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THE CHROMO-DIELECTRIC SOLITON MODEL: QUARK SELF ENERGY AND HADRON BAGS*

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Abstract

The chromo-dielectric soliton model (CDM) is Lorentz- and chirally-invariant. It has been demonstrated to exhibit dynamical chiral symmetry breaking and spatial confinement in the locally uniform approximation. We here study the full nonlocal quark self energy in a color-dielectric medium modeled by a two parameter Fermi function. Here color confinement is manifest. The self energy thus obtained is used to calculate quark wave functions in the medium which, in turn, are used to calculate the nucleon and pion masses in the one gluon exchange approximation. The nucleon mass is fixed to its empirical value using scaling arguments; the pion mass (for massless current quarks) turns out to be small but non-zero, depending on the model parameters.

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I. INTRODUCTION

The chromo-dielectric soliton model (CDM) [1] is a Lorentz- and chirally invariant low-energy effective field theory based on quantum chromodynamics (QCD). In order to simulate gluon condensates and other scalar structures (as, e.g., $q\bar{q}$ pairs) inside hadrons the QCD lagrangian density is supplemented by a scalar field σ that mediates the gluons through a color-dielectric function.

Following arguments first given by T.D. Lee, a suitably modeled color-dielectric function $\kappa(\sigma)$ guarantees absolute color confinement [2]. The assumed potential of the scalar field is quartic and has two minima, one at zero and a second, deeper minimum at a finite value identified as the vacuum value σ_v. In the absence of quarks, the normal state of the σ-field is at the vacuum value. In the presence of quarks and gluons, the σ-field finds a minimum in the vicinity of zero; the quarks and gluons dig a hole in the vacuum. This is the origin of confinement in the model.

The CDM differs from the original Friedberg-Lee (FL) nontopological soliton model [3] in the essential feature that there is no direct quark-sigma coupling term. Thus the model is chirally invariant for massless quarks. Krein *et al.* [4] showed that for a locally uniform dielectric medium, chiral symmetry is dynamically broken if the strong coupling constant or the inverse of the color-dielectric function exceeds a critical value. Consequently, the quarks acquire an effective ("constituent") mass. The Nambu-Goldstone boson corresponding to this symmetry breaking has been identified with the pion [5].

While the locally uniform model demonstrated *spatial confinement* and the emergence of the pion, it did not demonstrate *color confinement*. Furthermore, it was shown that the range of non-locality of the quark self-energy was of the order of the typical hadronic length scale and hence large compared with the soliton surface. Therefore, it was deemed essential to investigate the nonlocal quark self-energy for a realistic *and* self-consistent soliton.

This is the problem we address in the present paper. We first obtain the linearized (Abelian) gluon propagator in an inhomogeneous color-dielectric medium. Because of the Abelian approximation, the calculation is analogous to a problem in electrodynamics. The Schwinger-Dyson equation for the quark propagator is solved along the imaginary energy axis in order to avoid mass poles on the real energy axis. Quark wave functions are obtained by solving the Dirac equation with the self-energy playing the role of a nonlocal scalar potential which is analytically continued to the real energy axis.

The mutual interaction between quarks and – in the case of mesons – antiquarks in hadrons is treated in the one gluon exchange approximation (OGE). Corrections due to center of mass motion are taken into account approximately. Using scaling arguments, we fix the nucleon mass to its empirical value and calculate the pion mass as a function of phenomenological parameters. In the case of massless current quarks, the pion mass turns out to be small but non-zero. Since a vanishing pion mass is demanded by Goldstone's theorem, the calculated pion mass can be considered a test of the approximation schemes applied [6].

The paper is organized as follows. Sec. II introduces the basic features of the chromodielectric soliton model. After deriving the equations for the gluon-propagator in an inhomogeneous medium (Sec. III), Sec. IV addresses the formulation of the appropriate Schwinger-Dyson equation for the quark self-energy. This self-energy is used in Sec. V as an effective nonlocal quark potential in order to determine quark wave functions in a bag. Sec. VI contains details of the numerical solution of the corresponding equations and presents results for the self energy. Sec. VII describes the calculation of hadronic properties in the OGE-approximation and, finally, Sec. VIII sums up the main results and discusses future prospectives.

II. THE MODEL

The CDM lagrangian density is given by [4]

$$\mathcal{L}_{CDM} = \bar{q}(\mathrm{i}\gamma^\mu \partial_\mu + g_s \frac{1}{2}\,\lambda^a\, A^a_\mu\, \gamma^\mu - m_f)q - \kappa(\sigma)\frac{1}{4}\, F^a_{\mu\nu} F^{a\mu\nu} + \frac{1}{2}(\partial_\mu \sigma)^2 - U(\sigma) + \mathcal{L}'\,, \tag{1}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu\,, \tag{2}$$

where the color $SU(3)$ structure constants satisfy $\left[\lambda^a, \lambda^b\right] = 2if^{abc}\lambda^c$, q are the quark fields, A^a_μ are the gluon fields, σ is the effective scalar field which determines the effective color-dielectric function $1 \geq \kappa(\sigma) \geq 0$, and $\mathcal{L}'$ contains any necessary counter terms, gauge fixing term, or ghosts. It is evident that the model is gauge invariant. m_f is the quark flavor (current-) mass matrix. Throughout this paper we will set the current quark masses equal to zero, so that the model is also explicitly chirally invariant. The color-dielectric function $\kappa(\sigma)$ mediates the gluon field and is designed to guarantee color confinement. It has been shown [2] that the following assumptions must be satisfied: $\kappa(0) = 1, \kappa(\sigma_v) = \kappa'(\sigma_v) = \kappa'(0) = 0$. These constraints are satisfied, e.g., by

$$\kappa(\sigma) = 1 + \theta(x)\, x^n \left(nx - (n+1)\right), \qquad n > 2, \tag{3}$$

with $x = \sigma/\sigma_v$. The vacuum-value of the σ-field is denoted by σ_v. We choose $n = 3$ for simplicity so that $\kappa(\sigma)$ is continuous at $x = 0$.

Analogous to the FL-model, the potential of the σ-field is given by the quartic form

$$U(\sigma) = \frac{a}{2!}\sigma^2 + \frac{b}{3!}\sigma^3 + \frac{c}{4!}\sigma^4 + B\,. \tag{4}$$

The "bag constant" B is fixed in terms of the other model-constants so that $U(\sigma_v) = 0$. In the FL-model, $U(\sigma)$ is chosen to be quartic in order to make the model renormalizable. Although our model is not renormalizable (due to the presence of $\kappa(\sigma)$) we stick to this form in order to minimize the numbers of free parameters in the model. We identify $U''(\sigma_v) \equiv m_{GB}^2$ with the lowest 0^{++} glueball mass and require $U'(\sigma_v) = 0$ [2].

We now discuss the divergences of the model in more detail. The model exhibits both infrared and ultraviolet divergences. The origin of the *infrared* divergence is the same as for the MIT bag model. For a spherical bag, for example, the electric monopole term of the quark self-energy diverges as $r \to \infty$. This happens in the CDM, if the color-dielectric constant vanishes as $r \to \infty$. The infrared divergence is thus associated with *color confinement*. However, for a color singlet bag, no infrared divergence occurs since the self and mutual interaction terms cancel when ladder diagrams for the mutual interaction are properly calculated. The monopole term of the self energy is ignored in the MIT bag model. Since this term is the source of color confinement in our model we cannot neglect it. In our calculations we choose a Fermi function shaped spherically symmetric color-dielectric function as displayed in figure 1.

The *ultraviolet* divergence is associated with the point nature of quarks. It is shown in ref. [4] that the effective quark mass, which is generated because of dynamical chiral symmetry breaking, goes to infinity if the color-dielectric function approaches zero. This divergence is thus connected with *spatial confinement*. We handle this divergence by introducing an energy cutoff (asymptotic freedom). For numerical reasons, we regulate the infrared divergence by adjusting $\kappa_v = \kappa(\sigma_v)$ to a small, non-zero value, and discuss the limit $\kappa_v \to 0$.

III. THE GLUON PROPAGATOR

We assume that parts of the non-Abelian effects are effectively included in the σ-field. This allows us to approximate the gluon field by its Abelian part. Hence the gluon field equations are formally identical to Maxwell's equations in an inhomogeneous medium characterized by a time-independent color-dielectric function $\kappa(\mathbf{r})$. The field equations for the vector potential $A_\mu(\mathbf{r}, t)$ read (we follow here references [7] and [8]):

$$\partial^\mu \kappa(\mathbf{r})[\partial_\mu A_\nu - \partial_\nu A_\mu] = J_\nu(\mathbf{r}, t)\,. \tag{5}$$

Since the Abelian approximation destroys gauge invariance, the choice of gauge is part of the approximations. We choose the Coulomb gauge defined by

$$\boldsymbol{\nabla} \cdot (\kappa \mathbf{A}) = 0\,. \tag{6}$$

The $\nu = 0$ component of Eq. (5) yields

$$-\boldsymbol{\nabla}\kappa(\mathbf{r}) \cdot \boldsymbol{\nabla} A_0(\mathbf{r}, t) = J_0(\mathbf{r}, t)\,. \tag{7}$$

The time-time component of the Green's function, D^{00}, defined by

$$A_0(\mathbf{r}, t) = \int d^3r' D^{00}(\mathbf{r}, \mathbf{r}') J_0(\mathbf{r}', t)\,, \tag{8}$$

satisfies the equation

$$-\boldsymbol{\nabla}\kappa(\mathbf{r}) \cdot \boldsymbol{\nabla} D^{00}(\mathbf{r}, \mathbf{r}') = \delta^3(\mathbf{r} - \mathbf{r}')\,. \tag{9}$$

Note that $D^{00}(\mathbf{r}, \mathbf{r}')$ is instantaneous.

Now consider the $\nu = i$ components of Eq. (5)

$$\kappa \partial_t^2 \mathbf{A} - \nabla^2(\kappa \mathbf{A}) + \boldsymbol{\nabla} \times (\kappa \mathbf{A} \times \boldsymbol{\nabla} \ln \kappa) = \mathbf{J} - \kappa \boldsymbol{\nabla} \partial_t A_0 \equiv \mathbf{J}_{tr}\,. \tag{10}$$

The transverse current defined by Eq. (10) can be expressed in terms of $\mathbf{J}$ using the time-time Green's function:

$$\mathbf{J}_{tr}(\mathbf{r}\,,t) = \mathbf{J}\,(\mathbf{r}\,,t) - \kappa(\mathbf{r}\,)\boldsymbol{\nabla}\int d^3r' D^{00}(\mathbf{r}\,,\,\mathbf{r}\,')\partial_t J_0(\mathbf{r}\,',t)\,. \tag{11}$$

Using current conservation, $\partial_t J_0 + \boldsymbol{\nabla}\cdot\mathbf{J}\ = 0$, and performing a partial integration, we obtain

$$\mathbf{J}_{tr}(\mathbf{r}\,,t) = \mathbf{J}\,(\mathbf{r}\,,t) - \kappa(\mathbf{r}\,)\boldsymbol{\nabla}\int d^3r' \left(\boldsymbol{\nabla}' D^{00}(\mathbf{r}\,,\,\mathbf{r}\,')\right)\cdot\mathbf{J}\,(\mathbf{r}\,',t)\,. \tag{12}$$

We now Fourier transform the time dependence of $\mathbf{J}\,(\mathbf{r}\,,\,t)$ and $\mathbf{A}\,(\mathbf{r}\,,\,t)$ to $\mathbf{J}\,(\mathbf{r}\,,\,\omega)$ and $\mathbf{A}\,(\mathbf{r}\,,\,\omega)$. The Green's function corresponding to Eq. (10) satisfies

$$-\left[\nabla^2 + \omega^2 + \boldsymbol{\nabla}\times(\boldsymbol{\nabla}\ln\kappa)\times\right]\kappa\overleftrightarrow{D}(\mathbf{r}\,,\,\mathbf{r}\,') = \overleftrightarrow{\delta}_{tr}(\mathbf{r}\,,\,\mathbf{r}\,')\,, \tag{13}$$

where the components of the transverse delta function are given by

$$\delta^{ij}_{tr}(\mathbf{r}\,,\,\mathbf{r}\,') = \delta^{ij}\delta^3(\mathbf{r}\,-\mathbf{r}\,') - \kappa(\mathbf{r}\,)\partial^i\partial'^j D^{00}(\mathbf{r}\,,\,\mathbf{r}\,')\,. \tag{14}$$

In this paper we will restrict ourselves to spherical bags. In this case the Green's functions can be decomposed in terms of spherical harmonics [7]:

$$D^i{}_{i'} \to \overleftrightarrow{D}(\mathbf{r}\,,\mathbf{r}\,';\omega) = \sum_{jll'm_l} d_{jll'}(r,r';\omega)\overleftarrow{Y}_{jlm_l}(\Omega)\overrightarrow{Y}^*_{jl'm_l}(\Omega')\,, \tag{15}$$

$$D^{00}(\mathbf{r}\,,\mathbf{r}\,') = \sum_{lm_l} d^0_l(r,r')Y_{lm_l}(\Omega)Y^*_{lm_l}(\Omega')\,. \tag{16}$$

Some of the tensor components are shown in figs. 2 and 3. The $\kappa(r)$ parameters are again $R = 0.8$ fm, $A = 0.1$ fm, and $\kappa_v = 0.1$.

It should be noted that the Green's functions do not carry any color indices. This results from the fact that the medium is color-neutral so that $D_{\mu\nu}$ has a trivial (diagonal) color structure.

Details of the derivation and solution of equations (9) and (13) are given in [7], an important correction is reported in ref. [8].

IV. THE SCHWINGER-DYSON EQUATION IN THE QUARK-GLUON SECTOR

Being now in the possession of the gluon propagator in the cavity we can study the Schwinger-Dyson equation for a single quark in a cavity. In the course of this calculation we need not refer to the σ-field.

The Schwinger-Dyson equation reads (in $(\omega, \mathbf{r})$-space):

$$\Sigma(\mathbf{r}, \mathbf{r}'; \omega) = \mathrm{i}\alpha' \int_{-\infty}^{\infty} d\omega' D_{\mu\nu}(\mathbf{r}, \mathbf{r}'; \omega')\gamma^\mu S(\mathbf{r}, \mathbf{r}'; \omega - \omega')\gamma^\nu, \tag{17}$$

with $\alpha' = (4/3)g_s^2/2\pi$. In Eq. (17) we have already approximated the one-particle irreducible quark-gluon vertex Γ^μ by the bare one.

It is easy to show that both the gluon propagator $D_{\mu\nu}$ and the quark propagators G do not have poles off the real ω-axis. So according to the Schwinger-Dyson equation, the self energy Σ should have no pole of the real ω-axis as well. Thus we perform a Wick rotation, $\omega \to \mathrm{i}y$ and study the self energy first for imaginary ω. The "rotated" Schwinger-Dyson equation now reads:

$$\Sigma(\mathbf{r}, \mathbf{r}'; y) = -\alpha' \int_{-\infty}^{\infty} dy' D_{\mu\nu}(\mathbf{r}, \mathbf{r}'; y')\gamma^\mu S(\mathbf{r}, \mathbf{r}'; y - y')\gamma^\nu. \tag{18}$$

Simultaneously, the Dirac equation for the quark propagator has to be satisfied:

$$(\omega\gamma^0 - \boldsymbol{\gamma}\cdot\mathbf{p} - \Sigma)S = \delta^3(\mathbf{r} - \mathbf{r}'). \tag{19}$$

To simplify the notation we have used the shorthand ΣS for $\int d^3r_2 \Sigma(\mathbf{r}, \mathbf{r}_2; \omega) S(\mathbf{r}_2, \mathbf{r}'; \omega)$. Throughout this paper repeated spatial coordinates are integrated over.

We now define the following hermitian functions,

$$G = -S\beta, \quad V = \beta\Sigma. \tag{20}$$

Eqs. (19) and (17) then become

$$(-\omega + \boldsymbol{\alpha}\cdot\mathbf{p} + V)G = \delta^3(\mathbf{r} - \mathbf{r}'), \tag{21}$$

$$V(\mathbf{r}, \mathbf{r}'; y) = \alpha' \int_{-\infty}^{\infty} dy' D_{\mu\nu}(\mathbf{r}, \mathbf{r}'; y')\alpha^\mu G(\mathbf{r}, \mathbf{r}'; y - y')\alpha^\nu. \tag{22}$$

Here $\alpha^\mu \equiv (1, \boldsymbol{\alpha})$ is used for notational convenience only. It is obviously not a Lorentz vector.

From the coupled Eqs. (21, 22), the hermiticity of G and V can be verified:

$$G^\dagger(\mathbf{r}, \mathbf{r}'; \omega) = G(\mathbf{r}, \mathbf{r}'; \omega^*), \tag{23}$$

$$V^\dagger(\mathbf{r}, \mathbf{r}'; \omega) = V(\mathbf{r}, \mathbf{r}'; \omega^*). \tag{24}$$

The hermitian conjugation includes the interchange of the arguments $\mathbf{r}$ and $\mathbf{r}'$:

$$V_{ij}^\dagger(\mathbf{r}, \mathbf{r}'; \omega) \equiv V_{ji}(\mathbf{r}', \mathbf{r}; \omega)^*. \tag{25}$$

A. Angular decomposition of the quark propagator

In order to solve the coupled Eqs. (21, 22) numerically, we make an angular decomposition of the appropriate quantities. For spherically symmetric color-dielectric functions $\kappa(r)$, the hermitian properties G and V can be decomposed [9,10]

$$G(\mathbf{r}\,,\mathbf{r}\,';\omega) = \sum_\kappa \begin{pmatrix} g_\kappa^{11}(r,r';\omega)\pi_\kappa & g_\kappa^{12}(r,r';\omega)i\sigma_r\pi_{-\kappa} \\ -g_\kappa^{21}(r,r';\omega)i\sigma_r\pi_\kappa & g_\kappa^{22}(r,r';\omega)\pi_{-\kappa} \end{pmatrix}, \tag{26}$$

$$V(\mathbf{r}\,,\mathbf{r}\,';\omega) = \sum_\kappa \begin{pmatrix} v_\kappa^{11}(r,r';\omega)\pi_\kappa & v_\kappa^{12}(r,r';\omega)i\sigma_r\pi_{-\kappa} \\ -v_\kappa^{21}(r,r';\omega)i\sigma_r\pi_\kappa & v_\kappa^{22}(r,r';\omega)\pi_{-\kappa} \end{pmatrix}, \tag{27}$$

where the respective angular part is given by the 2×2 matrices

$$\pi_\kappa(\Omega,\Omega') \equiv \sum_\mu \mathcal{Y}_{\kappa\mu}(\Omega)\mathcal{Y}^\dagger_{\kappa\mu}(\Omega')\,. \tag{28}$$

The following reduction relationship holds:

$$\int d\Omega_2 \pi_\kappa(\Omega_1,\Omega_2)\pi_{\kappa'}(\Omega_2,\Omega_3) = \delta_{\kappa\kappa'}\pi_\kappa(\Omega_1,\Omega_3)\,. \tag{29}$$

$\mathcal{Y}_{\kappa\mu}(\Omega)$ are the usual two component spinor spherical harmonics. They are eigenstates of the operators J^2, L^2, J_z, and $K = (J+1/2)(-1)^{(J-L+1/2)}$ [2]

$$\mathcal{Y}_{\kappa\mu}(\Omega) = \sum_{m_l,m_s} < l_\kappa\, m_l, \frac{1}{2}\, m_s | j_\kappa\, \mu > Y_{l_\kappa m_l}(\Omega)\chi_{m_s} \tag{30}$$

and obey the orthonormality relation

$$\int d\Omega \mathcal{Y}^\dagger_{\kappa\mu}(\Omega)\mathcal{Y}_{\kappa'\mu'}(\Omega) = \delta_{\kappa\kappa'}\delta_{\mu\mu'}\,. \tag{31}$$

The radial functions g and v have the following symmetry properties

$$g_\kappa^{ij}(r,r';\omega) = g_\kappa^{ji}(r',r;\omega) = g_\kappa^{ij}(r,r';\omega^*)^*\,. \tag{32}$$

Inserting Eqs. (26) and (27) in the Dirac Eq. (21) for the quark propagator and using (31) and (29) yields

$$\left[\begin{pmatrix} -\omega & -1/r - \partial/\partial r + \kappa/r \\ 1/r + \partial/\partial r + \kappa/r & -\omega \end{pmatrix} + \begin{pmatrix} {v_\kappa}^{11} & {v_\kappa}^{12} \\ {v_\kappa}^{21} & {v_\kappa}^{22} \end{pmatrix}\right] \begin{pmatrix} {g_\kappa}^{11} & {g_\kappa}^{12} \\ {g_\kappa}^{21} & {g_\kappa}^{22} \end{pmatrix}$$

$$= \frac{\delta(r - r'\,)}{rr'}\,, \tag{33}$$

where vg denotes $\int r_2^2 dr_2 v(r, r_2; \omega) g(r_2, r'; \omega)$ for notational convenience.

Defining $\bar{g}(r, r'; \omega) = rr' g(r, r'; \omega)$ and $\bar{v}(r, r'; \omega) = rr' v(r, r'; \omega)$ Eq. (33) simplifies finally to

$$\left[\begin{pmatrix} -\omega & -\partial/\partial r + \kappa/r \\ \partial/\partial r + \kappa/r & -\omega \end{pmatrix} + \begin{pmatrix} \bar{v}^{11}_\kappa & \bar{v}^{12}_\kappa \\ \bar{v}^{21}_\kappa & \bar{v}^{22}_\kappa \end{pmatrix}\right] \begin{pmatrix} \bar{g}^{11}_\kappa & \bar{g}^{12}_\kappa \\ \bar{g}^{21}_\kappa & \bar{g}^{22}_\kappa \end{pmatrix} = \delta(r - r'\,)\,. \tag{34}$$

Details of the non-trivial numerical solutions of this equation are discussed in Sec. VI.

B. Radial part of the Schwinger-Dyson equation

Inserting (26) and (27) into Eq. (22), we can write

$$V(\mathbf{r}\,, \mathbf{r}\,'; y) = \begin{pmatrix} v^{11}_\kappa(r, r'; \omega)\pi_\kappa & v^{12}_\kappa(r, r'; \omega) i\sigma_r \pi_{-\kappa} \\ -v^{21}_\kappa(r, r'; \omega) i\sigma_r \pi_\kappa & v^{22}_\kappa(r, r'; \omega)\pi_{-\kappa} \end{pmatrix} \tag{35}$$

$$= \alpha' \int dy' D_{\mu\nu}(\mathbf{r}\,, \mathbf{r}\,'; y')\alpha^\mu G(\mathbf{r}\,, \mathbf{r}\,'; y - y')\alpha^\nu$$

$$= -\alpha' \int dy' d_{jll'}(r, r'; y') \boldsymbol{\sigma} \cdot \boldsymbol{\mathcal{Y}}_{jlm} \begin{pmatrix} g^{22}_\kappa(r, r'; \omega)\pi_{-\kappa} & -g^{21}_\kappa(r, r'; \omega) i\sigma_r \pi_\kappa \\ g^{12}_\kappa(r, r'; \omega) i\sigma_r \pi_{-\kappa} & g^{11}_\kappa(r, r'; \omega)\pi_\kappa \end{pmatrix} \boldsymbol{\mathcal{Y}}^*_{jl'm} \cdot \boldsymbol{\sigma}$$

$$+ \alpha' \int dy' d^0_l(r, r'; y') Y_{lm} \begin{pmatrix} g^{11}_\kappa(r, r'; \omega)\pi_\kappa & g^{12}_\kappa(r, r'; \omega) i\sigma_r \pi_{-\kappa} \\ -g^{21}_\kappa(r, r'; \omega) i\sigma_r \pi_\kappa & g^{22}_\kappa(r, r'; \omega)\pi_{-\kappa} \end{pmatrix} Y^*_{lm}\,. \tag{36}$$

From the Appendix we find

$$\sum_m \boldsymbol{\sigma} \cdot \boldsymbol{\mathcal{Y}}_{jlm}(\Omega)\pi_\kappa(\Omega, \Omega')\boldsymbol{\sigma} \cdot \boldsymbol{\mathcal{Y}}^*_{jl'm}(\Omega') = \sum_{\kappa'} \mathcal{A}^{\kappa'\kappa}_{jll'}\,\pi_{\kappa'}(\Omega, \Omega')\,, \tag{37}$$

$$\sum_m \boldsymbol{\sigma} \cdot \boldsymbol{\mathcal{Y}}_{jlm}(\Omega)\sigma_r\pi_\kappa(\Omega, \Omega')\boldsymbol{\sigma} \cdot \boldsymbol{\mathcal{Y}}^*_{jl'm}(\Omega') = \sum_{\kappa'} \mathcal{B}^{\kappa'\kappa}_{jll'}\,\sigma_r\pi_{\kappa'}(\Omega, \Omega')\,, \tag{38}$$

$$\sum_m Y_{lm}(\Omega)\pi_\kappa(\Omega, \Omega')Y^*_{lm}(\Omega') = \sum_{\kappa'} \mathcal{C}^{\kappa'\kappa}_l\,\pi_{\kappa'}(\Omega, \Omega')\,. \tag{39}$$

The self-energy coefficients $\mathcal{A}, \mathcal{B}$ and $\mathcal{C}$ are explicitly defined in the Appendix.

With these formulae the radial Schwinger-Dyson Eq. (36) reads

$$\bar{v}^{11}_\kappa(r, r'; y) = \alpha' \int dy' \Big[d^0_l(r, r')\bar{g}^{11}_{\kappa'}(r, r'; y')\mathcal{C}^{\kappa\kappa'}_l$$

$$- d_{jll'}(r,r';y')\bar{g}^{22}_{\kappa'}(r,r';y-y')\mathcal{A}^{\kappa-\kappa'}_{jll'}\Big], \tag{40}$$

$$\bar{v}^{12}_{\kappa}(r,r';y) = \alpha' \int dy' \Big[d^0_l(r,r')\bar{g}^{12}_{\kappa'}(r,r';y')\mathcal{C}^{-\kappa-\kappa'}_l + d_{jll'}(r,r';y')\bar{g}^{21}_{\kappa'}(r,r';y-y')\mathcal{B}^{-\kappa\kappa'}_{jll'}\Big], \tag{41}$$

$$\bar{v}^{21}_{\kappa}(r,r';y) = \alpha' \int dy' \Big[d^0_l(r,r')\bar{g}^{21}_{\kappa'}(r,r';y')\mathcal{C}^{\kappa\kappa'}_l + d_{jll'}(r,r';y')\bar{g}^{12}_{\kappa'}(r,r';y-y')\mathcal{B}^{\kappa-\kappa'}_{jll'}\Big], \tag{42}$$

$$\bar{v}^{22}_{\kappa}(r,r';y) = \alpha' \int dy' \Big[d^0_l(r,r')\bar{g}^{22}_{\kappa'}(r,r';y')\mathcal{C}^{-\kappa-\kappa'}_l - d_{jll'}(r,r';y')\bar{g}^{11}_{\kappa'}(r,r';y-y')\mathcal{A}^{-\kappa\kappa'}_{jll'}\Big]. \tag{43}$$

Here we have also used Eqs. (15) and (16) as well as the symmetry properties $d^0_l(r,r') = d^0_l(r',r)$ and $d_{jll'}(r,r';\omega) = d_{jl'l}(r',r;\omega)$ which hold for both pure real and imaginary ω because in Eq. (13) for the gluon propagator only ω^2 (and not ω) appears.

V. THE QUARK WAVE FUNCTION

By interpreting the nonlocal self-energy as an effective potential we can now determine the wave function $q(\mathbf{r})$ and energy eigenvalue ϵ of a single quark in the cavity. The corresponding Dirac equation reads

$$\boldsymbol{\alpha}\cdot\mathbf{p}\; q(\mathbf{r}) + \int d^3r_2 V(\mathbf{r},\mathbf{r}_2;\epsilon)q(\mathbf{r}_2) = \epsilon\; q(\mathbf{r}). \tag{44}$$

In spherical coordinates, $q(\mathbf{r})$ can be written in the form [2]

$$q(\mathbf{r}) = \sum_{\kappa\mu} \begin{pmatrix} u_\kappa(r)/r \\ -\mathrm{i}\sigma_r v_\kappa(r)/r \end{pmatrix} \otimes \mathcal{Y}_{\kappa\mu}(\Omega). \tag{45}$$

After angular decomposition, the radial part of Eq. (44) obeys

$$\begin{pmatrix} 0 & -\partial/\partial r + \kappa/r \\ \partial/\partial r + \kappa/r & 0 \end{pmatrix} \begin{pmatrix} u_\kappa(r) \\ v_\kappa(r) \end{pmatrix} + \int dr_2 \bar{V}_\kappa(r,r_2;\epsilon) \begin{pmatrix} u_\kappa(r_2) \\ v_\kappa(r_2) \end{pmatrix} = \epsilon \begin{pmatrix} u_\kappa(r) \\ v_\kappa(r) \end{pmatrix}. \tag{46}$$

VI. NUMERICAL CALCULATION

In our calculations we use a (modified) Fermi function shaped spherically symmetric color-dielectric function

$$\kappa(r) = \frac{1-\kappa_v}{1+e^{(r-R)/A}} + \kappa_v \,, \tag{47}$$

where R and A are the radius and the surface thickness respectively of the profile (see fig. 1). The small but non-zero vacuum value κ_v guarantees that, e.g., the energy of a single quark in the cavity remains finite. For color-singlet multi-quark systems the limit $\kappa_v \to 0$ can be performed as will be shown in Sec. VII.

Since our model is not renormalizable, an ultraviolet momentum cutoff is needed; this is consistent with asymptotic freedom. This cutoff should reflect the energy scale of the described physics; we choose $\Lambda_{CDM} = 5.0$ fm^{-1}. In terms of the variables used in this paper the ω'-integration in Eq. (17) is cutoff at $|\omega_{max}| = \Lambda_{CDM}$ and the necessary summations over angular momenta are limited by $l_{max} = R\ \omega_{max}$ with R from Eq. (47). A careful analysis of the renormalization problem for a nonlocal, spatially varying dielectric medium can be found in ref. [11].

With the gluon propagator derived in Sec. III, we solve the coupled equations (34) and (40 - 43) to obtain the full quark propagator and the quark self-energy on the imaginary ω-axis. Because of the absence of poles in this region the self energy is numerically stable and no oscillations occur. A Taylor-expansion method is subsequently applied to construct the quark self-energy on the real ω-axis:

$$v_\kappa(r,r';z) = v_\kappa(r,r';0) + z v'_\kappa(r,r';0) + \frac{z^2}{2} v''_\kappa(r,r';0) + \frac{z^3}{6} v^{(3)}_\kappa(r,r';0) + \frac{z^4}{24} v^{(4)}_\kappa(r,r';0) + \cdots \,, \tag{48}$$

where the derivatives are evaluated in terms of the discrete values of the functions along the imaginary ω-axis.

Eq. (34) is a coupled system of integro-differential equations. For its numerical solution we use matrix inversion. It is well known that the leap frog instability [12] (p. 342) appears in an equation like Eq. (34) when the first order derivative is replaced by a centered difference. Therefore we introduce a small second order derivative term to suppress the instability:

$$\left[\begin{pmatrix} -\omega & -\partial/\partial r + B\partial^2/\partial r^2 + \kappa/r \\ \partial/\partial r + B\partial^2/\partial r^2 + \kappa/r & -\omega \end{pmatrix} + \begin{pmatrix} \bar{v}^{11}_\kappa & \bar{v}^{12}_\kappa \\ \bar{v}^{21}_\kappa & \bar{v}^{22}_\kappa \end{pmatrix}\right]$$

$$\times \begin{pmatrix} \bar{g}^{11}_{B\kappa} & \bar{g}^{12}_{B\kappa} \\ \bar{g}^{21}_{B\kappa} & \bar{g}^{22}_{B\kappa} \end{pmatrix} = \delta(r - r'\,)\,, \tag{49}$$

where $B = b\ \Delta$ is a small number. Δ is the grid interval and $b \sim \pm 1$ for $\kappa = \mp 1$ (the sign is important to suppress the leap frog effect!). This additional regularizing term does not spoil the accuracy of the solution.

Numerically we find that $g_\kappa = g_{B\kappa}$ satisfies Eq. (34) very well if $B \sim \pm\Delta$. Therefore $g_{B\kappa}$ can be considered as a first approximation to g_κ. There must be a discontinuity in the Green's function for first order differential equations. In Eq. (34), this discontinuity occurs in its off diagonal elements [10]. We find numerically that the additional second derivative term smooths ot the discontinuity somewhat.

This approximation can be improved. This will be demonstrated first in general terms. Consider the following two Green's equations:

$$L_0 G_0 = \delta\,, \tag{50}$$

$$(L_0 + L_B)G = \delta\,. \tag{51}$$

After operating with G_0 on both sides of Eq. (51) and integrating, we have

$$\begin{aligned} G_0 &= G_0(L_0 + L_B)G \\ &= G + G_0 L_B G\,, \end{aligned} \tag{52}$$

or

$$G = G_0 - G_0 L_B G\,. \tag{53}$$

Similarly, by integrating both sides of Eq. (34) with $\bar{g}_{B\kappa}$, we have the exact relation:

$$\begin{aligned} &\bar{g}_\kappa(r, r'; \omega) = \bar{g}_{B\kappa}(r, r'; \omega) \\ &+ \int dr_2 \bar{g}_{0\kappa}(r, r_2; \omega) \begin{pmatrix} 0 & B\partial^2/\partial r^2 \\ B\partial^2/\partial r^2 & 0 \end{pmatrix} \bar{v}_\kappa(r_3, r_2; \omega) \bar{g}_\kappa(r_2, r'; \omega)\,. \end{aligned} \tag{54}$$

This equation can be solved by iteration. However, in this case the leap frog instability eventually creeps in again. We have thus carried out only one iteration.

Some of the results of the self energy calculation is shown in figs. 4, 5 and 6. The figures display $\bar{V}_\kappa^{mn}(\omega; r, r')$ for $\kappa = -1$, $\omega = 1\text{fm}^{-1}$ and $(mn) = (11), (22)$ and (12) respectively.

We note that the self energy is non-zero. This implies a dynamical breaking of chiral symmetry. The structure of the self energy reflects the nonlocal character of the interaction. However, the self energy is sharply peaked around $r = r'$ reflecting the dominance of the local contribution.

The self energy is inserted in the Dirac equation which is solved self-consistently for the ground state ($\kappa = -1$). The result is shown in fig. 7 for the $\kappa(r)$ profile of fig. 1.

The single-quark energy ϵ is shown in fig. 8 as a function of κ_v. It does not exhibit a sign of divergence as far as the calculation could be carried out (down to $\kappa_v = 0.05$). In fact, ϵ turns out to be quite insensitive to κ_v if κ_v is small. The presented results are thus gratifying.

We have tested our numerical calculation by varying the following numerical parameters:

(a) the number N_{max} of r grid points,

(b) the integral limit r_{max} of r and

(c) the number N_ω of ω grid points.

For the actual parameters we have chosen the physical observables are all insensitive to them.

VII. HADRONIC PROPERTIES

Having calculated the wave function and energy eigenvalue of a single quark in a cavity, we now investigate color-neutral composite systems of N_q valence quarks. Evidently, $N_q = 2$ for mesons and $N_q = 3$ for baryons. The energy of these systems is calculated in the one gluon exchange approximation. Finally, corrections due to the center-of-mass motion and to the σ-field are taken into account approximately. Using scaling relations we fix the mass of the nucleon m_N and study the pion mass m_π. Its deviation from zero is a measure of how good our approximations are since the pion should be massless according to Goldstone's theorem.

A. One gluon exchange approximation

The one gluon exchange interaction energy between quarks (of equal eigen-energy) is given by [2]

$$E_{ex} = \alpha' \int d^3r_1 d^3r_2 [j^0(\mathbf{r}_1) D^{00}(\mathbf{r}_1, \mathbf{r}_2) j^0(\mathbf{r}_2) - \mathbf{j}(\mathbf{r}_1) \cdot \overleftrightarrow{D}(\mathbf{r}_1, \mathbf{r}_2; 0) \cdot \mathbf{j}(\mathbf{r}_2)] \,, \tag{55}$$

with $\alpha' = \frac{1}{4} g_s^2 \sum_{i<j} <\boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j>$. The color matrix element $<\boldsymbol{\lambda}_1 \cdot \boldsymbol{\lambda}_2>$ has the value $-16/3$ for the pion and $-8/3$ for the nucleon [2]. Taking into account that that for both the pion and the nucleon, the quarks are in the ground state with $\kappa = -1, \mu = \pm 1/2$ the corresponding currents can be evaluated and the exchange energy is ready calculated.

The total energy of quarks and gluons in a hadron with N_q valence quarks is then given by

$$E_{q,g} = N_q \ \epsilon + E_{ex} \,. \tag{56}$$

B. Corrections and sigma contributions

Up to now the σ-field has been neglected. However, it contributes to the total energy of the bag. The σ-field can be reconstructed from $\kappa(r)$ and $\kappa(\sigma)$ given in Eqs. (47) and (3) respectively. Then the σ-field energy is given by

$$E_\sigma = \int \left[\frac{1}{2} (\nabla \sigma)^2 + U(\sigma) \right] d^3r \,, \tag{57}$$

with the potential $U(\sigma)$ given in Eq. (4). The total energy of the bag is then $E_{bag} = E_{q,g} + E_\sigma$.

We now address the hadronic center-of-mass energy. Since localization of the bag breaks Lorentz invariance, the bag acquires a non-zero total momentum that contributes to the total energy of the system. The easiest way to correct this effect is to use the following approximate formula (projection [13] would be better but more cumbersome):

$$m_h^2 = E_{bag}^2 - < P^2 >_{bag} \,, \tag{58}$$

with

$$< P^2 >_{bag} = N_q \ < P^2 >_q + < P^2 >_\sigma \ . \tag{59}$$

The momentum squared of one quark is given by

$$< P^2 >_q = \int d^3r |\nabla q|^2 \,. \tag{60}$$

In order to calculate $< P^2 >_\sigma$, the coherent state approximation [2] is used:

$$< P^2 >_\sigma = \int d^3k k^2 \omega_k f_k^2 \,. \tag{61}$$

f_k are the Fourier transforms of $\sigma(r)$, ω_k is the σ-field energy in the mode k.

For slowly varying ω_k we finally get:

$$< P^2 >_\sigma = (m_{GB}^2 + < k^2 >)^{\frac{1}{2}} \int d^3r (\nabla\sigma)^2 \,, \tag{62}$$

with the glueball mass m_{GB}.

C. Scaling and the nucleon mass

Scaling can be used to generate new solutions [14] from those presented so far. The equations are invariant under scale transformations where all lengths r are replaced by

$$r \;\rightarrow\; r' = \lambda r \,, \tag{63}$$

all energies and frequencies (including the cutoff Λ_{CDM}) are replaced by

$$E \rightarrow \; E' = E/\lambda \tag{64}$$

and

$$a \;\rightarrow\; a' = a/\lambda^2 \,, \quad b' \; \rightarrow b/\lambda \,. \tag{65}$$

c and α_s are invariant. The σ-field and the gluon field potentials scale as length^{-1}.

D. Numerical results

Throughout our calculations, we use a cutoff $\Lambda_{CDM} = \omega_m = 5$ fm^{-1}. With $l_m = R\,\omega_m$, the quark wave functions and energies depend on the two parameters R and A from the $\kappa(r)$ profile. The hadron masses additionally depend on the parameters a, b, c of the σ-field potential $U(\sigma)$. In order to minimize the number of free parameters, we assume that $U(\sigma)$ is universal in all hadrons. However, each hadron has a different $\kappa(r)$ profile reflecting the fact that the hadronic size is not universal.

The numerical procedure is as follows: We choose a potential $U(\sigma)$ and calculate the corrected nucleon mass according to Eq. (58). Using scaling relations we renormalize all dimensional properties by fixing the nucleon mass to its empirical value $m_N = 938$ MeV. With these renormalized properties we now calculate the pion mass as a function of R and A.

To this point, the σ-field is not self-consistent. We now vary the parameters of the $\kappa(r)$ profile in order to find an extremum in the energy. This is a first approximation to a fully self-consistent treatment. However, we expect the results to be reasonable since the proper shape of the σ-field is similar to a Fermi function.

We find that there is not always a minimum in A for a given $m_\pi(R)$. This may be due to the crude method used to correct the effects of the-center-of mass motion, to the form of $\kappa(r)$, or the point is an extremum, not a minimum. The resulting pion mass is small but non-zero.

VIII. SUMMARY AND PROSPECTIVES

Within the framework of the chirally-invariant chromo-dielectric soliton model, the Abelian gluon propagator is solved in configuration space for a color-dielectric function with two parameters. The quark self energy was obtained by solving the (nonlocal) Schwinger-Dyson equation in configuration space as a function of imaginary energy. Quark wave functions and real eigenvalues were obtained. Bag states were constructed for the pion and the nucleon including one gluon exchange mutual interaction between quark pairs. The parameters of the parameterized σ-field (or equivalently, the dielectric function $\kappa(r)$) were varied to extremize the bag energy. Approximate center-of-mass corrections are calculated. Employing scaling relations, the nucleon mass was set to its empirical value. The resulting pion mass was determined to be small (the actual value depending on the model parameters) but not zero, as demanded by Goldstone's theorem.

Extensions of the present work include the following:

a) Center-of-mass corrections based on variation after projection. This technique has been studied extensively by Lübeck *et al.* [13] for the Friedberg-Lee soliton and was found to give significant corrections. It is certainly more reliable than the prescription $m^2 \approx < H >^2 - < p^2 >$ used in the present paper.

b) A "more" self-consistent treatment of the soliton field by either solving the differential equation for the σ-field or by including more parameters in the functional form of $\kappa(\sigma)$ and $\kappa(r)$.

c) Calculation of the mutual gluon exchange between quark pairs by full summation of ladder diagrams.

d) A systematic adjustment of model-parameters to fit the properties of all low-lying hadrons. This is not as tedious a task as it might first appear. The parameters of the model are $a,\ b,\ c$ and α_s. The functional form of $\kappa(\sigma)$ also introduces a model dependence, but the results appear to be quite insensitive to that. Of the four, one is set by the nucleon mass using scaling from any given set. Results appear to be relatively insensitive to the "family" characterized by b^2/ac [2] but this is related to the glueball mass which is assumed to lie in the range of $1 - 2\ GeV$. The key parameters include the nucleon size, magnetic moments, g_A/g_V and the N-Δ mass splitting. Other hadronic spectra properties are then regarded as predictions of the model.

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APPENDIX A:

In this appendix the self-energy coefficients $\mathcal{A}, \mathcal{B}$ and $\mathcal{C}$ from Sec. IV B (Eqs. (37-39)) are explicitly evaluated.

We start with formula (5.9.15) of Edmonds' [15]

$$\sigma_q \chi_\nu = \sqrt{3} < 1/2,\ \nu,\ 1,\ q | 1/2,\ q + \nu > \chi_{q+\nu}\,. \tag{A1}$$

Note that we use throughout our calculations the phase convention of Edmonds. With the definitions

$$\mathcal{Y}_{\kappa\mu}(\Omega) \equiv \mathcal{Y}^{l_\kappa}_{j_\kappa\mu}(\Omega) \equiv \sum_{\nu m} < l_\kappa, m, 1/2, \nu | j_\kappa, \mu > Y_{l_\kappa m}\chi_\nu \, , \tag{A2}$$

$$\boldsymbol{\mathcal{Y}}_{ll'm}(\Omega) \equiv \sum_{qm'} < l', m', 1, q | l, m > Y_{l'm'}\boldsymbol{\epsilon}_q \, , \tag{A3}$$

$$\boldsymbol{\epsilon}_{\pm 1} = \mp \frac{\hat{\mathbf{x}} \pm \mathrm{i}\hat{\mathbf{y}}}{\sqrt{2}}, \quad \boldsymbol{\epsilon}_0 = \hat{\mathbf{z}} \, , \tag{A4}$$

we get

$$\begin{aligned}
&\boldsymbol{\mathcal{Y}}_{ll'm}(\Omega) \cdot \boldsymbol{\sigma}\mathcal{Y}_{\kappa\mu}(\Omega) \\
&= \sum_{m_1 q m_2 \nu} < l',\, m_1,\, 1,\, q | l,\, m > Y_{l'm_1}\sigma_q < l_\kappa,\, m_2,\, 1/2,\, \nu | j_\kappa,\, \mu > \chi_\nu Y_{l_\kappa m_2} \\
&= \sum_{m_1 q m_2 \nu L M \nu'} < l',\, m_1,\, 1,\, q | l,\, m > Y_{LM} < l_\kappa,\, m_2,\, 1/2,\, \nu | j_\kappa,\, \mu >< l',\, 0,\, l_\kappa,\, 0 | L,\, 0 > \\
&\times < l',\, m_1,\, l_\kappa,\, m_2 | L,\, M > \sqrt{\frac{(2l'+1)(2l_\kappa+1)}{4\pi(2L+1)}}\sqrt{3} < 1/2,\, \nu,\, 1,\, q | 1/2,\, \nu' > \chi_{\nu'} \\
&= \sum_{\mu' j L} (-1)\sqrt{(2l+1)(2j_\kappa+1)(2l_\kappa+1)(2l'+1)3/2\pi} \\
&\times < l,\, m,\, j_\kappa,\, \mu | j,\, \mu' >< l',\, 0,\, l_\kappa, 0 | L, 0 > \begin{Bmatrix} L & 1/2 & j \\ l' & 1 & l \\ l_\kappa & 1/2 & j_\kappa \end{Bmatrix} \mathcal{Y}^L_{j\mu'} \, .
\end{aligned} \tag{A5}$$

Here we have used the contraction formula for spherical harmonics (Edmonds (5.16)) and the definition of the $9j$-symbols (Edmonds (6.4.3)).

Similarly,

$$\begin{aligned}
&2\hat{\mathbf{r}} \cdot \boldsymbol{\mathcal{Y}}_{ll'm}(\Omega)\mathcal{Y}_{\kappa\mu}(\Omega) \\
&= \sum_{m_1 q m_2 \nu} 2 < l', m_1,\, 1, q | l, m > Y_{l'm_1}\sqrt{\frac{4\pi}{3}} Y_{1q} < l_\kappa,\, m_2,\, 1/2,\, \nu | j_\kappa,\, \mu > \chi_\nu Y_{l_\kappa m_2} \\
&= \sum_{m_1 q m_2 \nu} 2 < l',\, 0,\, 1, 0 | l, 0 > Y_{lm}\sqrt{\frac{4\pi}{3}\,\frac{(2+1)(2l'+1)}{4\pi(2l+1)}} < l_\kappa,\, m_2,\, 1/2,\, \nu | j_\kappa,\, \mu > \chi_\nu Y_{l_\kappa m_2} \\
&= \sum_{jL\mu'} (-1)^{1/2+l+l_\kappa+j}\sqrt{\frac{(2l_\kappa+1)(2j_\kappa+1)(2l'+1)}{\pi}} < l',\, 0,\, 1,\, 0 | l,\, 0 > \\
&\times < l, 0, l_\kappa, 0 | L, 0 > \begin{Bmatrix} j_\kappa & 1/2 & l_\kappa \\ L & l & j \end{Bmatrix} < l,\, m,\, j_\kappa,\, \mu | j,\, \mu' > \mathcal{Y}^L_{j\mu'}
\end{aligned} \tag{A6}$$

and

$$\begin{aligned}
&Y_{lm}(\Omega)\mathcal{Y}_{\kappa\mu}(\Omega)\\
&=\sum_{m_1 q m_2 \nu} Y_{lm} < l_\kappa,\, m_2,\, 1/2,\, \nu|j_\kappa,\, \mu > \chi_\nu Y_{l_\kappa m_2}\\
&=\sum_{jL\mu'}(-1)^{1/2-j_\kappa+l_\kappa+2j}\sqrt{\frac{(2l_\kappa+1)(2j_\kappa+1)(2l+1)}{4\pi}}\\
&\times < l, 0, l_\kappa, 0|L, 0 > \begin{Bmatrix} j_\kappa & 1/2 & l_\kappa \\ L & l & j \end{Bmatrix} < j_\kappa,\, \mu,\, l,\, m|j,\, \mu' > \mathcal{Y}^L_{j\mu'}\,.
\end{aligned} \tag{A7}$$

According to Eq. (A5), the expression $\sum_{m\mu}\boldsymbol{\mathcal{Y}}_{ll'm}(\Omega)\cdot\boldsymbol{\sigma}\mathcal{Y}_{\kappa\mu}(\Omega)\mathcal{Y}^\dagger_{\kappa\mu}(\Omega')\boldsymbol{\sigma}\cdot\boldsymbol{\mathcal{Y}}^*_{ll''m}(\Omega')$ is proportional to $\delta_{jj'}$ and $\delta_{\mu\mu'}$. Now L and L' have to be equal or differ by 1. However, $< l',\, 0,\, l_\kappa,\, 0|L,\, 0 >< l'', 0,\, l_\kappa, 0|L', 0 >$ vanishes if $|L-L'|$ is odd, since $l'-l''$ is even, so only terms with $L=L'$ (or $\kappa'=\kappa''$) contribute. Thus

$$\begin{aligned}
&\sum_{m\mu}\boldsymbol{\mathcal{Y}}_{ll'm}(\Omega)\cdot\boldsymbol{\sigma}\mathcal{Y}_{\kappa\mu}(\Omega)\mathcal{Y}^\dagger_{\kappa\mu}(\Omega')\boldsymbol{\sigma}\cdot\boldsymbol{\mathcal{Y}}^*_{ll''m}(\Omega')\\
&=\sum_{\kappa\mu'}\mathcal{A}^{\kappa'\kappa}_{ll'l''}\,\mathcal{Y}_{\kappa'\mu'}(\Omega)\mathcal{Y}^\dagger_{\kappa'\mu'}(\Omega')\,.
\end{aligned} \tag{A8}$$

The following symmetry relation $\mathcal{A}^{\kappa'\kappa}_{ll'l''}=\mathcal{A}^{\kappa'\kappa}_{ll''l'}$ holds.

Similarly, according to Eq. (A2), the expression $\sum_{m\mu}2\hat{\mathbf{r}}\cdot\boldsymbol{\mathcal{Y}}_{ll'm}(\Omega)\mathcal{Y}_{\kappa\mu}(\Omega)\mathcal{Y}^\dagger_{\kappa\mu}(\Omega')\boldsymbol{\sigma}\cdot\boldsymbol{\mathcal{Y}}^*_{ll''m}(\Omega')$ is proportional to $\delta_{jj'}$ and $\delta_{\mu\mu'}$. Now L and L' have again to be equal or differ by 1. However, $< l, 0,\, l_\kappa, 0|L, 0 >< l', 0,\, 1, 0|l, 0 >< l'', 0,\, l_\kappa, 0|L', 0 >$ vanishes if $|L-L'|$ is even, since $l'-l''$ is even, so only terms with $L=L'\pm 1$ (or $\kappa'=-\kappa''$) contribute. Thus

$$\begin{aligned}
&\sum_{m\mu}2\hat{\mathbf{r}}\cdot\boldsymbol{\mathcal{Y}}_{ll''m}(\Omega)\mathcal{Y}_{\kappa\mu}(\Omega)\mathcal{Y}^\dagger_{\kappa\mu}(\Omega')\boldsymbol{\sigma}\cdot\boldsymbol{\mathcal{Y}}^*_{ll''m}(\Omega')\\
&=\sum_{\kappa\mu'}\tilde{\mathcal{B}}^{\kappa'\kappa}_{ll'l''}\,\mathcal{Y}_{\bar{\kappa}'\mu'}(\Omega)\mathcal{Y}^\dagger_{\kappa'\mu'}(\Omega')\,.
\end{aligned} \tag{A9}$$

Similarly, we have

$$\sum_{m\mu}Y_{lm}\mathcal{Y}_{\kappa\mu}(\Omega)\mathcal{Y}^\dagger_{\kappa\mu}(\Omega')Y^*_{lm}(\Omega')=\sum_{\kappa\mu'}\mathcal{C}^{\kappa'\kappa}_{l}\,\mathcal{Y}_{\kappa'\mu'}(\Omega)\mathcal{Y}^\dagger_{\kappa'\mu'}(\Omega') \tag{A10}$$

Finally, the quark-gluon coupling coefficients $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ are given by

$$\begin{aligned}
\mathcal{A}^{\kappa'\kappa}_{ll'l''}=&\frac{3}{2\pi}\sqrt{(2l'+1)(2l''+1)}(2j_\kappa+1)(2l_\kappa+1)(2l+1) < l', 0,\, l_\kappa, 0|l_{\kappa'}, 0 >\\
&\times < l'', 0,\, l_\kappa, 0|l_{\kappa'}, 0 > \begin{Bmatrix} l_{\kappa'} & 1/2 & j_{\kappa'} \\ l' & 1 & l \\ l_\kappa & 1/2 & j_\kappa \end{Bmatrix}\begin{Bmatrix} l_{\kappa'} & 1/2 & j_{\kappa'} \\ l'' & 1 & l \\ l_\kappa & 1/2 & j_\kappa \end{Bmatrix}\,,
\end{aligned} \tag{A11}$$

$$
\begin{aligned}
&\tilde{\mathcal{B}}^{\kappa'\kappa}_{ll'l''} \\
&= (-1)^{-1/2+l+l_\kappa+j_{\kappa'}} \sqrt{3/2(2l'+1)(2l''+1)(2l+1)} \frac{(2l_\kappa+1)(2j_\kappa+1)}{\pi} \\
&\times < l'', 0,\, l_\kappa, 0 | l_{\kappa'}, 0 >< l', 0,\, 1, 0 | l, 0 >< l, 0, l_\kappa, 0 | l_{\bar{\kappa}'}, 0 > \\
&\times \begin{Bmatrix} j_\kappa & 1/2 & l_\kappa \\ l_{\bar{\kappa}'} & l & j_{\kappa'} \end{Bmatrix} \begin{Bmatrix} l_{\kappa'} & 1/2 & j_{\kappa'} \\ l'' & 1 & l \\ l_\kappa & 1/2 & j_\kappa \end{Bmatrix} ,
\end{aligned} \tag{A12}
$$

$$
\mathcal{C}^{\kappa'\kappa}_{l} = \frac{(2l_\kappa+1)(2j_\kappa+1)(2l+1)}{4\pi} < l, 0, l_\kappa, 0 | l_{\kappa'}, 0 >^2 \begin{Bmatrix} j_\kappa & 1/2 & l_\kappa \\ l_{\kappa'} & l & j_{\kappa'} \end{Bmatrix}^2 . \tag{A13}
$$

Furthermore,

$$
\begin{aligned}
&\sum_{m\mu} \boldsymbol{\mathcal{Y}}_{ll'm}(\Omega) \cdot \boldsymbol{\sigma} \sigma_r \mathcal{Y}_{\kappa\mu}(\Omega) \mathcal{Y}^\dagger_{\kappa\mu}(\Omega') \boldsymbol{\sigma} \cdot \boldsymbol{\mathcal{Y}}^*_{ll''m}(\Omega') \\
&= \sum_{m\mu} [2\hat{\mathbf{r}} \cdot \boldsymbol{\mathcal{Y}}_{ll''m}(\Omega) - \sigma_r \boldsymbol{\mathcal{Y}}_{ll'm}(\Omega) \cdot \boldsymbol{\sigma}] \mathcal{Y}_{\kappa\mu}(\Omega) \mathcal{Y}^\dagger_{\kappa\mu}(\Omega') \boldsymbol{\sigma} \cdot \boldsymbol{\mathcal{Y}}^*_{ll''m}(\Omega') \\
&= \sum_{\kappa\mu'} \mathcal{B}^{\kappa'\kappa}_{ll'l''}\, \sigma_r \mathcal{Y}_{\kappa'\mu'}(\Omega) \mathcal{Y}^\dagger_{\kappa'\mu'}(\Omega') \, .
\end{aligned} \tag{A14}
$$

In that very last step we have used Eqs. (A5) and (A6) as well as the identity $\sigma_r\, \mathcal{Y}_{\kappa\mu} = -\mathcal{Y}_{\bar{\kappa}\mu}$ and the identification

$$
\mathcal{B}^{\kappa'\kappa}_{ll'l''} \equiv -\mathcal{A}^{\kappa'\kappa}_{ll'l''} - \tilde{\mathcal{B}}^{\kappa'\kappa}_{ll'l''} \, . \tag{A15}
$$

Working out the hermitian conjugate of Eqs. (A8) and (A9) we get the following symmetry relations: $\mathcal{A}^{\kappa'\kappa}_{ll'l''} = \mathcal{A}^{\kappa'\kappa}_{ll''l'}$ and $\mathcal{B}^{\kappa'\kappa}_{ll'l''} = \mathcal{B}^{-\kappa'\ -\kappa}_{ll''l'}$.

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Figures

Figure 1: The color-dielectric function $\kappa(r)$ for $R = .8$ fm, $A = .15$ fm and $\kappa_v = .15$ (r in fm).

Figure 2: The tensor part of the gluon propagator in the transverse magnetic mode $d_{102}(r, r')$.

Figure 3: The tensor part of the gluon propagator in the transverse magnetic mode $d_{122}(r, r')$.

Figure 4: The quark self energy on the real ω-axis: $\bar{v}^{11}_{-1}(r, r')$.

Figure 5: The quark self energy on the real ω-axis: $\bar{v}^{22}_{-1}(r, r')$.

Figure 6: The quark self energy on the real ω-axis: $\bar{v}^{12}_{-1}(r, r')$.

Figure 7: The quark wave function. $ru(r)$ is the darker line, $rv(r)$ is the lighter one.

Figure 8: The single-quark energy ϵ (in fm^{-1}) as a function of κ_v

TABLES

TABLE I. Table of pion masses for $a = 39.9$, $b = -746.2$, $c = 4569.6$, $B = 0.03892$, $m_{GB} = 1310.8$

R (fm)	A (fm)	E_q (MeV)	$E_{q,g}$ (MeV)	$\sqrt{< P^2 >_Q}$ (MeV)	$\sqrt{< P^2 >_\sigma}$ (MeV)	m_π (MeV)
0.6	0.150	404.92	610.61	477.20	189.6	171.31
0.6	0.175	405.81	612.17	476.50	178.3	235.94
0.6	0.200	405.97	613.19	476.10	170.1	290.23
0.6	0.225	405.71	615.40	475.90	164.0	344.06
0.6	0.250	405.22	614.81	475.80	159.4	390.66
0.8	0.150	295.37	452.89	489.10	249.4	344.64
0.8	0.175	296.77	454.19	486.50	232.1	284.70
0.8	0.200	297.20	454.17	484.80	219.0	208.37
0.8	0.225	297.01	453.40	483.70	208.8	61.61
0.8	0.250	296.39	452.22	482.90	200.7	198.74
1.0	0.150	230.69	358.91	485.20	311.7	338.67
1.0	0.175	232.25	360.33	480.00	288.5	238.99
1.0	0.200	232.76	360.09	476.20	270.6	27.71
1.0	0.225	232.60	358.83	473.80	256.3	237.19
1.0	0.250	231.99	356.95	471.70	244.8	342.74
1.2	0.150	194.97	308.29	400.90	375.5	371.10
1.2	0.175	196.45	309.63	398.10	346.6	462.86
1.2	0.200	197.08	309.56	396.10	323.9	540.69
1.2	0.225	197.18	308.61	394.60	305.7	612.00