A Phenomenology of Race in Frege's Logic

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Abstract:

This article derives from a project attempting to show that Western formal logic, from Aristotle onward, has both been partially constituted by, and partially constitutive of, what has become known as racism. In the present article, I first discuss, in light of Frege's honorary role as founder of the philosophy of mathematics, Reuben Hersh's *What is Mathematics, Really?* Second, I explore how the infamous section of Frege's 1924 diary (specifically the entries from March 10 to April 9) supports Hersh's claim regarding the link between political conservatism and the (historically and currently) dominant school of the philosophy of mathematics, to which Frege undeniably belongs. Third, I examine Frege's attempt at a more reader-friendly introduction to his philosophy of mathematics, The Foundations of Arithmetic. And finally, I briefly analyze Frege's *Begriffsschrift* to see how questions of race arise even at the heights of his logical abstraction.

Keywords: Frege, logic, racism; Nazism; mathematics

> The insanity of such systems lies not only in their first premise but in the very logicality with which they are constructed. The curious logicality of all isms, their simple-minded trust in the salvation value of stubborn devotion without regard for specific, varying factors, already harbors the first germs of totalitarian contempt for reality and factuality.
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> Hannah Arendt, *Origins of Totalitarianism*

What is Logic, Really?

In his preface to *What is Mathematics, Really?*, Hersh (1999) calls his approach to his subject "humanist," and argues that "*from the viewpoint of philosophy*, mathematics must be understood as a human activity, a social phenomenon, part of human culture, historically evolved, and intelligible only in a social context" (xi). Hersh suggests that the current state of the philosophy of mathematics is "parallel" to "philosophy of science in the 1930s," before theorists like Thomas Kuhn and Paul Feyerabend turned the discipline on its head (xii). According to Hersh, the particular "dogmatism" under which the philosophy of mathematics still suffers is what Philip Kitcher terms "neo-Fregeanism" (xii). This framework, Hersh claims, is "out of touch with mathematicians, users of

mathematics, and teachers of mathematics" (xii). And as is also the case for my larger project, Hersh believes that his work "can assist educational reform," in his case "by helping mathematics teachers and educators understand what mathematics is" (xiii).

An important part of Hersh's self-described "subversive" and "radical" perspective on this issue is that neo-Fregeanism, along with the foundationism of which it a part, "are descendants of a centuries-old mating between mainstream philosophy of mathematics and religion/theology" (xiv). Accordingly, the reader will find many insights in Hersh's analysis that align closely with comparable insights from this article on (the species of mathematics called) logic and religion/theology. I will now offer a brief analysis of a few relevant sections of Hersh's text, beginning with the introduction.

Hersh early on claims that "The method of mathematics is 'conjecture and proof'" (5). In one of many examples of Hersh's humanization of mathematics, he asserts that the mathematician "needs reassurance and encouragement as she struggles with" a mathematical problem, and then "when she proposes a solution she needs agreement or criticism" (5). Proof according to Hersh is "the analogue to experiment in physical science," and refers to the event whereby "the argument convinces qualified, skeptical mathematicians" (6). As opposed to people whom Hersh terms "naive non-mathematicians," described as people who believe that "axioms come first," Hersh argues that "anyone who has done mathematics knows what comes first—a problem" (6).

Returning to the issue of different schools of the philosophy of mathematics, Hersh claims that "among mathematicians" the predominant two are "Platonism and formalism" (7). Platonism, "or realism, as it's been called," is important to my own investigation because it is the school to which Frege belongs (9). To Frege's credit, this places him, according to Hersh, with many professionals in the field. "An inarticulate, half-conscious Platonism is nearly universal among mathematicians" (11). On the other hand, Hersh adds, "most of this Platonism is half-hearted, shamefaced" (11). What, one might wonder, is the reason for shame? "Platonism without God," Hersh puts it dramatically, "is like the grin on Lewis Carroll's Cheshire cat...The grin remained without the cat" (12). Most mathematicians no longer endorse the kind of Christianity customary in a full-bodied Platonist theory. "Platonists explain mathematics," Hersh elaborates, "by a separate universe of abstract objects, independent of the material universe. But how do the abstract and material universes interact," he asks, without God?

I skip ahead now to Part Two of Hersh's book, which is an account of the history of the philosophy of mathematics. It begins with a discussion of the abovementioned "foundationism," a name given by Hungarian philosopher of mathematics Imre Lakatos to "Frege in his prime," along with three other thinkers, all of whom according to Hersh "were hooked on the same delusion: *Mathematics must have a foundation*" (91). "Behind Frege," Hersh places "Leibniz," "Aquinas, Augustine," and "the great grandfather of foundationism—Pythagoras" (91). As a consequence of

these religious ancestors, Hersh argues that one finds "that the roots of foundationism are tangled with religion and theology" (91). However, and of particular interest to the present investigation, "In Frege," this entanglement "is out of sight," by which Hersh means, not that the intertwining of math and religion has gone away, but that it has gone underground, under the surface, only to "pop up like a jack-in-the-box" in the rest of the foundationists that Lakatos names (91). It will be my project here to show that this entanglement is, on the contrary, actually quite visible in Frege if one reads him with sufficient care.

Hersh labels this entire genealogical line, from Pythagoras to Frege and beyond, "the Mainstream," according to whom "mathematics is superhuman—abstract, ideal, infallible, eternal" (92). The opposing camp, Hersh christens "the humanists and mavericks," those who "see mathematics as a human activity, a human creation," among whom he finds J. S. Mill. Hersh offer two detailed historical narratives, beginning with that of the former group. It is here, in "the Mainstream," that one finds Frege, making it the only narrative that will concern me here.

"The philosophy of mathematics," according to Hersh, "seems to start with Pythagoras (about 572-479 B.C. or a little later)," and the secret society founded in his name "combined mysticism and superstition with geometry and mathematics in a way incomprehensible today" (92). For example, "The number one, they argued, is the generator of numbers and the number of reason," and "the number two is the first even or female number, the number of opinion" (93). Note here the misogynistic assignment of mere opinion to the female, while vaunted reason belongs with the male. Just like their descendant, Leibniz, the Pythagoreans held that to "uncover the regulative mathematical forms in nature was to reveal the divine intelligence itself," and their "discovery that the harmonics of music were mathematical was regarded as a religious revelation" (93). Connecting this directly back to the issue of formal logic, Hersh writes that "Cornford says (p. 194): 'Parmenides, the discoverer of logic, was an offshoot of Pythagoreanism, and Plato [found] in Pythagoreanism the chief source of his inspiration" (95). And Aristotle, the founder of logic, was of course a student of Plato. Hersh summarizes this, Frege's Pythagorean heritage, as "an intimate blending of religion and reasoning, of moral aspiration with logical admiration of what is timeless" (95).

I skip ahead now to the point where Hersh's narrative reaches the nineteenth century, and thus the backdrop for the work of Frege. "Until the nineteenth century," Hersh observes, and in stark contrast to present-day Western commonsense, "geometry was regarded by everybody, *including mathematicians*, as the most reliable branch of knowledge" (137). But then "two crises" occurred, namely "the recognition that there's more than one thinkable geometry," and "the overtaking of geometrical intuition by analysis" (137). (Interestingly, many Kantians, including Frege, refused to accept non-Euclidean geometry as "really geometry") (263). Consequently, Hersh explains, "Loss of certainty in geometry threatened loss of all certainty" (137).

In the immediate aftermath of this crisis, however, nineteenth-century mathematicians "rose to the challenge" (137). Mathematics had always been considered the unassailable basis of all

Western thought, the method thought to distinguish Western thought from the allegedly imperfect and unreliable knowledge of cultures such as those indigenous to Africa. Thus, if the wall of mathematics were to fall, the hordes of subjectivism and cultural relativism, it was feared, would storm the city and raze it to the common ground. "Led by Dedekind and Weierstrauss," writes Hersh, "they replaced geometry with arithmetic as a foundation for mathematics" (137). These were the immediate predecessors of Frege, and the shapers of the edifice which he wished to preserve in his own way.

Hersh credits a contemporary of Frege, George Boole, as the one who "figured out how to make logic part of mathematics," while Bertrand Russell, follower of Frege, "claimed the opposite—that mathematics is nothing but logic" (140). This latter view is known as "logicism," and I will return to it shortly regarding Frege. The historical winner in this contest of interpretation seems to have been Boole. "Far from a solid foundation for mathematics," Hersh notes, "set theory/logic is now a branch of mathematics, and the least trustworthy branch at that" (140). According to Hersh, this fact is also borne out in contemporary science. "For today's mathematical logician, logic is just another branch of mathematics like geometry or number theory. He disowns philosophical responsibilities" (141). However, in philosophy (as always), things are not so simple. "In U. S. philosophy departments, on the other hand," Hersh notes, "'analytic philosophy,' a kind of left-over from logicism and logical positivism lingers on" (141).

The section immediately preceding Hersh's section on Frege is entitled "What is logic? What Should it Be?" It begins with the observation that "Everyday experience, and ample study, by psychologists, show that most of our thinking doesn't follow logic" (140). This might mean, Hersh ventures, that "the scope of logic is too narrow" (140). On the other hand, "Computing machines"—of which Leibniz invented one of the first—"do almost always obey logic" (140). This, in turn, suggests to Hersh the following answer to the titular question of the section: "Logic is the rules of computing machinery!" (140) One concrete example of this would be IBM's Hollerith punch-card machines. Further, Hersh adds, "Logic also applies to people when they try to be computing machines" (140). One concrete example here might be the Nazi officials who used the aforementioned IBM punch-cards to carry out their bureaucratized genocide.

Hersh begins his section on Frege by referring to him as "the first *full-time* philosopher of mathematics," noting that Frege "considered himself to be working entirely with the tradition of" thinkers such as Leibniz (141). The most important part of Frege's work, according to Hersh, was probably "the introduction of quantifiers," and "is considered the birth of modern logic" (141). Again, referring back to Leibniz, Hersh writes that Frege agreed with Leibniz that the truth of arithmetic is "analytic," or self-contained (142). Hersh further notes that the term "logic" for both Frege and Leibniz means "the intuitively obvious rules of correct reasoning" (142).

With regard to geometry, on the other hand, Frege departed from Leibniz in favor of Kant, claiming that the truth of geometry is "synthetic" or world-directed. As a result of this duality (namely, analytic arithmetic and synthetic geometry), Hersh argues that it makes no sense to "speak

of Frege's philosophy of mathematics. He had a philosophy of arithmetic, and a different philosophy of geometry. Arithmetic is logic; geometry is space intuition. Fitting them together is about as awkward as yoking an ape and an alligator" (142).

Hersh then begins a new section devoted exclusively to Frege's *Foundations of Arithmetic* (what Hersh dubs "Logicism's Koran"), and which I will analyze in detail below (142). In the *Foundations*, in brief, "Frege constructed the natural numbers out of logic," after first having tried "to demolish all previous definitions" of number (142). In this latter endeavor, "First Frege trounces [J. S.] Mill, who based arithmetic on empirical experience," a critique of Mill that Hersh condemns as unfair (143) With regard to Frege's second attack in the *Foundations*, a rejection of the view that numbers are ideas, Hersh objects that "Frege is confounding private and public sense of 'idea'," and "assumes that an idea resides only in one person's head (private ideas)" (144). However, Hersh objects, "ideas can be shared by several people, even millions of people (public ideas)" (144).

As for Frege's own proposal, namely that numbers are abstract objects, Hersh finds such a conception of number to evince "an astonishing kinship to Plato's Ideas" (145). On this basis, Hersh concludes that "Frege is a Platonist" (145). Returning to the connections among Plato, Pythagoras and religion, Hersh then reminds the reader that "By Frege's time, the association between Christian theology and mathematical Platonism had gone underground" (145). The results of Frege's work in this area are (in)famously flawed. Hersh puts it simply. "The axioms from which Russell and Frege attempted to construct mathematics are contradictory!" (148).

Frege's followers, in an attempt to resolve this problem, chose "to reformulate set theory to avoid the Russell paradox," but according to Hersh, "This patched up set-theory could not be identified with logic in the philosophical sense of 'rules for correct reasoning'" (149). Even more strangely, "Despite this philosophical failure, logico-set theoreticism dominates the philosophy of mathematics today" (149). And perhaps most strangely of all, logico-set theoreticism dominates the philosophy of mathematics today despite the fact that its founders turned their backs on the view during their own lifetimes. "Logicism never recovered," Hersh writes, "from the Russell paradox. Eventually both Frege and Russell gave it up" (149). As for Frege in particular, his final philosophical move was to abandon Leibniz altogether, and accept Kant's philosophy of mathematics wholesale. "In his old age," Hersh informs the reader, Frege "decided arithmetic too was based on geometry and space intuition" (150). Why then, one might wonder, is logicism still so dominant? And why is Frege still so highly respected and extensively studied? Perhaps Frege's diary entries will suggest a clue or two.

Questions of Race in Frege's Diary

These entries in Frege's diary were first translated into English and published in the philosophical journal *Inquiry*, volume 39, in 1996. They are preceded there by a brief editor's

note, which is also worth of consideration here. Alastair Hannay begins his "Editorial" by referring to the experience of discovering that "a cherished mentor" (in this case Frege) possessed what Hannay terms "questionable" views (301). Given the intensely anti-Semitic content of the text this editorial introduces, "questionable" seems itself a questionable choice. Even more questionable is Hannay's subsequent reference to the "peccadillos [sic]" of such a mentor (301). This second paragraph then ends by noting the similar flaws of other famous twentieth-century philosophers such as Russell, Wittgenstein, and Heidegger.

Hannay then concedes that one cannot defend Frege's "peccadilloes" on the grounds that "they were part of a wider humanistic vision," for the simple reason that "Frege's work was not the sort to accommodate such a vision – *no place there for any view of human possibility* let alone a mitigating one" (301, emphasis added). Hannay does not raise the question, however, as to whether the lack of a view of human possibility in a philosophy or way of thinking is itself a flaw. On the contrary, in a footnote to the very next sentence, Hannay shifts abruptly to a celebration of Frege's "integrity and grace," along with Frege's "dedication to the truth" (301). Further, Hannay's subsequent description of Frege's behavior as "almost superhuman" echoes Nietzsche's famous concept of the *übermensch*, infamously taken up into Nazi Aryanism.

Interestingly in this connection, Hannay concludes his editorial with the suggestion that "the bitter pill of Frege's diary may be sugared by surprise at someone with abstract interests having any politics at all" (302). Is Hannay suggesting here that if one is a philosopher, then it is better to be a Nazi than to be apolitical? He elaborates on this point as follows: "Faced with the often specious authority of philosophers in practical matters, and the crudeness of some 'efforts to dominate and be known' in philosophy in general, the fumblings of a Frege may even be preferred however much we regret the results" (302). Philosophers, Hannay seems to be saying, (a) often have no credible authority regarding national or international politics, and (b) their own professional politics are often bumbling, therefore (c) we are fortunate that a philosopher as technically formidable as Frege was an anti-Semite.

After Hannay's editorial, but before the diary proper, there is one more introduction which is worthy of consideration here. In his "Translator's Preface," Richard L. Mendelsohn defends Frege by (a) harshly criticizing Hans Sluga's attack on Frege as misleading and distorted, and pointing out both that (b) "Frege was quite old at the time of the writing of the diary," and also (c) "these later years were not easy for [Frege]" (304). Ultimately, Mendelsohn concludes, "What the diary shows more clearly than ever is how much Frege was a creature of his time" (305). The implication of Mendelsohn's remarks seems to be that if one is forced to concede that Frege was guilty of anti-Semitism, then everyone else of his era was guilty too.

But should we not hold philosophical geniuses to a higher standard than the average layperson? Was that not Hannay's reason for bringing up the sins of Russell, Wittgenstein and Heidegger? Or are these latter three thinkers to be excused as glibly as Mendelsohn seems to be excusing Frege? It is clear, in any event, that at least Heidegger is not currently being

thus excused. Does Mendelsohn's remark suggest that we should be reevaluating the case of Heidegger's fascism?

Medelsohn then concludes his own preface by himself appending yet another introductory text to Frege's diary, this time by one of the original editors of Frege's diary, Gottfried Gabriel. It begins to seem as though the diary were so dangerous and contagious that it had to be wrapped in multiple layers of protection. Gabriel argues, against Michael Dummett's accusation that the diary passages in question (from March 10 to April 9) were suppressed by the editors of Frege's writings, that those passages were instead "not included in the publication of the *Nachgelassene Schriften* on the grounds that the remaining reflections were of a political nature and 'could not be numbered among the scientific [*wissenshaftliche*] *Nachlaß*'" (306). I leave it to the reader to judge the scientific/philosophical relevance of the text which I can now finally begin to analyze directly.

Frege introduces the diary as consisting of "ideas which are perhaps worthy of being developed in the future" (308). The issue here then, contra Gabriel, concerns the realm of ideas from the beginning, ideas worthy of presumably scientific/philosophical development. The second entry indirectly refers to Adolf Stoecker, a theologian who was, according to the translator, "one of the most influential representatives of modern anti-Semitism in Germany," and whose anti-Semitism Frege "shares" (309n). Frege's ranting against political leftism and unions begins in the third entry, "12 March 1924," in which Frege concludes that the problem is at base one of "Two devils," namely "the devil of pride" and the "devil of envy" (310). This is the first move to the political right in the diaries, the extreme end of which movement lies explicit fascism.

In the entry from the next day, Frege's anti-union metaphors shift from demonology to pathology, in his reference to France's having "suffered from a dangerous illness," one which "had infected Germany" (311). This rhetoric returns in the "16 March 1924," in which Frege writes of the "socialist infection" (312).

In a footnote to the end of this same entry, Mendelsohn quotes Frege on the aforementioned Stoecker as follows: "I have nothing against Stoecker; he has for my part only the one failing as a politician, that he is a priest, and as a priest, that he pursues politics" (312n). Here Frege seems to be advocating indirectly for the separation of church and state. But the relationship between the two for Frege is more complicated, as the entry from two days later, "18 March 1924," begins to suggest. "A religious obligation," Frege writes, "is an obligation over whose fulfillment no human judge stands guard and judges" (313). It seems that there is a definite hierarchy in place here, according to which religion exists beyond the reach of the political. This, in turn, is the first of many appearances in the diary of what might be understood as that submerged and hidden religiosity which—according to Hersh—deeply informs Frege's thought in general.

It is in the April entries that Frege's anti-Semitism is first openly announced. In the "9 April" entry, he begins by bemoaning the fact that "Much of our misfortune has its cause in the

fact that we take so little pride in our national characteristics," having "so many people of different races," who "claim to be considered as Germans" (322). There is thus, for Frege, a fixed set of characteristics—like the members of set which constitute the extension of a concept in his mathematical logic—that make up the true German person, and any deviation or admixture of other characteristics is a blurring of the true concept, and therefore a social nightmare. Frege then turns to the Jewish "race" in particular, noting that "Up until the year 1866, Jews were generally not permitted to stay overnight in my native town of Wismar" (322). This policy, Frege remarks almost casually, "was obviously due to bad experiences" (322). He merely assumes here, without any evidence or analysis, that his native townspeople's prejudice towards another race of people was justified.

The last sentence of this paragraph then connects this anti-Semitic theme to Frege's earlier theme of the separation of church and state. "With the Jews," Frege asserts (again without apparent evidence), "religion is very closely tied to their national characteristics, and these especially, with their way of doing business" (322). To what nation is Frege referring—perhaps the historical nation of Israel? At any rate, he directly connects here the ills of socialism, along with their "devils" of pride and envy, to the Jewish people.

As to how to resolve these Jewish problems, though demurring that he does not "feel qualified to make proposals for the politics of the moment," Frege nevertheless asserts later in the same entry that "No doubt one needs youthful vigor to sweep away the people" (323, 324). Why would a philosopher as apparently rational, logical and cerebral as Frege, not only affirm his fellow citizens' (irrational) anti-Semitism, but also advocate the use of emotional propaganda to "sweep away" intentionally the rationality of Germany's people?

For whatever reason (or lack thereof), Frege further stipulates that this youthfully vigorous leader must also be a soldier. "A German Kaiser," Frege insists, "must be a military leader and have self-confidence" (324). After this further move to the political Right, Frege's next entry then criticizes former German ruler Otto von Bismarck for being "in some of his views rather liberal, for example, in his conviction of the necessity of a representation of the people or at least an influence of the people on the government" (325). Frege, this sentence makes clear, was a staunch advocate of monarchy, and more specifically for the God-like rule of one man over an entire nation. Frege then goes on to link monarchy's opposite, namely democracy, back to Jewishness. "What a misfortune," Frege complains on this note, "that Jews had such influence in the National Liberal Party" (325).

The next entry, "13 April," offers a more specific criticism on this point, against the democratic/representative "parliamentary process" in particular, specifically that it "is nothing peculiarly German, grown from German soil" (326). The person Frege is here criticizing for such parliamentary sympathies, however, was nothing but "peculiarly German" and born on German soil. Thus, "So long as Bismarck and a Wilhelm I watched over Germany, things went well because they were powerful. However, when a weaker person took hold of the reins, things went

downhill with Germany and we sank into a swamp" (326). Again, Frege's preferred solution is a strong and powerful single leader, born with correct (that is, German) characteristics and from the correct (German) soil. And again, the alternative is to end up with people with the wrong (Jewish) characteristics from the wrong (Jewish) soil.

Toward this end, Frege later makes more specific suggestions as to how to deal with the most offensive of these other kinds of people. After retelling the story of how Jewish people were prevented from staying overnight in Wismar, Frege makes the following prophetic suggestion: "If one wants to make laws against the Jews, one must be able to specify a distinguishing mark [*Kennzeichen*] by which one can recognize a Jew for certain. I have always seen this as a problem" (331). In a footnote to this sentence, Mendelsohn suggests the following link between this text and the *Foundations*: "Frege has touched on the topic of the connection between a correct definition and the specification of a distinguishing mark [*Kennzeichen*] in his *Grundlagen der Arithmetik*" (331). This suggestion, in turn, opens up the possibility that there is a connection between Frege's projects for, on the one hand, a logically pure foundation for mathematics, and on the other hand, a racially pure foundation for politics.

Buttressing this outright anti-Semitism is Frege's defense in the next entry of an essay by Dr. Friedrich Weber, which is itself a defense of Weber's own role in Hitler's attempted coup d'état (the "Beer Hall Putsch" of 1923). Despite Frege's previous condemnation of any violence against the government, Frege writes here that "The sentiments articulated there [in Weber's essay] have my full approval" (331). Apparently, it is only a Leftist overthrow powered by unions, rather than any violent revolution in general, that for Frege is never justifiable. In support of this interpretation, Frege claims in the same entry that "the destruction of Marxism, or at least its expulsion from the entirety of the fully enfranchised citizens, is a necessary precondition of the possible establishment of a strong Reich, which Dr. Weber strives for" (332).

In another reversal of a key position earlier in the diary, Frege opines in the "29 April" entry that "One must not mistake the desire for the well-being of the fatherland for a political program, especially if the desires are pious" (335). Apparently Frege considers only a Leftist mixture of church and state (as for example when priests advocate welfare for the poor) not admixture per se, unjustifiable. On the contrary, piety (of a certain kind) for Frege can actually purify an obviously political aim of all political essence, transforming such an aim into one of those (politics-trumping) religious obligations against which no civil rights can stand.

In the next day's entry, Frege (a) (again) bemoans that fact "that there are so many Jews in Germany," (b) explicitly mentions the word "Aryan," (c) repeats the concern over how to physically identify Jewish people, and (d) concludes that "one must be satisfied with fighting the ways of thinking [*Gesinnung*] which show up in the activities of the Jews" (336). This latter strategy can be accomplished, he notes, with "the loss of civil rights" (336). Put differently, since the concept "Jew" seems too vague to be workable, one should simply expand the concept to include anyone resembling the Jews (such as, perhaps, Communists and homosexuals). Far from

denying the prejudicial nature of this program, Frege on the contrary erects an apologetics for prejudice per se, asserting that "prejudice is necessary for patriotism" (337).

> The question here is not about a judgment in the sense of logic, not about considering something as true, but about one's feelings and inner attitude. Only Feeling [*Gemüt*] participates, not Reason, and it speaks freely, without having spoken to Reason beforehand for counsel. And yet, at times, it appears that such a participation of Feeling [*Gemüt*] is needed to be able to make sound, rational judgments in political matters (337).

Especially coming from such a devout proponent of logic, this passage seems entirely out of place. Why would Frege accept the abandonment of reason in the pursuit of politics? How can he justify smuggling the concepts of soundness, rationality, and judgment into the conclusion of a process from which he intentionally removed those concepts? How, in other words, can irrationality, or a-rationality, terminate in the rational?

Frege's irrationality certainly does not terminate in the rational, in that it is followed here by an expression of admiration for Hitler. "Adolf Hitler writes correctly," the entry begins chillingly, "in the April issue of *Deutschlands Erneuerung*, that Germany no longer had a clear political goal after the departure of Bismark" (339). What, one might wonder, would be Frege's answer to this alleged problem? "We urgently need," he insists, "a revival of religion" (341). Unconsciously prophesying again in this entry (in this case, in regard to Hitler's own repackaging of ancient racism), Frege claims that "We must have prophets proclaim something new to come that really is something old" (341). And it is to the explicitly philosophical work of Frege-the-prophet—alleged by Hannay to be radically divorced from his politics—that my investigation now turns.

Questions of Race in Frege's *Thoughtscript*

I begin this part with Frege's earlier work, the *Thoughtscript*, to which the later *Foundations* was intended as a more accessible introduction. In the editor's introduction, Heijenoort refers to it as "perhaps the most important single work ever written in logic" (1). He also notes that Frege was originally, and by training, an outsider to the field of logic, as Frege's degree and dissertation were in mathematics (1). The text was certainly not accessible to logicians of Frege's day, particularly in regard to his awkward symbolism, which, Heijenoort notes, Frege stubbornly refused to modify. "The notation that he introduces for the conditional has often been criticized, and it has not survived. It presents difficulties in printing and takes up a large amount of space. But as Frege himself (*1896*, 364) says, 'the comfort of the typesetter is certainly not the *summum bonum*'..." (2).

What was worse (and noted above in Hersh), Frege's awkward formality did not even accomplish its lofty goal of logical perfection. After having praised Frege for "the analysis of the proposition into function and argument(s) instead of subject and predicate" (i.e., the paradigmatic move in analytic philosophy from empirical linguistics to symbolic mathematics as philosophical

medium), Heijenoort notes that in the end "the difference between function and argument is blurred," causing Frege to ultimately "fall into the abyss" of Russell's paradox (2). According to this paradox, in Frege's work (as well as that of others such as Dedekind) there is a set (namely, the set of sets which are not members of themselves) which both is and is not a member of itself.

Despite Frege's collapsing failure, Heijenoort assures us that "This flaw in Frege's system should not make us lose sight of the greatness of his achievement" (3). My question is the following: if the purpose of the entire project is to move from supposedly inherently flawed and imperfect natural languages to the perfect precision of an artificial language, then how can the self-contradictoriness of that artificial language not constitute the failure of the entire project? And if the entire project does indeed fail, then perhaps that failure suggests that what are to be preferred for logic are in fact natural languages (as opposed to artificial languages), at least in light of natural languages' greater prevalence, accessibility, naturalness and ease of use.

Heijenoort himself almost concedes this point, insofar as he acknowledges toward the end of his introduction that "At times *Begriffsschrift* begs for a clarification of linguistic usage" (4). In other words, even Frege's own artificial language-creating text cannot do entirely without the natural language clarifications of an interpreter, and where this artificial language does employ natural language, the results are just as inadequate as the artificial language itself. And yet, Frege went to all this effort, according to Heijenoort, simply because "The imprecision and ambiguity of ordinary language led him to look for a more appropriate tool" (1). Frege obviously did not find it, but I am going to look after him, turning to the text of the *Thoughtscript* itself.

The first sentence of Frege's Preface to the *Thoughtscript* ends with the word "certitude," and the third sentence ends with the phrase "the most secure foundation," so in both the content and the tone of this discourse, what he is searching for seems clear (5). Shortly thereafter one finds, at the heart of Frege's solution, the concept of purity, which is then repeated throughout the text. Following Frege's own famous method of "sentential holism," I offer here the entire sentence in which the first (and adjectival) use of purity appears: "The most reliable way of carrying out a proof, obviously, is to follow pure logic, a way that, disregarding the particular characteristics of objects, depends solely on those laws upon which all knowledge rests" (5).

That this method is the most reliable, however, is not in fact obvious to everyone. Nor is it obvious, at least not to the present author, why such purity is advantageous per se. Moreover, it is troubling, especially in the context of Frege's aforementioned racist remarks in the diary, that he here encourages his reader to ignore the concrete, detailed differences among beings in the world, in favor of abstract laws descending from one-knows-not-where. In other words, racism advocates treating all members of a race in the same, negative ways based on the imposition of offensive stereotypes—which also requires ignoring the concrete, detailed differences among individual members.

Frege's next (and adverbial) example of purity, found in the very next sentence, refers to logic not as a body of knowledge, but as a method. (Although perhaps all logic is ultimately

merely formal in the end, or can be understood as always reducible to method). "Accordingly," Frege writes, "we divide all truths that require justification into two kinds, those for which the proof can be carried out purely by means of logic and those for which it must be supported by facts of experience" (5). Note the slippage here between purity as an intra-logical distinction and purity as the distinction between logic and the rest of the world. In a way, all logic is pure, but in another way, only part of logic is pure. Note also that—reminiscent of the Nazi-prophetic remarks in the diary—logic is being used here to divide the world into two groups or camps, based on a standard of purity.

Frege's next sentence clarifies that this purity denotes a state devoid of "any activity of the senses," followed a few sentences later by the imperative to "prevent anything intuitive [*Anschauliches*] from penetrating here unnoticed" (5). Ironically, other meanings of *Anschauliches* include "clear," "vivid" and "lively," so this phrase could also be read somewhat poetically and strategically as claiming that Frege wanted either (a) to prevent anything clear from appearing to the reader, that is, to keep the reader in uncertainty, confusion and ignorance; or (b), especially in light of the above reference to the senses, to prevent anything that pulses with life and vitality, anything lively, from entering into the matter—including the concrete details of actual living organisms. Remember, on this note, the enduring concern in Frege's diary with the inability to differentiate Jews based on appearance alone; they too were often "penetrating unnoticed" into pure German society. This rhetoric also recalls Frege's repeated discussions of infection in the diary, specifically in reference to political conflict that he links to Judaism.

In order to carry out this plan for purity, Frege remarks that he "had to bend every effort to keep the chain of inference free of gaps," and in "the strictest possible way" (5). Instead of interpreting the difficulty involved in his plan as a potential indicator of its inappropriateness, Frege instead chooses to condemn natural language, that ancient and universal instrument of humanity, for its "inadequacy" (5). Frege then returns to his cloak-and-dagger rhetoric to describe the ways in which intuition "tries to sneak in unnoticed," compelling him to be ever more rigorous "so that [intuition's] origin can be investigated" (6). Note the ease with which this rhetoric could be applied to Frege's concerns about Jewish people being indistinguishable from "pure Germans." This suggests another reason to suspect a connection between Frege's battles for both mathematical and political purity.

According to Frege here, when it became clear to him that such gap-free inference would be impossible, he instead "decided to forgo expressing anything that is without significance for the inferential sequence" (6). Frege offers no explanation, however, as to how he decided what belongs in the latter category, and what does not. Presumably, the reader is supposed to assume that this choice was as "obvious" as the one Frege made in favor of "pure logic" above. But if Frege is wrong about what has significance for the inferential sequence, then more of real life, "the particular characteristics of objects," gets thrown out with the bathwater.

In the next paragraph, Frege helpfully compares the relationship between his thought-script and ordinary language ("the language of life") to that "which the microscope has to the eye" (6). (Incidentally, one cannot help but wonder here if Frege suspected that Jewish people might be distinguishable at the microscopic, even if not at the optic, level). Interestingly, if one were to replace "thought-script" for "microscope," and "natural language" for "eye," in the next sentence, one would get the claim that natural language is "far superior to" the thought-script, and that the thought-script too "exhibits many imperfections" as a conceptual instrument.

When one comes to Frege's discussion of the specialized uses of the microscope and the thought-script, however, those of the thought-script, namely "certain scientific uses," are much vaguer than those of the microscope (6). And insofar as Frege understands science to be a true description of reality, it would seem that the thought-script would therefore be the language or code for all of reality. The implication here would be that the expertise of the thought-script, unlike that of the microscope, would know no bounds (6).

The following paragraph explicitly returns to Leibniz, who also "recognized—and perhaps overrated—the advantages of an adequate system of notation" (6). Frege refers to Leibniz's "idea of a universal characteristic" as "so gigantic that the attempt to realize it could not go beyond bare preliminaries" (6). If Frege sees himself as a direct inheritor of this ambition—and Heijenoort quotes Frege unequivocally to that effect—then wherein does Frege's estimation of the problem, and his attempted solution, differ from that of Leibniz? Frege seems to anticipate this question when he writes that "even if this worthy goal cannot be reached in one leap, we need not despair of a slow, step-by-step approximation" (6). In fact, for Frege this process has already long since begun, insofar as he claims that "the signs of arithmetic, geometry, and chemistry" can be understood as "realizations, for specific fields, of Leibniz's idea" (7). Frege sees his own project as one "that adds a new field, indeed," and unsurprisingly (given the typical ego of the innovator logician) "the central one, which borders on all the others" (7).

Frege does not hesitate to capitalize immediately on this foray into the metaphorical well of geopolitics, proceeding therefrom directly to empire. The thought-script, he writes, can from this strategically advantageous position, "fill the gaps in the existing formula languages, connect their hitherto separated fields into a single domain, and extend this domain to include fields that up until now have lacked such a language" (7). In other words, the thought-script can solidify and fortify the infrastructure of thought, unify its power in a centrally-located unified command center, and then embark on a campaign of conquest of new territories for this emboldened regime of thought. The potential resonance here with the diary's insistence on a newly purified and powerful German monarchy is striking.

The word "purity" recurs yet again, and in specific relation to this idea, at the beginning of the first subsection of the first section, in reference to "*the more comprehensive domain of pure thought in general*" (11). The second subsection then turns to the issue of judgment [*Urteil*],

and of the judgment-stroke [*Urteilstreich*]. This emphasis on judgment is one that pervades the entire *Thoughtscript*, and it is an interesting one in light of the previous connections I have suggested between the "political" content of the diary and the "philosophical" content of this text. Put more directly, Frege here imports a concept into the highly abstract field of logic and mathematics, the most important denotation of which lies in practical affairs, especially jurisprudence and politics.

Despite acknowledging all these unlikely connections, the reader will likely object, nevertheless, that my analyses are based on only weak and scarce evidence. To answer this objection, I now offer similar analyses of (similarly) interesting moments in the *Foundations of Arithmetic*. Because Frege's tone there is more conversational, it offers many more moments with which to buttress my analyses of the *Thoughtscript*.

Questions of Race in Frege's *Foundations of Mathematics*

Frege's introduction asserts boldly (and arguably contra Aristotle) that "Thought is in essentials everywhere the same: there is no question as to whether for different objects there are different kinds of laws of thought" (12). The merely apparent differences, Frege argues, returning to his arch-concept of purity, "consist only in the greater or less purity and independence of psychological influences, and on external aids to thought" (12). There is only one qualitative form of thinking, then, for Frege, but there are various quantities of purity within it. And if one were to object that activities such as basic arithmetical calculations constitute counterexamples to this claim, Frege would respond that he denies such calculations the status of thought at all. "One can of course use number signs mechanically," he writes, "just as one can speak by parroting words: but that may hardly be called thinking" (13).

Three paragraphs later, and perhaps thinking of J. S. Mill, Frege asserts that "arithmetic has absolutely nothing to do with sensations," nor with their "fluctuation and indeterminacy," nor with the "inner images" that they create in the mind of the mathematician (14). That there may be a deep-seated fear in Frege behind this rejection of sensation is suggested later in the same paragraph. "If in the existing flux of things there is nothing fixed," he writes, "eternity persists, then the knowability of the world would end and everything would plummet into confusion" (15).

That Frege might be drawing on a (perhaps unconscious) political idea here is suggested by a claim only a few sentences afterward in regard to the concept of purity. "[T]here is often success first in recognizing a concept in its purity, in peeling away the foreign wrappings that conceal it from the mind's eyes" (15). Remember again the "foreign wrappings" that in the diary prevent German Jews from being distinguished from other Germans, making them impossible to detect by mere visual inspection. Remember also Frege's analogy, according to which his thought-script can be thought of as a kind of microscopic thinking capable of searching out invisible truths.

Later in this same paragraph, Frege refers for the first time to J. S. Mill as his explicit opponent. Frege notes, again in regard to purity, that within Mill's conception, "Instead of finding there a special purity of the concept, where one believes their source to lie near, one sees everything blurry and indistinct as through a haze" (15). In addition to purity, this sentence also returns to the (genetically-resonant) concept of source, and hints at the metaphor popular in Christianity of bodily knowledge as "seeing as through a glass darkly." Views in the philosophy of mathematics such as that of J. S. Mill have been termed by their opponents "psychologism," and Frege asserts here that "mathematics must refuse to tolerate all assistance from the side of psychology" (16).

It is perhaps no surprise that someone with Frege's anti-Semitic views would be hostile to a discipline whose goal is to explore the hidden depths of the human mind. For Frege, however (as well as many of his analytic followers) such issues are irrelevant, as evinced in one of Frege's "basic principles," found only a few paragraphs later. "The psychological is to be sharply separated from the logical, the subjective from the objective" (17). Thus, Frege insists that the question for the text's definitions "is not whether they are natural, but whether they penetrate to the crux of the matter and are logically exceptionless" (17). Note here the violent rhetoric of penetration, the Christianity-resonant metaphor of the crux or cross, and the positing of a necessary condition which Frege's own work in this text (as Hersh notes) fails to satisfy.

One example of a potentially race-connoting moment in the *Foundations* is found in the first of four brief numbered sections at the end of the introduction. It appears in the context of Frege's insistence on "rigor," which is thereafter the most frequent and important concept in the text. "In arithmetic," he writes, "because of the Indian origin of many of its methods and concepts, a more lax way of reasoning arose than in geometry as developed by the Greeks" (18). Paraphrasing strategically, for Frege, the darker-skinned Indians did not have the rigor (or intellectual purity?) that the Greeks did. And yet, it has been suggested that all Greek philosophy and science derives ultimately from Indian sources.

Returning to the rhetoric of rigor, Frege writes in section two of "the movement directed toward greatest rigor" (18). Also in sympathy with Leibniz, but without Leibniz's explicitly religious justifications, Frege claims that the "simplification" resulting from the pursuit of fewer and fewer "primitive truths," "is a goal worth striving for in itself" (19). Is Frege assuming an in-depth knowledge of Leibniz in the reader, or does he merely consider it another obvious truth that "one should seek simplicity for its own sake"?

In the next major (post-introduction) section of the book, "I. Opinions of Several Writers on the Nature of Arithmetical Propositions," Frege again first sets his sights on J. S. Mill, and again criticizes him in the name of purity. "Mill always confuses applications that one can make of an arithmetical proposition," Frege complains, "with the pure mathematical proposition itself" (26). As a counterexample to what Frege has earlier termed Mill's "gingerbread or pebble

arithmetic," wherein (according to Frege) every number is the result of a concrete perception of a number of things, Frege resorts to a political example.

Something can be part of something else, Frege insists, in ways other than being a physical sub-heap taken from a larger heap, such as "political assassinations as a part of murder as a whole" (26). One could argue, however, that such a conception is only made possible by the fact that both political assassinations and murders in general are originally concrete events in the world. The axiologically-loaded content of these two concepts, assassination and murder, actually seems to reinforce this point.

Eventually Frege also discusses one of the abiding themes of the larger project from which this article is taken, namely the phenomenon of whiteness in logical texts, and does so in support of attaching objective meaning to whiteness. "One ordinarily thinks by means of 'white'," Frege writes, "a certain perception, which, of course, is entirely subjective; but even in ordinary language use, it appears to me, it frequently abounds in objective sense" (41). One ordinary language example that Frege offers in support of his objectivist view of color is as follows: "One might say: It now *appears* red, but it *is* white" (41). One might interpret this example as Frege again showing concern with deceptive appearances, this time in connection to a word he conceives of as "abounding" in "objective" meaning.

Before returning to the issue of color, Frege revisits the primary subject of the *Foundations*, one which Frege understands as similar to color in its objectivity, namely number. Frege notes that "the French '*uni*' means 'even', 'smooth'," and that "'unity' is also used in a similar way, when the political unity of a country, unity of a work of art, is spoken of" (45). Note here the explicit return to a political theme only a few pages after having elaborated on the objective meaningfulness of color. One page later, Frege (again) returns to the issue of color, and he (again) does so through an example involving deceptive appearances. This time, however, the example also involves differentiation, categorization, and perhaps even a suggestion of the political.

> If I, for example, in contemplation of a white and a black cat disregard the properties by which they are distinguished, then I might arrive at the concept 'cat'. If I now also bring them both under this concept and perhaps call them units, then the white one still remains white, and the black black. Also thereby, from the fact that I do not think about the colors, or myself undertake to draw no conclusions from their difference, the cats will not become colorless (47).

The point seems to be that defining characteristics never go away, even if one tries to think (or wish) them away. Applying this insight strategically to the content of the diaries, perhaps Frege felt that Jews also remain Jews, and that "pure" Germans remain "pure" Germans, whether one thinks of both Jews and Germans as humans or not.

Pursuing Frege's points further, "units" can also mean a certain quantity of military personnel, and a few pages later Frege in fact brings up a similar subject. "I also believe," he writes, "that one is justified to speak of 45 million Germans without previously having thought

of or set down an ordinary German 45 million times, which might be somewhat cumbersome" (53). Here Frege (seemingly) casually suggests the current might (in population) of the German people; and he does so for no (apparent) reason except than to offer a basic counterexample to his opponent, Hankel. Perhaps, though, there are other potentially race-connected reasons at work in this choice of example; maybe they are even linked to the previous example of the cats.

Frege returns to the theme of the German population again a few pages later in a rare poetic moment in the *Foundations*. He argues that "the concept 'inhabitant of Germany'" is in fact "a function of time," given the constant change in population. "Thus, in the concept itself," Frege points out, "there is already something flowing" (57). It seems that there might be something else "already flowing" here in Frege—perhaps a surge of nationalistic pride in his native Germany, whose population he has just paraded out for display a few pages earlier? In other words, might there be something in these ideas that is stirring to Frege—something which might explain why he suddenly waxes eloquent? Perhaps supporting this interpretation, Frege asserts on the next page that the "collecting power of a concept" is "superior" to that of a mere apperception, as evidenced in its ability to "join together the inhabitants of Germany into a whole" (58). Is this whole, one wonders, merely "Germany," or might it be something more, perhaps even something like the "pure" Germany envisioned by the Third Reich under Hitler?

Again with political overtones, Frege writes a few pages later that "the form of identity," such as a person's identity as a German, "is the most sovereign in arithmetic" (63). In addition to the political origin of the word "sovereignty," Frege goes on to give an example from actual political history. "For that matter," he writes, "in the name 'Columbus' there is nothing about discovery or about America, and yet the same man is known as Columbus and the discoverer of America" (63). Thus is added to "sovereignty" the connotations of conquest and tyranny, by way of the pathfinder for perhaps the most expansive and brutal trans-continental empire in human history.

When Frege finally offers his own definition of number here in the *Foundations*, other concepts and phrases with problematic dimensions keep appearing. First, Frege observes how fixed identities allow one to "repeatedly recognize something even though it is given in a different way" (70). Might this something also be a someone, such as perhaps a Jewish person in Nazi Germany?

Second, Frege recommends his own system to the reader because within it "there is therefore complete conformity" (71). One could hear in this reference a resonance with the absolute conformity required and enforced by the Nazi regime (71).

Third, the rhetoric of purity returns here with a vengeance, as with a description of his "relation-concept" as belonging to "pure logic," and of a definition of Leibniz as "purely logical" (73, 77).

Fourth, Frege further recommends his view on the basis of its being "organic," like a seed, and of its being concerned with the "laws of the laws of nature" (86, 85). Here again the natural world in all its concrete particularity is conquered by sovereign abstraction.

Fifth, Frege offers a complementary attack on alternatives to his view, on three separate occasions, merely on the grounds that they "bring something foreign" into the matter (93, 94).

And finally, on the last page of the book, Frege celebrates how his approach allows "the possibility of avoiding the interference of external things" (94). Might an unconscious referent of these "external things" include the despised Jews in Germany? In light of the diaries, and when viewed altogether, these moments appear to raise important questions.

Conclusion

I have tried to show in this article how Hersh's criticisms of the philosophy of mathematics in general, and of Frege in particular, can be illuminatingly extended to the philosophy of logic. For one thing, the reader has observed how various philosophers in the analytic philosophy branch of Hersh's "Mainstream" tradition, including Hannay, Mendelsohn, Gabriel and Heijenoort, have defended Frege from conservative and ultimately Christian perspectives. For another, the virulent racism and conservative Christianity in Frege's infamous 1924 diary is impossible to deny. Lastly, and perhaps most surprisingly, even Frege's driest and most abstract texts, from the philosophy of logic and mathematics, are peppered with potentially race-connoting moments.

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