Procedural Fairness in Exchange Matching Systems

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The move from open outcry to electronic trading added another responsibility to futures exchanges—that of matching orders between buyers and sellers. Matching systems can affect the level and speed of price discovery, the distribution of revenue, as well as the level of price efficiency of a given market. Whether the matching system is procedurally fair is another important consideration. I argue that while FIFO (First In First Out) is a fair procedure in principle and is perceived as the default matching system, it is not a fair procedure in practice. Likewise, while pro rata is a fair procedure in principle, it is not so in practice. Nevertheless, both FIFO and pro rata are relics of an open outcry system. Instead, I propose an alternative approach to matching systems that builds on the strengths of electronic trading—the ability to randomize in real-time. I introduce random selection for service (RSS) as a matching system that is procedurally fair both in principle and in practice.

1. Introduction

The move from open outcry to electronic trading added another responsibility to futures exchanges—that of matching orders between buyers and sellers.[1] In open outcry, the decision of whose limit order to fill was left to the trader executing a market order.[2] There were, of course,

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[1] This article specifically focuses on futures exchanges, rather than financial exchanges more generally, because futures exchanges tend to be relatively straightforward compared to most other financial instruments. Such a setting provides a more manageable context in which to address the fairness questions this article raises. Nevertheless, much of what is discussed in this paper can be generalized to any centralized financial market.

[2] A market order is an order to buy or sell a contract immediately at the available price, whereas a limit order is an order to buy or sell a contract at a specific price or better. This article focuses on matching between existing limit order and incoming market orders because any incoming limit order to, for example, buy a contract at a price at which there is a limit order to sell the contract simply acts as a market order. In essence, matching only occurs between sitting limit orders and incoming market orders that are at the same price, even if sometimes it might appear to happen instantaneously.

some general rules made by the exchanges, and there were officials supervising each pit to ensure that rules were followed. But open outcry relied heavily on norms and pit etiquette developed amongst traders. Generally, for products with more price volatility, traders would match trades on a 'first come, first serve' basis, or as it is generally known in the industry 'first in first out' (FIFO). For products with less price volatility, a more sophisticated system was used. Traders with a market order might give priority to the first trader who placed a limit order, but they would also prorate their market order in some way to let other traders fill at least some of their limit order as well. This system came to be known as 'pro rata'. Both procedures for matching orders (and their hybrids) were informally developed and existed as a social norm among traders.

The shift to electronic trading began in earnest when Deutsche Terminbörse (DTB) introduced electronic trading for the long-term German bond (Bund) in 1990 in order to compete with the London International Financial Futures and Options Exchange's (LIFFE) predominantly open outcry.[3] The "battle of the Bund" reached its crescendo in the late nineties, when DTB managed to displace LIFFE as the dominant market for Bund trading, an outcome that was seen as an important victory for electronic trading (Gorham & Singh 2009).[4] During that time, LIFFE

[3] For helpful discussions on this topic I thank Mark Ibbotson, who was the been Director of Market Operations of LIFFE at the time and was well positioned to be informed on how these events unfolded.

[4] A couple reasons that DTB won the Bund trading battle was the political push in Germany to repatriate trading in Bunds to "Finanzplatz Deutschland," and the fact that DTB was able to get its trading screens into the US before LIFFE. I thank an anonymous reviewer for these explanations.

developed the LIFFE CONNECT system to compete with DTB.[5] Despite losing its dominance with the Bund, LIFFE managed to remain the dominant exchange for short-term interest rates (STIRS) products like the Short Sterling.

By introducing electronic trading, both DTB and LIFFE had to take responsibility for how market orders were matched with limit orders by maintaining a continuous limit order book (CLOB).[6] In electronic trading, a pure FIFO matching system assigns timestamps to each limit order in the order it was received. When a market order is placed, the limit orders are filled in their entirety starting with the earliest limit order at the best price until the entire market order is matched. While DTB operated a FIFO matching system for its products, LIFFE implemented a FIFO matching system for its higher volatility products and a mostly pro rata matching system for its lower volatility products like its STIRS.[7] A pure pro rata matching system fills limit orders in accordance with their proportion compared to the entire quantity of limit orders at the price level when a market order is placed. Larger limit orders will receive a larger portion of a fill than will smaller orders.

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[5] LIFFE already had an electronic trading platform called APT (Automated Pit Trading) from 1989, but that platform was used primarily for after-hours trading.

[6] One could, in principle, envision some trading platform in which this decision is left to the traders, for example one in which traders see limit orders broken up into individual lots, but such a system seems prohibitively clunky and would suffer from a variety of drawbacks. For more on CLOBs, see (Haeringer & Melton 2020) section 2.1.

[7] LIFFE's STIRS matching system had a blend of pro rata and FIFO, since the first limit order that established a new best price would match first, followed by pro rata sharing for all orders from other limit orders that had joined that best new price. Field and Large (2008) argue that the "pro-rata algorithm has similarities to practices seen in long-standing futures trading pits, where, possibly because time priority is hard to establish, market orders are often shared-out among a number of distinct competing liquidity suppliers" (p. 11).

What matching system is implemented affects the level and speed of price discovery, the distribution of revenue, as well as the level of price efficiency of a given market (Angel & Weaver 1998, Panchapagesan 1997). Another important consideration that often arises when considering matching systems is fairness.

In this article, I argue that while FIFO is a fair procedure in principle and is perceived as the default matching system, it is not a fair procedure in practice. Likewise, while pro rata is a fair procedure in principle, it is not so in practice. Nevertheless, both FIFO and pro rata are relics of an open outcry system. Instead, I propose an alternative approach to matching systems that builds on the strengths of electronic trading—the ability to randomize in real-time. I introduce random selection for service (RSS), which randomly fills individual limit order contracts from all those in the book at the time a market order is placed as a matching system that is procedurally fair both in principle and in practice.[8]

I begin in §2 by establishing that FIFO seems, at first pass, to be a fair procedure to determine what limit order gets filled, but that there are several considerations that can give us reason to question whether FIFO is indeed fair in practice. In §3 I argue that pro rata can be considered a fair procedure in principle, but because it cannot be implemented in its pure form it will not be procedurally fair in practice. In §4, I propose an alternative approach to matching systems that builds on the strengths of electronic trading as a departure from open outcry—the ability to randomize in real-time. I introduce random selection for service (RSS) as a matching system that is procedurally fair in principle. In §5, I discuss some practical considerations against RSS, but argue that it would also be procedurally fair in practice. I conclude in §6. Ultimately, exchanges remain path dependent and have not changed their matching systems much since the

[8] I discuss RSS more in depth in §4.

initial floor-based adaptation. It is time to reevaluate how exchanges approach their order matching systems and make these matching systems fairer.[9]

2. Procedural fairness and FIFO

According to Boatright (2014), "[t]he fundamental ethical requirement of financial markets is that they be fair" (9). Formal, or procedural, fairness requires that we apply the same rules impartially and equally to each agent (Hooker 2005; Heath 2010; Angel & McCabe 2013). Procedural fairness, at least in the context of matching systems, requires impartiality—that the determination of the matching system not be influenced by which traders benefit or are harmed by the matching system (Gert 1995, 104)—, and equality—that the matching system treat all similar traders in a similar fashion (Aristotle, *Nicomachean Ethics*, V.3. 1131a10–b15).

As Heath (2010, 167) explains, one can also employ a concept of substantive fairness in the context of finance. However, any concept of substantive fairness will inevitably be both a more complex and contested concept than procedural fairness. It would require settling questions regarding the substance of the rules regarding what we owe to whom, based on issues like needs, desert, or prior agreements. Nevertheless, because the concept of procedural fairness provides sufficiently novel and interesting insights in the context of exchange matching systems, the focus in this article is solely on the procedural aspect of fairness.

FIFO, at least in principle, treats all limit orders placed at the same time similarly, where the similarity is in the time they are processed. This treatment is impartial, as it does not matter who the traders are or how much they stand to benefit or be harmed. FIFO can thus be considered a fair procedure in principle. This emphasis on speed as the core consideration of

[9] Haynes & Onur (2020) make the case that precedence rules have not been widely studied.

procedural fairness is echoed by Haeringer & Melton (2020) in the distinction they draw between what they call 'access fairness' and 'outcome fairness':

> [Access fairness] is based on the desire that among equally fast traders no trader should be advantaged over any other in the allocation of resource he or she receives… That is, equally fast traders should have the same probability to be ranked first, second, third,… Outcome fairness on the other hand is the standard notion of equal treatment of equals, which simply requires that two equally fast traders submitting identical orders should obtain the same expected outcomes. (5)

Despite going on to propose a system that does not do so, Haeringer & Melton's focus on fairly treating *equally fast* traders exemplifies the view that the relevant attribute for fairness is often taken to be equality among the equally fast.

This sentiment is not unique to futures exchanges and FIFO. 'First come, first serve' (FCFS) is a ubiquitous concept in daily life. We stand in line at the supermarket, we wait our turn to get on the bus, and we queue over the phone while waiting for our turn to talk to customer service. We often think queues are generally a fair mechanism for the distribution of scarce goods because we think that those who join the queue at similar times should be treated equally, and this entails that people should receive the good they want in the order in which they join the queue. If there is no underlying feature for why some traders systematically get in the queue sooner than others (an assumption that will be challenged below), queues strike us as a fair allocation mechanism.[10] Perry & Zarsky (2014) present empirical evidence that supports the claim that people view FIFO as a fair way of distributing a good, that people are more satisfied

[10] This is not to say that we do not accept as fair some deviations from FCFS in daily life. Supermarkets often have express lanes for those with smaller orders (e.g. 10 items or less), theme parks allow people to pay extra to jump the queue, and airport security has special lines for those that have been pre-vetted (TSA precheck or Global Entry in the US). We often also consider need or desert, among others possibilities, as justifications for jumping the queue.

when distribution is FIFO-based, that people object to violations of FIFO, and that people feel nervous when asked to violated FIFO themselves.[11] Compliance with FIFO has become a salient and undisputed determinant of people's sense of fairness (Perry & Zarsky 2014; 1606-7).

Recently, John & Millum (2020) have addressed the question of whether these widespread attitudes towards queuing are justified. While they do not find that waiting times have intrinsic moral significance, John & Millum argue that FCFS/FIFO tends to be fair for two reasons. First, an allocation based on waiting time "does not privilege certain people over others on the basis of morally irrelevant factors" (198). Second, they argue that FIFO expressively signals the equal moral standing of individuals because "[i]f everyone who wants a good must get into the same line for that good and follow the same waiting procedure, then everyone's claim is treated equally" (199).

Since queues are so pervasive in our lives, are generally efficient, and intuitively strike us as fair, it is no wonder that FIFO is considered a 'gold standard' for matching systems.[12] Like queues more generally, FIFO is solely focused on the order in which limit orders arrive in determining what limit orders get fulfilled and when. Nevertheless, while in principle FIFO is a fair procedure, in practice there are at least two reasons to question whether FIFO is able to deliver a fair matching system—the order in which limit orders are placed is derived from

[11] Perry & Zarsky (2014) cite (Zhou & Sorman 2008) in particular regarding empirical support.

[12] In many products, there is only one, or a small number of limit orders at any given price order. In reality, an aggressive market order is likely to match all those limit orders. For such cases, FIFO can be viewed as a way for the exchange to enhance efficiency by saving resources, since pro rata takes up more computing power. According to Haynes & Onur (2020), "Most markets use time as the secondary precedence rule, making the queue of the orders at the best price operate in a first-in-first-out (FIFO) manner" (p. 2).

unequal opportunity, and exchanges are not able to genuinely fill limit orders in the order in which they were placed.

First, while there are substantive benefits to getting into the queue quickly, in practice not everyone has an equal opportunity to do so.[13] To illustrate this point, consider an example from John & Millum (2020), where a refreshments table is set out outside a meeting room. The allocators do not know how people will arrange themselves inside the meeting room, and the allocators do not know in what order the people will file out to the table once the meeting is over. As such, the allocators have no reason to suspect that any particular person will be first in line.[14] Since there is no good reason to think that any particular person will be first out of the room, it might seem reasonable to think that everyone has an equal opportunity to join the refreshments queue. Although, of course, in practice this will not in fact be true. There will be some who are closer to the door, some who are quicker than others, some who are willing to fling elbows to get the coffee, and a host of other reasons one might get to the refreshments table first. A motivated audience member can easily 'game the system', for example by deliberately sitting near the door or getting up and leave a minute before the talk is over.

Part of the reason we might be willing to accept queues in the case of the refreshments queue is that not much hinges on how long people wait in that scenario. It does not much matter whether they receive their refreshments early or whether they need to wait a few minutes. Yet while the stakes are low in the refreshments case, they are quite high in the case of futures

[13] For a helpful discussion of equality of opportunity see (Arneson 2015).

[14] John & Milum argue that in such a case queues "tends towards perfect fairness" because it is epistemically equi-probable that each person will be first in line. In (Hersch & Rowe, under review), we argue against this in more detail.

exchanges. Institutions invest hundreds of millions of dollars to be faster than their competitors, and those without deep pockets find themselves at a disadvantage.[15] Much has been said about the ever-increasing attempts to shave off a few milliseconds, microseconds, and even nanoseconds off one's latency in order to be able to beat the competition (Jones 2013, Biais & Foucault 2014, O'Hara 2015, Menkveld 2016).

When speaking about matching systems in general, Perry & Zarsky (2014) raise the possibility that "more affluent participants may have the resources necessary to secure early arrival or to strategically adapt to any non-random method" (1609). This insight is very much clear in the speed arms races that are prevalent in today's exchanges. Since this speed race is so costly, it crowds out smaller traders with less resources. The playing field is not level and opportunities to join the queue are not equal, because one's access to resources becomes a dominant factor in determining whose orders get fulfilled first.[16] A variety of solutions offered include introducing a Pigouvian tax (Biais et al. 2015), adding a latency floor (Melton 2015), and shifting to frequent batch auctions (Budish et al. 2015), all as ways to prevent additional benefits of increasing speed.

Even if there were no problems with some getting in the queue faster than others due to a basic lack of equal opportunity, FIFO cannot be implemented as a fair procedure in practice

[15] See for example: Christopher Steiner, "Wall Street's Speed War", *Forbes Magazine*, September 27, 2010. Anton Troianovski, "Networks Built on Milliseconds", *Wall Street Journal,* May 30, 2012. Alexander Osipovich, "High-Frequency Traders Eye Satellites for Ultimate Speed Boost", *Wall Street Journal,* April 1, 2021.

[16] That one's wealth affects where one gets in the queue happens in a broad range of life contexts. One jarring example, since the good (lifeboats) was scarce, and those who did not get it lost their lives, is the way lifeboats on the Titanic were located. Lifeboats were placed either directly adjacent or close to First- and Second-Class cabins, and Third-Class passengers did not have dedicated lifeboats.

because it cannot guarantee a queue in which those who enter it earlier actually get their limit orders filled sooner. For FIFO to be fair, exchanges need to be able to guarantee that the queue is functioning properly. The problem is that exchanges are unable to genuinely process orders on a FIFO basis when they are designed to process orders as quickly as possible (Melton 2017, Melton 2018, Melton 2020, Haeringer & Melton 2020). A variety of practical technological limitations with networking equipment cannot guarantee equal latency on all ports at a nanosecond scale. This gives rise to problems such as infrastructure jitters, in which small non-constant variations in processing times arise (Mavroudis 2020). Under FIFO "hardware constraints imposed by the switch technology cannot guarantee fairness" (Haeringer & Melton 2020, 29).

While FIFO is a fair procedure in principle, it is not so in practice. In the next section I explore whether the existing alterative—pro rata—fares any better.

3. The pro rata alternative

Some form of pro rata has been used for some contracts since before electronic trading came to the fore, especially for low volatility short term interest rates (STIRS) like the Short Sterling. These products usually have much lower volatility than products that use FIFO.[17] Pro rata (often with the first order that established a new best price receiving priority) was usually used for such low-volatility products on the trading floor, and as such was an obvious choice for

[17] For a discussion of price volatility caused by HFT in FIFO systems see (Zhang 2010, Shabbir 2015).

an electronic matching system.[18] This kind of path dependency explanation can help see why pro rata (and its hybrid variants) is still used today.

We might think that, unlike FIFO, pro rata fails even in principle as a fair procedure because it gives preference to those who are in a position to place larger orders over those who only place smaller ones. But this is a mistake. While pro rata might *seem* to favor larger traders over smaller ones, it in fact equalizes between limit order *contracts*. In a pure pro rata system, when the sum of 1000 contract-limit order at a given price is hit by a market order for 100 contracts, each contract is given a 0.1 weight. If a trader has a ten-contract limit order, they will receive one contract, if they have a one hundred contract limit order-ten contracts, five hundred-fifty contracts, and so on. The ratio of order placed to orders filled treats each contract as having equal weight as every other contract. While those who place larger orders do receive a larger portion of the fulfillment, this portion is strictly proportional to the size of their order. There is nothing in a pro rata matching system that inherently favors larger traders.[19]

Pro rata thus does not exhibit any in principled lack of procedural fairness. Pro rata, like FIFO, treats, at least in principle, all limit orders similarly, in that similarly sized orders are filled in similar proportion to their size. This treatment is also impartial in that it does not consider traders' circumstances beyond limit order size.

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[18] Ray Chanman, chairman of Transmarket Group, speculates that pro rata emerged as an informal norm on the trading floor because traders preferred pro rating a market order to trying to keep track of any queue among those traders who had a standing limit order. Pro rata also preempts the possibility of traders arguing and fighting over who was before who in the queue.

[19] A significant downside with pro rata arises when distributions raise the need to dealing with fractions. This is addressed in §4.

Additionally, pro rata has the advantage that it avoids the social costs of the speed race that FIFO incentivizes. Attempts to reduce one's latency in order to be able to beat the competition is not only costly to those engaging in it, it can be viewed as a misallocation of resources that does not in anyway promote the social good. Unlike FIFO, pro rata does not arrange limit orders in a queue, so speed is irrelevant for getting in the limit order queue. Pro rata avoids the speed arms race by offering a different matching system that has nothing to do with speed (in pure pro rata systems), and by removing the incentive to place limit orders quickly.

One might push back against the claim that speed does not matter for pro rata systems. Osipovich (2019) discusses an incident in which two firms in the Eurodollar market raced each other to have the biggest limit order, by iteratively placing and canceling orders, each time increasing the size of their orders by a few contracts. The data processing this required was so great, there was a fear on part of the CME that it would overload their systems. While this might seem like a case in which a pro rata system incentivized a speed race, in this particular case the race is not a speed race, but rather a 'size race' and is due to the CME's system which at the time filled the order of the largest order 10-20 microseconds before the next bid.[20] But this is not inherent in a pure pro rata system, rather an aspect of how the CME managed its matching system.

Nevertheless, even in a pure pro rata system, speed can still be important to some extent, though not for determining what limit orders get filled. Faster traders can cancel limit orders faster when market conditions change or when they no longer want their orders filled, they can place market orders faster, and they can get market data and process it faster in order to make more timely trading decisions. Nevertheless, what matters for getting limit orders filled in a pure

[20] This information is based on a report by Brandon Richardson.

pro rata system is the size of the limit order, and the role speed plays does not create the social adverse incentives that FIFO does. There is simply no financial gain to be had by getting limit orders in earlier than other traders.

Pro rata also has some additional advantages over FIFO from the perspective of the exchanges. Pro rata incentivizes traders to place limit orders even if they join in late, which increases market liquidity. This is a particularly important issue for products with low price volatility. Additionally, since there is no benefit to getting early into the queue, pro rata does not incentivize traders to layer the books—the practice of placing multiple orders at different prices with the intention of only getting one fulfilled—and sit on orders—the practice of entering a limit order and keeping it for an extended period of time (e.g. many days). This is especially a problem with big institutions that have a direction view for the day, and they simply want to get the best price for that direction. They can either load the book on the buy or sell side at multiple price levels in order to make sure that whatever the price; they will be first in line.

While pro rata has some advantages, just like we have reason to doubt that FIFO can adhere to the standards of procedural fairness in practice, a similar charge can be leveled against its pro rata alternative. The main disadvantage with pro rata is that as a matching system it suffers from a lack of completeness that prevents it from being implemented as a pure matching system on its own without appealing to a secondary matching rule. Slightly tweaking the earlier example, we can see that complexity is introduced if the numbers do not work perfectly, and often they do not. When a trader has a 100-contract limit order out of 1100 contracts, under pure pro rata, a market order for 100 contracts will result in the trader being entitled to 9.09 contracts.[21] Since contracts are assigned as discrete units, dealing with fractions requires a

[21] In a pure pro rata system 1000+100=1100. 100/1100=1/11. 1/11*100=9.09.

secondary matching rule. When matching contracts using pro rata, exchanges need to determine whether to round up or round down a partial contract. In a pure pro rata, this order will get 9 contracts filled. The exchange might then round up the fraction of the contract so that the trader will get one more contract filled, leading to a total of ten. Alternatively, the exchange might round down, leading to a total of nine contracts filled.

Whatever secondary matching rule an exchange applies in such cases, there is an incentive to place appropriately sized limit orders in order for the trader to maximize the likelihood of their order being filled. If the exchange rounds up, then the trader placing a limit order for 100 contracts has an incentive to split their order into individual lots to maximize the likelihood that they will receive all 10 contracts. If the exchange rounds down, the trader placing the limit order for 100 contracts has an incentive to place a single large order, and then there is an advantage to size in practice. Janeček & Kabrhel (2007) argue that an optimal trade strategy for trading on the Time Pro-Rata system (specifically the one introduced by Euronext.LIFFE in 2007 for the short-term interest rate futures contracts) involves a high degree of order splitting of incoming limits orders. In line with this theoretical prediction, Aspris et al. (2015) found empirical support for Janeček & Kabrhel's theoretical claim that there is indeed an increase in the proportion of single contract additions and cancellations. Ultimately, these secondary rules open up space for traders to manipulate the matching system in a way that undermines pro rata's procedural fairness.[22]

[22] Another problem with pro rata is that it incentivizes placing artificially bigger orders than one actually hopes to fill. Since under pro rata which orders are filled and how many contracts are filled depends on the size of the limit order, traders are incentivized to place orders that are significantly larger than what they actually want to get filled

4. The principled case for Random Selection for Service (RSS)

In the previous two sections I have argued that while FIFO and pro rata are fair procedures in principle, they fail to be fair in practice. FIFO fails to be fair in practice because the opportunity to get sooner into the queue is unequal and because technological limitations make it the case that the exchanges cannot in practice process orders on a genuine FIFO basis. Pro rata, on the other hand, suffers from a lack of completeness and cannot in practice be implemented in its pure form, requiring exchanges to go beyond procedural fairness and commit to potentially contested notions of substantive fairness. In this section I propose an alternative that is able to avoid these problems both in principle and in practice—RSS.

The need to to deal with a reminder and the resulting need for secondary rules can be avoided by turning to an RSS matching system. All RSS provides are probabilities, rather than actual contracts and contract fractions. As a result, when a trader's limit order, as in our previous example, entails them to 9.09% share of the market order, this does not mean 9.09 contracts. It means 9.09% *probability* for getting each contract filled. When dealing with probabilities we have no problem working with fractions, and no secondary rule is needed. Being able to completely sidestep the need for secondary rules results in a less manipulatable system as well as avoids the need to take a stand on what substantive fairness requires. The fact that an RSS system is directly focused on distributing chances evenly, and on fills only indirectly, is its core strength from the perspective of procedural fairness.

(Field & Large 2008). This negatively effects the function of the market as an information generating system, since the information available through prices and order sizes is inaccurate.

To illustrate this, suppose there are two limit buy orders, one for ten contracts—trader 1, and one for 90 contracts—trader 2. A 20-contract market sell order comes in. Under FIFO, it would go to whoever was first. If trader 1 was first, they would get their full 10, and the remaining 10 would go to trader 2. If trader 2 was first, they would receive all 20. Under pro-rata, trader 1 would get 2 contracts, and trader 2 would get 18 contracts. Both options are entirely deterministic. Under RSS, for *each* of the 20 contracts there is a 10% chance is goes to trader 1 and a 90% chance it goes to trader 2. For trader 1, each individual limit order *contract* has a 10% chance of getting filled. If this was repeated a large enough amount of times, we should expect that on average trader 1 gets 2 contracts and trader 2 gets 18 contracts, just like pro-rata. However, that is only on average for a large number of iterations. For any particular trade the numbers can range from 10 contracts to trader 1 and 10 to trader 2, to 0 contracts for trader 1 and 20 contracts for trader 2. The probability that trader 1 gets at least one contract filled is about .88, and the probability that trader 2 gets all 20 contracts filled is about .12. The probability that trader 1 gets all 10 contracts filled is only approximately 0.000007 (or about seven in a million), whereas the probability that trader 2 gets at least 10 contracts filled approaches 1.[23]

When traders place bigger limit orders what they are in essence doing is buying more lottery tickets. Unlike buying lottery tickets, though, merely placing a limit order is free. The limiting factor in the case of lotteries is the cost of the tickets, and no one ever complains that they won too much money. In the limit order case, the limiting factor is how many contracts end up getting filled. If it were always better to get more contracts filled, traders could place ever larger limit orders. But for most trading strategies, one only actually wants a limited number of

[23] I thank an anonymous reviewer for presenting this in terms of probabilities.

fills. The risk of getting more fills than one wants is what limits the size of the limit orders that traders place under RSS.

Under RSS, traders cannot be too cavalier because they do not know if their limit order will get filled to an extent much larger than they would have wanted. In this respect, RSS, like FIFO, dis-incentivizes limit order inflation, which is a problematic aspect of pro rata.

Both Haeringer & Melton (2020) and Budish et al. (2015) offer their own alternatives, both different from my RSS. Haeringer & Melton propose a random serial dictatorship (RSD) to randomize the order in which incoming limit orders are processed. They propose such a system as a way of overcoming the specific challenge that exchanges face when trying to fully implement a pure FIFO, which faces technological challenges due the way exchanges process incoming orders through multiple switches. While also a form of randomization, it builds on the basic FIFO mechanism, adding the randomization to the ordering of the limit orders, to then be serially filled in full. Such a randomization overcomes the technological challenges of arriving closer to a fair FIFO mechanism, but it does not overcome the problems this article has highlighted with even a pure FIFO system. My proposal, by contrast, randomizes the likelihood of each contract in all standing limit orders getting filled, and in this regard resembles more a randomized pro rata mechanism than Haeringer & Melton's randomized FIFO mechanism.

Budish et al. propose a market design in which all trade requests received during the same interval (e.g. 100 milliseconds) are treated as having arrived at the same (discrete) time, and are then distributed pro rata (so a hybrid FIFO pro rata matching system). RSS, by contrast, entirely moves away from any focus on the time when limit orders were submitted, and only focuses on the relative limit order size of all existing limit orders at the moment the market order was placed (more similarly to a pure pro rata system).

Most cases in which we find queues particularly appealing are bottleneck cases.[24] These are cases such as supermarket checkout, airport security, or car traffic, where there is a delay between an individual demanding the good and them receiving it because it is not possible to give each individual the good immediately. In the case of futures trading, usually once limit orders at a given price start getting filled in earnest, most limit orders get canceled as this is an indicator of the market going against them. In this way queues in futures markets are different from most queues we are familiar with. It is not merely about getting one's limit order filled, but rather getting it filled before others.

Exchange matching systems are better understood as giving rise to cases of scarcity. In futures markets there is always a scarcity in the form of market orders to match the limit orders at any given price. When that scarcity disappears, it is merely because the market has moved either up or down to establish a new spread between bids and asks, for which the limit orders at each face a scarcity of market orders to meet them. Lotteries are particularly well suited for dealing with scarcity, and there is a vast literature advocating relying on lotteries in cases of scarcity,

[24] Elsewhere (Hersch & Rowe, under review), we argue that we can distinguish between cases of abundance, scarcity, and bottleneck cases. A good is abundant if demand for the good can be satisfied by the supply. For abundant goods demand (D) ≤ supply (S). Cases of scarcity arise when the demand for the good cannot be met by supply. For scarce goods D>S. Between these two types of cases exists a third type of case—bottleneck cases. Bottlenecks introduce an element of time. Bottleneck cases look like cases of scarcity at t_0, and like cases of abundance at t_n. The distinction between cases of scarcity and bottlenecks underpins reasons to use lotteries in cases of scarcity and queues in bottleneck cases. Within this framework, market orders should be viewed as scarce goods, rather than bottleneck goods. As such, for more general reasons that are beyond the scope of this article, lotteries, or RSS, is a fairer way to allocate market order fills among those who have placed limit orders.

often arguing that lotteries are a substantively fair way to allocate scarce goods.[25] A lottery assigns potential recipients a chance of receiving a good that can be distributed equally even if the scarce good cannot. This position is called the "distributive view" of lotteries (Wasserman 1996); when a proportionate allocation of the good between potential recipients is not possible, a lottery is able to divide what *can* be divided, namely the chance of receiving the good.

Why is it fair to equalize probabilities and not something else? First, the chance of getting a limit order filled is directly related to the good traders want (their order filled) and can be given as a temporary replacement. Such a chance is valuable to a trader because although it does not guarantee the limit order will be filled, it does allocate a real likelihood that it will be filled at the timespan between placing a limit order and a market order coming in. Second, having a limit order filled is already on a probabilistic scale. If the market order is large enough as to fill all the limit orders in the queue, there is a 100% chance that a trader's limit order will be filled as well. So, a chance that one's limit order filled that falls below this will still be of value, albeit of lesser value than a 100% chance. This is because if a trader values a 100% chance of having their limit order filled, then a 50% chance of the order being filled will still be of value to the trader, albeit half as valuable, and so on. Third, as Broome (1984, p.40) argues, "If a good or bad cannot be distributed equally, it sometimes seems a good idea at least to distribute it randomly. Randomness appears to be a way of bringing some fairness into an inherently unfair situation."

Conducting a lottery every time a limit order is filled by a market order in in-person trading would have been overly complicated and burdensome to the extent that trading would have been substantially slowed, if not outright impossible. However, since the shift to electronic

[25] Authors who argue that fairness requires the allocation of a chance of receiving a good in cases of scarcity include (Broome 1984, Broome 1990, Kamm 1993, Kornhauser & Sagar 1988, Saunders 2008, Stone 2011, Sher 1980).

trading, utilizing lotteries through randomization has become possible (Tkacik 2003). The core of an RSS matching system is that limit orders are neither filled based on a queue, nor based on a pro rata division. Instead, whenever a market order is placed, which limit-order contracts get filled is determined through a randomization process. All limit order contracts receive equal weight and probability of getting filled. Under RSS one does not merely get a proportionate partial limit order filled, but rather gets a proportionate chance of getting some of their limit order filled. Sometimes that will be nothing, sometimes some small part of their limit order, and other times it will be the whole thing.

Like FIFO and pro rata, RSS can also be considered a fair procedure in principle. RSS treats all limit order similarly, in that similarly sized orders are given the same chances of getting filled in similar proportion to their size. As I discuss in the next section, unlike FIFO and pro rata, RSS is also a fair procedure in practice.

5. The practical case for RSS

There are good reasons to think that, just like FIFO and pro rata, RSS is procedurally fair in principle. However, in this section I argue that it is also procedurally fair in practice. I discuss several practical considerations that could be raised against implementing RSS in a real-world context, one technical, one psychological, and one political, and argue that none of these give reason to doubt RSS's procedural fairness in practice.

First, while creating the semblance of randomness effectively enough to trick a human observer is easy enough, more genuine randomness in much more difficult. Generating anything approaching genuine randomness would require substantive computing power. Exchanges generally seek to minimize the computing power they employ, since computing power involves costs. Any attempt at shortcuts by exchanges with respect to randomizing opens the door for those traders with sufficient resources to employ algorithms that take advantage of the lack of complete randomness in the system. If there is money to be made by figuring out what aspect of the system is not completely random, exchanges should assume that someone will do so.[26] If traders manage to take advantage of this randomness shortcoming, then the practical advantage of RSS over the alternatives is lost.

One response is that while it might be difficult or costly to generate true randomness, generating true FIFO also faces substantive difficulties (Rao et al. 2020, Haeringer & Melton 2020). More importantly, exchanges should avoid shortcuts in making sure that their matching system is indeed random. Pseudo random number generators (PRNGs) seem to be sufficient for our purpose and need not be susceptible to adversarial players (Sunar et al. 2007). But even if PNRGs are not enough, hardware random number generators that are commonly used in cryptography offer a straightforward response to the randomness generation worry (Tkacik 2003).

A second worry is that the fact that the RSS process is randomized and non-deterministic might be unappealing to traders in and of itself. Given the varieties of uncertainties that anyone participating in the market faces, some might consider the non-deterministic nature of randomization as introducing further unnecessary and unwelcome uncertainty, even if the uncertainty is about something else. John & Millum (2020) explain that "allocating scarce

[26] Consider some strategies in the pit days, where despite there being the informal FIFO or pro rata systems, being a big guy, being loud, wearing colorful jackets, all helped in getting attention and circumventing the norms. That these behaviors/traits were adopted demonstrates that when it was possible to skew things in one's favor, traders would do so.

resources on the basis of waiting time optimizes distribution equality" (195). Queues in general, and FIFO in particular, have the advantage that they minimize the standard deviation of expected waiting time in a queue. It is generally reasonable to assume that consistency and predictability are valued by most people queuing. Futures markets specifically, according to the 'traditional price-insurance' theory, act as a form of price insurance to reduce uncertainty. This view goes back to the first days of the Chicago Board of Trade, the first to list a futures contract in 1868. As Levy (2012) writes:

> Corporations like the Chicago Board of Trade thus centralized, systematized, and socialized risk. This was a new argument [at the time]. Organized commodities futures exchanges first mounted an explicit social defense of financial speculation. Speculation was risk management. (p. 249)

Johnson (1960) explains that on the traditional theory dealers in 'actual' commodities who desire 'insurance' against the price risks they face turn to futures markets to hedge their risk, whereas speculators assume the risks that hedgers wish to transfer. Johnson notes that "[t]he futures market is visualized as a convenient mechanism through which price risk can be transferred from one group to another" (p. 140). Ultimately, futures markets enable traders to either increase or decrease their exposure to risk in an organized setting (Hawtrey, 1940, p. 203). Introducing uncertainty by randomizing what limit orders get filled could be viewed as undermining the risk-reduction motivation for hedgers to enter futures markets to begin with.[27]

While RSS adds another aspect of uncertainty, this worry misses the mark. It confuses the uncertainty hedgers wish to avoid about fundamentals (and thus the price) and the uncertainty coming from RSS as an allocation mechanism. For hedgers, futures explicitly reduce uncertainty about prices, something that they are indeed willing to pay a premium for in the form of the profits speculators enjoy as market makers. But uncertainty about whether a limit order will get

[27] This is also discussed in (Hersch 2020).

filled is of a different kind and arises for all matching systems, since it is never guaranteed that there will be sufficient market orders to match the standing limit order at a given price. When hedgers place limit orders rather than market orders, they are acting as de facto speculators by assuming risk that their order will go unfilled in any matching system they operate in. Insofar as hedgers wish to reduce uncertainty about fills, in any matching system, they simply need to place a market order.[28]

Third, any change in how exchanges operate results in winners and losers. Some trading algorithms that work well under other matching systems might not be as successful under RSS. Traders whose interests would be harmed by converting to an RSS matching system could lobby the exchange to prevent such a change from occurring.[29] Even if a matching system is imperfect, changing one that works is not necessarily an appealing option for fairly risk-averse publicly traded established exchanges. Furthermore, as Budish et al. (2019) argue, exchanges earn economic rents from the arms race for speed, and so it might be in their interest to maintain a FIFO style status quo. As a result, established exchanges would not find RSS practically appealing.

The same cannot be said for upstart exchanges that intend to act as market disrupters. RSS could be attractive to new exchanges that wish to differentiate themselves from their more established competition. More importantly, the Commodity Futures Trading Commission

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[28] I thank an anonymous reviewer for helping me with this response.

[29] Examples of such lobbying with the CFTC in regards to introducing a speedbump by ICE see 'Comments for Industry Filing IF 19-001' https://comments.cftc.gov/PublicComments/CommentList.aspx?id=2946, or with the SEC in regards to denying the use of speedbumps by the CBOE see 'Comments on CBOE EDGA Rulemaking' https://www.sec.gov/comments/sr-cboeedga-2019-012/srcboeedga2019012.htm.

(CFTC), which is the governmental agency in charge of overseeing futures trading, could unilaterally mandate the use of RSS. If the CFTC finds an RSS matching system to be superior to the alternatives, it is within its powers to mandate that exchanges use RSS as their matching system, thus deeming various exchange-level concerns moot.

Even if one accepts that in practice RSS can fairly allocate contracts among those racing to place limit orders, they might be concerned that RSS provides no help when market conditions change and there is race between those wishing to cancel their current limit orders and traders wishing to place market orders to fill those limit orders first before they get canceled.[30] However, while this might appear to count against RSS, neither standard FIFO and standard pro rata fare any better at minimizing a speed race in such a context. None of the matching systems examined here either aim to make such races fairer or have resources to do so. Moreover, profits from speed in the race to place limit orders is estimated to be at least an order of magnitude higher than profits due to the race between those wishing to cancel current limit orders and traders wishing to place market orders to fill those limit orders before they get canceled (Farmer & Skouras 2012, 7). One solution, suggested by Howorka et al. (2020) is to prioritize cancel orders. Indeed, insofar as mechanisms like Howorka et al.'s are meant and able to address fairness concerns in these contexts, there is nothing in RSS (nor in FIFO or pro rata), that precludes applying batch auctions or latency floors on top of one of these matching systems insofar as it is useful in addressing other fairness concerns.

Speed does remain a relevant factor among those aiming to integrate new information to adjust trading strategies and enable traders to place market orders to fill soon to disappear limit

[30] Howorka et al. (2020) describes these as three races, separating the second into two distinct races. These are described as maker-maker, maker-taker, and taker-taker races. I thank an anonymous reviewer for raising this concern.

orders. Yet speed is not in itself an evil to disincentivize wherever traders find it profitable to invest in. If financial markets reduced all motivation for speed in all aspects of trading, it would result in temporally inefficient markets. When market conditions change, speed in adjusting to said changes is not inappropriate. From the perspective of fairness, speed is only a problem insofar as it increases unfairness, something not inherent to speed itself.

6. Conclusion

Both FIFO and pro rata have historical roots dating back to open outcry trading. In faster moving markets FIFO was the norm, while less volatile markets tended to operate under a pro rata system. The specifics of these matching systems were not enforced by the exchange and allowed sufficient flexibility for each pit to self-optimize its way of working. To a large extent, when it came to the process of matching market orders with limit orders, exchanges allowed informal social norms to develop. When exchanges shifted from pit trading to electronic trading, what happened was a shift from informal matching norms that are socially enforced to formalized norms that are enforced by an authority (MacCormick 1998). The move to electronic trading required exchanges to take on an active role in determining how limit orders were to be filled.

While FIFO is a fair allocative procedure in principle, it fails to be procedurally fair in practice because of technical challenges and because traders do not all have an equal opportunity to enter the queue quickly. Pro rata, while also procedurally fair in principle, suffers in practice from a lack of completeness. With the benefit of hindsight and the privilege of time, we can ask whether there are any matching systems that might offer a fairer procedure in practice than either

FIFO or pro rata. In this article I argued that RSS would be a fair matching system procedure in principle as well as in practice.

The argument in favor of an RSS matching system is a special case of a more general argument related to the allocation of scarce goods when there are no morally relevant differences between potential recipients.[31] In another article, (Hersch & Rowe, under review), I develop a framework for distinguishing between cases in which queues (such as FIFO) should be used and cases in which lotteries, or random allocation (such as RSS) should be used. Exchange matching systems, and the type of scarcity they involve, make contracts the type of good that ought to be distributed through lotteries rather than through queues, because attempting to fairly distribute waiting times when only a subset of those waiting to get their limit order filled will get it entails that other traders will, in effect, be waiting forever. A lottery, by contrast, ensures the fair distribution of a desirable secondary good when it is not possible for all the limit orders to be filled—the chance of getting one's limit order filled.

RSS would have been prohibitively difficult to implement during the era of open outcry. However, now that trading is conducted electronically, RSS is attainable. Generally, technological advancements have opened up possibilities that contribute to more fairness. This does not mean that technological developments always lead to fairer allocations, and sometimes

[31] The assumption of a lack of morally relevant differences is central to this framework. Many times, such differences exist (Fumagelli 2021), as might be the case when two individuals requiring a kidney transplant can differ along a variety of aspects that might be morally relevant, e.g. whether they were heavy drinkers, their likelihood of relapsing, the likelihood of surgery success, age, social status, etc. In exchange matching systems, one might reasonably argue that some differences between traders are morally relevant to whether their limit order gets matched, one of these could be whether they command sufficient resources to allow them lower latencies.

they contribute to a reduction in fairness. But they do increase the *possibility* that we develop fairer procedures. Examples of technological developments that seem to have led to fairer procedures include the National Resident Matching Program that matches over thirty thousand medical students with residency spots, universities that employ electronic course enrollment systems to distribute access to popular courses, and, more recently, the states-level electronic allocation systems that were used to distribute COVID-19 vaccines to those who need them.

The case of exchange matching systems is no different. Electronic trading technology makes it possible to have a fairer procedure for matching fill orders than open outcry allowed. Overcoming path dependency is never easy, as the fact that I have typed this article on a QWERTY keyboard demonstrates (see (David 1985)). However, the downside of continuing to do things just because that is how they were always done is not a sufficiently good reason to avoid adopting a fairer matching system.

While this article makes headway in introducing a viable alternative to the standard approaches to matching systems that have dominated the electronic futures trading industry since it first developed, it leaves unanswered questions regarding different ways to operationalize specific RSS mechanisms, with each potentially having different strengths and weaknesses. Nevertheless, it does offer a step in the right direction.

7. References

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