Counterfactual knowledge, factivity, and the overgeneration of knowledge*

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Abstract

Antirealists who hold the knowability thesis, namely that all truths are knowable, have been put on the defensive by the Church-Fitch paradox of knowability. Rejecting the non-factivity of the concept of knowability used in that paradox, Edgington has adopted a factive notion of knowability, according to which only actual truths are knowable. She has used this new notion to reformulate the knowability thesis. The result has been argued to be immune against the Church-Fitch paradox, but it has encountered several other triviality objections. Schlöder in a forthcoming paper defends the general approach taken by Edgington, but amends it to save it in turn from the triviality objections. In this paper I will argue, first, that Schlöder's justification for the factivity of his version of the concept of knowability is vulnerable to criticism, but I will also offer an improved justification that is in the same spirit as his. To the extent that some philosophers are right about our intuitive concept of knowability being a factive one, it is important to explore factive concepts of knowability that are made formally precise. I will subsequently argue that Schlöder's version of the knowability thesis overgenerates knowledge or, in other words, it leads to attributions of knowledge where there is ignorance. This fits a general pattern for the research programme initiated by Edgington. This paper also contains preliminary investigations into the internal and logical structure of lines of inquiries, which raise interesting research questions.

1 Introduction

Antirealists, including idealists, verificationists and intuitionists, are of the view that all truths are knowable (Hart and McGinn, 1976; Hart, 1979; Dummett, 1991; Tennant, 1997). Call this the knowability thesis. The latter has been challenged by the so-called Church-Fitch paradox of knowability (Fitch, 1963), which starts from the aforementioned view and leads to the claim that all truths are known, which is

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utterly implausible. It has been questioned whether the concept of knowability that is used in the Church-Fitch paradox is a good one, since it is non-factive (i.e. knowable falsehoods are possible). It has been argued that our intuitive concept of knowability is factive (Brogaard and Salerno, 2006; Fuhrmann, 2014) or, in other words, that only truths are knowable. Moreover, Kvanvig (1995) has pointed out that, given that antirealists want to define truth as knowability, they can only accept a factive notion of knowability. In response, Edgington (1985), Fara (2010), Fuhrmann (2014) and Spencer (2017) have tried to defuse the Church-Fitch paradox by making use of a factive concept of knowability.[1] In what follows, I will focus on (Edgington, 1985).

In its simplest version, the idea of Edgington (1985) is that the antirealist thesis should be understood as the view that all *actual* truths are possibly known to be actually true. In symbols:

AK $A\phi \to \Diamond KA\phi$,

with A the actuality operator, $\Diamond$ the possibility operator and K the knowledge operator. The standard truth condition for possiblity sentences is the following (Kripke, 1963): sentence $\Diamond\phi$ is true at world w just in case ϕ is true at at least one world that is modally accessible from w. The modal accessibility relation is a binary relation on the worlds. The standard truth condition for knowledge sentences is the following (Hintikka, 1962): sentence $K\phi$ is true at world w just in case ϕ is true at all worlds that are epistemically accessible from w. The epistemic accessibility relation is a reflexive binary relation on the worlds. The standard truth condition for actuality sentences is the following (Kaplan, 1989): sentence $A\phi$ is true at world w if and only if ϕ is true at the actual world, w_0.

If one operates with the standard semantical truth conditions for those operators, then one can derive the disastrous conclusion that $A\phi \to KA\phi$ (Rabinowicz and Segerberg, 1994). Schlöder (2019) calls this the *simple triviality argument*. There is a technical solution for this problem: make use of a particular kind of two-dimensional semantics (Rabinowicz and Segerberg, 1994). Heylen (2020) argues that, even with this solution, there is another trivialisation issue lurking around the corner. One can derive in two-dimensional semantics from the hypothetical validity of AK and via the frame condition for AK that possibly every actual truth is known to be actually true. In other words, a consequence of Edgington's knowability thesis is the possible omniscience of actual truths. He argues that this consequence is very implausible as well. Consider, for instance, all the actual truths about all the real distances between all atoms in the universe. It follows from Edgington's thesis that all those are known in a single possible world.

Williamson (1987) raises another worry, namely: how can one have knowledge in a non-actual world about the actual world? After all, there is no causal contact

[1] Relatedly, Artemov and Protopopescu (2013) and Fischer (2013) have argued not for making knowability factive but for restricting the knowability thesis to certain classes of truths, namely so-called 'stable' and 'uniform' truths, respectively.

between worlds. Given that non-actual *de re* knowledge about the actual world is hard to make sense of, a natural suggestion is that knowledge in a non-actual world about the actual world is possible if the epistemic agents in that non-actual world have a description of the actual situation (i.e. an incomplete but correct description of the actual world). With α a description of the actual situation and with ϕ a sentence, Williamson considers two cases: one in which α materially implies ϕ and one in which α counterfactually implies ϕ. Let us discuss these in turn.

Taking up the first suggestion, one can reformulate AK to get the following (Schlöder, 2019):

α**K** $A\phi \to \Diamond K(\alpha \to \phi)$.

Williamson points out that, if α includes ϕ as a conjunct, then the knowability of ϕ in the form of $\Diamond K(\alpha \to \phi)$ amounts to the knowability of a trivial logical truth $((\alpha \wedge \phi) \to \phi)$. Here is another way to show that triviality lurks around the corner. Knowability is necessarily factive: if α is a description of the actual world and if $\Diamond K(\alpha \to \phi)$, then ϕ (is true in the actual world). In other words, it is a truth that

$$\Box(\Diamond K(\alpha \to \phi) \to (\alpha \to \phi)).$$

Given modal axiom scheme K, one can then derive that:

$$\Box\Diamond K(\alpha \to \phi) \to \Box(\alpha \to \phi).$$

Given modal axiom scheme 5, one can subsequently infer that:

$$\Diamond K(\alpha \to \phi) \to \Box(\alpha \to \phi).$$

So, if the knowability of ϕ based on the description α is necessarily factive, then α strictly implies that ϕ.[2] Note that, if the $\Diamond$ operator in αK is interpreted in a very weak sense (e.g., as a logical possibility operator), then the $\Box$ operator has to be interpreted in a corresponding very strong sense (i.c., as a logical necessity operator). Of course, it makes sense to interpret the $\Diamond$ operator in αK in a very weak sense, because it makes it harder to come up with counterexamples.

Taking up the second suggestion, we get the following principle (Schlöder, 2019):

α**K'** $A\phi \to \Diamond K(\alpha \mathrel{\Box\!\!\to} \phi)$,

with $\mathrel{\Box\!\!\to}$ the counterfactual conditional operator. The standard truth condition for counterfactual sentences is the following (Lewis, 1973): sentence $\phi \mathrel{\Box\!\!\to} \psi$ is true at world w just in case all ϕ-worlds closest to w are ψ-worlds. Principle αK' is more than a theoretical option for a defender of AK to deal with the problem of non-actual *de re* knowledge about the actual world, something along the lines of αK'

[2] One can show similarly that, if $\Diamond K\phi$ is supposed to be necessarily factive, then $\Box\phi$ follows from it.

can be extracted from (Edgington, 1985). When discussing the application of AK to examples Edgington (1985) has done two things. First, she uses a counterfactual conditional instead of a material conditional: the knower in a possible situation knows that, if α had happened, then ϕ would still be the case. Second, she has a proposal for how to specify α: α is the negation of whatever has been done in that possible situation to acquire knowledge that ϕ. A key example given by Edgington (1985, p. 565) is the following:

> A comet is returning shortly. The comet is in the process of breaking up, and this will be our last chance to observe it. A spacecraft is being sent to investigate it and collect samples, in the hope of providing answers to certain questions, for example, does the comet contain pre-biotic molecules? Suppose the true answer to this question is *p*; and suppose, for simplicity, that if everything goes according to plan, *p* will be known — either they will find them, or they will gather information which rules out there being any.

She continues by claiming that:

> If [the mission succeeds and it is known that *p*], it will be known that, had the mission failed, [...] it would never have been known that *p*.

If we focus for now only on the replacement of the material conditional in αK by a counterfactual conditional, then we end up with something in the vicinity of αK'. However, αK' is also subject to the same trivialisation issue. Williamson points out that, if α includes ϕ as a conjunct, then the knowability of ϕ in the form of $\Diamond K(\alpha \mathrel{\Box\!\!\rightarrow} \phi)$ amounts to the knowability of a trivial logical truth $((\alpha \wedge \phi) \mathrel{\Box\!\!\rightarrow} \phi)$. Again, one can also show that triviality lurks around the corner by assuming that knowability is necessarily factive: if α is a description of the actual world and if $\Diamond K(\alpha \mathrel{\Box\!\!\rightarrow} \phi)$, then ϕ (is true in the actual world). By the similar reasoning as above, one can once again derive that $\Box(\alpha \rightarrow \phi)$. Given modal axiom scheme 4, one can next infer that $\Box\Box(\alpha \rightarrow \phi)$. In the Lewis-style semantics for counterfactuals, the following is valid

$$\Box(\phi \rightarrow \psi) \rightarrow (\phi \mathrel{\Box\!\!\rightarrow} \psi), \tag{1}$$

and is its necessitation. Hence, $\Box(\alpha \mathrel{\Box\!\!\rightarrow} \phi)$. Here it is also the case that, if the $\Diamond$ operator in αK' is interpreted in a very weak sense (e.g., as a logical possibility operator), then the $\Box$ operator has to be interpreted in a corresponding very strong sense (i.c., as a logical necessity operator). Of course, it makes sense to interpret the $\Diamond$ operator in αK' in a very weak sense, because it makes it harder to come up with counterexamples.

Edgington (2010) has tried to argue that one can have knowledge about the actual situation in a non-actual situation if the latter shares a causal history up to a point of departure with the former. But Schlöder (2019) argues this move does not solve the trivialisation issue either.

To sum up, the antirealist thesis that all truths are knowable has been threatened by the Church-Fitch paradox of knowability, which involves a non-factive concept of knowability. Edgington has tried to save the thesis by reformulating it with the help of a factive concept of knowability, but her version of the thesis is vulnerable to other triviality issues.

In the next section I will give a brief exposition of the proposals made by Schlöder (2019), who takes up Edgington's αK' and develops it to make it immune against the triviality issues. A key concept employed by Schlöder is the (un)successful pursuit of a line of inquiry. In Section 3 I will put into doubt the justification that Schlöder offers for the factivity of his notion of knowability, but I will also amend his justification to fix the problem. The problem and the fix take into account the internal structure of lines of inquiries. In Section 4 and Section 5 I will argue that Schlöder's reformulated antirealist thesis is open to the criticism that it generates too much knowledge, which is due to the existence of alternative lines of inquiries (satisfying certain conditions). In Section 6 I will briefly go into the logical structure of lines of inquiries. It will be argued that, although appeal to 'disjunctive' lines of inquiries are *prima facie* promising ways of getting around the problem of the overgeneration of knowledge, in some cases this will not work due to the problem of counterfactual *de re* ignorance about alternative lines of inquiry. Finally, in Section 7 I will summarize everything and I will also draw three general conclusions about what is (not) fruitful to research.

The first general conclusion puts the problem of overgeneration of knowledge for Schlöder's knowability thesis in a wider perspective as part of a type of problem faced by the entire research programme that was started by Edgington. The second general conclusion emphasizes the relevance of factive notions of knowability, of which there are few that are made formally precise and that are non-trivial. The third general conclusion is that the preliminary investigation of the internal and logical structure of lines of inquiries invites further research.

2 Schlöder on knowability and antirealism

Schlöder (2019) thinks that a further development of Edgington's idea will lead to a version of the knowability thesis that avoids the Church-Fitch paradox of knowability and also the triviality issues raised by Rabinowicz and Segerberg (1994), Williamson (1987) and Heylen (2020). The guiding idea for reformulating the knowability thesis is in Schlöder (2019, Section 2.3)'s own words the following:

(*) Given some actually true *p*, there is a course of inquiry *i* that is not successfully pursued, but were it successfully pursued, would impart the knowledge that had it not been (i.e. in actuality), *p*.

Schlöder (2019, Section 2.3) gives the following example to illustrate his guiding idea:

> the procedure i *to look at the ceiling* which, if successfully pursued, would impart the knowledge that *hadn't I looked, there (still) would be a fly on the ceiling* (Edgington, 1985).

The starting point for making this more precise is αK'. I will explain how to incrementally proceed from the latter to Schlöder's full proposal. Schlöder proposes to the restrict counterfactual knowledge to those worlds in which a certain counterfactual condition, β, obtains:

$\beta\alpha$K' $A\phi \rightarrow (\beta \boxright K(\alpha \boxright \phi))$

Next, one needs to determine what the β-condition is and what α is. Here Schlöder will take his cue from the suggestive remarks made by Edgington on how to specify α in her discussion of some examples. Let us use *Inq* for the set of all possible courses of inquiry. Furthermore, if $i \in Inq$, let us use $sp(i)$ to express that course of inquiry i has been successfully pursued. Schlöder's proposal is then that α and β in $\beta\alpha$K' are the following:

- $\beta : sp(i)$,
- $\alpha : \neg sp(i)$,

for at least one $i \in Inq$. Note, first, that $\alpha = \neg\beta$. Second, in case ϕ is *unknown* in the actual world, this implies that no course of inquiry has been successfully pursued. Under that condition, α does indeed give an incomplete (yet correct) description of the actual world. Third, note that the possibility operator $\diamond$ has been replaced by $sp(i) \boxright \ldots$. The counterfactual is true if there are no $sp(i)$-worlds, but then there is a tension with the presupposition that i is a *possible* course of inquiry. So, the truth of the counterfactual and the restriction of i to the possible courses of inquiry, implies that there is at least one $sp(i)$-world. In effect, $sp(i) \boxright \ldots$ functions then as a restricted possibility operator. The result of the proposed substitutions for α and β in $\beta\alpha$K' is the following:

$$A\phi \rightarrow \exists i \in Inq(sp(i) \boxright K(\neg sp(i) \boxright \phi)).$$

Let us focus for the moment on the consequent of the above. Maybe there is a line of inquiry that has been successfully pursued or maybe there isn't. For the knowledge of $\neg sp(i) \boxright \phi$ to be about the actual world, the actual world needs to be a world at which it is true that line of inquiry i was not successfully pursued. So, the consequent of AK would have to be reformulated as follows:

$$\exists i \in Inq(\neg sp(i) \wedge (sp(i) \boxright K(\neg sp(i) \boxright \phi)))$$

But of course, it might also be that a line of inquiry has been successfully pursued at the actual world and that it results in knowledge. Therefore, the consequent of AK has to be reformulated as follows:

$$K\phi \vee \exists i \in Inq(\neg sp(i) \wedge (sp(i) \boxright K(\neg sp(i) \boxright \phi)))$$

Moreover, Schlöder does not only want the left-to-right direction but also the other direction, because the antirealist thesis is that truth is *equivalent* to knowability. So, one would get:

$$A\phi \leftrightarrow (K\phi \vee \exists i \in Inq(\neg sp(i) \wedge sp(i) \mathrel{\Box\!\!\rightarrow} K(\neg sp(i) \mathrel{\Box\!\!\rightarrow} \phi)))\,.$$

Finally, Schlöder drops the actuality operator, so that one gets the following:

ART $\phi \leftrightarrow (K\phi \vee \exists i \in Inq(\neg sp(i) \wedge (sp(i) \mathrel{\Box\!\!\rightarrow} K(\neg sp(i) \mathrel{\Box\!\!\rightarrow} \phi))))$

The reasons for dropping the actuality operator are likely the following. First, the right-hand-side of ART does not explicitly mention the actuality operator. Second, one does not need the actuality operator for the right-to-left direction of ART, i.e. it is not necessary for the factivity of knowability. Consider the first disjunct on the right-hand-side of ART, $K\phi$. By the factivity of *knowledge*, it follows that ϕ. Next, consider the second disjunct on the right-hand-side of ART. Schlöder appeals to the following principle:

Symmetry For any formula ϕ and ϕ-world w and $\neg\phi$-world w', if w is the closest ϕ-world to w', then w' is the closest $\neg\phi$-world to w.

More generally, for any formula ϕ and ϕ-world w and $\neg\phi$-world w', if w is among the closest ϕ-worlds to w', then w' is among the closest $\neg\phi$-worlds to w. Now consider a world w at which, for some $i \in Inq$, it is true that $\neg sp(i)$ and that $sp(i) \mathrel{\Box\!\!\rightarrow} K(\neg sp(i) \mathrel{\Box\!\!\rightarrow} \phi)$. Let world v be among the closest-to-w worlds at which $sp(i)$ is true. Then $K(\neg sp(i) \mathrel{\Box\!\!\rightarrow} \phi)$ is also true at v. By the reflexivity of the epistemic accessibility relation, it follows that $\neg sp(i) \mathrel{\Box\!\!\rightarrow} \phi$ is true at v. By symmetry and by the assumption that v is among the closest-to-w worlds at which $sp(i)$ is true, it follows that w is among the closest-to-v worlds at which $\neg sp(i)$ is true. By the truth of $\neg sp(i) \mathrel{\Box\!\!\rightarrow} \phi$ at v, it follows then that ϕ is true at w. So, the truth of the second disjunct of the right-hand-side of ART also entails ϕ. Therefore, in both cases it follows that ϕ. In this reasoning we did not have to appeal to the semantics of the actuality operator.

Importantly, Schlöder claims that ART avoids both Williamson's and Heylen's triviality objections. Williamson's triviality objection does not work against ART for the following reason. As noted above, $sp(i)$ functions as a *restricted* possibility operator. It is not in general the case that all the worlds in a model are worlds at which $sp(i)$ is true. The right disjunct of the right-hand side of ART entails that $K(\neg sp(i) \mathrel{\Box\!\!\rightarrow} \phi)$ and, hence, $\neg sp(i) \mathrel{\Box\!\!\rightarrow} \phi$ is true relative to all the $sp(i)$-worlds that are the closest-to-a-given world. This leaves open that there are other worlds in the model at which $\neg sp(i)$ and $\neg\phi$ are both true. Therefore, it is left open that there are worlds at which $\neg sp(i) \mathrel{\Box\!\!\rightarrow} \phi$ is false and, hence, $\Box(\neg sp(i) \mathrel{\Box\!\!\rightarrow} \phi)$ is false. Thus, it can be that $K(\neg sp(i) \mathrel{\Box\!\!\rightarrow} \phi)$ is possible in a restricted sense, while $\Box(\neg sp(i) \mathrel{\Box\!\!\rightarrow} \phi)$ is false. This crucially depends on $\neg sp(i)$ and $\neg\phi$ being compossible.

The condition that a line of inquiry not being successfully pursued can coincide with the falsity of a sentence that is a possible topic of inquiry is commonly satisfied. Let ϕ be expressing that there are cookies in the cupboard in the kitchen. And let the line of inquiry be walking into the kitchen, opening the cupboard, and looking into the cupboard and perhaps moving things around. Clearly, the line of inquiry can be unsuccessful (maybe one of your children is crying and you go and check on the child instead), while there are no cookies in the cupboard. Yet, sometimes the condition is not satisfied, notably if $\phi = \neg sp(i)$. In that case $\neg sp(i)$ and $\neg\phi$ ($= \neg\neg sp(i)$) are not compossible.

Schlöder's solution is to force compossibility by adding an extra clause to ART, namely

$$\neg\Box K(\neg sp(i) \to \phi)\,.$$

If $\neg\Box K(\neg sp(i) \to \phi)$ is true at a world w and if all worlds in W are modally accessible, then it follows that there is a $w' \in W$ such that $\neg K(\neg sp(i) \to \phi)$ is true at w', whence it follows that there is an epistemically accessible world $w'' \in W$ such that $\neg sp(i)$ and $\neg\phi$ are true at w'', which means that $\neg sp(i)$ and $\neg\phi$ are compossible. The reformulated antirealist thesis is then the following:

ART'

$$\phi \leftrightarrow (K\phi \vee \exists i \in Inq(\neg sp(i) \wedge \neg\Box K(\neg sp(i) \to \phi) \wedge (sp(i) \mathrel{\Box\!\!\rightarrow} K(\neg sp(i) \mathrel{\Box\!\!\rightarrow} \phi))))$$

This is then the final proposal in (Schlöder, 2019).

The antirealist thesis ART' does not succumb to the problem of possible omniscience (Heylen, 2020) either. There is an important preliminary point to be made here, to wit, there are plausibly no "super" procedures that can impart knowledge about everything. To repeat a previous example, given that there is actually a fly on the ceiling, there is the procedure to look at the ceiling which, if successfully pursued, would impart the knowledge that hadn't I looked, there (still) would be a fly on the ceiling. However, given that there is actually an ant on the floor, the procedure to look at the ceiling will not impart knowledge that hadn't I looked, there (still) would be an ant on the floor. The more coarse-grained procedure to look at something (in the room), whichever that is, can be successfully pursued by, for instance, looking at the floor, without imparting knowledge that, if I hadn't looked at anything, there (still) would be a fly on the ceiling. We will have to make do with multiple procedures that each can yield only limited knowledge.

There is no possible omniscience if and only if, for each of the courses of inquiry, at the worlds closest to the actual world at which one of those courses of inquiry has been successfully pursued, there is at least one sentence for which it is not known, that had that course of inquiry not been successfully pursued, it would (still) have been true. Let us say that w is the actual world and that w' is among the closest-to-w worlds where line of inquiry i has been successfully pursued. Furthermore, let's say that ϕ is some sentence (that is actually true but unknown). Then there has to a possible world, w'', that is epistemically accessible from w and it has to be false at w'' that, if i had not been successfully pursued, then it would (still) be the case that ϕ. Therefore, there has to be is at least one world w''' that is among the closest-to-w'' worlds at which i has not been successfully pursued and that is distinct from the actual world w (for, otherwise, ϕ would have to be true at w'''). Then there is no formal restriction on making ϕ false at w''' (assuming that $\neg sp(i)$ and ϕ are logically independent). Therefore, it is not a formal consequence of ART' that there is possible omniscience. To come back to the preliminary point, i is the procedure to look at the ceiling, w is a world in which there is an ant on the floor, whereas w''' is a world in which there is no ant on the floor.

A final remark is that the arguments for possible omniscience advanced by Heylen (2020) all make use of properties of the frame conditions of knowability theses such as Edgington (1985)'s AK. However, as Schlöder (2019, Section 2.3), points out: his knowability thesis is formulated with the help of counterfactuals, which do not have frame conditions. So, the argumentation strategy developed by Heylen cannot get traction.

So, ART' is safe from the issues pointed out by Williamson and Heylen. In the next two sections I will discuss two new problems, each related to one direction of his antirealist thesis. The right-to-left direction of that thesis says that only truths are knowable in Schlöder's sense, whereas the left-to-right direction of that thesis says that all truths are knowable in Schlöder's sense. The first problem pertains to right-to-left direction of ART' and is discussed in Section 3. The second problem pertains to the left-to-right direction of ART' and is discussed in Section 4 and Section 5.

3 The factivity of knowability

It is important that Schlöder's concept of knowability is factive or, in other words, that the right-to-left direction of ART' is true. As we have seen, the justification for this depends on Symmetry. However, there are reasons to doubt Symmetry.[3] Suppose that worlds correspond with the length of a particular person a. So, person a is 180 cm tall in world w_{180}, 179 cm tall in world w_{179} and 178 cm tall in world w_{178} (perhaps due to malnutrition in that person's childhood). Let ϕ express that the length of a is at least 180 cm. Naturally, the closest-to-w_{178} world at which ϕ is true is w_{180}, because the difference in length of a in w_{178} on the one hand and w_{180} on the other hand is the smallest for any of the ϕ-worlds. However, the closest-to-w_{180} world at which ϕ is false is naturally w_{179}, not w_{178}, because the difference in length of a in w_{180} on the one hand and w_{179} is the smallest for any of the $\neg\phi$-worlds. The Symmetry principle is confronted with a problem with formulas that contain gradable predicates. Of course, Schlöder uses Symmetry only for $\phi = sp(i)$, so the question is whether Symmetry restricted to those kind of formulas is confronted with a similar problem. I will argue that it is.

[3]The following example was given by Lorenz Demey and roughly the same example was given by an anonymous reviewer.

Consider a line of inquiry that has only been successfully pursued if a number of steps have consecutively been executed. A mundane example is to walk into the kitchen, to open the cupboard, to looking into the cupboard, to move things around and take them out of the cupboard, to look again. Even this homely example of a multi-step line of inquiry can be used to produce a counterexample to Symmetry.

Cookies Suppose that in the actual world the line of inquiry was not successfully pursued because something went amiss before even the first step could be executed. Before you are walking into the kitchen, one of your children is crying and you go and check on the child instead. Now consider a possible world w in which you have successfully pursued the line of inquiry. Arguably, the worlds that are closest to w in which the line of inquiry has not been successfully pursued do not include the actual world but rather the worlds in which something went amiss before the line of inquiry could be completed. For instance, consider a world in which you did walk into the kitchen, you did open the cupboard, and you did look at the objects in front of you, but you did not yet rummage through the cupboard to look at the objects hidden behind other objects, because in this world you hear the music of an ice cream van and you decide to abort your search for cookies and buy an ice cream instead.

Both the actual world and the world just described are worlds in which the line of inquiry has not been successfully pursued. However, it is clear that in the latter world, one has come closer to successfully pursuing the line of inquiry than in the former, and hence, the latter world is closer to w than it is to the former. So, if lines of inquiries are sometimes best conceived as step-by-step procedures, then one can object to restricted Symmetry because the pursuit of lines of inquires can in those cases be thought of as gradable.

Symmetry is problematic, but a tweak to Schlöder's account of knowability can guarantee factivity, even when we are acknowledging that lines of inquiries can involve a sequence of steps that all need to be executed. First, let us add vocabulary to express that successfully pursuing a line of inquiry is equivalent to successfully executing a certain number of steps. The predicate $se(i,n)$ expresses that step n of line of inquiry i has been successfully executed. Then we have the following:

$$\forall i \in Inq \exists n \in \mathbb{N}_{>0}(sp(i) \leftrightarrow se(i,n)) \tag{2}$$

Second, let us encode the idea that successfully executing a step in a line of inquiry is equivalent to successfully executing that step and all preceding steps.

$$\forall i \in Inq \forall n \in \mathbb{N}_{>0}(se(i,n) \leftrightarrow \forall m \in \mathbb{N}_{>0}(m \leq n \rightarrow se(i,m))) \tag{3}$$

Combining (2) and (3) gives us the following equivalences:

$$\forall i \in Inq \exists n \in \mathbb{N}_{>0}(sp(i) \leftrightarrow \forall m \in \mathbb{N}_{>0}(m \leq n \rightarrow se(i,m))) \tag{4}$$

$$\forall i \in Inq \exists n \in \mathbb{N}_{>0}(\neg sp(i) \leftrightarrow \exists m \in \mathbb{N}_{>0}(m \leq n \wedge \neg se(i,m))) \tag{5}$$

The above equivalences may suggest the following substitutions in the right disjunct of the right-hand side of ART or ART':

- replace $sp(i)$ everywhere by $\forall m \in \mathbb{N}_{>0}(m \leq n \rightarrow se(i,m))$ and
- replace $\neg sp(i)$ everywhere by $\exists m \in \mathbb{N}_{>0}(m \leq n \wedge \neg se(i,m))$,

with n the number of the last step that needs to be successfully executed for line of inquiry i to be successfully pursued. Unfortunately, that would not allow us to get factivity in a plausible way. Briefly, the reason is again that the worlds at which the line of inquiry has not been successfully pursued and that are closest to a world in which the line of inquiry has been successfully pursued are plausibly those worlds at which the penultimate step of that line of inquiry has not been successfully executed, whereas the actual world might be one at which an earlier step was not successfully executed. However, what would work is replacing $\neg sp(i)$ everywhere by $\neg se(i,m)$ in the right disjunct of the right-hand-side of ART or ART' and to give the restricted quantifier $\exists m \in \mathbb{N}_{>0} : m \leq n$ wide-scope. Here is the result for ART:

$$\exists i \in Inq \exists n \in \mathbb{N}_{>0} \exists m \in \mathbb{N}_{>0}(m \leq n \wedge \neg se(i,m) \wedge (se(i,n) \mathrel{\Box\!\!\rightarrow} K(\neg se(i,m) \mathrel{\Box\!\!\rightarrow} \phi)) \tag{6}$$

In addition, we need a more plausible symmetry principle. Here it is:

Symmetry-relative-to-steps For all $i \in Inq$, for all $n \in \mathbb{N}_{>0}$, for all $m \leq n \in \mathbb{N}_{>0}$, for any $se(i,n)$-world w and $\neg se(i,m)$-world w', if w is among the closest $se(i,n)$-worlds to w', then w' is among the closest $\neg se(i,m)$-worlds to w.

Clearly, the Symmetry-relative-to-steps principle does not succumb to the same type of objection as the Symmetry principle: a world in which all but the last step of a line of inquiry has been successfully executed is not even among the 'not-the-first-step-successfully-executed'-worlds that are closest to a world in which all steps of the line of inquiry have been successfully executed, whereas it would plausibly be among the 'not-successfully-pursued'-worlds that are closest to the latter world. Furthermore, the right disjunct of (6) in combination with the Symmetry-relative-to-steps principle entails ϕ, thus restoring factivity. I leave the details of the argument to the reader.

The solution proposed to the problem respects the spirit of Schlöder's account of knowability. It guarantees factivity. It relies on the symmetry-relative-to-steps principle. The latter is not vulnerable to the objection raised at the beginning of this section. Further refinements are possible. For instance, it is not necessary to work with a picture of all lines of inquiry being step-by-step procedures. But for those lines of inquiry that are step-by-step procedures and, therefore, are vulnerable to the kind of objection we started with, the solution outlined above seems to be promising. However, I won't be pushing this forward. Instead I want to focus on a problem for Schlöder's version of the antirealist thesis. In Section 4 I will lay the formal groundwork for the problem in the form of a theorem and in Section 5

I will put forward counterexamples to ART' that are based on the aforementioned theorem.

4 The overgeneration of knowledge: formal result

The attentive reader has probably noticed that I have not discussed Schlöder's analysis of Fitch sentences yet. What if ϕ in ART or ART' is a Fitch sentence, i.e. a sentence of the form $q \wedge \neg Kq$? It is claimed that it is consistent to claim that a Fitch sentence is knowable. According to Schlöder (2019, Section 2.2), in the following scenario a Fitch sentence is knowable:[4]

(Comet). Suppose there is a scientific experiment that can be performed exactly once, and failure to perform it results in that some knowledge remains unknown forever. To give a specific case, suppose there is a comet that is disintegrating, and we have precisely one shot at landing a probe on it to analyse its composition before it is gone. There is a chance of critical failure (say, the probe breaks on landing), which results in us never being able to analyse the comet (there is no time to send another probe before it disintegrates).

The analysis provided by Schlöder (2019, Section 2.3) is the following:

> As in (Comet), the knowability of a Fitch proposition $q \wedge \neg Kq$ can be witnessed by a course of inquiry i that imparts the knowledge that q without changing the truth value of q, plus the background knowledge that pursuing i is the only way to know q.

Here q describes the comet's composition and line of inquiry i consists in the probe landing on the comet, analysing its composition, and sending back the data.

Let us spell out Schlöder's analysis in a little bit more detail. Given that the course of inquiry i has imparted knowledge that q, the first condition is the following:[5]

$$sp(i) \boxright K(\neg sp(i) \boxright q). \tag{7}$$

Assuming that 'the background knowledge that pursuing i is the only way to know q' applies to those closest worlds where i has been successfully pursued, the second condition is the following:

$$sp(i) \boxright K(\neg sp(i) \boxright \neg Kq). \tag{8}$$

With the two conditions rendered formally, we are ready to prove some results, but first I will give an overview of what is to follow in this section.

First, it will turn out that (8) is a necessary condition for counterfactual knowledge of $q \wedge \neg Kq$ (Lemma 1). To repeat, the truth of (8) is based on 'the background

[4] This example is based on (Edgington, 1985, p. 565).

[5] I am assuming here that it is meant that the 'without changing the truth value of q' bit is within the scope of the knowledge that has been imparted by the course of inquiry i.

knowledge that pursuing i is the only way to know q', so line of inquiry i is a 'one shot'. However, in some cases, even in some very mundane cases, there is more than one line of inquiry that could have been successfully pursued to discover that q. Let's say that there is an alternative line of inquiry j. Furthermore, suppose that the following two conditions are met. If line of inquiry i were successfully pursued, it would remain the case that, if another line of inquiry, j, had been successfully pursued, then q would also be known. Moreover, if line of inquiry i were successfully pursued, it would be a close possibility that line of inquiry i wouldn't be successfully pursued but that line of inquiry j would be successfully pursued. Under those conditions, (8) is false (Lemma 2). It turns out that one can also derive the falsity of (8) under two other conditions, one of which is stronger and one of which is weaker. So, if there is an alternative line of inquiry j and some conditions are met than there is no counterfactual knowledge of $q \wedge \neg Kq$. Combined with the famous Church-Fitch lemma (Lemma 4), this leads then to the unknowability of $q \wedge \neg Kq$ (negation of the right-hand side of ART') and, hence, the falsity of $q \wedge \neg Kq$ (negation of the left-hand side of ART'). It is this result (Theorem 1) that will be used in Section 5 and that leads to the overgeneration of knowledge in certain cases, in which next to a line of inquiry i there is an alternative line of inquiry j meeting the above conditions.

The first lemma says that (7) and (8) are jointly sufficient and individually necessary conditions for a conjunct of the right disjunct of the right-hand side of ART'.

Lemma 1. *The conjunction of (7) and (8) is logically equivalent to:*

$$sp(i) \mathrel{\Box\!\!\rightarrow} K(\neg sp(i) \mathrel{\Box\!\!\rightarrow} (q \wedge \neg Kq)). \tag{9}$$

Proof. This follows from the following theorems. First, there are the following two theorems from the logic of counterfactuals:

$$(\phi \mathrel{\Box\!\!\rightarrow} \psi) \wedge (\phi \mathrel{\Box\!\!\rightarrow} \theta) \models \phi \mathrel{\Box\!\!\rightarrow} (\psi \wedge \theta); \tag{10}$$

$$\phi \mathrel{\Box\!\!\rightarrow} (\psi \wedge \theta) \models (\phi \mathrel{\Box\!\!\rightarrow} \psi) \wedge (\phi \mathrel{\Box\!\!\rightarrow} \theta). \tag{11}$$

Second, there is the following theorem of counterfactual logic:

$$\text{If } \psi \models \theta, \text{ then } \phi \mathrel{\Box\!\!\rightarrow} \psi \models \phi \mathrel{\Box\!\!\rightarrow} \theta. \tag{12}$$

Third, there are the following two theorems from the logic of knowledge:

$$K(\phi \wedge \psi) \models K\phi \wedge K\psi; \tag{13}$$

$$K\phi \wedge K\psi \models K(\phi \wedge \psi). \tag{14}$$

□ □

What Lemma 1 makes clear is that the condition (8), namely the counterfactual knowledge that pursuing i is the only way to know q, is a *necessary* condition for the counterfactual knowledge of $q \wedge \neg Kq$ (the right disjunct of the right-hand side of ART'). But more often than not, the former condition will fail. There are quite realistic circumstances under which these conditions hold. In fact, there are two pairs of conditions under which it is false at a world that $sp(i) \boxright K(\neg sp(i) \boxright \neg Kq)$.

Lemma 2. *Suppose that there is at least one $sp(i)$-world and that the following two counterfactuals are true at a world w, for some course of inquiry j:*

$$sp(i) \boxright (sp(j) \boxright Kq)\,; \tag{15}$$

$$sp(i) \boxright \neg(\neg sp(i) \boxright \neg sp(j))\,. \tag{16}$$

Then it is false at w that (8).

Proof. Suppose that there is at least one $sp(i)$-world and that (15)-(16) are true at a world w. Let's say that w' is among the closest-to-w $sp(i)$-worlds. By the first enumerated assumption, it follows that $sp(j) \boxright Kq$ is true at w'. By the second enumerated assumption, it follows that among the closest-to-w' $\neg sp(i)$-worlds there is at least one $sp(j)$-world. The combination of these two intermediary conclusions entails that among the closest-to-w' $\neg sp(i)$-worlds there is at least one Kq-world. Consequently, it is false at w' that $\neg sp(i) \boxright \neg Kq$ and, therefore, it is also false at w' that $K(\neg sp(i) \boxright \neg Kq)$. Since w' is among the closest-to-w $sp(i)$-worlds, it is false at w that $sp(i) \boxright K(\neg sp(i) \boxright \neg Kq)$. □ □

Lemma 3. *Suppose that there is at least one $sp(i)$-world and that the following two counterfactuals are true at a world w, for some course of inquiry j:*

$$sp(i) \boxright K(sp(j) \boxright Kq) \tag{17}$$

$$sp(i) \boxright \neg K(\neg sp(i) \boxright \neg sp(j)) \tag{18}$$

Then it is false at w that (8).

Proof. Suppose that there is at least one $sp(i)$-world and that (17)-(18) are true at a world w. Let's say that w' is among the closest-to-w $sp(i)$-worlds. By the first enumerated assumption, it follows that $K(sp(j) \boxright Kq)$ is true at w'. By the second enumerated assumption, it follows that $\neg K(\neg sp(i) \boxright \neg sp(j))$ is true at w'. Consequently, there is at least one world w'' such that w'' is epistemically accessible from w' and $\neg sp(i) \boxright \neg sp(j)$ is false at w''. Therefore, there is among the closest-to-w'' $\neg sp(i)$-worlds at least one $sp(j)$-world, w'''. Given that w'' is epistemically accessible from w', it follows that $sp(j) \boxright Kq$ is true at w''. Hence, w''' is a Kq-world. But then $\neg sp(i) \boxright \neg Kq$ is false at w'' and, consequently, $K(\neg sp(i) \boxright \neg Kq)$ is false at w'. This implies that $sp(i) \boxright K(\neg sp(i) \boxright \neg Kq)$ is false at w. □ □

Condition (15) in Lemma 2 is weaker than than condition (17) in Lemma 3, because the antecedent is in both cases the same but the consequent is weaker in the first case. But condition (16) in Lemma 2 is stronger than than condition (18) in Lemma 3, because the antecedent is in both cases the same but the consequent is stronger in the first case. Note that the condition that there is at least one $sp(i)$-world may be implicit in the fact that $i \in Inq$ or, in other words, that i is a *possible* course of inquiry. But I have opted for making it explicit.

Lemma 4 (Church-Fitch)**.** *It is logically true that* $\neg K(q \wedge \neg Kq)$*.*

Theorem 1. *Assume that ART' with $\phi = q \wedge \neg Kq$ is true at a world w, and that, for every $i \in Inq$ for which $\neg sp(i)$ is true at w, there is at least one $sp(i)$-world and, for some course of inquiry j, (15)-(16) or (17)-(18) are true at world w. Then it is false at w that $q \wedge \neg Kq$ or, in other words, it is true at w that $q \rightarrow Kq$.*

Proof. Suppose that, for every $i \in Inq$ for which $\neg sp(i)$ is true at w, there is at least one $sp(i)$-world and that (15)-(16) or (17)-(18) are true at world w as well. By Lemma 2 and Lemma 3, it follows that (8) is false at w. By Lemma 1, it follows that (9) is false at w. Since (9) is one of the conjuncts of

$$\neg sp(i) \wedge \neg \Box K(\neg sp(i) \rightarrow \phi) \wedge (sp(i) \;\Box\!\!\rightarrow K(\neg sp(i) \;\Box\!\!\rightarrow \phi)),$$

it follows that the right disjunct of the right-hand side of ART' is false at w. Moreover, by Lemma 4, it follows that $K(q \wedge \neg Kq)$ is false at w. So, the left disjunct of the right-hand side of ART' is also false at w. Therefore, the right-hand-side of ART' is false at w, because both of its disjunctions are false at w. Assuming the truth of ART', it follows that the left-hand-side of ART' is also false at w. In other words, it is false at w that $q \wedge \neg Kq$ or, in other words, it is true at w that $q \rightarrow Kq$. □ □

The above theorem comes with conditions. In the next section I will present counterexamples to ART' that are based on the above theorem.

5 The overgeneration of knowledge: cases

My main claim is the following. There are cases in which it is true that, for every possible line of inquiry that has been unsuccessfully pursued at a world, there is a possible world at which that line of inquiry has been successfully pursued and there is another line of inquiry for which conditions (15)-(16) or (17)-(18) are true at that world. Assuming ART' with $\phi = q \wedge \neg Kq$ and given Theorem 1, it follows that it is *false* that q is true but unknown or, in other words, it is true that, if q is true, then it is *known*. But in those cases it is *true* that q is true but unknown or, in other words, q is *unknown* despite being true. In other words, we obtain *overgeneration of knowledge*. Therefore, ART' is not in general true.

I will present two cases. The first case is quite literally a garden-variety case.

Strawberries My wife and I want to know whether there still are any ripe strawberries in the garden. We are coordinating between the two of us who will go and check. The other person is going to check whether we still have ice cream in the fridge. It is a close call but it is decided that I will go and check whether there are ripe strawberries in the garden. But then we receive a phone call with an invitation for a BBQ at a friend's place. We leave without checking whether there still are strawberries in the garden and whether there still is ice cream in the fridge. If the phone call had been made five minutes later, I would have discovered that there were indeed still a few ripe strawberries in the garden. But I would not have known that, if I had not come and checked, then the presence of strawberries in the garden would not have been known. For, if my wife had gone and checked, she would have come to know the same fact, and it is a live possibility that not I but she would have gone and checked. Or, I know that, if my wife had gone and checked, she would have come to know that there are ripe strawberries in the garden, and I don't know that, if I had not gone and checked, my wife wouldn't have gone and checked either. (This holds *mutatis mutandis* when our roles had been reversed.)

In the above scenario line of inquiry i consists of me going into the garden to check, while line of inquiry j consists of my wife going into the garden and check. They are the only relevant lines of inquiries in the sense that, if they were successfully pursued, it would be known that, if they hadn't, then q (namely, that there are ripe strawberries in the garden) would (still) be true — see (7). Conditions (15)-(16) are in this scenario the following: if my wife had gone and checked, she would have come to know the same fact, and it is a live possibility that she would have gone and checked if I hadn't. Conditions (17)-(18) are in this case the following: I know that, if my wife had gone and checked, she would have come to know that there are ripe strawberries in the garden, and I don't know that, if I had not gone and checked, my wife wouldn't have gone and checked either. Given that both (15)-(16) and (17)-(18) are true in this hypothetical scenario and that lines of inquiries i and j are the only relevant ones, the right-hand side of ART' is false for the Fitch sentence 'there are still ripe strawberries in our garden but it is not known that it is the case'. It then follows that the left-hand side of ART' is false for that same Fitch sentence. So, given the hypothetical truth that there are still ripe strawberries in our garden, it is *known* that there are still ripe strawberries in our garden. But nobody has visited our garden since we left. Hence, it is very reasonable to claim that it is *unknown* that there are still ripe strawberries in our garden.

The second case is a case of historical science fiction. Its main difference is that the alternative lines of inquiries are now not individuated based on people who know each other (husband and wife), their spatiotemporal locations (home and garden) and their ability to perceive objects (normal eyesight), but by groups who don't know each other, their spatiotemporal locations and the detection technologies at their disposal. The relevance of this difference will become apparent in the next section.

Iceberg Suppose that two countries, A and B, in the twentieth century have each developed technology with military use. Country A has developed sonar, a detection system using acoustic waves, and country B has developed radar, a detection system using radio waves. These technologies are kept secret and no other country has them. Country A has outfitted a submarine with sonar and country B has outfitted a ship with radar. Both the submarine and the ship are patrolling in the Arctic Ocean. Neither crew knows about the other vessel. There is an iceberg at a certain spot in the Arctic Ocean. The submarine is a bit to the southwest of the iceberg and the ship is a bit to the southeast of the iceberg. If the submarine had went northeast a bit, it would have detected the presence of the iceberg using sonar. If the ship had gone northwest a bit, it would have detected the iceberg using radar. As it happens, they each independently set a different course and they do not detect the presence of the iceberg. If the submarine had gone northeast, it would not have been the case that, had it not gone north east, it would not have been known that there is an iceberg at that spot. For, if the ship had gone northeast, it would also have been known that there is an iceberg at that spot, and it is a live possibility that not the submarine but the ship had picked an iceberg detecting course. (This holds *mutatis mutandis* when the ship had gone northeast.)

Following Schlöder's account of knowability, the Fitch sentence 'there is an iceberg at that spot in the Arctic Ocean but it is not known that there is one' is unknowable in the sense that the right-hand side of ART' is false. In combination with ART' this implies then that it is *known* that there is an iceberg at that spot in the Arctic Ocean if that is indeed the case. Yet, nobody has detected it. So, it is again very plausible to claim that it is *unknown* that there is an iceberg at that spot in the Arctic Ocean.

Note that in the case of Iceberg the conditions (15)-(16) mentioned in Lemma 2 are satisfied, but not the conditions (17)-(18) mentioned in Lemma 3. The submarine crew are unaware and unsuspecting of the ship and its secret detection technology, so in those cases they don't have any *de re* knowledge about them. Consequently, the submarine crew does not know that, if the ship had set course to the northwest and used its radar, it would be known that there is an iceberg at that spot in the Arctic Ocean. More formally, if there is no *de re* knowledge about line of inquiry j, then there is also no *de re* knowledge of the form $K(sp(j) \mathrel{\Box\!\!\rightarrow} Kq)$, whereas it it is true that $\neg K(\neg sp(i) \mathrel{\Box\!\!\rightarrow} \neg sp(j))$. So, if there is *counterfactual de re ignorance* about j, as in the case of the closest $sp(i)$-worlds, then condition (17) is false, even though the condition (18) is true. Generally speaking, positive epistemic (sub)formulas with a free variable ranging over lines of inquiry are not satisfied relative to assignments to that free variable about which there is *de re* ignorance, however, it does not imply that negative epistemic (sub)formulas with a free variable ranging over lines of inquiry are not satisfied relative to those assignments to that free variable. Note that the problem of counterfactual de re ignorance does not affect conditions (15)-(16), because they don't contain a free variable ranging over lines of inquiries within the scope of the knowledge operator.

The overgeneration of knowledge resulting from ART' is not as extreme as in the case of the conclusion of the Church-Fitch paradox. In the latter the thesis that all truths are knowable entails that all truths are known. This implies *maximum* overgeneration of knowledge: *all* truths are supposed to be known, despite of the fact that most of them are not known. In contrast, ART and ART' imply a *limited* overgeneration of knowledge: *some* truths are supposed to be known, despite the fact that they are not in fact known. But irrespective of the extent of the overgeneration of knowledge, we do get overgeneration of knowledge from ART'.

In the next section the logical structure of lines of inquiries will be discussed and it will be examined whether the logical structure of lines of inquiries can be exploited to save Schlöder's knowability thesis from the problem of the overgeneration of knowledge. The problem of counterfactual *de re* ignorance about alternative lines of inquiry will reappear.

6 The logical structure of lines of inquiries and the problem of *de re* counterfactual ignorance

In response to the problem of the overgeneration of knowledge, one might propose to include the 'disjunction' of two lines of inquiry into the set of lines of inquiry.[6] In the case of Strawberries there is then supposed to be a line of inquiry that consists of me *or* my wife going to the garden to check. In the case of Iceberg there is then supposed to be a line of inquiry that consists of the submarine going northeast and using sonar *or* the ship going northwest and using radar. Let's say that we are allowing 'disjunctive' lines of inquiry:

$$\exists k (k \in Inq \wedge k = dis(i, j)).$$

If anything is an analytic truth about disjunctive lines of inquiries, then surely the following are:

$$sp(i) \rightarrow sp(dis(i, j)); \tag{19}$$

$$sp(j) \rightarrow sp(dis(i, j)). \tag{20}$$

Therefore, the following are also analytic truths:

$$\neg sp(dis(i, j)) \rightarrow \neg sp(i); \tag{21}$$

$$\neg sp(dis(i, j)) \rightarrow \neg sp(j). \tag{22}$$

The analyticity can be used to argue against the following variations on condi-

[6]Julian J. Schlöder suggested an 'algebra of lines of inquiry' during discussion of his work in Leuven on 22 February 2019.

tion (16), with i replaced by $dis(i,j)$:

$$sp(dis(i,j)) \mathrel{\Box\!\!\rightarrow} \neg(\neg sp(dis(i,j)) \mathrel{\Box\!\!\rightarrow} \neg sp(i)); \tag{23}$$

$$sp(dis(i,j)) \mathrel{\Box\!\!\rightarrow} \neg(\neg sp(dis(i,j)) \mathrel{\Box\!\!\rightarrow} \neg sp(j)). \tag{24}$$

$$sp(dis(i,j)) \mathrel{\Box\!\!\rightarrow} \neg(\neg sp(dis(i,j)) \mathrel{\Box\!\!\rightarrow} \neg sp(dis(i,j))). \tag{25}$$

Let us assume (23)-(25) for a *reductio*. The argument proceeds as follows. First, apply necessitation to (21), (22), and the logical truth $\neg sp(dis(i,j)) \rightarrow \neg sp(dis(i,j))$ to put a first box operator in front of the material implications. Next, use modal axiom scheme 4 to put a second box operator in front of the first modal operator. Subsequently, use (1), necessitation to (1) and modal axiom scheme K to derive the following:

$$\Box(\neg sp(dis(i,j)) \mathrel{\Box\!\!\rightarrow} \neg sp(i)); \tag{26}$$

$$\Box(\neg sp(dis(i,j)) \mathrel{\Box\!\!\rightarrow} \neg sp(j)). \tag{27}$$

$$\Box(\neg sp(dis(i,j)) \mathrel{\Box\!\!\rightarrow} \neg sp(dis(i,j))). \tag{28}$$

Then use

$$(\phi \mathrel{\Box\!\!\rightarrow} \psi) \rightarrow (\Diamond\phi \rightarrow \Diamond\psi),$$

which is also valid in the Lewis-style semantics for counterfactuals, to derive from (23)-(25) that, if $\Diamond sp(disj(i,j))$, then (26)-(28) are false. Next, use contraposition to conclude that $\neg\Diamond sp(disj(i,j))$. Finally, recall that it was assumed throughout that $\Diamond sp(i)$ and $\Diamond sp(j)$, from which it follows in any normal modal logic by the necessitation of (19) and (20) that $\Diamond sp(disj(i,j))$. We have derived a contradiction.

Thus, relative to the disjunctive line of inquiry, there is no alternative line of inquiry for which all the conditions of Lemma 2, in particular condition (16), are met. A broadly similar argument can be lodged against the variations on condition (18), with i replaced by $dis(i,j)$. The only relevant difference is that it has to be assumed that as analytic truths they are not just necessarily true but also necessarily known. I leave it to the reader to fill in the details. Thus, relative to the disjunctive line of inquiry, there is no alternative line of inquiry for which all the conditions of Lemma 3 are met. The introduction of 'disjunctive lines of inquiry' promises to undermine the problem of the overgeneration of knowledge.

However, it is not because the earlier conditions for the lack of counterfactual knowledge are not satisfied relative to the disjunctive line of inquiry that there is indeed counterfactual knowledge relative to the line of inquiry. I will now argue that condition (9), with i replaced by $dis(i,j)$, is not in all cases satisfied:

$$sp(dis(i,j)) \mathrel{\Box\!\!\rightarrow} K(\neg sp(dis(i,j)) \mathrel{\Box\!\!\rightarrow} (q \wedge \neg Kq)) \tag{29}$$

Note that the variables i, j, which range over lines of inquiries, are occurring freely within a positive epistemic subformula, namely

$$K(\neg sp(dis(i,j)) \mathrel{\Box\!\!\rightarrow} (q \wedge \neg Kq)).$$

Consequently, for this positive epistemic subformula to be satisfied there has to be *de re* knowledge about i and j. Let us return now to the Iceberg case. No agent in this case has *de re* knowledge of *both* lines of inquiry i and j, because the agents on the submarine don't know about the ship, its spatiotemporal location and its secret detection technology, and the *mutatis mutandis* for the agents on the ship. Now among the closest worlds in which the disjunctive line of inquiry has been successfully pursued there will be worlds in which only one of the constituting lines of inquiries has been successfully pursued. There is no detection of the other vessel and its location, because the other vessel could have gone, say, south. So, among the closest worlds in which the disjunctive line of inquiry is successfully pursued, there remains *de re* ignorance about one of the constituting lines of inquiry.[7] The problem of the overgeneration of knowledge is not solved by appeal to disjunctive lines of inquiries because of the problem of counterfactual *de re* ignorance about alternative lines of inquiry.

7 Conclusions

Defenders of the antirealist thesis that all truths are knowable faced a tough hurdle in the form of the Church-Fitch paradox of knowability: if the antirealist thesis is accepted, then all truths are known. The paradox features a particular concept of knowability, namely possible knowledge. That concept is not factive: it is possible to know a falsehood. In response, Edgington (1985) has proposed a factive concept of knowledge, namely possible knowledge of actuality. She has used this concept to reformulate the antirealist thesis. The Church-Fitch paradox of knowability cannot be run against Edgington's version of the antirealist thesis. However, Williamson (1987) and Heylen (2020) have formulated several triviality objections. At this point Schlöder (2019) enters the debate. Building on Edgington's insights, he introduces a new factive notion of knowability and a new version of the antirealist thesis. He forcefully argues that the latter can deflect the triviality objections offered by Williamson (1987) and Heylen (2020).

In this article I have argued that both directions of Schlöder's version of the antirealist thesis are confronted with problems. The right-to-left direction of that thesis says that only truths are knowable in Schlöder's sense, whereas the left-to-right direction of that thesis says that all truths are knowable in Schlöder's sense. The problem with the right-to-left direction of the thesis is that its justification rests on an implausible principle. Yet, I have described a way to improve that justification. The problem with the left-to-right direction of the thesis is that it leads to attributions of knowledge where there is ignorance. This is the problem of the overgeneration of knowledge.

[7] By the way, it could also have happened that both vessels go north and that they detect the presence of each other (although that is not always the case). But even if there were a detection of the other vessel, it is not easy to find out about the detection technology on board of the other ship. There remains *de re* ignorance about the other line of inquiry.

In Schlöder's own conclusion he was quite willing to separate both directions of his own version of the antirealist theory of truth. He thinks that the left-to-right direction of the thesis, which says that all truths are knowable in Schlöder's sense, is false. However, he also thinks that the right-to-left direction of the thesis, which says that only truths are knowable in Schlöder's sense, is useful, because we need a factive concept of knowability. He was wise to separate the two. This brings us to the first two general conclusions.

First, the research programme initiated by Edgington (1985) seems to be "degenerating" in the sense that its various versions of the knowability thesis continue to run into serious theoretical problems. What is more, a pattern starts to emerge. The Church-Fitch paradox shows that the knowability thesis, with knowability understood as the possibility to know, entails omniscience: all truths are known. Edgington (1985)'s version of that thesis avoids actual omniscience, but it brings possible omniscience in: there is a possible state at which all truths are known. Schlöder (2019)'s version of the knowability thesis entails neither omniscience nor possible omniscience, but it still overgenerates knowledge. That is the pattern: whether it is actual or possible, maximal or limited, each of the proposals that were mentioned overgenerates knowledge. This seems to be the ghost of the Church-Fitch paradox that it is hard to expel.

Second, to the extent that philosophers such as Brogaard and Salerno (2006) and Fuhrmann (2014) are right about the claim that our intuitive concept of knowability is factive, it is indeed worth exploring factive concepts of knowability that are made formally precise. Brogaard and Salerno (2006, p. 261) ask us to consider the following imaginary dialogue between two colleagues, A and B:

> A: We could be discovered.
> B: Discovered doing what?
> A: Someone might discover that we're having an affair.
> B: But we are not having an affair!
> A: I didn't say that we were.

There is something very odd about using the phrase "could be discovered" in a non-factive way. It is to Edgington (1985)'s credit that she introduced a factive notion of knowability that was later made precise. The factivity of Edgington's notion of knowability came with triviality issues. Schlöder (2019) improved on Edgington's notion by managing to keep factivity while dodging the triviality issues. The work done in Section 3 is meant to be an improvement on the justification for the factivity of Schlöder's notion of knowability. This remains a fruitful line of research.

The third general conclusion is that we should investigate the internal and logical structure of lines of inquiries more. The internal structure of lines of inquiries, with some being step-by-step procedures, is something that played an important role in the discussion of the factivity of Schlöder's notion of counterfactual knowledge (Section 3). The logical structure of lines of inquiry, with some being 'disjunctive', is something that played a key role in the attempt to save Schlöder's knowabilitiy thesis from the problem of the overgeneration of knowledge (Section 6). Regarding both the internal and logical structure of lines of inquiries it should be emphasized that I have by no means developed complete theories about these. Can we think of all lines of inquiries as finite step-by-step procedures? If we are going to allow 'disjunctive' lines of inquiries, can we then also allow 'conjunctive' lines of inquiries, and so on for the other logical operators and perhaps even quantifiers? (If it turns out to be a bad idea to impose logical structure, be it closed under logical operations or not, then that is bad news for someone who tries to save Schlöder's knowability thesis from the problem of the overgeneration of knowledge.) These are interesting research questions.

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