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Safety First: Making Property Talk Safe for Nominalists

Abstract: Nominalists are confronted with a grave difficulty: if abstract objects do not exist, what explains the success of theories that invoke them? In this paper, I make headway on this problem. I develop a formal language in which certain platonistic claims about properties and certain nominalistic claims can be expressed, develop a formal language in which only certain nominalistic claims can be expressed, describe a function mapping sentences of the first language to sentences of the second language, and prove some facts about that function and facts about some sound logics for those languages. In doing so, I prove that, given some plausible metaphysical assumptions, a large class of sentences about properties of concrete objects are "safe" on nominalistic grounds. Whenever some true sentences about concrete objects and some sentences belonging to this class that are true according to platonists collectively entail a conclusion about concrete objects, some nominalistically acceptable sentences are true and entail the same conclusion. Because the proof can itself be formulated without abstract objects, it provides a nominalistic explanation of the success of theories that invoke properties of concrete objects.

1. Introduction

There are many platonistic arguments that are of great theoretical and practical utility. Physicists use mathematical arguments when doing physics, chemists use arguments about chemical properties when doing chemistry, and so on. Yet their usefulness poses a great problem for nominalists: how are nominalists to explain it?

The problem can be made more precise. Imagine a platonist made the following remark:

> You nominalists say that there are no abstract objects. How do you explain the fact that platonistic arguments are "safe" with respect to their conclusions about concrete objects? What I mean by this is as follows. Many platonistic arguments have conclusions about concrete things. Moreover, good platonistic arguments never lead to falsehoods about concrete things. Every logically valid platonistic argument is such that *if* (a) its premises exclusively about concrete things are true and (b) its premises about abstract objects are such that we platonists regard them as true, *then* (c) if its conclusion is exclusively about concrete objects, its conclusion is true. Us platonists can explain why platonistic arguments are safe in this way: the premises about abstract objects that we regard as true just are true, and whenever a logically valid argument has all true premises, the truth of the conclusion necessarily follows. But you nominalists do not regard the premises of abstract objects as true. So how do you explain the safety of good platonistic arguments?

I will call the task of explaining why valid platonistic arguments that meet (a) and (b) never yield false conclusions about concrete objects the *problem of inferential safety*. As our imagined interlocutor pointed out, platonists have an easy solution to the problem. They can accept the most straightforward explanation, namely that the premises that platonists regard as true really are true. But nominalists have work to do.

In this paper, I offer a nominalist solution to the problem of inferential safety for good platonistic arguments concerning properties of concrete objects. I do this by offering a nominalistically acceptable proof that such arguments never lead to falsehoods about concrete objects. I begin by discussing the problem of inferential safety in greater detail. I then point out that there is a need for a new nominalist solution to the problem of inferential safety, or perhaps several new partial solutions. Next, I provide an overview of the argumentative strategy I take in order to establish the proof I will offer and describe its structure. Afterwards, I describe two formal languages necessary to fill out the structure of the proof, along the way discussing how the metaphysics underlying the second language supports the proof. I then complete the proof. Finally, I conclude the paper by outlining how the proof can likely be extended to solve the problem of inferential safety for good platonistic arguments concerning relations that obtain among concrete objects and propositions about concrete objects.[1]

2. The Problem of Inferential Safety

[1] The business of proving safety results has a long history. John Divers (1999) provides a safety result for possible world talk, while Richard Woodward (2010) and Lukas Skiba (2019) have provided safety results for abstract object talk in general. Unfortunately, Woodward's proof and Skiba's proof suffer from a difficulty encountered by the proof of safety I discuss based on Cian Dorr's (2008) paraphrases of claims about abstract objects: they require that either nominalism is merely contingently true or that counterpossibles are non-trivial. As will be seen, the proof of safety I endorse does not face either limitation. However, it should be noted that both Woodward's proof and Skiba's proof have the advantage of applying to talk of composite objects. My proof of safety is of no help to mereological nihilists. (To clarify, Woodward's proof needs the existence of abstracta to be contingent to apply to talk of abstract objects and needs the existence of composites to be contingent to apply to talk of composite objects. By contrast, Skiba's proof applies in either case if the relevant phenomenon is contingent or counterpossibles are not trivial.)

The force of the problem of inferential safety will be more readily apparent if some example arguments that pose the problem are provided and inferential safety is contrasted with conservativity, a related notion that has been of concern to nominalists. I will turn to the examples first. Then I will offer a definition of inferential safety. Afterward, I will contrast safety with conservativity and argue that safety matters more for nominalists.

Consider the following logically valid argument, adapted from Hartry Field (1980, p. 23):

1. There are exactly twenty-one aardvarks.
2. On each aardvark, there are exactly three bugs.
3. Every bug is on exactly one aardvark.
4. There are exactly twenty-one aardvarks iff 21 exactly numbers the aardarvks.
5. On each aardvark, there are exactly three bugs iff for each aardvark, 3 exactly numbers the bugs on it.
6. For any *x* and *y*, *if x* exactly numbers the aardvarks; for each aardvark, *y* exactly numbers the bugs on it; and every bug is on exactly one aardvark, *then* the product of *x* and *y* exactly numbers the bugs.
7. The product of 21 and 3 is 63.
8. There are exactly sixty-three bugs iff 63 exactly numbers the bugs.
9. Therefore, there are exactly sixty-three bugs.

Premises (1)-(3) are exclusively about concrete objects: the adjectival use of the numerals can be defined purely logically, which means that they do not carry any special ontological commitments. However, every other premise is platonistic. Premises (4), (5), and (8) entail, in combination with their nominalistic antecedent, that 21, 3, and 63 exist. Premise (7) entails these numbers exist on its own. A nominalist should accept premise (6), since on their view it is vacuously true: a nominalist does not believe in numbers. Even so, it is not exclusively about concrete things.

Suppose that premises (1)-(3) are true. Then a few things should be noted. First, it is a good platonistic argument. Every other premise of the argument is true according platonists, and as mentioned, it is logically valid. Second, it is undoubtedly the case that (9) is true if (1)-(3) are true. The question is why. A platonist can say that every premise of the argument is true *simpliciter*, but a nominalist cannot give that explanation. So why is it that the truth of (1)-(3) guarantees the truth of (9), if it is not by means of a logical entailment from true platonistic principles? A nominalist could just say that it is a brute fact, but this would be highly unsatisfying. There is nothing special about this argument: parallel phenomena can be found by swapping out ‘aardvark’ and ‘bug’ for other common nouns and the numerals for different numerals (e.g. ‘21’, ‘3’, and ‘63’ for ‘7’, ‘5’, and ‘35’, doing likewise for the letter equivalents). It would be highly implausible to take it as a brute fact that for each modification of (1)-(3), the matching modification of (9) is necessitated by their truth. When the platonist can do so much better, nominalism appears virtually untenable.

Fortunately for nominalists, (1)-(3) alone are sufficient to prove (9), albeit only at great length. Thus the nominalist need not take it to be a brute fact that (1)-(3) necessitate (9). And in general, due to a result by Hartry Field (1980, 1992) to be discussed later, we can be sure that any time some combination of nominalistic sentences – that is, sentences that do not contain any vocabulary for abstract objects – and theorems of orthodox impure set theory prove a nominalistic sentence, there is a proof of the latter from the nominalistic sentences alone. However, it is not only mathematical arguments that raise this difficulty. The following logically valid argument involving properties serves to illustrate this point:

1. A and B have the same intensive properties and different extensive properties.
2. The property of being red is an intensive property.
3. The property of being 20 kg in mass is an extensive property.
4. Everything is such that it is red iff it has the property of being red.
5. Everything is such that it is 20 kg in mass iff it has the property of being 20 kg in mass.
6. A is red.
7. A is 20 kg in mass.
8. Therefore, B is red and B is not 20 kg in mass.

Assuming for convenience that there is some nominalistically acceptable way of interpreting ‘20 kg in mass’, premises (6) and (7) are exclusively about concrete things, but none of the others are. (2)-(5) are all true according to platonists. Whether (1) is true according to platonists depends on contingent facts about A and B. Platonists might not know enough about A and B to feel confident about the truth or falsity of (1), but truth according to platonists is not what platonists actually believe: truth according to platonists is what platonists would believe if they had all relevant nominalistic information about the world. Such information would be sufficient to determine a reasonable platonist's belief about the truth or falsity of (1). With that in mind, it is undeniably the case that if (6) and (7) are true and (1) is true according to platonists, then (8) is true as well. The question, once again, is why. The platonist has a straightforward answer: in that case, all the premises are true *simpliciter*. The nominalist needs some alternative account.

Having illustrated the problem with examples, I now define inferential safety. Doing so is easy with a small amount of formalism. Although in the introduction my imagined platonist interlocutor called arguments "safe", from now on I will apply the notion of safety to pluralities of sentences. This will greatly simplify matters. Let 'Χ' and 'Ψ' be schematic letters for plural terms for sentences. Let the plural analog of the set-theoretic operation of union be defined just as one would expect. Let 'χχ' be a plural variable for things among Χ, 'ψψ' be a plural variable for things among Ψ, and let 'ψ' be a singular variable for sentences among Ψ.[2] Then:

Χ are inferentially safe with respect to Ψ ≔

for all χχ, ψψ, and ψ, *if* $\chi\chi \cup \psi\psi \vdash \psi$ and ψψ are true, *then* ψ is true.

In keeping with the preceding discussion, it is an evident fact that sentences that are true according to platonists are inferentially safe with respect to sentences exclusively about concrete objects. This much we know inductively: time and time again, we see good platonistic arguments with true conclusions about the concrete world, and we never see any with false conclusions about concreta. The problem of inferential safety is to explain why.

[2] More accurately, in any schema instance, let 'χχ' be a plural variable for things among the denotation of the instance of 'Χ' in that schema instance, and likewise for the rest. This degree of precision is cumbersome, so I opted for something simpler, albeit strictly speaking incorrect.

It is worth noting that the notion of inferential safety can be extended beyond abstract objects. It is an equally evident fact, for example, that sentences about composite objects that are true according to classical mereologists are inferentially safe with respect to sentences exclusively about mereological simples. What I have called 'the problem of inferential safety' could be more perspicously titled 'the problem of inferential safety posed by sentences that are true according to platonists': there is also a problem of inferential safety posed by sentences that are true according to classical mereologists, and just as platonists can solve the first problem easily, so classical mereologists can solve the second problem easily. Mereological nihilists, by contrast, have work to do. Since my focus is on abstract objects, however, I will continue to use the simpler title for the problem posed by sentences that are true according to platonists.

Inferential safety can be contrasted with the more well-known notion of conservativity. Conservativity is typically defined as follows, where 'T' is a schematic letter for terms for theories and the remaining symbols are interpreted as above:

T is conservative with respect to Ψ ≔

for all $\psi\psi$, if $\psi\psi$ are consistent, then T $\cup$ $\psi\psi$ are consistent.

If T is thought of as the conjunction of X, this definition immediately entails the following definition, which serves as a better contrast to inferential safety:

X are conservative with respect to Ψ ≔

for all $\chi\chi$ and $\psi\psi$, if $\psi\psi$ are consistent, then $\chi\chi \cup \psi\psi$ are consistent.

Conservativity and safety are interestingly related. On the one hand, conservativity entails safety if Ψ are closed under negation. Suppose X is conservative with respect to Ψ. Now suppose that $\chi\chi \cup \psi\psi \vdash \psi$. Then $\chi\chi \cup \psi\psi \cup \neg\psi$ are not consistent. By conservativity and the closure of Ψ under negation, if $\psi\psi \cup \neg\psi$ are consistent, then $\chi\chi \cup \psi\psi \cup \neg\psi$ are consistent: $\psi\psi \cup \neg\psi$ are among

Ψ. So by *modus tollens,* $\psi\psi \cup \neg\psi$ are not consistent. This fact entails that if $\psi\psi$ are true, ψ is true. Therefore, if X is conservative with respect to Ψ and Ψ are closed under negation, the following holds: if $\chi\chi \cup \psi\psi \vdash \psi$ and $\psi\psi$ are true, then ψ is true. And that just is safety.

However, safety does not entail conservativity. It might be that for all $\chi\chi$, $\psi\psi$ and ψ, if $\chi\chi \cup \psi\psi \vdash \psi$ and $\psi\psi$ are true, then ψ is true, but $\psi\psi \cup \neg\psi$ are consistent. In the most obvious case, it might be that for all $\chi\chi$, $\psi\psi$, and ψ such that $\chi\chi \cup \psi\psi \vdash \psi$, if $\psi\psi$ are true, then $\chi\chi$ are true, but this connection between the truth of $\psi\psi$ and the truth of $\chi\chi$ does not hold because of any logical principle. In that case there need not be any logical entailment between $\psi\psi$ and ψ even though $\psi\psi$'s truth is sufficient for ψ's truth. But the most obvious case is not the only possibility in which safety obtains but conservativity fails. There might be some sentences Φ such that for all $\chi\chi$, $\psi\psi$, and ψ, $\chi\chi \cup \psi\psi \vdash \psi$ iff for some $\varphi\varphi$ among Φ, $\varphi\varphi \cup \psi\psi \vdash \psi$ and if $\psi\psi$ are true, then $\varphi\varphi$ are true. That would entail X are safe with respect to Ψ even if each of X is necessarily false. However, it need not entail that X are conservative with respect to Ψ: the connection between the truth-values of any $\varphi\varphi$ among Φ and any $\psi\psi$ need not be based on a logical relationship between them.

Historically, nominalists have been interested in conservativity results for what's true according to platonists. Field (1980, 1992) is the paradigm example: he proves that the claims of standard impure set theory are conservative with respect to nominalistic sentences. My view is that nominalists should seek after safety results rather than conservativity results. The reason is twofold. First, we can be confident on inductive grounds that sentences that are true according to platonists are safe with respect to nominalistic sentences, but we should not be confident that the former are conservative with respect to the latter as a whole. Field's conservativity result only applies to a fragment of the abstract objects we talk about: besides sets, there are properties, relations, propositions, abstract artifacts, fictional entities, and so on. Experience does not provide us with grounds for thinking that what platonists say about these other kinds of abstract objects are conservative, and absent such experience, I fail to see any grounds for thinking so at all. (Conservativity is especially doubtful in light of "mixed claims", described in the next section.) Second, if it turns out that sentences that are true according to platonists are not conservative with respect to nominalistic sentences, that need not concern the nominalist. All the nominalist needs to explain is the fact that good platonistic proofs never lead to falsehoods about concreta. While a conservativity result is one way to explain that, as will become apparent, it is not the only way.

3. The Need for a New Solution to the Problem of Inferential Safety

There are solutions to the problem of inferential safety already on offer. Nevertheless, there is a need for more: at least one, and perhaps several. Existing solutions either are too limited in scope to completely solve the problem or rely on problematic assumptions. Here I will examine the limitations and difficulties encountered by the solutions presently available. Along the way, I will also mention several features of my partial solution and advantages that it possesses.

It is best to begin with Field. As mentioned above, Field proves a conservativity result for impure set theory with respect to sentences exclusively about concrete objects. Since those sentences are closed under negation, by the connection between conservativity and safety noted above, Field also proves that impure set theory is inferentially safe with respect to those sentences. Although there have been concerns about Field having used set theory in his proof of conservativity and having utilized a model-theoretic notion of consistency in characterizing conservativity – sets, including models, are not nominalistically acceptable – Field later supplied a nominalistic proof of his conservativity result and account of consistency.[3] If there are any remaining doubts as to the nominalistic acceptability of Field's methods, I will not raise them here.

[3] Charles Chihara (1990, pp. 162-3) raises the point about Field using set theory in his proof, and Stuart Shapiro (1983) criticizes Field for using a model-theoretic notions in characterizing conservativeness. On the latter issue, see

Nevertheless, there are two major concerns with Field's approach. One is that Field's conservativity result is only for impure set theory. Even though the set theory for which he proves his result is enriched with predicates for concrete objects and assertions that various concreta are ur-elements (so as to entail that there is a set of all protons, for example), it is not clear that all theories of interest that apply mathematics to the concrete world can be neatly divided into theorems of his enriched set theory and nominalistic claims. Even if a theory only refers to mathematical entities and concreta, it might be that it cannot avoid making use of non-theorem mixed claims: claims that make reference to concrete things and mathematical entities but are not theorems of the enriched set theory. Yet Field's conservativity result doesn't apply to the union of his enriched impure set theory with such claims. Field did show how Newtonian gravitational theory can be articulated nominalistically, and the union of its nominalization and impure set theory has a clean divide. However, as David Malament (1982) has pointed out, the method Field uses for nominalizing Newtonian gravitational theory does not straightforwardly generalize to all scientific theories of interest. Without a universal procedure to cleave mathematized theories into nominalistic claims and theorems of the enriched impure set theory, it is uncertain that Field's conservativity result is enough to solve the problem of inferential safety for all applications of mathematics to the concrete world.

A second concern is that mathematical objects are not the whole of the abstract realm. As already noted, there are many other kinds of abstract objects. Even if Field's conservativity result is satisfactory for all applications of mathematics, Field has only offered a partial solution to the problem of inferential safety. There is still the question of why sentences about properties, relations, propositions, abstract artifacts, fictional entities, and so on that are true according to

Field (1985) for a reply and Chihara (1990, pp. 153-9) for discussion. Field (1992) provides a nominalistic proof of conservativity, and Field (1998) provides a nominalistic account of conservativeness.

platonists are inferentially safe. Nominalists either need a solution that encompasses all kinds of abstract objects at once, or they need to take a piecemeal approach and show that sentences about one kind of abstract object are safe, then show sentences about another kind of abstract object are safe, and so on until they have worked through all the kinds. Either way, there is still room for a new all-at-once solution or room for new partial solutions.

The second major concern is more pressing than the first for my purposes. I will not be attempting a safety result for mathematics here, so my proposal is not in competition with Field's. If his result is enough for all applications of mathematics, so much the better. However, the first major concern is still useful as a contrast to my approach. I will focus on proving a safety result for sentences about properties of concrete objects that does not require carving up theories into nominalistic claims and theorems of some property theory. That is, my safety result will apply even to mixed claims about properties and concreta that cannot plausibly be represented as theorems of some theory of properties alone (e.g. premise (1) of my second example argument in §2). Moreover, just as the early critics of Field insisted that a proof of conservativity must be nominalistic to serve nominalists' interests, I insist on nominalistically proving inferential safety. That said, my motivations differ. Field's original goal was only to show that platonists should concede that mathematics is conservative by their own lights. For that purpose, platonistic methods might well do. Moreover, since sentences exclusively about concrete objects are closed under negation, a proof of the conservativity of mathematics using platonistic methods should automatically lead platonists to concede it is inferentially safe. But that mathematics is inferentially safe is not at issue: that much is evident inductively. The issue is *why* it is safe. If a proof of safety is platonistic, then only platonists can cite that proof as an explanation, since only they believe its premises to be true. It offers the nominalist no explanation, and therefore, no solution to the problem of inferential safety. What goes for mathematics goes for what's true according to platonists generally, which is why I limit myself to nominalistic methods.

That ends my discussion of Field's approach. I now turn to a prospective all-at-once solution: appealing to what would be the case were abstract objects to exist. Cian Dorr's (2008, pp. 36-37) strategy for paraphrasing sentences about abstract objects provides a good case study for this approach. He proposes paraphrasing any sentence about abstract objects φ as follows:

(There are numbers, features, etc. $\wedge$ concrete things are how they actually are) $\Box\!\!\rightarrow \varphi$.

Alternatively, to ensure that the abstract objects follow the principles we postulate they follow according to our best theories of them, the paraphrase can be put as:

([Insert the various axioms of our best theories of abstract objects] $\wedge$ concrete things are how they actually are) $\Box\!\!\rightarrow \varphi$.[4]

This strategy for paraphrasing claims about abstract objects can be deployed in a proof of inferential safety as follows. Abbreviate '([insert the various axioms of our best theories of abstract objects] $\wedge$ concrete things are how they actually are)' as 'PLATONISM'. Let Γ be an arbitrary finite set of sentences comprising the premises of a good platonistic argument and let ψ be an arbitrary nominalistic sentence that Γ proves. Then let $\sigma(\Delta) = \{x \mid \exists y(y \in \Delta \wedge x = \ulcorner \text{PLATONISM} \Box\!\!\rightarrow y \urcorner)\}$ for any finite set of sentences comprising the premises of a good platonistic argument Δ and let $\sigma(\varphi) = \ulcorner \text{PLATONISM} \Box\!\!\rightarrow \varphi \urcorner$ for any sentence φ. It is trivial to show that if Γ proves ψ,

[4] Dorr's strategy requires that there be some nominalistic way of interpreting counterfactuals, which are standardly analyzed in terms of possible worlds. One way of doing so would be by a strict entailment view of counterfactuals. On this view, a counterfactual of the form $\phi \Box\!\!\rightarrow \psi$ really has the logical form of $\Box[(\phi \wedge \chi_1 \wedge \ldots \wedge \chi_N) \rightarrow \psi]$, where $\chi_1 \wedge \ldots \wedge \chi_N$ are supplied by context. I neither affirm nor deny this view, but I assume that some nominalistic way of interpreting counterfactuals is correct.

then $\sigma(\Gamma)$ proves $\sigma(\psi)$ in any counterfactual logic that contains deduction within counterfactual conditionals.[5] Now suppose the following principles hold:

A. For any nominalistic φ, φ is true iff $\sigma(\varphi)$ is true.

B. For any platonistic φ, if φ is true according to platonists, then $\sigma(\varphi)$ is true.

Both claims are quite plausible. Regarding (A), the antecedent of each counterfactual explicitly states that concrete objects would be how they actually are, so the nominalistic facts shouldn't change. Regarding (B), it seems undeniable that if the basic axioms and principles platonists accept were true, then anything the platonists in fact accept as true would be true. Since every nominalistic member of Γ is true and every platonistic member of Γ is true according to platonists, it follows from (A) and (B) that the members of $\sigma(\Gamma)$ are true. Since $\sigma(\Gamma)$ proves $\sigma(\psi)$, $\sigma(\psi)$ is true as well. And by (A), it follows that ψ is true. QED.

At first glance, it appears that we have a proof of safety that meets my desiderata. If talk of sets of sentences were replaced with plural talk of sentences, the proof would be entirely nominalistic. This is easily achievable. But unfortunately, there is serious problem with (A). It is plausible that if abstract objects do not exist, then necessarily, they do not exist.[6] Standardly, every counterfactual with an impossible antecedent is regarded as true. But then the right-to-left direction of (A) fails: $\sigma(\varphi)$ is true for every nominalistic φ, even when φ is false. (A) is thus false, and one can't infer ψ from $\sigma(\psi)$ in the proof without (A).[7] Dorr himself recognizes that nominalism is metaphysically necessary if true and rejects the orthodox position on counterpossibles. However,

[5] For an example of such a logic, see David Lewis (2001/1973, p. 132).

[6] Though widely held, this point is controversial. Several philosophers, such as Field (1993) and Mark Colyvan (2000), have maintained that the truth or falsity of nominalism is contingent (see Kristie Miller [2012] for discussion). Still, even if it is not settled that nominalism is either necessarily true or necessarily false, that there is controversy makes it best to have a solution to the problem of inferential safety that is consistent with nominalism being necessary.

[7] I thank Benjamin Middleton for making this point to me.

while the orthodox position has faced serious critique in recent philosophical literature, it also has prominent defenders.[8] It is best to be able to solve the problem of inferential safety without having to take a stand on counterpossibles if possible.[9]

Fortunately, a different proof of inferential safety can be given that follows a generalization of the proof's structure. Like Field, I will claim some sentences that are not in any way about abstract objects entail the conclusions of good platonistic arguments on their own. Like Dorr, in giving my proof of inferential safety, I make use of counterfactuals. But unlike Field, my safety result can be applied to theories that are not cleanly dividable into platonistic theorems and nominalistic claims, and unlike Dorr, my counterfactuals will have possible antecedents.[10] The counterfactuals I will use are slight modifications of the counterfactuals I used in the past to nominalistically paraphrase sentences about properties of concrete objects: the proof of safety I offer here stands to a modified version of my 2020 paraphrase strategy much as the proof of safety I offered on behalf of Dorr stands to Dorr's.[11] I will nominalistically prove that good platonistic arguments containing as their only premises about abstract objects the members of a wide class of sentences about properties of concreta never have false conclusions about concrete objects. In offering such a proof, I take myself to be giving a nominalistic explanation of why the sentences in that class are inferentially safe with respect to sentences exclusively about concreta. While the

[8] For an overview of arguments that non-vacuous counterpossibles are important for metaphysics, philosophy of mathematics, and philosophy of logic, as well as arguments for orthodoxy, see Alexander W. Kocurek (2021). For a recent defense of orthodoxy, see Timothy Williamson (2020).

[9] Additionally, if counterpossibles are not trivial, it is plausible that deduction within counterfactual conditionals is invalid. However, Skiba (2019) has addressed how a safety result can be obtained using Dorr's sort of strategy even without deduction within counterfactual conditionals.

[10] Note that Field is explicitly a mathematical fictionalist, and Skiba treats a Dorr-style paraphrase as constituting a fictionalist approach to abstract objects. My approach could be regarded as fictionalist as well, though I don't think much hangs on whether it counts as a kind of fictionalism or some other kind of nominalist theory.

[11] In my 2020 paper, I alluded to the fact that a safety result is likely available for the paraphrase strategy I offered, though I did not show that this is so (see fn. 24 of that paper). The current paper makes good on that claim for a slightly tweaked version of that paraphrase strategy.

scope of the proof is specific to platonistic arguments concerning properties, in the conclusion I will explain how the proof can be extended to nominalistically demonstrate that several other types of good platonistic arguments never have false conclusions about concreta. Regardless, I take it that if it is possible to provide a nominalistic proof in this case, that considerably raises the likelihood that it can be done in general.[12]

4. The Argumentative Strategy Taken to Prove Safety

The argumentative strategy underlying the proof of safety exploits facts about tokens of expressions like '*x* is an architect' or '*y* is an elk'. Properties are related to these inscriptions on the basis of the latter having as their satisfaction conditions the instantiation conditions of the former. For example, being an architect is both the ground of a thing's instantiating the property of being an architect and the ground of that thing's satisfying the tokens of '*x* is an architect'. It turns out that for the properties expressed by tokens, there is a bijective pairing between those properties and certain fusions of tokens based on the satisfaction condition of the parts of a given fusion being the instantiation condition of one of those properties. Simplifying matters somewhat, by way of illustration, the property of being an elk is paired up with the fusion of every token that is the same in meaning as tokens of '*y* is an elk': the instantiation condition of the former is the satisfaction condition of the parts of the latter. This suggests that if every property is expressed by at least one token, sentences about satisfying the parts of these fusions can play the same inferential role as sentences about instantiating properties, and sentences that classify these fusions according to their satisfaction conditions can play the same inferential role as sentences that classify

[12] Field and Dorr are not the only philosophers to propose nominalist systems. For example, Chihara has his own, and Geoffrey Hellman (1989) has developed one as well. It would take me too far afield to critique them all. I will note, however, that even if these alternative proposals are successful, they are generally aimed at mathematics. Property discourse has been much neglected by comparison, and part of what I am attempting to do is to fill in some of the gaps left by the focus on mathematics.

properties according to their instantiation conditions. Of course, many properties are not actually expressed by any token. That is where counterfactuals enter the picture.

Suppose S is a metaphysically possible scenario such that if S were to obtain, (i) all properties would be expressed by tokens and (ii) all the facts about actual concrete objects would be the same. Then (almost) any good platonistic argument carried out in terms of properties with a conclusion ψ exclusively about actual concrete objects has a parallel sound argument conducted with counterfactuals about S and the conclusion of which entails ψ.[13] To construct that parallel, first, from any premise φ exclusively about actual concrete reality in the original argument, infer $S \mathrel{\Box\!\!\rightarrow} \varphi$. This inference is sound because of (ii). Second, for any premise φ about properties in the original argument, infer $S \mathrel{\Box\!\!\rightarrow} \varphi^*$, where φ^* is the result of replacing talk of properties in φ with talk of the fusions corresponding to them mentioned in the previous paragraph. This is licensed by the bijective pairing mentioned above and the fact that every property would be expressed by some token were S to obtain: if φ is true according to platonists, then $S \mathrel{\Box\!\!\rightarrow} \varphi^*$ is true. The results of these two steps are counterfactual premises that (a) share S as their antecedent and (b) the consequents of which have the same logical forms and expressions for actual concrete reality as the original premises. Because of (b), the consequents of the premises of the new argument prove ψ. Because of that and (a), in standard counterfactual logic, the new premises prove $S \mathrel{\Box\!\!\rightarrow} \psi$. That is the new conclusion. Then, because of (ii), $S \mathrel{\Box\!\!\rightarrow} \psi$ entails ψ. All of this together amounts to a proof of safety for claims about properties that are true according to platonists.

The metaphysically possible scenario that I identify as meeting (i) and (ii) is the existence of a pluriverse. Letting 'universe' be a common noun for maximal fusions of spatiotemporally

[13] The arguments excluded from the scope of this claim are those that cannot be represented in the formal language L to be developed later in this paper.

related things, a pluriverse can be described as a fusion of universes such that every way a universe could possibly be is a way one of its universe parts is. Since every property is possibly expressed by some token and tokens are parts of universes, the existence of a pluriverse guarantees that every property is expressed by at least one token. It therefore meets (i). Moreover, because it is necessarily the case that universes are causally isolated from one another, all the facts about actual concrete reality would remain the same were there a pluriverse. Thus the existence of a pluriverse meets (ii) as well.

Of course, many details are left unspecified in this overview, and I have taken liberties with platonistic language. Filling out those details and making do with nominalistically acceptable resources is the task of the rest of this paper.

5. The Structure of the Safety Proof

The structure of the proof is as follows. First, I will introduce two languages, L and L+. L is a two-sorted, first-order language expressing claims about properties of concrete objects and (actual) concrete objects themselves, and L+ is a two-sorted, first-order language with a counterfactual operator that expresses claims about actual concrete objects and "o-fusions", the fusions of tokens mentioned in the last section. The sentences of L+ are nominalistic. I will define a function *f* from sentences of L to sentences of L+. Moreover, *f* has the following features, where 'proves' means 'proves under classical natural deduction rules for two-sorted, first-order languages' unless otherwise specified:

I. For any finite set Δ of sentences of L and sentence φ of L, if Δ proves φ, then $f(\Delta)$ proves $f(\varphi)$ in a weak counterfactual logic.

II. For any nominalistic sentence φ of L, φ is true iff $f(\varphi)$ is true.

III. For any platonistic sentence φ of L, if φ is true according to platonists, then $f(\varphi)$ is true.

So it follows that for any finite set Γ of sentences of L and nominalistic sentence ψ of L, if (a) Γ proves ψ, (b) the nominalistic members of Γ are true, and (c) the platonistic members of Γ are true according to platonists, then ψ is true for wholly nominalistic reasons. And this is so because:

- $f(\Gamma)$ proves $f(\psi)$ in a weak counterfactual logic [by I and (a)],
- every member of $f(\Gamma)$ is nominalistic [by the description of L+ and *f*] and true [II, (b), III, and (c)], and
- $f(\psi)$ is true iff ψ is true [by II].

This entails that the sentences of L that are true according to platonists are inferentially safe with respect to the nominalistic sentences of L. Despite the fact that the sentences about properties are literally false, one can show that many good platonistic arguments involving claims about them never lead to falsehoods about the concrete. And since sets and functions are ultimately dispensable from the proof – talk of sets can be replaced with plural quantification, and *f* can be replaced with the definition that underlies *f* – the demonstration is wholly nominalistic.[14]

6. The Language of Properties

Now that the structure of the proof of safety has been laid out, the first task is to introduce L. The purpose of L is to be able to represent sentences with several components formally. The first is:

(1) Singular terms for *actualities* and *sproperties*.

'Actuality' can be defined as follows, where '@' is the actuality operator:

x is an actuality := @(*x* is concrete).

[14] Of course, I have quantified over sentence-types rather than sentence-tokens in describing the structure of the proof, which might threaten its nominalistic status. I suspect that it would be easy to construct a notion of deductive consequence appropriate to tokens and recast the proof in terms of the possibilities for inscribing sentence-tokens. However, I shall not attempt this here. Goodman and Quine (1947) developed a proof system for tokens that serves as an illustration of how the first part of this task can be done.

Something is an actuality iff it is actually concrete. Note that every concrete object is an actuality. Note also that this is not a necessary truth, as it is possible for there to be vastly more zebras (say) than there are actual zebras, or even actual concrete things at all.[15] The definition of 'sproperty' is:

x is a sproperty := *x* is a property of concrete objects that does not concern other properties.

For example, the property of being red is a sproperty, while the property of having two fundamental properties is not. Sproperties are focused on in order to avoid complications.

Other components of the sentences that L seeks to represent include:

(2) Quantification over actualities or quantification over sproperties.

(3) The standard truth-functional connectives.

(4) Predicates of any finite arity that apply to and exclusively concern concrete objects.

(5) Monadic predicates that say what kind of property a given property is, such as 'is a neurological property' and 'is a chemical property'.

(6) An identity predicate for actualities (e.g. 'are identical actualities')

(7) An identity predicate for sproperties (e.g. 'are identical sproperties').

(8) A dyadic predicate for instantiation.

Say that some sentence is a *core sentence about properties of concrete objects* just in case each of its components either (a) falls under one of (1)-(8), (b) occurs exclusively in the sentence as a component of something falling under (1)-(8) [e.g. 'every' in 'every actuality instantiates a sproperty'], or (c) is a complex construction of things falling under (1)-(8).[16] The goal of L is to represent all and only the core sentences about properties of concrete objects. Note that the fact that quantification is over actualities and over sproperties specifically excludes sentences with

[15] At least, this is true in the sense of 'actual' captured by the actuality operator.

[16] Core sentences largely overlap with what I have elsewhere described as simple sentences about properties of concrete objects (2020, fn. 5), except that core sentences can also contain an identity predicate for sproperties. What I called simple sentences are some, but not all, of the core sentences.

generic quantification from qualifying as core sentences. For example, ‘every actuality instantiates a sproperty’ is a core sentence, but not ‘everything instantiates a sproperty’. This is what makes core sentences capable of being interpreted in a two-sorted language.

With this in mind, I can now turn to L. L contains the following elements, dropping quotation marks for convenience: (a) the truth-functional connectives $\wedge$, $\rightarrow$, $\vee$, $\neg$; (b) universal and existential quantifiers of two sorts, $\forall_{\mathrm{a}}$, $\exists_{\mathrm{a}}$ and $\forall_p$, $\exists_p$; (c) variables for two sorts, $x_{\mathrm{a}}, x_{1\mathrm{a}}, \ldots, y_{\mathrm{a}}, y_{1\mathrm{a}}, \ldots$, etc., and $x_p, x_{1p}, \ldots, y_p, y_{1p}, \ldots$, etc.; (d) predicates for two sorts, R_{a}, $\mathrm{R}_{1\mathrm{a}}, \ldots$ (monadic), R_{aa}, $\mathrm{R}_{1\mathrm{aa}}, \ldots$ (dyadic) and so on for any finite arity, *and* R_p, R_{1p}, R_{2p}, etc. (monadic only); (e) constants for two sorts, $\mathrm{a}_{\mathrm{a}}, \mathrm{a}_{1\mathrm{a}}, \ldots, \mathrm{b}_{\mathrm{a}}, \mathrm{b}_{1\mathrm{a}}, \ldots$, etc., *and* $\langle \mathrm{R}_{\mathrm{a}} x_{\mathrm{a}} \rangle_p$, $\langle \mathrm{R}_{1\mathrm{a}} y_{\mathrm{a}} \rangle_p$, and so on for each one-place open sentence of L containing only predicates of sort ‘a’; (f) two identity predicates, $=_{\mathrm{aa}}$ and $=_{pp}$; and (g) a special predicate, $\in_{\mathrm{a}p}$.

The sort ‘p’ is for sproperties, while the sort ‘a’ is for actualities. The constants of sort ‘p’ have their referents fixed by the following schema:

$\langle \Phi_{\mathrm{a}} \gamma_{\mathrm{a}} \rangle_p$ = the property of being a γ such that $\Phi\gamma$

For example, if ‘R_{a}’ is interpreted as ‘is red’, ‘$\langle \mathrm{R}_{\mathrm{a}} x_{\mathrm{a}} \rangle_p$’ should be read as ‘the property of being an x such that x is red’. It is important to note that the ‘a’-subscripts are dropped when reading the constant. ‘$\langle \mathrm{R}_{\mathrm{a}} x_{\mathrm{a}} \rangle_p$’ is not intended to be read as ‘the property of being an actuality x such that x is red’: the property that the former term picks out can be instantiated by any red thing, but the property that the latter term picks out can only be instantiated by actualities. This may seem like an oddity given the function of ‘a’-subscripts, but there is no formal barrier to it. The constants do not have any internal logical structure: filling in the brackets is only a convenient device for indicating which property is picked out by which constant.

Predicates of sort ‘a’ can be any predicate meeting condition (4) or condition (6). Predicates of sort ‘*p*’ are restricted to predicates meeting condition (5) or condition (7). ‘$\in_{ap}$’ is the predicate for instantiation. For example, ‘$\in_{ap}c_a\langle R_a x_a\rangle_p$’ should be read as ‘$c_a$ instantiates the property of being an *x* such that *x* is red’ – or, more simply, ‘c_a instantiates the property of being red’. Surely the property of being an *x* such that *x* is red just is the property of being red. More generally, I assume that the constants I introduced for properties are sufficient to denote every property associated with an open sentence of L, albeit how the terms should be read may be more elaborate than colloquial names for properties.

L contains exactly those linguistic items necessary to formally express and only formally express core sentences about properties of concrete objects. It is important to note that although core sentences of properties of concrete objects constitute only a fragment of the sentences we use about properties, it nevertheless constitutes an enormous fragment, and the fragment that is most crucial to ordinary discourse. For example, it would be a simple matter to represent many arguments made in biology or chemistry in L after making some minor adjustments to their premises: all one must do is be clear that biology- and chemistry-related concrete objects are actualities, as are all concrete objects, and that biological and chemical properties are sproperties.

7. The Nominalistic Language and Its Metaphysics

Having introduced L, I now turn to L+. In order to set up L+, I must introduce a few notions. I will first introduce ‘o-token’, which is to be defined as:

> *x* is an o-token := *x* is a one-place open sentence-token ∧ *x* is not a fusion of one-place open sentence-tokens.

One-place open sentence-tokens are the inscription equivalents of one-place open sentences. They can be categorized according to what they indicate of what satisfy them. For any inscription φ, let the result of inscribing a token of ‘*’, then φ, and another token of ‘*’ be a term-token referring to φ. Then just as ‘x is a hydrogen molecule’, ‘y is combustible’, and ‘z is a solid’ are chemical one-place open sentences, *x is a hydrogen molecule*, *y is combustible*, and *z is a solid* are chemical one-place open sentence-tokens (assuming this paper is printed on a hard copy). The variables of these open sentences and their tokens in the open sentence-tokens take the place of ‘it’ and tokens of ‘it’, respectively. Moreover, the tokens are o-tokens because they are not fusions of one-place open sentence-tokens themselves. This is not a trivial point: one could imagine a language in which each of the letter-token parts of some one-place open sentence-token *o* was itself composed of one-place open sentence-tokens of another language, in which case *o* would not be an o-token. The significance of this point will be apparent shortly.

The next notions to be defined are that of an o-fusion and f-satisfaction. They may be defined as follows:

> x is an o-fusion := x is a fusion of some o-tokens such that (i) each one is the same in meaning as each other one, and (ii) no o-token that is not among them is the same in meaning as one of them.
>
> x f-satisfies y := y is an o-fusion $\wedge$ $\forall z$(z is one of the o-tokens y fuses $\rightarrow$ x satisfies z).

To put it informally, an o-fusion is a maximal fusion of same-meaning o-tokens, and f-satisfaction is satisfaction of each of the o-tokens an o-fusion fuses. Since o-tokens are not fusions of o-tokens, o-fusions are uniquely decomposable into maximal pluralities of same-meaning o-tokens. This ensures that the condition for f-satisfying a given o-fusion is uniquely pinned down by a single maximal plurality of same-meaning o-tokens. I assume throughout this paper that composition is permissive enough that the existence of a maximal plurality of same-meaning o-tokens is sufficient for them to compose an o-fusion, but not so permissive that two o-fusions can exactly overlap.

The significance of o-fusions and f-satisfaction is that singular terms for and quantification over sproperties in L will be replaced with singular terms for and quantification over o-fusions in

L+, and the predicate for instantiation in L will be replaced with a predicate for a variant of f-satisfaction in L+. O-fusions are the fusions of tokens mentioned in the overview of my argumentative strategy in §4, while f-satisfaction is satisfaction of their parts.

The counterfactuals of L+ describe what would be the case were there a pluriverse. Like 'actuality', 'pluriverse' is readily understood, but unlike 'actuality', it is difficult to define.[17] In §4, I described a pluriverse as a fusion of universes such that every way a universe could possibly be is a way one of its universe parts is.[18] Unfortunately, nominalists cannot use this description as a proper definition: the description involves quantifying over ways that universes could possibly be, and nominalists do not believe in ways. Instead of providing a definition for 'pluriverse', I will explain its intended meaning with a schema. Every instance of the following schema is true, where 'Φx' is a schematic letter for an open sentence that lacks *de re* content, intrinsically characterizes spatiotemporal things, and in which 'x' and 'x' alone is free:

$$\Diamond\exists x\Phi x \leftrightarrow \Box\forall y[y \text{ is a pluriverse} \rightarrow \exists z(\Phi z \wedge z \text{ is part of } y)].^{19}$$

For example, the following instances of the schema are true:

$$\Diamond\exists x(x \text{ is a flying horse}) \leftrightarrow \Box\forall y[y \text{ is a pluriverse} \rightarrow \exists z(z \text{ is a flying horse} \wedge z \text{ is part of } y)].$$

$$\Diamond\exists x(x \text{ is composed of } 10^{(1,000)} \text{ spatiotemporally-related cats}) \leftrightarrow \Box\forall y[y \text{ is a pluriverse} \rightarrow \exists z(z \text{ is composed of } 10^{(1,000)} \text{ spatiotemporally-related cats} \wedge z \text{ is part of } y)].$$

In English, the first instance says that it's possible for there to be a flying horse iff necessarily, every pluriverse has a flying horse part, while the second instance says that it's possible for there to be something composed of $10^{(1,000)}$ spatiotemporally-related cats iff necessarily, every pluriverse

[17] In my 2020 paper, I offered a relatively simple definition of 'pluriverse' (§4). But that definition is not metaphysically neutral. Using that definition, if it turns out that necessarily, no more than one universe exists, then necessarily, every universe is a pluriverse. But this is clearly not satisfactory. While I do not in general believe that definitions must be metaphysically neutral, as a concession to the desire for neutrality I explicate the meaning of 'pluriverse' in a neutral way here.

[18] Note that José A. Benardete (1964, pp.149-154) and Sider (2002) use 'pluriverse' differently.

[19] Here '$\Diamond$' means 'it is metaphysically possible that' and '$\Box$' means 'it is metaphysically necessary that'.

has a part composed of $10^{(1,000)}$ spatiotemporally-related cats. In addition to providing the schema, I can say that it would continue to have only true instances even if English had arbitrarily strengthened expressive power. This provides additional information since in actual fact, English has all manner of expressive limitations. But the fact that the schema remains valid under any arbitrary extension of English means that, for example, necessarily, if there a pluriverse, then every spatiotemporal thing is a duplicate of part of it. Even without an explicit definition, the meaning of 'pluriverse' is made clear by the schema and this remark.[20]

The notion of a pluriverse is inspired by David Lewis' (1986) plurality of worlds. If a modal version of Lewis' recombination principle is true, then pluriverses are qualitatively exactly like Lewis' plurality of worlds (or rather, the fusion of Lewis' plurality), though they carry no associated reduction of possibility. By a modal version of Lewis' recombination principle, I have in mind the following. Let 'yy_u' be a plural variable for parts of multiple universes and 'xx_I', 'zz_I', 'vv_I', and 'ww_I' be plural variables for things that are parts of one universe. Then:

$$\Box(\forall xx_I \forall yy_u[\Diamond\exists zz_I(zz_I \text{ and } yy_u \text{ are equinumerous}) \rightarrow (\Diamond\exists vv_I[vv_I \text{ and } yy_u \text{ are equinumerous} \wedge \forall v(v \text{ is among } vv_I \rightarrow \exists ww_I[ww_I \text{ are duplicates of } xx_I \wedge ww_I \text{ compose } v])])]).$$

Informally, the principle says that necessarily, for any collection of parts of a single universe and collection of parts of multiple universes, if it is possible for there to be a universe with as many parts as there are members of the second collection, then it is possible for there to be a universe containing as many copies of the first collection as there are members of the second collection. This effectively constitutes a modal version of Lewis's idea that for any parts of a universe and cardinal k, if there is a possible world with k-many parts, then there is a possible world with k-many duplicates of the parts of the first universe. Lewis didn't need a modal version since he believed in concrete possible worlds, but the rest of us need one to capture the heart of Lewis'

[20] This could be explained at length in terms of tokens to make it nominalistically acceptable.

principle. The reason I don't quantify over cardinals in the modal version is that cardinals are not nominalistically acceptable. (I assume 'equinumerous' can be taken as a primitive rather than defined in terms of bijections.)

It is obvious that the modal recombination principle entails that pluriverses look like Lewis' plurality of worlds. What is more surprising is that if the modal recombination principle is false, pluriverses don't look that way. Both Lewis' original principle, in the context of modal realism, and the modal recombination principle, in a general context, express the thought that there are no necessary connections between distinct existences: that the only limits on recombining entities are the limits of space itself. If the modal principle fails, then pluriverses will fail to possess certain combinations of entities because some necessary connections preclude them. That goes against Lewis' picture. Nevertheless, they would still be pluriverses. Pluriverses are directly characterized in terms of what is possible for universes, not by a recombination principle governing their parts. Even if the modal recombination principle is false, however, pluriverses should still contain enough qualitative variety for my proof of safety to work. In my opinion the principle is true, but nothing much hangs on it.

It is important to note that no possible qualitative o-tokens or possible non-qualitative o-tokens concerning only actualities – qualitative or non-qualitative with respect to their meaning, that is – differ in meaning from every o-token part of a pluriverse.[21] Every possible meaning for a

[21] Regarding possible non-qualitative o-tokens, Neil Sinhababhu (2008) describes how immortal beings in other universes could uniquely describe actualities in ours, and they could likewise uniquely describe actualities in other universes if there are any. The immortal beings could use these descriptions to name actualities and inscribe non-qualitative o-tokens concerning them (e.g. such a being could write down an o-token that is the same in meaning as *x loved Caesar*, or *y is a better painting than the Mona Lisa*, or whatever non-qualitative o-token might be inscribed concerning an actuality.) And since every possibility for an immortal being doing so is an aspect of a way a universe can possibly be, every such possibility would obtain were there a pluriverse. Note that this reasoning assumes that the actualities coexist with the pluriverse. As indicated by the schema for 'pluriverse', a pluriverse is a fusion of universes such that every *qualitative* way a universe could possibly be is a way one of its universe parts is, and that the immortal beings are describing actualities depends on the *non-qualitative* fact that the actualities are present. No matter what descriptions they may create, immortal beings cannot describe actualities if actualities are

qualitative o-token or non-qualitative o-token concerning only actualities is expressed by an o-token in at least one way a universe can possibly be, so given that all these ways are exemplified in a pluriverse, the o-token parts of a pluriverse must collectively take on every possible meaning for o-tokens with the exception of those that concern merely possible individuals.[22] This is what ensures that were there a pluriverse, every qualitative sproperty or non-qualitative sproperty concerning only actualities would have a unique o-fusion corresponding to it, even those for which there is no corresponding actual o-token. Where there is an o-token with a given meaning, so there is a unique o-fusion of o-tokens with that meaning. This point will be returned to later.[23]

With these expressions introduced, I can now describe L+. L+ contains the following elements, dropping quotation marks for convenience: (a) the truth-functional connectives $\wedge$, $\rightarrow$, $\vee$, $\neg$; (b) universal and existential quantifiers of two sorts, $\forall_{a}$, $\exists_{a}$ and $\forall_{o}$, $\exists_{o}$; (c) variables for two sorts, x_{a}, x_{1a}, …, y_{a}, y_{1a}, …, etc., and x_{o}, x_{1o}, …, y_{o}, y_{1o}, …, etc.; (d) predicates for two sorts, $R+_{a}$, $R+_{1a}$, … (monadic), $R+_{aa}$, $R+_{1aa}$, … (dyadic) and so on for any finite arity, *and* R_{o}, R_{1o}, R_{2o}, etc. (monadic only); (e) constants for one sort, a_{a}, a_{1a}, …, b_{a}, b_{1a}, …, etc., (f) two identity predicates, $=_{aa}$ and $=_{oo}$; (g) a special predicate $O(\Phi_{a}\gamma_{a})_{o}$ for each open sentence $\Phi_{a}\gamma_{a}$ of L; (h) a definite description $[\iota z_{o}O(\Phi_{a}\gamma_{a})_{o}z_{o}]_{o}$ for every special predicate $O(\Phi_{a}\gamma_{a})_{o}$; (i) an additional special predicate, $\in_{ao}$; (j) a counterfactual operator, $\Box\!\!\rightarrow$; and (k) a single sentence letter, P.

not there to be described. However, since all the actualities would exist were there a pluriverse (a point for which I will argue later), this assumption is no real limitation.

[22] Not even in a pluriverse is it possible to refer to all possible individuals. Importantly, this point avoids Kaplan's-Paradox-like considerations that could be generated from non-qualitative properties pertaining to pluralities including merely possible objects. Thankfully, even platonists do not agree over whether such properties exist.

[23] Here I am discussing possible o-tokens and meanings, both of which are strange bedfellows with nominalism. But this is a choice of convenience rather than necessity. The point could be made by using a predicate for sameness of meaning and explaining how o-tokens cannot be so and so were there a pluriverse, but only laboriously.

As before, the sort 'a' is for actualities. Sort 'o' is for o-fusions. The 'O' predicates can be defined in the metalanguage through the following schema, where 'α_o' is a schematic letter for a term of sort 'o':

> $O(\Phi_a\gamma_a)_o\alpha_o$:= α_o is an o-fusion that fuses every o-token i such that for all γ, (i's free variable-token parts will be replaced with name-tokens for γ and i's other parts will not change in meaning $\Box\!\!\rightarrow$ the sentence-token that will result from the replacements will mean that $\Phi\gamma$).

Note that both the 'a' subscripts are dropped when interpreting the 'O' predicates. As with the constants of sort 'p' in L, this poses no formal difficulty: the predicates do not have any internal logical structure and are merely indexed by what occurs within the parentheses. This is also why it is fine to index them by predicates of L that are not predicates in L+. The definite descriptions are terms for the unique o-fusions satisfying the conditions corresponding to their associated 'O' predicates. For example, if 'R_a' is still interpreted as 'is red', '$[\iota z_o O(R_a x_a)_o z_o]_o$' should be read as 'the o-fusion that fuses every o-token i such that for all x, (i's free variable-token parts will be replaced with name-tokens for x and i's other parts will not change in meaning $\Box\!\!\rightarrow$ the sentence-token that will result from the replacements will mean that x is red)'. Such o-tokens would include, for example, **x* is red*, **y* is a red thing*, and **z* es rojo*. Each of these o-tokens has the following in common and would continue to have it common were there a pluriverse (the counterfactual point is a consequence of a thesis I will defend later): for any individual, were its free variable-token to be replaced with a name-token for that individual and its other parts not to change in meaning, the resulting sentence-token would mean that individual is red.[24]

[24] If there were a pluriverse, o-fusions would have parts in many different universes. This means that the 'will' used in the statement of what an 'O' predicate means must pick out, for each i, i's particular future. Objects in different universes are not only spatially isolated from each other, but temporally isolated as well. While this poses no special difficulty, it worth commenting on so as to make clear the intended meaning.

The inclusion of definite descriptions in L+ raises an important issue. In my proof of safety, I exploit facts about classical consequence that obtain among the consequents of some counterfactuals of L+ that have as their antecedent 'there is a pluriverse' when given their intended interpretation. One might expect that this is illegitimate given that L+ contains definite descriptions: classical quantifier rules are not valid in the logic of definite descriptions, which is a slight strengthening of free logic.[25] However, every definite description of sort '*o*' is such that were there a pluriverse, its associated 'O' predicate would uniquely apply to something. That would be sufficient for each definite description to refer. This can be established as follows:

> As mentioned earlier, no possible qualitative o-tokens or possible non-qualitative o-tokens concerning only actualities differ in meaning from every o-token part of a pluriverse. Therefore, were there a pluriverse, every open sentence indexing an 'O' predicate would be such that some o-token part of the pluriverse would have its meaning: the open sentences indexing the 'O' predicates are either qualitative or concern only actualities, each has a possible o-token (each could be tokened and is not composed of open sentences), and possible o-tokens share their meaning with the open sentences of which they are possible tokens. Since for each indexing open sentence there would be an o-token with its meaning, there would be a unique o-fusion that would fuse the o-tokens with its meaning. Necessarily, an o-fusion is such that an 'O' predicate indexed by a given open sentence applies to it iff the o-fusion fuses the o-tokens with the meaning of the indexing open sentence. (To help see this, consider the example of '$O(R_a x_a)$' and what the o-fusion to which it applies must be like.) Therefore, were there a pluriverse, every 'O' predicate indexed by an open sentence would apply to a unique o-fusion. Since every 'O' predicate is indexed by an open sentence, every 'O' predicate would apply to a unique o-fusion.

So all the definite descriptions would refer were there a pluriverse. Since, as I will argue, every actuality would exist as well and thus no constant of sort 'a' would become empty, there is nothing objectionable in exploiting facts about classical consequence that obtain among consequents of these counterfactuals in my proof. Every nearest pluriverse world is, in effect, a classical world with respect to everything L+ can describe, so it is safe to apply the classical quantifier rules for

[25] See John Nolt (2006, pp. 1039-1043) for discussion.

terms at them – even when those terms are definite descriptions. Classical consequence is truth-preserving at those worlds even if classical logic is not the correct logic for L+.

Every predicate of L+ has its meaning defined by the following condition: (i) if $\Phi_{a\ldots a}$ of L is an intrinsic predicate, $\Phi+_{a\ldots a}$ is the same in meaning as $\Phi_{a\ldots a}$; and (ii) if $\Phi_{a\ldots a}$ of L is an extrinsic predicate, $\Phi+_{a\ldots a}$ has the meaning that $\Phi_{a\ldots a}$ would have if the comparison indicated by $\Phi_{a\ldots a}$ were restricted in scope to actualities. Thus 'R_a' and '$R+_a$' are both interpreted as 'is red', but if 'W_a' is interpreted as 'is wide', '$W+_a$' is interpreted as 'is wide in relation to actualities'. I know of no systematic way to make this transition from extrinsic predicates to scope-restricted extrinsic predicates in the syntax of natural language, but that poses no difficulty here.

Predicates of sort '*o*' are restricted to predicates that say what kind of o-fusion a given o-fusion is, such is 'is a neurological o-fusion' or 'is a chemical o-fusion', and the identity predicate '$=_{oo}$'. For any predicate Φ_p of L, Φ_o is to be interpreted the same way except that 'property' is swapped for 'o-fusion'. '$\in_{ao}$' is a predicate for f-satisfaction in relation to the actualities, which I shall call f-satisfaction+. This is the aforementioned variant of f-satisfaction used in the proof. Just as one can say that a mountain satisfies **x* is wide* in relation to its immediate surroundings even if it is narrow compared to the mountains of its region overall, so one can say that something would satisfy **x* is wide* in relation to the actualities even were it to be narrow compared to enormously wide non-actualities. And if something would satisfy **x* is wide* in relation to the actualities even were that to be the case, so it would f-satisfy the o-fusion of every o-token that is the same in meaning as **x* is wide* in relation to the actualities. Note that if an o-fusion is a fusion of o-tokens that specify how something is intrinsically, such as the fusion of every o-token that is the same in meaning as **x* is ten-sided*, then something f-satisfies+ it iff it f-satisfies it *simpliciter*. After all, something is ten-sided in relation to the actualities iff it is ten-sided *simpliciter*.

Lastly, 'P' is interpreted as 'there is a pluriverse'.

8. Two Kinds of Substitution

I have now described the two languages, L and L+, mentioned in the outline of the proof I offer. To identify the function *f* that was mentioned in the outline, I must first describe two kinds of substitution between L and L+.

First, I define the *nominalization of the elements of L*, abbreviated as ⌜ nom(ϵ) ⌝ for any element of ϵ of L, as follows:

- For any truth-functional connective Θ of L, nom(Θ) = Θ
- For any predicate $\Phi_{a\ldots a}$ of L, nom($\Phi_{a\ldots a}$) := $\Phi+_{a\ldots a}$
- For any predicate Φ_p of L, nom(Φ_p) := Φ_o
- For any constant or variable γ_a of L, nom(γ_a) := γ_a
- For any variable γ_p of L, nom(γ_p) := γ_o
- For any constant $\langle\Phi_a\gamma_a\rangle_p$ of L, nom($\langle\Phi_a\gamma_a\rangle_p$) := $[\iota z_o O(\Phi_a\gamma_a)_o z_o]_o$
- nom('$\in_{ap}$') := '$\in_{ao}$'
- nom('$\forall_a$') := '$\forall_a$', nom('$\exists_a$') := '$\exists_a$', nom('$\forall_p$') := '$\forall_o$', and nom('$\exists_p$') := '$\exists_o$'

The *nominalization of a sentence φ of L*, or nom(φ), is the sentence that results from replacing, in order, every element of φ with its nominalization. Likewise, the *nominalization of a set Δ of sentences of L*, or nom(Δ), is the set of every nominalization of a sentence that occurs in Δ. The nominalization of any sentence of L is a sentence of L+.

It is a key fact that for every set of sentences Γ of L and sentence ψ of L, Γ proves ψ iff nom(Γ) proves nom(ψ). This is because from a classical perspective, nom(Γ) and nom(ψ) are notational variants of Γ and ψ, respectively. The only differences between them are as follows.

Every constant of sort '*p*' is swapped with a definite description of sort '*o*', which is treated as a classical constant of sort '*o*': see my earlier remarks on this point after definite descriptions of sort '*o*' were described. Every predicate $\Phi_{a\ldots a}$ is swapped with $\Phi+_{a\ldots a}$, except where it occurs in terms and 'O' predicates. These exceptions might seem to make trouble for the claim of notational variance, but remember that these predicates are only present in these items for indexing purposes anyway: these items do not possess internal structure that make them logically interact with other items in any special way. Finally, every remaining '*p*' subscript is swapped with an '*o*' subscript, which swaps variables for variables, quantifiers for quantifiers, and the instantiation predicate for the f-satisfaction+ predicate. In other words, constants, variables, predicates, and quantifiers of sort '*p*' – all the elements of Γ and ψ that are of sort '*p*' – are swapped with definite descriptions that are treated like constants, variables, predicates, and quantifiers of sort '*o*' – all the elements of nom(Γ) and nom(ψ) that are of sort '*o*' – and some predicates of sort 'a' and the instantiation predicate are swapped for different predicates of sort 'a' and the f-satisfaction+ predicate. This swapping preserves syntactic structure, at least with respect to what classical logic can see. While fairly obvious after the matter has been thought through, I draw attention to this fact because it will be useful later.[26]

[26] If there is any remaining doubt on this point, consider the following procedure. First, swap the constants of sort '*p*' with some unindexed dummy constants of sort '*p*' – 'a_p', 'b_p', and so on, adding these into L. Since the indexed '*p*' constants lack any internal structure, the resulting Γ′ and ψ′ are notational variants of Γ and ψ, and Γ proves ψ iff Γ′ proves ψ′. Then swap every predicate $\Phi_{a\ldots a}$ with $\Phi+_{a\ldots a}$ and every '*p*' subscript with an '*o*' subscript. The resulting Γ′′ and ψ′′ are obviously notational variants of Γ′ and ψ′, and Γ′ proves ψ′ iff Γ′′ proves ψ′′. Now swap every *x* that is a dummy constant of sort '*o*' – 'a_o', 'b_o', etc. – with the nominalization of the indexed '*p*' constant replaced by the '*p*' dummy constant from which *x* was arrived at via the subscript swapping. The result is nom(Γ) and nom(ψ). Given that the definite descriptions are treated like ordinary classical constants, nom(Γ) and nom(ψ) are notational variants of Γ′′ and ψ′′, and Γ′′ proves ψ′′ iff nom(Γ) proves nom(ψ). So Γ proves ψ iff Γ′ proves ψ′ iff Γ′′ proves ψ′′ iff nom(Γ) proves nom(ψ), which entails Γ proves ψ iff nom(Γ) proves nom(ψ).

With the notion of a nominalization of a sentence of L in place, I now turn to defining the notion of a *pluriverse substitution of a sentence φ of L*, abbreviated as ⌜ plur(φ) ⌝. The definition is simple. Let ‘S’ be a schematic letter for a sentence of L. Then:

plur(S) := ⌜ P □→ nom(S) ⌝

For example, if φ = ‘$\in_{ap}c_a\langle R_a x_a\rangle_p$’ and should be read as ‘c_a instantiates the property of being an *x* such that *x* is red’, then plur(φ) = ‘P □→ $\in_{ao}c_a[\iota z_o O(R_a x_a)_o z_o]_o$’ and should be read as ‘there is a pluriverse □→ c_a f-satisfies+ the o-fusion that fuses every o-token *i* such that for all *x*, (*i*’s free variable-token parts will be replaced with name-tokens for *x* and *i*’s other parts will not change in meaning □→ the sentence-token that will result from the replacements will mean that *x* is red)’. The *pluriverse substitution of a set Δ of sentences of L*, abbreviated as ⌜ plur(Δ) ⌝, is the set of every pluriverse substitution of a sentence that occurs in Δ. As with nominalizations, the pluriverse substitution of any sentence of L is a sentence of L+.

With the definition of pluriverse substitution in place, I can finally identify *f*. For every sentence φ of L, *f*(φ) = plur(φ): *f* is the function mapping sentences of L to their pluriverse substitutes. Having described L and L+ and identified *f*, I am ready to complete the proof of safety.

9. The Proof of Safety

Given the above identity of *f*, completing the following two tasks is sufficient to complete the proof of safety in §5. One is showing that for any finite set Γ of sentences of L and sentence ψ of L, if Γ proves ψ, then plur(Γ) proves plur(ψ) in a weak counterfactual logic. The other is establishing the following facts: (i) every nominalistic sentence of L is such that it is true iff its pluriverse substitute is true, and (ii) every platonistic sentence of L is such that it is true according to platonists only if its pluriverse substitute is true.

The first task is easy. As observed earlier, Γ proves ψ iff nom(Γ) proves nom(ψ). It is a standard principle of counterfactual logic that if some finite set of sentences Δ prove another sentence φ, then $\{x \mid \exists z(z \in \Delta \wedge x = \ulcorner \chi \;\Box\!\!\rightarrow z \urcorner)\}$ proves $\ulcorner \chi \;\Box\!\!\rightarrow \varphi \urcorner$: this is a corollary of deduction within counterfactual conditionals. Therefore, if nom(Γ) proves nom(ψ) and Γ is finite, then plur(Γ) proves plur(ψ) in a weak counterfactual logic. By biconditional elimination and hypothetical syllogism, it follows that if Γ proves ψ and Γ is finite, then plur(Γ) proves plur(ψ) in a weak counterfactual logic. QED.

The second task is more philosophical and difficult, but can be completed. First, consider (i): that every nominalistic sentence of L is such that it is true iff its pluriverse substitute is true. Every nominalistic sentence of L differs from its nominalization only in that its predicates are replaced with their '+' variants – predicates with very similar meaning, except that the scope of comparison is explicitly restricted to actualities. So the pluriverse substitute of any nominalistic sentence φ of L is a sentence in effect stating that however things are for actualities according to φ – and φ, being nominalistic, is entirely about actualities – is how they would be for actualities were there a pluriverse. For example, if a given sentence of L should be read as 'Mt. Everest is the tallest mountain', its pluriverse substitute should be read as 'were there a pluriverse, Mt. Everest would be the tallest mountain among the actualities'. Then to show that a nominalistic sentence of L is true iff its pluriverse substitute is, what must be established is (1) that nothing would change for actualities, either intrinsically or in how they relate to each other, were there a pluriverse (left-to-right direction), and (2) however the actualities would be were there a pluriverse, intrinsically or in relation to each other, is how they are in fact (right-to-left direction).

To establish (1), consider the following. In order for two spatiotemporal things to causally interact, they must be spatiotemporally related to each other. It wouldn't matter if there were a single electron spatiotemporally isolated from Earth or uncountably many of them: the history of Earth would have gone exactly the same as it has in fact. What goes for Earth goes for the rest of our solar system, Alpha Centauri, and anything at all in our universe. A consequence of this is that there are no causal relationships that cross the boundary of a single universe. Since everything is an actuality iff it is a fusion of actual spatiotemporal things (and itself spatiotemporal if there is only one universe, but it is an epistemic possibility that there are others), the fusion of the actualities is the fusion of every actual universe. And if there were a pluriverse, there would be many more universes, but none of them would interact with any of the actual universes. Actualities would be causally isolated from everything that would be added to reality were there a pluriverse. Since actualities would not be causally influenced by any of the new additions to reality the pluriverse would bring, they would not be any different intrinsically or in how they relate to one another.[27] So however actualities are in fact is how they would be intrinsically and in relation to each other were there a pluriverse. This establishes (1), and similar reasoning establishes (2). Therefore, a nominalistic sentence of L is true iff its pluriverse substitute is.[28]

So (i) is true. Now consider (ii): every platonistic sentence of L is such that it is true according to platonists only if its pluriverse substitute is true. Platonistic sentences of L can do a few things. One thing they can do is (a) describe what kinds of sproperties or specific sproperties

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[27] This might appear to depend on the assumption that all change involves causation, which can be challenged. For example, our universe is fine-tuned for carbon-based life. One might think that if there is in fact a pluriverse, then if our universe alone were to exist, it would not be fine-tuned: it would be highly improbable for there to be a fine-tuned universe in the absence of a pluriverse, and that fact in some way lowers the probability of our universe being fine-tuned if our universe alone exists. However, even if eliminating all but one of the universes from a pluriverse would change the properties of the remaining universe, it is difficult to see how adding the universes needed for there to be a pluriverse would make a difference to whatever universe is the starting point. No matter how the original universe is, the pluriverse must contain a universe qualititatively just like it. By the same rationale, even if the starting point contains multiple universes rather than a single one, the addition of the universes needed to make a pluriverse will not change the original universes. Given that fact, it seems unlikely that non-causal factors introduced by the addition of the pluriverse will make any difference to the actualities.

[28] I presented a more detailed explanation of the necessary causal isolation of universes and its impact on pluriverse counterfactuals in my paraphrase paper (2020, §4, especially fn. 26).

exist, and how kinds of sproperties or specific sproperties relate to other kinds of sproperties. This is the case with sentences that could be read as ‘there is a biological property’, ‘not every biological property is a mammalian property’ or ‘the property of performing photosynthesis is not a mammalian property’. Another thing they can do is (b) describe how sproperties relate to actualities, as with sentences that could be read as ‘every [actual] mammal instantiates mammalian properties’ or ‘no [actual] mammal instantiates the property of performing photosynthesis’ (the bracketed parts are often absent in natural language but entirely safe to add: every concrete object is an actuality. I make this point to stress just how comprehensive core sentences are). Finally, some of them also do what nominalistic sentences do while adding platonistic content: they (c) in part describe how actualities are. For example, a sentence that could be read as ‘every [actual] mammal instantiates the property of being warm-blooded iff it is warm-blooded and some [actual] mammal is warm-blooded’ does this in its right conjunct. Sometimes (a)-(c) will overlap. ‘There is a mammalian property such that every [actual] mammal instantiates it’ combines (a) and (b), for instance. Nonetheless, everything platonistic sentences of L do is a combination of (a)-(c). To establish (ii), I will argue that each of these three things that a platonistic sentence of L can do is such that if a particular sentence’s doing it is acceptable to a platonist, then the corresponding thing that sentence’s pluriverse substitute does is conducive to the truth of its pluriverse substitute. The argument could be presented more rigorously as an induction on sentences – assume there is a name for every sproperty and a definite description for every corresponding o-fusion, show that atomic platonistic sentences are acceptable to platonists only if their pluriverse substitutes are true, and then move on to logically complex sentences by justifying general claims about the connectives and quantifiers – but I shall confine myself to an intuitive justification.

Consider first sentences that do (c). Any part of a platonistic sentence of L that describes how actualities are does not differ from a nominalistic sentence that states what that part of the platonistic sentence states. Platonists and nominalists do not disagree with each other about concrete reality: an ideally-informed platonist and an ideally-informed nominalist will agree on how many trees there are, which people built which artifacts, and so on. Therefore, any part of a platonistic sentence describing how actualities are is acceptable to platonists iff it gets the relevant nominalistic facts about reality right. It gets those facts right iff were there a pluriverse, the actualities would be intrinsically and in relation to each other however they are in those nominalistic facts. To take 'every [actual] mammal instantiates the property of being warm-blooded iff it is warm-blooded and some [actual] mammal is warm-blooded' as an example, the right conjunct is true iff were there a pluriverse, some actual mammal would be warm-blooded. This is just an extension of the reasoning justifying (i).

Now consider sentences that do (a). Recall that no possible qualitative o-tokens or possible non-qualitative o-tokens concerning only actualities differ in meaning from every o-token part of a pluriverse. This means that were there a pluriverse, intuitively, there would be an o-fusion corresponding to every sproperty relevant to actualities – even those for which there is no corresponding actual o-token, an important fact since we sometimes make claims that depend on actually inexpressed properties (e.g. any two concreta with the same perfectly natural properties are perfect duplicates). Every sproperty is expressible, or at least every intrinsic sproperty and extrinsic sproperty solely concerning actualities is. That means that there would be o-tokens corresponding to each, and consequently one o-fusion corresponding to each. Moreover, just as properties can be categorized into kinds based on their instantiation conditions, so o-fusions can be categorized into kinds based on their f-satisfaction conditions. The two categorizations are parallel. For example, just as not every biological property is a mammalian property – the divide is based on what instantiates them – so not every biological o-fusion is a mammalian o-fusion – the divide is based on what f-satisfies them. The parallels in existence and categorization mean that sentences meeting condition (a) are true according to platonists only if their pluriverse substitutions are true: the existence and kind facts that supposedly obtain among properties find exact parallels in the existence and kind facts that would obtain among o-fusions were there a pluriverse. Thus ‘there is a biological property’, ‘not every biological property is a mammalian property’ and ‘the property of performing photosynthesis is not a mammalian property’ are true according to platonists iff ‘were there a pluriverse, there would be a biological o-fusion’, ‘were there a pluriverse, not every biological o-fusion would be a mammalian o-fusion’, and ‘were there a pluriverse, the property of performing photosynthesis would not be a mammalian o-fusion’ are true *simpliciter*.

Lastly, consider sentences that do (b). Recall that actualities would be exactly the same if there were a pluriverse, in themselves and with respect to each other, and that every sproperty would have a corresponding o-fusion that falls under o-fusion kinds parallel to the sproperty kinds the sproperty falls under. This means two things. First, whether a platonistic sentence of L expresses a general claim about sproperties (e.g. ‘Sam instantiates human properties’) or a claim about specific sproperties (e.g. ‘Sam instantiates the property of being a human’), its pluriverse substitute will describe a situation in which the corresponding o-fusions would be f-satisfied+ were there a pluriverse (e.g. ‘were there a pluriverse, Sam would f-satisfy+ human o-fusions’, ‘were there a pluriverse, Sam would f-satisfy+ the fusion of every o-token that is the same in meaning as *x is human*’). Second, every sproperty is such that it is in fact instantiated by an actuality iff some o-fusion correspondent of it would be f-satisfied+ by that actuality were there a pluriverse.

To see why, consider first intrinsic sproperties. No actuality would be intrinsically any different were there a pluriverse, and an actuality f-satisfies+ the correspondent of an intrinsic sproperty just in case it f-satisfies it. The instantiation condition of an intrinsic sproperty is the f-satisfaction condition of its correspondent, so an actuality instantiates an intrinsic sproperty iff were there a pluriverse, it would f-satisfy+ its correspondent. Now consider extrinsic sproperties. No actuality would differ in how it is related to other actualities were there a pluriverse. This means that an actuality instantiates an extrinsic sproperty iff were there a pluriverse, it would instantiate that sproperty in relation to the actualities. So the instantiation condition – as a matter of contingent fact, but no matter – of an extrinsic sproperty is the instantiation-relative-to-the-actualities condition it would have were there a pluriverse. But the latter is also the f-satisfaction+ condition its correspondent would have were there a pluriverse, so an actuality instantiates an extrinsic sproperty iff were there a pluriverse, it would f-satisfy+ its correspondent. Since every sproperty is intrinsic or extrinsic, this is enough to justify (b).

Having demonstrated (a)-(c), I have shown why (ii) is true. Since I have shown that (i) and (ii) are true, I have completed the second task necessary to establish the safety result. And since I have completed the first task earlier, I have established the safety result. There is a completely nominalistic explanation of why good platonistic arguments concerning properties of concrete objects never have false conclusions about concreta.[29]

10. Conclusion

In this paper, I have given a nominalistically acceptable proof that good platonistic arguments containing core sentences about properties of concrete objects never have false conclusions about

[29] Of course, in my justifications of (i)-(iii), I have talked as though properties exist. This is for ease of presentation. A more careful, nominalistic justification could be given by making clear that there is a parallel between facts about properties *according to platonists* and facts about o-fusions in pluriverses, being very careful to always affix "according to platonists" where necessary.

concreta. The proof I have offered makes no use of counterpossibles, instead relying on plausible principles regarding pluriverses, o-fusions, and f-satisfaction+. I will conclude by mentioning how the safety result could likely easily be extended to other kinds of sentences that are true according to platonists. In particular, a similar proof that good platonistic arguments containing core sentences about relations that obtain among concrete objects or propositions about concrete objects (extending the notion of a 'core sentence' in the obvious way to accommodate relations and propositions) never have false conclusions about concreta seems in the offing. One would add to L a sort for each adicity of relation and add to L+ a sort for each adicity of maximal, uniquely decomposable fusions of same-meaning open sentence-tokens (where the adicity is determined by the adicity of the open sentence-tokens it fuses), treating propositions as 0-place relations and sentence-tokens as 0-place open sentence-tokens for convenience. Analogs of f-satisfaction+ could be built to correspond to each adicity of bearing relation (the relation case) and truth (the proposition case). The metaphysical component of the proof would be much the same as here. The proof provided in this paper is only the lower limit of how far pluriverse counterfactuals can be used to establish nominalistic safety results.

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