ARISTOTLE'S ARGUMENT FROM UNIVERSAL MATHEMATICS AGAINST THE EXISTENCE OF PLATONIC FORMS*

PIETER SJOERD HASPER
https://orcid.org/0000-0002-8168-9641
University of Tübingen
Department of Philosophy
Bursagasse 1, 72074 Tübingen
Germany
pshasper@hotmail.com

Article info
CDD: 185
Received: 15.10.2019; Accepted: 21.10.2019
DOI: http://dx.doi.org/10.1590/0100-6045.2019.V42N4.PH

Keywords
Platonic Forms
Object of scientific knowledge
Ekthesis
Eudoxus
Mathematics

Abstract: In *Metaphysics* M.2, 1077a9-14, Aristotle appears to argue against the existence of Platonic Forms on the basis of there being certain universal mathematical proofs which are about things that are 'beyond' the ordinary objects of mathematics and that cannot be identified with any of these. It is

* This article has its origin in a larger paper about how to understand Aristotle's claim that *epistêmê* is of universals. In that form it has profited from comments from audiences in München, Berkeley, Bloomington, Oxford and Pittsburgh. In its present form it has improved after critical questions from an audience in Chicago and from Lucas Angioni. I dedicate the paper to the Philosophy Department of Indiana University at Bloomington, in gratitude.

a very effective argument against Platonism, because it provides a counter-example to the core Platonic idea that there are Forms in order to serve as the object of scientific knowledge: the universal of which theorems of universal mathematics are proven in Greek mathematics is neither Quantity in general nor any of the specific quantities, but Quantity-of-type-x. This universal cannot be a Platonic Form, for it is dependent on the types of quantity over which the variable ranges. Since for both Plato and Aristotle the object of scientific knowledge is that F which explains why G holds, as shown in a 'direct' proof about an arbitrary F (they merely disagree about the ontological status of this arbitrary F, whether a Form or a particular used in so far as it is F), Plato cannot maintain that Forms must be there as objects of scientific knowledge - unless the mathematics is changed.

§ 1. INTRODUCTION

Throughout his works, and in the saved fragments of *De Ideis* in particular, Aristotle discusses Platonist arguments for the existence of Platonic Forms, as well as arguments against these arguments and arguments against their existence. Often he is content with the observation that the Platonist argument does not establish what it purports to establish, and also that the Platonist argument establishes more than the Platonist would like. With the Third Man Argument, which features prominently in Aristotle's writings, we have an argument directly refuting the claim that there are Platonic Forms. Needless to say, all of these arguments have been extensively studied. In this article I want to add an argument to this catalogue of arguments, one which has escaped notice thus far, but which in the context of Greek science provides a very powerful objection against the whole theory of Forms – or so I hope to show.

The most explicit presentation of the argument I will be concerned with we find in the following argument against there being Platonic Forms:

> Further there are some universal things dealt with in proofs by mathematicians beyond those substances [those substances being Forms of numbers and other mathematical objects] (γράφεται ἔνια καθόλου ὑπὸ τῶν μαθηματικῶν παρὰ ταύτας τὰς οὐσίας). Then there will be also this other substance in between, separated from the Forms and the intermediates, which is neither a number nor points nor a magnitude nor a period of time (ἔσται οὖν καὶ αὕτη τις ἄλλη οὐσία μεταξὺ κεχωρισμένη τῶν τ' ἰδεῶν καὶ τῶν μεταξύ, ἣ οὔτε ἀριθμός ἐστιν οὔτε στιγμαὶ οὔτε μέγεθος οὔτε χρόνος). But if that is impossible, clearly it is also impossible that those things [namely, numbers and the other mathematical objects] are separated from the perceptible things (εἰ δὲ τοῦτο ἀδύνατον, δῆλον ὅτι κἀκεῖνα ἀδύνατον εἶναι κεχωρισμένα τῶν αἰσθητῶν). (*Met.* M.2, 1077a9-14)

This argument is not immediately clear, but its structure can easily be discerned:

(1) There are mathematical theorems about all kinds of mathematical objects universally, not confined to numbers, point-units, magnitudes or periods of time alone.
(2) Each mathematical theorem needs a Form to be true of. (Implicitly assumed for reduction)
(3) Therefore there must be a Form which is not a number or point-units or a magnitude or a period

of time.

(4) That is impossible.

(5) Therefore (2) is false.

(6) Therefore there cannot be a Form for any mathematical theorem to be true of. (It must rather be true of something else.)

There is almost universal agreement that the kind of theorems Aristotle refers to are those which belong to what I shall call 'universal mathematics' – which consists basically of the theorems belonging to Eudoxus' general theory of proportion as we know it from Euclid, *Elements* 5.[1] Aristotle's primary example of such theorem is the alternation of proportional, that is:

$a : b = c : d$ if and only if $a : c = b : d$.

This theorem is not true of numbers or magnitudes alone, but of all types of quantity – that is why it belongs to universal mathematics.

Thus we can understand (1): there are theorems about all types of quantity together. Now it seems obvious that a Platonist would be committed to something like (2), for it is a common idea of Platonism that one, or perhaps even the, function of the Forms is to serve as the object of scientific

---

[1] W.D. Ross, *Aristotle's* Metaphysics. *A Revised Text with Introduction and Commentary* (Oxford, 1924) II, 413; T. Heath, *Mathematics in Aristotle* (Oxford, 1949) 223; J. Lear, 'Aristotle's Philosophy of Mathematics', *Philosophical Review* 91 (1982) 161-192; J. Cleary, *Aristotle and Mathematics. Aporetic Method in Cosmology and Metaphysics* (Leiden, 1995) 291; only J. Annas, *Aristotle's* Metaphysics *Books M and N* (Oxford, 1976) 143, does not really attempt to make sense of the details of the argument, but seems satisfied with some general remarks that Aristotle's example of universal mathematics is a good example to state the problem how a higher and a lower science can share the same subject-matter.

knowledge. Clearly, theorems belonging to universal mathematics are part of scientific knowledge, so with (2) the Platonist is also committed to holding that (3) there is a Form for theorems of universal mathematics to be true of.

The remaining part of Aristotle's argument, however, is more difficult to understand. In (4) he claims that it is impossible that there is a Form for universal mathematics to be true of. Why is that? There are only two answers available in the literature, and both suggest that the impossibility rests on considerations held by historical Platonists, and thus not necessarily part and parcel of the Platonist hypothesis of (2) itself. Jonathan Lear remarks: 'Neither are there any obvious candidates, as squares, circles, and numbers are obvious candidates for geometry and arithmetic, nor would the Platonist be happy to "discover" a new ideal object',[2] implying a little later that there would be such a universal object for such theorems to be true of, namely *magnitude* as it is used in Euclid, *Elements* 5, which is 'applicable to spatial magnitudes, numbers and times'.[3] Thus Lear seems to hold that the impossibility of (4) is not of a mathematical kind, and that the Platonist, even if he knew about Euclidean *magnitude*, must have had peculiar reasons to deny that it could serve as the universal object and thus as the Form for the theorems of universal mathematics to be true of.[4]

---

[2] Lear, 'Philosophy of Mathematics' 166-167

[3] Lear, 'Philosophy of Mathematics' 167

[4] Thus Lear claims the Aristotle's argument here is *ad hominem* (166). It should also be noted that the Platonist Syrianus appears to accept the existence of such universal objects which are 'more simple and more universal and more comprehensive than the others, and because of this closer to intellect and clearer and more knowable than the particular ones' (*In Metaphysica* 90.4-7), even though he restricts this to '*logoi* in the soul' – but that does not seem to make a difference to his point. In fairness, though,

John Cleary comments more extensively on Aristotle's argument here, but he too assumes that the Platonist who is Aristotle's opponent here has contingent reasons for rejecting the object of theorems in universal mathematics to be a Form. First of all, Cleary refers to Aristotle's characterisation of the universal of which the alternation of proportionals holds as not being 'a single thing having a name' (ὠνομασμένον τι … ἕν – *AnalPost* 1.5, 74a21): 'for Plato it was at least a necessary if not a sufficient condition for the positing of a Form that there be some corresponding common name; cf. *Rep.* 596a.'[5] Secondly, he refers to Aristotle's statement that the Platonists refused to posit Forms for series which are ordered by priority (*EN* 1.6, 1096a17-19), and suggests that the series point, line, plane and solid would form such a series.[6]

Needless to say, such interpretations make for a rather weak argument on Aristotle's part. Moreover, they do not seem to fit very well with Aristotle's language: 'if this is impossible, …', which suggests more than an *ad hominem* impossibility. But the worst consequence is that they make it virtually impossible to come up with an answer to the question why Aristotle thinks he is entitled to draw the final conclusion (6). For if it is impossible for there to be a Form for theorems belonging to universal mathematics because of reasons peculiar to Platonists of Aristotle's days, then these Platonists can easily resist the general conclusion Aristotle wants to draw, that the normal mathematical objects, like numbers, lines, planes and solids cannot be Forms either: they can claim that these peculiar considerations just do not hold for them.

---

one should also acknowledge that Syrianus probably did not know what he was talking about.

[5] Cleary, *Aristotle and Mathematics* 292

[6] Cleary, *Aristotle and Mathematics* 292

It has to be acknowledged, though, that also if one were to assume that there are less *ad hominem* reasons for denying that the object of universal mathematics can be a Form, a similar worry exists (though, as we shall see, it is one which can be dispelled). For how can Aristotle derive the conclusion that no mathematical theorem is true of a Form from the point of the *reductio* that in the single case of universal mathematics the Platonic hypothesis (2) that for every theorem there is a Form for it to be true of? What is more, it seems as if Aristotle should have been aware of this worry, because in the next chapter of *Metaphysics* M he appeals again to theorem in universal mathematics, but this time draws a weaker conclusion:

> For just as also the universal [proofs] in mathematics are not about separated things in addition to magnitudes and numbers, but are about them, not, however, in so far as they are such as to have magnitude or be divisible, it is clear that it *is possible* too that both the definitions and proofs [in mathematics] are about perceptible magnitudes, not, however, in so far as [they are] perceptible, but in so far as [they are] like *this* (δῆλον ὅτι ἐνδέχεται καὶ περὶ τῶν αἰσθητῶν μεγεθῶν εἶναι καὶ λόγους καὶ ἀποδείξεις, μὴ ᾗ δὲ αἰσθητὰ ἀλλ’ ᾗ τοιαδί). (1077b17-22)

Of course, once Aristotle has refuted Platonism for mathematical objects, he does not need the stronger conclusion any more. But the passage does show that he can draw the distinction between it being impossible for mathematical theorems to be true of Platonic Forms and it being possible for mathematical theorems not to be true of Platonic Forms. Thus the worry might be rephrased as how Aristotle thinks he can get from the latter to the former.

In this paper I want to provide answers to both these questions. I will first argue that the reason why it should be impossible for there to be a Form for universal mathematics is of a mathematical kind: in the context of Greek mathematics it is impossible that the universal of which theorems like the alternation of proportionals are true of is a Platonic Form, given the minimal requirement of separation and priority in account for Platonic Forms. In the second half of the paper I will argue that this mathematical impossibility, together with some assumptions shared by Platonists and Aristotle about the logical structure of proofs and their explanatory import, makes it impossible for the Platonist to defend his hypothesis that Forms are there in order to be objects of scientific knowledge.

§ 2. WHY CAN THERE NOT BE FORMS FOR UNIVERSAL MATHEMATICS TO BE TRUE OF?

That it seems unlikely that Aristotle would have thought that the reason why there cannot be a Form for theorems in universal mathematics is peculiar to the Platonists of his day, also appears from arguments in which he himself presupposes that there are problems in the case of such theorems. One example is the following aporetic argument from *Posterior Analytics* 1.24:

> Further, if (1) a universal is not something in addition to the particulars, and (2) a proof instils the opinion that this in virtue of which one proves, is something (εἶναι τι τοῦτο καθ' ὃ ἀποδείκνυσι), that is, that that exists as some nature among the things there are (καί τινα φύσιν ὑπάρχειν ἐν τοῖς οὖσι ταύτην), for example, [some nature] of *triangle* in

> addition to particular ones (τριγώνου παρὰ τὰ τινὰ), and of *shape* in addition to particular ones, and of *number* in addition to particular numbers, and (3) a better proof is one which is about something which is rather than something which is not, that is, one by which one will not be deluded, rather than by which one will be, and (7) a universal proof is such (for taking the next step (4) they will prove as in the case of proportion, for example that whatever is such – which is neither a line nor a number nor a solid nor a plane, but rather something in addition to those – will be proportional [προϊόντες γὰρ δευκνύουσιν ὥσπερ περὶ τοῦ ἀνὰ λόγον, οἷον ὅτι ὃ ἂν ᾖ τι τοιοῦτον ἔσται ἀνὰ λόγον ὃ οὔτε γραμμὴ οὔτ' ἀριθμὸς οὔτε στερεὸν οὔτ' ἐπίπεδον, ἀλλὰ παρὰ ταῦτά τι]); if (5) this [proof], though more universal, is less about something which is than a particular one and instils a false opinion, then (7) a universal proof would be worse than a particular one. (*AnalPost* 1.24, 85a31-b3)

The structure of this argument is as follows:

(1) A universal is not something in addition to the particulars.
(2) A proof that F qua F is G instils the opinion that F is something in addition to the particular Fs and thus that F is some nature.
(3) A proof about x which is a proof about something which is and thus does not instil a false opinion that x is something which is, is better than a proof which is a proof about something which is not, and

thus does instil a false opinion.

(4) A proof about proportions, and thus of universal mathematics, is of something *y* which is in addition to a line, a number, a solid and a plane, and which is none of them.

(5) This *y* is something which is to a lesser degree than a particular thing.

(6) Therefore, by (2), (4) and (5), a proof of universal mathematics instils a false opinion.

(7) Therefore, at least by (6), (2) and (3), every universal proof is such and worse than a particular proof.

It is a somewhat complicated argument, because it may seem that strictly speaking, with (1) in hand, Aristotle does not need the reference to universal mathematics – I could perhaps also have said that (7) is derived from (1), (2) and (3). However, there is a difference between 'not being [something]' and 'not being something in addition to the particulars'[7] – a difference which Aristotle is going to exploit in his solution of this aporetic argument –, and thus the reference to universal mathematics is very helpful, given the formulation in (3) in terms of merely being or not being something.

It should be noted how strikingly similar the argument (2)-(7) is to the argument from universal mathematics in *Metaphysics* M.2, with (2) corresponding to the Platonic hypothesis (2) in that argument, (4) to proposition (3) in that argument, and (5) to the impossibility claim (4) in that argument. One would even run into the same worry regarding the generality of the final conclusion (7), if it were to be exclusively based on the case of universal

[7] Note that the specification 'in addition to the particular things' in (2) only shows up in the examples provided, and not in the general statement preceding them.

mathematics. Of course, there is also an important difference: whereas in *Metaphysics* M.2 Aristotle invokes theorems of universal mathematics against Platonic Forms, here he uses them, aporetically, against a position which is dear to his own heart, namely that universal proofs are better than particular ones. But the conclusion drawn from the case of universal mathematics is in both arguments very similar, namely that the universal of which universal mathematics is true of, is not of the right type. This makes it almost impossible that the reason why Aristotle claims in *Metaphysics* M.2 that it is impossible there to be a Form in the case of universal mathematics is based on specific views held by the Platonist.

This is confirmed if we look somewhat more closely at how Aristotle deals with the aporetic argument. In his solution of it Aristotle says the following:

> And further, there is no necessity to assume that because it refers to one thing, this [universal] is something in addition to those, [at least] nothing more so than with the other things which do not signify something, but rather [something] qualified or in relation to something or acting. Therefore, if [this were assumed], not the proof would be the ground, but the hearer. (*AnalPost* 1.24, 85b18-22)

The point Aristotle makes here is that (2) as phrased is not quite correct: a proof that *F* in virtue of *F* is *G* should correctly instil the opinion that *F* is something and is a nature (the second half of (2) as stated above), but not the opinion that *F* is something *in addition to* the particular *F*s (the first half of it as stated above). Since (2) is crucial for the aporetic argument, the argument cannot conclude that *every* universal proof is deluding us and thus worse than a particular proof. But this solution leaves the part of the aporetic argument which concerns universals mathematics, (4)-(6), intact; in *Analytica Posteriora* 1.24 Aristotle thus does not say anything to indicate that he would consider that part to make an incorrect point.[8]

There is positive evidence that Aristotle accepts the part (4)-(6) of the aporetic argument in some further remarks he makes about universal mathematics. Most important is a question he raises in *Metaphysics* E.1:

> One could raise the problem whether ever first philosophy is universal or concerns some genus or some single nature – for not even in mathematics the same way [applies], but rather geometry and astronomy are concerned with some nature, while the universal [type of mathematics] is common to all. (1026a23-27)

Clearly, being universal and common to all, which is true of universal mathematics, is contrasted with being concerned with some domain and some single nature. Thus Aristotle himself denies that universal mathematics is true of a single nature. Thus in terms of the aporetic argument of *Posterior Analytics* 1.24, he would agree that proofs in universal mathematics, by being a proof that F in virtue of F is G

[8] In the context of *AnalPost* 1.24, it may seem that there is even positive evidence that Aristotle accepts the point made in (4)-(6), because he seems to limit his arguments in favour of universal demonstrations to universals which have 'the same account' and do not apply 'in virtue of homonymy' (85b10-11, cf. 15-16) – and as we shall see below, he does not think the universal in the case of universal mathematics has a single account. However, as appears from *AnalPost* 2.17, he also does not think the universal in the case of universal mathematics is homonymous. Therefore it seems more likely that in *AnalPost* 1.24 he does not say anything to dispel possible confusions in the specific case of universal mathematics.

(just like any proof), instil the false opinion that F is some (single) nature.

Thus there is sufficient evidence that Aristotle himself thinks that the universal of which universal mathematics is true of is of a strange type. That, however, is not the most common account of the object of universal mathematics. It is a persistent idea, which one can also recognise in the remarks by Lear, that there is a normal universal for universal mathematics to be true of. Lear's suggestion is *magnitude* as it appears in Euclid's *Elements* 5, while others have suggested *quantity*. This idea is connected with a certain interpretation of the most informative passage in Aristotle about the proof of the alternation of proportionals:

> Also [primary-universal seems to the proof] that the proportion also alternates, *qua* numbers and *qua* lines and *qua* solids and *qua* periods of time, in the way it once used to be proved separately, when it was possible to be proved of all [types of quantity] with a single proof. But because these [types of quantity] – numbers, magnitudes, solids, periods of time – are together not a single thing bearing a name and differ in form from each other, it used to be established separately.
> Nowadays, however, it is proved universally. For it did not belong *qua* lines or *qua* numbers, but *qua* this, which they hypothesise to belong universally. (*AnalPost* 1.5, 74a17-25)

A common interpretation of this passage is the following. There used to be a time that the alternation of proportionals was proved in a case by case way, for each type of quantity separately. However, with the discovery of Eudoxus' general theory of proportionality, a single general proof became available. Thus when in the old days the alternation of proportionals was proved of *line* and of *plane* and of *solid* and of *number* and, remarkably enough, of *period of time*, after Eudoxus' discovery it was proved of another universal, namely of *quantity* in general, just as it is proved in Euclid, *Elements* 5.16 of *magnitude*.

As I have argued elsewhere, however, this interpretation cannot be correct.[9] The first reason is that before Eudoxus' general theory of proportionality, with its hallmark new definition of proportion,[10] it was impossible to give separate proofs for each type of quantity. If there was any theory of proportion worthy of the name, it must have been a theory employing the concept of *anthyphairesis*: two ratios are the same if they have the same anthyphairesis, an algorithm yielding for each ratio between two quantities a (possibly infinite) series of integers obtained by listing how many times the smaller may be subtracted from the larger, how many times the remainder of the larger after that subtraction may be subtracted from the smaller, then how many times the remainder of the smaller after the previous subtraction may be subtracted from the remainder of the larger of the subtraction before the previous one, and so forth, either to infinity (in the case of incommensurable quantities) or until there is no remainder left. It can be

[9] The next four paragraphs summarise my argument in 'Sources of Delusion in *Analytica Posteriora* 1.5', *Phronesis* 51 (2006), 252-284, at 263-266; for references to further literature, see that discussion.

[10] Euclid, *Elements* 5, def. 5: 'Magnitudes are said to be in the same ration, the first to the second and the third to the fourth, when, if any equimultiples whatever be taken of the first and third, and any equimultiples whatever of the second and fourth, the former equimultiples alike exceed, are alike equal to, or alike fall short of, the latter equimultiples respectively taken in corresponding order.'

shown that with such a theory one can establish the alternation of proportionals in the case of lines, over a construction involving ratios between plane figures, but that is all. In the cases of numbers, periods of time and solids it would just have been impossible.

If one reads carefully, one can also see that Aristotle's account is inconsistent with such an interpretation. Aristotle implies that when separate proofs used to be given, it was already possible to prove it with a single proof, that is, proof procedure, which can only have been one in accordance with Eudoxus' definition of proportion. He also uses the past tense 'did belong' to state the fact that it the alternation of proportionals is true of some more general universal than each of the separate types of quantity.

The second reason the popular account is incorrect is related to the fact that Aristotle is very specific in his description of the universal involved in the alternation of proportionals. We find it in *Posterior Analytics* 2.17:

> [I]f [a problem] is as if in a genus (ὡς ἐν γένει), [the middle term] will have a similar character. For example, on what ground does a proportion alternate? For there is a different ground in the case of lines and of numbers and yet the same, different *qua* line, but the same *qua* having an increment such as *this* (ᾗ δ' ἔχον αὔξησιν τοιανδί). (99a6-11)

That the universal which the theorem of the alternation of proportionals is true of, is not *quantity* or *magnitude* (as used by Euclid in *Elements* 5), but rather *having an increment such as this*, is based on mathematical grounds. In Greek mathematics it is impossible that there is a ratio between different types of quantity, because *ratio* is defined in terms of exceeding on multiplication:

> Magnitudes are said to have a ratio in relation to each other when they are able, when multiplied, to exceed each other. (Euclid, *Elements* 5, def. 4)

Thus the quantities involved in a ratio must be of the same type, for multiplying a number will not result in exceeding a line, or multiplying a line in exceeding a plane figure. This homogeneity condition on ratio excludes the possibility that the alternation of proportionals is true of just any four quantities, or even of two pairs of quantities, each pair being of the same type. For in the latter case, 'after' the alternation one still ends up with two ratios between non-homogeneous quantities.

Thus theorems in universal mathematics cannot be true of *quantity* in general, for in that case alternation should already be possible if each of the four quantities in a proportion meets the condition of being a quantity, for example if the ratio between two numbers is the same as the ratio between two lines. On the other hand, theorems in universal mathematics cannot be true of a particular type of quantity, such as *number*, either, for then there would not be one general theorem. They must rather be true of *quantity of type x*, with *x* being a variable ranging over the different types of quantity. Postulating such a universal as the universal theorems in universal mathematics are true of, ensures both that they are general, covering all quantities, and that they apply only if the proportionals are all four of the same type.

The universal as specified by Aristotle, *having an increment such as this*, refers with 'increment' presumably to the multiplication element in the Eudoxean definition of *ratio*, but for the rest it is of exactly the same form as *quantity of type x*, equally, with '*this*', featuring a variable ranging over types of quantity. Aristotle is aware that this makes for a universal of a rather special kind. He expresses this by saying that this universal does not have a proper name,[11] that it is not a single thing,[12] that it does not have a single nature,[13] that it belongs to an 'as if genus'[14] and does not pertain to a real genus,[15] and that for the proofs of universal mathematics the types of quantity 'differ from each other in form'.[16]

It is easy to understand Aristotle's characterisations of this universal. They all refer to point that this universal features a variable ranging over the different types of quantity, and thus for its content is dependent on these types of quantity and on each separately: as concerning the one type of quantity its content is different than as concerning the other type of quantity. This universal is a variable universal.

Now it is this feature of the universal which proofs in universal mathematics are true of, and which is of a purely mathematical nature, which makes it impossible that there is a Form for proofs in universal mathematics to be true of. Platonic Forms are separated from the particulars over which they 'range', and they are prior in account to Forms which are less general than them and subsumed under them: each Form is what it is in a completely independent way, being a single nature on its own. It is because of their

---

[11] *AnalPost* 1.5, 74a8 and 21

[12] *AnalPost* 1.5, 74a21

[13] *Met.* E.1, 1026a23-27

[14] *AnalPost* 2.17, 99a7

[15] *Met.* E.1, 1026a23-27; see also *Met.* K.7, 1064b8-9 and compare *SE* 11, 172a11-13, which, once one recognises it, contains a clear reference to universal mathematics.

[16] *AnalPost* 1.5, 74a8-9 and 22

very nature as Forms that Forms are like this. The universal which proofs in universal mathematics are true of, however, does not meet this requirements for being a Form: it is not separated from the types of quantity it ranges over and it is not prior in account to less general universal subsumed under it; it does not have a single nature by its own.

§ 3. HOW DOES THE ARGUMENT FROM UNIVERSAL MATHEMATICS SHOW THAT THERE CANNOT BE ANY FORM FOR SCIENCE TO BE TRUE OF?

Now that we know why Aristotle can claim, in (4) of the argument from *Metaphysics* M.2, that there cannot be a Form for universal mathematics to be true of, we must turn to the more difficult question why Aristotle feels justified to draw the conclusion, in (6), that there cannot be any Form for mathematics to be true of. For there is nothing in Aristotle's argument thus far which would make it impossible that arithmetic, say, is true of number. After all, *number*, according to Aristotle, is a genus with a single nature, has a single account and is always the same.

In this section, however, I will argue that if one appreciates the full context of the debate between the Platonist and Aristotle, it will be clear that Aristotle, by pointing to the problematic case of universal mathematics, has raised a very powerful objection to the theory of Forms, even on the Platonist's own terms.

To see how problematic the case of universal mathematics is for the Platonist, we should start with the obvious observation that Aristotle's argument from universal mathematics in (1)-(4) in *Metaphysics* M.2 shows that the Platonist hypothesis (2) that every universal *F* which is the object of a theorem, that is, a bit of scientific knowledge, is a Form, is not universally true: for some universals this is false. Thus the Platonist needs to explain why this hypothesis should apply to some *F*, while it does not apply to the universal in the case of universal mathematics. The only way the Platonist can do this is by claiming that the Platonist hypothesis holds in the case of normal universals *F*, but does not hold in the case of universals which are not normal. In the case of the latter the theorem in question may be true of some non-standard universal *F*, but this is because it is true of a series of parts G_i of *F*, each of which is more fundamental than the whole *F*. In fact, this is the old position which Aristotle refers to in his explanations at *Posterior Analytics* 1.5, 74a17-25: the mathematicians used to prove the theorem of the alternation of proportionals separately for each type of quantity, even though they used one single proof procedure. It may well be that this old position was inspired by ontological qualms.

Moreover, the Platonist could argue that this claim that each of the G_i is more fundamental is not merely an *ad hoc* one, since he could point out that even Aristotle in some sense agrees. For does not Aristotle himself admit, in *Posterior Analytics* 2.17, that for the alternation of proportionals 'there is a different ground in the case of lines and of numbers and yet the same, different *qua* line, but the same *qua* having an increment such as *this*'? And does not Aristotle himself, in *Posterior Analytics* 1.24, restrict the claim that the universal 'would be [something], not in any lesser degree than some particular things, but rather even more so, to the extent that the imperishable things are among them, while the particular things are more perishable' to the case of universals which constitute 'some *single* and non-homonymous account'?

Aristotle recognises that the Platonist is on solid ground with the distinction between normal universals and non-standard universals. However, Aristotle does not accept this as a good ground for distinguishing between them as far as the applicability of the Platonist hypothesis is concerned. His reason for rejecting the distinction as a ground for limiting in a non-arbitrary way the domain of application of the Platonist hypothesis is that he assumes that the Platonist hypothesis is based, not on ideas about the object for mere knowledge to be true of, but rather on ideas about the object of *explanatory* knowledge, as providing the explanatory ground for all kinds of general facts.[17] But if that is the case, then the Platonist way out, by returning to the old situation, before the introduction of the universal *having an increment such as this*, becomes problematic, because even in the old situation for each of the types of quantity the same proof procedure was used – since a demonstration is an explanation, the fact that the alternation of proportionals is demonstrated in exactly the same way leads to the conclusion that the explanatory ground must be the same as well, and cannot consist of a series of universals G_i.

For Aristotle this idea that the object of scientific knowledge is only an object of scientific knowledge if it constitutes the explanatory ground for the universal fact to be explained and demonstrated is codified in his requirement that a proof should be primary-universal. He explains this requirement in *Posterior Analytics* 1.4-5: a proof is only a real proof if it shows that some fact that F is G holds in virtue of F itself and in so far as it is F, and there is no less informative universal than F in virtue of which it

[17] I use the term 'object of scientific knowledge' in the strict sense of the grammatical object x in sentences of the type 'knowing x', as in 'knowing x that it is F', which in the case of scientific knowledge amounts to 'knowing x that it *qua* x is F'. In this sense it is always an item, never a proposition or fact, which is the object of scientific knowledge.

holds.[18] The conception of the logical structure of such a proof involved here is that of what one may call a *direct proof*: in a proof that *F* is *G*, some particular *F* is only to be considered in so far as it is *F*, and in the course of the proof, while strictly adhering to this limitation, it is shown that this particular *F* is *G*, thus licensing the conclusion that *F* is *G* in virtue of *F* itself.[19]

In *Posterior Analytics* 1.24 Aristotle provides arguments for this requirement. The argument which presupposes least of Aristotle's technical vocabulary and concepts is the following, and thus seems the most fundamental Aristotle has on offer,[20] is the following:

> Again, we see the reason why up to this point, and we think we have knowledge then, when it is not because some other [thing] is either becoming or being this. For the ultimate [thing] is already in this way an end and limit. For example, for the sake of what did he go? In order to get the money, and this in order to return what he owed, and that in order that he would not act against justice. And proceeding in this way, we claim [him] to go and [this] to be and to become because of that as a goal, when it is not any more because of something else or for the sake of something else, and in

[18] As I argue in 'Sources of Delusion' 279-284, this requirement is first formulated at *AnalPost* 1.4, 73b32-74a3, and made most explicit at *AnalPost* 1.5, 74a32-b4.

[19] 'Sources of Delusion' 276-277

[20] A little earlier, at 85b23-27, Aristotle presents an argument based on the universal being primary and therefore explanatory, but it is not exactly clear what goes into this argument and how many of his own ideas Aristotle presupposes in it.

> that case to know most of all because of what he went.
> If, then, things are similar in the case of all grounds and reasons why, and in the case of things which are in this way grounds, as for the sake of which, we have in this way most of all knowledge, we therefore also have then most of all knowledge in the other case, when this is not the case any more because something else is. (85b27-38)

Based on this argument Aristotle immediately presents a next argument, which contains the criterion for a primary-universal proof we already know from *Posterior Analytics* 1.4-5:

> When, then, we know that the external [angles] are equal to four [right angles] because it is an isosceles, there is still left [the question] because of what the isosceles [has such external angles] – because it is a triangle, and that, because it is a straightlined shape. If that is not any more [the case] because something else [is], then we know most of all. And then we know a universal. Therefore the universal [demonstration] is better. (*AnalPost* 1.24, 85b38-86a3)

By itself this evidence should suffice for understanding why *Aristotle* claims that because of the argument from universal mathematics, it is impossible that there be Platonic Forms to serve as objects for the other parts of mathematics as well. Assuming the requirement that a proof be of the most abstract universal possible, Aristotle does not think the Platonist can limit the damage done by the argument from universal mathematics against principle

(2). However, I want to go further and make a case that even the Platonist should agree with Aristotle's verdict, because the Platonist shares the underlying idea of proofs isolating the explanatory ground, as well as the accompanying conception of the logical structure of an explanatory proof.

There is some evidence that Plato thought that knowledge is explanatory. One dialogue in which this is made clear is the *Meno*:

> [T]rue opinions, as long as they remain, are a fine thing and all they do is good, but they are not willing to remain long, and they escape from a man's mind, so that they are not worth much until one ties them down by an account of the explanation (αἰτίας λογισμῷ). And that, Meno my friend, is recollection, as we previously agreed. After they are tied down, in the first place they become knowledge, and then they remain in place. (97e-98a)

where, of course, the dialogue's example of recollection and thus the tying down by an account of the explanation is a very simple mathematical theorem.

Also in *Republic* 10 the difference between *epistêmê* and *doxa*, which in *Republic* 5 is aligned with the distinction between Forms and particulars as their objects,[21] is most

[21] I suspect there is a case to be made that the key difference between *epistêmê* and *doxa* in *Republic* 5, namely that the former is, and the latter is not, infallible, is in fact nothing more than the difference between explanatory and non-explanatory knowledge: the former isolates the universal feature which connects it with other features, and this connection always holds, whereas the latter can merely identify the extension of a feature – but the

easily understood in terms of explanatory versus non-explanatory knowledge: whereas the flute-player has knowledge of what constitutes a good or a bad flute, because he can relate its qualities to the function of a flute, the flute-maker merely has correct *doxa*, because he lacks this further knowledge, and is just acquainted with the qualities of good and bad flutes.[22]

In Aristotle's reports of Platonist arguments for the existence of Forms, one finds a similar emphasis on explanation:

> If every science does its job referring to something which is one and the same and not to any of the particulars (εἰ πάσα ἐπιστήμη πρὸς ἕν τι καὶ τὸ αὐτὸ ἐπαναφέρουσα ποιεῖ τὸ αὑτῆς ἔργον καὶ πρὸς οὐδὲν τῶν καθ' ἕκαστον), [for example, a geometer [does his job] referring to some single triangle and not to an individual drawn, and similarly the other sciences,] there would be in accordance with each science something else in addition to the perceptible things which is eternal and a paradigm for the things coming to be in accordance with each science – and such is the Form. (Aristotle, *De Ideis*, in: Alexander of Aphrodisias, *In Metaphysica* 79.5-8 [*recensio vulgata*, with an addition from the *recensio altera*])

And what else is the job of a science than to explain? This is made more explicit in the next argument from the

---

extension of a feature is changeable, and thus not stable, which makes the cognitive state set over them fallible.

[22] *Republic* 10, 601b-602b

sciences in the *recensio altera*:

> Further, if medicine is not science of this health, but of health without qualification, health-itself will be something. And if the geometer does not have scientific knowledge of this commensurate [thing] or of this equal [thing], but of equal without qualification and of commensurate without qualification, by referring the other things to which, he proves that these [other things] are such and such things (εἰ ὁ γεωμέτρης οὐ τοῦδε τοῦ συμμέτρου ἢ τοῦδε τοῦ ἴσου ἐπιστήμην ἔχει, ἀλλὰ ἁπλῶς ἴσου καὶ ἁπλῶς συμμέτρου, πρὸς ἅ τὰ ἄλλα ἀναφέρων ἀποδείκνυσι τάδε τινὰ εἶναι), there is therefore equal-itself and commensurate-itself. Therefore, these things are Forms. (Aristotle, *De Ideis*, in: Alexander of Aphrodisias, *In Metaphysica* 79.10-14 [*recensio altera*])

Moreover, also Aristotle reports that the Platonist wants to distinguish between sciences and mere crafts as far as these arguments are concerned.[23] This implies that the Platonist draws a distinction between the job done by a science (which includes branches of expertise like medicine) and the job done by a craft – the only possible way, it seems, is to deny that crafts involve explanatory knowledge.

Thus we have evidence to ascribe to the Platonist the view that Forms are only objects of explanatory knowledge, and not of other types of knowledge. In terms of existence arguments this amounts to the Platonic hypothesis that there are Forms only to serve as the object of explanatory

[23] *De Ideis*, in: Alexander of Aphrodisias, *In Metaphysica* 79.20-89.7

knowledge, and thus as explanatory grounds in scientific explanations. This evidence, I will argue next, is confirmed by the evidence for the claim that the Platonist also shares with Aristotle the conception of a direct proof as constitutive of scientific knowledge. For with the conception of a direct proof, in which the feature *F* which is made out to be responsible for other features *G* is isolated from other features, scientific knowledge is naturally identified as knowledge of this explanatory ground *F* as explanatory of other features *G*.

A first piece of evidence that the Platonist shares this conception of a direct proof as constitutive for scientific knowledge appears at the heart of Plato's exposition of the different types of cognition and their correlated objects, in the discussion of the divided line in *Republic* 6. There Socrates claims:

> I think you know that those dealing with kinds of geometry, calculation and the like hypothesize *odd* and *even* and the shapes and three kinds of angles and all the other things kindred to them for each science, and, having made them, as if they know them, hypotheses, do not deem it necessary to give any account of them any more, neither to themselves nor to others, as if they were clear to everyone, but rather started out from them and going through the then remaining things ended up in an agreed manner at that for whose investigation they set out. …
>
> Then you also know that they use visible forms and produce arguments about them, not thinking about them, but rather concerning those things which they resemble, producing arguments for the sake of the square itself and the diameter itself, and not

> for the sake of the one they draw, and thus with the other things. The things themselves which they form and draw, of which there are both shadows and images in waters, using these in turn as images, in seeking to see those things themselves which one cannot see in any other way but by thought. (510c2-511a2)

Clearly 'ending up in an agreed manner at the things under investigation' refers to proofs in which a demonstration consisting of a number of unobjectionable steps is offered of some other features. Now these proofs, according to Socrates, are apparently concerned with visible objects, but this appearance is misleading: the scientist does not think about these perceptible *F*-objects, but rather about the *F* itself, which is the real object of the demonstration, and of which the perceptible *F*-objects are mere 'images'.

On the basis of this passage alone we may already ascribe to Plato a specific conception of a direct proof: the feature *F* which is isolated in the proof and made responsible for 'the things under investigation' is the Form *F* itself; the conclusion that *F* is *G* holds of the perceptible *F*-things, because these are 'images' of the real *F*.[24] But

[24] It has been argued by M.F. Burnyeat, 'Platonism and Mathematics. A Prelude to Discussion', in: A. Graeser (ed.), *Mathematics and Metaphysics in Aristotle* (Bern, 1987) 213-240, at 229-230, that Forms cannot fulfil this function of being the thing appearing in the *ekthesis*, because they cannot undergo geometrical operations and cannot appear several times, as is required by some proofs (for example about two circles). Instead, he proposes, the things appearing in the *ekthesis* are the intermediates, which are intelligible entities, like the Forms, but come in numbers. This objection presupposes that the thing set out is very much like the particular diagram used by mathematicians. If, on the other hand, the thing set out is to be the feature which is isolated in a direct proof, then there does not

there is more evidence for this ascription in reports by Aristotle, in which he claims that the Platonists hold a certain interpretation of *ekthesis*.

Ekthesis is a concept originating in the practice of Greek mathematics. At the beginning of a real proof an individual *F*-object is introduced, or 'set out' (the verb is *ektithesthai*), which in the course of the proof is going to serve as a representative object for all the *F*-things. Having conducted the proof in the case of this individual *F*-object, the mathematician concludes at the end to a universal statement of the form 'all *F*s are *G*'. Thus *ekthesis* is inseparably linked to the conception of a direct proof.

Aristotle has an abstractionist interpretation of *ekthesis* – in fact, for him the term '*ekthesis*' seems rather to refer to the purpose of the introduction of the *F*-object, that it is to serve as a representative object for all *F*-things, rather than to the introduction strictly speaking of the *F*-object itself. There is evidence for this abstractionist view in *Posterior Analytics* 1.4:

> I call 'universal' that which belongs *both* as of a whole *and* in virtue of itself and *qua* it (καθ' αὑτὸ καὶ ᾗ αὑτό). ... 'In virtue of itself' and '*qua* it' are the same, for example point and *straight* belong to a line in virtue of [the line] itself – for they also [belong] *qua* line – and two right angles [belongs] to triangle *qua* triangle – for a triangle is also equal to two right angles in virtue of itself.
>
> And the universal belongs then, when it is shown in the case of an arbitrary and primary thing. For example, having two right angles is not universal for a figure, even though it is

seem to be anything untoward to the idea that the Form is the thing set out.

> possible to show of a figure that it has two right angles, but not an arbitrary figure, nor does one make use of an arbitrary figure in showing [that]. For the square is a figure, but it does not have [angles] equal to two right ones. On the other an isosceles which is arbitrary [has angles] equal to two right ones, but it is not primary; rather, a triangle is prior. (73b26-39)

In this passage Aristotle explains that if in an argument a particular *F*-thing is made arbitrary and merely considered in so far it is *F*, the argument establishes that all the features *G* shown to hold of the particular *F*-thing hold of *F qua F* and in virtue of *F*. Using the verb *ektithesthai*, he seems to have the same idea in mind in *Prior Analytics* 1.41:

> One should not think that an absurdity follows because something is set out (παρὰ τὸ ἐκτίθεσθαί τι). For we do not use at all in addition that it is an individual (οὐδὲν γὰρ προσχρώμεθα τῷ τόδε τι εἶναι); rather, [we behave] in the way the mathematician says that the line is one foot long and this one straight and without breadth, while they are not, but does not use them in such a way as to deduce on the basis of these things. (49b33-37)

The thing set out is an individual, but in the *ekthesis* we do not use it as an individual, but as something universal. In a way that amounts to engaging in a falsehood,[25] because it is

[25] There are other well-known passages where Aristotle uses the same comparison with what the mathematician does – for the full list and a discussion of the mathematical side of the comparison,

not a universal, but this falsehood does not enter into the premisses of the proof and is therefore inconsequential.[26]

In several passages, however, Aristotle ascribes to the Platonists a view on *ekthesis* as well, one according to which the individual set out is not a particular made arbitrary and universal by abstracting away from all features irrelevant for the proof, but rather the Form itself. One passage we encounter in *Metaphysics* M.9:

> [The Platonists], supposing that if there are to be some substances over and above those which are perceptible and in flux, then these must be separate, did not have any others and set out these universally said substances (ταύτας δὲ τὰς καθόλου λεγομένας ἐξέθεσαν). Hence it all but follows that the universal objects and the particular ones are of the same nature. (1086b7-11)

Another one occurs in *Sophistical Refutations* 22:

---

see F. Acerbi, 'In What Proof would a Geometer use the *ΠΟΔΙΑΙΑ*?', *Classical Quarterly* 58 (2008) 120-126.

[26] There is also evidence for this abstractionist account of *ekthesis* in Aristotle's own use of arguments with *ekthesis* in his own logic. Most explicit is the following passage:

[W]e must set out something to which each of them do not belong and produce the deduction of that. For the deduction will be necessary in the case of these things. And if it is necessary of the thing set out, it is also necessary of *some that*. For the thing set out is precisely what *some that* is. (*AnalPr* 1.8, 30a9-13)

However we are going to interpret this passage exactly, it is clear the thing set out stands for 'some that' without being identical to it.

> But it is not the setting out that produces the third man, but rather the agreeing that what is set out is what an individual is (ὅπερ τόδε τι), for it will not be possible for precisely what man is, just like precisely what Callias is, to be something individual (οὐ γὰρ ἔσται τόδε τι εἶναι, ὅπερ Καλλίας, καὶ ὅπερ ἄνθρωπός ἐστιν). And it will not make any difference [namely, to the ekthesis] if someone should claim that what is set out is not precisely what some individual is, but precisely what a quality is (ὅπερ ποιόν), for what is beside the many things will be one thing, for example, man. (179a3-8)

Aristotle's critical point in both passages seems very similar: by claiming that the thing set out, which Aristotle and the Platonist both assume to be a universal, is a substance rather than a feature, the Platonist runs into problems. In *Sophistical Refutations* 22 Aristotle explicitly identifies these problems with the Third Man Argument, in *Metaphysics* M.9 he alludes to it, as the point that the universal belongs to the same class of things as the particulars is the crucial premiss of the Third Man Argument. In the *Sophistical Refutations* Aristotle offers an alternative account of *ekthesis*, presumably his own, that the thing set out – which must be the thing resulting after the abstraction and thus the universal – is not a substance, but rather a feature in one of the other categories.

Now we saw that Aristotle makes a similar point in *Posterior Analytics* 1.24, in the passage in which he tries to escape from the aporetic argument that universal proofs are misleading as to their ontological consequences. To quote the passage again:

> And further, there is no necessity to assume

> that because it refers to one thing, this [universal] is something in addition to those, [at least] nothing more so than with the other things which do not signify something, but rather [something] qualified or in relation to something or acting. (85b15-21)

Thus Aristotle responds in providing exactly the same solution, that the universal involved in a proof does not 'signify something', that is, that it is not a substance, but is or signifies rather a quality, a quantity or something else of that sort. This suggests that the Platonist view on *ekthesis*, that the object isolated in a direct proof is the Form, is nothing else than the Platonist hypothesis that each proof needs a Form to be true of. This is confirmed by the following passage, in which a Platonist conception of *ekthesis* is rejected partly by appeal to proofs in universal mathematics:

> And what seems to be quite easy, to show that all things are one, does not occur (ὃ τε δοκεῖ ῥᾴδιον εἶναι, τὸ δεῖξαι ὅτι ἓν ἅπαντα, οὐ γίγνεται·), for by the setting out all the things do not become one, but [merely] something itself becomes one thing (τῇ γὰρ ἐκθέσει οὐ γίγνεται πάντα ἓν ἀλλ᾽ αὐτό τι ἕν), if one grants everything; and not even that is the case, unless one is to grant that the universal is a genus – but that is impossible in some cases. (*Met.* A.9, 992b9-13)

This passage occurs as part of the list of arguments against Platonism in *Metaphysics* A.9, where it seems to be an isolated argument. Now that we know Aristotle's views about the nature of the universal in the case of universal mathematics, it is easy to see that with the last clause

Aristotle refers to such proofs. We may also recognise Aristotle's point that in normal cases the universal a proof is concerned with is one thing: the universal isolated in the *ekthesis* is the single nature of which it is proved that something holds of that universal in virtue of itself. The Platonist alternative, which Aristotle in *Posterior Analytics* 1.24 describes as holding that this universal *F* is one thing *in addition to* the particular *F*s, and in *Sophistical Refutations* 22 and *Metaphysics* M.9 as holding that this universal *F* is a substance of the same type as the perceptible *F*-things, he describes here in terms of all the perceptible *F*-things being one thing in the *ekthesis*. Probably the best way to understand this description is in terms of the passage quoted from the divided line in *Republic* 6: all *F*-things are merely images of the *F*-itself, and by way of the proof this is shown to be the case, as it shows that all these *F*-things are G in virtue of the *F*-itself.

Thus the Platonist hypothesis reduced to absurdity in the argument from universal mathematics in *Metaphysics* M.2 is nothing more than the Platonist conception of *ekthesis*, that the universal isolated in a direct proof is the Form *F*-itself. Therefore the Platonist subscribes to Aristotle's conception of the logical structure of scientific proofs, including the concomitant requirement that in such direct proofs the real explanatory universal be isolated, because of which in virtue of itself all *F*s are *G*.

Thus the argument from universal mathematics as we find it in *Metaphysics* M.2 and also, now, in *Posterior Analytics* 1.24 and *Metaphysics* A.9, poses a real problem for the Platonist, not because Aristotle has foisted upon him some Aristotelian assumptions, but because the Platonist accepts, quite naturally, the logical structure encountered in proofs of Greek mathematics, and its interpretation in terms of isolating the explanatory ground in virtue of which the theorem is shown to hold. Somehow the Platonist should account for the fact that with universal mathematics there is one single proof which covers all types of quantity without being true of *quantity* in general, but rather of *quantity of type x*. But he cannot do this without endangering his Platonist hypothesis.

The nature of the Platonist predicament can be explained in yet another way in the form of a dilemma. In order to account for proofs in universal mathematics being single proofs concerned with a single universal, there are only two options for the Platonist: either he gives up the Platonist hypothesis immediately or he tries to distinguish between universals for which the hypothesis does hold and universals, like *having an increment such as this*, for which it does not hold. Because he shares Aristotle's conception of direct proofs and their explanatory interpretation, he is forced to draw this distinction while finding a way to make sense of proofs in universal mathematics which does justice to the different nature of the universal involved. The only way, it seems, that this can be done is to adopt an abstractionist account of *ekthesis* for the case of universal mathematics: the different types of quantity are real Forms, while the universal which they have in common in the context of such proofs, is merely constructed by way of abstraction. But if that is the case, and such an abstractionist account works in the case of universal mathematics, the Platonist must also explain why he does not wish to adopt such an account in the case of proofs for which he would like so very much to posit Forms as their object. This seems an impossible task. Thus Aristotle seems to be justified, also on the terms of the Platonist himself, to draw the conclusion that the argument from universal mathematics shows that there cannot be Forms for the other objects of mathematics either.

§ 4. CONCLUDING REMARKS

Now that it has been shown that the impossibility Aristotle refers to in (4) of the argument from *Metaphysics* M.2 is of a mathematical kind, and that Aristotle is entitled to his conclusion (6) that there cannot be any Form for mathematics to be true of, given the conception of proofs and explanation shared between him and the Platonist, I want to conclude this paper with some more general remarks.

The first thing to remark upon is that there is nothing in the argument from universal mathematics which, as in *Metaphysics* M.2, limits its conclusion (6) to Forms in mathematics; it may just as well be applied to all Forms which would serve as the object of scientific knowledge. This seems also how Aristotle understands the argument, since he refers to it in the context of perfectly general points about universal versus particular demonstrations (in *Posterior Analytics* 1.24) and against the Platonist conception of *ekthesis* (in *Metaphysics* A.9). As it happens, the context of *Metaphysics* M.2 is limited to mathematical objects, and that seems the only reason why Aristotle phrases the conclusion in a more limited way. However, it may be that the argument originated in a mathematical context, as may be suggested by the report in *Posterior Analytics* 1.5, if my suggestion is correct that the way Eudoxus' theory of proportion was first used, as applying to each type of quantity separately, reflects some of the same ontological concerns as are taken advantage of by Aristotle in the argument from universal mathematics. Moreover, Platonists of Aristotle's day seem to have understood mathematics as a kind of super-science, to which all other sciences are subordinated, so that an argument targeting mathematical objects alone would already suffice for a strike at the heart of Platonism.

One may wonder whether it would be possible for a

Platonist to escape from the argument from universal mathematics if mathematics were different – or in other words, how solid the mathematical impossibility is that there cannot be a Form in the case of universal mathematics. Now it seems only possible to get around the mathematical impossibility if one can argue that there is in fact only one type of quantity, and that there can be ratios between the different sub-types. Such claims were being made for the first time from the end of the 16th century onwards, by mathematicians like Simon Stevin and John Wallis. They extended the concept of number to fractions, including irrational fractions, thus insisting on the similarity between the domains of arithmetic and geometry. In the same vein they could assign numbers to magnitudes of different dimensions and thus accept ratios between magnitudes of different dimensions. Thus they posited an abstract conception of number which is shared by numbered groups (the Greek conception of number) and geometrical magnitudes alike. Theorems like the alternation of proportionality could then be proved of these abstract numbers in a direct proof, very much in the way Eudoxus' had introduced.[27]

What such developments achieved was in fact a realignment of the theory of scientific explanation and knowledge with ontology in that the hypothesis that for every proof there is a single unified object to be true of was reinstated. It is the mismatch between the theory of scientific explanation and knowledge with ontology which

[27] For some introductory information about the history of universal mathematics from this perspective, see P. Cantù, 'Aristotle's Prohibition Rule on Kind-Crossing and the Definition of Mathematics as a Science of Quantities', *Synthese* 174 (2010) 225-235. A more extensive discussion of these developments one can find in K. Neal, *From Discrete to Continuous. The Broadening of Number Concepts in Early Modern England* (Dordrecht, 2002).

is used by Aristotle against the Platonist with such powerful effect. A Platonist who builds his case on the idea that everything knowable in science should be really real will not accept such a mismatch; a philosopher like Aristotle, for whom scientific explanation need not be about what is really real in the first place, because only particular substances belong to that category, can be rather sanguine about it.

REFERENCES

ACERBI, F. 'In What Proof would a Geometer use the *ΠΟΔΙΑΙΑ*?', *Classical Quarterly* 58 (2008) 120-126

ANNAS, J. *Aristotle's* Metaphysics *Books M and N* (Oxford, 1976)

BURNYEAT, M.F. 'Platonism and Mathematics. A Prelude to Discussion', in: A. Graeser (ed.), *Mathematics and Metaphysics in Aristotle* (Bern, 1987) 213-240

CANTÙ, P. 'Aristotle's Prohibition Rule on Kind-Crossing and the Definition of Mathematics as a Science of Quantities', *Synthese* 174 (2010), 225-235

CLEARY, J. *Aristotle and Mathematics*. *Aporetic Method in Cosmology and Metaphysics* (Leiden, 1995)

HASPER, P.S. 'Sources of Delusion in *Analytica Posteriora* 1.5', *Phronesis* 51 (2006), 252-284

HEATH, T. *Mathematics in Aristotle* (Oxford, 1949)

LEAR, J. 'Aristotle's Philosophy of Mathematics', *Philosophical Review* 91 (1982) 161-192

NEAL, K. *From Discrete to Continuous. The Broadening of Number Concepts in Early Modern England* (Dordrecht, 2002)

ROSS, W.D. *Aristotle's* Metaphysics. *A Revised Text with Introduction and Commentary* (Oxford, 1924)