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The Train Paradox

Jeremy Gwiazda

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Abstract When two omnipotent beings are randomly and sequentially selecting positive integers, the being who selects second is almost certain to select a larger number. I then use the relativity of simultaneity to create a paradox by having omnipotent beings select positive integers in different orders for different observers.

Keywords paradox · relativity of simultaneity

Imagine that two omnipotent beings, Sam and Max, decide to hold a competition to see who can choose the largest number, each randomly selecting a positive integer (i.e., when a selection is made, each positive integer has an equal chance of being selected). Sam has won a coin toss, and is thereby able to decide whether to select first or second. Sam should randomly select his number second. Max, selecting first, will select some number, M. What is the probability that Sam will then select a number larger than M? The probability is *almost certain*,[1] as there are infinitely many numbers greater than M, but only finitely many numbers less than or equal to M. When two omnipotent beings are randomly and sequentially selecting positive integers, the being who selects second is at a remarkable advantage.

Next imagine that an observer, Beth, encounters Sam and Max. Max selects integer M at 10:00, but does not immediately show M to Beth. Sam selects S at 10:01, and immediately shows S to Beth. Max shows M, selected at 10:00, to Beth at 10:02. Beth is almost certain to find that S is greater than M, as S was selected after M. I believe that this reasoning is correct, that is, that the order that the numbers were *selected* in is the crucial consideration,

[1]Certainty minus an infinitesimal.

J. Gwiazda (✉)
Department of Philosophy, Graduate Center, City University of New York, 365 Fifth Avenue,
New York, NY 10016, USA
e-mail: jgwiazda@gc.cuny.edu

and not the order that the numbers were *observed* in. Others have argued that the order of observation is the crucial consideration.[2] In what follows, I use the relativity of simultaneity to create a paradox by having omnipotent beings (Sam and Max) select positive integers in different orders for different observers. I shall construct the example so that the order of selection[3] matches the order of observation, i.e., the order two numbers are selected in is the order they are observed in. Thus the paradox applies also to those who hold that the order of observation is the crucial consideration. (Variants of the paradox can be devised by inverting the order of selection and observation; however, I hold that the order of selection is the crucial consideration. To reiterate, the number selected at a later time is almost certain to be the larger of two numbers randomly selected from the positive integers at different times.)

I make use of and assume that the reader has some familiarity with Einstein's thought experiment demonstrating the relativity of simultaneity, which involves lightning strikes at each end of a train.[4] I replace each lightning strike with an omnipotent being and a Light Emitting Diode (LED) display, labeled Sam and Max below. There are three observers: Art, Beth, and Cal. On the train are Sam, Beth, and Max. Art is moving to the right relative to the train. Cal is moving to the left relative to the train. Each omnipotent being *creates* a number, i.e., each omnipotent being randomly selects a positive integer and displays it on the LED display. When the omnipotent beings create the numbers (in Beth's frame of reference), the three observers are equidistant from the omnipotent beings.

In Beth's frame of reference, the omnipotent beings create the numbers at the same time (and Beth observes the numbers at the same time).[5] Beth concludes that Sam will create a larger number than Max about half of the time and vice versa, because each omnipotent being is performing the same action at the same time.

In Art's frame of reference, Max creates a number before Sam (and Art observes Max's number before observing Sam's number). Art reasons that Sam, who creates after Max, is almost certain to create a larger number than Max.[6]

In Cal's frame of reference, Sam creates a number before Max (and Cal observes Sam's number before observing Max's number). Cal reasons that Max, who creates after Sam, is almost certain to create a larger number than Sam.

Which observer is correct? That is, imagine running the experiment many times. Will each omnipotent being create a larger number than the other about half of the time, as Beth expects; or will Sam almost always create a larger number than Max, as Art expects; or will Max almost always create a larger number than Sam, as Cal expects? Each observer would appear to possess sound reasoning, though (at most) only one can be correct.[7]

[2] In informal conversations.

[3] Selection will be replaced by *creation*, to be defined as selection and display.

[4] Albert Einstein, *Relativity: The Special and the General Theory – A Popular Exposition* (New York: Crown Publishers, 1961). In particular, see chapter 9, though my set-up differs slightly from Einstein's.

[5] As discussed above, the order of creation (selection and display) matches the order of observation.

[6] As argued above.

[7] I thank Mr. Cormier and Professor Levin for helpful, engaging discussion.