# Doing the Math: Comparing Ontario and Singapore Mathematics Curriculum at the Primary Level

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## Abstract

This paper sought to investigate the fundamental differences in mathematics education through a comparison of curriculum of 2 countries—Singapore and Canada (as represented by Ontario)—in order to discover what the Ontario education system may learn from Singapore in terms of mathematics education. Mathematics curriculum were collected for Grades 1 to 8 for Ontario, and the equivalent in Singapore. The 2 curriculums were textually analyzed based on both the original and the revised Bloom's taxonomy to expose their foci. The difference in focus was then compared and discussed to find the best ways to improve the Ontario mathematics curriculum. With one of the best education systems in North America, the Ontario mathematics curriculum would only need to refocus its attention towards a more balanced approach, with greater focus on understanding through practices. Ontario would benefit greatly from a deeper research into the Singaporean math curriculum.



*Keywords*: Confucianism, effective education, curriculum, teaching strategies, mathematic curriculum

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# CHAPTER ONE: INTRODUCTION

In the past decades, Western societies have been considered more advanced for their superior education systems (Tan, 2017; Yelland, 2012). Most educators attributed the exceptional quality in education in the West to its scientific approach to inquiry. With education being an intrinsic part of a society, it is reasonable to say that the dominant philosophical ideas of a community determine the system of education. As a result, the Eastern education system is very different from that of the West. In the Western paradigm, the main influence comes from Aristotle's Realism and Plato's Idealism. Plato proposed an education system consisting of developing personal morals through questioning, discussion, and lectures to expose the underlying truth of matters (Abiddin et al., 2011; Hassan & Jamaludin, 2010). The Aristotelian school of thought, developing from Plato, believes the truth, or reality, is objective and observable, and can be scientifically quantified. The curriculum, in this philosophical realm, should emphasize the physical, scientific, and mathematical world. In tandem with mastery, students need to master critical thinking skills and demonstrate ability to conduct scientific experiments and observations. Education in East Asia, in particular, is heavily influenced by Confucianism, which views education as a means of perfecting a person's multiple roles in the society (Tan, 2017; Vuong et al., 2018). With such a different underlying philosophical base, education in Eastern cultures was perceived to be inferior in quality compared to Western schools of thought (Kember, 2016). The perception is prevalent both among educators in the Western paradigm as well as students from the East. This is evident in the flow of students from Eastern countries to the West for all levels of study, especially from developing nations, expecting to benefit from better education. In Canada alone, there were 494,525 international students in different levels in 2017, a 34% increase from 2014 (Global Affairs Canada, 2018). Also, the Canadian Bureau for International Education (2018) details the top hosting country for international students as the U.S.A., with more than one million international students in 2017.

Despite the perception of inferiority, Eastern education has been receiving attention from the international community. Singapore and other East Asian countries have scored high in international studies conducted over the years, and the strength of these nations appears to lie in their teaching of science and mathematics. According to the OECD through PISA, most Asian students outperform their North American counterparts and are close to surpassing most European students. The latest comprehensive PISA survey in 2018 (shown in Figure 1.1) ranked a majority of Asian countries as top performers, with China (591), Singapore (569), Macao (558), Hong Kong (551), Chinese Taipei (531), and Japan (527) being the highest performers, while the highest ranking country within the OECD is Estonia (523) and the top performer for North America is Canada with 512 (Schleicher, 2019).

Rankings in PISA have changed over the past three surveys (2012, 2015, and 2018), but the top performers remained the same. One of the top performers amongst the Western nations is Canada; however, Canadian students' performance in mathematics still fell short compared to Singapore (Roser et al., 2018). With more focus on mathematics and science, the Trends in International Mathematics and Science Study (TIMSS) reported in 2015 that the top performing countries in terms of mathematics score were Singapore (618), Hong Kong (615), Korea (608), and Chinese Taipei (597); the U.S.A. averaged 539, and Ontario, Canada, fell short at 512 (Provasnik et al., 2016).

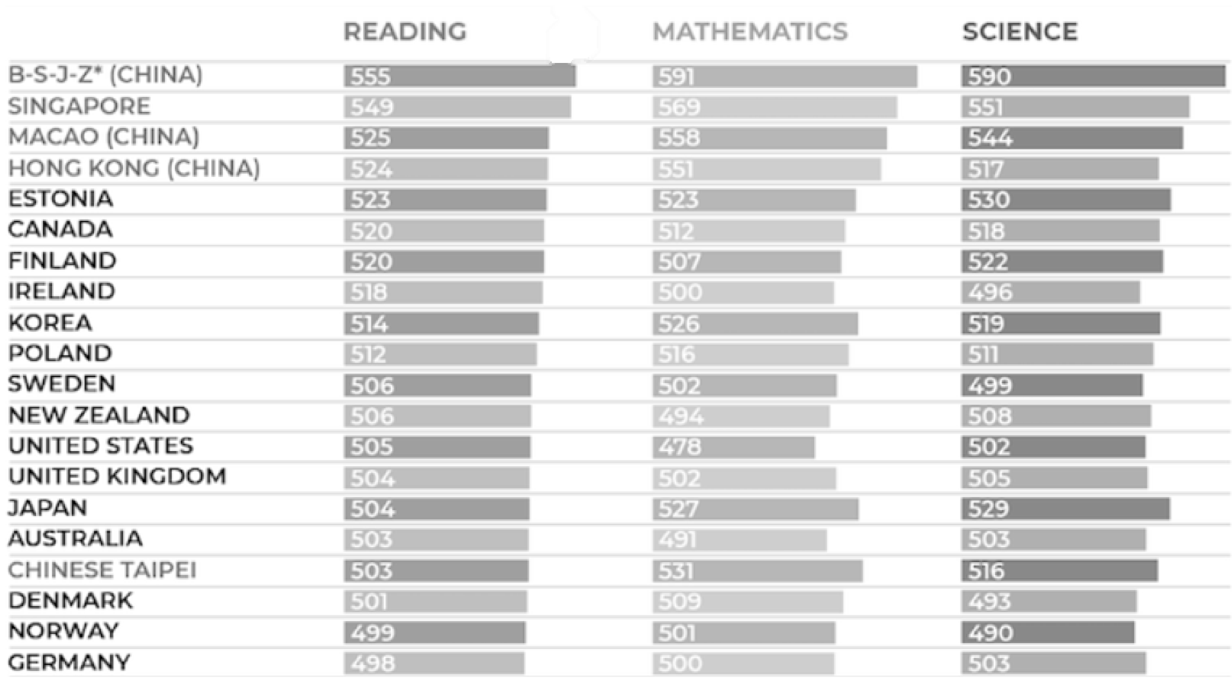


*Figure 1.1.* PISA 2018 average scores, extracted from OECD (2018)

This suggests that Singapore has not only a superior mathematics education system compared to Canada and the U.S.A., but also questions the long-considered superior pedagogic philosophy of the West. Evidently, there are qualities in the Eastern education system that have been overlooked. In fact, following the TIMMS results in 2015, various countries have been adopting the Asian mathematics curriculum, with Australia being one of the first to consider adopting “Beijing Mathematics.” More notably, the Singaporean mathematics materials including books and guides are being used in some school in both the U.S.A. (Hu, 2010) and the U.K. (Warrell, 2017).

Thus, this paper investigates the fundamental differences in mathematics education through a comparison of the curriculum’s goals and objectives of the two countries—Singapore and Canada—in order propose potential improvements to mathematics education in Canada. While Singapore has a national education system, the Canadian system is provincial. However, with Ontario being the most populous province and among the leaders of education and pedagogy in Canada (The Conference Board of Canada, 2014), this paper will focus on Ontario as representative of Canada.

## Research Objectives

In contemporary education, one of the issues is the apparent outperformance of East Asian students compared to their Western counterparts. With evidence of higher student performance in Eastern mathematics and science, an obvious question to ask is what these high-performing countries have in common in terms of characteristics. Seeking answers to these questions may provide some clues about the factors that lead to such performance. The common perception amongst educators has been that Asian students are more prone to employ rote learning strategies, which had been associated with poor learning outcomes in Western educational research (Yelland, 2012). Despite such perception, Asian students perform just as well as their Western peers in international comparative studies (Hassan & Jamaludin, 2010; Kember, 2016; Provasnik et al., 2016). Therefore, it is appropriate to re-examine the existing research about learning in Asia to ascertain whether the differences in learning approaches have any effects on performance. This paper revisits the research into differences between Western education philosophy and Eastern education culture that is heavily influenced by Confucianism. It reconsiders the importance of relevant education objectives for creating effective curriculum that enhances high performance in students and attempts to provide a different perspective to the Bloom's taxonomy of education objectives. This research aims to bring a different point of view to curriculum and provides a more effective comparison between the chosen representatives of each education philosophy.

## Research Questions

This paper seeks responses to three specific questions: (a) What are the educational goals and objectives and how they are outlined in the curriculum? (b) What is the intersection between the two curricula—the similarities, differences, and features in common? (c) What can both education systems learn from each other?

## Methodology

This paper follows a comparative research methodology to answer the stated research questions. The curriculum of each country will be explored and analyzed with focus on education objectives. The result will be compared for similarities, as well as differences. These differences will then be discussed to find ways that students in Ontario may improve their performance. The comparative research method is optimal for the purpose of this research, since it allows direct comparison of the educational objectives.

## Scope

The two main international testing and survey systems referenced in this paper are PISA and TIMMS. While the PISA included all students between 15−16 years of age, the TIMMS tests students in Grade 4 (9-year-olds) and Grade 8 (13-year-olds). Ideally, the scope of the study should include all curriculum from Grade 1−9 inclusively to cover both primary and secondary education levels. However, due to the Singaporean curriculum diverging into separated streams from Secondary 3 (equivalent to Grade 9 in Ontario), consistent comparison can only be made within the first 8 years of school. The scope of this study only includes curriculum for children ages 6 to 13. At this age range, the grade equivalent in Ontario would be from Grade 1 to 8. At the same time, the equivalent in Singapore would include Primary 1 to 6, and Secondary 1 and 2.

## Structure

First, existing literature is explored in Chapter 2, aiming to answer the three main theoretical bases: (a) the difference between the Eastern education paradigm and the Western education approach, (b) the construct of an effective curriculum and the role of education objectives, and (c) unique considerations in mathematics education. These are combined to create the theoretical background for the study's later comparison. Next, the research methodology of this study is presented and evaluated in Chapter 3. Chapter 4 presents the case that was collected from secondary sources, including an in-depth description of each curriculum, and the actual comparative analysis highlighting the similarities and differences between the chosen curricula. Chapter 5 then presents implications of this study for Ontario, regarding curriculum objectives and focus. The final chapter discusses the conclusions as well as the limitations of the study.

# CHAPTER TWO: LITERATURE REVIEW

While this paper looks specifically into the mathematics curriculum, no comparison could be done before exploring the fundamentals of the two differing education systems. This chapter will start by examining the different underlying philosophies by comparing their approaches to inquiry itself. The Western paradigm will be explored first to provide a base for comparison. The Confucius education paradigm will then be explained in-depth to attempt to explain both the Chinese learners' paradox and bridge the existing gap in understanding. Curriculum design and effective education will then be discussed to provide background on why having the relevant objectives within the curriculum is the basis for more productive outcomes. The review of literature ends with a brief discussion of the challenges of math education.

## The Western Approach to Inquiry and Learning

In general terms, the Western philosophical base for education is founded on two schools of thought: traditional and modern. The traditional philosophy stems from Athens, Rome, and Judeo-Christianity, while Eastern philosophy develops from Islam, Confucianism, Taoism, and Mahayana Buddhism (Hassan & Jamaludin, 2010; Merriam & Kim, 2008). These philosophies fundamentally influence the world view and certainly create their education system (Kim & Kim, 2013; Sadovnik et al., 2013). In the Western education paradigm, the main influence comes from Aristotle's Realism and Plato's Idealism. In *The Republic*, Plato proposed a "duality of mind and body" in which the physical world is separated from the human's consciousness. Within such philosophy, the goal of human enlightenment is to seek for the universal truth that is "orderly and regular." The goal of education within this realm of philosophy consists of developing personal morals through questioning, discussion, and lectures to expose the underlying truth of matters. The education process in Plato's Idealism also includes learning from history and imitating past actions to positively affect present problems. Developed from Plato's Idealism, the Aristotelian school of thought believes the truth, or reality, to be objective and observable, and scientifically quantified. The curriculum, in this philosophical realm, should emphasize the physical, scientific, and mathematical world. At the same time, students are expected to learn through demonstration and recitation to master facts and knowledge. In tandem with mastery, students must be able to master critical thinking and demonstrate ability to conduct scientific experiments and observations.

When looking into contemporary education, Westerners stress active learning for their learners; thus, students within the Western paradigm appear as, and are encouraged to be active learners in the classroom, as teaching and learning focus not on teacher-centered strategies but become more learner-centric (Kember, 2016; Yelland, 2012). The idea of active learning includes encouraging students to actively take part in learning and planning. Through this process, the students become the main drive in creating an effective learning environment, and not the teacher (Abiddin et al., 2011). For example, students are encouraged to critically think and express their views in classrooms' activities such as group discussions or in-class assignments. Following such discussion, the students are normally asked to present their findings to the entire class. This sequence of activities enables the students to be actively involved in the learning process, both within and outside the classroom (Hassan & Jamaludin, 2010). Apart from that, through the effort of the teachers in assigning materials before class, the students are also given an opportunity to express their ability and talents by analyzing and solving problems on their own. The topics will be discussed during class time the following days. By doing this, every student is given a chance to learn and practice researching skills, as well as critical thinking. In fact, children develop their critical thinking ability on the basis of the problem-solving process (Lawson, 2000; Parker, 2015). Therefore, Education development in the West could be said to includes critical thinking, communication skills, collaboration, and problem solving skills (Abiddin et al., 2011; Hassan & Jamaludin, 2010), and are part of 21st century learning skills.

Furthermore, schools based on a Western education philosophy also support and encourage students in self-management and in controlling their own learning process (Hassan & Jamaludin, 2010). This means describing the teachers' roles to lead and guide the students by giving them guidelines rather than direct instructions. In the classroom, children share more responsibility in their leaning process, assessment, and evaluation. Assessments are conducted to determine their capability to self-manage and also focus on the application of knowledge. In the Western education philosophy, students are encouraged to learn by understanding. Students do not only memorize what they have learned; they are also encouraged to understand what they are learning (Abiddin et al., 2011; Yelland, 2012). This gives the students opportunities to understand the topic that they are learning in a deeper way and to develop critical thinking skills. In addition, they also have more time to have group discussions with their fellow classmates and also to do research on the topic assigned. The lifeless textbooks are replaced, and students are taught to take notes and ask questions when the teachers are teaching (Hassan & Jamaludin, 2010; Parker, 2015). Thus, students are able to pay attention to the teachers who more likely guide them to understand what has been taught (Parker, 2015).

There are both benefits and challenges to this approach to teaching. To begin with, the Western education philosophy emphasizes active learning that involves students' active participation in class and group discussions. Such education builds students' confidence to speak in front of the class, which results in creating outspoken and confident individuals who do not fear to voice their thoughts (Abiddin et al., 2011). It also aims to encourage students to understand and accept others' opinions, as well as developing their interpersonal skills as they exchange ideas with each other (Abiddin et al., 2011; Hassan & Jamaludin, 2010). In this way, students listen to the ideas of others, learn from their peers, and at the same time are encouraged to express their own perspectives. While active learning is a time-consuming process and may result in the teacher's inability to complete the syllabus, it is beneficial in providing different kinds of learning experiences to the children (Aksit, Niemi & Nevgi, 2016). This is a good initiative to be implemented in education as it helps to build positive self-concept and boost self-confidence among students.

With positive self-concept, students are more likely to have positive thoughts about themselves and the world. Once they believe that they can do it, they will work hard to achieve the goal (Abiddin et al., 2011; Hassan & Jamaludin, 2010), the Western education paradigm has long been considered superior in quality for its output of highly motivated, creative, and knowledgeable students into the workforce (Kember, 2016; Yelland, 2012). This is apparent in the influx of students from the developing nations in Asia to the developed Western world. Globally, the number of international students continues to increase, a rebound from the slowdown due to the 2008 global financial crisis (Global Affairs Canada, 2018). The U.S.A. and the U.K. take in about one-third of all internationally mobile students (Canadian Bureau for International Education, 2018). Of the four major English-speaking international student destinations, international student enrollment increased 42% between 2008 and 2014 in the U.S.A. Figure 2.1 further illustrates the trend in the number of international students in selected number of host countries.

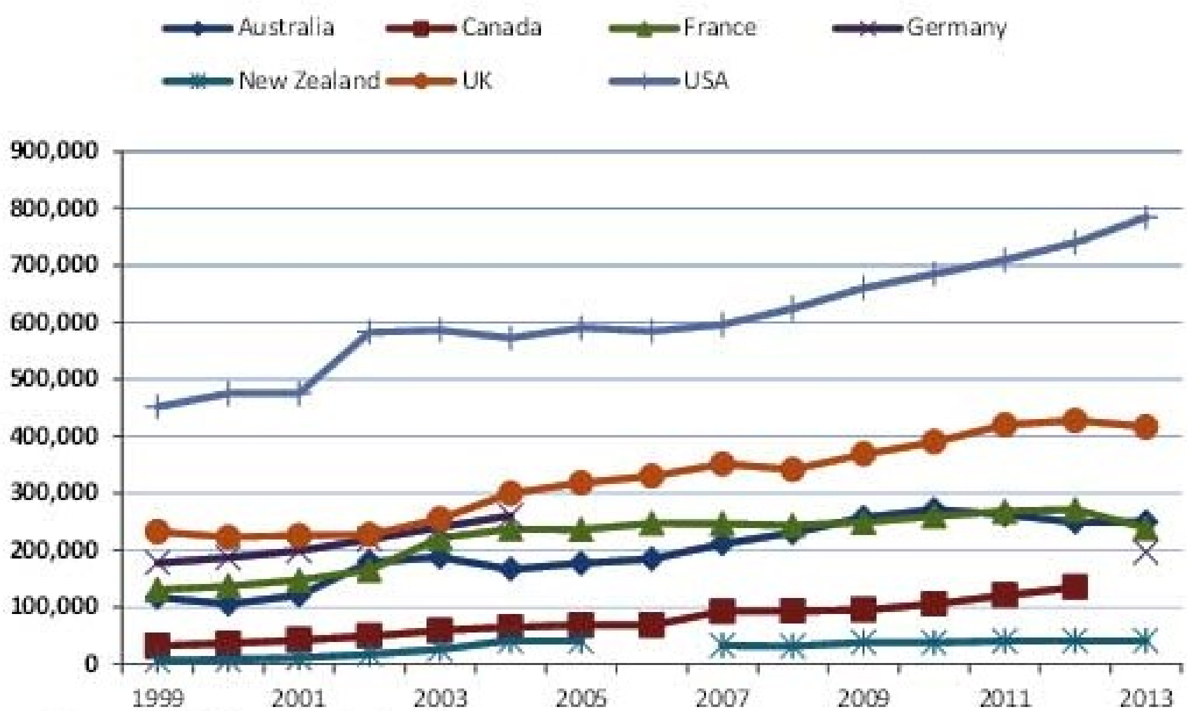


*Figure 2.1*. Intake of international students in developed nations. Source: Global Affairs Canada (2018).

According to the latest Institute of International Education (IIE) Open Doors data, international students in higher education brought in US$39 billion to the U.S. economy in 2018, creating more than 455 thousand jobs (National Association for Foreign Student Affairs, 2019). Similar to the U.S.A., Australia is the third most popular destination for international students, attracting nearly 7% of the world's international students (Canadian Bureau for International Education, 2018). The Australia Council of Private Education and Training (ACPET) report in 2013 estimated that international students contributed AU$15.4 billion to the Australian economy. Export of international education services (including postsecondary and other non-postsecondary enrollments) is considered the fourth largest export from Australia. Evidently, education export is contributing a considerable amount to the developed economies. With Canada being the fourth option, maintaining and improving quality, output, and student performance in all school levels could expand such demand and positively impact the economy.

## The Chinese Learners' Paradox

In the last part of the 20th century, a major research theme focused on trying to explain what became known as the "paradox of the Chinese learner" (Kember, 2016; Leung, 2002). There were widespread perceptions that Chinese students had a propensity for rote learning, which have been found to be associated with poor learning outcomes in Western educational research (Kember, 2016; Tran, 2013; Yelland, 2012). Yet comparative studies had found that Chinese students performed at least as well as their Western counterparts (as shown in Chapter 1) via multiple international standardized test results. Research into the paradox produced two contributing explanations:

1. Research in Hong Kong and China uncovered evidence of a set of approaches to

learning, intermediate between pure surface and deep approaches, which combine memorization and understanding (Kember, 2016; Leung, 2002). Observations of Chinese students apparently attempting to memorize material could have been misinterpreted as rote memorization, when in fact the memorization was combined with attempts to reach understanding, and was, therefore, not a surface approach (Kember, 2016).

2. When Chinese students do employ a surface approach, it is likely to be a response to perceptions of contextual factors in the teaching and learning environment, rather than as a characteristic of a cultural group or a predominant regional trait (Kember, 2016; Tran, 2013).

Since the early 1990s when international education researchers started to study the Asian paradigm, comparative international testing has advanced in “ambit and rigor” (Kember, 2016, p. 2). The performance of Chinese or East Asian students has become noteworthy. The paradox was originally formulated as suggesting that there was no evidence of the inferior performance which might be expected from a preponderance of rote learning (Kember, 2016; Tran, 2013). Recent results from the testing by PISA, reviewed previously, would suggest that there is now clear evidence of superior performance (Roser et al., 2018). It therefore seems timely to re-examine the body of research into the approaches to learning of Chinese learners in an attempt to see whether it has any part to play in explaining the superior performance.

Research into the paradox of the Chinese learner was prompted by a commonly advanced perception that Chinese learners were more prone than their Western counterparts to employ rote learning (Kember, 2016). The observation had been widespread in anecdotal form, but affirmations in print are also quite common. The following quotation, from the official minutes of a course planning committee in a university in Hong Kong, is sufficient to establish how entrenched negative perceptions of Chinese students were at the time systematic investigations of the perceived phenomenon started.

> Students in Hong Kong ... expect lecturers to teach them everything that they are expected to know. They have little desire to discover for themselves or avail themselves of the facilities which are available to them within the teaching institution. They wish to be spoon fed and in turn they are spoon fed. Lecturers are under pressure to feed the student with a certain amount of academic and community needs information and the simplest way to do it … is to adopt the old and traditional approaches to teaching. (Minutes of the [...] Course Planning Committee, 1989, p. 13, as cited in Kember, 2016, p. 3)

In the past, literature has reported extensively on how memory is overused in Eastern education. As explored above, within the Confucius educational philosophy, understanding is one of the major goals which is aided by mastery through either memorization or practice. While memorization is a major part of education in Asia, personal efficacy and mastery are the goals of such drills. Especially in mathematics education, the previous impression that Western educators have when studying Eastern education philosophy is flawed. Historically speaking, Western educators interpreted mathematics drills as repetition that enhances only memory. This misinterpretation of drills in repetition is shown in TIMMS results, with Asian students regarding memorization as insignificant. Singaporean students were especially among those considering memorization as not important at all, second to only Slovenia.

Rote learning is seen as an undesirable approach to learning, which when adopted by Western students has tended to be associated with poor learning outcomes (Tran, 2013). Rote learning has been associated with a surface approach to learning, which is normally envisaged as less desirable than a deep approach, particularly in higher education (Tran, 2013; Yelland, 2012). Abundant research into approaches to learning has shown that such an approach to learning should lead to poor learning outcomes (Yelland, 2012; Parker, 2015). Such contradiction between theory and actual performance make the claim difficult to ignore. Thus, the following section explores Eastern education philosophy in-depth from an insider's perspective.

## The Influence of Confucianism in Eastern Education

Eastern students normally acquire knowledge directly from teachings of their religion, such as Islam, Buddhism, Confucianism, Hinduism, and Taoism (Merriam & Kim, 2008; Tan, 2017). As explored by Hassan and Jamaludin (2010), the Eastern paradigm is more of a one-way transmission of knowledge. The teaching and learning process through Eastern education philosophy stresses the major outcomes are from the teachers through the teacher-directed learning process. This means that teachers are fully responsible for the class effectiveness by preparing and planning all the activities for their students (Cheng & Wong, 1996; Leung, 1998). Thus, there is no doubt that a good teacher needs to be hardworking in delivering ideas, teaching, and maintaining a good relationship with their children (Hassan & Jamaludin, 2010; Tan, 2017). For instance, the students are not trained or required to do anything before class, since all materials are provided by the teachers (Tran, 2013). Usually, they are not required to carry out any research on the topics that are going to be discussed in the next lesson and are responsible only to receive input from their teachers (Tran, 2013). Thus, compared to the Western educational model, Eastern education philosophy holds on to the concept of teaching where students receive full knowledge from the teachers inside the classroom and the students are considered to be passive learners (Hassan & Jamaludin, 2010). Within the Western research body, it is widely accepted that in the Eastern education the students practise the concept of memorizing, as this philosophy focuses mainly on book learning and memorization within the teaching and learning process (Cheng, 1998; Yelland, 2012). However, such generalization shows where the majority of existing literature inadequately explains. A key question confronting a Confucian conception of education is whether such a paradigm is able to nurture critical and creative thinkers who are empowered to critique prevailing worldviews and effect social changes. Recent textual analysis of the “Analects” (*Lunyu*) and “Records of Learning” (*Xueji*) by Tan (2017) revealed the opposite to what was expected of a surface-learning education system.

**Confucian Education Philosophy In-Depth**

Confucianism itself is not an education system but rather a philosophy created by Confucius to advise kings and nobles on ways of life. Despite not being an education model, a prominent theme in Confucianism is education. In fact, Confucius himself devoted his whole life to teaching his disciples and persuading the political leaders of his time to enact his educational ideals (Tan, 2017). Tan (2017) explored the Confucian canon known as the Four Books and Five Classics (*sishu wujing*). To understand the significance of education in Ancient China, a researcher also has to understand the history of the country. Throughout all the major dynasties, China practised extreme nepotism, where children of officials or kings were heirs to the same office positions (Tan, 2017). While the system filled the majority of office positions, there were bound to be shortages of actual skills and knowledge. Thus, every year, a number of civilians were chosen to be "promoted" through a series of examinations (Tan, 2017). Thus, Confucius wrote Xueji as study materials for students looking to take part in the national contests. Tan explains that by the time of Xueji (one of the last books in the series), an educational system comprising schools in the villages and a national academy in the capital already existed. Although Xueji was written specifically for students preparing for office positions, the educational principles discussed are applicable to all learners and reflect the essence of Confucian education (Tan, 2017). The central place of education in Confucianism is stated in the opening passage of Xueji:

> If a ruler desires to transform the people [and] perfect [their] customs, [the ruler] can only do so through education! … The goal is to radically change the people by refining their conventional ways of thinking and doing. The reference to transformation and perfection in the above verse signifies that the scope is extensive, going beyond skills training and cognitive advancement to paradigm shift and character development. The actualization of this aim of education naturally requires a normative standard to guide the ruler in knowing whether and when the people have been transformed and their customs perfected. This standard is revealed in Xueji II to be "dao" (Way), which is the object of learning. "Dao" is the Way of Heaven (tian) or "guiding discourse." To realize "dao" is to

> understand and experience the "vision of human excellence." (As cited in Tan, 2017, Aim of Education section, para. 1)

The term curriculum as used in this article refers to the totality of learning experiences provided to students. This means that the curriculum includes not just the contents to be studied, but also all planned activities, programs, events, and functions that take place in a variety of learning sites (Tan, 2017). Following the aim of education to realize and broaden "dao," Confucian curriculum should be holistic, broad based, and integrated (Tan, 2017). A rounded education affirms a Confucian mandate for students to transcend theoretical knowledge of "dao" by appreciating and abiding in it; mere head knowledge is rejected by Confucius.

Confucius reiterates the deficiency of mere intellectual knowledge in another passage when he asks rhetorically:

> [If a person can] recite three hundred poems but is incapable of performing an entrusted official duty and exercising [one's] initiative when sent abroad, what good are the many poems [to that person]? (As cited in Tan, 2017, Curriculum section, para. 2)

Within the Confucius paradigm, individuals are encouraged to reinforce and put into practice what they have learned through self-cultivation and social interaction (Tan, 2017). Tan (2017) goes on to explain that a person self-cultivates by gradually and steadfastly appropriating the symbolic resources and sharable values from "dao" that form the basis of a Confucian curriculum. Self-cultivation presupposes that the realization of "dao" depends ultimately on oneself. That success is obtained through nurture (self-cultivation) rather than nature is taught by Confucius, who observes that "human beings are similar in their nature, but differ as a result of their practice" (as cited in Tan, 2017, Curriculum section, para. 4).

Besides being holistic, the curriculum is also broad-based. The Analects stresses the primacy of learning widely and broadening oneself with culture (Tan, 2017). The "culture" mentioned is the normative tradition of "dao" that is encapsulated in the Zhou dynasty (Tan, 2017; Vuong et al., 2018). A broad-based curriculum, therefore, introduces learners to varied defining aspects of the Zhou culture, such as its literature, arts, and ceremonies (Kim, 2013; Tan, 2017). These domains of learning or subjects are part of the six arts (*liuyi*) in Ancient China that consist of rituals, music, archery, charioteering, calligraphy or writing, and mathematics (Tan, 2017). Tan (2017) explains that the third characteristic of a Confucian curriculum is integration. The six arts are not unrelated and discrete disciplines, nor are they taught theoretically without real life application. Instead, the six arts are interconnected, mutually reinforcing, and practice-oriented (Tan, 2017; Tran, 2013). To facilitate the synthesis of subjects, the curriculum should be well-structured and progressive. Xueji V outlines a 9-year program that systematically introduces students to a value- centered, rounded, and comprehensive curriculum (Tan, 2017). It can be observed that the curriculum is structured in such a way that the students learn by "accumulating [what one has learned]" (*Xueji* IX) by consolidating and adding to the knowledge base (as cited in Tan, 2017, Curriculum section, para. 6). Tan explains that the idea of widening and deepening one's learning from a solid foundational knowledge is also propounded by Confucius; he reminds learners "not to forget what one has acquired monthly" (Analects 19.5) and "to keep alive the old in order to know the new" (Analects 2.11; as cited in Tan, 2017, Curriculum section, para. 6).

With the Confucian paradigm explored thoroughly, there are apparent agreements and disagreements with the existing Western paradigm. It is timely to investigate what makes a curriculum effective, and by doing so to narrow this study's focus.

**Confucianism in Singapore**

Singapore is one of Asia's most diverse countries, both culturally and religiously (Tan, 2012). Historically speaking, Singapore is a relatively new nation, granted independence from the Federation of Malaysia in 1965 (Han, 2007). Over the years, the first Prime Minister of Singapore, Lee Kwan Yew, has used the term "Asian values" and "Confucian values" interchangeably in his debates as well as philosophical values that was established to foster the country's racial and cultural harmony. The term ''Asian values'' was first used in the 1970s by academics attempting to explain the seemingly miraculous economic growth of the East Asian states of Japan, Singapore, South Korea, Taiwan, and Hong Kong by referring to the cultural values found in those societies (Hill, 2000; Teik, 1999; Xu, 2005). Lee Kuan Yew—who also used the term "Confucian values"—spoke in terms of "hard work, strong family ties, sacrifice for the future" as well as "respect for education, learning, and entrepreneurial spirit, filial piety, respect for elders… and freedom in an orderly society" (Xu, 2005, as cited in Han, 2007, p. 5). One way in which the ideals of "Asian" values are promoted in Singapore is through the education system (Han, 2007). The influence of Chinese culture in Singapore (the most influential being Confucianism) is expected considering the population's composition. Figure 2.1 shows a majority of the population being Chinese at 74%. As the dominant ethnic group, Chinese culture and values are prevalent in Singapore, including philosophical and cultural values from Confucianism.

Specifically, the education system in Singapore is centralized, with textbook makers having to follow rigorous standards set out by the Singaporean government (Han, 2007). It is reasonable to expect the education system in Singapore would also be heavily affected by Confucianism and other East Asian values.

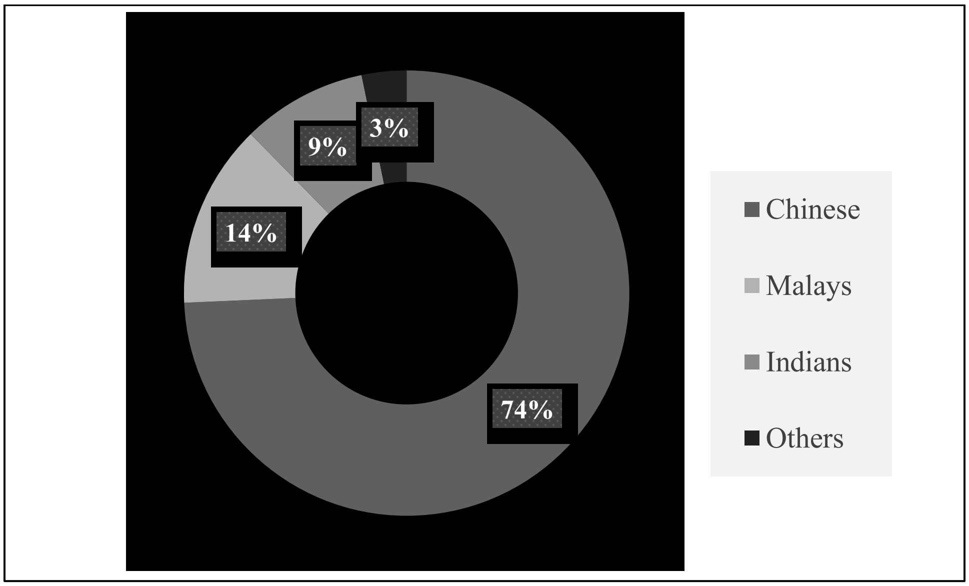


*Figure 2.2.* Singapore population by ethnic group.

## Defining Effective Education

John Abbott (1994) defines learning as the process that includes reflective activity which enables the learner to draw upon previous experience to understand and evaluate the present, to shape future action, and to formulate new knowledge. In this definition, learning and education include an active process in which the learner relates new experience to existing meaning and may accommodate and assimilate new ideas (Abbott, 1994; Abiddin et al., 2011). This definition also suggests that the past, present, and future are connected, implying un-learning and re-learning although a linear connection is not assumed. Dennison and Kirk (1990) describe four elements in a learning process, drawing on the model by Kolb (1984). Figure 2.2 displays the model and its involved processes.

This cycle highlights activity in learning (Do), the need for reflection and evaluation (Review), the extraction of meaning from the review (Learn), and the planned use of learning in future action (Apply; Dennison & Kirk, 1990). The model describes the process for an individual learner who is actively making sense of a learning occasion, or for a group of learners involved together (Dennison & Kirk, 1990). Whatever the overall time scale, time is required for individuals to reflect, make meaning, and move forward. While the model above lays out sufficiently the activities within a classroom, the model does not specify prior conditions (e.g., how learners select what to learn, the beliefs which the learner brings) or the context in which learning happens. Indeed, this definition does not refer to other people in the context: teachers, facilitators, peers, et cetera. Other elements are included in a model developed by Biggs and Moore (1993; see Figure 2.4).

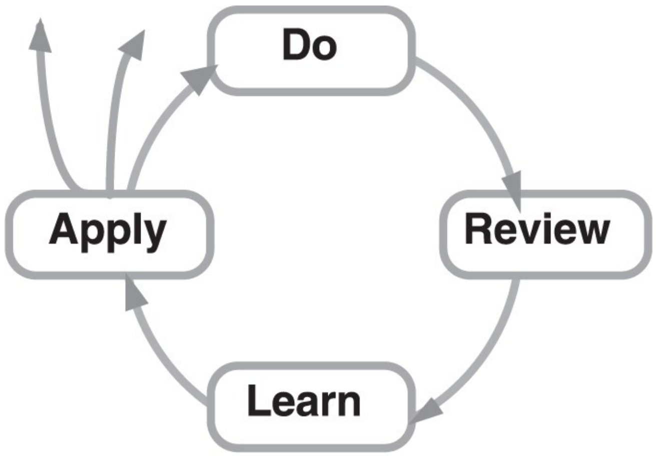


*Figure 2.3.* The Dennison and Kirk learning cycle. Source: Dennison and Kirk (1990).

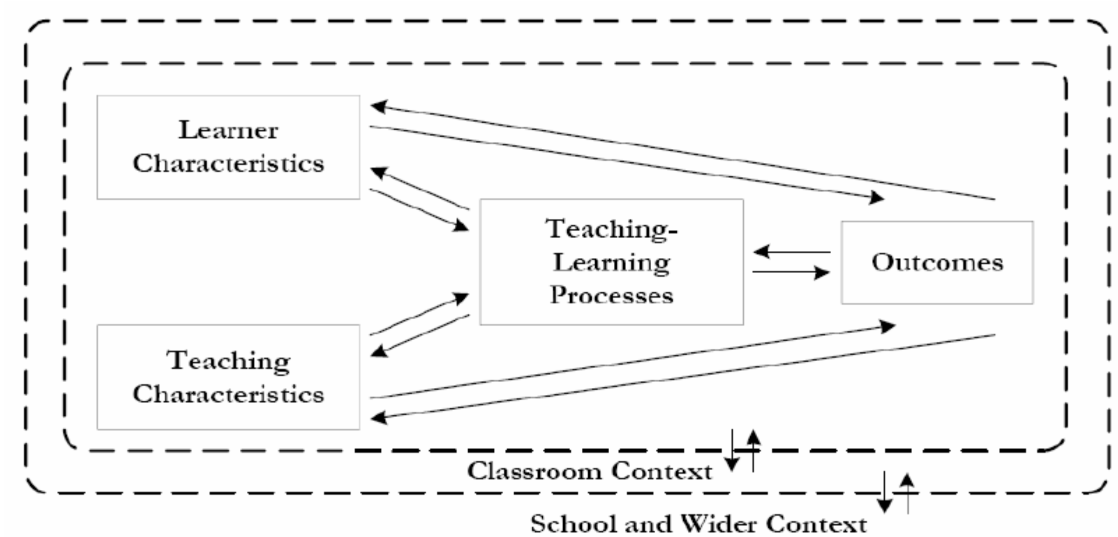


*Figure 2.4.* The process of learning. Source: Biggs and Moore (1993).

Although the term "effective" has been widely used, it only makes sense when context and goals are specified (Biggs & Moore, 1993). As explored by Biggs and Moore (1993), within a defined context, effectiveness can easily be measured by time, target, and outcome. The contemporary educational context, however, has changed since the inception of the above model. The knowledge base in society is increasing rapidly, and now doubles every 373 days; information is also not the possession of a few "experts" but rather the majority is being processed by a wider range of the population (Watkins et al., 2002). With the two mentioned characteristics, the landscape of learning is much wider and richer, involving multiple contexts, modes, and sources. While the learner, context, and wider context cannot be influenced by the educators, teachers can change teaching and the actual learning process to facilitate better learning outcomes (Watkins et al., 2002). Watkins et al. (2002) explain that "teaching characteristics" mean features of curriculum, assessment, and conceptions of teaching. Even when external aspects of these are poorly designed, classrooms and schools can employ effective learning to address them. Curriculum which addresses big ideas and which gives learners the big picture is most engaging, according to Watkins et al.s' findings. Therefore, curriculum with coherence should be crafted for the learner and the ability to make connections in different contexts should also be considered (Watkins et al., 2002). With more focus on the curriculum, a larger state-wide curriculum should focus on creating effective and well-defined outcomes and goals (Confrey & Stohl, 2004; Leung, 2002).

Effective curriculum goals and objectives have two major outcomes. First, students should acquire the same knowledge from the same courses, regardless of the teacher (Confrey & Stohl, 2004). Second, the students should finish a course ready to move on to the next course, year, or life stage. In this sense, the learning goals and objectives aim to standardize learning objectives for all students and simplify outcomes for teachers (Confrey & Stohl, 2004). Curriculum goals are generally defined as broad statements that lead towards long-term outcomes (International Bureau of Education, 2020). These goals are typically designed to be met by students after a longer period of time such as a year of schooling or a series of courses in a discipline. Specifically, goals are always farther-reaching than objectives, and as such are usually based on the idea that they lead students towards being better able to be productive members of their societies (International Bureau of Education, 2020). Goals can incorporate a variety of processes, including skills to be learned, attitudes to be adopted, or concepts to be understood. Objectives, on the other hand, are related to goals in that they are specific methods through which students can demonstrate their understanding or application of goals (International Bureau of Education, 2020). Some objectives should be longer-term, with students needing to work towards mastery over a year or semester. Other objectives, on the other hand, should be completed over a shorter period such as one week or unit. While creating goals and objectives is difficult in itself, determining how and what will be beneficial is even more so (Confrey & Stohl, 2004; International Bureau of Education, 2020). Bloom's taxonomy of educational objective was crafted to aid in this process.

## A Closer Look at Curriculum

Before going deeper into mathematics education, it is beneficial to define in detail the term "curriculum" used in this paper. It is also important to be fully aware of relevant research on comparative studies in mathematics curriculum done in the past, so that the present study can benefit from these past studies and build upon the knowledge accumulated so far.

According to Taba (1962), the foundations for a comprehensive theory of curriculum planning were only laid in the 1930s, although studies related to the curriculum have a much longer history. Before the 1930s, educators had very vague notions about the term “curriculum.” It was roughly used to refer to the course of study offered by an educational institution, and a course of study was usually meant the content of the instruction. In fact, many educators at that time did not distinguish between a curriculum and a syllabus. Ralph Tyler was perhaps the first educator to systematically study this concept of curriculum and insisted that it was more than mere content plus instruction. In his classic book *Basic Principles of Curriculum and Instruction*, Tyler (1949) listed four basic aspects to be considered in connection with a curriculum: purpose, instruction method, organization, and assessment. The model conceived as a result of this components was a linear model, as shown in Figure 2.4. The model specifies not only the components of a curriculum but also the relationship between the components. The linear, downward model represents a chronological relationship from top to bottom. Thus, according to this model, we should first of all determine the aims and objectives of the educational programme. Based on the aims and objectives, the content of instruction is chosen, which in turn determines how the content is to be organised. Lastly, to complete the process, clear specification of how the aims and objectives are to be evaluated has to be laid down. Assessments, according to Tyler (1949), should be designed to evaluate students’ level of understanding in relative to the material, rather than evaluation of the curriculum itself.

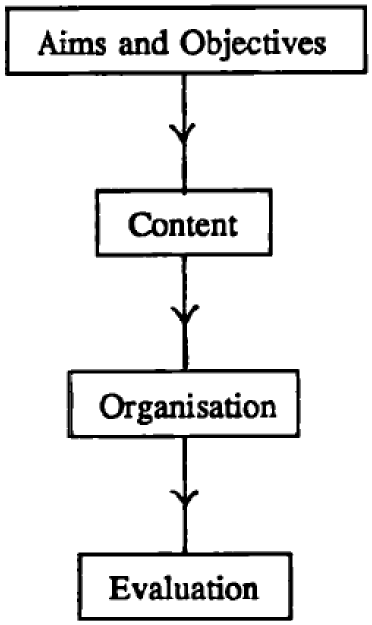


*Figure 2.5.* Tyler's model of curriculum design. Source: Tyler (1949).

Other curriculum models could be considered derivatives of Tyler's original model. For example, Wheeler's model shown in Figure 2.5 has nearly identical components compared to that of Tyler's linear model. Wheeler (1967) , however, suggested that the curriculum design process is a cycle, instead of a linear finite process. The cyclical model proposed by Wheeler suggests that the curriculum is seen not as a static structure but as the product of an ongoing, developmental exercise. The evaluation phase should inform and determine the aims and objectives in the next cycle, which would in turn lead to new selection of learning experiences and content.

While Wheeler's model creates a cycle for curriculum design, there was a lack of relationship between each component that to Kerr (1968) was insufficient to create an effective curriculum. Kerr (1968) proposed an interconnected model for curriculum development that stresses the relationship between all components. All the components of a curriculum, in Kerr's view, affect each other. The curriculum objectives, for example, affect not only the knowledge chosen in the instruction, but also the learning experience to be organized for the students and the curriculum evaluation. Evaluation, on the other hand, does not only inform the aims and objectives but also affects both the knowledge taught and the organization of the learning experience for students, including methods of teaching used. While the abovementioned models work with any subject fields, the Second International Mathematics Study (SIMS), which was the predecessor of the quoted TIMMS result in the introduction, proposes a mathematic curriculum-specific model. The conceptual model used in the SIMS conceives a curriculum as being made up of three level: the intended, the implemented, and the attained levels (Travers, 1980). The intended curriculum refers to intention at the educational system level. This information can be obtained from government officials and from officially published documents such as syllabuses, curriculum guides, and circulars. The implemented curriculum refers to the curriculum as transacted in the classroom, and there are various reasons that prevent this from being identical to the intended curriculum. The third level pertains to the curriculum as attained by the students. Figure 2.6 demonstrates the SIMS model.

The SIMS model, unlike the models designed for curriculum planning reviewed so far, is essentially a model developed for the purpose of research and evaluation and contributes in highlighting the importance of looking at three important but distinct levels in the curriculum, a fact that is often overlooked by some researchers (Leung et al., 2014). Often, in conducting curriculum research, researchers use the same label to discuss things at quite different levels (e.g., using the word “content” without making clear whether it is the intended content or the implemented content). The SIMS model has clarified the conception of this complex notion of a curriculum (Leung et al., 2014).

Frederick (1992) proposed a revised, more holistic model to be used in future curriculum research, shown in Figure 2.7. In the model Frederick (1992) suggested, there are three components at the (system) intended level. They are the aims and objectives as specified in the official documents, the intended content to be covered as set out in the official syllabus, and the officially intended methods to be use. In this model, teachers’ beliefs and intentions are perceived as mediation between the intended and implemented curricula, and so are placed between the intended and the implemented levels (Frederick, 1992). By mediation, it is meant both when the teacher interprets the intention of the system and when the teacher holds a view that is different from the intention of the system.

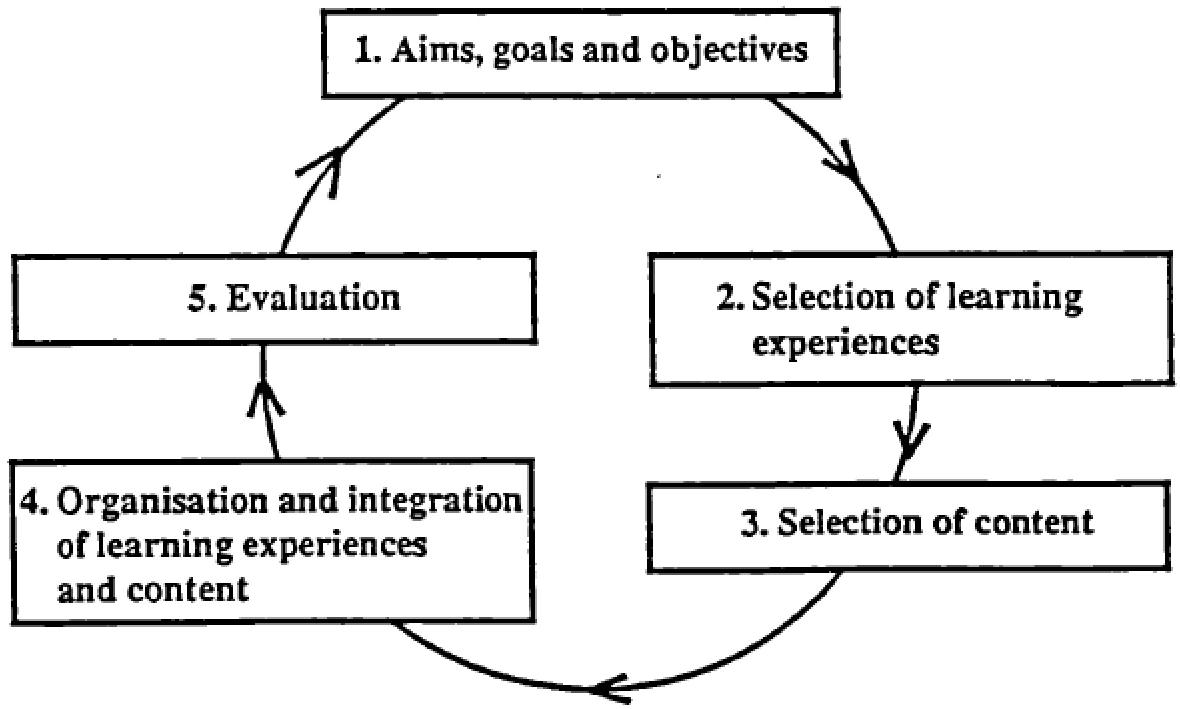


*Figure 2.6.* Wheeler's model of curriculum design. Source: Wheeler (1967).

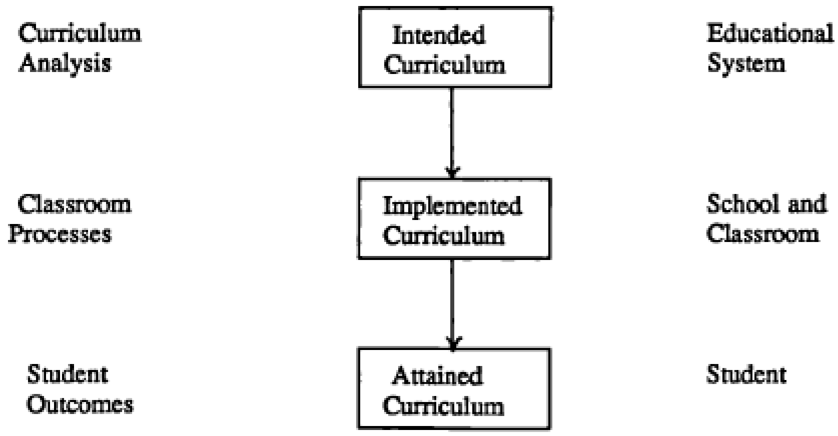


*Figure 2.7*. The SIMS model. Source: Travers (1980, p. 189).

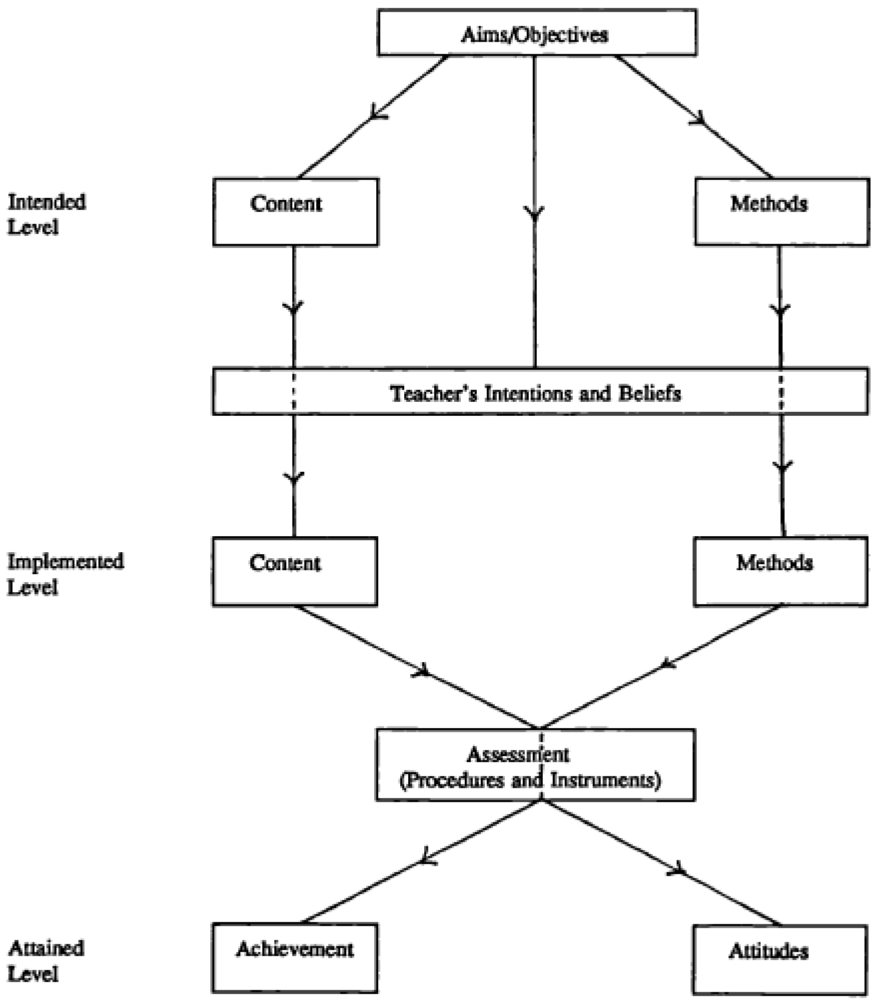


*Figure 2.8.* Frederick's model for curriculum study. Source: Frederick (1992).

In either case, the influence of the intended curriculum on the implemented curriculum is only indirect. The attained curriculum comprises student achievement and attitudes, and these are affected by the teaching methods and content at the implemented curriculum level. Studying the assessment procedures and test instruments is important both in understanding the intentions of the system and in interpreting test results of achievement and attitudes (Frederick, 1992).

For the purpose of this research, the materials that are reviewed focus mainly on the first level: intended level. Since the attained level is highly varied and is readily available through comparative studies conducted by international organizations, it is redundant to delve deeper into students' performance data. Furthermore, the attained level is the final product of a combination of factors through which the intended level manifests.

**Bloom's Taxonomy of Educational Objectives**

As learners, we know from experience that some learning tasks are more difficult than others. To take an example from elementary school, knowing our multiplication tables by rote requires a qualitatively different type of thinking than does applying our multiplication skills through solving ‘‘word problems.’’ In both cases, a teacher could assess students' knowledge and skills in either of these types of thinking by asking them to demonstrate those skills in action, in other words, by doing something that is observable and measurable. With the publication in 1956 of the *Taxonomy of Educational Objectives: The Classification of Educational Goals*, an educational classic was born that powerfully incorporated these concepts to create a classification of cognitive skills (Armstrong, 2020; Krathwohl, 2002). Bloom's taxonomy contains six categories of cognitive skills ranging from lower-order skills that require less cognitive processing to higher-order skills that require deeper learning and a greater degree of cognitive processing. The differentiation into categories of higher-order and lower-order skills arose later; Bloom himself did not use these terms (Krathwohl, 2002).

As Armstrong (2020) explains, knowledge is the foundational cognitive skill and refers to the retention of specific, discrete pieces of information like facts and definitions or methodology, such as the sequence of events in a step-by-step process. Knowledge can be assessed by straightforward means; for example, multiple choice or short-answer questions that require the retrieval or recognition of information (Armstrong, 2020; Krathwohl, 2002). Health professionals, for example, must have command of vast amounts of knowledge such as protocols, interactions, and medical terminology that are committed to memory, but simple recall of facts does not provide evidence of comprehension, which is the next higher level in Bloom’s taxonomy (Krathwohl, 2002). Learners show comprehension of the meaning of the information that they encounter by paraphrasing it in their own words, classifying items in groups, comparing and contrasting items with other similar entities, or explaining a principle to others (Krathwohl, 2002). Comprehension requires more cognitive processing than simply remembering information and learning objectives that address comprehension will help learners begin to incorporate knowledge into their existing cognitive schemas by which they understand the world (Armstrong, 2020). This allows learners to use knowledge, skills, or techniques in new situations through application, the third level of Bloom’s taxonomy.

Many teachers use this level as part of their performance-based activities for students, such as script writing after book review, storyboarding for a recent film, or construct a model for the lesson (Armstrong, 2020). Moving to higher levels of the taxonomy, learning objectives relate to analysis. Here is where the skills that are commonly thought of as critical thinking enter. Distinguishing between fact and opinion and identifying the claims upon which an argument is built require analysis (Armstrong, 2020; Krathwohl, 2002). Following analysis is the level of synthesis, which entails creating a novel product in a specific situation. Finally, the pinnacle of Bloom's taxonomy is evaluation, which is also important to critical thinking (Armstrong, 2020). When instructors reflect on a teaching session and use learner feedback and assessment results to judge the value of the session, they engage in evaluation (Krathwohl, 2002).

The taxonomy is useful in two important ways. First, use of the taxonomy encourages teachers to think of learning objectives in behavioural terms and considering what the learner can do as a result of the instruction (Krathwohl, 2002). A learning objective written using action verbs indicates the best method of assessing the skills and knowledge taught. Lists of action verbs that are appropriate for learning objectives at each level of Bloom's taxonomy are widely available on the internet (Armstrong, 2020). Second, considering learning goals in light of Bloom's taxonomy highlights the need for including learning objectives that require higher levels of cognitive skills that lead to deeper learning and transfer of knowledge and skills to a greater variety of tasks and contexts (Armstrong, 2020).

Based on findings of cognitive science following the original publication, a later revision of the taxonomy changes the nomenclature and order of the cognitive processes in the original version. In this later version, the levels are remember, understand, apply, analyze, evaluate, and create (Krathwohl, 2002). Krathwohl (2002) and other researchers propose reorganization that places the skill of synthesis rather than evaluation at the highest level of the hierarchy. Furthermore, this revision adds a new dimension across all six cognitive processes. It specifies the four types of knowledge that might be addressed by a learning activity: factual (terminology and discrete facts); conceptual (categories, theories, principles, and models); procedural (knowledge of a technique, process, or methodology); and metacognitive (including self-assessment ability and knowledge of various learning skills and techniques; Krathwohl, 2002).

**Bloom’s Taxonomy and Mathematics Education**

Given the prevalence of testing in mathematics and the regular use of mathematics as a context for studying student reasoning and problem solving, Bloom’s taxonomy has been applied and adapted by mathematics educators since its publication (Thompson, 2008). The terms and goals, however, are defined differently specifically for mathematics education, depending on mathematical topic being taught. Figure 2.9 outlines sample questions based on different levels of the taxonomy.

Resnick (1987) noted that thinking skills resist precise forms of definition, but lower- and higher-order thinking can be recognized when each occurs (as cited in Thompson, 2008). Lower-order thinking (LOT) is often characterized by the recall of information or the application of concepts or knowledge to familiar situations and contexts. Thompson (2008), citing Schmalz (1973), notes that LOT tasks requires a student “to recall a fact, perform a simple operation, or solve a familiar type of problem. It does not require the student to work outside the familiar” (p. 97). In contrast, Resnick

(1987) characterized higher-order thinking (HOT) as “nonalgorithmic”; in general, HOT involves solving tasks where an algorithm has not been taught or using known algorithms while working in unfamiliar contexts or situations (as cited in Thompson, 2008).

| | New Terms | Actions | Learning Activities |
|---|---|---|---|
| Higher-order thinking | **Creating**<br><br>(Putting together ideas or elements to develop an original idea or engage in creative thinking). | Designing<br>Constructing<br>Planning<br>Producing<br>Inventing<br>Devising<br>Making | ***Creating:*** *(*Generating new ideas, products, or ways of viewing things) *How could we determine the number of pennies in a jar without counting them? Apply and integrate several different strategies to solve a mathematical problem. Design a new monetary system or an experiment for establishing ...*<br>Designing, constructing, planning, producing, inventing.<br>*Invent a machine to do a specific task. Develop a menu for a new healthy foods restaurant.* |
| | **Evaluating**<br><br>(Judging the value of ideas, materials and methods by developing and applying standards and criteria). | Checking<br>Hypothesizing<br>Critiquing<br>Experimenting<br>Judging<br>Testing<br>Detecting<br>Monitoring | ***Evaluating:*** (Judging the value of a product for a given purpose, using definite criteria) *Develop a proof ... and justify each step ..., Using a definition ... determine ...* Justifying a decision or course of action, checking, hypothesizing, critiquing, experimenting, judging<br>*What criteria would you use to evaluate if your answer is correct? Prepare a list of criteria to judge ... Evaluate expressions.* |
| | **Analyzing**<br><br>(Breaking information into its component elements to explore relationships). | Comparing<br>Organizing<br>Deconstructing<br>Attributing<br>Outlining<br>Structuring<br>Integrating | ***Analyzing:*** *(*Breaking information into parts to explore understandings and relationships) *Given a math word problem, determine the strategies that would be necessary to solve it. Write a paragraph describing the relationship ..., How does ... compare to ...*<br>Comparing, organizing, deconstructing, interrogating, finding<br>*Design a survey to find out ... Graph your results. Use a Venn Diagram to show how two topics are the same and different. Translate between visual representations, sentences, and symbolic notation. Make predictions based on experimental or statistical data.* |
| Lower-order thinking | **Applying**<br><br>(Using strategies, concepts, principles and theories in new situations). | Implementing<br>Carrying out<br>Using<br>Executing | ***Applying:*** (Using information in concrete situations) *Compute the area of actual circles. Use the given graph to ..., Choose and describe the best method to ...*<br>Using information in another familiar situation, implementing, carrying out, using, executing<br>*Draw a diagram which shows these fractions or take photographs of the fractions. Determine measures of central tendency and dispersion Write a journal entry. Write an explanation about this topic for others.* |
| | **Understanding**<br><br>(Understanding of given information). | Interpreting<br>Exemplifying<br>Summarizing<br>Inferring<br>Paraphrasing<br>Classifying<br>Comparing<br>Explaining | ***Understanding:*** (Grasping the meaning of material) *Given the mathematical formula for the area of a circle, paraphrase it using your own words. Select the graph that illustrates*<br>Explaining ideas or concepts Interpreting, summarizing, paraphrasing, classifying, explaining<br>*Find items that you can use to show the fractions. Retell or write in your own words... Report to the class ... Write a summary report of the event.* |
| | **Remembering**<br><br>(Recall or recognition of specific information). | Recognizing<br>Listing<br>Describing<br>Identifying<br>Retrieving<br>Naming<br>Locating<br>Finding | ***Remembering:*** (Remembering previously learned material) *State the formula for the area of a circle. State the rule of ..., Explain and use the procedure for ...*<br>Recalling information, recognizing, listing, describing, retrieving, naming, finding, locating<br>*List the fractions you know and can show. List the attributes of your shape. Make a concept map of the topic. Make a chart showing...* |

*Figure 2.9*. The revised Bloom's taxonomy adapted for mathematics. Source: Thompson (2008).

The thinking skills in BT considered LOT include knowledge and comprehension, while the thinking skills of analysis, synthesis, and evaluation are considered HOT, Application often falls into both categories. In BT, for a test item to be at the level of application or higher, a “new situation” (to the student) is required. Bloom emphasized in his original work in 1956 and subsequent discussions on this issue (Bloom et al., 1971, Furst 1981) that application and higher levels in the taxonomy do not refer to test items where only minor changes are made, but otherwise, the procedure was the same to that practiced in class. As Bloom et al. (1981) stated:

> By “new problems and situations” we mean problems and situations which are likely to be new to the student. These are similar to those which were included in the instruction but have some element of newness or unfamiliarity for the student. Students should not be able to solve the new problems and situations merely by remembering the solution to or the precise method of solving a similar problem in class. It is not a new problem or situation if it is exactly like others solved in class except that new quantities or symbols are used (as in mathematics or physics). (p. 233)

## Current Challenges in Mathematics Education

Mathematics and mathematics education possess unique features and challenges compared to other subject matters. This section will explore the meaning of mathematics education, and also look at some current challenges in the new digital age.

Mathematical literacy is defined by the United Nations Educational, Scientific and Cultural Organization (UNESCO) as the fundamental priority objective of mathematics taught during basic education. Numeracy enables the development of mathematical knowledge and competencies necessary for integrated and active participation in a given society and for adaptation to foreseeable changes in that context. Mastery of basic numeracy and measurement, which has long constituted the rudiments of mathematical knowledge required for participation in society, no longer suffices today (UNESCO, 2012). Such change in demand could be attributed to the digital culture that is increasingly embedded in our contemporary societies. Traditionally, the content of mathematics education for all has consisted of numeracy, the decimal system, arithmetic operations, and the capacity to solve elementary arithmetic problems such as problems of proportionality, knowledge of systems of magnitudes, and knowledge of common two- and three-dimensional geometrical forms. These remain the core bases of mathematical literacy (UNESCO, 2012).

With the changing environmental and technological context, mathematics education faces drastic changes in approach, assessment, technological, research, diversity, and educators. Since this paper focuses on the quality of teaching, the education approach and educators is the scope of exploration. Investigating challenges in these aspects gives a general picture of how curriculum could change to address and improve upon these hurdles. Research on classroom practices in the context of teaching and training studies and surveys conducted by international institutions show that, for the moment, basic mathematics has not changed much in approach or teaching practices (Robinson, 2006). The current practices in the West, according to Robinson (2006), are considered boring for their formality, vagueness, lack of real-life application, lack of relevance to technology, and tack of creativity in practices.

As in many other disciplines, teachers are the key to the positive and sustainable development of education systems, especially in mathematics. In fact, they now constitute the principal challenge to quality mathematics education for all (UNESCO, 2012). The problems are manifold, quantitative, and qualitative. In some countries, the profession of basic education teacher has a good social image—wages are acceptable, if not attractive, and working conditions are good. All of this helps to make the profession attractive (UNESCO, 2012). Unfortunately, this situation is far from being the norm, even in developed countries, as shown by the serious teacher recruitment and retention problems experienced in a number of such countries (Dorfler, 2003; OECD, 2005). The declining interest in mathematics study at university makes the problem more acute, generating a vicious circle (Holton, 2009).

The second challenge is in quality, because in many countries the quality of teacher education is far from satisfactory, even when there is no quantitative problem. As stressed earlier, expectations of basic education have risen substantially (Dorfler, 2003).. To meet these growing demands, teachers must be well trained mathematically and pedagogically. However, as found by a UNESCO (2012) report, most basic education teachers have experienced difficulties in their own mathematics education and have a negative image of the discipline. They are often general-purpose teachers and their credit hours of science education and, above all, mathematics education account for only a fraction of their training. This context makes their initial and in-service training all the more problematic (Dorfler, 2003; UNESCO, 2012).

The qualitative aspect of teachers' knowledge was explored in a research study by An et al. (2004), who found a clear discrepancy within the unique mathematics pedagogical skills of teachers themselves, regardless of actual mathematics knowledge. It was made clear that in respect to mathematics, educators in the U.S.A. and China differed greatly not only in approach, but also in understanding of students' varying performance level. The Chinese system emphasizes gaining correct conceptual knowledge by reliance on traditional, more rigid development of procedures, while educators in the U.S. emphasize a variety of activities designed to promote creativity and inquiry to develop concept mastery, but often has a lack of connection between manipulatives and abstract thinking, and between understanding and procedural development (An et al., 2004). An et al. focused not only on approach, but also on the pedagogical content knowledge of the teachers, stating that an effective teacher must also possess a deep and broad knowledge of teaching and curriculum or profound pedagogical content knowledge apart from being the subject master.

There are major differences between Ontario, Canada and Singapore, starting from education philosophy itself. However, as Cheng (1998) has explained, values in education can be borrowed and transposed to complete a different philosophy. While underlying beliefs could not be shifted, methodologies and strategies could be borrowed. The following chapters will investigate one of the main ways that a curriculum could adapt to better itself through creating more optimal objectives. With mathematics education possessing unique characteristics and challenges, it is important to investigate what is effective, as well as the underlying reasons to fully understand and implement.

## Challenges in Canadian Mathematics Education

Looking into mathematics education in Canada, the curriculum is also perceived to be underperforming both by parents and analysts (Flanagan, 2018; Little, 2017). Both problems outlined by Flanagan and by Little are consistent with the overall current challenges set out by the United Nations for mathematics education. Flanagan (2018) claims that the current system of "discovery math" is a controversial system that overly focuses on inquiry-based learning strategies while neglecting memorization and practices that are considered lower-order skills. The University of Toronto's Mary Reid also notes that the two education philosophies mentioned are complementary rather than contradictory (as cited in Flannigan, 2018). To investigate the current problem in mathematics education, the Education Quality and Accountability Office (EQAO) administers standardized math tests in Ontario. These tests main goals are "to assess how well Ontario's public education system is developing students' reading, writing and math skills. EQAO provides reliable and useful information that is used to help improve student achievement and ensure the accountability of school boards" (Source, Date, p. X). Questions may arise from the results. The EQAO's latest data, released in August 2018, stated that 61% of Grade 3 students and 49% of Grade 6 students met the provincial standard for math. Both numbers have fallen steadily through the 2010s. Grade 6 students saw the bigger drop, peaking at 63% in 2009, while Grade 3 scores topped out at 71% in 2010 (Flanagan, 2018).

The secondary problem outlined by Little (2017) was that of skill shortage of the educators themselves. The author quoted in the report existing statistics from a previous study that saw more than 80% of Grade 3 and Grade 6 teachers possessing no postsecondary education in mathematics (Little, 2017). Similar to the problem found by the United Nations quoted earlier, the bottleneck in competent mathematics teachers creates huge barriers for improvements and requires more attention and investment to rectify.

# CHAPTER THREE: RESEARCH METHODOLOGY

Comparative research entails studying two or more similar groups, individuals, countries, events, or conditions by comparing them with respect to specific characteristics. Through such comparisons, comparative research offers a mechanism to understand and evaluate the factors that shape and change our world. It can provide insight into world events, a greater understanding of the governments and systems that exist throughout the world, a means for learning from past mistakes, and a greater understanding of other cultures. To some extent, all research is comparative in nature and comparative research offers many benefits and advantages (Creswell, 2007).

Comparative analysis performs several important functions that are closely interlinked. More specifically, comparative analysis enhances the understanding of one's own society by placing its familiar structures and routines against those of other systems (understanding) (Eisner, 1991; Phillips & Scheisfurth, 2014); comparison heightens our awareness of other systems, cultures, and patterns of thinking and acting, thereby casting a fresh light on our own political communication arrangements and enabling us to contrast them critically with those prevalent in other countries (awareness) (Esser & Vliegenthart, 2017); comparison allows for the testing of theories across diverse settings and for the evaluating of the scope and significance of certain phenomena, thereby contributing to the development of universally applicable theory (generalization) (Phillips & Scheisfurth, 2014; comparison prevents scholars from over-generalizing based on their own, often idiosyncratic, experiences and challenges claims to ethnocentrism or naïve universalism (relativization) (Phillips & Scheisfurth, 2014); and comparison provides access to a wide range of alternative options and problem solutions that can facilitate or reveal a way out of similar dilemmas at home (alternatives) (Phillips & Scheisfurth, 2014).

What distinguishes comparative research from simple border-transgressing kinds of (international/transnational) research is that comparativists carefully define the boundaries of their cases. This can be accomplished in a variety of ways based on structural, cultural, political, territorial, functional, or temporal qualities (Phillips & Scheisfurth, 2014). Thus, it is not only territories that can be compared. That said, if territories are compared, the comparison can occur at many levels above and below the nation-state and can incorporate other relevant social, cultural, and functional factors. Macro-level cases, however defined, are assumed to provide characteristic contextual conditions for a certain object that is investigated across cases (Phillips & Scheisfurth, 2014). Different contextual conditions (factors of influence) are used to explain different outcomes regarding the object under investigation (embedded in these contexts and hence affected by them), while similar contextual conditions are used to explain similar outcomes (Esser & Vliegenthart, 2017).

## Selection Rationale

To answer the research questions, this study follows a content analysis approach and carefully examines the mathematics curriculum of Singapore and Ontario. The decision to choose Singapore is for its consistent performance among the top of PISA and TIMMS ranking.

While Singapore has a national education system, the Canadian system is provincial. However, with Ontario being the most populous province and among the leaders of education and pedagogy in Canada (The Conference Board of Canada, 2014), this paper focuses on Ontario as representative of the Canadian education.

## Evaluation of Methodology

Being a natural and elementary function of the human mind, the act of comparing like and unlike phenomena is one of the most profound and generative perceptual processes on which much of our reasoning rests. Comparison is so fundamental to our cognition that thinking without comparison is almost unthinkable (Esser & Vliegenthart, 2017). Yet, naturalness in itself is far from being a reliable enough property to guarantee production of scientific knowledge and unless comparative mode of analysis is disciplined according to the principles of production of science its results do not qualify as scientific (Creswell, 2007; Phillips & Schweisfurth, 2014). A comparative study, as to any other research, possesses both strengths and weaknesses, especially when it comes to scientific validity and efficiency.

### Strength

Most of the data were collected from already existing sources, reducing procedural efforts and many of the ethical concerns (Esser & Vliegenthart, 2017). Variables are not manipulated, and treatments are not applied, again simplifying steps of the research process. In comparative research, the effect of the variable has already occurred, and the goal is to examine the impact or effect of the independent variable on the dependent variable. It is a commonly chosen method when variables cannot be manipulated due to ethical or practical reasons (Esser & Vliegenthart, 2017).

### Weaknesses

The widespread use of comparison can easily cause the impression that this method is a firmly established, smooth, and unproblematic mode of analysis, which due to its unquestionable logical status can generate reliable knowledge once some technical preconditions are met satisfactorily (Esser & Vliegenthart, 2017). Yet comparison is a demanding method strategy that requires reflection and careful consideration.

Indeed, there are a number of severe limitations and constraints associated with comparison that, calling for serious attention, should warn against and prevent any easy-minded uncritical adoption of this mode of analysis (Esser & Vliegenthart, 2017; Phillips & Schweisfurth, 2014). Any comprehensive and detailed discussion of these limitations and constraints would, however, require a treatment that exceeds the scope of this paper. Therefore, in this section only a few elementary issues are discussed while other important aspects of the question are left out.

Guba and Lincoln (1994) define the concept of reliability as a criterion by which to judge qualitative research as belonging to the positivist or post-positivist paradigm. They mention that those working from a constructivist paradigm would prefer the concept "dependability." Bechger et al. (1999) define comparative validity as "appropriateness, meaningfulness and usefulness of comparative inference made from test" (p. 19). Dogan (2004, pp. 324–340) discusses the limits of quantitative (statistical) methods:

1. Cross-national comparisons use national averages. Most countries are characterized by important internal diversity, either regional or vertical in terms of social strata.

2. Survey research in comparative studies include errors generated by sampling procedures, weighing of data, unclear questions, insufficient training of interviewers.
3. Worldwide statistical comparisons may be invalid because of discrepancy between the quality of statistical data for the advanced countries and for the developing ones; that is material of unequal accuracy.

The problem that remains unsolved is the following: To what extent is it possible for a researcher to penetrate into a different context, get acquainted with the universe of meaning pertinent to that context, and acquire the relevant insights that are sufficiently deep to put them in the position of comparing the unfamiliar worlds of others with his or her own?

The problem of asymmetric understanding is in itself an old epistemological problem that has been discussed by many and in a variety of ways (Azarian, 2011). Sufficient to mention here is that, unlike the case with the natural sciences, the subject matter of humanities and social sciences are purposeful, intentional. and meaning-creating actors and any account of what comprise their social lives; that is, their perceptions, actions, relations, and activities requires an adequate understanding of the meanings produced and mediated thereby (Creswell, 2007; Esser & Vliegenthart, 2017). In other words, when researchers examine phenomena that belong to fields far from home, they should have the ability to take head of the particularities of the terrain under their feet, have the sensitivity that enables them to penetrate beneath the apparent similarities and grasp the context-specific meanings attached to the phenomena under observation (Creswell, 2007; Esser & Vliegenthart, 2017).

The point to be stressed here is that such an understanding is an extremely demanding and time-consuming task that should be sought after with sufficient amount of seriousness. Furthermore, the problem of asymmetric understanding is indeed one of the primary concerns of comparative analysis with important implications that are relevant not only for asymmetric kind of comparison but for all comparative strategies and especially for the comparative approaches designed to find general, cross-contexts-valid causal explanations. Robert Anderson et al. (1986, as cited in Azarian, 2011), for instance, drew on Peter Winch's (1958) problematization of the very concept of "sameness" and the difficulties associated with discerning the same phenomena across what he called various "forms of life." On this basis, the authors argue that, in pursuing to compare like with like, comparative research requires the particular ability of recognizing sameness or similarity of phenomena across various contexts.

Especially when working with a cross-national comparative study, gaining access to comparable data may be an issue. In some cases, the comparable data from a particular country may not exist or may have been destroyed. It may be necessary to form an international team comprised of members from all countries involved in order to gain access to the needed information.

## Data Analysis Procedure

With analyzing the Ontario and Singapore mathematics curriculum, a frequency table was created with the most-used words. The researcher then supplied lists of words within each category for the program to match with the established frequency table. Then a text analysis tool matched the action verbs matching each category within the Bloom's taxonomy and tallied the total appearance of all matched verbs within each category. A simple frequency table was then created, with the total being the total numbers of matched verbs. Then a distribution of each category was created by division of frequency within each category. The same process was repeated with the revised taxonomy in order to provide two sets of comparable data within each country, as well as validating analysis result of each data set by contrasting them against each other. Only the final distribution graphs were included in the following sections.

# CHAPTER FOUR: CURRICULUM ANALYSIS

Differing curricula have different foci based on their intended purposes. For mathematics education, the overall purpose should be understanding and fluency (Chevallard, 2015). However, prior to analysis of the curriculum itself, an overview of each will be examined, looking into the overall intended focus from their wordings. The actual focus of each will then be investigated through textual analysis of each curriculum using frequency as stated in the methodology chapter. This ensures that this paper does not take the curriculum's wording at face value and is intended to expose the actual focus by analyzing the respective distribution of objectives.

## Overview of the Ontario Mathematics Curriculum

The Ontario curriculum recognizes the diversity that exists among students who study mathematics and is based on the belief that all students can learn mathematics and deserve the opportunity to do so. It recognizes that all students do not necessarily learn mathematics in the same way, using the same resources, and within the same time frames. It supports equity by promoting the active participation of all students and by clearly identifying the knowledge and skills students are expected to demonstrate in every grade. It recognizes different learning styles and sets expectations that call for the use of a variety of instructional and assessment tools and strategies. It aims to challenge all students by including expectations that require them to use higher-order thinking skills and to make connections between related mathematical concepts and between mathematics, other disciplines, and the real world.

This curriculum is designed to help students build the solid conceptual foundation in mathematics that will enable them to apply their knowledge and further their learning successfully. It is based on the belief that students learn mathematics most effectively when they are given opportunities to investigate ideas and concepts through problem solving and are then guided carefully into an understanding of the mathematical principles involved. At the same time, it promotes a balanced program in mathematics. The acquisition of operational skills remains an important focus of the curriculum. Attention to the processes that support effective learning of mathematics is also considered to be essential to a balanced mathematics program. Seven mathematical processes are identified in this curriculum document: problem solving, reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing, and communicating. The curriculum for each grade outlined in this document includes a set of "mathematical process expectations" that describe the practices students need to learn and apply in all areas of their study of mathematics.

The development of mathematical knowledge is considered a gradual process. A continuous, cohesive program throughout the grades is necessary to help students develop an understanding of the "big ideas" of mathematics—that is, the interrelated concepts that form a framework for learning mathematics in a coherent way. The fundamentals of important concepts, processes, skills, and attitudes are introduced in the primary grades and fostered through the junior and intermediate grades. The program is continuous, as well, from the elementary to the secondary level.

The *Ontario Curriculum Grades 1 to 8: Mathematics, 2005* (Ontario Ministry of Education, 2005) identifies the expectations for each grade and describes the knowledge and skills that students are expected to acquire, demonstrate, and apply in their class work and investigations, on tests, and in various other activities on which their achievement is assessed and evaluated. Two sets of expectations are listed for each grade in each strand, or broad curriculum area, of mathematics:

- The overall expectations describe in general terms the knowledge and skills that students are expected to demonstrate by the end of each grade.
- The specific expectations describe the expected knowledge and skills in greater detail. The specific expectations are grouped under subheadings that reflect particular aspects of the required knowledge and skills and that may serve as a guide for teachers as they plan learning activities for their students.

In addition to the expectations outlined within each strand, a list of seven "mathematical process expectations" precedes the strands in each grade. These specific expectations describe the key processes essential to the effective study of mathematics, which students need to learn and apply throughout the year, regardless of the strand being studied. Teachers should ensure that students develop their ability to apply these processes in appropriate ways as they work towards meeting the expectations outlined in all the strands. When developing their mathematics program and units of study from this document, teachers are expected to weave together related expectations from different strands, as well as the relevant mathematical process expectations, in order to create an overall program that integrates and balances concept development, skill acquisition, the use of processes, and applications.

The five strands are Number Sense and Numeration, Measurement, Geometry and Spatial Sense, Patterning and Algebra, and Data Management and Probability. Number

Sense and Numeration refers to a general understanding of number and operations as well as the ability to apply this understanding in flexible ways to make mathematical judgments and to develop useful strategies for solving problems. In this strand, students develop their understanding of number by learning about different ways of representing numbers and about the relationships among numbers. They learn how to count in various ways, developing a sense of magnitude. They also develop a solid understanding of the four basic operations and learn to compute fluently, using a variety of tools and strategies.

A well-developed understanding of number includes a grasp of more-and-less relationships, part–whole relationships, the role of special numbers such as 5 and 10, connections between numbers and real quantities and measures in the environment, and much more.

Measurement concepts and skills are directly applicable to the world in which students live. Many of these concepts are also developed in other subject areas, such as science, social studies, and physical education. In this strand, students learn about the measurable attributes of objects and about the units and processes involved in measurement. Students begin to learn how to measure by working with non-standard units, and then progress to using the basic metric units to measure quantities such as length, area, volume, capacity, mass, and temperature. They identify benchmarks to help them recognize the magnitude of units such as the kilogram, the litre, and the metre. Skills associated with telling time and computing elapsed time are also developed. Students learn about important relationships among measurement units and about relationships involved in calculating the perimeters, areas, and volumes of a variety of shapes and figures.

Spatial sense is the intuitive awareness of one's surroundings and the objects in them. Geometry helps us represent and describe objects and their interrelationships in space. A strong sense of spatial relationships and competence in using the concepts and language of geometry also support students' understanding of number and measurement. Spatial sense is necessary for understanding and appreciating the many geometric aspects of our world. Insights and intuitions about the characteristics of two-dimensional shapes and three-dimensional figures, the interrelationships of shapes, and the effects of changes to shapes are important aspects of spatial sense. Students develop their spatial sense by visualizing, drawing, and comparing shapes and figures in various positions. In this strand, students learn to recognize basic shapes and figures, to distinguish between the attributes of an object that are geometric properties and those that are not, and to investigate the shared properties of classes of shapes and figures. Mathematical concepts and skills related to location and movement are also addressed in this strand.

One of the central themes in mathematics is the study of patterns and relationships. This study requires students to recognize, describe, and generalize patterns and to build mathematical models to simulate the behaviour of real-world phenomena that exhibit observable patterns. Young students identify patterns in shapes, designs, and movement, as well as in sets of numbers. They study both repeating patterns and growing and shrinking patterns and develop ways to extend them. Concrete materials and pictorial displays help students create patterns and recognize relationships. Through the observation of different representations of a pattern, students begin to identify some of the properties of the pattern. In the junior grades, students use graphs, tables, and verbal descriptions to represent relationships that generate patterns. Through activities and investigations, students examine how patterns change, in order to develop an understanding of variables as changing quantities. In the intermediate grades, students represent patterns algebraically and use these representations to make predictions. A second focus of this strand is on the concept of equality. Students look at different ways of using numbers to represent equal quantities. Variables are introduced as “unknowns” and techniques for solving equations are developed. Problem solving provides students with opportunities to develop their ability to make generalizations and to deepen their understanding of the relationship between patterning and algebra.

The related topics of data management and probability are highly relevant to everyday life. Graphs and statistics bombard the public in advertising, opinion polls, population trends, reliability estimates, descriptions of discoveries by scientists, and estimates of health risks, to name just a few. In this strand, students learn about different ways to gather, organize, and display data. They learn about different types of data and develop techniques for analysing the data that include determining measures of central tendency and examining the distribution of the data. Students also actively explore probability by conducting probability experiments and using probability models to simulate situations. The topic of probability offers a wealth of interesting problems that can fascinate students and that provide a bridge to other topics, such as ratios, fractions, per cents, and decimals. Connecting probability and data management to real-world problems helps make the learning relevant to students.

**Distribution of Educational Objectives Within Bloom’s Taxonomy**

Using the action verbs by categories within the original Bloom’s taxonomy shows an even distribution of objectives in the Ontario’s mathematics curriculum within the first five categories. It is evident in Figure 4.1 below that when analyzed within the original taxonomy, the Ontario mathematics curriculum has a higher focus on evaluation, with a noticeable margin separating it and the closest category (25% vs. 17% for application).

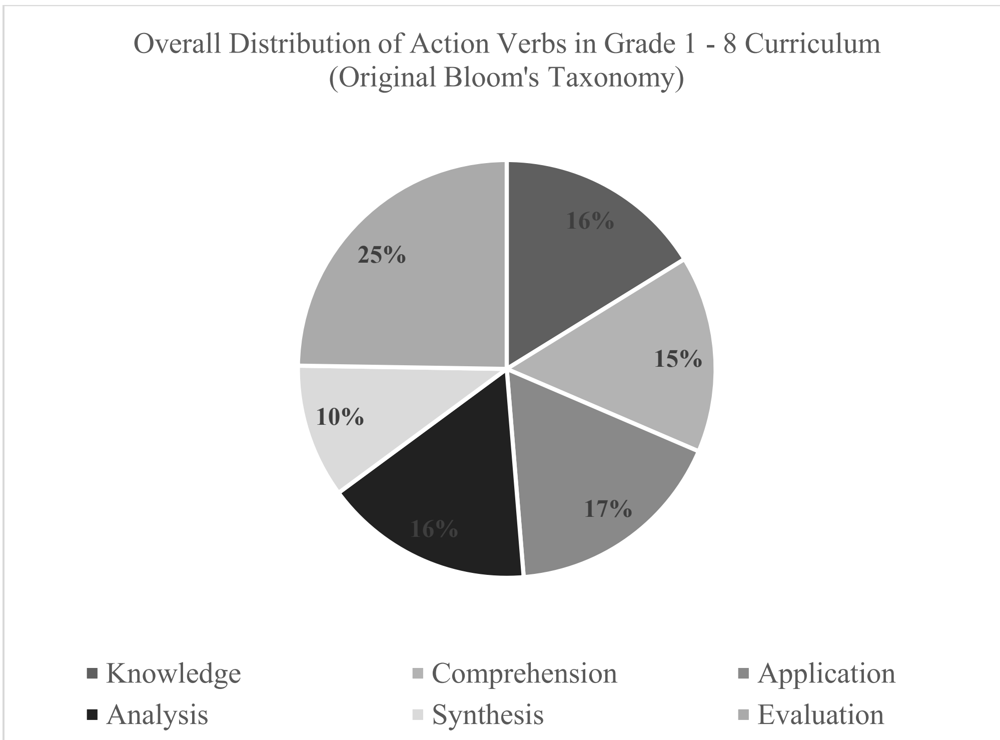


*Figure 4.1*. Action verb distribution in Ontario’s curriculum using the original Bloom’s taxonomy.

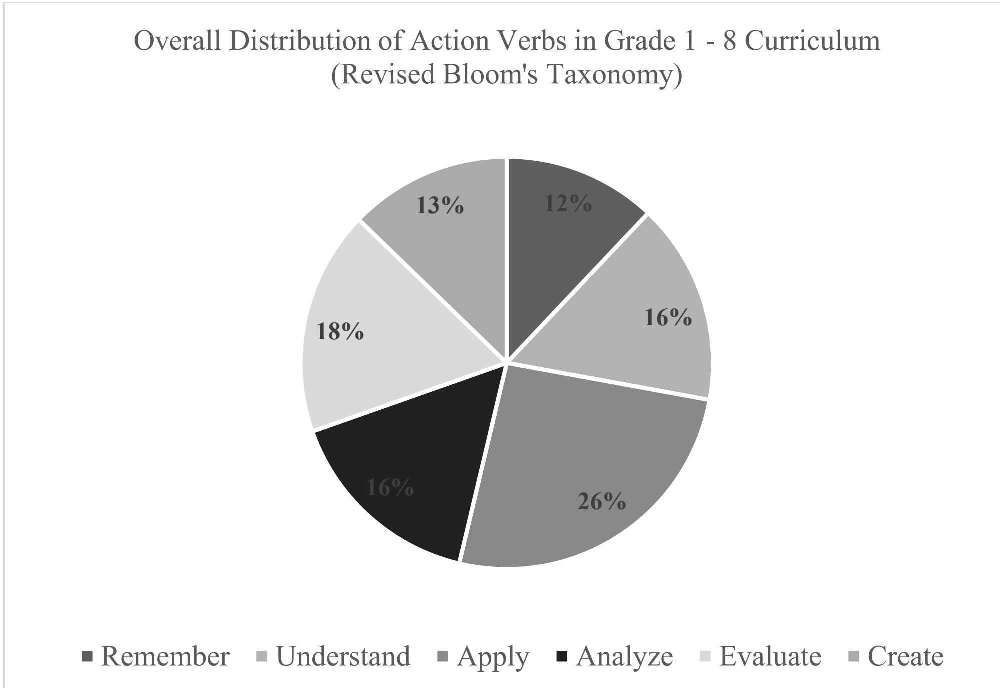


*Figure 4.2*. Action verb distribution in Ontario’s curriculum using the revised Bloom’s taxonomy.

While analyzing with action verbs suggested by the original taxonomy results in high focus in evaluation, the results differ when using the revised taxonomy for analysis. With the supplied word list within the revised taxonomy, the analysis shifts from a high focus on evaluation to a clear concentration on application or apply as the focused action verb. The margin between "apply" and the next category "evaluate" was also a noticeable 8%.

The differing results make the choice of using both datasets useful in creating a clearer picture of the overall focus of the Ontario mathematics curriculum. While the position and distribution changes, the same two categories were within the top in both the original and revised taxonomy. In the original taxonomy, the top two categories were Evaluation and Application. Similarly, analysis using the revised taxonomy yields similar top two categories, "apply" and "evaluate." With these results, it may be concluded that while looking at the overall Grade 1-8 curriculum with both the original and revised taxonomy as a framework, the focus of mathematics education in Ontario is twofold: application and evaluation. The students are encouraged towards higher critical thinking skills, as well as application of the learnt material for better understanding.

**Overview of the Singapore Curriculum**

The overarching goal of the Singaporean mathematics curriculum is to ensure that all students achieve a level of mastery of mathematics that will serve them well in life, and for those who have the interest and ability, to pursue mathematics at the highest possible level. The broad aims of mathematics education in Singapore are to enable students to:

- acquire and apply mathematical concepts and skills;
- develop cognitive and metacognitive skills through a mathematical approach to problem solving; and

- develop positive attitudes towards mathematics.

The mathematics curriculum comprises a set of syllabuses spanning 12 years, from primary to pre-university, and is compulsory up to the end of secondary education. Each syllabus has its own specific set of aims to guide the design and implementation of the syllabus. The aims also influence the choice of content, skills, as well as context to meet the specific needs of the students at the given level or course. Each syllabus expands on the three broad aims of mathematics education differently to cater for the different needs and abilities of the students.

Mathematics is largely hierarchical in nature. Higher concepts and skills are built upon the more foundational ones and have to be learned in sequence. A spiral approach is adopted in the building up of content across the levels.

The mathematics curriculum consists of a set of connected syllabuses to cater to the different needs and abilities of students. This section gives an overview of the syllabuses and their connections so that teachers are better able to appreciate the mathematics curriculum as a whole.

The Primary Mathematics syllabus assumes no formal learning of mathematics. However, basic pre-numeracy skills such as matching, sorting and comparing are necessary in providing a good grounding for students to begin learning at Primary 1 (P1) to Primary 6 (P6). These levels are equivalent to Ontario's Grade 1-6.

The P1−4 syllabus is common to all students. The P5−6 Standard Mathematics syllabus continues the development of the P1−4 syllabus whereas the P5−6 Foundation

Mathematics syllabus revisits some of the important concepts and skills in the P1–4 syllabus. The new concepts and skills introduced in Foundation Mathematics is a subset of the Standard Mathematics syllabus.

The O-Level Mathematics syllabus builds on the Standard Mathematics syllabus for Secondary 1–6 (equivalent to Grades 6–12 in Ontario). The N(A)-Level Mathematics syllabus is a subset of O-Level Mathematics, except that it revisits some of the topics in Standard Mathematics syllabus. The N(T)-Level Mathematics syllabus builds on the Foundation Mathematics syllabus.

The O-Level Additional Mathematics syllabus assumes knowledge of O-Level Mathematics content and includes more in-depth treatment of important topics. The N(A)-Level Additional Mathematics syllabus is a subset of O-Level Additional Mathematics syllabus. O-Level Additional Mathematics together with O-Level Mathematics content provides the prerequisite knowledge required for H2 Mathematics at the pre-university level.

At the pre-university level, mathematics is optional. The H1 Mathematics syllabus builds on the O-level Mathematics syllabus. H2 Mathematics assumes some of the O-Level Additional Mathematics content. H3 Mathematics is an extension of H2 Mathematics.

The Mathematics Framework has been a feature of the Singaporean mathematics curriculum since 1990 and is still relevant to date. The central focus of the framework is mathematical problem solving; that is, using mathematics to solve problems. The framework sets the direction for and provides guidance in the teaching, learning, and assessment of mathematics at all levels, from primary to pre-university. It reflects also the 21st century competencies.

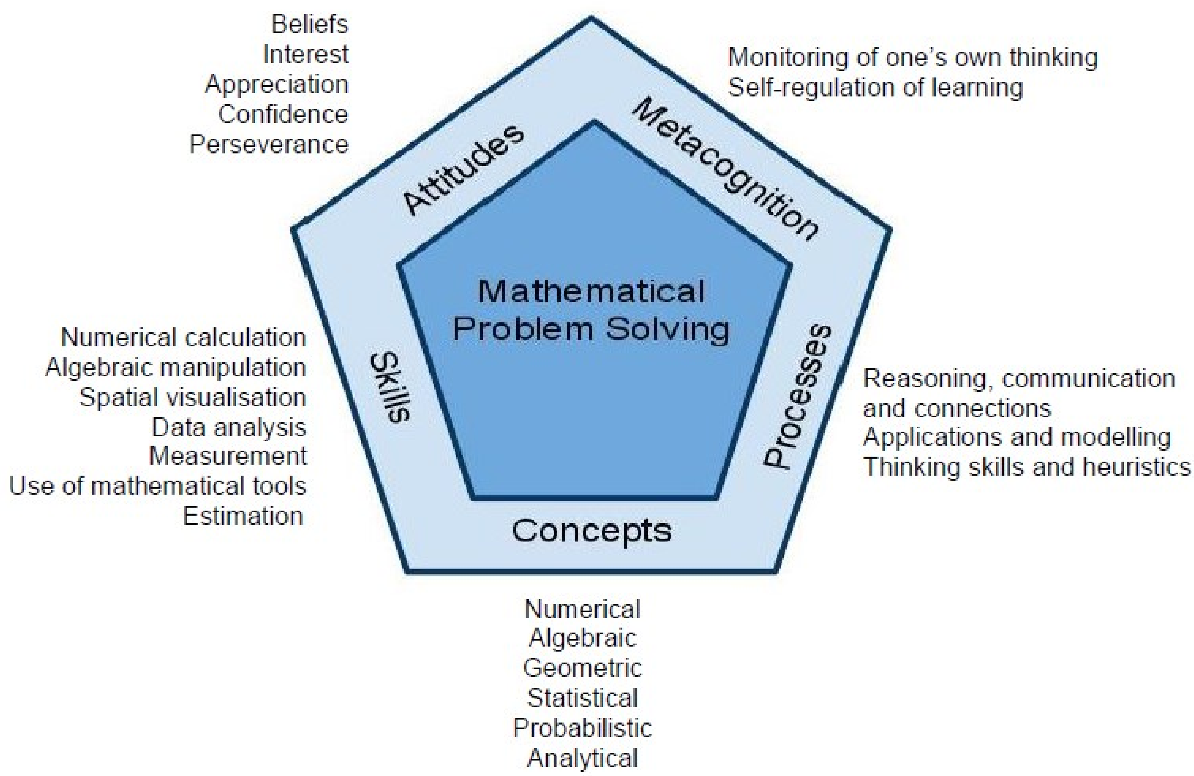


*Figure 4.3*. The Singaporean mathematics framework.

The framework stresses conceptual understanding, skills proficiency and mathematical processes, and gives due emphasis to attitudes and metacognition. These five components are interrelated.

Mathematical concepts can be broadly grouped into numerical, algebraic, geometric, statistical, probabilistic, and analytical concepts. These content categories are connected and interdependent. At different stages of learning and in different syllabuses, the breadth and depth of the content vary. To develop a deep understanding of mathematical concepts, and to make sense of various mathematical ideas as well as their connections and applications, students should be exposed to a variety of learning experiences including hands-on activities and use of technological aids to help them relate abstract mathematical concepts with concrete experiences.

Mathematical skills refer to numerical calculation, algebraic manipulation, spatial visualisation, data analysis, measurement, use of mathematical tools, and estimation. The skills are specific to mathematics and are important in the learning and application of mathematics. In today's classroom, these skills also include the abilities to use spreadsheets and other software to learn and do mathematics. To develop proficiencies in mathematics skills, students should have opportunities to use and practise the skills. These skills should be taught with an understanding of the underlying mathematical principles and not merely as procedures.

Mathematical processes refer to the process skills involved in the process of acquiring and applying mathematical knowledge. This includes reasoning, communication and connections, applications and modelling, and thinking skills and heuristics that are important in mathematics and beyond.

Applications and modelling allow students to connect mathematics that they have learned to the real world, enhance understanding of key mathematical concepts and methods, as well as develop mathematical competencies. Students should have opportunities to apply mathematical problem-solving and reasoning skills to tackle a variety of problems, including open-ended and real-world problems. Mathematical modelling is the process of formulating and improving a mathematical model to represent and solve real-world problems. Through mathematical modelling, students learn to deal with ambiguity, make connections, select and apply appropriate mathematics concepts and skills, identify assumptions and reflect on the solutions to real-world problems, and make informed decisions based on given or collected data.

Thinking skills and heuristics are essential for mathematical problem solving. Thinking skills are skills that can be used in a thinking process, such as classifying, comparing, analyzing parts and whole, identifying patterns and relationships, induction, deduction, generalizing, and spatial visualization. Heuristics are general rules of thumb of what students can do to tackle a problem when the solution to the problem is not obvious. These include using a representation (e.g., drawing a diagram, tabulating), making a guess (e.g., trial and error/guess and check, making a supposition), walking through the process (e.g., acting it out, working backwards), and changing the problem (e.g., simplifying the problem, considering special cases).

The syllabus is organized along three content strands with a listing of mathematical processes that cut across the three strands. Mathematical processes refer to the process skills involved in the process of acquiring and applying mathematical knowledge. This includes reasoning, communication and connections, applications, and thinking skills and heuristics that are important in mathematical problem solving and beyond.

At the primary level, students develop these process skills through problem solving. They learn to lay out their work logically; communicate their thoughts clearly both in written and oral forms; and reason inductively by observing patterns, similarities and differences. They make connections among mathematical ideas, and between mathematics and everyday life. Through solving problems in a real-world context, students see the relevance of mathematics in everyday situations. They formulate methods and strategies to solve problems and develop the habit of checking the reasonableness of their answers against the real-world context. Most importantly, they develop reasoning and problem-solving skills that are essential to lifelong learning. The teaching of process skills should be deliberate and yet integrated with the learning of concepts and skills. Students should be exposed to heuristics and use problem-solving approaches such as the Polya's model in class. Teachers could "think aloud" to give attention to these processes and make them visible to students. Through practice, students will develop habits and strategies that will help them be better and more independent learners.

At the secondary level, students gradually move from inductive arguments to deductive arguments and justifications (e.g. to ju,stify a mathematical statement using a short chain of logical reasoning or to disprove a statement by a counter example). They will continue to make connections among mathematical ideas, and between mathematics and the real world. They should be able to read and critique arguments that are supported by mathematics.

Greater attention will be given to applications and modelling at the secondary level, as students become more mature and aware of their immediate environment and phenomena. Besides learning standard mathematical models, students should, under teacher guidance, develop an awareness and understanding of the mathematical modelling process. They work on real-world problems either individually or in groups. They would need to understand the real-world situation, make assumptions about the situation, devise a mathematical model to solve the problem, and interpret the solution in the context of the situation. The process of mathematical modelling widens and deepens students' understanding of mathematics, and helps them develop important 21st century skills, including collaboration, creativity, communication, and critical thinking. Students should be given opportunities to work in groups and use ICT tools for modelling tasks. ICT tools empower students to work on problems which would otherwise require more advanced mathematics or computations that are too tedious and repetitive. Through practice, students will develop habits and strategies that will help them be better and more independent learners.

**Distribution of Educational Objectives Within Bloom's Taxonomy**

As expected, when exploring the Singaporean curriculum framework, skill proficiency, mathematical processes, and understanding were the major foci. This is in line with the findings of the data analysis. Not only does Singapore's curriculum not have an even distribution of action verbs, the overall distribution chart shows and interesting position where there are three clear major foci, with the highest one being application. As mentioned, understanding, mathematical processes, and proficiency were the three explored foci of the curriculum framework and translate directly to the three major foci in the curriculum's wording itself, with comprehension (understand), application (skill proficiency), and mathematical processes (evaluation) being the highest repeating categories, with 18%, 34%, and 20%, respectively, while knowledge and synthesis only takes up a total of 14% within the overall distribution.

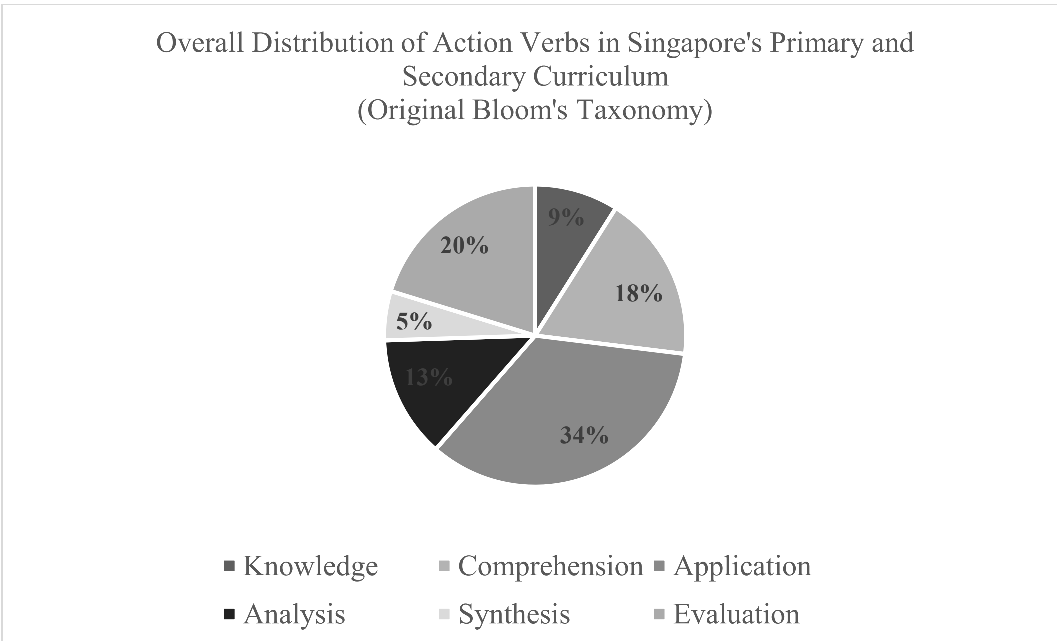


*Figure 4.4*. Action verb distribution in Singapore's curriculum using the original Bloom's taxonomy.

The result differs when it comes to analysis within the revised taxonomy. While the largest component of the curriculum remained the same with “apply” making up 30% of the total matched action verbs, the other categories’ shares are more even. However, similarly to the situation of the Ontario’s curriculum, the same top shares are consistent between the two datasets.

Comparing the results within the two sets of data, the result is consistent with the explored curriculum framework. It is very clear that the Singaporean curriculum focuses on application, with other foci being understanding and evaluation.

## Comparative Analysis

The comparison of the two curricula’s objectives could be made through overviews provided by the dataset. It is very clear that the major focus of the two curricula is not completely different. Both countries look to encourage practices through application and foster critical thinking through evaluations. This is in line with the explored benefits from encouraging high-order thinking (HOT) within Bloom’s taxonomy (BT).

The major differences are the level of focus in understanding, and overall distribution in action verbs within the curriculum wordings themselves. The Singaporean curriculum concentrates on application with more than 30% of its action verbs matching with the category, while the Ontario curriculum fluctuates within the 20% of distribution. It is also important to point out that the two-fold focus is a result of two separate datasets that are not consistent from one to another in highest focus. At the same time, the Singaporean curriculum is consistent between the two datasets when it comes to the most important objective. In order to simplify the data and visualize the differences, the datasets of the corresponding categories between the original taxonomy and the revised taxonomy were averaged to make an overall distribution table for the two country. A radar chart was then created make visual the differences between the two (Figure 4.6).

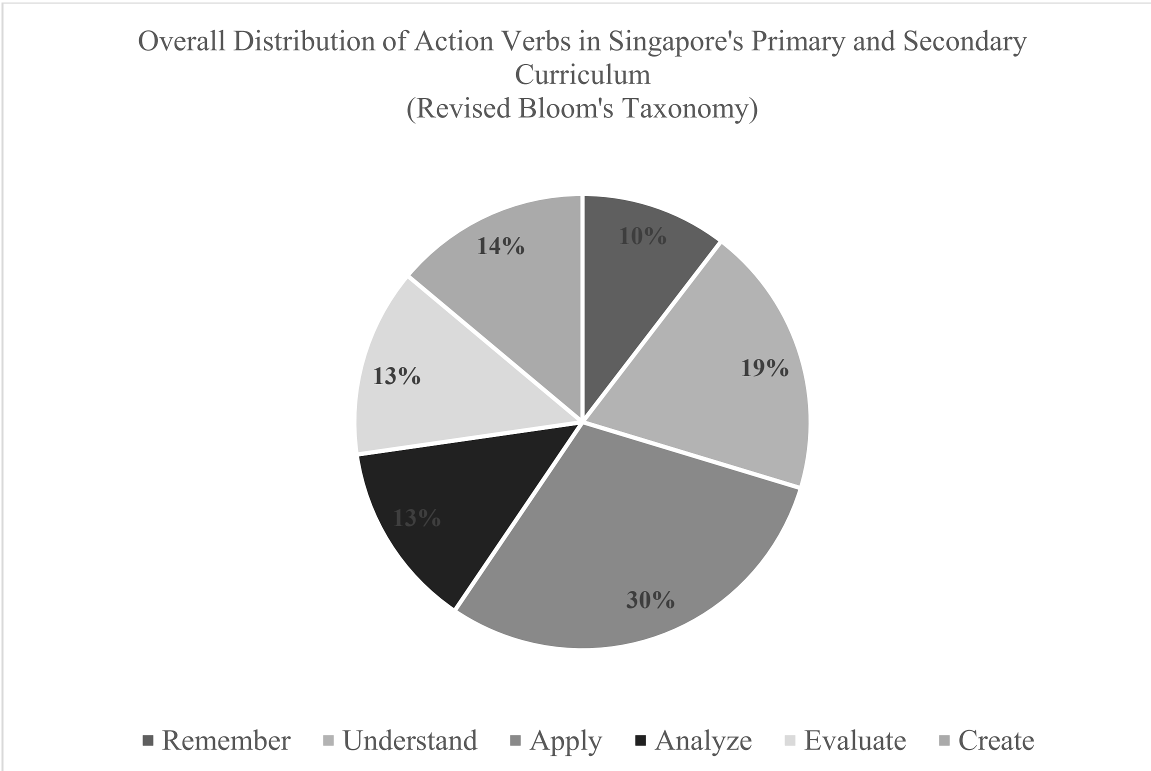


*Figure 4.5*. Action verb distribution in Singapore’s curriculum using the revised Bloom’s taxonomy.

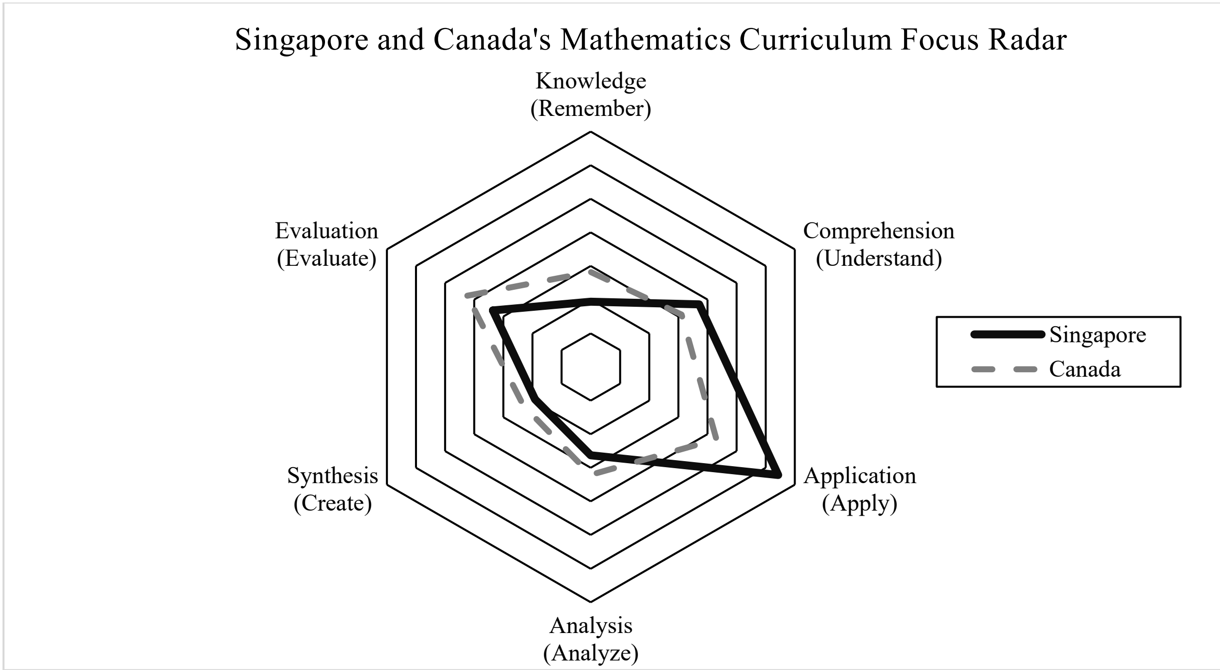


*Figure 4.6*. Focus radars of Singapore's and Ontario's mathematics curriculum.

Figure 4.6 illustrates the difference in distribution of the action verbs within the explored curricula. With clear radar of the distribution of action verbs within each curriculum, the differences in focus are apparent. It is quite clear that in the quest for higher critical thinking, the Ontario's mathematics curriculum focuses on Evaluation (Evaluate) and Knowledge (Remember), rather than paying more attention to Application (Apply).

The Singaporean curriculum, on the other hand, closely followed the curriculum's framework in its wording and objectives, as explored previously. This is apparent through the radar chart with Singapore having two categories that have considerably higher focus than in Ontario: Application (Apply), and Comprehension (Understand). It is debatable that the focus on comprehension and application is solely responsible for the widely difference in performance in TIMMS and PISA score. However, since mathematics education is unique in its characteristics, it is important to discuss this result in the following chapter.

## CHAPTER FIVE: DISCUSSION

As explored in the literature, mathematics as a subject is perceived differently from other subjects in school, especially when it comes to teaching and studying strategies. For example, to be a good English student, to read and understand novels, or poetry, students need to have memorized the meanings of many words. But no English student would say or think that learning about English is about the fast memorization and fast recall of words. This is because we learn words by using them in many different situations—talking, reading, and writing. English teachers do not give students hundreds of words to memorize and then test them under timed conditions. A very common misconception is the perception that the goal of mathematics is getting the right answer, not interpreting or making meaning. In fact, the core of mathematics is reasoning—thinking through why methods make sense and talking about reasons for the use of different methods (Boaler, 2015). In the analysis of the curriculum documents, it is apparent that both countries recognize the importance of application in mathematics. When looking at the focus radar individually, both countries pay similar attention to application when teaching mathematics.

In practice, however, application and comprehension often go in tandem. As argued in Tan (2017), the perception that mathematical drills done in Asian schools are solely for the purpose of memorization is false, but rather done in tandem with explanation as a form of application. This is consistent with the results presented in Chapter 4 when looking at Singapore's focus radar. While comprehension and comprehension are mostly considered low-order thinking skills (LOT), they provide theoretical and skills basis for higher order tasks. As found by Flannagan (2018), the

Canadian classrooms often skip this step to focus more on other “discovery” tasks that promised to allow higher-order critical thinking skills. While this focus is consistent with the HOT and LOT division, the approach should be more balanced, since the essential structure of the taxonomy was a cumulative hierarchy (Kastberg, 2003). The taxonomy is a hierarchy because the classes of objectives are arranged in order of increasing complexity, and cumulative because each class of behaviours was presumed to include all the behaviours of the less complex classes (Kreitzer & Madaus 1994, as cited in Kastberg, 2003). This means HOT tasks can only be effectively fulfilled if LOT is at a sufficient level. By lowering focus on understanding but being overly focused on HOT such as evaluating or creating, the Ontario curriculum has potentially been depriving students of understanding core knowledge that is fundamentals to more complex mathematical concepts.

## Implications for Ontarian Educators

With the above results, Ontario could implement other strategies in mathematics education to improve the overall outcome. A paradigm shift in focus within the curriculum itself could be a start. While it seems that the wording of a curriculum may not be significant, setting clear and effective objectives may make a significant difference in both teaching and learning, as well as in teaching materials. One proposal is the introduction of a more balanced approach to mathematics education that revaluates the current over focus on HOT tasks. Along with that, teachers and students should also be active in defining and changing the curriculum to fit their learning and teaching styles, and not be constrained by a single method or approach. At the same time, visits to

Singapore, or having experts participating in classrooms, could return ideas for more effective teaching strategies.

Another step for Ontario and Canada going forward is investment in not only teaching materials but also on training for teachers themselves. As explored in the literature, a shortage of skilled educators may have negative implications for the education system in the long term. Having an enthusiastic, developed, and skilled teaching force is highly beneficial, especially with mathematics being fundamental to other sciences. This would potentially shift the attitudes in favour of mathematics from students and families, since they would be benefiting from a better education experience. This could reverse the mentioned vicious cycle in the UN listed challenges in mathematics education and benefit other aspects of education.

# CHAPTER SIX: CONCLUSION

In past decades, Western societies have been considered more advanced for their superior education systems (Tan, 2017; Yelland, 2012). Most educators attributed the exceptional quality in education in the West to their scientific approach to inquiries. This system of scientific and abstract thinking is grounded in the Realism philosophical realm of Aristotle and Idealism of Plato (Abiddin et al., 2011; Hassan & Jamaludin, 2010). With education being an intrinsic part of a society, it is reasonable to say that the dominant philosophical ideas of a community influence the system of education. As a result, the Eastern education system is very different from that of the West. East Asia, in particular, is heavily influenced by Confucianism, which views education as a means of perfecting a person's multiple roles in the society (Tan, 2017; Vuong et al., 2018).

Canada, as one of the top performers among Western education systems (Mullis et al., 2016), could learn from Singapore to improve its performance in mathematics. With the explored differences in not only the curriculum content but also curriculum objectives, Canada could take several steps to make education more efficient. Firstly, a thorough study of the Singaporean curriculum would be the logical step forward. The mathematics framework of the Singaporean curriculum would be a great tool since the framework provides the overall focus for the actual curriculum. Understanding the philosophy behind the curriculum would help Canadian educators. Thus, further research and analysis of the Singaporean mathematics framework and curriculum would be desirable.

From the preliminary comparison made in the previous chapters, a second step that Ontarian and Canadian mathematics educators could take is revaluating the current curriculum and shift its focus to more practices and understanding through practices. As mentioned, while it could be understood that mathematics drills and exercises benefit only memory, understanding from practice is a major part of the Singaporean effective curriculum. Similarly to the United Kingdom's and the U.S.A.'s adoption of the Singaporean curriculum stated in the introduction, with the proven methods available for use, educators from Ontario and Canada could adopt it immediately or test the method in one classroom or school.

Canada, with an already strong education system, would need only to tweak its focus to make mathematics performance higher in primary and middle school in order to prepare students for higher scientific studies in higher education levels. While there are obvious cultural differences between the two nations, education values could be used, borrowed, and transferred from one culture to another in increments (Cheng, 1998). This process may benefit both the originating and receiving countries. For Canada, the most obvious benefit is having its students being more competent in mathematics and potentially in other sciences. For Singapore, the use of its curriculum in foreign soil would act as an educational "export" that would not only allow Singapore to validate its philosophy and study the effect of its curriculum in another culture to improve it further, but also to benefit the economy though sales of materials.

## Limitations and Future Study Recommendations

This research, while analyzing the curriculum quite scientifically, could be flawed when analyzing the curriculum's objective wordings as a whole. This may have left out key words, or not taken into account context of each key repeated words. This may result in skewed statistics in categories that are closely related.

Second, while it is true that the curriculum plays a major role in shaping performance and attitudes of students and educators, it cannot be denied that the culture of a country affects all stakeholders, including not only roles within a school, but also the surrounding environment. Considering mathematics education, differences in culture between Canada and Singapore could factor into the difference in performance. Future studies could spend more time and resources looking into the cultural aspects of the two countries and pinpoint differences that could potentially benefit mathematics studies.

Third, this study only investigated mathematics education as a stand-alone subject. However, in the school context, many subjects can be taught through the process of integration, and hence the overall focus of the entire education system should be investigated to see if mathematics is considered important amongst other subjects. The result of that would also shed some light on how and why certain countries have higher performing mathematics curriculum than the other. Finally, future larger-scale studies could be done to explore mathematics education in countries other than the two researched countries. With a larger sample size, results would be more generalizable and could contribute to a shifting education paradigm and not just in one province.

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