Representational indispensability and ontological commitment

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Much recent work in the philosophy of mathematics is guided by the view that Platonism's prospects depend on mathematics' explanatory role in science. If mathematics plays an explanatory role, and in the right kind of way, this carries ontological commitment. Conversely, this thought goes, if mathematics merely plays a representational role then our world-oriented uses of mathematics fail to commit us to mathematical objects. Against this assumption, I argue that our representational practices *prima facie* carry ontological commitment.

In section one I set out the dialectical context. In section two I provide what I take to be our current best theory of mathematical representation. In section three I argue that this theory looks ontologically committing twice over, opening up an argument for Platonism that grants that mere quantification does not entail ontological commitment *and* that mathematics only ever plays a representational role. In section four I distinguish this from superficially similar arguments and explore the prospects for nominalist responses to this representationalist argument.

1. Ontology, explanation and representation

The once-dominant Quine-Putnam indispensability argument is widely taken to be unconvincing, on the basis of its reliance on a faulty confirmational holism.[1] This argument's reliance on a link between quantifying over a (term that purports to refer to) an object and commitment to that object is broken by our appeal to idealized, or fictional, objects and by the fact that holism does not cohere with scientific practice (Maddy 1992; Maddy 1997; Leng 2002; Leng 2010). There is consensus amongst both Platonists and nominalists that in order to discern what our world-oriented uses of mathematics ontologically commit us to, we must look at *what* we do with mathematics and *how* we do it. Do mathematical objects have to exist for us to be able to do what we do by quantifying over (terms that purport to refer to) them?

On a recent Platonist line, one of the things we do with mathematics is explain. In particular, we use mathematics to explain *non*-mathematical phenomena, in much the same way that unobservable physical objects are posited to explain such phenomena. Those who are realists about unobservable physical entities commonly appeal to our explanatory practices that involve them – such entities feature in our best explanations, and this gives us reason to affirm their existence. Platonists press that there are cases of mathematical entities featuring in our best explanation of some physical phenomena. So, inference to the best explanation (endorsed in the arguments for realism about unobservables) can be deployed to infer to the existence of mathematical objects. This move is what leads to the enhanced indispensability argument (EIA) (Baker 2005; Baker 2009; Baker 2016; Baker & Colyvan 2011; Bangu 2012; Lyon & Colyvan 2008: Lyon 2012). As Baker, arch explanationist Platonist, says:

> Despite their opposing sympathies, both authors [Melia, a nominalist and Colyvan, a Platonist] agreed that it is not enough – for the purposes of establishing platonism – that mathematics be

[1] The content of confirmational holism and its role in the Quine-Putnam argument is subject to debate, which I bracket here (Morrisson 2011; Field 2016).

> indispensable for science; it has to be indispensable *in the right kind of way*. Specifically, it needs to be shown that reference to mathematical objects sometimes plays an *explanatory role* in science. (Baker 2009: 613)

One natural nominalist response is to claim that mathematics plays a more limited role in our scientific theories than talk of unobservable physical objects: whilst unobservable physical objects *explain*, the role of mathematics in our scientific theories is non-explanatory. One way of fleshing out this picture is to think of mathematics as something like a language, or framework, in which a theory's content is stated. Much as we might express a proposition or thought by using a natural language, we express content about the world using mathematics (Field 2016: P-33). A second way of understanding this role is to appeal to the notion of indexing (Daly & Langford 2009). On this line, mathematics plays the role of pointing to physical facts, or to parts of the non-mathematical world. This contrasts with the role played by physical objects, which do more than index: they constitute facts, stand in causal relations, and explain why bits of the world are as they are. A third way of understanding the role is by characterising it as representational (Saatsi 2011). On this way of understanding the role, whilst unobservable physical objects stand in causal and explanatory relationships to the world, the mathematics stands only in representational relationships. There is reason to think that these are ways of coming to understand the same phenomena: indexing seems to have much in common with denotation, which in turn has much in common with the idea of mathematics being used as a language with which non-mathematical facts are expressed (in as much as we take the words of a language to denote their referents) – it is standard, also, to talk of language as representing the world. No matter how this notion is spelled out, there seems to be agreement that *if* mathematics' power is merely expressive (rather than additionally being explanatory), then the use of mathematics in science fails to be ontologically committing. Hence, the focus on the possibility of explanatory mathematics.

Platonists and nominalists alike endorse the following assumption: if mathematics plays a merely representational (or expressive, or indexing) role, then our world-oriented uses of mathematics do not justify Platonism. Call this the *ontological innocence assumption*. The thought is that it is possible for mathematics to play a representational role even if mathematical objects do not exist (Liggins 2014; Melia 2000; Melia 2002; Saatsi 2011; Yablo 2001; Yablo 2012). This thought motivates the Platonists' attempts to find cases of explanatory mathematics and the nominalists' attempts to defend what *mathematical representationalism*, the view that mathematics only plays a representational role.[2]

Nominalists who combine the ontological innocence assumption with mathematical representationalism often accept that mathematics is representationally indispensable. Although Yablo is not, if we are being careful, a nominalist, he captures this aspect of the view well:

> Numbers enable us to make claims which […] we […] would otherwise have trouble putting into words. (Yablo 2002: 230)

For the representationalist nominalist, our ultimate account of the world may well necessarily be expressed mathematically but this lacks ontological ramifications.

Should we accept the ontological innocence assumption? It is difficult to find explicit defence of the assumption. This is perhaps explained by the focus on the EIA: in order to disarm the argument, the thought goes, it is sufficient to demonstrate that mathematics only plays a representational role. If mathematics is never explanatory, then the EIA can be rejected. Nevertheless, even *if* this strategy for rejecting the EIA can be vindicated, one may have lingering

[2] Mathematics also plays roles in facilitating inferences, generating predictions and so on. The mathematical representationalist should say that these uses are derivative, dependent on representation.

worries. Why think that it is safe to infer from the fact that mathematics only plays a representational role to the claim that our world-oriented uses of mathematics does not justify any form of Platonism? Demonstrating the truth of representationalism does not suffice for justifying *this* claim.

Take the following passage from Baker and Colyvan's discussion of Daly and Langford, who defend mathematical representationalism:

> According to [Daly and Langford's indexing account], mathematical modelling works in much the same way as map making or any other representational strategy. The basic idea is nicely illustrated in simple cases where mathematics is used to stand proxy for physical properties. The account works well in cases such as those Melia used to motivate it, several of which involve facts expressing distance relations, for example "a is 63 centimetres from b". The indexing strategy takes as its starting point the very natural thought that the above fact does not hold *in virtue* of the relation between *a*, *b* and the number 63; the fact in question is taken to hold by virtue of the spatial relationship between *a* and *b*, and this is all there is to it; this relationship is indexed by the number 63 but the number 63 does not enter into the relationship. (Baker & Colyvan 2011: 324)

Yablo expresses a similar sentiment to the one ascribed to Daly and Langford:

> The metaphysical issue of whether physical circumstances demand mathematical objects is to be distinguished from the representational issue of what it takes to *state* those physical circumstances. Numbers and functions might indeed be indispensable for this purpose. But so what? (Yablo 2012: 1013)

Let's assume that mathematical representationalism is true. On this line, all cases of applied mathematics are like the above, where 63 does not enter into the fact being expressed: only *a* and *b* and (physical) relations between them. But what about *the fact that* 63 plays a role in representing *a* and *b* and the relations between them? Does 63 enter into *this* fact, a fact about representation?

The facts expressed by the map (for example, a fact about two roads being parallel to each other) do not hold in virtue of the map (only in virtue of facts about the roads) just as, for the nominalist, the facts being expressed using mathematics hold only in virtue of how things are with the physical circumstances.[3] But what we are interested in is *the fact that* the map expresses this fact, even if the fact being expressed does not hold in virtue of the map. If pressed to explain the relationship between the map and the terrain being represented, we may tell some story about the map and the terrain being similar in various senses: it is these facts about the map and the terrain, and the relations between them, that makes it the case that the map represents the terrain and makes it the case that we can learn facts about the roads from studying the map.

If indexing or representing is supposed to be analogous in this sense, then the Platonist may claim that we are forced into saying that 63 plays the role gestured at in the above passages in virtue of some of the relations it stands in. The fact being expressed using 63 does not involve 63, but *the fact that* 63 is used to express this fact involves 63. It is important not to slip between these two facts. One is a fact about the world being represented by mathematics, the other is a fact about representation.

Some appear to find it so clear mathematics' representational capacity generates no ontological commitments that they do not offer a defence of the claim. Yet, it seems that the truth of the ontological innocence assumption seems to currently turn on vague analogies between mathematics and maps *and* on what objects must be like in order to stand in the kind of

[3] Apart from the heavy duty Platonist (who holds that magnitudes consist in a mathematical object being related to a physical object), this is also part of the Platonist picture. See Knowles 2015 for a recent discussion of heavy duty Platonism.

relationships involved in indexing and representing. In order to adjudicate in a way that is both enlightening and non-question begging, we need to know more about the relationships involved in indexing/representing. In virtue of *what* does 63 express the fact about *a* and *b* and distance relations? Can mathematics play this kind of role without the existence of mathematical objects?

In a recent intervention into the debate about mathematics' *explanatory* role, Saatsi presses the point that we can't know whether mathematics' explanatory role generates ontological commitment until we know more about how mathematics explains, if it indeed does (Saatsi 2016). The same lesson applies here: whether or not it is safe to make the ontological innocence assumption turns on how mathematical representation functions. What is needed to adjudicate this discussion is an account of mathematical representation.

2. Mathematical representation

Recent work on mathematical representation has been primarily descriptive: the aim is to get at a better understanding of how our mathematical representational practices work. Nevertheless, these current best theories of mathematical representation can be assessed for ontological commitment.

There are two prominent accounts of mathematical representation: Pincock's mapping account (Pincock 2004; Pincock 2007; Pincock 2012) and Bueno and Colyvan's inferential conception (Bueno & Colyvan 2011).[4] The extent to which the accounts are in genuine disagreement is a delicate question (one touched on briefly below), but they share a common core– and it is this idea that is relevant for the current purposes. I will refer to both accounts as "the mapping account(s)".

[4] This conception is also discussed in more detail in Bueno & French 2018.

The basic idea appealed to by both accounts is a simple one. Take a given example of a mathematical representation (say, the Lotka-Volterra equations). The mathematical vehicle as a whole is taken to denote the non-mathematical target system and parts of the vehicle are taken to denote parts of the target. In some minimal sense of representation *qua* denotation, the mathematical vehicle now represents the non-mathematical target. But what is interesting about the representation is not just that that the Lotka-Volterra equations *denote* or *are about* predator-prey populations, but the fact that we can learn about such populations using the equation – what is interesting is mathematical representation's status as epistemic representation.[5] According to the mapping account(s), the existence of a structure-preserving mapping between the mathematical and non-mathematical domains is what explains a mathematical representation's status as a faithful, or successful, epistemic representation. The mathematical vehicle qualifies as a (partially) faithful epistemic representation in virtue of there being a structural relation between the structure of the mathematical vehicle and the assumed structure of the non-mathematical target system. The standard notion of structure takes a structure S to be a pair, consisting of a set of objects D of the structure (or the domain or universe of the structure) and a set of relations extensionally defined on D.

Pincock sums this up when he says that we can identify the content of a mathematical representation by asking:

1. What mathematical entities and relations are in question?

[5] Consider the London Underground logo and a map of the underground system. There is a sense of 'representation' in which both the logo and the map represent the underground system in the same way – representation *qua* denotation. But there is a sense in which the map represents the system in a way that the logo does not. We can learn about the system by reasoning about the map, and so it is an *epistemic* representation – and because at least one of the inferences we perform using the map is an inference to a truth, it is a partially faithful epistemic representation. See Contessa 2007 for details.

2. What concrete entities and relations are in question?
3. What structural relation must obtain between the two systems for the representation to be correct? (Pincock 2012: 27)

Nguyen and Frigg note that this sort of core account of mathematical representation is implicit in the semantic view of theories and also suggest that the inferential conception also relies "crucially on mappings to and from target systems and mathematical structures, and in this sense is an advanced version of, rather than an alternative to, the mapping account" (Nguyen & Frigg 2017: 6). Eliding the two accounts may be too quick. There do appear to be substantive disagreements. For example, there is apparent disagreement about whether accuracy conditions for a representation should be explained in terms of the inferences that a user can perform or the other way around (Bueno & Colyvan suggest the former (Bueno & Colyvan 2011: 352), Pincock the latter (Pincock 2012: 28) and whether the kinds of structural relations that hold between the mathematical structure and the (assumed) physical structure can be captured by appealing to morphisms of various kinds or whether the relations are themselves mathematical in nature (again, Bueno & Colyvan suggest the former and Pincock the latter).

There's reason to think that many of the perceived disagreements between the two accounts result from the fact that Bueno & Colyvan discuss Pincock's earlier, simpler, presentation of the mapping account: he has since developed the account in Pincock 2012, in a way that accommodates many of the worries that Bueno and Colyvan have. This is reflected in the little that Pincock says about the relationship between the two accounts, spelling out his motivation for adopting his approach in terms of convenience (Pincock 2012: 28). Regardless, what we have is a minimal best theory of mathematical representation. It is this that should be assessed, in order to ascertain the truth of the ontological innocence assumption.

3. Returning to the ontological innocence assumption

Recall Yablo's statement of the assumption:

> Numbers and functions might indeed be indispensable for this purpose [representing physical circumstances]. But so what? (Yablo 2012: 1013)

We can now see what's what. Worryingly for the nominalist, it seems that mathematical representation comes with ontological commitments. It is committing *twice over*. Not only does our current best theory of mathematical representation, when read at face value, commit us to the objects (or structures) that populate the mathematical domain in question, it also seems to commit us to the existence of *another* mathematical object – the structure-preserving mapping between the two domains. Recall that best understanding of mathematical representation says that a mathematical representation successfully (or faithfully) represents some non-mathematical target if there is a structural mapping between the mathematical structure and the structure that the physical target is taken to instantiate. The holding of the morphism *accounts for* the success of the representation.

This opens the way to a Platonist argument that grants the nominalist *both* that mere quantification over mathematical objects is insufficient to generate ontological commitment *and* the controversial claim that mathematics only ever plays a representational role in science and is never explanatory. On this line, if we are to make sense of our representational practices using mathematics, it is necessary to posit objects to populate the mathematical domain and structure-preserving mappings between the (structures of) the two domains. The existence of mathematical objects falls out of our best account of mathematical representation. The Platonist can agree that (as Yablo says) numbers and functions are indispensable for the purpose of representing physical circumstances but demure in claiming that this purpose is not ontologically innocent. Indeed, it only seems to make sense if there are mathematical objects. This is a representationalist route to Platonism, sidestepping the (interesting though vexed) questions about mathematical explanation.

It is common ground that mathematics plays this representational role – indeed, it could hardly be denied. One route for the nominalist is to develop an account of mathematical representation that significantly differs from the mapping account. However, the notion of structural similarity is sufficiently central to the notion of mathematical representation that this is a difficult task. Another route is to argue that the mapping account is, in fact, ontologically innocent.

4. Nominalist responses

In this section I discuss routes that the nominalist might take in order to respond to the representationalist argument but suggest that work needs to be done to make good on them.

One thought is that the existence of structure-preserving mappings, at least, can be resisted by noting that these structural relations can be reconfigured in second-order logic, "and so they need not presuppose mathematical objects" (Bueno 2016: 2602). However, to assume that this is so is to assume that one of the remaining areas of seeming disagreement between the mapping account and the inferential conception has been settled: whether the relations between the mathematical domain and the physical domain are exhausted by morphisms of various kinds, or whether they will be relations that mathematics is required to state. It also raises the question of the ontological commitments of second-order logic: so, for the nominalist at least, the bump merely reappears elsewhere in the carpet.[6]

[6] It is for this reason, also, that it would not be straightforward to adopt the kind of stance familiar from the literature on fictionalism about models (see, paradigmatically, Frigg 2010) – for the account of mathematical representation also

Concerning the existence of the objects populating the mathematical domain, there is a response that is also, on closer inspection, lacking. Bueno and Colyvan claim that the inferential conception can be endorsed by nominalists (Bueno & Colyvan 2011: 366-367). They suggest that the nominalist can distinguish quantifier and ontological commitment and claim that "quantification is not enough for ontological commitment" (*ibid*), following Azzouni (Azzouni 2004).[7] The idea here is that the fact that our account of mathematical representation quantifies over the objects in the mathematical domain is insufficient for commitment (this is mere quantifier commitment, which comes apart from ontological commitment). This, of course, isn't the only way to flesh out the line that one can adopt the mapping account of mathematical representation whilst maintaining that the mathematical objects that make up the vehicle domain fail to exist – but, rather, is representative.[8]

No matter how the proposal is spelled out, there is a unified rejoinder. It would be a mistake for the Platonist to say that we are committed to mathematical objects merely because they are *quantified over* in our account of mathematical representation. This would be a retrograde step – we're now all agreed, so the story goes, that merely quantifying over an object does not secure ontological commitment. This justifies the focus on mathematical explanation and representation and the move away from questions about whether quantification over mathematical objects is indispensable *simpliciter*. The representationalist argument is instead an answer to the question that

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appeals to the existence of the *mappings*. Adopting fictionalism about the *mappings* also seems to strip the account of its ability to explain the faithfulness of a given representation.

[7] See also Bueno's suggestion along these lines (Bueno 2016: 2600)

[8] The ontological commitments of our world-oriented uses of mathematics are also discussed in Bueno & French 2018, in which the inferential conception is spelled out in more detail than in Bueno & Colyvan 2011. However, there the main discussion of ontological issues and the Platonism/nominalism debate concerns the enhanced indispensability argument and the prospects of defending mathematical representationalism (Bueno & French 2018: 156-173). When discussing mathematical *representation*, Bueno and French claim that "those who are inclined towards a nominalist understanding of mathematics, it is still possible to provide a fictionalist reading of the framework as well" (Bueno & French 2018: 193), and refer the reader to general discussions of fictionalist accounts of our world-oriented uses of mathematics. These, however, are susceptible to the response spelled out in this section.

arises from the rejection of holism: what do we do with mathematics and what would have to be the case in order for us to do it? According to the argument, what grounds the success of a faithful mathematical representation is the holding of a mapping between the (structures of) the mathematical and physical domains. In response to the nominalist, the Platonist can argue that denying the existence of the mathematical domain and the mapping rules out endorsing this story about how we are able to successfully represent the non-mathematical world using mathematics. Denying the existence of mathematical objects threatens to render our successful representational practices mysterious. The mere fact that our best theory of mathematical representation quantifies over mathematical objects ought to do no heavy lifting for the Platonist.

One final concern is that the argument is just the familiar argument that in asserting mixed mathematical-physical statements, we are committed to the existence of the mathematical objects involved. *This* argument can be undermined by, for example, arguing that one can utter a mixed mathematical-physical statement and then 'take back' the mathematical commitments of the statement (Melia's weaseling strategy (Melia 2000; Knowles & Liggins 2015)), or by endorsing the Azzouni line previously discussed, challenging the idea that the truth of such statements requires mathematical objects at all. However, the representationalist argument claims that, according to our best current theory of how mathematical representation works, it is the *existence* of a structural mapping between the objects of the two domains that accounts for the success of a given mathematical representation. Commitment to such objects cannot be walked back, or denied, without depriving us of our answer to the questions about how mathematics can represent.

5. Conclusion

The debate over mathematical ontology has entered the explanatory epicycle. This stage in the debate gets one thing right: that to assess the ontological commitments of applied mathematics, we must look carefully at what it is that we do with mathematics. However, it errs in thinking that explanation ought to be the focus. The Platonist and nominalist alike agree that mathematics plays a representational role. I have argued here that our best account of mathematical representation seems to require the existence of mathematical objects: both those that populate the vehicle domain and those structural relations that the mathematical and physical domain stand in. Demonstrating that this is not the case is an underappreciated task for those with nominalist inclinations.

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