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THE LOGICAL RECONSTRUCTION OF PURE EXCHANGE ECONOMICS: ANOTHER ALTERNATIVE

INTRODUCTION

The structuralist metascience approach of Sneed (1971) and Stegmüller (1976, 1979) has recently been applied to Walrasian pure exchange economics (PEE) by Balzer (1982a) and Haslinger (1983). The second of these papers (Haslinger) was written in order to rectify at least two perceived inadequacies of the earlier paper (Balzer). First, Haslinger argued that "utility" was not as essential to PEE as Balzer had initially claimed. Second, Haslinger argued that Balzer's reconstruction "did not do justice to the standard texts" (p. 115) of general equilibrium economics. This latter inditement is particularly important since both authors agree that textual fidelity is a necessary (though not sufficient) condition for an adequate structuralist reconstruction.

In this paper we will argue that while most of Haslinger's criticisms of Balzer (presented in Section I of his paper) are valid, he too fails to do justice to the standard texts in a variety of ways.[1] For one thing, Haslinger's reconstruction *still* places unnecessary emphasis on utility and utility functions. For another thing, Haslinger's characterization of equilibrium is inadequate (actually Balzer's approach is favored on this issue). And finally, Haslinger overstates the magnitude of the qualitative comparative statics information available from PEE as well as the importance of the gross substitute assumption. These criticisms constitute Section I of our paper.

In Section II we offer our own reconstruction of PEE which uses market excess demand as the fundamental concept and provides a more accurate portrayal of what appears in the standard economic texts and theoretical publications than the reconstructions of either Balzer or Haslinger. In Section III we point out how our reconstruction of PEE differs (particularly with respect to empirical hypotheses) from structuralist reconstructions in physical science (such as Sneed (1971) and Moulines (1975))

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and discuss some other authors who have also made the same realization. The closing paragraphs of Section III discuss the metascientific implications of these differences.

I. HASLINGER'S RECONSTRUCTION OF PEE

An important part of any structuralist recostruction of a scientific theory is the classification of the terms and functions of the reconstructed theory into those which are "theoretical" and those which are "non-theoretical".[2] For Balzer "the central and crucial concept" (p. 23) of the *utility function* is PEE-theoretical (p. 32). In his reconstruction, the central axiom of PEE specify the "maximization of utilities" (pp. 26–27), and applications (or uses) of the theory are or are not "empirical" depending on exactly the way in which utility is used (pp. 34–35).

Haslinger considers both the importance and the theoreticity of utility to be a negative aspect of Balzer's reconstruction. Haslinger considers the *demand function* (a concept totally neglected by Balzer), rather than utility, to be the most important PEE-theoretical concept of his reconstruction (p. 123). The demand function simply describes the quantities of each good which the utility maximizing consumer desires to purchase at various prices and initial endowments. This concept is extremely important in Haslinger's reconstruction since it allows the non-theoretical concepts such as price, utility, and initial endowments to be combined in what Haslinger considers to be the three fundamental axioms of PEE (p. 123).

In order to clarify the relationship between these two concepts, the utility function and the demand function, it is helpful to examine neoclassical consumer choice theory in more detail. Consider a pure exchange (no production) economy with H individuals (or households) and n commodities. We will index the individuals by $h = 1, 2, \ldots, H$ and the commodities by $i = 1, 2, \ldots, n$. Each individual h has a utility function $U^h: R^n_+ \rightarrow R$ which associates a real number with each "bundle" of the n commodities.[3] Each individual also has an initial endowment of the n commodities which is given by $\omega^h = (\omega^h_1, \omega^h_2, \ldots, \omega^h_n) \in R^n_{++}$. if the prices in the economy are $p = (p_1, p_2, \ldots, p_n) \in R^n_{++}$, then the budget set (or affordable set) for individual h is given by,

$$B^h(p) = \{x \in R^n_+ \mid px \leqq p\omega^h\}.$$

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Given this budget set, the choice problem for the individual is simply to choose the bundle of commodites within this set which maximizes his or her utility. This optimal choice, written as a function of the prices, and parameterized by the initial endowment, is the *demand function* for the individual. Thus, individual h has the demand function,

$$x^h(p;\omega^h) \;=\; (x_1^h(p;\omega^h),\;.\;.\;.\;,\;x_n^h(p;\;\omega^h)).$$

Often the endowment parameter is surpressed and the demand function for individual h is written simply as $x^h(p)$.

While Haslinger is critical of Balzer for his emphasis on utility, Haslinger's own reconstruction also depends on utility in an essential way. Since the individual demand function is merely the result of the constrained maximization of the utility function, working with such individual demand functions does not remove the fundamental dependency of the theory on the utility concept. Haslinger's second axiom of PEE (p. 123) is particularly telling in this regard. The axiom requires the *actual* demands of individuals (observable and nontheoretical) to actually *be* demand functions (i.e., derived from utility maximization). This axiom makes Haslinger's no less dependent on utility than Balzer's.[4]

If we intend to capture what is in the standard textbooks and what practicing general equilibrium theorists actually do, it is necessary to remove utility even more than Haslinger has done. Even a cursory examination of what general equilibrium theorists actually do when they do research in PEE, will demonstrate that neither *individual* utilities nor *individual* demands are very important. This is because PEE is a theory about prices in *competitive markets*. In competitive markets prices are *not* determined by individuals or individual demands. As the above sketch of the relationship between utility and demand clearly shows, individuals do *not* take prices as choice variables. Individual demand functions are the result of maximizing utility for *given* prices, these prices are not directly determined by individuals. In PEE both individual utilities *and* individual demands are one-step-removed from what "really" matters. What really matters is *market* demand and *market* supply. It is these *market* functions which should be the fundamental concept in any reconstruction which is true to the economic literature.[5]

Return to the above symbolism, the *market demand* for good i is given by,

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$$x_i(p) = \sum_{h=1}^{H} x_i^h(p).$$

The market demand function is then, $x(p) = (x_1(p), \ldots, x_n(p))$. Since we are considering only a pure exchange exonomy, the market supply is fixed by the total available endowment and is thus independent of prices. We can therefore write the supply of good i as,

$$\omega_i = \sum_{h=1}^{H} \omega_i^h,$$

and the total market supply as $\omega = (\omega_1, \ldots, \omega_n)$. The measure of the difference between demand and supply is *excess demand*. Using the symbol z for excess demand, we have that,

$$z_i(p) = x_i(p) - \omega_i,$$

is the excess demand for good i, and $z(p) = (z_1(p), \ldots, z_n(p))$ is the *excess demand function* for the economy. It is this excess demand function which actually determines prices in PEE. If $z_i(p) > 0$ then p_i will increase and if $z_i(p) < 0$ then p_i will decrease. If a $p^* = (p^*_1, \ldots, p^*_n)$ is reached such that $z_i(p^*) = 0$ for all $i = 1, 2, \ldots, n$ then p^* is the general equilibrium price vector.

Thus, the market excess demand function captures both market demand amd market supply. It determines the way in which prices change outside of equilibrium and fully characterizes the equilibrium when reached. For this reason, it is market excess demand, rather than either utility or individual demands, which should be the fundamental concept in any reconstruction of PEE. Certainly excess demand is treated as the fundamental concept by practicing general equilibrium theorists. The traditionally important research questions of PEE have been the "existence", "uniqueness", and "stability" of the general equilibrium price vector p^*. For these traditional questions economic theorists have employed mathematical structures with market excess demand as the fundamental concept. Utility or even individual demands do not enter this theoretical work at all.[6]

While any reconstruction of PEE which is true to text and theoretical practice should start from the concept of market excess demand, it is also true that utility maximization must "lie behind" these excess demand

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functions.[7] How is it possible to reconstruct PEE in a way which makes *the market* (and thus excess demand) primary, and still account for the underlying utility maximization? This apparent dilemma can be resolved by using the theoretical results of Debreu, Mantel, McFadden, MasColell, Richter and Sonnenschein.[8] Basically, this theoretical literature concludes that any function which has the characteristics usually attributed to an excess demand function by general equilibrium theorists can be generated from an economy of utility maximizing individuals. A reconstruction which utilizes these results (and explains them further) is contained in Section II below. At this point we will only register our complaint against the neglect of excess demand and the emphasis no individual utility by both Balzer and Haslinger.

Another aspect of Haslinger's reconstruction which does not seem to fit the text and practice of PEE is his characterization of *equilibrium*. Haslinger's third axiom of PEE (p. 123) requires that the economy "always achieves a state of Walrasian equilibrium" as long as no "outside" force prevents it "from adjusting toward it". He states that equilibrium theorists "restrict their attention to Walrasian states only" and that "all 'laws' of PEE exclusively apply to this subject of states". This is certainly not true to the standard texts or the practice of general equilibrium theorizing.

Much of the literature on general equilibrium theory concerns the "tâtonnement" or other adjustment scheme whereby prices change in response to excess demand.[9] The standard position of general equilibrium theorists is that unless the equilibrium price vector is "stable", i.e., prices converge to p^* from an initial nonequilibrium position, then the comparative statics which Haslinger (pp. 124–125) considers the principal use of PEE (and we discuss further below) are not possible. Demonstrating various conditions under which such stability does or does not hold has constituted a great portion of the writings on Walrasian general equilibrium theory. If Haslinger's Axiom 3 is correct then all of this literature would be outside of the theory. An adequate reconstruction of PEE must allow for certain types of disequilibrium *within* PEE, and it must characterize the laws of the theory so that they hold in such disequilibrium states. Again, we attempt to provide such a reconstruction in Section II below.[10]

This brings the discussion to Haslinger's Axiom 4. This axiom is the gross substitute (GS) assumption. It requires that an increase in the price

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of any good increases the excess demand of all others, thus $\partial z_i/\partial p_j > 0$ for all $i \neq j$.[11] Haslinger is quite correct when he states (p. 124) that this is an additional assumption which is not implied by utility maximization.[12] He is also correct in stating that the GS assumption guarantees the uniqueness and global stability of p^*. He is *not* correct to consider this assumption an axiom of the theory. During the history of general equilibrium theory many special excess demand restrictions have been proposed which guarantee that uniqueness, stability, or certain comparative statics results can be obtained. GS is *but one* of these special restrictions.[13] To count this one special condition as an axiom of the theory would (again) exclude all of the important work which utilizes other restrictions (or none at all). Such special restrictions constitute *specializations* of the fundamental theory and should be reconstructed as such.

Our final comment, before turning to our own reconstruction, concerns Haslinger's characterization of the qualitative comparative statics properties of PEE. This topic is particularly important since comparative statics is *the* fundamental tool of every equilibrium theory in economics (not just PEE). The comparative statics method is as follows. Start with a system in equilibrium, disturb this equilibrium by a change in some parameter, and then record the new equilibrium position when it is reached. Information about the system is obtained by comparing the first equilibrium with the second – hence the name, comparative statics. As Haslinger points out (p. 124) it is often impossible to obtain all of the information necessary for even an approximate quantitative characterization of the two equilibria. Thus, it is often necessary to use *qualitative* comparative statics which compares the two equilibria in terms of direction only (not magnitude). For instance, suppose a pure exchange economy is in equilibrium with price vector p^* and consumers change their "taste" in favor of good 1 and away from good 2. What can we say about the *direction* of the price changes for not only goods 1 and 2 but also all of the other goods in the economy? Qualitative comparative statics is used to answer such questions.

Haslinger is quite optimistic regarding the ability of PEE (as he has reconstructed it) to provide such qualitative comparative statics information. He claims there is always a function L which provides this information. "If an economy changes – in the sense that the number of

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commodities and/or households alter and/or some (or all) individual utility and/or initial endowments change — then L shows how the equilibrium of the 'new' economy changes relative to the 'old' one" (Haslinger, p. 123).

This picture of the quantitative comparative statics power of PEE is *unrealistically rosy, even if* we were to accept Haslinger's reconstruction of PEE (which includes the GS assumption). The "even if" is important here since we have argued above that Haslinger's characterization of PEE is too restrictive; that it excludes much of the important work in general equilibrium theory and considers what are actually specializations to represent the general theory. As restrictive as Haslinger's reconstruction is, it is still *not* restrictive enough to generate the comparative statics results which he claims are available. If PEE is reconstructed more broadly (as we do below) the situation is even worse. There are essentially no comparative statics results available for the more general case.

It is true that under the GS assumption certain qualitative comparative statics information (the so-called three Hicksian laws of comparative statics) can be obtained for a limited class of disturbances. In particular, if there is an increase in the aggregate demand for one good at the expense of the numeraire commodity and the demand for all other commodities does not shift, then the direction of price changes can be predicted.[14] But this is an extremely narrow class of disturbance. For the general case which Haslinger discusses (p. 123), where "the number of commodities and/or households alter and/or some (or all) individual utility and/or initial endowments change" we have *no idea* what will happen to the equilibrium prices, even under the (restrictive) GS assumption.

II. AN ALTERNATIVE RECONSTRUCTION OF PEE

In this section we present our own reconstruction of PEE which avoids the problems we have noted in the reconstructions of other authors. The fundamental "primitive" notion in our reconstruction is *market excess demand* (although individual utility maximization is always behind these excess demands). The reconstruction accommodates what appears in the standard texts and research publications, including the literature on disequilibrium adjustment. Equilibrium, as well as special excess demand

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restrictions, are treated as core specializations rather than as a basic part of the theory. As is consistent with the actual theoretical literature, no comparative statics information is available from the theory without extremely restrictive additional assumptions.

Our first definition characterizes the possible models of the theory.

DEFINITION D1. x is a possible model of PEE ($x \in M_p$) if there exists a structure consisting of an n, N, z, p, and a D such that,

(1) $x = \langle n, N, z, p, D \rangle$,

(2) $N = \{1, 2, \ldots, n\}$ where n is a positive integer,

(3) $D \subseteq R^n_{++}$,

(4) $p \in D$,

(5) $z: D \to R^n$ where z is a continuous function.

The verbal explanation of the symbols in D1 should be apparent from the discussion in Section I. There are n different types of commodities and N is the set of commodity indices. D is the price domain and only strictly positive prices are considered. An even more general definition of PEE might consider prices on the boundary of R^n_+. z if the excess demand function, it assigns the excess demand (positive, negative, or zero) to each commodity for every p in D. We assume that z is continuous though weaker restrictions could be specified. The continuity facilitates the proof of the standard theorems which we present below.

Recall that according to the structuralist view of scientific theories the possible models are the things for which the theory *might* be true. The *models* of the theory, on the other hand, are the structures which actually satisfy the axioms of the theory. PEE has two such axioms. These are given in the following definition.

DEFINITION D2. x is a model of PEE ($x \in M$) if there exists a structure consisting of n, N, z, p, and a D such that,

(1) $x \in M_p$,

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(2) $z(p) = z(\lambda p)$ for all $p \in D$ and for all $\lambda \in R_{++}$,

(3) $pz(p) = 0$ for all $p \in D$.[15]

In Definition D2, condition (2) requires that excess demand functions be homogeneous of degree zero. If all prices are "scaled up" or "scaled down" by the same amount then excess demand remains unchanged. This condition implies that only relative prices, or price ratios, matter for excess demand. Had we started with individual utility maximization, rather than market excess demands, then this homogeneity condition would follow automatically from the way that the budget sets ($B^h(p)$) are constructed. Since market excess demand functions are the primitives in our reconstruction it is necessary to specify (2) as a separate axiom of the theory. Condition (3) is called Walras Law. It requires that the total value of all positive excess demands are exactly offset with the total value of all negative excess demands for any price vector. Again, as with (2), this condition would follow from the budget constraint of each individual trader. Finally, notice that D1 and D2 imply that $M \subseteq M_p$, as required by the structuralist view.

While neither D1 nor D2 mention utility or utility maximization, the theoretical results discussed in Section I (Debreu, Mantel, MasColell, Richter and Sonnenschein) guarantee that utility maximization always lies "behind" any model of PEE. From the particular result in Debreu (1974) we know that if $x \in M$ there will always exist a set of n individual traders, each with a well-behaved utility function, such that the utility maximizing choices of these traders add up to the excess demand function z. This becomes our first theorem for PEE.

THEOREM T1. For all $x \in M$ there exists a set of n traders whose maximization of well-behaved utility function generates z.

Proof. Debreu (1974).

Another important concept for PEE which was not mentioned in either D1 or D2 is *equilibrium*. After all, PEE is a general equilibrium theory: how equilibrium is characterized is extremely important. Thus far, all of the restrictions imposed by D1 and D2 hold for all price vectors in the

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domain D. We have not made any special restrictions regarding the particular p^* which clears all markets (i.e., $z_i(p^*) = 0$ for all $i = 1, 2, \ldots, n$). In fact, nothing defined so far seems to guarantee that such an equilibrium price vector even exists!

Actually it is a standard result in general equilibrium theory that under the conditions specified for $x \in M$, such a p^* always exists. This is our second theorem for PEE.

THEOREM T2. For all $x \in M$ there exists a p^* such that $z(p^*) = 0$.

Proof. This is a standard existence result, see for instance Arrow and Hahn (1971, p. 28).

While T2 guarantees that an equilibrium will always exist for any model of PEE, it does not require practicing general equilibrium theorists to focus exclusively on p^*. As argued above, in areas such as the analysis of the disequilibrium adjustment process, what happens outside of equilibrium is at least as important as what happens at equilibrium. Despite certain claims to the contrary (Haslinger, p. 123), models in equilibrium do not exhaust the concerns of general equilibrium theorists. The equilibrium price set p^* (set since p^* may not be unique) merely constitutes a *core specialization* of PEE.[16] We will call this the *equilibrium specialization* of PEE and it constitutes our third definition.

DEFINITION D3. x is an equilibrium specialization of PEE ($x \in M(P^*)$) if there exists a structure consisting of an n, N, z, p, D, and a P^* such that

(1) $x \in M$.

(2) $P^* \subseteq D$,

(3) $z(p^*) = 0$ for all $p^* \in P^*$.

Special restrictions on excess demand functions can also generate core specializations of PEE. For instance, the gross substitute case generates the following specialization.

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DEFINITION D4. x is a gross substitute specialization of PEE ($x \in M_{GS}$) if there exists a structure consisting of an n, N, z, p, and a D such that,

(1) $x \in M$,

(2) z is differentiable on D,

(3) $\partial z_i / \partial p_j > 0$ for all $i \in N, j \in N$, and $i \neq j$.

During the 1950s and 1960s (the heyday of theoretical work in PEE) much of the theoretical literature focused on the GS specialization. GS was found to be an extremely interesting restriction for PEE, one which guaranteed a number of desirable properties. One of these is the following uniqueness theorem.

THEOREM T3. If $x \in M_{GS}$ then P^* contains only a single element.
Proof. Arrow and Hahn (1971, p. 223).

Many times in the preceding discussion we have alluded to disequilibrium dynamics within PEE, we will now discuss this question more formally. Of the many adjustment mechanisms which have been suggested during the history of mathematical general equilibrium theory the most common is the continuous time Walrasian tâtonnement. The characterization which initiated with Samuelson (1941) specifies the price adjustment mechanism as a system of first order autonomous ordinary differential equations. Our definition D5 gives one of the many specific formulations which have appeared in the literature.

DEFINITION D5. x is a tâtonnement specialization of PEE ($x \in M_t$) if there exists a structure consisting of an n, N, z, p, D, and an h such that,

(1) $x \in M$,

(2) $h: R^n \to R^n$ where h is differentiable,

(3) $h' > 0$ and $h(0) = 0$,

(4) $\dot{p}_i = h_i(z_i(p))$ for all $i \in N$.[17]

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Notice that on the basis of D5 the price of a good with positive excess demand will increase while the price of a good with negative excess demand will fall. At equilibrium ($z_i(p^*) = 0$ for all $i \in N$) D5 implies the adjustment process will stop since $\dot{p}_i = 0$ for all $i \in N$.[18] The system of differential equations in (4) generates a price path $p(t|p(0))$ in D for any initial price vector $p(0) \in D$. Much of the literature of general equilibrium theory during the last thirty years has been concerned with the "stability" of this tâtonnement adjustment mechanism (i.e., convergence of the price path to the equilibrium price vector p^*). One of the many results in this area is the following.

THEOREM T4. If $x \in M_{GS}$ and $x \in M_t$ then the unique equilibrium price vector p^* is globally stable, that is, $\lim_{t\to\infty} p(t|p(0)) = p^*$ for all $p(0) \in D$.
Proof. Arrow and Hahn (1971, p. 288).

This brief discussion of the stability of the adjustment process concludes our reconstruction of PEE. In the next section we turn to the methodological implications of our reconstruction, and in particular, to the differences between out reconstruction of PEE and existing structuralist reconstructions of theories in physical science.

III. METASCIENTIFIC IMPLICATIONS

As a structuralist reconstruction our reconstruction of PEE has both advantages and disadvantages. On the positive side, our reconstruction is a much more accurate portrayal of what actually appears in the standard economic texts and theoretical publications than any reconstruction previously offered in the structuralist literature. This is of course an empirical claim, but one which we are extremely confident will withstand the test of future research.

On the negative side, there is not very much which is specifically *structuralist* about our reconstruction. The purpose of a structuralist reconstruction is to help us understand the relationship between the formal (mathematical) structure of a scientific theory and its empirical claims. Structuralist reconstructions in physical science are concerned with "the fundamental question of what distinguishes a *theory of mathematical*

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physics from a mere *mathematical* theory" (Stegmüller, 1979, p. 7) – structuralist reconstructions in social science should have similar concerns. Our reconstruction of PEE says nothing about the empirical claims of PEE.

According to the structuralist program, scientific theories are characterized not only by a formal mathematical structure such as our reconstruction provides, but also by a *set of intended applications* (I) which contains those "parts of reality to which we want to apply our theoretical apparatus" (Balzer, 1982b, p. 20). Where is the set of intended applications for our reconstruction? The set I, according to the structuralist position, is a subset of the set of *partial possible models* (M_{pp}). This set M_{pp} is probably the most important aspect of the structuralist views of theories since M_{pp} is generated from the set of possible models (M_p) by "lopping off" (Stegmüller, 1979, p. 25) the theoretical terms from each element of M_p. Without the distinction between M_p and M_{pp} it is unclear what separates scientific theories with testable empirical hypotheses from "only their mathematical skeletons" (Stegmüller, 1979, p. 13). In our reconstruction D1 characterizes the elements of M_p, but we offer *no* definition of the important set M_{pp}.

Why the absence of M_{pp} and I in our reconstruction? The fact is that given the narrow structuralist definition of these sets, they cannot be adequately specified for PEE. Consider the notion of excess demand. As we have reconstructed PEE, excess demand functions are PEE-theoretical. What would be left of the possible models defined in D1 if all the PEE-theoretical functions were "lopped off"? Just prices? There are similar difficulties with the set of intended applications. Where do we find this set? As Balzer (p. 41) states, "in economics we cannot point out a single real, concrete system which is commonly accepted by economists to be a standard example of PEE". Even if we could find something like a set I, how would we know that $I \subseteq M_{pp}$ as required by the structuralist program, since it is unclear how M_{pp} should be specified?

We are certainly not the first to point out that it is very difficult to eliminate PEE-theoretical terms or find the set of intended applications (in the structuralist sense) for a general equilibrium reconstruction which is at all consistent with the field's professional practice. Both Balzer and Haslinger point out this difficulty, but then they both waffle a bit on its

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implications . Balzer makes the statement quoted above but then goes on to say that while general equilibrium theorists do not appear to practice anything like empirical science, it "at least should be possible" (p. 41). Haslinger responds in a similarly ambivalent way. As stated above Haslinger places a great emphasis on the (possibly empirical) comparative statics results of the theory. On the other hand, both demand functions and equilibrium are PEE-theoretical on Haslinger's view, so it is unclear just how empirical these results would be even if they were available to the degree which Haslinger seems to believe.

Other structuralist authors who have examined theories closely related to PEE, particularly Händler, have been much less ambivalent than either Balzer or Haslinger. Händler simply states (1980a, p. 50; 1980b, pp. 154–155; 1982, p. 75) that all general equilibrium theories in economics are *pure theories*. Where "a pure theory does not intend to speak about reality. A pure theory is just a (sometimes very complex) picture of a possible world which does not actually exist" (Händler, 1982, p. 75). While Händler's view may appear rather extreme it does seem to be supported by the fact that we were unable to specify either M_{pp} or I and still maintain the textual fidelity of our reconstruction.

One approach to defending the empirical integrity of general equilibrium theory against claims such as Händler's is to cite examples of "applied" general equilibrium theory. For instance, there has been extensive literature on general equilibrium theory being applied to questions such as taxation and international trade policy.[19] While this literature is based on production rather than pure exchange economies, there are also some cases of real world pure exchange economies where (at least some of) the predictions of PEE seem to apply. Radford's (1945) classic study of the economics of a prisoner of war camp is one such case.[20] While it may be possible to construct a convincing evidential defense on the basis of this applied literature, such an approach is not the one which we will take. In fact, our approach, while still a defense of the empirical character of general equilibrium theory, starts from just the opposite position.

Our position is that *even if* we accept Händler's view and concede that $I = \emptyset$ for PEE, it is not necessary to reach the same conclusions as Händler and other structuralist authors have regarding the total empirical emptiness of PEE and related general equilibrium theories. Our position

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is that accepting Händler's argument (an argument, remember, which seems to be supported by our own reconstruction) does *not* imply that general equilibrium theory is not empirical in *any sense*; it only implies that general equilibrium theory is *not empirical in the structuralist sense*. All that can be implied is that mathematical structure of PEE (or any other general equilibrium theory) does not relate to real economies *in exactly the same way* as Sneed and others found the mathematical structure of physics to relate to real physical systems. In fact, in order to make this important distinction more clear we suggest the terminology of "pure theory" be dropped altogether. Instead of calling something a pure theory merely because it does not relate to its empirical domain in exactly the way the structuralist school argues certain physical theories relate to their empirical domains, we suggest such theories be called *non-S-empirical* (non-structuralist-empirical). While we are willing to concede that our reconstruction supports the view that PEE is non-S-empirical, we are not willing to concede that it supports the view that PEE has nothing at all to say about economic reality (as the term "pure theory" implies).

Those who believe that argreement with the structuralist program's view of the relation between mathematical structure and empirical domain is a necessary condition for empirical science, will not find the distinction between "pure theory" and "non-S-empirical" to be very useful. We suspect this will be the case for the authors discussed above; Balzer, Händler, and Haslinger. Obviously, this is not our interpretation. We argue that it is entirely possible for a mathematical economic theory such a PEE to be empirical "in some sense" while being non-S-empirical.

How is this possible? More specifically, in what sense could PEE be empirical even if it is non-S-empirical? Rather than provide a single answer to this question we will sketch a number of responses. In particular, we will briefly survey the views of Hausman (1981a, 1981b), Varian and Gibbard (1978), and Weintraub (1985) which provide three different answers to the above question. It will not be necessary for us to advocate any one of these views against the others, or even suggest that these three exhaust the possible alternatives. The point is only to demonstrate that there are a variety of ways in which general equilibrium theories might say something interesting about real economies without being empirical in the exact way the structuralist school claims theories are empirical.

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Hausman (1981a, 1981b) argues that while general equilibrium theories are "virtually without explanatory power" (1981b, p. 17) they relate to real empirical economies in at least two separate ways. First, they are of "heuristic value" because "they have in fact helped in developing valuable empirical economic theories" (1981b, p. 26). Second, they provide theoretical reassurance that less abstract economic theories which share a similar analytical structure "are on the right track" (1981b, p. 28). For instance, the zero degree homogeneity assumption in PEE implies that only "relative" prices matter, similar assumptions are often made in macroeconomic theories with testable empirical hypotheses.

Varian and Gibbard (1978) on the other hand, argue that while some economic theories generate empirical hupotheses which are approximately true, others (and PEE would be in this set) are only *caricatures*. Caricature theories (Varian and Gibbard use the term models) are not designed to generate hypotheses which are even approximately true of empirical entities, instead they are designed "to distort reality in a way that illustrates certain aspects of that reality" (1978, p. 676). For instance, the tâtonnement specialization of PEE is a caricature of the way in which surpluses and shortages influence the prices of goods. The tâtonnement is specified "not to approximate reality, but to exaggerate or isolate some feature of reality" (1978, p. 673).

Another view of general equilibrium is Weintraub (1985), who argues that PEE is not a "theory" at all but rather one aspect of the much more general neoWalrasian research program. The neoWalrasian program is a progressive scientific research program in the sense of Lakatos with a rich history of testable (and confirmed) empirical hypotheses. Two of the hard core propositions of the neoWalrasian program are optimizing individual behavior and equilibrium. If the program is to generate anything except nonsense we must know that these two propositions are not inconsistent. It is Weintraub's position that the long process of confirming the consistency of these two hard core propositions generated the structure which we have reconstructed as PEE. On this view, what we have characterized as PEE is not a theory at all but rather the product produced during the "hardening" of the hard core of the (empirically relevant) neoWalrasian research program.

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While all three of these views are very different, and to a certain extent mutually exclusive, they all demonstrate various ways that PEE might say something potentially interesting about real observable economic phenomena even if the mathematical structure of PEE does not generate empirical hypotheses or have a set of intended applications which exactly fits the structuralist view of scientific theories. In the final analysis it must be remembered that the structuralist view of scientific theories was formulated exclusively on the basis of the structure of mathematical physics. Reconstructions of PEE which find the set M_{pp} (or the set I) empty, or which do not consistently draw a line between PEE-theoretical and non-PEE-theoretical terms, do not demonstrate that PEE is a pure theory - they only demonstrate that mathematical economics is not mathematical physics.[21]

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NOTES

[1] For the purpose of this paper, if Balzer's name is used without a reference date it refers to Balzer (1982a), and if Haslinger's name is used without a reference date it refers to Haslinger (1983).

[2] Recall the structuralist definition of theoretical.

"What does it mean to say that a quantity (function) f of a physical theory T is T-theoretical? Roughly speaking, it amounts to a brief story contained in the following two statements. In order to perform an empirical test of an empirical claim containing the T-theoretical quantity f, we have to measure values of the function f. But all known measuring procedures (or, if you like, all known theories of measurement of f values) presuppose the validity of this very theory T" (Stegmüller, 1979, pp. 17–18).

[3] The standard assumptions on $U^h(\cdot)$ are that the function be monotonic, strictly quasi-concave, and differentiable (or at least continuous). Such utility functions are considered "well-behaved" by economists.

[4] It should be mentioned that Haslinger requires utility maximization at "any given prices" (p. 119) rather than only at "equilibrium" (as Balzer has it). Haslinger is true to standard economic usage on this point.

[5] Whether such market functions are or are not PEE-theoretical is an important question as we will see below.

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[6] See for instance Chapters 9, 10, 11, and 12 of Arrow and Hahn (1971) and Chapters 3, 5, and 6 of Quirk and Saposnik (1968).
[7] Thus utility *is* discussed in the early chapters of the references in note 6.
[8] The principal papers are Debreu (1974), Mantel (1977), McFadden, *et al.* (1974) and Sonnenschein (1973). A summary of this literature is given in Shafer and Sonnenschein (1982).
[9] See for instance Chapters 11 and 12 of Arrow and Hahn (1971) and Chapter 5 of Quirk and Saposnik (1968). Haslinger (p. 125) admits that such literature exists, but dimisses it as not important.
[10] It should be noted that Balzer (p. 43), unlike Haslinger, makes the Walrasian equilibrium position p^* a "specialization" of the theory rather than an axiom. Our reconstruction is closer to Balzer than to Haslinger in this regard.
[11] This assumption when combined with the zero degree homogeneity of the excess demand function (which is discussed below) implies that $\partial z_i/\partial p_i < 0$ for all $i = 1, 2, \ldots, n$.
[12] There have only been a few attempts to even determine what utility functions would need to look like for GS to hold. One of these is Fisher (1972).
[13] Two others include the "dominant diagonal" condition which is implied by GS but does not imply it, and the so-called "Morishima sign conditions" which include gross complements ($\partial z_i/\partial p_j < 0$ for $i \neq j$) in a particular way. For the dominant diagonal condition see Arrow and Hahn (1971, pp. 233–235) and for the Morishima conditions see Quirk and Saposnik (1968, pp. 213–215).
[14] See Quirk and Saposnik (1968, pp. 210–213). Most comparative statics results for general equilibrium systems are only local (i.e., differential changes). This particular GS result was extended to the global case (i.e. discrete changes) by Morishima (1964). Arrow and Hahn (1971, p. 252) and Cornwall (1984, pp. 48–50) consider the slightly more general case where the demand for one good increases and the demands for each of the other goods either decrease or remain the same.
[15] $pz(p) = \Sigma_{i=1}^{n} p_i z_i(p)$.
[16] Stegmüller (1979, p. 26).
[17] Dot over a variable indicates the time derivative.
[18] A bit more sophistication is required in specifying (4) if boundary behavior is considered. We have simplified things considerably by only considering strictly positive prices.
[19] Shoven and Whalley (1984) provide a recent survey of this literature. It is also discussed in Chapter 3 of Cornwall (1984).
[20] I need to thank Daniel Hausman for reminding me of this classic paper.
[21] The author provides a more global critique of the structuralist program in economics in Hands (1985).

BIBLIOGRAPHY

Arrow, K. J. and Hahn, F. H.: 1971 *General Competitive Analysis*, Holden-Day, San Francisco.

Balzer, W.: 1982a, 'A Logical Reconstruction of Pure Exchange Economics', *Erkenntnis* **17**, 23–46.

Balzer, W.: 1982b, 'Empirical Claims in Exchange Economics", in W. Stegmüller, W. Balzer, and W. Spohn (eds.), *Philosophy of Economics*, Springer-Verlag, New York, 16–40.

Cornwall, R. R.: 1984, *Introduction to the Use of General Equilibrium Analysis*, North-Holland, Amsterdam.

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Debreu, G.: 1974, 'Excess Demand Functions', *Journal of Mathematical Economics* **1**, 15–21.
Fisher, F. M.: 1972, 'Gross Substitutes and the Utility Function', *Journal of Economic Theory* **4**, 82–87.
Händler, E. W.: 1980a, 'The Logical Structure of Modern Neoclassical Static Microeconomic Equilibrium Theory', *Erkenntnis* **15**, 33–53.
Händler, E. W.: 1980b, 'The Role of Utility and of Statistical Concepts in Empirical Economics: The Empirical Claims of the Systems of Aggregate Market Supply and Demand Functions Approach', *Erkenntnis* **15**, 129–157.
Händler, E. W.: 1982, 'The Evolution of Economic Theories: A Formal Approach', *Erkenntnis* **18**, 65–96.
Hands, D. W.: 1985, 'The Structuralist View of Economic Theories: The Case of General Equilibrium in Particular', *Economics and Philosophy* **1**, forthcoming.
Haslinger, F.: 1982, 'Structure and Problems of Equilibrium and Disequilibrium Theory', in W. Stegmüller, W. Balzer, and W. Spohn (eds.), *Philosophy of Economics*, Springer-Verlag, New York, pp. 63–83.
Haslinger, F.: 1983, 'A Logical Reconstruction of Pure Exchange Economics: an Alternative View', *Erkenntnis* **20**, 115–129.
Hausman, D. M.: 1981a, *Capital, profits, and Prices*, Columbia University Press, New York.
Hausman, D. M.: 1981b, 'Are General Equilibrium Theories Explanatory?', in J. C. Pitt (ed.), *Philosophy in Economics*, D. Reidel, Dordrecht, Holland, pp. 17–32.
Mantel, R. R.: 1977, 'Implications of Microeconomic Theory for Community Excess Demand Functions', in M. D. Intriligator (ed.), *Frontiers of Quantitative Economics III A*, North-Holland, Amsterdam, pp. 111–126.
McFadden, D., Mas-Colell, A., Mantel, Rd., and Richter, M. K.: 1974, 'A Characterization of Community Excess Demand Functions', *Journal of Economic Theory* **9**, 361–374.
Morishima, M.: 1964, *Equilibrium, Stability, and Growth*, Oxford University Press, London.
Moulines, C. U.: 1975, 'A Logical Reconstruction of Simple Equilibrium Thermodynamics', *Erkenntnis* **9**, 101–130.
Quirk, J. and Saposnik, R.: 1968, *Introduction to General Equilibrium Theory and Welfare Economics*, McGraw-Hill, New York.
Radford, R. A.: 1945, 'The Economic Organization of a P.O.W. Camp', *Economica*, 189–201.
Samuelson, P. A.: 1941, 'The Stability of Equilibrium: Comparative Statics and Dynamics', *Econometrica* **9**, 97–120.
Shafer, W. and Sonnenschein, h.: 1982, 'Market Demand and Excess Demand Functions', in K. J. Arrow and M. D. Intriligator (eds.), *Handbook of Mathematical Economics: Volume II*, North-Holland, Amsterdam, pp. 671–693.
Shoven, J. B. and Whalley, J.: 1984, 'Applied General Equilibrium Models of Taxation and International Trade', *Journal of Economic Literature* **22**, 1007–1051.
Sneed, J. R.: 1971, *The Logical Structure of Mathematical Physics*, D. Reidel, Dordrecht, Holland.
Sonnenschein, H.: 1973, 'Do Walras' Identity and Continuity Characterize the Class of Community Excess Demand Functions?', *Journal of Economic Theory* **6**, 345–354.
Stegmüller, W.: 1976, *The Structure and Dynamics of Theories*, Springer-Verlag, New York.
Stegmüller, W.: 1979, *The Structuralist View of Theories*, Springer-Verlag, New York.
Varian, H. R. and Gibband, A.: 1978, 'Economic Models', *The Journal of Philosophy* **75**, 664–677.

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Weintraub, E. R.: 1985, *General Equilibrium Analysis: Studies in Appraisal*, Cambridge University Press (forthcoming).

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