Latent Structural Analysis for Measures of Character Strengths: Achieving Adequate Fit

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Abstract

The VIA Classification of Strengths and Virtues is the most commonly used model of positive personality. In this study, we used two methods of model modification to develop models for two measures of the character strengths, the VIA Inventory of Strengths-Revised and the Global Assessment of Character Strengths. The first method consisted of freeing residual covariances based on modification indices until good fit was achieved. The second was residual network modeling (RNM), which frees residual partial correlations while minimizing a function that penalizes more complex models. Models based on both strategies were developed for the two questionnaires. The resulting structural models were then applied to four other samples. Though both modification procedures achieved good fit in the sample used to develop the models, only RNM resulted in adequate model fit for both measures in all cross-validation samples. This finding suggests RNM is more robust against overfitting than traditional practices. Moreover, the result supports the validity of the three-factor model of character strengths with replicability.

Keyword: Character strengths; VIA Inventory of Strengths; Confirmatory Factor Analysis; Residual network modeling; Cross-validation

Data availability statement: The data that support the findings of this study are openly available in the Open Science Framework at https://osf.io/gtxb9/

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One of the consequences of the emergence of positive psychology was a growing interest in studying the concept of positive personality. A seminal contribution to this work was the development of the VIA Classification of Strengths and Virtues (Peterson & Seligman, 2004).[1] The VIA Classification was the product of a three-year effort involving more than 50 representatives from multiple disciplines with expertise in various aspects of positive human functioning. The result was a list of 24 dimensions, called character strengths, that was thought to provide a comprehensive perspective on positive personality. As a starting point for identifying an overarching set of dimensions for conceptualizing the domain, the authors also suggested the character strengths reflected six higher-order dimensions that were developed conceptually. Based on the assumption that these more abstract dimensions should mirror social conventions associated with positive human functioning, they reviewed moral texts from various traditions--Islam, ancient Greece, Judeo-Christianity, Hinduism, Buddhism, Confucianism, and Taoism--and identified six themes they considered universal across their sources (Dahlsgaard et al., 2005). These themes were labeled Wisdom and Knowledge, Courage, Humanity, Justice, Temperance, and Transcendence. Given their cultural status, Peterson and Seligman referred to their broader themes as virtues. They then associated each of the 24 strengths with one of the virtues on conceptual grounds. The resulting two-level model is summarized in Table 1.

Peterson and Seligman (2004) also introduced the VIA Inventory of Strengths (VIA-IS) as a measure of the 24 strengths. Its development spurred research on the model, with

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[1]VIA originally stood for "Values in Action" but is now an orphaned acronym.

approximately 1,300 academic works to date investigating the VIA Classification[2]. This includes evidence of measurement invariance across 16 countries (McGrath, 2016), supporting the cross-cultural significance of the model, at least across countries with a substantive tradition of personality research. The development of the VIA-IS also spurred the applied measurement of character strengths. The VIA-IS has since been completed millions of times by individuals around the world, in a variety of contexts including career development, personal coaching, and life planning.

One topic of this research has been exploratory latent structural analysis of the VIA Inventory. Unfortunately, none of these studies with the original instrument replicated the six-factor structure (McGrath, 2014), with most studies settling on 3-5 factors. The developers of the VIA Classification recognized that subsequent empirical analysis might not support their six-virtue model and left open the possibility of its modification.

Subsequent research found that a three-factor model was particularly reliable across populations, measurement instruments, and analytic strategies (McGrath, 2015; McGrath et al., 2018). A recent study has demonstrated that the three-factor model can be well cross-validated across different samples with advanced factor analytic methods (McGrath et al., 2021). McGrath et al. (2021) demonstrated that the three-factor model of the revised version of the VIA-IS was consistently valid as shown in prior research. They freed several residual covariances in the model that achieved adequate fit across multiple datasets. These factors have been called Caring, Inquisitiveness, and Self-Control, to distinguish them from other constructs in the VIA

[2] As of August 17, 2021, a total of approximately 1,300 entities were found from Google Scholar when “VIA Classification” & “Values in action” & “Character strengths” was entered to its search form.

Classification. It has been suggested that these three themes merit being considered cultural virtues, especially as they overlap substantially with the original six (only Transcendence is not effectively encompassed by the smaller set). This model is summarized in Table 2.

More recently, the VIA Inventory of Strengths was revised. A full discussion of the reasons for revision may be found in McGrath (2019), but the most important included the following:

- At 240 items (10 items per strength scale), the scale was too long for practical use in many situations.
- All 240 items were positively keyed.
- Some items were considered overly specific or asked about sensitive issues or protected health information.
- Several of the original scales combined items representing very different contents, making interpretation of scores on those scales difficult.
- No scales were ever developed to represent the six conceptual or three empirical virtues.

After extensive research using a number of different statistical strategies, the VIA Inventory of Strengths-Revised (VIA-IS-R) was introduced. The number of items was reduced to 192 (8 per strength), and each scale is a combination of positively and negatively keyed items. Scales were also developed to represent the virtues. Two short forms of the VIA-IS-R were developed at the same time called the VIA-IS-P, indicating only positive keyed items are included, and the VIA-IS-M, which includes a mix of positively and negatively keyed items (McGrath, 2019). Each consists of 96 items, 4 per strength.

Two new measures of the VIA Classification were also developed: the Global Assessment of Character Strengths and Signature Strengths Survey. These new measures in combination with the VIA-IS-R and its short forms are referred to as the VIA Assessment Suite for Adults (McGrath, 2019). The GACS was used in addition to the VIA-IS-R in the present study. The rationale for this measure was based on another concept introduced by Peterson and Seligman (2004), referred to as signature strengths. These are defined as strengths that the individual identifies as particularly central to their identity.

Interviews with individuals about their signature strengths suggested these strengths were experienced as an essential part of who they are, as natural and effortless to express, and as uplifting or energizing to express (McGrath, 2019). To capture these three attributes in a questionnaire, a measure called the GACS was developed. The GACS begins by providing descriptions of the 24 strengths, then asks the respondent to rate their agreement with 24 items asking about the degree to which each of the strengths is an essential part of who they are. These items are followed by items representing the second and third signature strength attributes. The result is a 72-item instrument, three items per strength.

Validation of the VIA Inventory of Strengths

Various considerations are considered essential to ensuring that a measurement instrument meets acceptable standards. In her seminal work on this topic, Loevinger (1957) identified three phases in the construct validation of an instrument, called the substantive, structural, and external. The first is inherent to the development of the instrument, having to do with the degree to which the targeted constructs have been adequately defined, and the degree to which items have been identified that are reflective of those constructs. This is followed by evaluation of the instrument's structural validity. More recently, Hussey and Hughes (2020) outlined four lines of evidence important to establishing the structural validity of an instrument: internal consistency, test-retest reliability, confirmatory factor structure, and measurement invariance. Finally, external validation has to do with the extent to which items and scales are related to other variables in a manner consistent with the conceptual understanding of the underlying construct.

Multiple sources of data converge to suggest the VIA-IS-R meets most standards for construct validation. Substantive validity has been addressed in several ways. As described above, the conceptual model underlying the VIA Inventory was the result of an intensive process involving input from numerous subject matter experts (Niemiec, 2013). In terms of congruence between items and those constructs, one source of data used in item selection for the VIA-IS-R item pool was prototypicality ratings of the items as reflections of the scale targets (Fehr, 1988). In terms of structural validity, the internal consistency of scores on the VIA-IS-R scales has been found acceptable in four samples, and three-month test-retest reliability statistics were high in one sample (McGrath, 2019; McGrath et al., 2020; McGrath & Wallace, 2021). Preliminary evidence has also been provided for measurement invariance across gender and race (McGrath et al., 2020), though limitations of these analyses will be noted shortly. Finally, substantive validity has been demonstrated and replicated. The final scales have been found to show expected relationships with behavioral criteria in three samples (McGrath, 2019; McGrath & Wallace, 2021).

The only component of the validation process as described above that is omitted from this list is confirmatory factor structure. So far, an adequate structure has not been identified for the VIA-IS-R as a whole (McGrath et al., 2020). This failure can be the result of at least two factors. It is notoriously difficult to achieve adequate fit in a confirmatory factor analysis (CFA) for highly multi-dimensional systems (Floyd & Widaman, 1995). This problem could be exacerbated by the emphasis on comprehensiveness rather than simple structure during the development of the VIA Classification, with the result that some of the character strengths are not effectively captured by shared latent variables. For example, strengths such as humility and humor are poorly reflected in the three commonly emerging factors of Caring, Inquisitiveness, and Self-Control. Third, though the 24 strengths are conceptually distinct, they all represent desirable characteristic and so in practice tend to correlate fairly strongly and positively with each other.

Two previous attempts have achieved some success in developing an adequate confirmatory factor structure for the 24 strengths in light of these obstacles.[3] The first focused on the issue of high multi-dimensionality and strengths by the common factors. Berger and McGrath (2018) attempted to identify a subset of strengths for which the three-factor structure demonstrated good fit. They settled on a set that included nine of the strengths, three per factor: gratitude, kindness, and love for Caring; creativity, curiosity, and learning for Inquisitiveness; and perseverance, prudence, and self-regulation for Self-Control. The adequacy of this solution has since been cross-validated (Lamade et al., 2020; McGrath et al., 2020), though it has the obvious weakness of accounting for very few of the 24 strengths. McGrath and Walker (2016) were also able to achieve adequate fit by using modification indices to loosen restraints on residual covariances between strengths. However, their solution was developed for adolescent measures of the VIA Classification, which tend to produce a different factor model, so it cannot be generalized to adults.

[3]Ng et al. (2017) were able to generate a solution of adequate fit for the original VIA Inventory, but they were addressing a different question, i.e., whether it was possible to identify a subset of items for which a bifactor model with 24 specific factors was acceptable.

Psychometric Approaches to Achieving Confirmatory Structure

In traditional CFA models, correlations between item residuals are set to zero. As noted previously, the resulting model often fails to meet traditional standards for good fit. This problem is typically addressed by freeing model constraints, in particular the residual covariances between observed variables, based on modification indices (Jorgensen, 2017). Although this practice can improve model fit, it has been criticized on at least two grounds. First, it has the potential for overfitting and capitalization on sample-specific covariation, resulting in model modifications that are not relevant to the population as a whole (e.g., MacCallum et al., 1992).

Second, modification often continues until standards for good fit are achieved, defined by meeting pre-established values on various indices of model fit such as the RMSEA. Though this strategy has been widely utilized in previous studies examining measurement models, it has been criticized. These cutoff values may not be equally applicable in all contexts (Chen et al., 2008). Different authors have also suggested different cutoff values, and even combinatorial alternatives (Hu & Bentler, 1999). Therefore, the stopping point for model modification using this strategy can be perceived as arbitrary (Hermida, 2015).

A recent alternative approach is called residual network modeling (RNM; Epskamp et al., 2017). Instead of freeing residual covariances in a stepwise manner as is typically done with modification indices, RNM computes partial correlations between pairs of observed variables controlling for all others. In *lvnet*, an R package that implements RNM, these correlations are freed with the goal of minimizing the least absolute shrinkage and selection operator (LASSO; Tibshirani 1996). LASSO adds a penalty for each freed residual correlation term, thereby reducing the risk of overfitting. As a result, LASSO models tend to demonstrate better performance during cross-validation, which suggests greater resistance against overfitting (Han

& Dawson, 2021a; McNeish, 2015). A description of RNM is provided by Epskamp et al. (2017), and a brief explanation is provided in the Supplementary Materials.

The Present Study

The present study was conducted to develop and cross-validate a model of adequate fit for adult measures of the Classification, with particular interest in the VIA-IS-R. Achieving this goal involved evaluating different approaches that have been found successful for achieving good fit. These include exploratory structural equation modeling, traditional CFA, CFA with bifactor structure, and two different strategies for identifying residual covariances to estimate: freeing residual covariances using modification indices, and freeing residual partial correlations using RNM.

Method

Participants

The present study used five samples, which we will refer to as the derivation, VIA cross-validation, Mechanical Turk cross-validation, representative, and college student cross-validation samples. The first four samples had completed all the scales of the VIA Assessment Suite for Adults (McGrath, 2019). Additional demographics for each of the first four samples can be found in the references cited.

Derivation Sample

The derivation sample consisted of 4,286 individuals who accessed the website of the VIA Institute on Character (www.viacharacter.org) between October 2015 and March 2016 and completed the English language version of a 120-item shortened version of the original VIA Inventory of Strengths (McGrath, 2019). There is no charge for completing the inventory at the site, and upon completion respondents receive personal feedback on their results. After receiving their feedback, they were asked if they would be willing to complete additional questionnaires for research purposes. Unfortunately, it is not possible to determine how many people rejected the request to participate. Those who continued were administered 309 additional items developed as candidates for inclusion in the VIA-IS-R as well as several other measures, including the GACS. This is the dataset from which the items of the VIA-IS-R were ultimately selected. For this study, scores for the VIA-IS-R were generated using the combination of items retained from the 120-item version and additional items from the 309-item set.

The sample was 77.67% female and 22.33% male. Educational level was quite high, as is typical of individuals who approach the VIA website. Only 5.70% had not attended college, and 40.35% had gone to graduate school. The most common country of origin was the United States (50.91%), followed by Australia (10.87%), Canada (7.36%), and the United Kingdom (6.01%). The remaining 24.85% were from a variety of countries. Mean age was 45.55 years (*SD* = 13.11). No compensation was provided for participation.

VIA Cross-Validation Sample

The VIA cross-validation sample consisted of a second group of adults who approached the VIA Institute website between August and October 2017 to complete the 120-item version of the VIA Inventory (McGrath & Wallace, 2021) and the GACS. These participants similarly responded to a request to volunteer for a research project after receiving feedback on their results. Again, the response rate is indeterminate. The sample consisted of 631 residents of the United States who completed all questionnaires and passed an attention check. The sample was 76.9% female, and 94.0% had attended at least some college. Mean age was 41.9 (*SD* = 13.1). No compensation was provided for participation.

Mechanical Turk Cross-Validation Sample

In June 2017 a sample was recruited through Amazon Mechanical Turk (mTurk) to complete a study of character strengths (McGrath & Wallace, 2021). The sample included 743 individuals who completed all questionnaires, including the VIA-IS-R and GACS, and passed an attention check. Participants resided in the United States and were fluent in English. This sample was 49.0% female, and 13.2% had not attended college. Mean age was 34.4 (*SD* = 10.2). Participants received $7 for completing the portion of the study that generated the data used in the present project.

Representative Sample

This sample was recruited in collaboration with the survey company Qualtrics. It approximated Census data for the U.S. adult population on the variables gender, age, education, race, and region of the country. The sample consisted of 1,765 individuals who completed all questionnaires and passed an attention check. Data were collected in October and November 2019. The sample was 51.8% female, and 40.0% had never attended college. Mean age was 46.5 (*SD* = 17.0). All were compensated, though amounts varied depending on the difficulty of recruiting within different demographic categories.

College Student Cross-Validation Sample

The final sample consisted of 471 college student participants recruited from a public university located in the Southern United States between April 2019 and November 2020. They completed only the GACS. The sample was 86.42% female, and the mean age was 22.2 (*SD* = 6.58). The participants were provided with a course credit upon the completion of the survey.

Measures

VIA Inventory of Strengths-Revised

The VIA-IS-R (McGrath, 2019) consists of 192 items, 8 items per scale, of which 81 are negatively keyed. Items are completed on a scale from 1 (*Very Much Unlike Me*) to 5 (*Very Much Like Me*). Coefficient alpha values were generated for VIA-IS-R strength scale scores in all four samples. The lowest value was .69, and only two of 96 estimates were < .70.

Global Assessment of Character Strengths

As noted above, the GACS is a 72-item questionnaire, three items per strength. For each strength, one item addresses how essential a part the strength is to who they are, one how natural and effortless it is to express the strength, and one item how uplifting or energizing they find expressing that strength. Responses are provided on a 7-point scale from *Very Strongly Disagree* to *Very Strongly Agree*. All 120 reliability estimates for the GACS across the five samples were ≥ .77.

Procedure

The first four samples completed both the VIA-IS-R (with the scores for the derivation sample computed by extracting the VIA-IS-R items from the larger set of items they completed) and the GACS. The college student cross-validation sample completed only the GACS. The Mechanical Turk cross-validation sample completed seventeen attention items distributed across the questionnaires. Participants were excluded if they answered at least four attention items (approximately 1/4) incorrectly. Excluded participants were still reimbursed for their time. All R source code and environment files used in the present study are shared via the Open Science Framework project page https://osf.io/gtxb9/.[4]

Exploratory Factor Analysis

Data collection for the first four samples was approved by the Institutional Review Board for Fairleigh Dickinson University; data collection for the final sample was approved by the Institutional Review Board for the University of Alabama. The five samples generated four sets of VIA-IS-R strength scores and five sets of GACS scores. The analysis proceeded as follows. First, two methods were applied to each of the nine data sets to determine the number of factors to retain in subsequent analyses. Parallel analysis involved generating 1000 data matrices of random normal data equal in size to the original data matrix, using principal components analysis to generate eigenvalues for each data matrix, and comparing the eigenvalues for the actual data to those for the random data. The number of factors to retain was based on the number of data-based eigenvalues that exceeded 95% of the corresponding eigenvalues from the random data (Glorfeld, 1995).

The minimum average partial procedure involved sequentially partialing each principal component from the data correlation matrix and computing the mean value for the resulting squared partial correlation matrix. Extraction stops when the mean squared partial correlation reaches a local minimum, suggesting further partialing is removing unique rather than shared

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[4]The first four of the five datasets were gathered under contract with the VIA Institute on Character, which requests data not be posted to the Internet. However, an Excel file with data for the questionnaires used in this study can be requested of the second author at REDACTED@ REDACTED.edu.

variance (Velicer, 1976). Velicer et al. (2000) suggested the accuracy of the procedure could be improved by raising the partial correlations to the fourth rather than second power.

Factor retention strategies were implemented using the RAWPAR and MAP functions in the *EFA.dimensions* package in R (O'Connor, 2020). The latter function generates results based on both variants of the minimum average partial procedure, so three estimates of the number of factors to retain was available for each of the nine datasets. Based on these results, iterative principal axis factor analyses with promax rotation (power = 4) were generated for each data set as a final step in determining the number of factors to retain. These analyses were also conducted using the *EFA.dimensions* package.

Confirmatory Factor Analyses

As the starting point for attempting to achieve good fit, we focused on the representative sample, for two reasons. First, it was a relatively large sample, the largest after the derivation sample. At the same time, population matching on the basis of demographics, combined with the administration of the VIA-IS-R as an integrated instrument (in contrast to the derivation sample members, who completed some of the VIA-IS-R items during administration of an earlier version and others as part of a larger set of new items) meant results from this sample would potentially demonstrate more generalizability than the larger derivation sample.

Loadings from the three-factor VIA-IS-R and GACS exploratory factor analyses for the representative sample were used to generate exploratory structural equation models using the *lavaan* package in R (Rosseel, 2012) with robust maximum likelihood estimation. For this and subsequent analyses, adequate fit was defined as CFI ≥ .90, TLI ≥ .90, RMSEA < .08, and SRMR < .08. Good fit was defined as CFI ≥ .95, TLI ≥ .95, RMSEA < .05, and SRMR < .05.

The next step involved more traditional CFA and bifactor CFA without cross-loadings on specific factors. Specifically, for each strength the loading was freed for that factor which loaded most highly on the strength in the representative sample. Finally, two model-building strategies were implemented. Both began using the factor loading matrix from the CFA model. As described previously, the first strategy was an iterative process in which one residual covariance was estimated in each step based on the largest modification index in the previous step until good fit was achieved (see Supplementary Materials for additional computational details). The second was the more automated process of estimating partial correlations based on LASSO likelihood function minimization as described by Epskamp et al. (2017) and implemented using the *lvnet* package in R (Epskamp, 2019). Once a residual covariance structure was developed using modification indices and a residual network of partial correlations was estimated via the RNM, we cross-validated the resulting models by applying them to the remaining samples and evaluating goodness of fit.

Results

Exploratory Factor Analysis

Across 12 tests of the number of factors to retain for the VIA-IS-R, three suggested three factors, six suggested four factors, and three suggested five factors. The results suggesting five factors only emerged in the derivation sample, the three-factor solutions only in the two samples not gathered through the VIA website. For the GACS, the college student sample generated one outcome suggesting one and another suggesting two factors. Of the remaining 13 tests, five suggested three factors, five suggested four, and three suggested five. Based on these results, exploratory factor analyses for each dataset were conducted retaining three, four, and five factors, for a total of 27 analyses. Pattern matrix loadings with absolute values ≥ .40 were considered evidence of a meaningful relationship between variable and factor.

For the VIA-IS-R, all four three-factor analyses replicated the three factors described previously, with loadings of .40 or higher on the relevant factors in all cases except one loading of .39 for perseverance. For the GACS, the mTurk sample generated a factor dominated by teamwork, leadership, and zest rather than a self-control factor, but in all other cases the standard three-factor solution was evident. The three-factor solution consistently emerged in solutions retaining four or five factors. The most common additional factor for the GACS was most closely associated with strengths involved in teamwork (8 of 10 analyses). Five of eight VIA-IS-R analyses generated a factor in which leadership was central, but teamwork was not. No other pattern emerged consistently. Based on these findings, the three-factor solution of Caring, Inquisitiveness, and Self-Control was deemed to have been replicated as the most reliable emergent structure relevant to both instruments.

However, using the loadings from the three-factor exploratory solutions to generate exploratory structural equation models in *lavaan* did not meet criteria for good fit (see Tables 3-4), though the three-factor solution for the GACS in the representative sample came close. This was also true for CFA and bifactor CFA solutions with three specific factors. For initial traditional and bifactor model CFAs, we also found that none of samples (including both the VIA and GACS) reported at least adequate model fit (see Tables 3-4).

Model Modification and Cross-Validation

As described above, two approaches to model modification were implemented. The first involved the use of modification indices one at a time in conjunction with the loadings from the three-factor CFA until good fit was achieved. This involved estimating 78 residual covariances for the VIA-IS-R and 51 for the GACS in the representative sample. However, when cross-validation was conducted, the modified models reported inadequate fit with several samples (see Tables 3-4). Tables S2 (for the VIA-IS-R) and S3 (for the GACS) in the Supplementary Materials report the estimated correlation coefficient for each freed residual term.

We then used RNM with LASSO for model improvement. The minimization function includes a tuning parameter, v, that controls the size of the penalty associated with freeing residual partial correlations (see the Supplementary Materials for details). By default, *lvnet* tests a range of values for this parameter from .01 to .50. Due to a convergence error, we increased the lowest value for the tested range to .10 for the GACS and .1175 for the VIA-IS-R, but retained the cap of .50. When the VIA-IS-R was examined, RNM identified $v = .17$ as the best LASSO tuning parameter value. In the case of the GACS, $v = .14$ was identified as the best value for the parameter.

The resulting residual network included 107 non-zero partial correlations for the GACS and 90 non-zero partial correlations for the VIA-IS-R. Both models were associated with adequate to good model fit in the representative sample (see Tables 3-4 for the full results). Tables S4 and S5 provides the list of partial correlations freed in each model. Readers who are interested in performing CFA with their VIA-IS-R or GACS data may use the list of freed covariances (in the case of modification via modification indices) or freed residual partial correlations (in the case of modification via RNM). The structural model was then applied to each of the other datasets. In all cases the results suggested adequate to good model fit.

Discussion

Prior research has not produced a satisfying latent structural model of the VIA character strengths in adults. In the present study, we employed both a well-established and a relatively new data-driven method for model modification to improve fit of the common three-factor solution for both the VIA-IS-R and GACS. The RNM improved the model fit for both instruments by identifying a network of residual partial correlations to be freed based on minimization of a likelihood function that penalizes complexity. The results showed that the RNM outperformed all other methods for measurement modeling, including exploratory structure equation modeling, traditional and bifactor CFA, and modification via modification indices. Furthermore, when compared with modification based on modification indices, RNM-based models were associated with better fit in all cross-validation samples. It suggests that the residual network estimated with the RNM can provide a generalizable residual correlation structure that is robust against overfitting, and superior to the traditional strategy based on modification indices identifying residual covariances to free.

Both strategies involved freeing a substantial number of residual terms: 28% of possible covariances were freed for the VIA-IS-R, 18% for the GACS. The numbers were even higher for RNM, which involved freeing 33% of possible correlations for the VIA-IS-R and 39% for the GACS. The larger number for the RNM is perhaps to be expected. Where the traditional approach using modification indices is terminated as soon as good fit is achieved, RNM will continue to free residual terms so long as the likelihood function continues to shrink. Also as expected, comparison of values in Table S2-S5 in the Supplementary Materials indicates that when freed covariances are standardized as correlation coefficients, they are consistently larger than the partial correlations estimated by RNM.

The fact that the RNM results consistently cross-validated, and that three of four goodness of fit indices include penalties for more complex models (only SRMR offers no such penalty), the large number of freed terms would seem not to be attributable to overfitting.

Instead, the finding probably reflects the reality of the character strengths. As noted earlier, as elements of positive personality the character strengths tend to correlate quite strongly with each other, beyond what can be accounted for by common factors alone. For example, previous studies have found the first principal component accounts for more than 40% of variability in the strength scales (McGrath, 2015; McGrath et al., 2010), and correlations between strength scales often exceed .50 (McGrath et al., 2010). Moreover, the smaller estimated correlation coefficients resulting from shrinkage and regularization by LASSO in RNM also perhaps contributed to prevention of overfitting.

Traditional CFA models, which limit the basis for scale inter-correlations to factor loadings and correlations between factors, are likely to result in a suboptimal model. We therefore recommend that future research on the latent structure of the character strengths, including investigations into measurement invariance, begin with the residual network outlined in the Supplementary Materials. Specifically, factor loadings would be freed as indicated in Table S1, and residual partial correlations would be freed as indicated in Tables S4 (for the VIA-IS-R) and S5 (for the GACS). Note that these are correlations rather than covariances, and so would require computation using *lvnet* or some other package that instantiates RNM rather than the more familiar CFA packages such as *lavaan*.

The findings from the present study provide several implications for future studies. First, the three-factor model was supported across diverse populations through data-driven factor analysis method. So far, there have been debates about whether the VIA model can produce a consistent, reliable, and valid structural model. Previous studies conducted with conventional psychometrical methods have reported a variety of measurement model solutions (Han, 2019). In the present study, we applied the RNM and then showed that the validity of the three-factor model was replicable across multiple independent datasets. At the conceptual level, the three-factor model can be proposed as the best structure for the 24 character strengths across populations and measurement tools. Thus, it will be able to inform future studies on how character strengths are organized and structured in human psychology, such as research on virtues and values (Han, 2019). Given researchers interested in the topic have been concerned about whether the VIA model can be employed as a reliable and valid tool for their research due to the inconsistent factor models reported across different studies, the finding from the present study might contribute to addressing their concerns and establishing the conceptual and theoretical basis of such research.

Second, from at the practical level, the data-driven RNM method provides additional insights into the measurement of latent personality structure across diverse datasets. A model developed through a data-driven approach, which was employed in the present study, is more likely robust against overfitting and less biased (Han, 2021; McNeish, 2015). The improved cross-validation resulting from freeing error covariances is consistent with concerns that have been raised about the poor fit of CFA models that only attempt to account for correlation between observed variables across factors only through factor intercorrelations when applied to personality variables such as character strengths (Hopwood & Donellan, 2010). In fact, we also demonstrated that the RNM-identified model was better cross-validated across multiple datasets compared with the models identified by conventional factor analysis methods. Hence, researchers who are concerned about producing a measurement model that can be well generalized across diverse populations and datasets may consider employing data-driven methods that allow for secondary relationships between variables (Han & Dawson, 2021b), such as the RNM.

Limitations and Future Directions

The study and methodology used here has some limitations that should be mentioned. Four of the samples were collected online, and the fifth consisted of college students. Accordingly, even the representative sample was restricted to individuals with a certain level of computer expertise. However, the reality is that this limitation generally applies to samples used in character strength research. Moreover, the data was collected within Western countries, primarily the United States, so cross-cultural and cross-language validation should be addressed in future studies. Additionally, it would be worth noting that in several datasets, the VIA cross-validation and college student cross-validation datasets in particular, the majority of the participants were females. The use of compensated participants in the other samples was specifically intended to offset demographic limitations of the self-selected samples. Of course, in the whole dataset, sufficient number of responses were collected from both male and female participants, so the aforementioned point in the two specific datasets would not be a major issue in the present study in general.

It should also be noted that the iterative RNM process requires substantial computational resources. The RNM estimations for the present study took approximately 2-3 hours to run on a computer equipped with a multicore processor. Furthermore, when a convergence issue occurred, the RNM procedure was trapped in an infinite loop and could not complete. We had to identify the issue and adjust the tuning parameter manually. Users who do not have sufficient background knowledge in computer programming and computational statistics might find the effective use of *lvnet* daunting.

Currently, the developer of *lvnet* has discontinued technical support of the package (S. Epskamp, personal communication, January 6, 2021). He recommended use of an alternative package going forward, *psychonetrics* (Epskamp, 2020). However, unlike *lvnet*, *psychonetrics* does not apply LASSO in regularizing estimated residuals, so we used *lvnet* in the current study because one of our main goals was to avoid overfitting. Future research could incorporate continuing developments in model building by examining how *psychonetrics* performs relative to our findings. However, we hope new packages will emerge that incorporate the very valuable strategy of adding a penalty for model complexity to the likelihood function. It is also worth noting that Pan et al. (2017) have developed an algorithm that combines model-building based on modification indices and a standard based on a Bayesian LASSO. Also, in a recent personal correspondence with Epskamp (S. Epskamp, personal communication, August 12, 2021), he expressed his interest in implementing LASSO in the RNM and *psychonetrics* in a long term. This is an area in which new options are swiftly emerging, and alternative strategies will merit consideration, though we believe that the success of our cross-validation across multiple samples suggests the value of the models we propose here for these two measures.

Declarations

Funding

No funding was received for conducting this study.

Ethics Approval

Ethical approval for this study was obtained from the Institutional Review Board of Fairleigh Dickinson University and the University of Alabama. Informed consent was obtained from all participants in this study.

Conflicts of interest/Competing interests

REM is a Senior Scientist for the VIA Institute on Character, which is the copyright holder for the two instruments used in this study. Four of the samples used in this study were collected with support from the VIA Institute on Character.

Availability of data and material

The data that support the findings of this study are openly available in the Open Science Framework at https://osf.io/gtxb9/

Authors' contributions

HH and REM designed research; HH and REM collected and analyzed data; HH and REM wrote the paper.

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Table 1

The VIA Classification of Character Strengths and Virtues

Virtues	Character Strengths
Wisdom & Knowledge	Creativity [originality, ingenuity]
	Curiosity [interest, novelty-seeking, openness to experience]
	Judgment & Open-Mindedness [critical thinking]
	Love of Learning
	Perspective [wisdom]
Courage	Bravery [valor]
	Perseverance [persistence, industriousness]
	Honesty [authenticity, integrity]
	Zest [vitality, enthusiasm, vigor, energy]
Humanity	Capacity to Love and Be Loved
	Kindness [generosity, nurturance, care, compassion, altruistic love, "niceness"]
	Social Intelligence [emotional intelligence, personal intelligence]
Justice	Teamwork [citizenship, social responsibility, loyalty]
	Fairness
	Leadership
Temperance	Forgiveness & Mercy
	Modesty & Humility
	Prudence
	Self-Regulation [self-control]
Transcendence	Appreciation of Beauty and Excellence [awe, wonder, elevation]
	Gratitude
	Hope [optimism, future-mindedness, future orientation]
	Humor [playfulness]
	Religiousness & Spirituality [faith, purpose]

Note. Adapted from *Character Strengths and Virtues: A Handbook and Classification* (pp. 29-30), by C. Peterson & M. E. P. Seligman, 2004, American Psychological Association/Oxford University Press. Copyright 2004 by the VIA Institute on Character. Adapted with permission. Terms in brackets are variants of the character strength according to Peterson and Seligman (2004).

Table 2

The Empirically Derived Virtues Model

Virtues	Character Strengths
Caring	Fairness
	Gratitude
	Kindness
	Capacity to Love and Be Loved
	Teamwork
	Forgiveness & Mercy
	Appreciation of Beauty and Excellence
	Leadership
	Humor
	Religiousness & Spirituality
Inquisitiveness	Creativity
	Curiosity
	Perspective
	Bravery
	Judgment & Open-Mindedness
	Love of Learning
	Zest
	Appreciation of Beauty and Excellence
	Hope
	Humor
	Social Intelligence
Self-Control	Honesty
	Judgment & Open-Mindedness
	Perseverance
	Prudence
	Modesty & Humility
	Perspective
	Self-Regulation
	Fairness

Note. McGrath et al. (2018) examined loadings from 12 data sets where factor analyses of the VIA strengths retained three factors in earlier measures of the strengths. A strength is associated with a virtue in this table if the relevant loading was ≥ .40 in at least 3/4 of the data sets. Within a virtue, strengths are listed in relative order of number of loadings that were .40 or higher. Five strengths cross-load: appreciation of beauty and excellence, fairness, humor, judgment & open-mindedness, and perspective.

Table 3

Model Fit Statistics for the VIA Inventory of Strengths-Revised

Method and sample	RMSEA	SRMR	CFI	TLI	Fit
ESEM					
Representative	.086	.072	.860	.857	Inadequate
Derivation	.101	.080	.753	.747	Inadequate
VIA	.142	.167	.504	.493	Inadequate
mTurk	.153	.221	.612	.604	Inadequate
CFA					
Representative	.104	.078	.809	.789	Inadequate
Derivation	.115	.086	.703	.670	Inadequate
VIA	.117	.095	.716	.686	Inadequate
mTurk	.128	.090	.765	.739	Inadequate
Bi-factor CFA					
Representative	.095	.062	.856	.826	Inadequate
Derivation	.107	.075	.762	.712	Inadequate
VIA	.110	.083	.767	.718	Inadequate
mTurk	.118	.079	.817	.778	Inadequate
Modification with modification indices					
Representative	.050	.040	.970	.952	Good
Derivation	.070	.055	.924	.877	Inadequate
VIA	.066	.060	.937	.898	Inadequate
mTurk	.080	.056	.936	.897	Inadequate
Modification with RNM					
Representative	.046	.026	.976	.959	Good
Derivation	.060	.041	.948	.909	Adequate
VIA	.065	.051	.944	.903	Adequate
mTurk	.070	.044	.955	.922	Adequate

Note. ESEM = exploratory structural equation modeling; CFA = confirmatory factor analysis; RNM = residual network modeling. VIA refers to the VIA cross-validation sample. Good fit: RMSEA < .05, SRMR < .05, CFI ≥ .95, TLI ≥ .95. Adequate fit: RMSEA < .08, SRMR < .08, CFI ≥ .90, TLI ≥ .90. Inadequate fit: RMSEA ≥ .08, SRMR ≥ .08, CFI < .90, TLI < .90.

Table 4

Model Fit Statistics for the Global Assessment of Character Strengths

Method and sample	RMSEA	SRMR	CFI	TLI	Fit
ESEM					
Representative	.074	.063	.917	.915	Adequate
Derivation	.087	.065	.822	.819	Inadequate
VIA	.090	.077	.798	.793	Inadequate
mTurk	.079	.068	.874	.872	Inadequate
College student	.074	.061	.912	.910	Adequate
CFA					
Representative	.092	.055	.883	.870	Inadequate
Derivation	.111	.083	.732	.703	Inadequate
VIA	.118	.098	.678	.644	Inadequate
mTurk	.101	.076	.812	.791	Inadequate
College student	.097	.056	.863	.848	Inadequate
Bi-factor CFA					
Representative	.084	.049	.911	.892	Inadequate
Derivation	.103	.071	.788	.743	Inadequate
VIA	.104	.080	.770	.722	Inadequate
mTurk	.092	.068	.857	.827	Inadequate
College student	.091	.053	.887	.864	Inadequate
Modification with modification indices					
Representative	.050	.032	.973	.962	Good
Derivation	.077	.056	.899	.859	Inadequate
VIA	.081	.071	.880	.832	Inadequate
mTurk	.069	.051	.931	.903	Adequate
College student	.066	.040	.949	.929	Adequate
Modification with RNM					
Representative	.022	.010	.996	.993	Good
Derivation	.057	.030	.960	.922	Adequate
VIA	.062	.037	.950	.902	Adequate
mTurk	.048	.026	.976	.953	Good
College student	.064	.024	.967	.935	Adequate

Note. ESEM = exploratory structural equation modeling; CFA = confirmatory factor analysis; RNM = residual network modeling. VIA refers to the VIA cross-validation sample. Good fit: RMSEA < .05, SRMR < .05, CFI ≥ .95, TLI ≥ .95. Adequate fit: RMSEA < .08, SRMR < .08, CFI ≥ .90, TLI ≥ .90. Inadequate fit: RMSEA ≥ .08, SRMR ≥ .08, CFI < .90, TLI < .90.

Supplementary Materials

Algorithm for Model Modification with Modification Indices

The procedure used to add residual covariances based on modification indices proceeded in a stepwise manner. After performing confirmatory factory analysis (CFA), modification indices were obtained by using *modindices* function provided by *lavaan* (Rosseel, 2012). The CFA model was modified until good model fit (RMSEA < .05, SRMR < .05, CFI ≥ .95, TLI ≥ .95) was achieved as described in the pseudo code below:

```
COMPUTE CFA with no covariances freed
WHILE RMSEA ≥ .05 OR SRMR ≥ .05 OR CFI < .95 OR TLI < .95
    COMPUTE modification indices
    ADD residual covariance with the largest modification index to model
    COMPUTE CFA with revised model
ENDWHILE
```

Residual Network Modeling with LASSO[5]

Where y indicates a vector of observed variable values, η are latent factor scores, Λ are factor loadings, and ε residual terms in a measurement model,

$$y = \Lambda\eta + \varepsilon.$$

Then, Σ, a variance-covariance matrix that contains the variance of each observed variable in its diagonal and the covariances between those variables in its off-diagonal elements can be estimated as follows:

$$\Sigma = \Lambda\Psi\Lambda^T + \Theta,$$

[5] See Epskamp et al. (2017) for further details.

where Ψ is the variance-covariance matrix of latent factor scores, Var(η), and Θ is the variance-covariance matrix of residuals, Var(Θ). CFA attempts to minimize the difference between a sample variance-covariance matrix, S, and a variance-covariance matrix based on the assumed model, Σ. In general, maximum likelihood (ML) estimation is utilized to estimate Σ. ML estimation attempts to minimize -2 log(*likelihood*),

$$-2\log(likelihood) = \log|\Sigma| + Tr[S\Sigma^{-1}] - \log|S| + P$$

where P indicates the number of observed variables. In the ideal case, $\Sigma = S$. The goal is to minimize -2 log(*likelihood*) through ML estimation.

In RNM, residual relationships between observed variables are estimated using the partial correlation between the two variables after controlling for all other variables (see Epskamp et al., 2018, for further details). The partial correlation coefficient between variables i and j can be symbolized by:

$$w_{ij} = w_{ji}.$$

RNM implemented in the *lvnet* package in R (Epskamp, 2019) also differs from traditional CFA in that it uses the LASSO, which penalizes unnecessarily complicated models with the goal of reducing overfitting. LASSO searches for the Σ that minimizes the following term:

$$log|\Sigma| + Tr[S\Sigma^{-1}] - \log|S| + P + vPenalty.$$

The LASSO penalty is:

$$Penalty = Sum(|w_{ij}|)$$

and v is a tuning parameter that adjusts the size of the penalty. As a result, improvement in agreement between the matrices Σ and S must exceed the penalty resulting from the addition of a non-zero partial correlation to the model to justify freeing the correlation. Note that if v is set to

0, the formula matches the traditional minimization function. The larger v is set (approaching ∞), the fewer partial correlations will be freed.

lvnet evaluates a range of values for v based on minimization of an Extended Bayesian Information Criterion value. By default, this range is set to [.01, .50], meaning the penalty is iteratively set to 1/100th the sum of the freed residual partial correlations and increased in 20 increments until it is half that sum.

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Supplementary Tables

Table S1

Freed Factor Loadings

Factor	Associated Strengths
Caring	Beauty
	Fairness
	Forgiveness
	Gratitude
	Honesty
	Hope
	Humility
	Kindness
	Love
	Spirituality
Self-Control	Bravery
	Leadership
	Perseverance
	Prudence
	Self-Regulation
	Teamwork
	Social Intelligence
	Zest
Inquisitiveness	Creativity
	Curiosity
	Learning
	Perspective
	Humor
	Judgment

Note. Freed associations between character strengths and factors were based on the largest loading for each strength in the representative sample.

Table S2

Freed Covariances via Modification Indices for the VIA Inventory of Strengths- Revised

Variable 1	Variable 2	Correlation Coefficient
Beauty	Creativity	.18
Beauty	Curiosity	.31
Beauty	Forgiveness	-.16
Beauty	Hope	-.25
Beauty	Learning	.27
Beauty	Perseverance	-.22
Beauty	Self-regulation	-.14
Beauty	Zest	-.15
Bravery	Creativity	.24
Bravery	Humility	-.15
Bravery	Leadership	.34
Bravery	Prudence	-.17
Creativity	Curiosity	.27
Creativity	Humility	-.21
Creativity	Leadership	.31
Creativity	Learning	.23
Creativity	Perspective	.12
Curiosity	Learning	.34
Curiosity	Social intelligence	-.12
Fairness	Forgiveness	.27
Fairness	Honesty	.25
Fairness	Hope	-.28
Fairness	Humility	.23
Fairness	Judgment	.10
Fairness	Kindness	.19
Fairness	Love	-.14
Fairness	Zest	-.24
Forgiveness	Bravery	-.15
Forgiveness	Creativity	-.16
Forgiveness	Honesty	.14
Forgiveness	Humility	.15
Forgiveness	Learning	-.13
Forgiveness	Spirituality	.09
Gratitude	Curiosity	.22
Gratitude	Honesty	.22

Variable 1	Variable 2	Correlation Coefficient
Gratitude	Humility	.25
Gratitude	Spirituality	.33
Gratitude	Teamwork	-.18
Honesty	Humility	.15
Hope	Curiosity	.39
Hope	Humor	.25
Hope	Kindness	-.42
Hope	Love	-.16
Hope	Perseverance	.13
Hope	Spirituality	.15
Hope	Teamwork	-.20
Hope	Zest	.20
Humility	Judgment	.12
Humor	Learning	-.15
Humor	Perspective	-.13
Humor	Self-regulation	-.14
Judgment	Prudence	.43
Judgment	Self-regulation	.17
Kindness	Honesty	.34
Kindness	Humility	.14
Kindness	Zest	-.24
Leadership	Humility	-.25
Leadership	Perspective	.22
Leadership	Social intelligence	.15
Learning	Judgment	.12
Learning	Social intelligence	-.27
Love	Curiosity	.17
Love	Humility	-.12
Love	Judgment	-.12
Love	Social intelligence	.17
Perseverance	Self-regulation	.21
Perspective	Judgment	.20
Perspective	Prudence	.21
Prudence	Self-regulation	.31
Social intelligence	Prudence	.11
Spirituality	Curiosity	.10
Spirituality	Honesty	.13
Teamwork	Bravery	-.14
Teamwork	Zest	.08

Variable 1	Variable 2	Correlation Coefficient
Zest	Curiosity	.38
Zest	Humor	.17
Zest	Perseverance	.26
Zest	Self-regulation	.17

Note. Values for the correlations were derived using the representative sample. These values were drawn from the "std.all" column in the *lavaan* output with the "standardization = TRUE" option.

Table S3

Freed Covariances via Modification Indices for the Global Assessment of Character Strengths

Variable 1	Variable 2	Correlation Coefficient
Beauty	Creativity	.27
Beauty	Curiosity	.25
Beauty	Forgiveness	-.21
Beauty	Gratitude	.13
Beauty	Learning	.22
Bravery	Leadership	.22
Bravery	Perspective	.18
Bravery	Prudence	-.13
Creativity	Curiosity	.41
Creativity	Learning	.21
Curiosity	Judgment	.16
Curiosity	Learning	.34
Fairness	Honesty	.20
Fairness	Kindness	.27
Fairness	Love	-.15
Fairness	Teamwork	.14
Forgiveness	Gratitude	-.19
Forgiveness	Honesty	-.12
Forgiveness	Prudence	.15
Forgiveness	Spirituality	.16
Gratitude	Hope	.17
Gratitude	Kindness	.13
Gratitude	Self-regulation	.11
Honesty	Kindness	.17
Hope	Spirituality	.32
Hope	Zest	.13
Humility	Love	-.19
Humility	Prudence	.33
Humility	Self-regulation	.19
Kindness	Leadership	-.10
Kindness	Love	.26
Leadership	Humor	.13
Leadership	Judgment	.14
Leadership	Perspective	.23
Leadership	Social intelligence	.23

Variable 1	Variable 2	Correlation Coefficient
Leadership	Teamwork	.39
Learning	Humor	-.21
Perseverance	Judgment	.19
Perseverance	Perspective	.20
Perseverance	Social intelligence	-.17
Perseverance	Zest	-.20
Perspective	Humor	-.18
Perspective	Judgment	.25
Prudence	Judgment	.17
Prudence	Self-regulation	.36
Self-regulation	Judgment	.16
Social intelligence	Judgment	.20
Social intelligence	Perspective	.31
Spirituality	Prudence	.13
Teamwork	Social intelligence	.22
Zest	Learning	-.13

Note. Values for the correlations were derived using the representative sample. These values were drawn from the "std.all" column in the *lavaan* output with the "standardization = TRUE" option.

Table S4

Freed Residual Partial Correlations via RNM for the VIA Inventory of Strengths-Revised

Variable 1	Variable 2	Partial Correlation Coefficient
Beauty	Creativity	.05
Beauty	Curiosity	.23
Beauty	Forgiveness	-.05
Beauty	Gratitude	.17
Beauty	Hope	-.04
Beauty	Kindness	.09
Beauty	Learning	.14
Beauty	Love	.10
Beauty	Perseverance	-.16
Beauty	Self-regulation	-.08
Beauty	Teamwork	.11
Beauty	Zest	-.07
Bravery	Creativity	.18
Bravery	Forgiveness	-.12
Bravery	Humility	-.01
Bravery	Leadership	.30
Bravery	Perspective	.10
Bravery	Prudence	-.16
Bravery	Teamwork	-.17
Creativity	Curiosity	.19
Creativity	Forgiveness	-.09
Creativity	Humility	-.11
Creativity	Humor	.13
Creativity	Leadership	.19
Creativity	Learning	.21
Creativity	Perspective	.13
Creativity	Self-regulation	.00
Curiosity	Hope	.17
Curiosity	Learning	.31
Curiosity	Self-regulation	-.07
Curiosity	Zest	.23
Fairness	Forgiveness	.26
Fairness	Honesty	.12
Fairness	Hope	-.07
Fairness	Humility	.12

Variable 1	Variable 2	Partial Correlation Coefficient
Fairness	Judgment	.07
Fairness	Kindness	.14
Fairness	Learning	.10
Fairness	Prudence	.03
Fairness	Zest	-.15
Forgiveness	Humility	.03
Forgiveness	Leadership	.00
Forgiveness	Perspective	-.09
Forgiveness	Spirituality	.07
Gratitude	Honesty	.12
Gratitude	Humility	.15
Gratitude	Spirituality	.30
Gratitude	Teamwork	-.16
Gratitude	Zest	-.07
Honesty	Humility	.09
Honesty	Kindness	.20
Honesty	Zest	-.07
Hope	Kindness	-.19
Hope	Leadership	-.01
Hope	Love	-.08
Hope	Social intelligence	-.03
Hope	Spirituality	.10
Hope	Teamwork	-.14
Humility	Humor	-.03
Humility	Judgment	.12
Humility	Kindness	.05
Humility	Leadership	-.20
Humility	Love	-.09
Humility	Self-regulation	.10
Humility	Zest	-.10
Humor	Self-regulation	-.23
Humor	Spirituality	-.11
Judgment	Learning	.12
Judgment	Love	-.09
Judgment	Perspective	.11
Judgment	Prudence	.40
Judgment	Zest	-.05
Kindness	Learning	.09
Kindness	Self-regulation	-.13

Variable 1	Variable 2	Partial Correlation Coefficient
Kindness	Social intelligence	.11
Kindness	Zest	-.15
Leadership	Perseverance	.11
Leadership	Perspective	.19
Leadership	Prudence	-.04
Leadership	Social intelligence	.19
Leadership	Spirituality	-.03
Leadership	Teamwork	.09
Learning	Perspective	.08
Learning	Teamwork	.07
Love	Social intelligence	.15
Perspective	Prudence	.19
Perspective	Social intelligence	.15
Perspective	Zest	-.06
Prudence	Self-regulation	.25
Prudence	Zest	-.04

Note. Values for the correlations were derived using the representative sample.

Table S5

Freed Residual Partial Correlations via RNM for the Global Assessment of Character Strengths

Variable 1	Variable 2	Partial Correlation Coefficient
Beauty	Creativity	.16
Beauty	Curiosity	.09
Beauty	Fairness	-.09
Beauty	Forgiveness	-.14
Beauty	Honesty	-.10
Beauty	Learning	.08
Beauty	Teamwork	-.04
Bravery	Creativity	.05
Bravery	Perspective	.10
Curiosity	Creativity	.35
Fairness	Honesty	.11
Fairness	Kindness	.11
Fairness	Love	-.24
Fairness	Teamwork	.14
Fairness	Zest	-.18
Forgiveness	Curiosity	-.01
Forgiveness	Honesty	-.10
Forgiveness	Judgment	-.08
Forgiveness	Learning	-.04
Forgiveness	Perspective	-.09
Forgiveness	Teamwork	.06
Gratitude	Beauty	.13
Gratitude	Bravery	-.06
Gratitude	Forgiveness	-.11
Gratitude	Social intelligence	-.11
Gratitude	Zest	-.10
Honesty	Perseverance	.12
Honesty	Perspective	.06
Hope	Fairness	-.06
Hope	Gratitude	.18
Hope	Honesty	-.08
Hope	Humility	-.07
Hope	Judgment	-.05
Hope	Kindness	-.11
Hope	Perseverance	.15

Variable 1	Variable 2	Partial Correlation Coefficient
Hope	Perspective	-.07
Hope	Prudence	-.08
Hope	Teamwork	.07
Hope	Zest	.08
Humility	Forgiveness	.04
Humility	Learning	-.07
Humility	Love	-.18
Humility	Perseverance	.07
Humor	Curiosity	.13
Humor	Forgiveness	-.06
Humor	Gratitude	.07
Humor	Leadership	.16
Humor	Prudence	-.05
Humor	Self-regulation	-.12
Judgment	Curiosity	.16
Kindness	Bravery	-.07
Kindness	Love	.13
Kindness	Perseverance	-.06
Kindness	Zest	-.16
Leadership	Bravery	.24
Leadership	Creativity	.07
Leadership	Judgment	.03
Leadership	Kindness	-.07
Leadership	Perspective	.14
Leadership	Social intelligence	.13
Leadership	Teamwork	.39
Leadership	Zest	.09
Learning	Creativity	.11
Learning	Curiosity	.32
Learning	Judgment	.04
Love	Curiosity	-.07
Perseverance	Bravery	.21
Perseverance	Judgment	.13
Perseverance	Perspective	.16
Perspective	Creativity	.02
Perspective	Curiosity	.04
Perspective	Judgment	.22
Perspective	Learning	.21
Prudence	Forgiveness	.08

Variable 1	Variable 2	Partial Correlation Coefficient
Prudence	Humility	.25
Prudence	Judgment	.10
Prudence	Kindness	-.07
Prudence	Leadership	.08
Prudence	Love	-.04
Prudence	Perseverance	.04
Self-regulation	Humility	.04
Self-regulation	Judgment	.10
Self-regulation	Kindness	-.14
Self-regulation	Love	-.11
Self-regulation	Perseverance	.11
Self-regulation	Prudence	.30
Social intelligence	Honesty	-.11
Social intelligence	Judgment	.10
Social intelligence	Perspective	.24
Social intelligence	Zest	.04
Spirituality	Curiosity	-.06
Spirituality	Fairness	-.09
Spirituality	Forgiveness	.09
Spirituality	Hope	.27
Spirituality	Humor	-.07
Spirituality	Judgment	-.06
Spirituality	Kindness	-.06
Spirituality	Learning	-.02
Spirituality	Prudence	.12
Teamwork	Curiosity	-.04
Teamwork	Honesty	-.05
Teamwork	Perspective	-.08
Teamwork	Social intelligence	.21
Teamwork	Zest	.11
Zest	Bravery	.09
Zest	Creativity	.12
Zest	Learning	-.07

Note. Values for the correlations were derived using the representative sample.