}) \rightarrow \{T, F\}$ is to know all inhering properties of the object. If Abul Fazl were allowed to study the object for an indefinite length of time, to inspect it from every possible angle, noting all its tinniest flaws and intimate details – observations conducted from *within* the scheme – everything that could be recorded is given by $\mathrm{tp}(u) : B(\mathcal{A}) \rightarrow \{T, F\}$. Two objects of the same type $\mathrm{tp}(u) = \mathrm{tp}(v)$ are the same from every angle, share the tinniest flaws and most infinitesimal details, and are indiscernible or equivalent under classification.

Assuming an identity of indiscernibles. Given a possible world as a model $\mathcal{U}$ of $\mathcal{A}$, the domain of $\mathcal{U}$ becomes a subspace of $S(\mathcal{A})$ under $\mathrm{tp}(\cdot) : u \to \mathrm{tp}(u)$. Another documented world determines another subspace. In other words the points of $S(\mathcal{A})$ are related to *possible objects*. Given a point $\mathfrak{p} \in S(\mathcal{A})$ its definition as $\mathfrak{p} : B(\mathcal{A}) \to \{T, F\}$ tells us every property an object of type $\mathfrak{a}$ will have should it become actual. Any property ψ of $\mathfrak{p}$ is answered by the value $\mathfrak{p}([\psi])$. Thus $\mathfrak{p} : B(\mathcal{A}) \to \{T, F\}$ places a possible object as an intersection of properties. With a point of $S(\mathcal{A})$ given, a possible object is fully disclosed through an exhaustive list of properties, all the smallest details that the scheme is capable of expressing.

3. Semantics

We should remark that because the signature $\mathcal{L}$ and the theory $\mathcal{A}$ are syntactic, the construction of $S(\mathcal{A})$ – which proceeded from $\mathcal{L}$ and $\mathcal{A}$ as mathematical objects – is also syntactic. We can see this by imagining another *type* of world classified by the logical structure of our conceptual scheme following a semantic substitution. Where our “organic” is sent to their property-O, our “synthetic” is sent to their property-S, and so on, while preserving the all logical relations. One might recall what Hilbert said of geometry: That the axioms may hold as well for tables, chairs and beer-mugs as they do of points, lines and planes. Likewise, the same conceptual scheme that would organize the contents of my experience, and would identify all the objects within a lounge and their mutual relations, might, with like suitability and economy, organize the contents of a geometrical world of pure mathematical objects. Imagining both the lounge $\mathcal{U}$ and the geometric world $\mathcal{V}$ as $\mathcal{L}$-structures modeling $\mathcal{A}$, their respective existing objects embed as subspaces (assuming Leibniz’s law) of $S(\mathcal{A})$. Where it becomes possible that the same point $\mathfrak{p} \in S(\mathcal{A})$ be identified both as by a geometric object and as lounge paraphernalia, and it is impossible to preference either interpretation syntactically.

But intuitively a conceptual scheme would classify consistent experiences. Or a constant *type* of world. Organizing human experience from a particular perspective, a perspective which does not jump to geometric worlds and back again. In Davidson’s language, conceptual schemes are “ways of organizing experience; they are systems of categories that give form to the data of sensation; they are points of view from which individuals, cultures, or periods survey the passing scene” (Davidson, 1973, p. 1). We can imagine these *scenes* of Davidson passing into and out of existence, while the mode of surveying, the point of view, is a constant. Each scene is surveyed and its contents organized by fixed meanings within the scheme. The concept “apple” selects-out all apples within that scene, the relation “heavier than” finds all pairs of objects standing in that relation, and so on. *When the next scene is surveyed* those concepts perform the same work.

Davidson relates that, “Strawson invites us to imagine possible non-actual worlds, worlds that might be described, using our present language, by redistributing truth values over sentences in various systematic ways. The clarity of the contrasts between worlds in this case depends on supposing our scheme of concepts, our descriptive resources, to remain fixed” and that this “requires a distinction within language of concept and content: using a fixed system of concepts (words with fixed meanings) we describe alternative universes. Some sentences will be true simply because of the concepts or meanings involved, others because of the way of the world. In describing possible worlds, we play with sentences of the second kind only" (Davidson, 1973, p. 9). We accept Strawson's invitation (Strawson, 1998). The *contents* of experience flux and are passing, concepts have fixed meanings which perform a classificatory work on whatever comes.

Even if the causal freeplay of the Universe voids the extension of a concept, even if all apples were to vanish, the concept's place within the scheme is secure. Of course, the absurdity of maintaining apples even when apples are gone, perhaps gone forever, is an argument for the revisability of the conceptual scheme. But to restate our purpose: A scheme is held constant long enough to understand its full consequences. That is not to say these concepts are *innate*, as some wondering through experience or a certain process of socialization is needed to acquire them, but once they are framed-so in the mind (and assuming the scheme is fixed) they obtain a transcendence to experience as an organizer of experience. Such might be described in terms of the above mentioned scheme-content dualism. Conceptual meanings obtain a certain transcendence (not completely unlike Kant's use of transcendental) above the flux of experience.

With fixed meanings, $S(\mathcal{A})$ become the possible object-types of a consistent *type* of world. Where above the $\mathcal{L}$ theory $\mathcal{A}$ could model a geometrical plane or a lounge, and therefore a point $\mathfrak{p} \in S(\mathcal{A})$ has two interpretations, by fixing the meanings of a scheme we mean *only* geometrical planes or *only* lounges (but not one then the other), so that each point has one coherent interpretation.

A point $\mathfrak{p} \in S(\mathcal{A})$ is interpreted as the possible object-type given by the values $\mathfrak{p}([\psi])$ where ψ is given a meaning respective to the fixed meanings of the scheme. With meanings attached to every element of the conceptual scheme, each point of $S(\mathcal{A})$ is given an interpretation, call $S(\mathcal{A})$ so interpreted $\mathbb{U}$.

4. Possible Objects

With meanings assigned to formulas, an element $\mathfrak{p} \in \mathbb{U}$ represents a certain logically formal object-type described by a family of properties $\mathfrak{p} : B(\mathcal{A}) \to \{T, F\}$. The ontology $\mathbb{U}$ contains the types of possible objects, since a possible world $\mathcal{U}$ models $\mathcal{A}$ and as above $u \to \mathrm{tp}(u)$ is a well-defined map $\mathcal{U} \to \mathbb{U}$ (mapping together indiscernibles). Any object Abul Fazl gathers in a passing scene is classified somewhere in $\mathbb{U}$. Thus $\mathbb{U}$ is ontologically exhaustive as the space which contains the object-types of every $\mathcal{U}$.

Even so, we should pause to ask whether an abstract object of $\mathbb{U}$ should be given the dignity of being thought about, or whether the better part of $\mathbb{U}$ is what Quine (1948) would call an "overpopulated universe" and a "disorderly slum of possibles" (p. 2). Mathematically, $\mathbb{U}$ exists by virtue of sound construction, much as the distributions of Schwartz exist or the algebraic closure of a field exists (whether these formal constructions are real or mathematical fiction is another question). Further, the objects of $\mathbb{U}$ meet Quine's minimal standard as elements of a set which can be quantified over.

Still, our universe seems something of a Plato's beard, which for Quine is "frequently dulling the edge of Occam's razor" (ibid, p. 1). But given that our possibles have proceeded from exact mathematical construction, rather than whatever phantasmogoria the imagination can conjure, we can reply to many of Quine's objections:

> Take, for instance, the possible fat man in that doorway; and, again, the possible bald man in that doorway. Are they the same possible

> man, or two possible men? How do we decide? How many possible men are there in that doorway? Are there more possible thin ones than fat ones? How many of them are alike? Or would their being alike make them one? Are no two possible things alike? Is this alike to saying it is impossible for two things to be alike? Or, finally, is the concept of identity simply inapplicable to unactualized possibilities? (ibid, p. 2)

Our definition of a possible object-type is $\mathfrak{p} : B(\mathcal{A}) \to \{T, F\}$. This definition allocates properties automatically and keeps these properties from self-contradiction. Responding to "unless the round square cupola on Berkeley College were, it would be nonsense to say that it is not" (ibid, p. 3). There are no round-square-cupolas, as such a possible object would satisfy $\mathfrak{p}(\text{round}) = T = F$. There are no round-square-cupolas or squared-circles to be found in $\mathbb{U}$.

Indeed, an object-type $\mathfrak{p} : B(\mathcal{A}) \to \{T, F\}$ must respect the relations of meanings native to the scheme. If properties have meanings which exclude one another (or p and q are such that $\exists x (p(x) \wedge q(x))$ is inconsistent with $\mathcal{A}$) then there is not an object in $\mathbb{U}$ satisfying those properties. This is so because $\exists x (p(x) \wedge q(x))$ is equated to 0 in $B(\mathcal{A})$ and therefore is empty in the Stone space. Above a possible object-type is described as a listing of all possible properties that would inhere in the object – but arbitrary lists will not specify possible objects, only those lists respecting the logical relations of meanings internal to the scheme.

Similarly, if we denote baldness by a predicate $b(x)$, and a possible bald man by u and a possible fat man by v, then (presuming the fat-man is not also bald) both are distinct possible objects, since $b(u) \neq b(v)$. Can two possible things be alike? Two possible objects can agree on a set of properties, but if they agree on all predicates then they are indiscernible and identify the same type in $\mathbb{U}$. This answers Quine's objection "Is the concept of identity simply inapplicable to unactualized possibles? But what sense can be found in talking of entities which cannot meaningfully be said to be identical with themselves and distinct from one another?" (ibid, p. 2).

Are there more possible bald-men than fat-men? At first glance the question is nonsense (the sort of question that would circulate within ontological slums), but even this nonsense might be resolved. Remember $\mathbb{U}$ is a topological space and has a natural Borel algebra Σ. For a unary formula $\psi(x)$ let $\text{tp}(\psi)$ be the representation of ψ in the Stone space in $\mathbb{U}$ (ie, the set of object-types satisfying ψ). With a measure μ on Σ, it is possible to compare $\mu(\text{tp}b)$ to $\mu(\text{tp}f)$ and determine which is larger. Such measurements are speculative in the extreme, I only wish to convey that because $\mathbb{U}$ is a mathematically consistent construction, it is sound and perfectly amenable to other mathematical tools (as the measure μ).

Consider Pegasus. Is Pegasus somewhere in $\mathbb{U}$? It depends on the conventions of the language. If w symbolizes "winged" and h symbolizes "horse," Pegasus is an object a such that $w(a) \wedge h(a)$. If $\exists x (w(x) \wedge h(x))$ is consistent with $\mathcal{A}$ then the formula $\exists x (w(x) \wedge h(x)) \not\sim 0$ in $B(\mathcal{A})$ and is represented by a non-empty subset of the Stone space $\mathbb{U}$. Conversely, if the meanings of "winged" and "horse" exclude one another a priori, so that $\exists x (w(x) \wedge h(x))$ is an analytic contradiction, then the formula is extensionless in the Stone space.

A question arises: Are all these possible objects *physically possible*? This depends on the existence of a posteriori necessary propositions. Using Kripke's example: The morning star is the evening star, but their identity is not knowable a priori. Since our universal ontology contains all objects that are consistent with the schemes conventions, there would exist distinct objects *Hesperus* and *Phosphorus* in $\mathbb{U}$. Similarly, Pegasus is in $\mathbb{U}$ provided the meanings of "winged" and "horse" are not exclusionary; however, a prolonged trans-modal investigation by Abul Fazl may verify that Pegasus does not exist in any possible world. Such would occur when the meanings of "winged" and "horse" are not analytically exclusionary – yet some physical law or principle of biological evolution prohibits the existence of a winged horse. The non-existence of Pegasus is then an *empirical discovery* (which is a posteriori necessary).

For this reason certain possible objects of $\mathbb{U}$ are physically impossible. To clarify the distinction call the objects of $\mathbb{U}$ *logically possible objects* (or just possible, unless the prefix of logical is needed for clarity) and the objects that appear in some possible world *physically possible objects* (all physically possible objects are logically possible, but not all logically possible objects are physically possible).

The distinction can be given a mathematical explanation. Recall that $\mathcal{Q}$ are all necessary propositions. There exist models of $\mathcal{A}$ that are not models of $\mathcal{Q}$ (logically possible worlds that are not physically possible). For $\mathcal{L}$ -formulas ψ and φ, notice that $\mathcal{A} \vdash \forall x\,(\psi\,(x) \to \varphi\,(x))$ implies $\mathcal{Q} \vdash \forall x\,(\psi\,(x) \to \varphi\,(x))$ (but not the converse). Therefore there is a surjective homomorphism of Boolean algebras $h : B\,(\mathcal{A}) \to B\,(\mathcal{Q})$. Because the Stone functor sends surjections to injections,

$$\mathrm{St}\,(h) : S\,(\mathcal{Q}) \to S\,(\mathcal{A})$$

is a topological embedding. Physically possible objects embed as a subspace of logically possible objects.

It might be protested that $S\,(\mathcal{Q})$ is the better choice for a universal ontology. It is true that, as to what could exist, physically possible objects are the better-fitting choice. Yet the advantage of logically possible objects is their *a priori* existence. Once the meanings of the scheme are framed, all possible object-types are populated through the form $\mathfrak{p} : S\,(\mathcal{A}) \to \{T, F\}$. They are a priori consequences of the structure of the scheme. Conversely, a physically possible object cannot be reckoned a priori. It is only through empirical investigation that $\mathcal{Q}$ might be known and consequently the scope of $S\,(\mathcal{Q})$. It might be required that Abul Fazl travel to the last possible world to verify the necessary non-existence of Pegasus. As we progress it will become clear that an ontology of logically possible objects – those objects that belong to the scheme – is the better choice of universe.

Curiously, a deductive gap between $\mathcal{Q}$ and $\mathcal{A}$ (the existence of a posteriori necessary truths) is equivalent to an ontological gap $S\,(\mathcal{A}) - S\,(\mathcal{Q})$. Since if $S\,(\mathcal{A}) = S\,(\mathcal{Q})$ then the continuous map $\mathrm{St}\,(h) : S\,(\mathcal{Q}) \to S\,(\mathcal{A})$ is both injective and surjective. But a continuous injection between compact Hausdorff spaces is a homeomorphism from between domain and image. Therefore $\mathrm{St}\,(h)$ is a homeomorphism between $S\,(\mathcal{Q})$ and $S\,(\mathcal{A})$. Applying the Stone functor h is an isomorphism between $B\,(\mathcal{A})$ and $B\,(\mathcal{Q})$ as Boolean algebras (from which it follows that $\mathcal{A}$ and $\mathcal{Q}$ coincide). Conversely, when $\mathcal{A}$ and $\mathcal{Q}$ coincide, then $B\,(\mathcal{A}) = B\,(\mathcal{Q})$ and $S\,(\mathcal{A}) = S\,(\mathcal{Q})$.

5. A Copernican Revolution

Objects (more exactly object-types) and conceptual schemes are *mathematically dual* in exactly the same way that a Stone space and a Boolean algebra are dual. Once a scheme has been framed and its meanings set, the full space of $\mathbb{U}$ is implicit.

The form $a : B(\mathcal{Q}) \to \{T, F\}$ gives an object-type as a unique positions within the classifications of the scheme. Positioned as an intersection of properties. But to say an object is classified and an object *is* an intersection of properties has become exactly the same. Classified objects and intersections of properties have been placed in a bijective correspondence. In a sense there can be no surprises: As the scheme classifies the content of experience, what it uncovers are not exotic relics, but *the productions of its own inner logical machinery.*

No matter how much Fazl observes his records may only combine objects into observed facts $\psi(a_1, \ldots, a_n)$ where $\text{tp}(a_1), \ldots, \text{tp}(a_n) \in \mathbb{U}$. Were he permitted to chronicle indefinitely, and explore every empirical corner, the scope of his observations can only excavate a subspace of $\mathbb{U}$. Yet this $\mathbb{U}$ was fashioned a priori before his work began. Everywhere he travels is a charted course through constructions, never exceeding certain Kantian boundaries laid by the schematic structure.

Following Quine's advice, the scheme itself ought to be revisable through experience. In that case the ontological universe $\mathbb{U}$ is also subject to revision, but that this new frontier of ontological possibilities is now *dual* to the revised scheme. This $\mathbb{U}'$ is mathematically dual to $\mathcal{A}'$. Modifying the scheme modifies objects, but the co-relativity of objects and schemes cannot be modified.

I propose this duality as a reconstruction of Kant's Copernican revolution:

> Thus far it has been assumed that all our cognition must conform to objects. On that presupposition, however, all our attempts to establish something about them a priori, by means of concepts through which our cognition would be expanded, have come to nothing. Let us, therefore, try to find out by experiment whether we shall not make better progress in the problems of metaphysics if we assume that objects must conform to our cognition.—This assumption already agrees better with the demanded possibility of an a priori cognition of objects–i.e., a cognition that is to ascertain something about them before they are given to us. The situation here is the same as was that of *Copernicus* when he first thought of explaining the motions of celestial bodies. Having found it difficult to make progress there when he assumed that the entire host of stars revolved around the spectator, he tried to find out by experiment whether he might not be more successful if he had the spectator revolve and the stars remain at rest. Now, we can try a similar experiment in metaphysics. (Kant, 1996, p.21)

With the important distinction that our revolution has a linguistic basis. Where Kant's objects *revolved* about the subject in orbits determined by cognitive faculty, ours are settled into linguistic paths. While we have not reconstructed Kant's revolution in all its details, the "primary hypothesis" is in tact: Objects revolve about schemes and not the reverse. Concepts are not abstracted from patterns within an ontology that is prior and given; the object descends from a production-line of classifications and appears in consciousness as shaped by the mode of classification.

But there remains an important counter-revolutionary objection: The possibility that *constructions* correspond (are isomorphic with) objects at their most objective and observer-independent state. The realist might concede that experiences are representational while adding that those representations are structurally isomorphic to the things in-themselves. We see in perspective, and of course the tree on the hill is not many scale-factors smaller than the tree an arms length away, but the tree in perspective can be claimed to be isomorphic with the tree itself. Even with the above mathematical duality, it might yet be that a distinguished scheme is isomorphic with reality, such that its constructions and natural objects happen to be in correspondence.

Notice that this objection is logically related to the possibility of translating between schemes. Were there only one conceptual scheme (equivalently, if all schemes are inter-translatable), then the possible objects of all schemes are common. In that case, there is but *one* duality, and to assert that concepts conform to objects or that objects conform to concepts are both consistent. Further, concepts conforming to objects would be the better interpretation, since there is one $\mathbb{U}$ that all conceptual schemes conform to. Thus our Copernican revolution will depend upon the existence of an alternate and non-translatable conceptual scheme.

Part 2. The Structure of Reality

6. Possible Worlds and Indiscernibles

Abul Fazl documents each "passing scene" as an $\mathcal{L}$-structure $\mathcal{U}_\alpha$, where $\mathcal{U}_\alpha \models \mathcal{A}$ for all $\alpha \in I$ (since observation is conducted *through* the scheme). Thus we might heuristically intuit possible worlds as possible models of $\mathcal{A}$. This intuition has technical problems that have already been discussed: First, a model of $\mathcal{A}$ might interpret the structure with an alien semantics (the possible *lounge-world* is interpreted as a *geometrical-world*). Our modal-scope should not include such inversions and departures of meaning. Secondly, as a consequence of the upward Skolem theorem: Any theory with an infinite model has a model at any cardinal, and these are intuitively too large to be a possible world.

The first problem is solved by *fixing a meaning* to each element of the scheme so that its formulas are articulations respective to a consistent *type* of world. A relation $R(a, b)$ between beer mugs, is always between beer mugs, and never transitions its meaning to Platonic solids. The second problem may be solved by bounding the size of the models. A hint derives from model theory, where it is known that it is only necessary to verify a sentence s is true in all finite and countably infinite models of a theory T to conclude that $T \vdash s$. In our case: To test whether s is *a priori* and a deductive consequence of a scheme, it would be enough to verify s in all finite and countably infinite worlds. Fortunately, *countably infinite* is fitting for the size of a possible world.[2]

With these qualifications possible worlds should include finite and countably infinite models of $\mathcal{A}$ whose object-types are a subset of $\mathbb{U}$. In this view, any such scattering of $\mathbb{U}$-object-types observing the analytic conventions of the scheme is a logically possible world.

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[2]At most countably infinite is a good fit for schemes which correspond to natural languages . There may be cause to think of larger worlds (for example, where points in space are all objects). But for our purposes, objects are to be the objects of Abul Fazl's representations, such as tables, chairs, cats, mats, etc. These objects would be at most countably infinite. Bounding the size of worlds in this way also ensures that the number of indiscernibles to any object are at most countably infinite, and this facilitates a tidy mathematical presentation of an object as a_n where a is the type of the object and n indexes which indiscernible. In other words, a world is contained in $\mathbb{U} \times \mathbb{N}$, which greatly simplifies the topic. Admittedly, there are reasons to allow more general sizes, but the resulting complexity only hinders the present discussion.

Definition. A *possible world* $\mathcal{U}$ is a model $\mathcal{U} \models \mathcal{A}$ which is at most countably infinite and whose object-types are a subset of $\mathbb{U}$ (up to indiscernibility).

A physically possible world $\mathcal{V}$ is a possible world such that $\mathcal{V} \models \mathcal{Q}$ where $\mathcal{Q}$ is the category of necessary truths (both analytic or a priori necessary and a posteriori necessary). Similar to the discussion of *possible objects* vs *physically possible objects*, a physically possible world is only knowable through empirical observation, while *logically possible worlds* are implicit once the structure of the scheme known and are enumerable *a priori.* The definition of a possible world allows for indiscernible objects. The reason for allowing a violation of Leibniz's law is to have sufficiently many models for the following theorem:

Proposition. *All truths necessary to logically possible worlds are a priori.*

Proof. It is enough to test whether $\mathcal{U} \models s$ for all finite and countably infinite models $\mathcal{U}$. Given an at most countably infinite model $\mathcal{U}$, order formulas $\psi(x)$ and $\varphi(x)$ by $\mathcal{U} \models \forall x(\psi(x) \to \varphi(x))$, this produces a Boolean algebra $B(\mathcal{U})$, and since consequences of $\mathcal{A}$ are modeled by $\mathcal{U}$ there is a surjection $B(\mathcal{A}) \to B(\mathcal{U})$. The Stone functor sends surjections to injections, thus there is an injective continuous map:

$$f : \mathrm{St}(B(\mathcal{U})) \to S(\mathcal{A})$$

Given $a \in \mathcal{U}$ representing a type $\mathrm{tp}(a) \in S(\mathcal{A})$, there are at most countably many b where $\mathrm{tp}(b) = \mathrm{tp}(a)$. By choosing an enumeration of indiscernibles $a^0, a^1, a^2, \ldots$ we can produce the injections:

$$\mathcal{U} \to S(\mathcal{A}) \times \mathbb{N}$$

by sending $a^k \to (a, k)$.

Further each point of $S(\mathcal{A})$ has an interpretation as a potential object, and so there is a bijection $S(\mathcal{A}) \times \mathbb{N} \to \mathbb{U} \times \mathbb{N}$. Composing with this last map produces an injection $h : \mathcal{U} \to \mathbb{U} \times \mathbb{N}$. Now use the domain $\mathcal{W} = h(\mathcal{U})$, and interpret each function f into $\mathcal{W}$ through $f^{\mathcal{W}}(h(a_1), \ldots, h(a_n)) = h\left(f^{\mathcal{U}}(a_1, \ldots, a_n)\right)$, each relation R as $R^{\mathcal{W}} = (h, \ldots, h)\left(R^{\mathcal{U}}\right)$, and each constant c as $c^{\mathcal{W}} = h\left(c^{\mathcal{N}}\right)$. Then $\mathcal{W}$ is a model of $\mathcal{A}$ which is a possible world and is isomorphic to $\mathcal{U}$.

A sentence s is true in $\mathcal{U}$, if and only if, s is true in the corresponding possible world $\mathcal{W}$. Therefore $\mathcal{A} \vdash s$, if and only if, s is true for all possible worlds. □

Curiously, for the proof to work, the modal scope must include possible worlds with indiscernible objects. Possible worlds with Leibniz's law are too few to make an appeal to the above variation of completeness. Thus, the possibility of indiscernibles has, at least, this purely formal logical evidence.

Notice the proof employed the domain $\mathbb{U} \times \mathbb{N}$. When a world $\mathcal{W}$ satisfies Leibniz's law we can view $\mathcal{W}$ as a subset of $\mathbb{U}$ and so, for these worlds, $\mathbb{U}$ is sufficient as a universal domain. When $\mathcal{W}$ has indiscernible objects an ontological embedding into $\mathbb{U}$ is impossible since indiscernibles must identify the same potential object type in $\mathbb{U}$. But for a class $\left[a^0\right]$ of indiscernible objects, enumerating $\left[a^0\right]$ as $\left\{a^0, a^1, a^2, \ldots\right\}$ views the class as $\left[a^0\right] \times \mathbb{N}$ contained in $\mathbb{U} \times \mathbb{N}$. Of course the choice of enumeration is not unique, and there are several embeddings of $\mathcal{W}$ into $\mathbb{U} \times \mathbb{N}$. But this non-uniqueness does not implicate the integrity of the semantics (exactly because the another choice of embedding only exchanges indiscernibles). The ontology of any possible world is contained in $\mathbb{U} \times \mathbb{N}$. In modal logic this is also called a *constant domain* semantics.

The goal is to define a constant and universal ontology as a mathematically imminent consequence of the conceptual scheme, which becomes ontological at the moment the meanings are arranged; thus all *facts* will combine objects that existed prior to the fact, like hands of cards drawn from a deck . That perspective will be developed bellow, but before doing so it is necessary to address the issue of indiscernibility in more detail. Our theorizing would have gone much easier assuming Leibniz's Law and your author was tempted to simply assume it. For example, assuming Leibniz's Law any domain of objects $\mathcal{U}$ becomes a straight-forward subspace of $\mathbb{U}$ and no further discussion is needed. The primary reasons for not assuming Leibniz's law was the theorem above and the simple observation that in other first-order systems (for models of other theories) indiscernibles are just a "fact of life" and eliminating them is mathematically misguided. Moreover, it is easy to conceive of indiscernibles respective to our linguistic conceptual schemes. Supposing a scheme had little expressive power (if ninety-nine percent of concepts in English were removed) then, simply because there are so few properties to distinguish objects and opportunities for discernment, indiscernibles become likely.

A possible world $\mathcal{U}$ has an embedding $h : \mathcal{U} \to S(\mathcal{A}) \times \mathbb{N}$ sending $a \to (\text{tp}(a), k)$, supposing that objects were relabeled so that $a \to (\text{tp}(a), n)$ $(n \neq k)$ it should not to matter since this amounts to exchanging indiscernibles. That is intuitive for *intrinsic* indiscernibility but what about *extrinsic* indiscernibility? Two objects sharing all the same properties need not stand in the same relations. For example, imagine two indiscernible motor vehicles a and b (they are the same dimensions, weight, model, etc. down to the smallest detail), if a is parked next to vehicle c does that imply that b is parked next to c?—No, b might be anywhere else in the world. The question becomes: Are free choices of labels consistent with these higher relations? Luckily, this is not an issue. The embedding sends objects to $\mathbb{U} \times \mathbb{N}$ and so tuples are taken from $(\mathbb{U} \times \mathbb{N})^n$. As in the proof above the embedding is built in a way to produce an isomorphism of models h where $h : \mathcal{U} \to \mathbb{U} \times \mathbb{N}$ and $h(\mathcal{U}) = \mathcal{W}$,

$$\mathcal{W} \models \psi(h(a_1), \ldots, h(a_n)) \text{ if and only if } \mathcal{U} \models \psi(a_1, \ldots, a_n)$$

Therefore whether vehicle b is parked next to c is resolved in the original model and there is no inconsistency. Moreover any choice of embedding can be made consistent (in that case the isomorphism is adapted to this alternative labeling).

The idea of a possible world being *a topological subspace* can also be rescued. Define the function $g : \mathbb{U} \times \mathbb{N} \to \mathbb{U}$ by mapping together indiscernible objects. The topology induced on $\mathbb{U} \times \mathbb{N}$ by g is generated by $g^{-1}(\text{tp}\psi)$ where ψ is a unary formula and $\text{tp}\psi$ is the representation of ψ as a clopen subset of the space of 1-types. This makes $\mathbb{U} \times \mathbb{N}$ a genuine topological space, but with indistinguishable points (the indiscernible objects). In other words, the space violates the separation axiom T_0, or is non-Kolmogorov. Any topological space has a Kolmogorov quotient identifying indistinguishable points, in our case the quotient map is g. The topological perspective is salvageable by allowing topologically indistinguishable points, then a possible world $\mathcal{W}$ embeds into $\mathbb{U} \times \mathbb{N}$ and can be viewed as a topological space under its subspace topology.

Having developed this perspective, the full mathematical power of model theory can be leveraged with surprising results. Proposition 4.1.3 from Marker (2002) states:

> Let $\mathcal{M}$ be an $\mathcal{L}$-structure, $A \subset \mathcal{M}$, and p an n-type over A. There is $\mathcal{N}$ an elementary extension of $\mathcal{M}$ such that p is realized in $\mathcal{N}$.

Here the n-types are constructed respective to the model $\mathcal{M}$ (By adding constants A and using $\mathcal{M} \models \forall x_1, \ldots, x_n \left(\psi\left(x_1, \ldots, x_n\right) \rightarrow \varphi\left(x_1, \ldots, x_n\right)\right)$ to order formulas into a Boolean algebra). In our case, if $\mathcal{U}$ is a world and $\mathrm{Th}\left(\mathcal{U}\right)$ are all its true sentences (which includes analytic truths, a posteriori necessary truths, and contingent truths), and p is a possible type of object consistent with $\mathrm{Th}\left(\mathcal{U}\right)$ and missing from $\mathcal{U}$, then there exists a strictly *larger world* $\mathcal{W}$ (ie, having all the objects of $\mathcal{U}$), so that: For all $(a_1, \ldots, a_n) \in \mathcal{U}^n$ and $\mathcal{L}$-formulas ψ:

$$\mathcal{U} \models \psi\left(a_1, \ldots, a_n\right), \text{ if and only if, } \mathcal{W} \models \psi\left(a_1, \ldots, a_n\right)$$

In other words, the extended world $\mathcal{N}$ keeps all objects in their present arrangements (if the "cat is on the mat" in $\mathcal{U}$ then that *same cat* is on the *same mat* in $\mathcal{W}$), while simultaneously realizing an object of the specified missing type.

Possible worlds have a resemblance to field extensions. If a solution to a polynomial is missing from the base field ($x^2 + 2 = 0$ is missing from the rational field) there exists an extended field that includes a solution (a $\sqrt{2}$), but which also agrees on all arithmetical facts of the base field ($1/2 + 1/2 = 1$ is still the case in the extension). Analogously: There exists a world agreeing with all the facts of this world (the populations of cities are the same, wars concluded on the same dates, etc.) but somewhere within exists your near twin (only one foot taller, with different colored eyes, or as close as can be approached respective to $\mathrm{Th}\left(\mathcal{U}\right)$). Even stranger, the near-twin is not quarantined to a realm of mathematical abstraction (as exotic mathematical objects often are), since $\mathrm{Th}\left(\mathcal{U}\right)$ *includes* every scientific, historic, sociological, and psychological law. The twin is more worldly than Pegasus or Plato's beard. Fitting well into the world's common senses.

It is also possible to omit types. Theorem 4.2.4 from Marker's text states:

> Let $\mathcal{L}$ be a countable language, and let T be an $\mathcal{L}$-theory. Let X be a countable collection of non-isolated types over $\emptyset$. There is a countable $\mathcal{M} \models T$ that omits all the types $p \in X$.

Here types are respective to the theory T (dual to formulas ordered by T). We can choose T to be the conventions of the language $\mathcal{A}$ or include necessary a posteriori truths $\mathcal{Q}$. In either case for a countable set X of non-isolated types there is a world $\mathcal{U}$ (modeling the theory) which omits them. For instance: The planet Earth and all objects within. Or the Milky Way Galaxy. Either might be *deleted*.

My intention is not to write a detailed mathematical investigation, but to illustrate how the mathematical tools allow a rigorous exploration of old speculations.

7. The Facts in Logical Space are the World

Let us finally define a conceptual scheme formally,

Definition. A *conceptual scheme* $\mathscr{U}$ is a signature $\mathcal{L}$, a set of sentences $\mathcal{A}$ representing linguistic conventions, and a universal domain of object-types $\mathbb{U}$ (characterizing meanings fixed to the scheme's concepts). That is, a triple $\mathscr{U} = (\mathcal{L}, \mathcal{A}, \mathbb{U})$.

Given a conceptual scheme $\mathscr{U} = (\mathcal{L}, \mathcal{A}, \mathbb{U})$, since $\mathbb{U} \times \mathbb{N}$ is a constant domain, it is possible to list all formulas in permutations of potential objects: For an $\mathcal{L}$-formula $\psi(x_1, \ldots x_n)$, all $\psi(a_1, \ldots, a_n)$ across $(a_1, \ldots, a_n) \in (\mathbb{U} \times \mathbb{N})^n$. For example, where $R(x, y)$ is the relation "is an acquaintance of," and x and y ranges over individual people, it becomes possible to enumerate, $R(a, b)$ across *potential* pairs of people $(a, b) \in (\mathbb{U} \times \mathbb{N})^2$. A possible world $\mathcal{U} \models R(a, b)$ describes a world where person a and person b are acquaintances. We might do the same with "is to the east of" or "is taller than."

Interpreting a free-standing formula $\psi(a_1, \ldots, a_n)$ as a *state-of-affairs* and a formula which obtains $\mathcal{U} \models \psi(a_1, \ldots, a_n)$ as a *fact*, the resulting combinatorics recalls the Tractatus (Wittgenstein, 1999), since all possible states-of-affairs (as combinations of objects) stand in logical space, and facts (true-states-of-affairs) are a sub-set of these affairs which happen to hold in the world, and which in an important sense *are* the world.

From our position there is an immediate interpretation of:

> (1.1) The world is the totality of facts, not of things.
> (1.11) The world is determined by the facts, and by these being *all* the facts.

Since after the domain $\mathbb{U}$ is given, all combinations of objects within formulas is mathematically imminent. All possible states of affairs gather as a set,

$$\mathrm{SA}(\mathscr{U}) = \{\psi(a_1, \ldots, a_n) : \psi \text{ is an } n\text{-ary } \mathcal{L}\text{-formula for some } n \text{ and } (a_1, \ldots a_n) \in (\mathbb{U} \times \mathbb{N})^n\}$$

while facts $\mathrm{F}_{\mathscr{U}}(\mathcal{U})$ (respective to $\mathcal{U}$) is the subset of $\mathrm{SA}(\mathscr{U})$ where $\mathcal{U} \models \psi(a_1, \ldots, a_n)$. There is a closely related construction from model theory. If $\mathcal{M}$ is an $\mathcal{L}$ -structure, and $\mathcal{L}_{\mathcal{M}}$ is the language obtained by adding $\mathcal{M}$ as constant symbols. The *elementary diagram Diag*$(\mathcal{M})$ of $\mathcal{M}$ is,

$$\{\psi(a_1, \ldots, a_n) : \mathcal{M} \models \psi(a_1, \ldots, a_n),\ \psi \text{ is an } \mathcal{L}\text{-formula}\}$$

The reason for the added constants is to make $\psi(a_1, \ldots, a_n)$ into sentences which can be modeled as a theory, otherwise the construction amounts to the same as $\mathrm{F}_{\mathscr{U}}(\mathcal{U})$. By Lemma 2.3.3 from Marker (2002), if $\mathcal{N}$ is an $\mathcal{L}_{\mathcal{M}}$-structure and $\mathcal{N} \models \mathrm{Diag}(\mathcal{M})$; then, viewing $\mathcal{N}$ as an $\mathcal{L}$-structure, there is an elementary embedding of $\mathcal{M}$ into $\mathcal{N}$. In the case where diagrams overlap (worlds that agree on all facts) then $\mathcal{M}$ is a substructure of $\mathcal{N}$ and $\mathcal{N}$ is a substructure of $\mathcal{M}$ and models are isomorphic. Therefore *all the facts* determine the world up to isomorphism. Up to isomorphism, since applying a model theoretic automorphism to the world preserves the elementary diagram while permuting he objects.

Though it should be noted that, in the Tractatus, a state-of-affair is not identical with a logical formula "as it stands printed on paper" (4.011). For Wittgenstein reality has a logical structure, this structure is conserved in thought, and subsequently from thought to printed propositions. So there are three movements, but when we speak of $\psi(a_1, \ldots, a_n)$ as a state-of-affairs we are already at the printed proposition and are silent on the logical structure of reality (indeed, since our thesis is that propositions involve construction). A helpful idea here is that of *isomorphism*. Wittgenstein does not use the word isomorphism in the Tractatus, but the notion is apparent in the text through statements such as:

> (2.12) The picture is a model of reality.

(2.15) That the elements of the picture are combined with one another in a definite way, represents that the things are so combined with one another.

Recall that Wittgenstein's was inspired to write the Tractatus after hearing of traffic courts where accidents are reenacted with scale-models. For the reenactment to be faithful, it should preserve the structure of the true event. The elements are arranged so that velocities of the toys correspond to velocities as they happened, "to the east of" in the model corresponds to "to the east of" in the event, and so on. The event and the reenactment are logically isomorphic.

The primary movements of the Tractatus can expressed diagrammatically through the isomorphisms:

$$\text{(reality)} \to \text{(picture)} \to \text{(proposition)}$$

in category theory isomorphisms are composible and invertible, and that should be the case here, therefore (reality) $\to$ (proposition) and (proposition) $\to$ (reality).

We can use $\psi(a_1, \ldots, a_n)$ as states-of-affairs (without commenting on the structure of reality) and reproduce propositions from the Tractatus *because* for Wittgenstein there is an isomorphic correspondence between reality and the proposition. States-of-affairs in logical space *are* propositions in logical space. If "facts in logical space are the world" (1.13), then committing those facts to propositions and arranging them in logical space reproduces the world (as an isomorphic copy). A significant amount of the Tractatus is interpretable in this way. I am not a Wittgenstein scholar, and my project is not to reconstruct the Tractatus, but I will present some results in this direction. Going forward it should be noted that Wittgenstein's logical atomism cannot be justified within our system and the "atomic" of the "atomic fact" will have to be dropped from the declarations. Wittgenstein uses atomic fact to denote facts which cannot be analyzed further into sub-facts. The fact "it is snowing outside and the roads are slippery" might be analyzed into "it is snowing outside" and "the roads are slippery," and these have an analysis into still smaller facts, at some point the analysis concludes and the remaining logical particles are the atomic-facts. These atoms do not exist in our system. The states-of-affairs generated by the conceptual scheme are *interconnected*; and, if there is reduction, then everything reduces into everything else.

I will give a mathematical argument for the non-existence of logical atoms. To start, it is not clear how Wittgenstein understands the logical atom, a best guess is a *subset* of $\mathrm{F}_{\mathcal{U}}(\mathcal{A})$ which generates all facts through logical combinations (under connectives). I will show there exist no such atoms in the one dimensional case (the higher dimensional case is analogous)

Proposition. *There does not exist a unique set of atoms.*

Proof. Let $A \subset F_{\mathscr{U}}(\mathcal{U})$ be a set of unary atoms determining unary facts. Knowing A must determine $F_{\mathscr{U}}(\mathcal{U})$ which in the one-dimensional case means that for an object $u \in \mathcal{U}$, knowing $a(u)$ across atoms determines $\psi(u)$ for general formulas. To know an object it is enough to know its *type* since, if b is indiscernible to a, then both agree on all unary formulas. For any unary formula ψ the topological position tells us if $\mathrm{tp}(a) \in \mathrm{tp}(\psi)$ for all formulas ψ. Thus a logical determination of a is fully given by the position of $\mathrm{tp}(a)$ in $S_1^{\mathcal{U}}(\emptyset)$ (here we use the types of $\mathcal{U}$). The existence of logical atoms can be restated: There are formulas A so that $\mathrm{tp}(A) = \{\mathrm{tp}(a) : a \in A\}$ generates the topology $S_1^{\mathcal{U}}(\emptyset)$. Note that the types represented by $\mathcal{U}$ in $S_1^{\mathcal{U}}(\emptyset)$ give a dense subspace, therefore atoms with respect to the domain $\mathcal{U}$ must become generators of the full space $S_1^{\mathcal{U}}(\emptyset)$. Now apply a homeomorphism h to $S_1^{\mathcal{U}}(\emptyset)$ where $h(A) \neq A$. If $\mathrm{tp}(\alpha)$ are atoms, then the pre-images $h^{-1}(\mathrm{tp}(\alpha))$ are as atomic. The original atom α therefore has a logical decomposition to with respect to this transformed set of atoms. This is not unlike applying an invertible linear transformation to a vector space, one basis is exchanged for another, and it is impossible to privilege either.

Where $u \to \mathrm{tp}(u)$ maps $\mathcal{U}$ into $S_1^{\mathcal{U}}(\emptyset)$ (not surjectively), if $A \subset F_U(\mathcal{A})$ are atoms, then knowing when $\mathrm{tp}(u) \in \mathrm{tp}(a)$ across $a \in A$ determines the topological position of $\mathrm{tp}(u)$. But that must also hold with the same map $u \to \mathrm{tp}(u)$ after replacing A with $h(A)$. □

A possible exception occurs when every homeomorphism h is such that $h(\mathrm{tp}A) = \mathrm{tp}A$. But this will be unlikely given the character of logical topologies. For example, supposing the Tarski algebra of $\mathcal{U}$ is countable and atomless (a likelihood for conceptual schemes corresponding with natural languages), then the space of types $S_1^{\mathcal{U}}(\emptyset)$ is the Cantor set. In that case: For *any* two non-empty clopen subsets $K, N \subset S_1^{\mathcal{U}}(\emptyset)$ there exists a homeomorphism $h : K \to N$ so that $h(K) = N$. The space is so self-symmetrical that you cannot even detect when a formula is more basic.

Moving on,

> (2.01) A fact is a combination of objects.

This combinatorial understanding of the fact is confirmed by our approach. A combination of objects $(a_1, \ldots, a_n)$ obtains in the fact. Handfuls of objects are pulled from a constant and universal ontology and, in the fact, settle into mutual coherence. States-of-affairs are like all possible hands drawn from this deck.

> (2.012) In logic nothing is accidental: if a thing can occur in a fact, the possibility of that fact must already be prejudged in the thing.
> (2.1123) If I know an object, then I also know the possibilities of its occurrence in facts.

Such is especially true for unary-properties. Objects (types of objects) are identical with order-theoretic homomorphisms from the algebra of logical formulas into $\{T, F\}$. An object *just is* its point-like position amid overlapping properties. To state an object-type as $\mathfrak{p} : B(\mathcal{A}) \to \{T, F\}$ is to know all of its intrinsic properties; conversely, to put one's finger *somewhere* at the intersection of properties defines an object-type.

Higher-dimensional facts are more complex. Analogous to the unary case: Ordering n-ary formulas under logical equivalence gives a Boolean algebra, its corresponding Stone space is the space of n-types $S_n(\mathcal{A})$ (it should be noted that $S_n(\mathcal{A})$ *is not* $S_1(\mathcal{A})^n$). If we treat the tuples $(a_1, \ldots a_n)$ as the object, then knowing its type, as in the one-dimensional case, completely determines its occurrence in n-ary facts. However, if $(a_1, \ldots, a_n)$ and $(b_1, \ldots, b_n)$ are indiscernible at each component, that is insufficient to show $\mathrm{tp}(a_1, \ldots, a_n) = \mathrm{tp}(b_1, \ldots, b_n)$. For this reason, there is a way in which an object a, perfectly known, is indeterminate with respect to higher facts.

> (2.0122) The thing is independent, in so far as it can occur in all *possible circumstances.*

In the sense that given a tuple of objects $(a_1, \ldots, a_n)$ and a formula ψ the pairing $\psi(a_1, \ldots, a_n)$ is well-formed. That is to say that a tuple applies to all circumstances.

> (2.02) Objects are simple.

Objects in $\mathbb{U}$ are *not* composite. The object has already been narrowed to a *point* in the logical topology; and, following Euclid, a point is "that without parts." Sub-objects in the extensional sense (fingers on the hand) are, in the logical topology, points unto themselves (at a certain geometric distance from the super-object).

> (2.022) It is clear however different form the real one an imagined world may be, it must have something common - a form - in common with the real world.
> (2.023) This fixed form consists of objects.

Indeed, an imagined world *draws* from the same reservoir of objects (from the universal ontology of $\mathbb{U}$); and, viewed from our world, rearranges them. Our world and a possible world are separated by a combinatorial permutation. But I only add that possible worlds must must first model the conventions of the language (also a common form). Objects are mathematically dual to these conventions.

> (2.03) In the fact objects hang one in another, like links in a chain.
> (2.032) The way in which objects hang together in the fact is the structure of the fact.

The formula ψ is the way in which the objects $(a_1, \ldots, a_n)$ hang together as $\psi(a_1, \ldots, a_n)$. This notion of structure will become very important in what follows, since it is the structure to be preserved under isomorphism. Supposing we agree with Wittgenstein that there is an isomorphism between reality and thought (the picture) where "The picture is a model of reality" (2.12) and "To the objects correspond in the picture the elements of the picture" (2.13). Objects $a_1, \ldots, a_n$ must map to elements $a'_1, \ldots, a'_n$ such that the structure of ψ is preserved.

In model theory, a map $j : \mathcal{M} \to \mathcal{N}$ between $\mathcal{L}$-structures is an isomorphism when for all formulas ψ,

$$\mathcal{M} \models \psi(a_1, \ldots, a_n), \text{ if and only if, } \mathcal{N} \models \psi(j(a_1), \ldots, j(a_n))$$

and this appears to be the correct mathematical formalism of the Wittgensteinian intuition. The structural feature of reality ψ and the objects $a_1, \ldots, a_n$ must map onto the pictorial feature ψ and its elements $j(a_1), \ldots, j(a_n)$. Our most important questions will come to depend on the existence or non-existence of such isomorphisms.

We have confirmed much of the *metaphysical side* of the Tractatus: Facts in logical space are the world, alternative worlds are alike to a reshuffling of objects, objects hang together in logical structure, etc. This metaphysics, however, is *printed on the page*. On the proposition side of Wittgenstein's (reality) $\to$ (picture) $\to$ (proposition). We have yet to eclipse the chronological project of Abul Fazl – being led by the hand across recordings and discovering this metaphysics at the horizon of record. With everything observed, between the bindings of his text: *Facts in logical space are the world.* Still unanswered is whether this space of facts, of true states-of-affairs, corresponds with the in-itself as it looks upon its own affairs. Does logical space correspond with the inner-space of reality?

8. The Existence of Untranslatable Schemes

A conceptual scheme is a *means of representation.* This is seen in the chronicles of Abul Fazl, in which propositions linguistically image a world-state:

"The Sultan reclines."

"The court musician, Tansen, begins to sing and play..."

"The instrument is made of..."

"A breeze has begun to blow and ruffles a curtain directly north-east of Tansen."

"The curtain is made of..."

The finished chronicle is a Wittgenstein-like picture of a world-state wherein "the proposition is a picture of reality" which "models reality as we think it is" (4.01). The question becomes: Is this representation unique (more properly, unique up to isomorphism)? For Wittgenstein reality has a logical structure which propositions *display.* There is an isomorphism from the logical structure of reality to the logical structure of propositions, where "We must not say, the complex sign $R(a,b)$ says 'a stands in a certain relation R to b'; but we must say, that a stands in a certain relation to b says that $R(a,b)$" (3.1432). The R is written into the real and the proposition can only display this.

Such a structural isomorphism is plausible, but should not be assumed. That is to say, we should not presume that all our analytic taxonomizing and grid-making activities are mirrored in the Universe as it follows its own courses. That the great Hericlitian tide washes ashore in these little symbols R for the sake of treasure hunting analytic philosophers to discover afterwards. That takes much for granted.

By assumption, any world models the conventions of the scheme, and for this reason the conventions of the scheme satisfy the common idea of *objectivity* since its anticipations and the patterns of nature (recorded through the scheme) reciprocally confirm. But do these conventions valorize as objective because they are intrinsic to reality, or because observations are conducted *through* the scheme in observance of its conventions? To use a Kantian example: Proclus' lemma holds of all experience and its negation is unobservable *because* Euclidean geometry is rule-like of representation. Observations always confirm any theorem of Euclid because Euclidean geometry is the framework through which experiences appear.

With reference to Wittgenstein, consider a traffic accident observed from two sides. Or it is better to say: Consider *an event*, which a western observer represents as a traffic-accident, approached by a second observer from their angle of observation. I believe it is possible to think of this *scene* – approached from two sides – as a *world* in the Wittgensteinian sense. Everything that was said of our observer Abul Fazl, who, recall, carried his recording tools into every corner of the Indian subcontinent, holds equally when he carries those same tools everywhere within the scene of the event. Here is a microcosm which also splits apart into facts in logical space. The scene models the conventions of the language – but, so it does from two sides, as it also models the conventions of the second observer.

Hence we have two "worlds" $\mathcal{U} \models \mathcal{A}_1$ and $\mathcal{V} \models \mathcal{A}_2$ and the fundamental question can be stated: Is there an isomorphism $f : \mathcal{U} \to \mathcal{V}$?

Intuitively no. Since the observing of a fact $\mathcal{U} \models \psi(a_1, \ldots, a_n)$ depends upon requisite linguistic resources being present. There must exist some combination of known concepts which produces ψ. Thus if we ask Abul Fazl to approach from his angle, and yet with all the power of his linguistic resources and given all possible linguistic combinations of his concepts, he simply cannot interpret industrialized urban landscapes, since red-lights, traffic-signs, and motor-vehicles, then he cannot reproduce the meaning of ψ.

Before we assume all meanings are discoverable through finite linguistic combinations, there exist clear counter-examples: Produce the meaning "good" using only descriptive terms. Goodness is transcendental to description – separated by a Humean gap. Hence transcendental or unreachable meanings are possible. It seems to me that Abul Fazl, dropped by time-machine into this alien landscape where metal beasts that we call vehicles roam and towers we call skyscrapers loom, will have a great many gaps before him. Of course, with exposure to experience, his conceptual scheme should adapt to tame the empirical chaos. But that is not our question. The question is: Given his scheme *as it stands*, does there exist a translation? Surely not. What we observe as a traffic-accident is to Abul Fazl a confusion as blooming and buzzing as that of James. *As it stands*, he simply does not have the tools to resolve the manifold of experience as we do.

There is still a way out for the conceptual chauvinist, since it is possible to argue that, although a concept is presently unreachable, objects stand and wait on the observer to integrate them into the appropriate concepts. Davidson (1973) admits that an alternate conceptual scheme could contain predicates whose extensions have no match, but the insists that the detection of this matchlessness depends on a mutual ontology. How do we know an extension is incongruent if not through common ontology of objects to signal where the incongruence occurs?

To this Wittgenstein provides a hint, "Objects form the substance of the world" (2.021). All of the tuples $(a_1, \ldots, a_n)$ which could be configured $\psi\,(a_1, \ldots, a_n)$ are mathematically dual to the scheme. If objects are the substance of facts, and the totality of facts is the world, then an alternative ontology is the substance of an alternative world (in representation). As the many things that might be molded from Descartes' wax would be *other things* when that wax is substituted for another substance. Davidson (1973) claims "Different points of view make sense, but only if there is a common coordinate system on which to plot them; yet the existence of a common system belies the claim of dramatic incomparability" (p. 6). But to press this geometric analogy: The coordinates are tuples of objects $(a_1, \ldots, a_n)$ hanging together in the fact $\psi\,(a_1, \ldots, a_n)$. To be certain: The affine n-space of the complex numbers, the real numbers, and a finite field, cannot be plotted in a common coordinate system.

Because ontologies and schemes are mathematically dual, when schemes diverge sufficiently, their respective ontologies decohere and the substance which fills extensions becomes incompatible. If we imagine the passing scene of Davidson, like a traffic-accident, the parts of the scenery that catch and become objects are contingent upon the scheme. A street sign catching and rending itself from the manifold is a production of the scheme. Another observer, without the concept of signage, might not recognize the sign as distinct from what is materially contiguous to it. There is no impetus to separate the sign from the concrete at its base or a nearby parking meter. The same passing scene may induce, in two observers, distinct parades of passing objects. If the alternate scheme is sufficiently departed from our own, then the corresponding ontology lacks the objects needed to replicate an extension. In other words, street-signs are not roaming or wondering objects that wait on us to notice the signage they have in common and to integrate them into their concept. To the other observer, scheme street signs are not objects at all and there is nothing there to be integrated.

Locke (1847) explains concept creation as a process of abstraction where the mind extracts patterns found in existing objects. The particular white observed in chalk, snow, milk, etc. is abstracted into the general "white." This is necessary, since if the mind were responsible for tracking each object with a distinct name it would quickly become overburdened. Such a sequence of mental operations is plausible but depends upon a hidden assumption: Objects are given, and the task of the mind is to discover patterns within. Chalk, snow, and milk are given, and among these the mind notices the same whiteness. However, a given ontology presumes, secretly, that a particular organization of the manifold of experience is so given (since again, objects and schemes are mathematically dual). Davidson's common ontology is logically equivalent to a common conceptual scheme (granted in a way that is mathematically obscure) and so his argument is circular.

A straightforward way to witness ontologies diverging is to position two schemes, one as base and the other as extension. This can be said to occur when $\mathscr{U}_1 = (\mathcal{L}_1, \mathcal{A}_1, \mathbb{U}_1)$ and $\mathscr{U}_2 = (\mathcal{L}_2, \mathcal{A}_2, \mathbb{U}_2)$ where $\mathcal{L}_1 \subset \mathcal{L}_2$ and for unary $\mathcal{L}_1$-formulas $\psi(x), \varphi(x)$, $\mathcal{A}_2 \vdash \forall x (\psi(x) \to \varphi(x))$, if and only if, $\mathcal{A}_1 \vdash \forall x (\psi(x) \to \varphi(x))$. In other words: $\mathscr{U}_2$ introduces new concepts to $\mathscr{U}_1$ while preserving the original conceptual relations. The expansion of a dictionary so to speak. When this occurs, there is an injection $i : B(\mathcal{A}_1) \to B(\mathcal{A}_2)$, and since the Stone functor sends injections to surjections, there is a surjective map $\mathrm{St}(i) : \mathbb{U}_2 \to \mathbb{U}_1$ which is explicitly the restriction map that sends $a : B(\mathcal{A}_2) \to \{T, F\}$ to $a|_{B(\mathcal{A}_1)}$. It is simple to show that when the extension is proper (ie, $B(\mathcal{A}_1)$ is a proper sub-algebra of $B(\mathcal{A}_2)$) that $\mathrm{St}(i)$ is not injective. This is consistent with the intuition that fewer boundaries made within the data of sensation implies fewer opportunities for objects to distinguish as object. Here we see that the granularity of a scheme conditions the ontology. A very fine scheme and a very coarse scheme must have incommensurate ontologies (and thus an incommensurate "substance").

There need not exist a isomorphic reconstruction of the fact $\mathcal{U} \models \psi(a_1, \ldots, a_n)$ in another mode of representation. Firstly, because the logical form of ψ is contingent upon the formation of meanings; secondly, because ontologies are incommensurate such that no sensible correspondence between objects exists. This answers Davidson.

9. The "Fittedness" of Schemes and Pragmatism

Let us distinguish between two categories of schemes: Scientific models and natural languages. The former tends towards mathematical exactness, uses a deeper layer of data mined through scientific instruments, and has a stronger claim to grasping the structure of reality. The later are the types of schemes that have been considered so far where ordinary language is used to make descriptions. Call a scheme corresponding to a natural language a *natural scheme* (also a natural representation) and call schemes corresponding to scientific models *formal schemes* (also formal representations).

In this section I wish to consider two natural schemes which resolve the same event (say a traffic-accident) into *untranslatable* facts, and whether preferencing either as better fitted to reality is justified.

Subjectivity is inseparable from natural schemes. This is shown by observing the universal tendency for linguistic communities (who intersubjectively agree on a scheme) to use those conceptual tools optimally *adapted* to the challenges effacing that community. Every natural representation is adapted to a short-band of challenges native to a specific somewhere.

The traffic accident provides an illustrative case. Where is the hint from nature that we *ought* to separate out the panel of the traffic sign as the object? Why not the panel and its pole? Or the panel, the pole and a small island of surrounding concrete? When we expand too far, where are the cries of nature against this abuse? A scheme should be efficient in prediction – what then is worth predicting? Arriving at the scene of an accident, the first philosophical question is not "Who is at fault?" but why these peculiar configurations metal are worth noticing at all. Surely adjudicating traffic accidents is adaptation to post-industrial life where all perpetually commute an urban labyrinth.

The conceptual chauvinist might yet argue that, while all schemes have signs of being adapted, certain evolutionary forces have pressurized his own linguistic community into a uniquely intimate correspondence with reality. The blooming and buzzing confusion which effaced Abul Fazl was, in fact, the glare of reality. As the initial shock diminishes, Abul Fazl becomes adapted to the chauvinist's reality (which is also *the* reality).

But the idea that reality changes in correspondence with the human striving of a particular historic epoch feels too convenient. How kind of nature to change its inner-self with our attempts to master it. Quine states that:

> Hence it is meaningless, I suggest, to inquire into the absolute correctness of a conceptual scheme as a mirror of reality. Our standard for appraising basic changes of conceptual scheme must be, not a realistic standard of correspondence to reality, but a pragmatic standard. Concepts are language, and the purpose of concepts and of language is efficacy in communication and prediction. Such is the ultimate duty of language, science, and philosophy, and it is in relation to that duty that a conceptual scheme has finally to be appraised.
>
> Elegance, conceptual economy, also enters as an objective. (Quine, 1950, p. 632)

Before Abul Fazl is dismissed as merely confused, we should acknowledge how our own parochial concerns influence what to us is economical and elegant. That, as the manifold is tamed and collapsed into form and resolves into the traffic accident, the style of construction is motivated by our pragmatic concern. We *need* to see these objects. Is the structure of reality changing or is language adapting to meet historically evolving challenges? In the later case, whatever is the sub-straight of the real (the in-itself, quantum-fields, or whichever) is simply of-itself-so and quite indifferent to the races we run upon it.

Between two human observers untranslatability must increase with divergent social and historic pressures. Note that this divergence occurs *with respect to* common faculties. Diverging *despite* evolution endowing all human subjects with a common perceptual apparatus. The divorce becomes all the more radical imagining subjectivities shaped by different evolutionary pressures. After all, human representation is optimized to solve the trouble of being human. Evolutionary, human faculties are all wired and adapted to measure a small island of earthbound experiences and exploit bands of information falling within the not-too-large, the-not-too-small, the-not-too-fast, and so on.

Thus when we imagine what it is like to be a bat:

> Now we know that most bats (the microchiroptera, to be precise) perceive the external world primarily by sonar, or echolocation, detecting the reflections, from objects within range, of their own rapid, subtly modulated, high-frequency shrieks. Their brains are designed to correlate the outgoing impulses with the subsequent echoes, and the information thus acquired enables bats to make precise discriminations of distance, size, shape, motion, and texture comparable to those we make by vision. But bat sonar, though clearly a form of perception, is not similar in its operation to any sense that we possess, and there is no reason to suppose that it is subjectively like anything we can experience or imagine. This appears to create difficulties for the notion of what it is like to be a bat. We must consider whether any method will permit us to extrapolate to the inner life of the bat from our own case, and if not, what alternative methods there may be for understanding the notion...
>
> If anyone is inclined to deny that we can believe in the existence of facts like this whose exact nature we cannot possibly conceive, he should reflect that in contemplating the bats we are in much the same position that intelligent bats or Martians would occupy if they tried to form a conception of what it was like to be us. The structure of their own minds might make it impossible for them to succeed, but we know they would be wrong to conclude that there is not anything precise that it is like to be us: that only certain general types of mental state could be, ascribed to us (perhaps perception and appetite would be concepts common to us both; perhaps not). We know they would be wrong to draw such a skeptical conclusion because we know what it is like to be us. And we know that while it includes an enormous amount of variation and complexity, and while we do not possess the vocabulary to describe it adequately, its subjective charater is highly specific, and in some respects describable in terms that can be understood only by creatures like us. The fact that we cannot expect ever to accommodate in our language a detailed description of Martian or bat phenomenology should not lead us to dismiss as meaningless the claim that bats and Martians have experiences fully comparable in richness of detail to our own...
>
> This brings us to the edge of a topic that requires much more discussion than I can give it here: namely, the relation between facts on the one hand and conceptual schemes or systems of representation on the other. My realism about the subjective domain in all its forms implies a belief in the existence of facts beyond the reach of human concepts. Certainly it is possible for a human being to believe that there are facts which humans never will possess the requisite concepts to represent or comprehend. Indeed, it would

> be foolish to doubt this, given the finiteness of humanity's expectations. After all, there would have been transfinite numbers even if everyone had been wiped out by the Black Death before Cantor discovered them. But one might also believe that there are facts which could not ever be represented or comprehended by human beings, even if the species lasted forever-simply because our structure does not permit us to operate with concepts of the requisite type. This impossibility might even be observed by other beings, but it is not clear that the existence of such beings, or the possibility of their existence, is a precondition of the significance of the hypothesis that there are humanly inaccessible facts. (After all, the nature of beings with access to humanly inaccessible facts is presumably itself a humanly inaccessible fact.) Reflection on what it is like to be a bat seems to lead us, therefore, to the conclusion that there are facts that do not consist in the truth of propositions expressible in a human language. We can be compelled to recognize the existence of such facts without being able to state or comprehend them. (Negel, What it is like to be a Bat, pp. 438-441)

Applying Nagel's "what is it like to be?" to organisms other than ourselves evokes radically untranslatable representations. The quickest way to recognize these breakdowns is to remember that objects and schemes are correlative. The objects which "hang together" in the fact are themselves dependent on the structure of representation. The objects assembling and dissembling in bat-phenomenology are not an adoptable alternative to human objects, but rather *incoherent* to human representation. Mobius strips and Klein bottles and other weavings of unreason. The Martian, whose faculties of perception evolved to exploit information that is extra-spectral or cleanly beyond human detection, conceives in shapes of the beyond, and in "things" unreckonable. Their Wittgenstinian pictures are a gallery of Lovecraftian twistings and what is simply and only beyondness.

At issue are the pragmatic interests exerting a pressure upon the structure of the scheme. Its architecture becomes tailored to the challenges which beset the subject, and the in-itself resolves into representation, not disinterestedly or in shapes of ultimate-reality, but into those shapes serviceable to problem solving. Wherever the observer is found so is a representation optimized to those puzzles and problems local to the site of observation; to bring another observer into our way of seeing is to bring them into our struggle, and that explains a diminishing bloom of confusion as much as anything.

Anywhere on earth (and perhaps beyond it) the subject is equipped with conceptions that problem solve and manage a struggle for existence. This is not unlike Marx's conception of history: Ideology is always super-structural to the economic struggle of a historically given context. It would a coincidence on the order of overlapping lightning strikes for Feudal-ideology to map onto the structure of the cosmos and apply to solar systems lightyears from the economic pressures which generated the ideas.

Granted, mastering nature is advantageous to the meeting of needs – so there is an incentive to figure nature at its more elemental. This cannot change that the scheme was not designed to understand nature disinterestedly, but to solve it respective to an interest. The problem must first stand apart as problematic before the scheme has a chance to analyze it. Within our linguistic pictures of nature linger artifacts of *what we wanted* from nature.

Above the idea of a scheme-extension was introduced (where an extension adds concepts while preserving the conceptual relations of the base scheme). A scheme-extension (increasing analyticness) has an intuitive epistemological legitimacy. There is a feeling that we are in correspondence with reality presently and that correspondence is preserved by adding new details. But if every refinement is epistemologically legitimate, so should be the reverse: A forgetting of details. Since how can we tell that this current correspondence is not the refinement of an earlier correspondence, and that earlier correspondence also has a prequel, and so on. At the limit of coarseness is a conceptual scheme with two concepts *everything* and *nothing*, and a single object, something like the Parmenidian sphere. A sequence of conceptual extensions connects this Parmenidean scheme with our language. How can we tell whether the path of refinements should have concluded with the panel as the sign or the panel and its pole? Or the panel, its pole, and some surrounding concrete? It cannot be coincidence that our current refinement *picks-out* exactly those objects which currently have the most utility.

As far as natural schemes corresponding with reality: The problem are the many subjective inputs which influence the position of meanings. It would be too convenient for the pursuit of our needs to guide up the path of ultimate reality so that subjective desire and absolute truth meet in the center.

The evidence is already here, and additional reports are rolling in from physical sciences, that human perception and more fundamental representations of reality are divorced. Paradoxes of the very small, the very fast, and the very energetic. It cannot be a coincidence that these paradoxes arise just beyond the horizon of events which impacted human evolution. Paradoxes like those from special relativity (traveling near the speed of light and then turning your headlights on) are not paradoxes respective to the math – they are the contradictions that follow bringing all your natural intuitions with you outside the band of information which they were designed to model. Human representation breaking down outside its own limits (a very Kantian notion).

10. The Logical Structure of Reality

The above arguments given, there remains a final way to salvage conceptual chauvinism: Reality has a logical structure to which a privileged conceptual scheme corresponds isomorphically. In that case, there might exist a plurality of representations where all but one are at some divergence with reality. Moreover, the pragmatic and subjective inputs into a scheme's structure are no issue, since these very forces (the drive for predictive power and explanatory elegance) press the observer deeper into a correspondence with reality. Davidson (1973) describes this as the *fitting* metaphor of scheme-content dualism, where the conceptual scheme conforms to sensory promptings after facing the tribunal of experience. If reality has an intrinsic logical structure, it is possible that a privileged scheme is fitted to it.

In the view of the Tractatus there is an isomorphism between the structure of reality and the structure of representation, and for this reason "we must not say $R(a, b)$ means a stands in relation R to b, but rather $R(a, b)$ therefore a relates to b." Reality has an intrinsic $R(a, b)$ to which the proposition stands in isomorphism.

In the same way that the cyclic group with $(p-1)$-elements (where p is a prime) is the group of $(p-1)$-roots of unity, or the group of multiplicative units of $\mathbb{Z}/p\mathbb{Z}$, each is the same structure under the guises of new symbols. There is then some $R(a,b)$ written into the real, perhaps not in those English letters or any names we know, but such that our labels and notations track the elements of the real in isomorphic correspondence. This is also called a *structural correspondence* which is distinguished from the weaker *correlative correspondence.* A *correlative correspondence* tells us that "snow is white" corresponds to the true state-of-affairs of snow being white, but not necessarily in terms of structural isomorphism. A *structural correspondence* tells us both that "snow is white" and that "snow" and "white" are natural kinds.

Because a scheme and its objects are mathematically dual, an intrinsic logical structure corresponds to an ontology of natural objects. Such would evade the "different substances of facticity" argument for conceptual relativism.

By positing a privileged correspondence with reality, the rival observer – the cultural other, the bat, the Martian – can be seen as misled by a false light of reason to the summit of an erroneous path, where they find their Mobius strips and Klein bottles and arrange these into logical pictures. But any arrangement of mistakes must be mistaken. Their pictures, even if logically formed, contain unnatural elements. Since it is possible to construct representations that function (that are *correlative*) but which miss completely the features of nature's true face.

Assuming reality has an intrinsic logical structure which can be "reached up to," then any two structurally correspondent schemes must be translatable. Since mathematical isomorphisms preserve structure up to a change of symbols, assuming an agreement with reality, Plutonian will translate (while preserving structure) into the symbols of Saturnian. This can be expressed in basic category theory:

$$\text{(Saturnian)} \to \text{(Reality)} \to \text{(Plutonian)}$$

The invertibility of the arrows (isomorphism) implies,

$$\text{(Saturnian)} \approx \text{(Plutonian)}$$

The task is to discover whether these morphisms are justified. It must be stressed that *correspondence* (verifying "snow is white" through an observation) is insufficient to prove a structural isomorphism. Moreover, it is seemingly impossible to determine from within the linguistic structure whether correspondence is structural or merely correlative. We showed this through thought experiments involving the Mughal court historian Abul Fazl. By employing the linguistic resources of his conceptual scheme, Abul Fazl can construct representations as logical as any other. A formula $\forall x\,(p\,(x) \to \exists y\,(q\,(x,y)))$ is as logically pure as it will ever become. Moreover, any of his chronicles, as a totality of facts, determines a world. By agreeing that the facts are "all that is the case" it is impossible for Abul Fazl to be more factually oriented or to accommodate facticity further. We found that his linguistic conventions are analytic and will be verified in all possible experiences (and so are as "objective" as any proposition could be). We also noted a category of a posteriori necessary truths. Discoverable scientific truths which hold necessarily of experience. Discoveries that are as much discoveries as any other. Supposing the formula $\forall x\,(p\,(x) \to \exists y\,(q\,(x,y)))$ is a posteriori necessary (a scientific discovery), one cannot tell, or see the angle, from which it might be dismissed as less scientific than our own cherished discoveries. By assumption, this *law* will be repeatedly corroborated and replicated, will never be falsified, and presents itself with perfect logical exactness.

Wittgenstein and Abul Fazl approach the same motor vehicle accident from "points of view from which individuals, cultures, or periods survey they passing scene." Each choosing a vantage from which to survey. In both representations "the world divides into facts." From within both languages, it cannot be determined which of these reports is more reality-oriented. None of the scientific merits (factuality, logical clarity, reproducibility, ...) preferences either representation.

A scientific merit (say reproducibility) does not require structural correspondence but arises through precision in language and conceptual construction. Both Abul Fazl and Wittgenstein, or Saturnian and Plutonian, might make predictions that are latter confirmed everywhere. The categories of analytic and a posteriori necessary truths are everywhere confirmed, but both of these categories are relative to a scheme and its arrangement of meanings.

For all schemes (used precisely) correlative correspondence is assured. A scheme's native propositions and the reality *represented through the scheme* as facts arrayed in logical space are *automatically* in correspondence. We cannot help but see the proposition mirrored *out there* within a conceptually processed experience.

Now we see the issue: A finely engineered conceptual scheme deployed without error will always confirm its own expectations and is at least *correlatively correspondent*; additionally, it is impossible to decide, from within the language, when a correspondence is correlative and when it is structural. Supposing the correspondence is denoted as a mathematical function $f : A \to B$, no signal is given when $a \in A$ corresponds correlatively or structurally. Consider how the sensation that force fields are structural and that goodness is structural overlap phenomenologically, and it is perhaps this overlapping confidence that motivates some philosophers to pursue the promise of a scientific ethics as if uncovering a field theory.

Of course, one could cite some darling of science that is verified everywhere and calculated down the thousandth decimal place; but as we have already seen, precision does not depend upon a *structural isomorphism* but upon a precise application of language. *Any* conceptual scheme could be verified everywhere and calculated to the thousandth decimal place.

Davidson's final attack against conceptual relativism relies on an over-extension of Tarski's semantic conception of truth (Beillard, 2008). Davidson claims that the meta-language, where object-language propositions are assigned truth-values, is a "language we know." That is not necessary. Indeed, we have already specified a theory of representational truth that satisfies Tarski's schema: p is true in $\mathcal{U}$ when $\mathcal{U} \models p$. There is some debate as to whether Tarski's conception of truth reduces to, or was a precursor to, truth in a model (Raatikainen, 2007). Regardless, the inductive building-up of model theoretic truth satisfies the T-schema ($\mathcal{U} \models p$ and $\mathcal{U} \models q$ implies $\mathcal{U} \models p \wedge q$ and so on) where the meta-language is set theory. In Model Theory, the fully theory of $\mathcal{U}$, the diagram of $\mathcal{U}$, the elementary diagram of $\mathcal{U}$, are all meta-linguistic constructions.

If we are willing to use set theory as our meta-language: Both Plutonian and Saturnian enumerate their respective facts $\mathrm{F}_{\mathscr{P}}(\mathcal{P})$ and $\mathrm{F}_{\mathscr{S}}(\mathcal{S})$. In a way, this is translation into "a language we know" – since we know set theory. Using set theory we can interrogate the diagram of an alien scheme. We could calculate the cardinality of their diagram. We could define a sigma-algebra onto their diagram and look at the Borel hierarchy. Or decompose it into group-orbits. Any number of mathematical tricks. But no amount of analysis will give the meanings of the alien language. We can only organize encrypted names of sentences in the meta-language. Though we see a proposition in their diagram, and recognize it as a signification, we will never know anything internal about it. Any conceptual scheme produces a correspondence amenable to a Tarskian theory of truth. Showing a structural isomorphism would require a type of evidence that is not rooted in correspondence (verification, repeatability, etc.) We cannot use the correspondence between "snow is white" and the snow being white as evidence that "snow" and "white" are structural.

Here is an example of what might qualify as structural evidence. Consider Euclidean geometry. Why might Euclidean geometry be structural? Euclidean geometry might be thought to result from a particular way of assigning distances between points through the Pythagorean theorem. Between two points (a_1, a_2, a_3) and (b_1, b_2, b_3) is assigned the distance $\left((a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2\right)^{1/2}$. Someone with enough mathematical imagination might ask: Why does 2 appear there? Why not another number? Why not 3? Or 1? Or why not a number so close to 2 that it should not make a difference? It turns out that any of those choices are valid ways of assigning distance. In fact substituting any $1 \leq p \leq \infty$ for 2 produces mathematically consistent distances. So we might ask: Why would the universe use two and not any of the infinitely many choices between one and infinity?

There are a couple of mathematically convincing reasons. For any metric $d(a, b)$ which assigns the distance between points a and b within a space S, a natural question of interest are distance preserving transformations (isometries) of the space. That is $f : S \to S$ such that $d(a, b) = d(f(a), f(b))$. It is easy to verify that if f and g are isometries so is g followed by f, or $f \circ g$. If f is an isometry, then its inverse f^{-1} (reversing the transformation) is an isometry. Finally, the identity map $\mathrm{id}_S(x) = x$ is clearly an isometry. Thus the set of all isometries form a mathematical group. But, with respect to the above, it is not clear that any choice of p would have the same symmetry group. The first piece of mathematical evidence that the universe ought to use 2 is that the group of isometries respective to that choice is quite a bit more lovely and important than the rest (for example, in its relation to the special orthogonal group).

Second to my mind is that this choice of $p = 2$ lines up distances with the nicest way of measuring volumes of space. Generally for a measure μ on a space S and a measure λ on a space T there is product measure $\mu \times \lambda$ on the cartesian product $S \times T$ defined by:

$$(\mu \times \lambda)(a \times b) = \mu(a)\lambda(b)$$

As there would be a product measure on $S \times T \times R$ and so on (this is an abstraction of the grade-school mantra "length times width times height"). Once more, there exist infinitely many ways of assigning measurements to (measurable) subsets of three-dimensional space. But only *one* (the nicest) starts by assigning as lengths of segments $l(a, b) = |a - b|$, and then builds up through "length times width..." to measuring squares and cubes, and then to more complex shapes which have something like a decomposition into little cubes, and so on. But we should like all of this to play nicely with distances within the space. If I apply a rotation to a cube, without changing its dimensions, the rotated cube ought to have the same volume, but now we have new edges to and potentially new distances. The nicest volumes (built up from absolute values between points) are preserved under rigid transformations at the choice of $p = 2$.

Something else rather special occurs at the choice of $p = 2$. At $p = 2$ alone, and not any other choice, not even for a choice of p so close to 2 as to differ on the trillionth decimal place, the space becomes a Hilbert space with an inner-product, which might be considered intuitively as a way of calculating angles between points. All choices of p have produce nice spaces in terms of distance, but only at the choice of $p = 2$ is there a way to think about angles.

This is what I picture as *structural evidence*. Evidence as to why the concepts *should be*. Such mathematical evidence does not depend on verification, consistency, predictiveness – as these are possible for any scheme – but is evidence that the *structure itself* is exceptional. Still, this feeling of exceptionalism resolves into purely subjective appraisals of beauty, elementariness, "conceptual economy," etc. Even pre-Einstein, the above would not constitute a *proof* of structural isomorphism. At most it is a clue that Euclidean geometry (and generalizations therefrom) could be more than just representation.

Pursuing this idea: There could be structural isomorphism between the most fundamental physical constituents of the Universe and our physical models of them. At the level of gravity, photons, fields etc. – these constituents might qualify as natural kinds. The reason being that Quantum mechanics has more than just correspondence to support it, since it involves rich mathematical structures. There is some cause to believe that the structures themselves are uniquely exceptional. Though the same retorts against the structurality of Euclidean geometry apply equally here. The impression of exceptionalism resolves into subjective feelings of beauty, elegance, etc. Additionally, the same constituent can have a plurality of mathematical perspectives. Is the most basic building block a particle or a wave? Perhaps the building block is a particle through the lens of mathematical tools that particularize, and a wave likewise. Further, it seems to me that both particles and waves are metaphors borrowed from ordinary human experience. The mathematical particle began as the phenomenological "tiny object" where it was isolated and pursued formally. There was a lead in everyday experience. The same can be said of waves. The little billiard ball applied to a sub-atomic particle is only metaphor. Or a "it must be like" prompted by experience and developed into a formal edifice. Are such developments not possible elsewhere? Would the sentient bat discover a "it must be like" within their phenomenology? Or the martian? Do they develop metaphors into edifices?

Secondly, while there might be a structural isomorphism at the most fundamental building blocks, the correspondence would be structural *only* at that level. Since we do not "survey the passing scene" in particle physics, any structural correspondence of particles cannot be lifted onto the outlook of our daily experiences. All levels above that base – ethics, aesthetics, politics, religion, etc. – then become constructed. It could prove that ultimate reality is like a giant encyclopedia of atomic events, where every item within is like a code for an elementary particle interaction – and all subjects that come to ultimate understanding agree on this text. Still, subjective utility and pragmatic considerations will decide how this text becomes *interpreted* under natural representation. A structural isomorphism, if existent, cannot be *extended* or *lifted* to that natural representation.

Of course fundamental constituents would determine the facts of a natural representation. Constructed facts such as "if someone has undermined the foundation of his house, we say that he could have known a priori that the house would cave in." (Kant, 1996, p. 45), are determined by all near-to particles. If all the atomic events were to recur, they would reproduce the same macroscopic objects and the representation would reappear. Both are connected. In earlier drafts I conceived of fundamental physical models as *substructural* to higher representations. So that the linguistically equipped subject might fill a blackboard with mathematical equations describing a quantum field, and on another blackboard write down a news headline, and the later would stand *over* the former. I came to recognize this as an error. There is a *connection* which will be explained shortly, but it is not one of sub-structure and super-structure.

Perhaps there are parallels to the Buddhist tradition. Certain schools admit that dharmas (elementary facts, particle events, irreducible phenomenological experience, etc.) have ultimate reality while facts involving aggregates (like a chariot or the self) are constructed. While a thinker like Nagarjuna rejects even dharmas as ultimately real and says all is construction (Nagarjuna's Middle Way, Siderits pp 13-17). In our case, that "particle at (x, y, z, t)" is ultimately real (structurally correspondent) is not enough to lift that *ultimate reality* onto natural representations involving chariots or selves or traffic signs.

We must ask: Where is the mathematical support for Wittgenstein's representation of the motor vehicle accident? When so much mathematical distinction is not proof that "line" and "plane" are structural – how much less justification is there for "steering wheel" and "traffic light"? If there were a structural correspondence at the level of fundamental constituents – at the level of the dharmas so to speak – that would not excuse appropriating the bearing of "ultimate reality" onto natural representations. Appropriating confidences and certainties from the mathematical world. Where we *pretend* that our everyday concepts came about as the solutions to a system of differential equations. I imagine a stereotypical subscriber to what is pejoratively called "scientism." The type who pictures themselves as stridently accepting of reality with a mind pruned and trimmed of all fantasy. Of possessing a psychic-hotline to ultimate empirical reality. We can overhear this type proclaiming "Particle-physics advances towards the final structure of reality through reason, I cheer-on these advances, further I tell myself I'm rational constantly and wear lab-coats in my self-image, therefore my outlooks are advancing – or have advanced – to the final structure of reality." Clearly an over-extension. Appropriating a certain flattering feeling that comes from viewing the Universe all settled and analyzed into mathematical equations for a personal worldview.

Natural schemes have tell-tale signs of construction. Every element of natural language admits of a repositioning, an expansion, a retraction, a bifurcation, etc. This is plain by phenomenological inspection. Whenever I ask, "What if there were no distinction made between light-bulb and candle-flame?" Or "What would happen if I invented a hundred categories of door-handle?" The answer is always that my efforts would generate another list of facts, neatly arranged and verifiable, logically perfect and no less factual. Much like the boarders of nations are shaped by chance historical events – so for the concept, chance events in the history of ideas have pushed and pulled them into their current shape. I can imagine an alternative unfolding of collective thought, where the constellation of concepts have settled into new positions, while maintaining the integrity of the subject to the object. The in-itself is as before – much as the Earth would be the Earth had the Spanish reconquest failed to take Granada.

Saussure in his course on linguistics (2011) examines the evolution of languages as transformations of a system of values (related to *meanings* as I use it but not identical) diachronically (that is, across time). He shows that meanings (values) are never fixed in their positions. The constellation of ideas is forever re-equilibrating disturbances in the placements of its meanings "It is as if one of the planets that revolve around the sun changed its dimensions and weight: This isolated event would entail general consequences and throw the whole system out of equilibrium" (ibid, pp. 84-85). A further analogy is made between the state state of a language and the state of a chess board where the value of a term corresponds to a position on the board. The moving of a single piece does not induce a general corruption of the system, yet has repercussions on the whole, and it is impossible for the player to predict the end of those consequences. In a game of chess, the present state of the board is independent of the previous state in the sense that an alternative sequence of moves might have produced that same conclusion and one could come across the game in its present state and know it as well as those playing from the beginning. With a variety of natural languages as case studies, Saussure carefully documents the diachronic moves of specific values, using abundant empirical evidence to demonstrate they do indeed move across the board. What is important from his research is that it answers the problem of an evolving conceptual scheme by demonstrating these changes have already happened: We are here after the change. It is just an empirical fact, shown in historical data, that languages evolve. Considering a fixed conceptual scheme (that is, considered synchronically) as a state of the chess board, we have already shown that each such state has a correlative ontology: What are the chances that one of those states (and one of those ontologies) is the correct one? Accepting that a scheme is capable of organizing the contents of experience across evolutionary time, and at any syncronic moment the integrity of the subject-object relation is maintained (regardless of whether the knight is on B4 or D5), we are in post-(Copernican) revolutionary state (at least for natural schemes).

Take the concept of *justice*. The concept has certainly held taken up a variety positions in the plane of thought. We observe a gulf between the preliminary definition forwarded in the Republic ("helping your friends and harming your enemies") and the definition offered in the Sermon on the Mount ("love your enemies"). Different positions on the board. Independent of these positions the in-itself stays itself (what else could it be?).

There are two spheres of linguistic activity:

(1) Formal representations of fundamental constituents. Of particles, forces, fields, and so on (and possibly higher scientific subjects).
(2) Natural representations. The linguistic activity of daily social life under ordinary conditions. Of sidewalks, traffic signals, justice, beauty, and so on.

A structural isomorphism of at the level of fundamental constituents need not lift to a structural isomorphism between daily linguistic activity and reality. Correlation is not evidence of structural correspondence. Facticity, verifiability, repeatability, etc. are not evidence of structural correspondence. A type of meta-linguistic or meta-representational evidence – the nature of which I cannot exactly state – is required for the suggestion (though not proof) of structural correspondence. It can be suggested that Quantum theory is structural; on the other hand, there is a total absence of structural evidence for the traffic signal.

Two cases: Formal representations are correlative or structurally correspondent. If formal representations are correlative, then natural representations are certainly correlative. If formal representations are structurally isomorphic, then, regardless, natural representations are still correlative.

11. The In-Itself

A problem was introduced: How do formal representations implicate natural representations? The relation should be one of *determination*: Since were all the particle-states to *re*-state themselves, the recurrence would induce the previously seen macroscopic objects and link them together in the same chain. Yet the determination is not reciprocal, since the natural representation is not sufficient to reconstruct the formal representation.

Let us evoke Laplace's demon: Knowing the positions and momentums of all particles is to know, with sufficient computing power, the positions and momentums of those same particles at any future time. Let $\mathscr{P}$ denote this Laplacian scheme, so that a particle world $\mathcal{P}_t$ at time t models its conventions. Further, let $\mathscr{U}$ denote a natural scheme so that $\mathcal{U}_t$ is the world at time t modeling its conventions. The facts $\mathrm{F}_{\mathscr{P}}(\mathcal{P}_t)$ (the factual positions and momentums of of the objects – ie, the particles) determines $\mathrm{F}_{\mathscr{U}}(\mathcal{U}_t)$. Which is so whether $\mathcal{U}_t$ was constructed under the conceptualizations of Mughal India, or early 20th century Vienna, or by any natural language. But we also know that the granularity of a conceptual scheme has a subjective arbitraity: The particle-world continues to determine natural representations even the natural scheme looses all its resolution and begins to represent the world in the most pixelated way. Clearly a reciprocity of determination is lacking. As resolution decreases, and information is lost, it becomes increasingly impossible to reconstruct particle-facts through natural-facts.

Particles *determine* the facts of natural representation (but not the converse). Further, time-evolving the mechanical system will determine a time-evolution of natural representations. Laplace's demon makes this intuitive: $\mathcal{P}_t$ determines $\mathcal{U}_t$, so that evolving $\mathcal{P}_t$ to $\mathcal{P}_\tau$ determines the evolution of $\mathcal{U}_t$ into $\mathcal{U}_\tau$.

Write an arrow $e : \mathcal{P}_t \to \mathcal{P}_\tau$ when $t \leq \tau$ and $\mathcal{P}_t$ time evolves into $\mathcal{P}_\tau$. This forms a category (a categorical representation of causality). What has been discovered is that morphisms $\mathcal{P}_t \to \mathcal{P}_\tau$ determine morphisms $\mathcal{U}_t \to \mathcal{U}_\tau$. The determination $R : \mathcal{P}_t \to \mathcal{U}_t$ gives a *functor* from the Laplacian-scheme to the scheme of a natural language.

$$\begin{array}{ccc} \mathcal{P}_t & \xrightarrow{e} & \mathcal{P}_\tau \\ \downarrow & & \downarrow \\ \mathcal{U}_t & \xrightarrow[R(e)]{} & \mathcal{U}_\tau \end{array}$$

A yet more fundamental structure may lie beneath the Laplacian-scheme which determines the sequence of Laplacian representations. It may be that particles are structural, or it could be that particles are constructed abbreviations which round-off the still more fundamental. In the later case: There is a functor from the fundamental representation $\mathcal{F}_t$ to particles $\mathcal{P}_t$ and from particles to natural representations, thus a composition of functors:

$$\begin{array}{ccc} \mathcal{F}_t & \xrightarrow{e} & \mathcal{F}_\tau \\ \downarrow & & \downarrow \\ \mathcal{P}_t & \xrightarrow{R_1(e)} & \mathcal{P}_\tau \\ \downarrow & & \downarrow \\ \mathcal{U}_t & \xrightarrow[R_1 \circ R_2(e)]{} & \mathcal{U}_\tau \end{array}$$

What brings this together is causality. The use of "causality" here may be contentious; since, at least in the Kantian approach to conceptual schemes (following Hume), causality is *representational.* Above, causality is meta-representational (it orders representations and brings $\mathcal{P}_t$ into correspondence with $\mathcal{U}_t$), and this is a large metaphysical ask depending on the choice of philosophical authority. If nothing else, causes and effects need not *inhere* in events, causality might only be "this follows that" (the Humean idea of causality); or more primitively still, just a category of formal arrows whose greater physicality is unknown.

The critical thing is this: A causal functor $R : \mathcal{P} \to \mathcal{U}$ need not preserve the structure of $\mathcal{P}$. Such is clear when $\mathscr{P}$ is the Laplacian-scheme and $\mathscr{U}$ are natural representations (as perceiving a traffic accident). The particle-facts determine the facts of the accident, but the structure of particles is not to be found in the higher representation. A pointed analogy for any reader with an understanding of mathematics: The fundamental group *measures* or *records* something about a topological space at a point, but is not itself spatial.

The notion of the in-itself is notoriously problematic, since it is posits itself as inscrutable while simultaneously asserting its own existence and connection to phenomena. It is both here and not here. Much of German philosophy following Kant is a struggle to reconcile that tension. We might attempt a definition as: The content organized by the conceptual scheme and that which appears through the model theoretic structure. This does not make the in-itself unstateable. A noumenal-state is I so that a causal functor sends I to a representation $R(I)$. A noumenal-state can be stated, but its *structure* is not found in the representation $R(I)$. It is measured by representation, but not isomorphically. Such does not evoke a contradiction of existing while non-existing. The in-itself exists and is para-linguistic.

Were Wittgenstein correct that propositions *display* the logical structure of reality, then any two conceptual schemes which agree with reality "reach up" to that structure and are mutually translatable:

$$R_1(I) \approx I \approx R_2(I)$$

When correspondence is only correlative, it is possible for the noumenal-state to appear as $R_1(I)$ and $R_2(I)$, but where $R_1(I) \not\approx R_2(I)$. Still, the noumenal-state is a coherent *something*, because observations conducted through a shared conceptual scheme always *agree* on matters of fact. "Snow is white," for all observers who look out their window with a shared conceptual apparatus. The in-itself resolves as facts in logical space identically for all such observers. Indeed, were the Universe classical, classical particles might qualify as the in-itself, in which case the Laplacian scheme successfully accesses the in-itself, and the functor $R : \mathcal{P} \to \mathcal{U}$ is an example of a noumenal-state $\mathcal{P}_t$ appearing as $R(\mathcal{P}_t) = \mathcal{U}_t$. Similarly, our current state-of-the-art quantum models might access the in-itself; then again, present results may only be the in-itself *appearing through* state-of-the-art structures; and, moreover, there is no way to detect whether the correspondence is structural or correlative. This does not regulate representational truths to uncertainty or imprecise subjectivity. Phenomena can be as Newtonian clockwork machined down to the thousandth decimal place. This is because a time-evolution between noumenal-states,

$$e : I_1 \to I_2$$

implies,

$$R_1(e) : R_1(I_1) \to R_1(I_2)$$

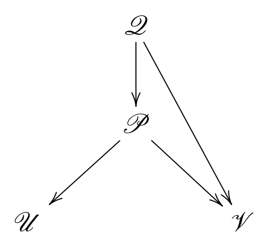
so that the prevailing of R_1-facts is as nonnegotiable as the sternest word of any God. Recalling Kant's example, "A house with an undermined foundation will fall," it is pulled down by $R_1(e) : R_1(I_1) \to R_1(I_2)$ with an arrow like an iron hand. Yet an alien observer might represent the same time-evolution as $R_2(e) : R_2(I_1) \to R_2(I_2)$, with their own iron-handed arrows, none of which recognize *houses*. All the clock-like regularity of the representational world is thus possible without the in-itself possessing the parts of a clock. The logos which feels so present at every joint in the relay of causality and whose fingerprints are so apparent throughout the order of the cosmos simply need not be there. A noumenal-state is the *it* which resolves as facts in logical space. A central something which portrays itself across representations. A subject-independent judicial body passing verdicts over representational dilemmas. "A house with an undermined foundation will fall," it is this sovereignty which rules so. The two appearances – the house with an undermined foundation, and the collapsed house – are deterministically bridged by the passage of noumenal-states. The "point of view," the angle of observation, which decomposes the noumenal-state state to include the undermined house among its facts, will afterwards – looked at again with the same angle – include the collapsed house. With unerring regularity. Yet "house" and "foundation of a house" are certainly not natural kinds. There is some inner turning of the in-itself that necessitates a representational series; but internally, the in-itself does not know the parts we make, nor which lies up and which down.

The linguistic version of conceptual schemes has the advantage of including representations that transcend the psychic. Kant ascribes Newtonian Mechanics to the constructive activity of the mind which feels intuitively absurd. Given the elegant dance of celestial bodies in their trillions, how could my mind make such sophisticated and encompassing clockwork? A linguistic scheme can be a mathematical model which has extended the reach of observation with scientific instruments. So it is possible that Newtonian mechanics is not structural, and the in-itself, *represented through* the mathematical structure, takes the appearance of inconceivably complex clockwork. These linguistic representations are also compatible with precise and immutable scientific law. Constructions have the potential to work the same each time and to travel scientifically delineated paths eternally. There is no tension with this and the common sense notion of *objectivity.*

For a conceptual scheme $\mathscr{U} = (\mathcal{L}, \mathcal{A}, \mathbb{U})$ define a *mode of representation* to be a category (also denoted $\mathscr{U}$) whose objects are possible worlds and morphisms are arrows $\mathcal{U}_1 \to \mathcal{U}_2$ when $\mathcal{U}_1$ causally evolves into $\mathcal{U}_2$. Above these were parameterized by a real variable representing time, but it may be efficacious to think of these in the unparameterized abstract. A *causal functor* (also determination) is a functor between modes of representation $\mathscr{U}$ and $\mathscr{V}$. Meaning that a $\mathscr{U}$-evolution $e : \mathcal{U}_1 \to \mathcal{U}_2$ is sent to a $\mathscr{V}$-evolution $R(e) : R(\mathcal{U}_1) \to R(\mathcal{U}_2)$. Intuitively these morphisms would be unique. There is at most one causal evolution $e : \mathcal{U}_1 \to \mathcal{U}_2$ between worlds $\mathcal{U}_1$ and $\mathcal{U}_2$. Likewise, if $\mathscr{U}$ determines $\mathscr{V}$ there is at most one determination $R : \mathscr{U} \to \mathscr{V}$ (there is only one way for a particle world to determine a natural representation). However, since math bellow does not require the assumption of uniqueness, the constructions will be left at their more general.

Two modes of representation $\mathcal{U}$ and $\mathcal{V}$ are isomorphic when there exists an invertible casual functor $R : \mathscr{U} \to \mathscr{V}$, in which case they are co-determinant. I am unsure whether co-determinancy implies translatability (through a model-theoretic isomorphism) or whether the modes of representation may be untranslatable but of equal predictive power. Form a category of representations Γ (a category of categories) whose objects are modes of representations and morphisms are causal functors.

We now have a mathematical order, something like a tree, of representational modes. At one position is the Laplacian-scheme $\mathscr{P}$, and beneath it, two natural representations connected by functors: $R_1 : \mathscr{P} \to \mathscr{U}$ and $R_2 : \mathscr{P} \to \mathscr{V}$. A particle-world $\mathcal{P}$ comes to be represented as $R_1(\mathcal{P}) = \mathcal{U}$ and $R_2(\mathcal{P}) = \mathcal{V}$, but where $\mathcal{U}$ is not translatable to $\mathcal{V}$ nor are both co-determinant. But then *above* $\mathscr{P}$ is a still more fundamental $\mathscr{Q}$ and a causal functor $R_0 : \mathscr{Q} \to \mathscr{P}$. So that a $\mathscr{Q}$-representation $\mathcal{Q} \in \mathscr{Q}$ is constructed as $(R_1 \circ R_0)(\mathcal{Q}) = \mathcal{U}$ and $(R_2 \circ R_0)(\mathcal{Q}) = \mathcal{V}$. Yet also, observers who deploy the natural representation $\mathcal{V}$ may be ignorant of particle physics, where, from their position, it is more proper to claim they construct $\mathscr{V}$ directly from $\mathscr{Q}$. The entire state of things looks something like:



Perhaps a physical model, a promised theory of everything, will someday "reach up" and know reality as it knows itself. This representation will capture the Universe at its most fundamental and will be called $\mathscr{F}$. Then for for any other mode of representation $\mathscr{U}$ there will exists a casual functor $\mathscr{F} \to \mathscr{U}$ and the theory of everything will have the top position of the above tree. Assuming that causal functors are unique, $\mathscr{F}$ satisfies a universal property in Γ as an initial object. If such an $\mathscr{F}$ does not exist, then the in-itself is truly unknowable (structurally) and this is the much more interesting case. Since while the in-itself (denoted $\mathscr{I}$) is structurally unknowable, its *action* on representations (the appearance generated by $\mathscr{I}$) is knowable, and this can be described using Category Theory.

Let us review Yoneda's lemma and the Yoneda embedding. Start with $\mathcal{C}$ a locally small category, and to any object $A \in \mathcal{C}$ associate the covariant functor into the category of sets defined by,

$$h_A = \mathrm{Hom}\,(A, -)$$

Given a $\mathcal{C}$-morphism $f : X \to Y$, we obtain $h_A\,(f) : \mathrm{Hom}\,(A, X) \to \mathrm{Hom}\,(A, Y)$ by sending $g \in \mathrm{Hom}\,(A, X)$ to $f \circ g \in \mathrm{Hom}\,(A, Y)$. Yoneda's lemma states that for any covariant functor F from $\mathcal{C}$ to (Set), that the natural transformations from h_A to F are in one-to-one correspondence with the elements of $F\,(A)$. Recalling that bijections are isomorphisms in the category of sets,

$$\mathrm{Nat}\,(h_A, F) \approx F\,(A)$$

Supposing that F is representable, there exists an object $B \in \mathcal{C}$ so that $F \approx h_B$, and the lemma takes the form:

$$\mathrm{Nat}\,(h_A, h_B) \approx \mathrm{Hom}\,(A, B)$$

Let $[\mathcal{C}, (\mathrm{Set})]$ denote the category of functors from $\mathcal{C}$ to (Set). Associate to a morphism $f : X \to Y$ the natural transformation $f^* : h_Y \to h_X$, so that given $g : A \to B$:

$$\begin{array}{ccc} \mathrm{Hom}(Y, A) & \xrightarrow{h_Y(g)} & \mathrm{Hom}(Y, B) \\ \downarrow & & \downarrow \\ \mathrm{Hom}(X, A) & \xrightarrow{h_X(g)} & \mathrm{Hom}(X, B) \end{array}$$

where the components are defined by pre-composition along f.

Defining $Y : \mathcal{C} \to [\mathcal{C}, (\mathrm{Set})]$ via $Y\,(A) = h_A$ and $Y\,(f) = f^*$ gives a contravariant functor which is fully faithful by the Yoneda lemma.

For example, in the category of topological spaces, $h_{S^1}\,(X) = \mathrm{Hom}\,\left(S^1, X\right)$ is all loops in X, and S^1 can be identified as corresponding to the functor which sends topological spaces to spaces of loops. While the topological space p consisting of a single point can be identified with the functor $h_p\,(X) = \mathrm{Hom}\,(p, X)$ which gives all points in X. Such is possible for any space. This allows for a generalization where a functor $F : (\mathrm{Top}) \to (\mathrm{Set})$ sends topological spaces to "shapes of type F" but where that type might not be representable. For a more thorough discussion on the Yoneda Lemma and the Yoneda Embedding refer to Mac Lane (2010).

In our case, beginning with the category Γ of representations, any mode of representation $\mathscr{U}$ can be thought of in terms of its action on other modes of representation $\mathscr{V} \to h_{\mathscr{U}}\,(\mathscr{V})$. For example, the Laplacian-scheme can be thought of as (partly) determined by its way of appearing through natural representations.

Embedding Γ into $[\Gamma, (\text{Set})]$ conserves the order of determination contra-variantly (ie, the Laplacian-mode still determines a natural-mode), but there can exist objects in $[\Gamma, (\text{Set})]$ *which are not representable* by objects in Γ. In other words, there can exist an object $\mathscr{I} \in [\Gamma, (\text{Set})]$ which causally determines while not being fully captured by any mode of representation (ie, there does not exist a mode-of-representation $\mathscr{F} \in \Gamma$ so that $\mathscr{I} \approx h_{\mathscr{F}}$).

The behavior of $\mathscr{I}$ should act like $\mathscr{V} \to \text{Hom}\,(\mathscr{I}, \mathscr{V})$ but where $\mathscr{I}$ is not actually in Γ. We know how this should look by observing the action of a given representation. A mode of representation is a category consisting of causal evolutions $e : \mathcal{U}_1 \to \mathcal{U}_2$, and this representation acts on another representation $\mathscr{V}$ via determinations $R\,(e) : R\,(\mathcal{U}_1) \to R\,(\mathcal{U}_2)$ which collectively give $\mathscr{V} \to \text{Hom}\,(\mathscr{U}, \mathscr{V})$, and this is an element of $[\Gamma, (\text{Set})]$. The in-itself $\mathscr{I}$ is a category consisting of noumenal-states, where an arrow $e : \mathcal{I}_1 \to \mathcal{I}_2$ is an evolution of the noumenal-state $\mathcal{I}_1$ into $\mathcal{I}_2$. But we know that the in-itself exhibits through representations, as we have immediate experience with appearances. Given a mode of representation $\mathscr{U}$ and an evolution $e : \mathcal{I}_1 \to \mathcal{I}_2$ there must occur a $\mathscr{U}$-evolution $R\,(e) : R\,(\mathcal{I}_1) \to R\,(\mathcal{I}_2)$. Therefore define $\text{Hom}\,(\mathscr{I}, \mathscr{U})$ as such functors between $\mathscr{I}$ and $\mathscr{U}$. With this it is possible to send $\mathscr{U} \to \text{Hom}\,(\mathscr{I}, \mathscr{U})$ and this is a functor from Γ to (Set), since given a determination $H : \mathscr{U} \to \mathscr{V}$ we obtain:

$$I_H : \text{Hom}\,(\mathscr{I}, \mathscr{U}) \to \text{Hom}\,(\mathscr{I}, \mathscr{V})$$

via $g \in \text{Hom}\,(\mathscr{I}, \mathscr{U})$ being sent to $H \circ g$. Which works because an evolution of noumenal-states $e : \mathcal{I}_1 \to \mathcal{I}_2$ commutes through representations:

$$\begin{array}{ccc}
\mathcal{I}_1 & \xrightarrow{e} & \mathcal{I}_2 \\
\downarrow & & \downarrow \\
\mathcal{U}_1 & \xrightarrow{g(e)} & \mathcal{U}_2 \\
\downarrow & & \downarrow \\
\mathcal{V}_1 & \xrightarrow{(H\circ g)(e)} & \mathcal{V}_2
\end{array}$$

Next consider Γ^* as the sub-category of $[\Gamma, (\text{Set})]$ generated by $Y\,(\Gamma)$ and $\mathscr{I}$ (which amounts to adding the in-itself as a limit to Γ). In natural science (and physical description more generally) there is a staircase of fundamentality: Particles are more fundamental than natural representations, but sub-particles are more fundamental than particles, and so on. Either this quest terminates with a theory of everything in which case Γ possesses this limit, or the in-itself is truly inaccessible, in which case noumena occupies the position of a limit in Γ^*. If morphisms from $\mathscr{I}$ are unique, then $\mathscr{I}$ is a terminal object (since the embedding was contra-variant) in Γ^*.

Phenomenologists in the tradition of Husserl practice an elimination of things-in-themselves by unraveling the object into its series of appearances. In the opening of *Being and Nothingness* Sartre describes this position nicely:

> By reducing the existent to the series of appearances that manifest it, modern thought has made considerable progress. The aim was to eliminate a number of troublesome dualisms from philosophy and to replace them with the monism of the phenomenon. Has it succeeded?

> In the first place, we have certainly got rid of the dualism that opposes the existent's inside to its outside. The existent no longer has an "outside," if by that we mean some skin at its surface that conceals the object's true nature from view. And if this "true nature" is, in turn, to be the thing's secret reality—something that we can anticipate or assume but that we can never reach, because it is "inside" the object in question—that does not exist either. The appearances that manifest the existent are neither internal nor external: they are all of equal worth, each of them refers to other appearances, and none of them has priority. Force, for example, is not a metaphysical *conatus* of some unknown kind, concealed behind its effects (accelerations, deviations, etc.); it is the sum of these effects. Similarly, an electric current has no secret other side: it is nothing but the collection of physiochemical actions (electrolytic processes, the incandescence of a carbon filament, the movement of the galvanometer's needle, etc.) that manifest it. None of these actions is sufficient to reveal it. But it does not point to anything *behind* it; each action points to itself and to the total series. (Sartre, 1943, pp. 1-2)

The above Yoneda perspective confirms that the object is not concealed by a skin. The secret reality of the thing *is* simply all its external sides as appearances (a nuemenal state was equated to an action across modes of representation). No inside remains after the object has been observed from all sides. Yet, every observation leaves a "secret other side." The representation *measure*s the nuemenal-state (as the fundemental group measures a topological space). The neumenal-state is objectified through representation, and how the objects combine into facts discloses a side of the in-itself. A measurement, a certain temperature, is recorded using the tools of the language. Yet there must be something unmeasured or a remaining interior. After all, Sartre's total series of appearances – after committing the phenomenological picture to proposition – is just the mathematical definition of the object's type as a order-theoretic homomorphism $B(\mathcal{A}) \to \{T, F\}$. But schemes and objects are correlative; therefore, absent a structural isomorphism (which we assume not to exist here), the total series points to a construction which is not the inside of anything. The in-itself is all its exterior presentations – yet some of these are known only to Negel's Martian or Bat. A "total series" underdetermines when serialized by a single conceptual scheme.

Suppose one hopes to know a traffic-light through an exhaustive series of observations and writes a series for as long as language allows. But this leaves a remainder. Another conceptual scheme must be adopted and the observer must look again. But objects and schemes are correlative – new lens is put-on and the traffic-light *vanishes*. So what was there from the start?

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