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Spatial Perception and Geometry in Kant and Helmholtz¹

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1. The Mind as an Object of Empirical Study

Can there be an empirical science of the mind? Opinions are divided. If within the purview of 'mind' we include the phenomena that are associated with such words as 'reasoning', 'understanding', and 'judging', then many philosophers would take the answer to be an obvious no. As these philosophers might say, understanding and judgment are "normative" notions; they are not processes, but achievements, and trying to investigate them empirically would make as much sense as trying to determine the meaning of the words 'just' and 'right', or of the term 'good art', by tape-recording random snatches of conversation and cataloguing the instances of these words and their contexts. But not everyone is so pessimistic. Indeed, tens of millions of dollars currently are being wagered on the chance that just like the solar system or the chemical elements, the mind can be made the object of empirical study.

My chief purpose here is not to try to answer or even to respond directly to my opening question; it is rather to investigate the question itself through the philosophical analysis of an historical episode involving Kant and Helmholtz.

But prior to engaging history, the question itself requires further specification. First, concerning what it would be for the study of the mind to be empirical. Presumably, we may agree that an empirical investigation is one in which the investigators pose questions that nature is made to answer--the crucial point being that the behavior of the natural phenomenon under investigation is the determining factor in answering an empirical question. The first sticking point to an empirical investigation of the mind is then whether "the mind" is a natural phenomenon. Can mental phenomena be captured by a language that leaves out irreducibly mental predicates? If "mental activity" can be seen as just another natural phenomenon, if it can be described and explained in a vocabulary that connects with the vocabulary of the natural sciences, then one might safely assume that it will be as amenable to empirical investigation as is, say, the weather.

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Another way to develop an empirical science of the mind would be to attempt a direct empirical assault on such mentalistic notions as judgment and understanding. In attempting to carry out this assault, however, I believe that one would soon find one's self drawn away from the field of the empirical. By this I mean that as the investigation proceeded, nature (i.e., experimental results) would not be answering the question of what a judgment is, or what it is to have grasped or understood something. The investigator must come prepared with the notions of judgment and understanding in order to decide which behaviors constitute the objects of her or his investigation. And once the criteria are set, then as long as the criteria are met by the experimental subjects, their attendant behavior is irrelevant to the idea that, say, a correct judgment has been rendered. One may investigate whether, in solving a mathematical problem involving division, a subject uses ordinary long division or a method of trial and error estimation; one may collect reaction times and check error frequencies under differing conditions of environmental "noise" or distraction; but whether the subject uses one method of division or another, whatever may be the reaction time, as long as his answers come out consistently right we shall ordinarily be willing to call what he does 'division'. The empirical tests may tell us something about the strategies that different individuals use in performing a division, but they won't inform us about division itself, or illuminate what it is to perform a correct division.

The question of whether there can be an empirical science of the mind thus divides into (at least) two questions. One pertains to whether the mental can be "naturalized". The other asks whether, failing that, the mental is itself amenable to empirical investigation, or remains in the domain of those topics that must be investigated "philosophically", or through the analysis, interpretation, and criticism of our cognitive practices and fundamental concepts. The period of Kant through Helmholtz, roughly 1750 to 1900, is an appropriate period on which to focus in investigating these questions, for it was during this period that there first arose a separation between philosophy and empirical psychology, or between a concern with knowledge and justification on the one hand, and an empirical science of mental phenomena, including cognition, on the other. Since 1900, there has been fairly wide agreement among philosophers that notions such as knowledge and justification, and the "laws of thought" themselves--where these are equated with logic--are not amenable to empirical investigation. (Quine 1969 may be an exception.) Frege (1884) in the late nineteenth century had suggested that what makes the laws of thought the laws of thought is their independence from the so-called thought-processes, the psychological states, of the individual. Frege conceived these "laws" as being worked out not by paying attention to what goes on in the mind of a reasoner, but by examining the common structure of objective judgment. The investigative work centers around systematizing objectivity of judgment; it is not an empirical, but a logical, investigation. As another example, Carnap in the 1930s could without hesitation set his logical investigations into the syntax of language apart from the psychological investigation of the relationship between "the words and expressions of a language" and the "actions and perceptions" of human beings (1934, p. 5). It is taken as "common knowledge" among many philosophers today that knowledge, judgment, and understanding are notions that are sharply divided from whatever the

cognitive psychologist studies empirically.

Prior to the eighteenth century, the question of the empirical study of the mind was not taken seriously, but for somewhat different reasons. There were two barriers to the empirical investigation of the mind. The first was simply that the idea of a generally empirical approach to nature as a whole was just being worked out in a practical way during the seventeenth century. The second was that the mind itself was considered to be partly or wholly outside of nature. It is true that in the Aristotelian tradition the soul as the form of the animal might be investigated as it manifested itself in vital, sensory, and rational activity. But this last, the cognitive side of the mind--rational intuition, or the intellect--was regarded as partaking of or expressing the divine. And while it might be appropriate to use one's experience of nature to lead toward an understanding of the divine, one surely couldn't expect to fully comprehend the divine through nature. Insofar as thought--mature judgment, understanding--was considered as participating in the divine, its normative character remained unproblematic, and the idea of naturalizing it, or treating it as just another object of empirical investigation, remained outside the domain of serious possibility.

The question of treating the mental as an empirical object was first directly addressed by thinkers with a firm grasp on the idea of empirical science in the period running from Kant through Helmholtz. Some of these thinkers were committed to applying empirical techniques to the mind. Others maintained that such application is impossible. David Hume stands out as producing an early formulation of the project to produce a "science of man"--basically, of the mental and moral life of human beings--on the basis of "experience and observation" (1739, p. xx). There can be little doubt that this was Hume's stated project; the subtitle to his *Treatise of Human Nature* characterizes the work as "An Attempt to introduce the experimental Method of Reasoning into moral Subjects," and the Introduction to that work contains a barely veiled comparison of his project with that of Newton. And in fact, if, leaving Hume's moral philosophy aside, one looks to the conception of science and the explanatory structure embodied in Hume's "science of man", one finds them to be properly Newtonian. As to the conception of science, just as Newton in his *Principia* disavowed any claims to discover the nature or essence of gravity or of matter, so Hume disavows any claims to uncover the essence or nature of the mind or of the understanding. As to the explanatory structure, Hume characterizes his endeavor as that of rendering "all our principles as universal as possible, by tracing up our experiments to the utmost, and explaining all effects from the simplest and fewest causes" (1739, p. xxi). In practice, this amounted to engaging in a "mental geography", or a cataloguing of mental phenomena, in order to uncover the elements of the mind, together with the attempt to discover universal principles, or general laws, that account for the combinations and dynamic relations among the mental elements (see also Hume 1777, sec. 1). Hume's picture is of simple ideas--e.g., for vision, punctiform sensations--combined with one another or entering into chains of succession with one another in accordance with three so-called "laws of association", the laws of resemblance, contiguity, and cause and effect. As it turned out, Hume's attempt to resolve the mind into these bare elements and laws hit a sticking point when he had to admit a primitive operation of perceiving

"relations" among ideas, an operation that he could not reduce to the laws of association. This primitive operation remained the last vestige of "rationalism" or of the operation of intellect in Hume's system; otherwise, he claimed to have reduced thinking to a series of (as it were) mechanical interactions among mental elements. Hume's vision of a mechanics of the mind driven by associative laws reflects the standard for attempts to develop a science of the mind in both Britain and Germany until the late nineteenth century.

The positions of Kant and Helmholtz can be set out in relation to Hume. Kant may be seen as pulling back from Hume's associationism on the ground that it fails to account for the cognitive judgments that actually are made--it fails because it cannot account for the claims to universal validity that our judgments carry with them. Helmholtz, by contrast, may be seen as carrying forward Hume's attempt at a thoroughgoing reduction of thought to associative laws--but now with the attention to system and to experimental detail of one of Germany's premier scientists. Moreover--and this heightens his interest in the present context--Helmholtz sought to apply this form of explanation to a set of problems that he took to be of central importance in Kant's writings on knowledge, involving spatial perception and geometry. Thus, in Kant and Helmholtz we have two powerful minds engaging the problem of whether there can be a science of the mind, with opposite results and yet while working (to some extent) on common ground.

2. Kant on Geometry and the Mind

Kant is notorious for setting his investigations in the Critique of Pure Reason apart from what he termed 'empirical psychology'. He was concerned in the Critique with illuminating the conditions for the possibility of knowledge and with determining the limits to knowledge. He was not concerned with establishing the empirical circumstances in which knowledge is obtained, or with investigating the individual peculiarities--such as effects of memory or of habit--that influence the actual judgments or assertions that might be tendered by an individual knowing subject. In the Introduction, when discussing the divisions of transcendental philosophy, Kant put the point as follows: "What has chiefly to be kept in view in the division of such a science, is that no concepts be allowed to enter which contain in themselves anything empirical, or, in other words, that it consist in knowledge wholly a priori." (A14 = B28).²

It is clear that whatever Kant was doing in the Critique, it wasn't empirical psychology, either in his sense of the term or in ours. But this fact does not resolve all questions about the relationship between Kant's project and a scientific psychology. For in developing his account of the possibility and limits of knowledge, Kant introduced a whole host of psychological-sounding terms, such as 'sensibility', 'sensory manifold', 'imagination', 'apperception', and so on. He posited a whole set of activities, such as 'synthesis', that allegedly lie behind and account for our experience. Kant's account of the cognitive faculties and their operations is sometimes called his "transcendental psychology", by way of indicating that it is distinct from empirical psychology but nonetheless constitutes or implicitly contains an analysis of the knower in terms of cognitive faculties and their distinctive operations.

It might well be asked what relationship there is between transcendental psychology and psychology considered as an empirical science. A common response has been that whereas scientific psychology is concerned with various individual minds whose peculiarities can be known only through empirical sampling, Kant is giving an account of Mind with a capital 'M', which ignores the peculiarities of particular minds (Ward 1883, pp. 162-166; cf. Lewis 1929, pp. 98-100). Kant, as it is sometimes said, was aiming to reveal the common structure in all thought and the universal conditions for knowledge; he was unconcerned with deviations from this structure or these conditions owing to local perturbations.

Taken as an appeal to the universality of the cognitive faculties and operations posited by Kant, this appeal to Mind with a capital 'M' does not successfully set Kant's project apart from psychology regarded as a theory-driven empirical science. The theoretical psychologist might, in fact, eagerly embrace the project of uncovering the universal cognitive mechanism that is only imperfectly or "noisily" manifested in individual minds. If Kant's aim is distinctive only by its universality, the psychologist could bring to bear an analogy with the laws of physics as idealized universal laws whose operations are never cleanly manifested in our sublunary world.

Nonetheless, Kant was doing something quite different from revealing the universal cognitive machine. As others have recognized, the key to this difference lies not in stressing the "universal" in his project for uncovering the universal conditions for knowledge, but rather in stressing that this is an inquiry into the conditions for knowledge. Kant himself, in a passage often cited in this regard, emphasizes this characterization of his project. The passage comes from the preface to the first edition of the Critique; in it, Kant describes the core of his investigation into the "rules and limits" of the understanding as having two sides: objective and subjective.

The one refers to the objects of pure understanding, and is intended to expound and render intelligible the objective validity of its a priori concepts. It is therefore essential to my purposes. The other seeks to investigate the pure understanding itself, its possibility and the cognitive faculties upon which it rests; and so deals with it in its subjective aspect. Although this latter exposition is of great importance for my chief purpose, it does not form an essential part of it. For the chief question is always simply this:--what and how much can the understanding and reason know apart from all experience? not:--how is the faculty of thought itself possible? (Axvi-xvii).

Here Kant sets the investigation of "cognitive faculties" and of "the faculty of thought itself" apart from the question of the objective validity of certain kinds of claims to knowledge. One way of restating the point might be to say that the "conditions for knowledge" that Kant seeks are not to be found in causes or processes, but in conditions of validity, and that these conditions are decided by appeal to the justification of a claim to knowledge, not to its causal origin.

The above passage with its seemingly clear separation of a concern with the validity of knowledge claims from a concern with causal

processes has not by any means led to agreement among Kant's interpreters on the role of psychology in the *Critique*. The passage itself shows Kant's own ambivalence: he says that the "subjective" side of his investigation is "of great importance" but not "essential" to his chief purpose. And in the continuation of the above passage, Kant writes that the search for causes "is somewhat hypothetical (though, as I shall show elsewhere, it is not really so)." (Axvii).

As Robert Paul Wolff has pointed out in his book on *Kant's Theory of Mental Activity* (1963), the above ambiguity and tentativeness reflect Kant's own uncertainty about just how deeply his investigation into knowledge is dependent upon psychological considerations. Wolff pursues the promising strategy of seeking to answer the question by turning to the *Critique* itself and determining whether its chief arguments and conclusions turn upon psychological or upon logical and conceptual considerations. Wolff's conclusion, based upon a painstaking exposition and analysis of Kant's "Transcendental Deduction of the Pure Concepts of Understanding", is that Kant employs both sorts of arguments (1963, p. 176). However, on Wolff's reading, Kant did not wish to recognize this, and in fact he would have liked to think "that only logical arguments are necessary for his purposes" (pp. 176-177). Wolff arrives at a contrary conclusion, that Kant's work in the Deduction depends absolutely upon his "theory of mental activity", including especially his account of transcendental synthesis. On this basis Wolff contends that "so far is Kant from demonstrating the separability of logic and psychology, as has long been fashionable to assert, that he actually demonstrates their complete inseparability" (p. 177).

Although I like Wolff's method of appealing to the structure of Kant's arguments in order to decide whether they are psychological or not, I cannot agree with his conclusion. Moreover, the source of the disagreement comes from the sorts of considerations that he appeals to regarding Kant's text. Wolff rests his case on the type of descriptive terms used by Kant in his analysis: it is alleged that 'transcendental synthesis' must refer to a psychological process. I would like to suggest the use of a different sort of evidence: not the language in which Kant couches his arguments and considerations, but the source from which they derive their force. If his arguments derive their force from an appeal to psychological considerations--whether established by introspection or introduced as a speculative hypothesis about causal processes--then they must be counted as psychological. If the considerations to which he appeals pertain only to questions of the actual or possible justification of knowledge, then matters stand differently, and I see no reason to view Kant's arguments as psychological even if some of the language he uses sounds psychological. After all, in this century many words, such as 'perception' and 'cognition', are used by both the philosophical and the psychological communities without there needing to be any overlap between the types of investigation being carried out.

I propose to examine the sorts of arguments and considerations that Kant introduces in support of his doctrine that Euclidean space is the form of human sensibility. In the introduction to the *Critique* Kant suggests that this doctrine be considered as part of his answer to the question, "How is pure mathematics possible?" (B 20). The question itself might be read as either psychological or epistemological; that

is, Kant might be asking what psychological abilities and processes are necessary for the development of mathematical knowledge, or he might be asking for an account of how the practices of pure mathematics can be justified. I think that nearly everyone would grant that in the context of the Critique, the epistemological question is the one that Kant is asking. The question of moment is whether in answering this epistemological question, Kant appeals to psychological considerations.

The core of Kant's answer to his epistemological question comes in the section of the Transcendental Aesthetic entitled "The Transcendental Exposition of the Concept of Space." Kant first explains the purpose of the section: "I understand by a transcendental exposition the explanation of a concept, as a principle from which the possibility of other a priori synthetic knowledge can be understood. For this purpose it is required (1) that such knowledge does really flow from the given concept, (2) that this knowledge is possible only on the assumption of a given mode of explaining the concept." (B40). The "transcendental exposition" of space will allow us to understand how synthetic a priori knowledge of space--in the form of Euclid's geometry--is possible. It will do this by introducing an explanation that is both necessary and sufficient for explaining the possibility of the type of knowledge in question.

It will be useful to quote Kant's exposition at length. The first paragraph runs as follows:

Geometry is a science which determines the properties of space synthetically, and yet a priori. What, then, must be our representation of space, in order that such knowledge of it may be possible? It must in its origin be intuition; for from a mere concept no propositions can be obtained which go beyond the concept--as happens in geometry (Introduction, V). Further, this intuition must be a priori, that is, it must be found in us prior to any perception of an object, and must therefore be pure, not empirical, intuition. For geometrical propositions are one and all apodeictic, that is, are bound up with the consciousness of necessity; for instance, that space has only three dimensions. Such propositions cannot be empirical or, in other words, judgments of experience, nor can they be derived from any such judgments. (Introduction, II). (B40-41).

The first thing to be noted is that Kant takes the object that is to be explained in this exposition--the science of geometry--to be given. Its synthetic a priori status was discussed in the Introduction and bolstered in the previous section of the Aesthetic, in which Kant argued that space has the status of an intuition rather than a concept. But in the Introduction (B20) and here as well Kant takes geometric knowledge, or the science of geometry, to be actual. He is not out to ground geometric knowledge, but rather to discover and to explain the grounding that he thinks it obviously must have.

In showing that his explanation reveals the necessary condition for the possibility of geometric knowledge, Kant considers three possible bases for geometry: it might be based on the analysis of concepts, i.e., analytic; it might be based on experience, i.e., synthetic a posteriori; or it might be synthetic a priori. In the above passage, Kant rules out

the first of these possibilities, that geometry is analytic, by invoking his earlier contention that basic propositions in geometry "go beyond the concept." On Kant's conception of an analytic/synthetic distinction, the claim that propositions in geometry contain material in their predicates that is not "contained" in the concept of the subject, or as we sometimes say, is not comprised within the meaning of the subject term, is sufficient to establish the proposition as synthetic. Kant further explicates his claim that geometry cannot be analytic at A47-48 = B65, where he claims that from the concept of two straight lines alone one could never derive the proposition that two straight lines cannot enclose a space; material in the predicate, "cannot enclose a space" does not follow conceptually from the subject concept, "two straight lines." Whether Kant here is allowing for the conceptual possibility of non-Euclidean geometry I will not attempt to decide. It is clear that he is appealing to the idea that Euclid's geometry cannot rest upon concepts alone, and that a given concept of Euclid's geometry becomes fully determinate only through the procedure of constructing a figure in accordance with the concept (A712=B740 to A738=B766). This procedure of construction requires a spatial extent, which may be supplied on paper or held in the imagination. The important point is that Kant has argued against the classification of geometric knowledge as analytic by appealing to the conditions under which a geometric proposition can be justified--e.g., only through a process of construction.

From Kant's point of view, to say that geometric propositions depend upon a basis beyond concepts, upon the apprehension of an extended figure, is to say that they depend upon intuition. So now the question becomes whether the intuitions that justify geometry are posteriori or a priori. Of course, Kant argues for the latter. In our passage from the Transcendental Exposition, he supports this claim by appealing to the epistemic status of geometrical propositions--their alleged absolute certainty--and then asking for the conditions under which a proposition can be established with absolute certainty on a basis of intuition. This leads Kant to the conclusion that the intuitional basis for geometry must be a priori, which means, as he puts it, that "it must be found in us prior to any perception of an object, and must therefore be pure, not empirical, intuition." This conclusion is reached on the argument that only in this way could the possibility of geometric knowledge be fully explicated.

Once we have been told that geometrical knowledge is based upon "pure intuition", the question naturally arises as to what we have been told. Notoriously, Kant's phrasing, that the intuition of space "must be found in us prior to any perception of an object", is very psychological-sounding. Being "in us" prior to perception sounds very much like being innate. And indeed, when one turns to the rest of the Transcendental Exposition for clarification, this impression is only reinforced. Kant continues the exposition by asking for the basis of an a priori intuition: "How, then, can there exist in the mind an outer intuition which precedes the objects themselves, and in which the concept of these objects can be determined a priori? Manifestly, not otherwise than in so far as the intuition has its seat in the subject only, as the formal character of the subject, in virtue of which, in being affected by objects, it obtains immediate representation, that is, intuition of them; and only in so far, therefore, as it is merely the form of outer

sense in general." (B41). The language here speaks concretely of the characteristics of the knower. It asks how a certain type of intuition can exist in the subject prior to experience, and answers that it can do so by having its seat in the subject, in such a way that when the subject is affected, it obtains intuitions only of a certain character. The subject's sensory capacities are limited, or are positively determined, in a certain way, that is, so that the subject can have spatial intuitions only of the kind that are described by Euclid's geometry.

A standard philosophical response to this part of Kant's doctrine is to say that in making these concrete ascriptions regarding the subject's sensory capacities, Kant was--or should have been--placing a logical rather than psychological limitation on the subject, and that he must or should have meant his talk of the priority of spatial intuition to be logical rather than temporal priority. I think that nearly everyone will--or should--grant the plausibility of shading Kant's exposition in this direction. Moreover, Kant himself, in the conclusion of this subsection of the Aesthetic, makes just this kind of claim for his argument. He concludes his argument by claiming that his explanation "is thus the only explanation that makes intelligible the possibility of geometry, as a body of a priori synthetic knowledge." (B41). Kant's formulation of his conclusion in terms of the possibility of geometrical knowledge reads as a return to the idiom of explicating the logical conditions for the justification of a certain type of knowledge.

If now we apply the criterion that I proposed above for deciding whether Kant's arguments are psychological or not, the result is that they must be regarded as logical and conceptual, or, since the subject matter is knowledge, epistemological. For the purpose of establishing Kant's conclusion about the possibility of geometric knowledge, it is irrelevant to his argument whether space as the form of intuition is inborn or acquired through experience. What Kant purports to establish is not how representations of space come into existence, but rather, that the representations of space that do come into existence must be of a certain character and no other. Must be, and cannot be otherwise. And he purports to establish this conclusion not through a theory of spatial perception and its development, but on the grounds that in this way and in this way only can the knowledge of geometry that is (or that he took to be) actual be explicated or shown to be possible.

Now we are back on the familiar and comforting ground of Kant as transcendental epistemologist rather than transcendental psychologist. But there are, I believe, still further considerations that do and should threaten this comfort, that do and should problematize the sharp division between epistemology and psychology in the case of Kant's doctrine of spatial intuition and geometry. I am not speaking here of the passages "left over" from my analysis above, in which Kant, despite the purpose of his argument and the actual source of its force, states that the intuition of space must be "found in us" prior to any particular perception of objects. This phrase may simply be a clumsy way of saying that, as a point of the "logic" of epistemology, the knowing subject must be regarded as determined independent of any particular perception or temporal sequence of development to have only one kind of spatial intuition. No, the source of our discomfort should not be concern that Kant's arguments are psychological, because they so

clearly derive their purported force from epistemological (or "transcendental") considerations. Rather, the problem is that Kant's conclusion contains an element of the psychological, insofar as it ascribes space as a form of intuition to the knowing subject.

In order to explain this point, I will briefly compare the status of Kant's conclusion that space is the form of outer intuition with Kant's claim that the twelve categories constitute the "form" of the understanding. According to Kant's doctrine, the twelve categories must be postulated in order to account for the transcendental unity of perception in any finite intelligent being. His claim for them is that they constitute the universal conditions for the possibility of a unified consciousness--that is, they underlie the very possibility of understanding or of thought itself. Not so with space as a form of intuition. Truly, from the perspective of accounting for the possibility of absolute certainty in geometry, Kant maintained that Euclidean space must be postulated as the necessary and universal form of human outer sense. But from the perspective of ourselves as finite intelligences, it is merely contingent that we experience things in space. Without here going into the details of Kant's doctrine, I shall simply assert that according to Kant it was an inscrutable aspect of the human knower that it has space as its form of outer intuition. Thus, even though the necessity that all of our spatial intuitions should have a certain character is grounded in epistemological considerations, the fact that we have spatial intuition at all, rather than some other form of intuition, has the status in Kant's system of a brute fact, which tempts one to say that it has the status of a brute fact of human psychology (but not one that would be studied empirically or justified by appeal to specific empirical results).

Having discussed the status of Kant's remarks on spatial intuition in relation to psychology, it is important in looking forward to Helmholtz to discuss one of the uses that Kant makes of his doctrine of sensibility. Kant transfers his doctrine on the status of space as a form of intuition to space regarded as an object of investigation in physics. Recall Kant's wording from above: "Geometry is a science which determines the properties of space synthetically, and yet a priori." (B40). It was a part of Kant's conception of geometry that the object of geometry is space, including physical space as given through empirical intuition. For once Kant had established to his own satisfaction the doctrine that space is a form of human sensibility, the notion of the faculty of sensibility itself ensured that all intuitions, pure or empirical, would be conditioned by this form. This was Kant's way of resolving the dispute between Newton and Leibniz over the status of space, by admitting with Leibniz that space is ideal and comprises relations among elements in a manifold (Kant's "matter"), while accommodating Newton by maintaining that the possible relations among these elements are conditioned by the form of human sensibility, which provides a basis for the intuition of space as infinite and as not constituted by its parts. Human sensibility is Kant's home for the homogeneous, infinite Euclidean space of Newtonian physics.

3. Helmholtz on Kant, Geometry, and the Psychology of Perception

Now let us turn to Helmholtz and his attempts to bring empirical science to bear on Kant's doctrine of geometrical intuition.

Helmholtz was a physiologist for the first part of his career, focusing upon the physiology and psychology of sight and hearing, and he spent the second part of his career as one of Germany's leading physicists. Throughout his life he maintained an interest in the philosophical presuppositions and implications that he found connected with his scientific work. One of his most long-standing intellectual pursuits was the problem of spatial perception and spatial vision, which he sometimes considered in connection with the question of the status of the axioms of geometry. Helmholtz regarded many of his results in this area as deriving from his work on the physiology of the senses, and as having its basis in his theory of visual perception.

In papers from the 1860s and 1870s, Helmholtz specifically sought to revise Kant's conclusions about the status of Euclid's geometry, both as a description of physical space and as a description of visual space. Helmholtz contended that the applicability of Euclid's geometry to physical and to visual space is an empirical question, and that in appealing to spatial intuition Kant unwittingly had appealed to something that is empirically conditioned.

Helmholtz's argument proceeded in two stages. First he questioned the idea that Euclid's geometry is the only conceivable geometry. This he did by invoking the work of Riemann (1867) as well as his own work on n -dimensional manifolds (Helmholtz 1870, pp. 11-17, and Helmholtz 1868). He carried out this part of his argument by appeal only to analytical geometry (rather than to diagrams or spatial images), in order to avoid any appeal to intuition (in Kant's sense). By considering a class of n -dimensional manifolds for which there exists a definite value for the distance between any two arbitrarily chosen points, Helmholtz took it as manifest that he was considering spatial manifolds. He went on to show that such manifolds can be defined that do not accord with Euclid's axioms, but that nonetheless yield a consistent geometry; these included manifolds with both positive and negative curvature, in Riemann's sense. On this basis, Helmholtz concluded that other consistent geometries besides Euclid's are indeed conceivable--that is to say, are conceivable in the sense that they can be described within the resources of analytic geometry.

In the second stage of his argument, Helmholtz went on to maintain that not only are non-Euclidean geometries conceivable (in the above sense), but that it also is the case that one can imagine the sorts of spatial intuitions one would have in a non-Euclidean space. Here he appealed especially to the work of the Italian mathematician Eugenio Beltrami (1868). Beltrami provided an account of the spatial intuitions that would be available to an observer moving through a space of negative curvature. Helmholtz, using Beltrami's device of mapping this "pseudospherical space" into the interior of a Euclidean sphere, provided an account of what it would be like for a person with a body, eyes, and habits such as ours (i.e., a person used to treating space as Euclidean) to move about in a non-Euclidean space (1870, pp. 21-22). He showed that the successive experiences that one would receive upon moving about would contradict Euclidean expectations but would be in conformity with what is expected in a pseudospherical space of constant curvature (on the assumption that there are "fixed" or rigid bodies--perhaps our own--and that these remain rigid as they move about; 1870,

pp. 15-17 and 24).⁴ For example, as the observer followed two lines that appeared to converge at the horizon, say, some 100 feet away, he would discover upon traversing the 100 feet that the lines were separate at that position but appeared to converge further on, and this experience would be repeated for as far as he followed the lines; lines that appeared to be parallel would diverge as he followed them along. Helmholtz also provided an account of the experiences of an observer who is imagined to enter a space with constant positive curvature, which he termed a "spherical space" (1870, pp. 22-23).

On the basis of this two-stage argument, Helmholtz concluded that Kant was wrong in holding that the axioms of Euclid's geometry apply to physical and visual space with necessity. He maintained that the applicability of the axioms must be decided empirically, and that our ordinary acceptance that space is Euclidean derives from limits placed upon our imagination by the structure of our sense organs and the character of the perceptions that they yield us. In arriving at these conclusions, Helmholtz did not argue that the geometry of n -dimensional manifolds is itself empirically derived or supported. Rather, he invoked Riemann's distinction between mathematically defined geometrical structures on the one hand, and their fit to physical space on the other (Helmholtz 1870, p. 12). Not the axioms of geometry considered purely mathematically, but the axioms of geometry as descriptions of space must be subject to empirical test according to Helmholtz. The question of which axioms describe physical space thus cannot be decided with apodictic certainty, and it takes on the status of any other empirical claim of physics.

Helmholtz certainly was on solid ground in his criticisms of Kant for accepting as certain what can receive only empirical support. But Helmholtz went on to suggest that it was within the framework of the physiology and psychology of the senses that he had achieved his conclusions, and to contend that the thought processes associated with spatial perception, and indeed, with scientific thought as well, could straightforwardly be made the object of empirical investigation (1870, pp. 25-26; 1878, pp. 118-129). Here he was not merely asserting that Kant was in error with regard to the status of human knowledge of space; his position carried the implication that Kant's enterprise was carried out in a wholly improper manner.

In order to evaluate Helmholtz's claim to have empiricized Kant's project, I will examine his theory of visual space perception. As is well known, the central feature of Helmholtz's theory of vision was the doctrine of unconscious inference. He maintained that our visual experience of a three-dimensional world results from a process of inference, from "data" provided by stimulation of the sense organs to the structure of distant objects. The "data" for such inferences might include the current pattern of retinal stimulation together with the innervation of the ocular nerves (the latter can be regarded as information about eye position). General premises, themselves established on the basis of past experience, would then be applied to this "data". These general premises would include rules for projecting each retinal location in a given direction from the eye, such that stimulation of, say, the left side of the retina leads to the projection of a small area of light off to the right. Other premises would give rules for determining depth from stereoscopic cues, and distance from

yet other cues, such as accommodative muscle sensations or atmospheric perspective. Application of these general premises to occurrent sensory stimulation would yield as a "conclusion" the imagistic experience of a spatial layout.

The question that obviously arises with respect to a theory of this sort is how to characterize the process of inference. Early in his career, this problem stumped Helmholtz. In his 1855 lecture on "Human Vision" at the Kant memorial celebration, Helmholtz toyed with the idea of characterizing these processes as the result of "mechanical . . . involuntary connections among ideas" (1855, p. 112). Yet he stopped short of endorsing this conception of the process, preferring instead, as he put it, to take a step together with Kant, and to recognize that an inference results from an act of judgment, which Helmholtz then left as an unanalyzed factor.

From the 1860s onward, Helmholtz adopted the position he had toyed with in 1855, and attempted to reduce the unconscious inferences of vision to the "laws of association", which he regarded as the fundamental "laws of thought". The flavor of this type of account may be given through an example of the build up of one such associative law (1878, pp. 123-136). Helmholtz maintained that when a given retinal location is stimulated, the resulting sensation has two components. The first component corresponds to the color and intensity of the light striking that retinal location. The second is a sensation, or "local sign", unique to that retinal location, which "labels" the first sensation as resulting from a specific retinal locus. Local signs initially have no spatial meaning (they merely label the retinal element), but through a process of association--say, between visual sensation and the locations of objects as discerned through the sense of touch--a rule is learned associating each local sign with a direction in space. Such rules are among the "general premises" that enter into the "unconscious inferences" of perception.

When one runs through the above example, the question arises of how the percipient chooses a sound rule for assigning spatial location to local signs from out of the mass of possible associations. This amounts to the task of describing a mechanical process of induction. Helmholtz was sensitive to this need and made his initial response as follows:

When the traces of like kind which are left behind in our memory by often repeated perceptions reinforce one another, it is precisely the law-like which repeats itself most regularly in like manner, while the incidental fluctuation is erased away. For the devoted and attentive observer, there grows up in this way an intuitive image of the typical behaviour of the objects which have interested him, and he knows as little afterwards how it arose as the child can give an account of the examples whereby it became acquainted with the meanings of words. (1878, p. 131).

Here Helmholtz appeals to the regularity of law-like associations, suggesting that their lawfulness will be enough to bring them to the foreground of thought, washing out the merely incidental.

This appeal may strike one as a little quick. Imagine the situation of the learner, faced with the flux of sensory stimulation and armed

only with some general laws of association--e.g., associative laws of "similarity" and "spatio-temporal contiguity". Imagined in this way, the situation appears hopeless. If the representations to be associated are images with complex elements, then what is to count as similarity? An indefinite number of similarities might be defined, and to give the associative laws a "head start" in selecting certain patterns is to abandon an associative account.

German physiologists, psychologists, and philosophers in the nineteenth century made an heroic effort to overcome this problem. We have seen the result of the effort in Helmholtz's espousal of the doctrine of local signs. The effort consisted in regarding the primary elements of psychological analysis as aspatial or punctiform sensations defined by variation in quality and intensity alone (1878, pp. 119-125). With such simple elements, the associative laws might have but a single dimension of similarity over which to associate. The problem of picking out the "relevant" dimension of similarity might thereby be avoided.

Once this effort has been made, the theoretician is still faced with giving an account of how the percipient gets from the flow of punctiform sensations to the construction of a spatially ordered world. Helmholtz seems to have recognized that a merely passive collocation of associated elements would not do the trick. He thus added a new feature to his account--the active testing of our hypotheses about the lawlike in experience.

We do not merely have alternating sense impressions which come upon us without our doing anything about it. We rather observe during our own continuing activity, and thereby attain an acquaintance with the enduring existence of a lawlike relationship between our innervations and the becoming present of the various impressions from the current range of presentables. Each of our voluntary movements, whereby we modify the manner of appearance of the objects, is to be regarded as an experiment through which we test whether we have correctly apprehended the lawlike behaviour of the appearance before us, i.e. correctly apprehended the latter's presupposed enduring existence in a specific spatial arrangement. (1878, pp. 135-136).

The operative notions are that through voluntary movements we "test" whether we have "correctly apprehended the lawlike". There has been a shift in the analysis, from pure association to the assumption of certain cognitive abilities, such as the ability to judge whether a certain putative law of the appearances is correct. What started out as an attempt to resolve thought into association has ended with an appeal to brute abilities of thought.

A similar shift of analysis occurs in Helmholtz's discussion of the case of scientific thought, which he considered to be explicable by the same type of associative processes as perceptual inference. Initially, he describes the position of the scientist as one in which the lawlike is simply given: "What we can find unambiguously, and as a fact without anything being insinuated hypothetically, is the lawlike in the phenomena. From the first step onwards, when we perceive before us the objects distributed in space, this perception is the acknowledgement of a lawlike connexion between our movements and the therewith occurring

sensations. Thus even the first elementary representations contain intrinsically some thinking, and proceed according to the laws of thought." (1878, p. 138). The lawlike presents itself in perception, without conscious hypothesis, only as the result of the unconscious processes of perception. Scientific apprehension of the lawlike is simply an extension of the most basic thought processes of all, such as occur in the simplest spatial perception.

That scientific apprehension of the lawlike should depend upon the inferential processes of perception need not be fatal to the project of an associative analysis of thought (including both perception and scientific thought). The problem is that perception itself has previously been described by analogy with scientific or experimental procedures. One is still waiting for the reduction of thought to mechanical association. But the wait will not be rewarded by Helmholtz. His excavations into the "laws of thought" reach bottom not at certain laws of association, but with an echo of the Kantian notion of the "law of causality" as a fundamental cognitive given.

Every inductive inference is based on trusting that an item of lawlike behaviour, which has been observed up to now, will also prove true in all cases which have not yet come under observation. This is a trust in the lawlikeness of everything that happens. However, lawlikeness is the condition of comprehensibility. Trust in lawlikeness is thus at the same time trust in the comprehensibility of the appearances of nature. While: should we presuppose that this comprehensibility will come to completion, that we shall be able to set forth something ultimate and finally unalterable as the cause of the observed alterations, then we call the regulative principle of our thought which impels us to this the law of causality. . . . The law of causality is an a priori given, a transcendental law. A proof of it from experience is not possible, since the first steps of experience, as we have seen, are not possible without employing inductive inferences, i.e. without the law of causality. (1878, p. 142).

The attempt to resolve thought into mechanical processes of association has ended with the statement that at least one cognitive principle must be presupposed at the outset. The attempt to resolve thought into blind laws for combining simple representations has ended with thought still resisting the analysis.

How do matters stand with respect to Helmholtz's attempt to use the empirical study of perception to refute Kant's theory of intuition? Not well. The attempt to explain perceptual inference by appeal to "mechanical" processes has not worked. If this attempt is regarded as part of a more general attempt to show that questions Kant had regarded as requiring an analysis of the justification of cognitive claims could be answered by an analysis invoking only "mechanical" laws of thought, then one must regard this broad attack upon Kant as a failure. This need not imply, however, the failure of Helmholtz's specific project of refuting Kant's views on the status of Euclid's geometry as a description of physical space and of the necessary form of human spatial intuition through his (Helmholtz's) theory of the senses. This more special project would be a success if Helmholtz's claim that Euclidean space is an acquired, contingent form of spatial intuition could be

shown to derive from his empirical work on vision.

When, however, we recall Helmholtz's arguments for the possibility of non-Euclidean intuitions, we find that they do not depend upon his theory of vision. The arguments sketched above all presuppose that the eyes and perceptual habits of the observers who experience non-Euclidean physical space are the eyes and habits of observers who are used to Euclidean space. For the purpose of these arguments, the Euclidean habits of these observers might be either innate or acquired--in either case the arguments would go through. Rather than arguments from the psychology of vision providing the basis for the possibility of experiencing non-Euclidean space, the arguments work the other way around. It is on the basis of showing that non-Euclidean space can be imagined that Helmholtz can contend that if spatial perception is indeed an acquired ability, the character of the ability that we acquire is contingent in that it depends upon the character of our own space (which, given the assumption of "fixity" as described above, is a matter of empirical fact). On the supposition that our visual habits are formed through experience, it may be supposed that they would be different in a non-Euclidean space (on this, see Reichenbach 1928, ch. 1, sec. 11).

Nonetheless, Helmholtz's arguments against Kant's teaching on the status of Euclid's geometry as a description of physical space are effective. The real work in Helmholtz's disagreement with Kant comes from his making it seem plausible that we could receive experiences that would indicate that the structure of objects in space is non-Euclidean--that non-Euclidean geometries are not only mathematically possible, but could possibly constitute the geometry of physical space. In basing his denial of Kant's doctrine upon the possibility that a sequence of experiences might be obtained indicating the non-Euclidean character of physical space, he was not doing psychology but was investigating the logical or epistemological status of claims about the geometrical structure of physical space. The imagined sequences of experience are significant as potential evidence for a claim in physics. What Kant had regarded as a brute but necessary fact of human psychology (pertaining to the form of outer sense), Helmholtz has shown to be more plausibly regarded as a fact about physical space that is contingent in the sense that it must be empirically determined. Kant is left open to this response on the basis of physical possibility because he had himself linked the question of the form of human outer sense with the character of physical space in his doctrine of outer sense. Kant needed this linking to support his claim that we can have knowledge of space in an a priori manner. Helmholtz effectively denied that a a priori knowledge of the structure of physical space is possible. But rather than it being the case that this conclusion was based on his work in psychology, it is more accurate to say that the plausibility of his claim in psychology that spatial perception might be non-Euclidean is based on an examination of the sorts of evidence that might be brought to bear on certain claims in physics. In arguing for the possibility of evidence for non-Euclidean physical space, Helmholtz takes the evaluative stance of the physicist examining the relation between theory and evidence, not the descriptive stance of the psychologist charting the "laws of thought".

4. Concluding Remark

What have I been doing? I've been examining a certain historical episode in order to get a grip on questions about the relationship between what I have called "philosophical questions" and questions that may be approached through the techniques of empirical psychology. I don't claim to have settled or to have solved the question of the relationship between the two (the question remains open). But I do claim to have portrayed a certain kind of slippage that characteristically goes on when one claims to empiricize questions of knowledge and justification by regarding knowledge as the product of a psychological process. This slippage occurred in Helmholtz's attempt to bring an empirically based psychology directly to bear on Kant's theory of knowledge. It was only by reading the characteristic modes of reasoning of the scientist into the fundamental processes of perception that Helmholtz could claim to get them back out again. Helmholtz was effective in his dispute with Kant when he brought arguments about the possibility that empirical evidence could determine our assignment of a particular geometrical structure to physical space to bear on Kant's claim to have discovered the necessary geometrical structure of physical space in the constitution of human sensibility.

Helmholtz did not succeed in developing a purely empirical approach to questions of cognition. This, of course, does not show that others cannot develop an empirical science of the mind. But insofar as the kind of slippage I have revealed here recurs, to that extent the ambition of subsuming the study of thought itself wholly within the purview of natural science will have eluded its pursuers. The normative character of thought will have eluded empiricization.

Notes

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²All citations to the Critique are to the marginal page numbers in the Kemp Smith edition (see Kant 1965); 'A' and 'B' refer to the first and second editions as printed in the Akademie edition of Kant's works.

³I develop this point more fully in a work in progress, entitled "Mind and Space from Kant to Helmholtz."

⁴This qualification about "fixed" or rigid bodies should be assumed in all subsequent contexts in which the empirical determination of the geometry of physical space is discussed.

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