The Philosophical Quarterly Vol. 00, No. 0 *2019*
ISSN 0031-8094 doi: 10.1093/pq/pqz016

WEAK ASSERTION

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We present an inferentialist account of the epistemic modal operator might. *Our starting point is the bilateralist programme. A bilateralist explains the operator* not *in terms of the speech act of rejection; we explain the operator* might *in terms of weak assertion, a speech act whose existence we argue for on the basis of linguistic evidence. We show that our account of* might *provides a solution to certain well-known puzzles about the semantics of modal vocabulary whilst retaining classical logic. This demonstrates that an inferentialist approach to meaning can be successfully extended beyond the core logical constants.*

Keywords: epistemic modals, inferentialism, bilateralism, assertion, rejection.

I. INTRODUCTION

In this paper, we address two *prima facie* unrelated problems. The first problem is that of explaining the meaning of epistemic vocabulary in a way that provides a satisfactory solution to certain linguistic puzzles (Yalcin 2007; Schnieder 2010; Dorr & Hawthorne 2013; Moss 2015; Russell & Hawthorne 2016).

The second problem is that of establishing inferentialism as a serious contender in the theory of meaning. We take Timothy Williamson to express a common sentiment when he writes that

> [i]f you want an explicit theory of how some particular linguistic construction contributes to the meanings of sentences in which it occurs, the inferentialist is unlikely to have one.(Williamson 2010, 23)

Inferentialists must do better. As Greg Restall puts it, if we truly want to make advances in proof-theoretic semantics, we need from inferentialists

> more work on the range of applications in the theory of meaning for speech acts beyond assertion and concepts beyond the core logical constants.(Restall 2016)

We make a start on the project of extending inferentialism beyond the core logical constants by developing an inferentialist account of epistemic modals.

As we shall see, this account provides a natural and elegant solution to the epistemic puzzles. This, we hope, shows that the prospects for vindicating inferentialism are brighter than usually assumed.

Our starting point is the bilateral framework (Smiley 1996; Rumfitt 2000; Incurvati & Schlöder 2017). This framework has been used to explain the operator *not* through the speech act of *rejection*. In this paper, we extend the bilateral framework to a *multilateral* one, and use the result to explain the operator *might* through the speech act of *weak assertion*. The upshot is a theory of epistemic modality which is both inferentialist and expressivist, in that it accounts for the meaning of *might* in terms of its *inferential* connections to speech acts *expressing* certain attitudes.

We begin by presenting the epistemic puzzles about *might* and introducing our preferred understanding of the bilateralist approach. Next, we provide evidence for the existence of a distinctive speech act of weak assertion and show how to embed it in the bilateral framework. This gives rise to the novel *epistemic multilateral logic*, which we use to inferentially explain *might* and solve the epistemic puzzles.

II. EPISTEMIC PUZZLES

A number of linguistic puzzles have been taken to show that a satisfactory account of epistemic modals requires *extensive failures* of classical logic (e.g. Willer 2013), unless one adopts complex pragmatic machineries (Dorr & Hawthorne 2013). We review these epistemic puzzles. Later in the paper, we give a natural account of *might* which solves the puzzles while retaining classical logic.

Yalcin's puzzle. Seth Yalcin (2007) observed that epistemic *might* differs in its embedding behaviour from self-reported ignorance of the contrary.

(1) #a. It is raining and I don't know that it is raining.
b. Suppose it is raining and I don't know that it is raining.
c. If it is raining and I don't know that it is raining, ...
#d. It is raining and it might not be raining.
#e. Suppose it is raining and it might not be raining.
#f. If it is raining and it might not be raining, ...

(1a) is a version of Moore's paradox. The typical explanation of its infelicity is *pragmatic*: asserting that *it is raining* pragmatically entails *I know that it is raining*, which contradicts *I don't know that it is raining*. This pragmatic entailment is suspended under *suppose* and *if*. The typical explanation therefore predicts that (1b) and (1c) should be felicitous, as is indeed the case.

Comparing (1a,b,c) with (1d,e,f) shows that the explanation for the infelicity of (1d) cannot be the usual pragmatic one. For the pragmatic explanation predicts that (1e) and (1f) should be felicitous, which they are not. Thus, a desideratum for a theory of epistemic modality is to explain why *p and it might not p* sounds infelicitous even in certain embedded contexts. However, if we explain this by taking *p and it might not p* to be semantically *contradictory*, classical logic entails that *if might p, then p*, trivialising *might*. This hints at an underlying problem, recently spelled out by Jeffrey S. Russell and John Hawthorne (2016).

Triviality. Say that an epistemic state *S rules out* a proposition *p* if updating *S* with *p* results in an absurd state. Then the following are plausible assumptions regarding the dynamics of *might* and *not*:

(MIGHT) Any epistemic state not ruling out *might p* does not rule out *p*.
(NOT) Updating any epistemic state with *not p* yields a state that rules out *p*.

However, Russell & Hawthorne (2016, 326) show, if we accept (MIGHT) and (NOT), we appear to have committed ourselves to *might p* entailing *p*. The proof goes as follows.

Suppose for *reductio* that some epistemic state, when updated with *not p*, yields a state *S* that does not rule out *might p*. By (MIGHT), whenever *might p* is not ruled out, *p* is not ruled out. Thus, *S* does not rule out *p* either. This contradicts (NOT). By *reductio*, updating any state with *not p* yields a state that rules out *might p*. That is, when a state is updated with *not p* and then *might p*, it results in an absurd state. However, the Commutativity of update tells us that updating with *A* and then with *B* has the same effect as updating with *B* and then with *A*. Hence, updating a state with *might p* and then with *not p* results in an absurd state. That is, updating any state with *might p* rules out *not p*. Now in dynamic logic, *A* entails *B* just in case updating any state with *A* results in a state that rules out *not B*. Therefore, *might p* entails *p*.

Russell & Hawthorne (2016, 336) conclude that if we want to hold on to (MIGHT) and (NOT), 'the only real option is to go in for some account that violates Commutativity'. Doing so is in fact typical for dynamic accounts (Veltman 1996; Willer 2013). However, these accounts constitute major departures from classical logic.

Modal disagreement. In broad outline, the orthodox approach to epistemic modality takes *might p* to say that *p* is compatible with some contextually determined body of knowledge (Kratzer 1977; DeRose 1991). Now consider the following dialogue.

(2) *Alice*: I can't find the keys.
Bob: They might be in the car.
Alice: No, they are not in the car. I just checked.

Alice and Bob appear to disagree. However, it is difficult to locate a proposition about a single body of knowledge that Bob is warranted in asserting and

Alice is warranted in rejecting. Several writers have concluded that to account for modal disagreement cases such as (2) we need to modify or supplement the orthodox account with unorthodox features (Egan *et al.* 2005; MacFarlane 2014).

III. BILATERALISM

Bilateralism emerged as an attempt to reconcile the inferentialist approach to meaning with classical logic. Here we show that, in addition, it can be regarded as a form of *inferential expressivism*: a view which combines features of the inferentialist and expressivist programmes.

Inferentialism and bilateralism. *Inferentialism* is the view that the meaning of an expression is given by its inferential role. Those who accept inferentialism about *logical* expressions typically take their inferential role to be captured by their introduction and elimination rules in a natural deduction system. The result is *logical inferentialism*: the view that the meaning of the logical constants is given by their introduction and elimination rules (Gentzen 1935; Prawitz 1965; Dummett 1991).

A major challenge for logical inferentialism is that not every pair of introduction and elimination rules seems to confer a coherent meaning on the constant involved (Prior 1960).[1] One way of meeting this challenge is to require a certain balance between introduction and elimination rules, known as *harmony* (Dummett 1991; Tennant 1997). As Dummett (1991) pointed out, however, whilst the harmony requirement rules out the known problematic cases, it also appears to sanction intuitionistic logic, since the rules for classical negation in a standard natural deduction system do not seem harmonious.

According to bilateralists, appearances are deceiving: it is the restriction of standard natural deduction systems to *asserted* content that prevents one from giving an inferentialist explanation of classical negation. Once *rejected* content is countenanced—as it should be—one can formulate harmonious rules for classical logic (Rumfitt 2000). Thus, bilateralism holds that the meaning of the logical constants should be given by conditions on both assertion and rejection.

Assertion and rejection are *speech acts*, which express *attitudes towards* propositions: assertion expresses *assent*, rejection expresses *dissent*. A fundamental property of these two attitudes is that they are *incompatible*: it is *absurd* to simultaneously have them towards the same proposition. This is not to say that it is *impossible* to assent and dissent from the same proposition. But someone knowingly doing so would be incoherent and experience this incoherence

[1] Prior's example is the connective *tonk*, characterized by the introduction rule 'From A infer A *tonk* B' and the elimination rule 'From A *tonk* B infer B'. Since these rules trivialize the logic, *tonk* is usually taken to be incoherent (but see Warren 2015 for a different take on the matter).

as cognitive dissonance (which may be resolved by compartmentalizing one's incompatible attitudes).

Following Gottlob Frege (1919), Timothy Smiley (1996) argues that assertion and rejection can be realised by answering *yes* or *no* to polar questions posed to oneself.

(3) a. Is it the case that *p*? Yes!
 b. Is it the case that *p*? No!
 c. Is it the case that *not p*? Yes!

Smiley understands (3a) as a linguistic realisation of asserting *p* and (3b) as a realisation of rejecting *p*. Thus, *p* is the *content* of a speech act *indicated* by the words *yes* and *no*, i.e. *yes* and *no* are *force-indicators* (Incurvati & Smith 2009). Rejection as in (3b) and negative assertion as in (3c) are treated as distinct phenomena (Smiley 1996).

On Smiley's account, (3b) and (3c) are still *inferentially equivalent*: in a fixed context, the same inferences can be drawn from an utterance of (3b) and (3c). But there is substantial debate about whether this is correct and what the attitude of dissent is (compare Rumfitt 1997, 2000; Dummett 2002; Dickie 2010). Imogen Dickie points out that whilst one may reject a proposition on grounds of falsity, there are many other reasons for doing so. In the former case, the rejection is *strong*, i.e., it is inferentially equivalent to a negative assertion. In the latter case, it need not be: it may be *weak*. In previous work (Incurvati & Schlöder 2017), we give the following as an example of a weak rejection.

(4) Is Franz here? No, not as far as I know.

Here, the speaker is expressing dissent from *Franz is here* on grounds of ignorance, not of falsity, and it would be mistaken to infer that she assents to *Franz is not here*.

Thus, if rejections can be weak, it is erroneous to treat (3b) and (3c) as inferentially equivalent: (3c) is potentially more informative. To make sense of this, we take *no* as a force-indicator for *weak* rejection and dissent to consist in finding a proposition *unassertible* (see Incurvati & Schlöder 2017). The fact that *no* often conveys *strong* rejection is explained by appealing to a general pragmatic pressure to maximize contrariness (Horn 1989): falsity is contrary to assertibility (whereas unassertibility is its contradictory), so, typically, a weak rejection is pragmatically strengthened to a strong one (see Incurvati & Schlöder 2017, §7).

In our extension of bilateralism to weak rejection, classical logic is the *logic of assent*. That is, assent to $A_1, A_2, \ldots, A_n$ bilaterally entails assent to B just in case B is classically derivable from $A_1, A_2, \ldots, A_n$. However, whilst standard bilateralist calculi for *strong* rejections are harmonious (Smiley 1996; Rumfitt 2000), harmony is lost in the system we presented in (Incurvati & Schlöder 2017). One of the strengths of the account we will develop is that it is faithful to the linguistic phenomena by allowing for weak rejection, but it restores harmony by including the speech act of *weak assertion* to be introduced below.

Expressivism and bilateralism. *Expressivism* about a class of linguistic terms holds that the meaning of these terms is to be explained via their putative connections to speech acts expressing attitudes. *Traditional* expressivism takes this connection to be *direct*: the meaning of terms from the relevant class is explained by taking those very terms to express an attitude. For instance, traditional ethical expressivism takes *it is good that p* to express moral approval towards *p* (Ayer 1936; Stevenson 1937).

Bilateralism is a form of expressivism in that it explains the meaning of the logical constants via their connections to assertion and rejection (expressing, respectively, assent and dissent). However, bilateralism should be distinguished from traditional expressivism. According to bilateralism, the meaning of the logical constants does not *consist* in the expression of assent and dissent but *derives* from it. We can illustrate this difference by considering how bilateralism deals with the Frege–Geach problem besetting traditional expressivism (see Schroeder 2008b for an overview).

Traditional expressivism about negation takes *not* to indicate rejection, expressing dissent. The Frege–Geach argument against this view now goes as follows (Frege 1919; Geach 1965). Consider the following seemingly valid inference.

(5) a. If not *p*, then not *q*.
b. Not *p*.

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c. Not *q*.

The *not p* in (5a) cannot indicate the rejection of *p*, since somebody uttering (5a) might *assent* to *p*. Thus, the *not* in (5a) must *modify* the proposition *p* instead of expressing an attitude *towards* it. Suppose we insist that in unembedded contexts such as (5b) *not p* expresses dissent towards *p*. Then, the content of (5b) does not coincide with the antecedent of (5a), and (5) cannot be validated by *modus ponens*. For the inference to be an instance of *modus ponens*, the *not* in (5b) must modify *p*. But then it cannot express dissent.

The bilateralist solution is straightforward: *not* is a compositional operator and hence modifies *p* in both (5a) and (5b), so (5) is just a case of *modus ponens*. The bilateralist account is still expressivist because although *not* is a compositional operator, it is *inferentially explained* in terms of the speech act of rejection, which expresses the attitude of dissent.

More generally, the bilateralist takes the meaning of any compositional operator to be given by its introduction and elimination rules; but, she contends, adequate meaning-conferring rules for negation must be framed in terms of conditions on rejection as well as assertion. For instance, bilateral systems include a rule allowing one to pass from the assertion of *not p* to the rejection of *p*.

Thus, according to our preferred understanding of bilateralism, the connection between the meaning of the logical constants and speech acts is *inferential*. For this reason, bilateralism is best regarded as an instance of *inferential expressivism*, the view that the meaning of an expression is explained in terms of its *inferential* connections to speech acts *expressing* attitudes.

Mixed inferences and Frege–Geach. Recall that the bilateralist takes *no* to indicate rejection. This means that *no* and *not* have different functions: the former is a force-indicator, the latter is a compositional operator. Nonetheless, *no* and *not* inferentially interact: the rules for *not* are formulated using the speech act indicated by *no*. Hence, *no* can be used to inferentially explain the meaning of *not*.

However, taking *no* to be a force-indicator gives rise to a revenge version of the Frege–Geach argument. Consider the following inference involving yes-or-no answers to polar questions.

(6) a. Is it the case that if not *p*, then not *q*? Yes!
b. Is it the case that *p*? No!

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c. Is it the case that *q*? No!

Intuitively, (6) is valid. As before, *not* in (6a) must *modify* the proposition *p*, but (6b) is taken to express dissent from *p* via the use of the force-indicator *no*. Thus, (6) cannot be validated by *modus ponens* unless *no* in (6b) also modifies *p* to *not p*, in which case it is not a force-indicator.

However, to validate (6) one need *not* consider it a direct application of *modus ponens* (Smiley 1996). Instead, the inference can be validated by taking it to involve an additional inferential step from rejection to negative assertion (Rumfitt 2000) or different inferential machinery altogether (Incurvati & Schlöder 2017).

According to bilateralists, the inference (6) is *mixed* in that it involves both assertion and rejection. Examples of other intuitively valid mixed inferences are not hard to come by.

(7) a. Is it the case that if *p*, then *q*? Yes!
b. Is it the case that *q*? No!

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c. Is it the case that *p*? No!

Someone who identifies rejection with negative assertion would consider this inference a case of *modus tollens*. But one need not theorise about negation and its relation to rejection to recognize (7) as valid. The bilateralist, for her part, can account for mixed inferences such as (7) by *coordinating* assertion and rejection.

To this end, note that if one asserted and rejected the same content, one would be both assenting and dissenting from that content. But this is absurd, since assent and dissent are incompatible attitudes. This sanctions the following *coordination principles*.

- Rejection principle: It is absurd to both assert and reject *p*.
- Smileian *reductio*: If it is absurd for someone to assert *p* that must be because they were already committed to rejecting *p*; and if it is absurd for someone to reject *p*, that must be because they were already committed to asserting *p*.

We will clarify the role of commitment and give a detailed justification of these principles in Section V, when implementing them in the formal system developed there. For now, note that using these principles one can explain the validity of (7). For suppose that someone asserts *if p then q* and rejects *q*. The fact that they have asserted *if p then q* means that should they also assert *p* they would be committed, via *modus ponens*, to asserting *q*. But the Rejection principle tells us that this would be absurd, given that they have rejected *q*. By Smileian *reductio*, we conclude that they were already committed to rejecting *p*.

This explanation of the validity of (7) uses the Rejection principle. One might therefore worry that whilst solving the conditional version of the Frege–Geach problem, inferential expressivism falls prey to its *negation* version (Unwin 1999; Schroeder 2008a). For in using the Rejection principle, inferential expressivism seems to *assume* that assent and dissent are incompatible. But this cannot be assumed: it needs to be *explained*.

Here is an outline of our response, developed in ongoing work. It is commonplace to explain the incompatibility of assent and dissent from *p* on the basis of the inconsistency of *p* and *not p* (perhaps in turn explained on the basis of a basic incompatibility between truth and falsity). Inferential expressivism *reverses* this order of explanation: it is the incompatibility of assent and dissent from *p* which explains the inconsistency of *p* and *not p*.

On this account, the incompatibility of assent and dissent is basic. But this is no reason to fault inferential expressivism, just as one should not fault the standard approach for taking the inconsistency of *p* and *not p* as basic or explaining it on the basis of a fundamental incompatibility between truth and falsity.

This is not to say that one cannot provide reasons for thinking that assent and dissent *are* incompatible. In particular, these attitudes are intimately connected with the speech acts expressing them. It is therefore possible to infer properties of assent and dissent from the way assertion and rejection are interpreted. And a speaker answering positively and negatively to the same polar question is normally taken to have changed their mind, on pain of incoherence.

Beyond the core logical constants. Inferential expressivism retains the appeal of expressivism whilst avoiding the pitfalls of its traditional versions. In particular, inferential expressivism takes the meaning of certain expressions to be suitably grounded in attitudes while providing a natural solution to the Frege–Geach problem. As a form of inferentialism, however, it must address a major challenge faced by inferentialist approaches to meaning: to account for a wide range of expressions and not only the core logical constants (i.e., the connectives, the quantifiers, and identity).

We take a step towards meeting this challenge by presenting an inferential expressivist account of *might*. This account takes *might* to be a compositional operator inferentially explained in terms of the speech act of *weak assertion*. As we shall argue, weak assertion can be performed using *perhaps* in certain suitable contexts. Thus, just as a bilateralist explains *not* in terms of *no*, we explain *might* in terms of *perhaps*.

IV. WEAK ASSERTION

It is widely agreed that *might* must *modify content* because of its embeddability behaviour (von Fintel & Gillies 2007; Swanson 2010; MacFarlane 2014). We concur, but it does not follow that *every* piece of epistemic vocabulary modifies content. Indeed, we will now present linguistic data in support of the claim that *perhaps* is a *force-modifier*, i.e., its occurrence in an utterance modifies the speech act that would otherwise be performed with that utterance, but not its content. Analogous data could be presented in defence of the view that *maybe* is a force-modifier, but we restrict attention to *perhaps*.

perhaps and might. It seems that *perhaps* and *might* have been conflated at times. For instance, Benjamin Schnieder (2010) uses examples involving *perhaps* to comment on Yalcin's puzzles about *might*, and Joshua Crabill (2013) makes use of Schnieder's insights, but replaces *perhaps* with *might* again. At first glance, *perhaps* and *might* are rather similar, aside from some syntactic differences.

(8) a. Perhaps it is raining.
    b. It might be raining.

(8a) and (8b) can be justifiably uttered in exactly the same circumstances. We take this to show that they are inferentially equivalent, i.e in a fixed context, the same inferences can be drawn from an utterance of (8a) and an utterance of (8b). Moreover, *perhaps* and *might* both lead to Yalcin's puzzle.

(9) #a. Suppose it is raining and perhaps it is not raining.
    #b. Suppose it is raining and it might not be raining.

Schnieder (2010) seeks to explain (9a) as a special case of the fact that *perhaps* does not embed under *suppose* at all.

(10) # Suppose perhaps it is raining.

(10) sounds rather odd. Hence, Schnieder argues, it is not surprising that (9a) sounds odd as well. Schnieder takes the fact that *perhaps* fails to embed under *suppose* to provide evidence that its function is to express an attitude. This is in agreement with what we argue below.

But what does this mean for *might*? To start with, the fact that *Perhaps it is raining* and *It might be raining* are inferentially equivalent does not mean that *might* and *perhaps* have identical embedding behaviour. For instance, while (10) is infelicitous, (11) appears to be fine.

(11) Suppose it might be raining.

Crabill (2013) claims that (11) is infelicitous, but we suspect this is due to his identification of *might* with *perhaps*. Here are two natural cases of embeddings of *might* under *suppose*.

(12) a. Biologists supposed it might be a gene like the one causing Burkitt's lymphoma that made cells lose control of their proliferation ...
(The New York Times, *Geneticists' Target: Immortal Cells*, 22 December 1992)
b. The standard model ... is presumably closer to the truth about fundamental particles than [earlier theories]. At least, it makes sense to suppose that it might be.
(Stanford Encyclopedia of Philosophy, *Truthlikeness*, Oddie 2016)

These examples provide evidence that (11) is generally acceptable. It follows that Schnieder and Crabill cannot solve Yalcin's puzzle. For if their strategy to explain the infelicity of (9a) applied to (9b) as well, (11) should be as infelicitous as (10).

Moreover, *perhaps* does not embed in conditional antecedents, whereas *might* does.

(13) #a. If perhaps it is raining, I'd better take an umbrella.
b. If it might be raining, I'd better take an umbrella.

Finally, Eric Swanson (2010) argues against expressivist treatments of *might* by observing that it embeds under quantifiers. In this respect too, *perhaps* differs from *might*.

(14) a. Every day it might be raining.
#b. Every day perhaps it is raining.

Because of its embedding behaviour, *perhaps* is sometimes taken to belong to the category of *speaker-oriented adverbs* like *frankly*, *fortunately,* or *evidently*, which speakers use to comment on their utterances (Mittwoch 1977; Bellert 1977; Ernst 2009).

However, it is not correct to say that *perhaps* is used to *comment* on the performance of a speech act. If one says *frankly p*, *fortunately p*, or *evidently p*, all the effects of asserting *p* still obtain, but this is not the case for *perhaps p*. For instance, on a commitment account of assertion (Brandom 1983), uttering *frankly p* commits one to *p*, and on a knowledge norm account (Williamson 2000), uttering *frankly p* requires one to know that *p*. But if one says *perhaps p*, one is *not* thereby committed to *p* or required to know that *p*.

Moreover, speaker-oriented adverbs cannot co-occur. For instance, *frankly fortunately it is raining* and *fortunately evidently it is raining* sound bad. However, speaker-oriented adverbs *can* co-occur with *perhaps*. Here are two examples.

(15) a. Frankly, perhaps Route 4 isn't what Ms. Milby needs to investigate. (The Washington Post, *Commuter Advice From Several Directions*, 6 December 2001)
b. Frankly it's perhaps now too late.
(BBC, *George Low stabbing: Cyprus murder suspect 'set free'*. 6 July 2017)

Thus, *perhaps* fits with neither speaker-oriented adverbs (such as *frankly*) nor compositional operators (like *might*). But there is a third option. Whilst *frankly* is used to *comment* on the performance of a speech act, *perhaps* is used to *modify* the speech act performed. Hence, in (15b), *frankly* serves to comment on the performance of the speech act obtained by modifying with *perhaps* an assertion of *it's now too late*.

One might object that *perhaps* cannot be a force-modifier because it embeds in conditional *consequents*.

(16) If it is going to rain, perhaps we should stay in.

However, such an embedding is compatible with *perhaps* not being a compositional operator. For instance, *frankly* also embeds in this way.

(17) If it is going to rain, frankly we should stay in.

But nobody would conclude that *frankly* is not a speech act adverb. Instead, conditionals such as (17) are best analysed as *conditional performances* of speech acts (Edgington 1995; Schnieder 2010). This is compatible with the occurrence of a force-modifying expression in the consequent of (16): *perhaps* modifies the speech act that is being conditionally performed.

perhaps as a force-modifier. Our evidence for *perhaps* being a force-modifier is the following: (i) *perhaps* exhibits the embedding behaviour that one would expect of a particle operating exclusively at the speech act level; (ii) the role of *perhaps* cannot be reduced to that of commenting on one's performance of a speech act; (iii) in polar questions, *perhaps* appears not to modify the core proposition; (iv) in commands, *perhaps* appears to modify force.

We have illustrated (i) and (ii) and now turn to (iii) and (iv). First, consider a natural use of *perhaps* in a polar question.

(18) Is it perhaps [made of] resin?
(British National Corpus, file KCV, line 4908)

Examining the potential positive answers to the question in (18) reveals that *perhaps* does not modify its core proposition.

(19) a. Is it perhaps resin?
b. Yes, it is.
?c. Yes, perhaps it is.
#d. Yes, but perhaps it is something else.

If *perhaps* were to modify the core proposition in (18), the appropriate positive answer to (19a) would be (19c). But the proper answer is (19b).[2] Moreover, (19d) indicates that *yes* here targets *it is resin*, since it cannot be felicitously followed by *perhaps it is something else* (whereas *perhaps it is resin* could be). We conclude that *perhaps* in (19a) affects the question's force, but not its content: it seems to make it a biased (Bellert 1977) or tentative question instead of a neutral polar question.

What about uses of *might* in polar questions? If *might* is a compositional operator, we should expect it to modify a question's core proposition. This prediction appears to be borne out.

(20) a. Might it be resin?
?b. Yes, it is.
c. Yes, it might be.
d. Yes, but it might be something else.

The preferred answer to the question in (20a) seems (20c).[3] This indicates that the question concerns a core proposition modified by *might*. Accordingly, (20d) is felicitous: if *yes* targets *it might be resin*, it should be compatible with *it might be something else*, as is indeed the case.

Now, consider an utterance in imperative mood containing *perhaps* and contrast it with the same utterance without *perhaps*.

(21) Perhaps check with the Seahawks.
(The New York Times, *Seahawks Beckon, and U.S.C. Cringes*, 8 January 2010)

(22) Check with the Seahawks.

These two utterances seem to express the same *content*, but with different *forces*: (22) is a command, whereas (21) seems more of a suggestion. Thus, in (21) *perhaps* appears to modify force.

[2] While (19c) does not strike us as downright infelicitous, it appears to be mockery by repetition. The appropriate answer using *perhaps* appears to be *I don't know—perhaps it is.*

[3] Again, (20b) is not downright infelicitous, but it seems to *overanswer* the question.

We conclude that *perhaps* modifies force rather than content. It modifies polar questions to biased or tentative questions and commands to suggestions, and has an analogous function when applied to assertions.

Now recall Smiley's suggestion that an assertion can be realised by posing a question to oneself and answering *yes*, as in (23a): *perhaps* can be used to modify such an answer, as in (23b), so as to perform the speech act we call *weak assertion*. For clarity's sake, we will henceforth call *strong assertion* what is usually referred to as *assertion*.

(23) a. Is it raining? Yes.
b. Is it raining? Perhaps yes.

(24) a. Perhaps it is raining.
b. Is it raining? Perhaps.

The forms in (24) are linguistic variants of (23b): in (24a), *it is raining* would otherwise be a strong assertion, but *perhaps* modifies this to a weak assertion; (24b) is like (23b) with an elided *yes*.

Thus, (23b), (24a), and (24b) all serve to perform the weak assertion of *it is raining*. This is a different speech act than the strong assertion of *it might be raining*. There is, however, a close connection between *might* and *perhaps*: as mentioned, they seem interchangeable in non-embedded contexts. This means that the weak assertion of *it is raining* and the strong assertion of *it might be raining* are inferentially equivalent. In what follows, we shall exploit this inferential equivalence to give an account of *might* in terms of weak assertion.[4]

Weak and strong assertion. We have gathered evidence for the existence of the speech act of weak assertion, but still need to explain what this speech act *is*.

To this end, consider Stalnaker's model of conversation (Stalnaker 1978, 2002). According to this model, conversation takes place against a backdrop of shared presuppositions, the *common ground*. In Stalnaker's view, the essential effect of an assertion is a proposal to update the common ground. Such a proposal may be *accepted* by all conversation participants, in which case the common ground is updated accordingly. However, not every update proposal is acceptable to all conversation participants. So any complete account of how the common ground is managed must include a mechanism by which to *prevent* such an update.

The speech act of *weak rejection* provides such a mechanism. Updating the common ground requires unanimous assent. But since assent and dissent

[4] Our linguistic observations about *perhaps* also seem to apply to *probably*. We conjecture that one can give a force-modifier analysis of *probably* and explain probability operators such as *likely* in terms of *probably*. One issue is that probabilistic adjectives come in all sort of degrees, but this aspect might be dealt with by augmenting the logic developed below with a theory of probability, inferentially grounded. We leave this to future work.

are incompatible attitudes, a weak rejection of *p* indicates that there is *not* unanimous assent to *p*. So, by performing a weak rejection, one prevents a common ground update (Incurvati & Schlöder 2017). Of course, one might also attempt to prevent an update with *p* by *asserting* something that excludes assent to *p*, e.g., *not p*. But then something *more* than preventing the update has happened. Thus, it would be mistaken to dispense with weak rejection because one has negative assertion.

To see where weak assertion fits into this model of conversation, consider the following dialogue, which is based on an example of Paul Grice (1991, 82) and which in (Incurvati & Schlöder 2017) we presented as a case involving a weak rejection.

(25) *Alice*: X or Y will win the election.
Bob: No, X or Y *or* Z will win.

Bob is here dissenting from both *X or Y will win* and *neither X nor Y will win*. Dissent from *X or Y will win* is expressed with the particle *no*, as per the bilateralist account. By contrast, dissent from *neither X nor Y will win* is *conveyed* via a pragmatic implicature: if Bob had not intended to dissent, he would have explicitly assented to *Z will win*. Indeed, this implicature can be cancelled.

(26) *Alice*: X or Y will win the election.
Bob: No, X or Y *or* Z will win. In fact, Z will.

We argue that *weak assertion* is the mechanism by which one can express dissent from *neither X nor Y will win*. Compare (27), where this effect of uttering *perhaps*—that one is dissenting from *neither X nor Y will win*—is non-cancellable.

(27) Perhaps X or Y will win the election. #In fact, neither of them will.

Thus, weakly asserting *X or Y will win* excludes assent to the negative of *X or Y will win* (and nothing more). That is, the essential effect of the weak assertion of *p* is to exclude the strong assertion of *not p* or, equivalently, exclude the *strong rejection* of *p*. (Recall that the strong assertion of *not p* is inferentially equivalent to the strong rejection of *p*.) Hence, just as the purpose of weak rejection is to exclude strong assertion (expressing assent), weak assertion works to exclude strong rejection (expressing assent to a negative).

This gives us four speech acts with the corresponding essential effects. By performing one of these speech acts, the speaker takes a public stance on the admissibility of a proposition into the common ground.

- By strongly asserting *p*, one proposes to add *p* to the common ground (or accepts a previous proposal to this effect).
- By strongly rejecting, *p* one proposes to add *not p* to the common ground (or accepts a previous proposal to this effect).

- By weakly asserting *p*, one prevents *not p* from being added to the common ground.
- By weakly rejecting *p*, one prevents *p* from being added to the common ground.

Jointly, these speech acts allow precise management of what is accepted into the common ground.

It is worth comparing this analysis of *perhaps* with some data about *might*. It has been observed that *might* can be used to reject a negative (Khoo 2015; Bledin & Rawlins forthcoming).

(28) *Alex*: It is not raining.
Becky: (No,) it *might* be.

Our analysis straightforwardly explains this piece of data. For the strong assertion of *it might be raining* is inferentially equivalent to the weak assertion of *it is raining*, which excludes assent to *it is not raining*. Hence, (28) is predicted to be a rejection move.

One might object that *might* can also be used to reject the positive *it is raining*.

(29) *Alex*: It is raining.
Becky: (No,) it *might* be.

However, the rejection in (29) is *pragmatic*: *might* conversationally implicates *not surely*, just as *some* in (30) implicates *not all* (Khoo 2015; Schlöder & Fernández 2015). This is evinced by the fact that these implicatures can be cancelled as in (31). Thus, (29) should not be mistaken as evidence regarding the *semantic* contribution of *might*.

(30) *Alex*: Alicia ate all the cookies.
Becky: (No,) she ate *some* cookies.

(31) a. She ate some cookies—in fact, she ate all of them!
b. It might be raining—in fact, it is raining!

might and the Frege–Geach problem. We mentioned earlier that traditional expressivist analyses of *might* are generally taken to be undermined by its embeddability behaviour. One way to appreciate this point is to observe that the embeddability of *might* under conditional antecedents can be used to run a version of the Frege–Geach argument (von Fintel & Gillies 2007; MacFarlane 2014).

(32) a. If *might p*, then *might q*.
b. *might p*.

---

c. *might q*.

Since this inference appears to be valid, the Frege–Geach argument can be performed as usual to establish that *might* in (32b) must modify content. Inferential expressivism takes *might* to be a compositional operator and hence sees (32) as a straightforward application of *modus ponens*, thereby avoiding this instance of the Frege–Geach problem.

We argued, however, that *perhaps* is a force-modifier. This gives rise to a revenge version of the Frege–Geach argument, just as taking *no* to be a force-indicator did. Consider:

(33) a. If *might p*, then *might q*.
b. Perhaps *p*.

---

c. Perhaps *q*.

This argument seems valid. Now on our account *perhaps* in (33b) does not modify *p*, whereas *might* in the antecedent of (33a) does. But then (33) is not an instance of *modus ponens*, and our account, it seems, cannot validate it.

Similarly to when we discussed *not*, this argument rests on the assumption that (33) *must* be a direct application of *modus ponens*. But we can validate (33) using, besides *modus ponens*, the fact that the weak assertion of *p* is inferentially equivalent to the strong assertion of *might p*. In particular, from the weak assertion of *p* (performed uttering *Perhaps p*), we can infer the strong assertion of *might p*. This, together with the strong assertion of *if might p, then might q* delivers, by *modus ponens*, the strong assertion of *might q* and hence the weak assertion of *q*.

The inference (33) involves weak and strong assertion, and is therefore mixed. When considering *no* and rejection, we presented a mixed inference which can be recognized as valid without theorising about embeddable operators. One can find analogous cases involving *perhaps*.

(34) a. If *p*, then *q*.
b. Perhaps *p*.

---

c. Perhaps *q*.

This inference cannot be validated simply by appealing to the inferential equivalence between *perhaps p* and *might p*. To validate this inference, one needs to *coordinate* strong and weak assertion, similarly to what happened in the case of strong assertion and weak rejection. We will show how to do this below.

From bilateralism to multilateralism. Bilateralism takes the meaning of the logical constants to be given by conditions on *strong assertion* and *weak rejection*. We argue that bilateralism should be extended to *multilateralism* by encompassing *weak assertion*. In the next section, we codify conditions on weak assertion, strong assertion, and weak rejection in a multilateral logic. These conditions give the meaning of *not* and *might*.

Weak assertion is therefore more basic than *not* and *might*: the basic speech acts of common ground management are prior to any embeddable operator. However, our logic does not include a sign for strong rejection, since the three other speech acts suffice to explain *not*, and a strong rejection is equivalent to the assertion of a negation. But we are not committed to *identifying* strong rejection with negative assertion, and one could introduce a sign for strong rejection via rules allowing one to pass from a strong rejection to a negative assertion and *vice versa*.

V. MULTILATERAL LOGIC FOR EPISTEMIC MODALITY

Our multilateral logic is cast in a language with constants $\wedge$, $\neg$, $\Diamond$, and $\rightarrow$, formalizing *and*, *not*, *might,* and conditionals.[5] We use upper-case letters to denote sentences and prefix them with *force-markers*: $+$ for strong assertion, $\ominus$ for weak rejection, and $\oplus$ for weak assertion. We give our inference rules in a natural deduction calculus. In the interest of brevity, we present a minimal set of rules for $\wedge$, $\neg$, $\rightarrow$, and $\Diamond$. The remaining cases can be derived using the rules provided.

Inference. We should first clarify what we mean by *inference* in the context of inferential expressivism. The immediate explanation is that inference preserves *attitude*: some attitudes towards some propositions *require* further attitudes towards further propositions. While intuitive, this explanation requires additional clarification.

Suppose we validate the inference from $+A$ to $+B$. Surely this does not mean that whoever strongly asserts A also strongly asserts B (Dutilh Novaes 2015). In terms of attitudes, it is implausible to say that whoever assents to A also assents to B (see Restall 2005). For assent requires awareness of what is being assented to, but infinitely many propositions follow from any given A, and nobody can be said to have assented to them all.

Restall's problem can be avoided by taking $+B$ to follow from $+A$ just in case whoever strongly asserts A is *committed to assenting* to B. To be committed to assenting to B does not require being aware of B. Rather, it requires assenting to B once this is pointed out.

For instance, suppose a speaker strongly asserts both *if A, then B* and A, but does not assent to B. In this situation, the speaker is nonetheless required to assent to B *once this is pointed out to them*—or admit to a mistake and retract an

[5] As noted when discussing conditional performances of speech acts, not all natural language occurrences of *if …then* correspond to an embeddable operator. In ongoing work, we further investigate which natural language *if …then* are formalised by our $\rightarrow$, but we set the issue aside here.

earlier assertion (Brandom 1983; MacFarlane 2010). This is what we mean by *being committed to assenting* to a proposition.

This attitude-theoretic account of inference sanctions the bilateral rules for asserted conjunction.

$$(+\wedge\text{I.})\ \frac{+A \qquad +B}{+(A \wedge B)} \qquad (+\wedge\text{E.}_1)\ \frac{+(A \wedge B)}{+A} \qquad (+\wedge\text{E.}_2)\ \frac{+(A \wedge B)}{+B}$$

A speaker who strongly asserts A and strongly asserts B is committed to assenting to A *and* B. Similarly, someone who strongly asserts A *and* B is committed to assenting to A and B individually.

The account is similar to Catarina Dutilh Novaes's (2015) proposal that if B follows from A and you have granted A, you are required to grant B. To be required to grant B does not mean to *have* granted B—rather to do so once B is put forward. The notion of *granting*, however, is married to assertion, whereas we aim to validate inferences also involving rejections and weak assertions.

Thus, we prefer to speak of *being committed to having certain attitudes*. This follows Incurvati & Schlöder (2017). There we explain implicit assent as *implicit commitment* in the context of a commitment account of assertion (Brandom 1983, 1994; Lascarides & Asher 2009), but generalize that account by encompassing attitudes other than assent.

The attitude-theoretic account can be used to validate inferences involving the other speech acts we are concerned with. For instance, it sanctions as valid the inference (7) discussed earlier, which has it that if one asserts *if* A *then* B and rejects B, one is committed to dissenting from A.

In what follows, we give inference rules that are justified in that they preserve the attitudes one is committed to having. Thus, our logic computes which attitudes a single speaker is committed to, given the attitudes they have displayed. In speech act terms, our logic computes which stances on what is admissible into the common ground a speaker is committed to, given the stances they have publicly taken.

Bilateral coordination principles. In Section III we described two principles coordinating assertion and rejection. Using $\perp$ as a sign for absurdity, we can now formalize these principles.[6]

$$(\text{Rejection})\ \frac{+A \qquad \ominus A}{\perp}$$

(Rejection) expresses the idea that it is absurd to both strongly assert and weakly reject the same proposition, i.e., $+A$ and $\ominus A$ are incompatible. This is a central property of strong assertion and weak rejection as the speech acts expressing the incompatible attitudes of assent and dissent (Dickie 2010).

[6] As usual in bilateral systems, $\perp$ is considered a punctuation mark and is therefore not prefixed by a force-marker (Tennant 1999; Rumfitt 2000).

The justification for the two halves of Smileian *reductio* goes as follows.

$$(\mathrm{SR}_1)\ \frac{\begin{array}{c}[+A]\\ \vdots\\ \bot\end{array}}{\ominus A} \qquad (\mathrm{SR}_2)\ \frac{\begin{array}{c}[\ominus A]\\ \vdots\\ \bot\end{array}}{+A}$$

The conditional proof in (SR_1) says that strongly asserting A leads to absurdity. This means that the speaker is committed to having an attitude towards A that is incompatible with assent to A. That is, A is unassertible to the speaker. Since to dissent is to find a proposition unassertible, it follows that the speaker is committed to dissenting from A. The converse argument justifies (SR_2): if rejecting A leads to absurdity, the speaker is committed to having an attitude that excludes dissent from A. This attitude must be assent to A.

Recall that besides expressing attitudes, assertion and rejection also serve to negotiate common ground updates. When read focusing on this role, (SR_1) says that if it is absurd for someone to propose a common ground update, that must be because their public stances on what is admissible into the common ground already commit them to preventing that update. Similarly, (SR_2) states that if it is absurd for someone to prevent a proposed update of the common ground with p, that must be because their public stances on what is admissible into the common ground already commit them to accepting a proposal to add p.

might and not. We now turn to the rules for $\Diamond$. As argued in Section IV, when *might* and *perhaps* take scope over the same non-embedded clause, they can be interchanged without affecting the inferential meaning of the sentence. Thus, $\Diamond$s can be introduced by moving from weak to strong assertion, and can be eliminated symmetrically.

$$(+\Diamond\mathrm{I.})\ \frac{\oplus A}{+\Diamond A} \qquad (+\Diamond\mathrm{E.})\ \frac{+\Diamond A}{\oplus A}$$

Two additional rules for $\Diamond$ account for the fact that *perhaps it might be raining* is inferentially equivalent to *perhaps it is raining*.

$$(\oplus\Diamond\mathrm{I.})\ \frac{\oplus A}{\oplus\Diamond A} \qquad (\oplus\Diamond\mathrm{E.})\ \frac{\oplus\Diamond A}{\oplus A}$$

These rules imply that iterating *might* does not affect the compositional content of an utterance (see also Yalcin 2007; Willer 2013). However, this only applies when the context does not indicate that multiple occurrences of *might* are to be understood with reference to different common grounds. A case based on (DeRose 1991, 584–5) will clarify the situation. A medical test has been run but the results are not known yet. A negative result rules out John having the disease; a positive result leaves that possibility open. Responding to a friend asking for information, John's partner Jane says:

> ANSWER-1. We haven't got the results yet. It might be the case that John might have the disease. We'll know whether he might have it when we get the results.

Jane asserts that *it might be the case that John might have the disease* but seems unwilling to assert that *John might have the disease*. This answer seems entirely appropriate. However, it seems equally appropriate for Jane to assert that *John might have the disease* (DeRose 1991, 583).

> ANSWER-2. John might have the disease. He has some of the symptoms. We won't get the test results until tomorrow.

In ANSWER-2, it is clear that *might* is to be understood with reference to the current common ground cg_c. This suggests that in ANSWER-1 Jane is unwilling to assert that *John might have the disease* because this occurrence of *might* is understood with reference to a *different* common ground. Since the fact that the common ground will be updated with the test results is very salient, this is naturally taken to be cg_a, the common ground after the results are known. Thus, in ANSWER-1, Jane asserts that *it might*^{cg_c} *be that John might*^{cg_a} *have the disease*. In so doing, she prevents from making it common ground that, once the results are known, she will prevent from making it common ground that *John does not have the disease*. This seems a correct reading of her utterance and does not imply that *John might*^{cg_a} *have the disease*, even in the presence of the $\oplus\Diamond$ rules.

The next rules give the meaning of negation. In Section IV we argued that the essential effect of the weak assertion of A is to exclude assent to $\neg A$, which is the same essential effect as that of the weak rejection of $\neg A$. Accordingly, $\oplus A$ and $\ominus\neg A$ are inferentially equivalent. Analogous arguments show that weakly asserting a negative is inferentially equivalent to rejecting the positive. Thus we also have rules allowing us to pass from $\oplus\neg A$ to $\ominus A$ and *vice versa*.

$$(\ominus\neg\text{I.})\ \frac{\oplus A}{\ominus\neg A} \qquad (\ominus\neg\text{E.})\ \frac{\ominus\neg A}{\oplus A}$$

$$(\oplus\neg\text{I.})\ \frac{\ominus A}{\oplus\neg A} \qquad (\oplus\neg\text{E.})\ \frac{\oplus\neg A}{\ominus A}$$

The rules for $\Diamond$ and $\neg$ are obviously in harmony, since the elimination rules are the direct inverses of the introduction rules. Hence, these rules confer a coherent meaning on *might* and *not*.

Conditionals and the specificity problem. Formulating the rules for $\rightarrow$ requires some care. In (Incurvati & Schlöder 2017) we argue that the Deduction principle ought to fail in bilateral logics in which rejections are weak due to the *specificity problem* (implicit in Dickie 2010). Consider the principle of classical negation introduction (CNI) and compare it with the also classically valid (CR).

$$\text{(CNI)}\ \frac{\begin{array}{c}[+A]\\ \vdots\\ \bot\end{array}}{+\neg A} \qquad\qquad \text{(CR)}\ \frac{+(A \to (B \wedge \neg B))}{+\neg A}$$

As Dickie (2010) observes, (CNI) must fail in bilateral logics in which rejections are weak. For the conditional proof in (CNI) establishes that asserting A leads to absurdity, i.e. that A is dissented from, whilst the conclusion of (CNI) says that $\neg A$ is assented to. But assenting to a negative is only *one* way of expressing dissent. Thus, (CNI) reduces all possible ways of dissenting from a proposition to assent to its negation. That is, (CNI) allows one to go from *unspecific* dissent to a *specific* way of dissenting, *viz*. assent to a negative. Therefore, (CNI) does not preserve attitude and is therefore not valid.

We can make things more concrete by presenting an invalid natural language instantiation of (CNI), using an example already encountered. Consider a situation in which one dissents from the proposition b that *X or Y will win the election* but assents to the proposition c that *X or Y or Z will win*. If one were to assent to the proposition a that *Z will lose the election*, one would hold incompatible attitudes. Formally, $\ominus b, +c, +a \vdash \bot$. By (CNI), it would follow that one is committed to assenting to the proposition $\neg a$ that *Z will not lose the election*. But this is a mistake, since one may also dissent from $\neg a$, if, say, one ascribes equal chances to all the candidates.

However, (CR) is not similarly defective. It states that $\neg A$ can be inferred from assent to a determinate, *specific* piece of information, *viz*. the proposition that any situation in which A holds is a contradictory situation. In the above example, assent to *Z will lose the election* leads to absurdity, but this does not entail assent to *if Z will lose, there is a contradiction*. Thus, the premiss of (CR) is logically stronger than the conditional proof in (CNI) because it is more informative. Hence, in particular, (CR) is a *weaker* principle than (CNI).

But then the Deduction principle ($+\to$I.) must fail in bilateral logics.

$$(+\to\text{I.})\ \frac{\begin{array}{c}[+A]\\ \vdots\\ +B\end{array}}{+(A \to B)}$$

Since Smileian *reductio* encodes a form of explosion, the conditional proof in (CNI) entails $+A \cdots +(B \wedge \neg B)$. Given ($+\to$I.), we could then obtain the premiss of (CR). Hence (CR) would entail (CNI)—but this is a mistake, since we determined (CR) to be strictly weaker than (CNI). Thus, ($+\to$I.) is invalid.

In (Incurvati & Schlöder 2017) we deal with the issue by *restricting* ($+\to$I.) to derivations that exclusively employ strongly asserted premisses. Let us write $\overset{+}{\vdots}$ for a subderivation which only uses premisses marked by $+$ and in which all remaining undischarged assumptions are also marked with $+$. Then, we can formulate our earlier restriction of $(+\rightarrow\text{I.})$ as follows:

$$(+\rightarrow\text{I.*})\ \frac{\begin{array}{c}[+A]\\ +\vdots\\ +B\end{array}}{+(A \rightarrow B)}$$

Since the language of our earlier system only includes the standard Boolean connectives, this solves the problem: in such a language, the only way to express dissent from A by *only* strong assertion is to assent to the negation of A or to the fact that A entails an absurdity. Thus, $(+\rightarrow\text{I.*})$ derives no more instances of (CNI) from (CR) than are already validated by (CR): whenever one derives dissent from A from only asserted Boolean premisses, the attitudes one is committed to already amount to the premiss or the conclusion of (CR) (Incurvati & Schlöder 2017).

This solution will not do in the current situation, since epistemic vocabulary adds a new way of expressing dissent by means of a strong assertion, namely $+\Diamond\neg A$. This is a strong assertion which is sufficient to infer dissent from A, but it amounts to *neither* asserting that A is false *nor* that A entails an absurdity.

To solve the problem posed by epistemic vocabulary, we further restrict $(+\rightarrow\text{I.})$. Deriving dissent from A (i.e. $\ominus A$) from $+\Diamond\neg A$ requires eliminating a $\Diamond$. We exclude such derivations from the Deduction principle.

$$(+\rightarrow\text{I.**})\ \frac{\begin{array}{c}[+A]\\ +\vdots\\ +B\end{array}}{+(A \rightarrow B)}\ \text{if } (+\Diamond\text{E.}) \text{ and } (\oplus\Diamond\text{E.}) \text{ were not used to derive } +B.$$

In effect, this restriction ensures that epistemic grounds to dissent are eliminated from the Deduction principle.

The usual bilateral version of $(+\rightarrow\text{E.})$ remains valid in our logic and is in harmony with $(+\rightarrow\text{I.**})$ according to the *levelling peak* criterion (Dummett 1991).[7]

$$(+\rightarrow\text{E.})\ \frac{+(A \rightarrow B) \qquad +A}{+B}$$

[7] A *peak* is an application of an introduction rule immediately followed by the corresponding elimination rule. To *level* a peak is to eliminate the successive introduction/elimination. A peak for $\rightarrow$ can be levelled by applying the derivation of $+B$ from $+A$ in $(+\rightarrow\text{I.**})$ to the minor premiss $+A$ of $(+\rightarrow\text{E.})$.The rules for strongly asserted $\rightarrow$ are, as they stand, not harmonious according to the general elimination harmony criterion (Read 2000). However, one can obtain a version of $(+\rightarrow\text{E.})$ which *is* general-elimination harmonious with $(+\rightarrow\text{I.**})$ by imposing the restriction of $(+\rightarrow\text{I.**})$ on $+A$ in $(+\rightarrow\text{E.})$. All results in this paper still go through under this restriction.

Multilateral coordination principles. The principles (Rejection) and Smileian *reductio* coordinate strong assertion and rejection. But our logic includes a force-marker for weak assertion. To properly integrate this speech act into the logic, we give two additional coordination principles. Our first coordination principle guarantees that strong assertion is a special case of weak assertion.

$$\text{(Assertion)}\ \frac{+A}{\oplus A}$$

The weak assertion of A excludes assent to $\neg A$. (Assertion) ensures that so does the strong assertion of A.

Our second coordination principle says that weak assertion is preserved under entailment.

$$\text{(Weak Inference)}\ \frac{\begin{array}{c}[+A]\\ +\vdots\\ +B\end{array} \qquad \oplus A}{\oplus B}\ \text{if } (+\Diamond\text{E.}) \text{ and } (\oplus\Diamond\text{E.}) \text{ were not used to derive } +B.$$

How can (Weak Inference) be justified? By Smileian *reductio*, the derivation of $+B$ from $+A$ means that assent to *not B* entails dissent from A. And the premiss $\oplus A$ excludes assent to *not A*. Thus, if it were absurd to exclude assent to *not A* whilst dissenting from A, we could conclude that assent to *not B* is excluded. That is, we could conclude $\oplus B$, as desired. But it is plainly *not* absurd to exclude assent to *not A* whilst dissenting from A. To wit:

(35) Is it raining? Perhaps it is, perhaps it is not. #It is.

In this example, assent to *not A* is excluded, but so is assent to A, which is therefore dissented from. A weak assertion of A excludes assent to *not A*, i.e. it excludes a *specific* way of dissenting from A. However, it does not exclude *general* dissent from A, such as that expressed through a weak rejection of A or assent to *might not A*.

Hence, to be able to conclude $\oplus B$ in (Weak Inference), we must have that assent to *not B* entails specific dissent from A. Formally, this is obtained by restricting the subderivation in (Weak Inference) in the same way as we did with the Deduction principle.

In Section III, we noticed that coordination principles help validate mixed inferences. Crucially, this is the case for (Weak Inference). In particular, recall the following inference involving strong and weak assertion.

(36) a. If p, then q.
b. Perhaps p.

―――――

c. Perhaps q.

This inference is immediately validated once (Weak Inference) is added to our logic.

Epistemic multilateral logic. This concludes the exposition of the rules of our logic. That is, we let *epistemic multilateral logic* be the natural deduction calculus consisting of the three rules for $\wedge$, the four coordination principles, the eight rules for $\Diamond$ and $\neg$, and the two rules for $\rightarrow$, *viz.* ($+\rightarrow$E.) and ($+\rightarrow$I.**). As we show in a technical companion to this paper (Incurvati and Schlöder ms), the logic is consistent, since it can be proved sound (and indeed complete) for an embedding into the modal logic S5.

Epistemic multilateral logic preserves the bilateralist defence of classical negation.

Proposition V.I. The following are theorems of epistemic multilateral logic:

- $+(A \rightarrow \neg\neg A)$.
- $+(\neg\neg A \rightarrow A)$.
- $+((A \rightarrow B) \rightarrow (\neg B \rightarrow \neg A))$.

Proofs are in the Appendix. Now it is easy to check that ($+\rightarrow$I.**) and ($+\rightarrow$E.) jointly entail the standard axioms for $\rightarrow$.[8] Together with Proposition V.I, this delivers Frege's axioms for the classical propositional calculus. Moreover, one can show that $A \wedge B$ with $\wedge$ characterized by ($+\wedge$I.) and ($+\wedge$E.) is equivalent to $\neg(A \rightarrow \neg B)$.

Therefore, the *logic of strongly asserted content* extends classical logic: on strongly asserted premisses we have all classical theorems and classical *modus ponens*. Thus we in fact validate classical *inference*: epistemic multilateral logic sanctions as valid all classically valid arguments. This, *nota bene*, applies to all substitution-instances of these arguments, including ones containing $\Diamond$s.

VI. RESOLVING THE EPISTEMIC PUZZLES

Epistemic multilateral logic is motivated independently of the epistemic puzzles described in Section II. We now show that it yields natural solutions to those puzzles.

Yalcin's puzzle. We explain Yalcin's puzzle by showing that the sentence *p and it might not p* gives rise to *incompatible attitudes*. The following derivation demonstrates that assenting to *p and it might not p* immediately reduces to both assenting and dissenting from *p*.

[8] That is, $+((A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C)))$, $+((A \rightarrow (B \rightarrow C)) \rightarrow (B \rightarrow (A \rightarrow C)))$, and $+(A \rightarrow (B \rightarrow A))$.

$$\dfrac{\dfrac{+(p \wedge \Diamond\neg p)}{+p}\,(+\wedge\text{E.}) \qquad \dfrac{\dfrac{\dfrac{+(p \wedge \Diamond\neg p)}{+\Diamond\neg p}\,(+\wedge\text{E.})}{\oplus\neg p}\,(+\Diamond\text{E.})}{\ominus p}\,(\oplus\neg\text{E.})}{\bot}\,(\text{Rejection})$$

Since assent and dissent are incompatible attitudes, to suppose that *p and it might not be p* is to suppose something knowingly absurd. But apart from indirect proof contexts, to suppose something knowingly absurd is incoherent. As a result, *suppose p and it might not be p* is infelicitous.

Some may find *suppose I assent to p and dissent from p* less infelicitous (if infelicitous at all) than *suppose p and might not be p*.[9] This difference, we submit, is due to the fact that in uttering *I assent to p and dissent from p* one is *reporting* the fact that one assents and dissents from *p*, whereas uttering *p and it might not be p* inferentially reduces to *expressing* assent and dissent from *p*. And while expressing assent and dissent from the same proposition is absurd, it need not be absurd to report that one both assents and dissents from it. As an analogy, *suppose that p and not p* is infelicitous because *p* and *not p* express incompatible attitudes, whereas *suppose that I assent to p and assent to not p* may sound more felicitous because *I assent to p and assent to not p* merely reports that one has incompatible attitudes.

It is worth comparing our explanation of Yalcin's puzzle to Moore's paradox. It seems that one *should not* express assent to *p* without knowing *p*. This is, in effect, the knowledge norm of assertion (Williamson 2000). However, it is not *absurd* to express assent to *p* without knowing *p*. Accordingly, it is perfectly fine to *suppose* that one has violated the norm, which explains the felicitousness of Moore sentences under supposition.

In sum, according to our explanation, Yalcin's puzzle is based on a conflict of attitudes emerging from the semantic properties of *might*. This clearly distinguishes it from Moore's paradox, which is based on a violation of a pragmatic norm.

Triviality. Russell & Hawthorne (2016) prove their triviality result in a dynamic setting, but epistemic multilateral logic is static. Nonetheless, one can reproduce in epistemic multilateral logic the crucial part of the triviality argument, *viz.* the one showing that if a state is updated with *might p*, it rules out *not p*. It becomes a proof that assent to *not p* and assent to *might p* are incompatible attitudes.

[9] We thank an anonymous referee for this observation.

$$\dfrac{+\neg p \qquad \dfrac{\dfrac{+\Diamond p}{\oplus p}\,(+\Diamond\text{E.})}{\ominus\neg p}\,(\ominus\neg\text{I.})}{\bot}\,(\text{Rejection})$$

According to Russell and Hawthorne's argument, one can conclude that assent to *might p* entails assent to *p*. This requires applying classical *reductio* (i.e. (CNI)) to derive $+p$ from the fact that $+\neg p$ leads to absurdity. However, epistemic multilateral logic does not sanction (CNI), although it validates a restricted version thereof (proof in Appendix).

$$(\text{CNI*})\ \dfrac{\begin{matrix}[+A]\\ +\vdots\\ \bot\end{matrix}}{+\neg A}\ \text{if } (+\Diamond\text{E.}) \text{ and } (\oplus\Diamond\text{E.}) \text{ were not used to derive } \bot.$$

But the derivation above eliminates a $\Diamond$ and hence does not license application of (CNI*). Indeed, there are models of epistemic multilateral logic in which $+\Diamond p$ holds but $+p$ does not (see Incurvati and Schlöder ms). So any purported proof of $+p$ from $+\Diamond p$ *has* to fail.

Thus, *contra* Russell and Hawthorne, rejecting Commutativity is not the only real option once we accept that if a state is updated with *not p*, it rules out *might p*. On our account, assent to *not p* and assent to *might p* are incompatible attitudes (so, in dynamic terms, we do have that if a state is updated with *might p*, it rules out *not p*). Yet, triviality is avoided: the culprit is the notion of *entailment* used to conclude that assent to *might p* entails assent to *p*.

Modal disagreement. Our account explains the phenomenon of modal disagreement. Consider again:

(37) *Alice*: I can't find the keys.
Bob: They might be in the car.
Alice: No, they are not in the car. I just checked.

On our account, there is indeed a proposition that Bob is warranted in asserting and Alice is warranted in rejecting, namely the proposition that *the keys might be in the car*. This means that Alice and Bob's disagreement is not about some particular body of knowledge, but about whether *not p* should be added to the common ground.

In particular, let *p* be the proposition that *the keys are in the car*. Then Bob asserts *might p*, which is inferentially equivalent to the weak assertion of *p*. Thus, he is committed to preventing *not p* from being added to the common ground. Alice, on the other hand, thinks that *not p* should be added to the common ground. As a result, she disagrees with Bob and rejects *might p*. This is predicted by our logic, since $\ominus\Diamond p$ is derivable from $+\neg p$.

VII. CONCLUSION

Traditional expressivism about *might* fails: because of its embeddability behaviour, *might* should be treated as a compositional operator. However, linguistic analysis reveals that *perhaps* is a force-modifier, in line with what traditional expressivists contend. In particular, in otherwise strongly assertoric contexts, *perhaps* serves to perform the speech act of weak assertion. When embedded into a multilateral logic, this speech act can be used to inferentially explain the meaning of *might*. This vindicates *inferential* expressivism about epistemic modality.

The resulting account of *might* has several attractive features. While expressivist, it solves the Frege–Geach problem. While inferentialist, it respects harmony but retains classical logic as part of the logic of strongly asserted content. Finally, the account deals with the two problems we set out to address: it provides a solution to the epistemic puzzles and, in so doing, demonstrates by example that inferentialism can be fruitfully extended beyond the core logical constants.

Inferentialism has often been associated with a revisionary approach to logic. Our findings show that, when augmented with resources from the expressivist programme, it can in fact be used to preserve classical logic even where current model-theoretic approaches typically recommend radical departures from it.[10]

REFERENCES

Ayer, A. J. (1936) *Language, Truth and Logic*. London: Gollancz.
Bellert, I. (1977) 'On Semantic and Distributional Properties of Sentential Adverbs', *Linguistic Inquiry*, 8/2: 337–51.
Bledin, J. and Rawlins, K. (forthcoming) 'Resistance and Resolution: Attentional Dynamics in Discourse', *Journal of Semantics*.
Brandom, R. (1983) 'Asserting', *Noûs*, 17/4: 637–50.
—— (1994) *Making it Explicit*. Cambridge, MA: Harvard University Press.
Crabill, J. D. (2013) 'Suppose Yalcin is Wrong About Epistemic Modals', *Philosophical Studies*, 162/3: 625–35.
DeRose, K. (1991) 'Epistemic Possibilities', *Philosophical Review*, 100/4: 581–605.

[10] This work has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No 758540) within the project *From the Expression of Disagreement to New Foundations for Expressivist Semantics*. For comments and discussion, we would like to thank Maria Aloni, Justin Bledin, Daniel Cohnitz, Cian Dorr, Salvatore Florio, Melissa Fusco, Chris Gauker, Angelika Kratzer, Carlo Nicolai, Una Stojnic, Frank Veltman, and two referees for this journal. Earlier versions of this material were presented at the Theoretical Philosophy Colloquium at Utrecht University, the Philosophy of Language and Mind (PLM) Masterclass at the University of Salzburg, and the New York Philosophy of Language Workshop. We are grateful to the members of those audiences for their valuable feedback.

Dickie, I. (2010) 'Negation, Anti-Realism, and the Denial Defence', *Philosophical Studies*, 150/2: 161–85.
Dorr, C. and Hawthorne, J. (2013) 'Embedding Epistemic Modals', *Mind*, 122/488: 867–913.
Dummett, M. (1991) The Logical Basis of Metaphysics. Cambridge, MA: Harvard University Press.
—— (2002) ' 'Yes','No' and 'Can't Say' ', *Mind*, 111/442: 289–95.
Dutilh Novaes, C. (2015) 'A Dialogical, Multi-Agent Account of the Normativity of Logic', *Dialectica*, 69/4: 587–609.
Edgington, D. (1995) 'On Conditionals', *Mind*, 104/414: 235–329.
Egan, A., Hawthorne, J. and Weatherson, B. (2005) 'Epistemic Modals in Context', in G. Preyer and G. Peter (eds.) *Contextualism in Philosophy*, 131–69. New York: OUP.
Ernst, T. (2009) 'Speaker-Oriented Adverbs', *Natural Language & Linguistic Theory*, 27/3: 497–544.
von Fintel, K. and Gillies, A. (2007) 'An Opinionated Guide to Epistemic Modality', in T. S. Gendler and J. Hawthorne (eds.) *Oxford Studies in Epistemology*, vol. 2, 32–62. Oxford: OUP.
Frege, G. (1919) 'Die Verneinung: Eine logische Untersuchung', *Beiträge zur Philosophie des deutschen Idealismus*, 1/2: 143–57.
Geach, P. T. (1965) 'Assertion', *The Philosophical Review*, 74/4: 449–65.
Gentzen, G. (1935) 'Untersuchungen über das logische Schließen I', *Mathematische Zeitschrift*, 39/1: 176–210.
Grice, H. P. (1991) *Studies in the Way of Words*, Cambridge. MA: Harvard University Press.
Horn, L. (1989) A Natural History of Negation. Chicago, IL: University of Chicago Press.
Incurvati, L. and Schlöder, J. J. (2017) 'Weak Rejection', *Australasian Journal of Philosophy*, 95/4: 741–60.
—— (ms) 'Epistemic Multilateral Logic', manuscript.
Incurvati, L. and Smith, P. (2009) 'Rejection and Valuations', *Analysis*, 70/1: 3–10.
Khoo, J. (2015) 'Modal Disagreements', *Inquiry*, 58/5: 511–34.
Kratzer, A. (1977) 'What 'Must' and 'Can' Must and Can Mean', *Linguistics and Philosophy*, 1/3: 337–55.
Lascarides, A. and Asher, N. (2009) 'Agreement, Disputes and Commitments in Dialogue', *Journal of Semantics*, 26/2: 109–58.
MacFarlane, J. (2010) 'What is Assertion?', in H. Cappelen and J. Brown (eds.) *Assertion. New Philosophical Essays*, 79–96. Oxford: OUP.
—— (2014) *Assessment Sensitivity: Relative Truth and its Applications*. Oxford: OUP.
Mittwoch, A. (1977) 'How to Refer to One's Own Words: Speech-Act Modifying Adverbials and the Performative Analysis', *Journal of Linguistics*, 13/2: 177–89.
Moss, S. (2015) 'On the Semantics and Pragmatics of Epistemic Vocabulary', *Semantics and Pragmatics*, 8/5: 1–81.
Oddie, G. (2016) 'Truthlikeness', in E. N. Zalta (ed.) *The Stanford Encyclopedia of Philosophy*. Winter 2016 edn. ⟨https://plato.stanford.edu/archives/win2016/entries/truthlikeness/⟩ accessed 11 April 2019.
Prawitz, D. (1965) *Natural Deduction. A Proof-Theoretical Study*. Uppsala: Almqvist & Wicksell.
Prior, A. N. (1960) 'The Runabout Inference Ticket', *Analysis*, 21/2: 38–9.
Read, S. (2000) 'Harmony and Autonomy in Classical Logic', *Journal of Philosophical Logic*, 29/2: 123–54.
Restall, G. (2005) 'Multiple Conclusions', in P. Hájek, L. Valdés-Villanueva and D. Westerståhl (eds.) *Logic, Methodology and Philosophy of Science: Proceedings of the Twelfth International Congress*, 189–205. London: Kings College Publications.
—— (2016) 'Review of *Advances in Proof-Theoretic Semantics*', *Notre Dame Philosophical Reviews*. ⟨https://ndpr.nd.edu/news/advances-in-proof-theoretic-semantics/⟩ accessed 11 April 2019
Rumfitt, I. (1997) 'The Categoricity Problem and Truth-Value Gaps', *Analysis*, 57/4: 223–36.
—— (2000) "Yes' and 'No'", *Mind*, 109/436: 781–823.
Russell, J. S. and Hawthorne, J. (2016) 'General Dynamic Triviality Theorems', *The Philosophical Review*, 125/3: 307–39.
Schlöder, J. J. and Fernández, R. (2015) 'Pragmatic Rejection', in *Proceedings of the 11th International Conference on Computational Semantics*, 250–60. London: Association for Computational Linguistics.

Schnieder, B. (2010) 'Expressivism Concerning Epistemic Modals', *The Philosophical Quarterly*, 60/240: 601–15.
Schroeder, M. (2008a) 'How Expressivists Can and Should Solve Their Problem with Negation', *Noûs*, 42/4: 573–99.
—— (2008b) 'What is the Frege-Geach Problem?', *Philosophy Compass*, 3/4: 703–20.
Smiley, T. (1996) 'Rejection', *Analysis*, 56/1: 1–9.
Stalnaker, R. (1978) 'Assertion', in *Syntax and Semantics*, Vol. 9, 315–32. Cambridge, MA: Academic Press.
—— (2002) 'Common Ground', *Linguistics and Philosophy*, 25/5–6: 701–21.
Stevenson, C. L. (1937) 'The Emotive Meaning of Ethical Terms', *Mind*, 46/181: 14–31.
Swanson, E. (2010) 'On Scope Relations Between Quantifiers and Epistemic Modals', *Journal of Semantics*, 27/4: 529–40.
Tennant, N. (1997) *The Taming of the True*. Oxford: OUP.
Tennant, N. (1999) 'Negation, Absurdity and Contrariety', in D. Gabbay and H. Wansing (eds.) *What is Negation?*, 199–222. Dordrecht: Kluwer.
Unwin, N. (1999) '*Quasi*-Realism, Negation and the Frege-Geach Problem', *Philosophical Quarterly*, 49/196: 337–52.
Veltman, F. (1996) 'Defaults in Update Semantics', *Journal of Philosophical Logic*, 25/3: 221–61.
Warren, J. (2015) 'Talking with Tonkers', *Philosopher's Imprint*, 15/24: 1–24.
Willer, M. (2013) 'Dynamics of Epistemic Modality', *Philosophical Review*, 122/1: 45–92.
Williamson, T. (2000) Knowledge and its Limits. Oxford: OUP.
—— (2010) 'Review of *Reason in Philosophy: Animating Ideas*', *Times Literary Supplement*, 5579: 22–3.
Yalcin, S. (2007) 'Epistemic Modals', *Mind*, 116/464: 983–1026.

APPENDIX: PROOFS

Proof of $+(A \to \neg\neg A)$ and $+(\neg\neg A \to A)$.

$$\dfrac{\dfrac{\dfrac{[+A]^1 \qquad \dfrac{\dfrac{[\ominus\neg\neg A]^2}{\oplus\neg A}\,(\ominus\neg\text{E.})}{\ominus A}\,(\oplus\neg\text{E.})}{\bot}\,(\text{Rejection})}{+\neg\neg A}\,(\text{SR}_2)^2}{+(A \to \neg\neg A)}\,(+\to\text{I.}^{**})^1
\qquad\qquad
\dfrac{\dfrac{\dfrac{[+\neg\neg A]^1 \qquad \dfrac{\dfrac{[\ominus A]^2}{\oplus\neg A}\,(\oplus\neg\text{I.})}{\ominus\neg\neg A}\,(\ominus\neg\text{I.})}{\bot}\,(\text{Rejection})}{+A}\,(\text{SR}_2)^2}{+(\neg\neg A \to A)}\,(+\to\text{I.}^{**})^1$$

Proof of $+((A \to B) \to (\neg B \to \neg A))$.

$$\dfrac{\dfrac{\dfrac{\dfrac{[+\neg B]^1 \qquad \dfrac{\dfrac{\dfrac{[+(A \to B)]^2 \qquad [+A]^3}{+B}\,(+\to\text{E.}) \qquad \dfrac{[\ominus\neg A]^4}{\oplus A}\,(\ominus\neg\text{E.})}{\oplus B}\,(\text{Weak Inference})^3}{\ominus\neg B}\,(\ominus\neg\text{I.})}{\bot}\,(\text{Rejection})}{+\neg A}\,(\text{SR}_2)^4}{+(\neg B \to \neg A)}\,(+\to\text{I.}^{**})^1}{+((A \to B) \to (\neg B \to \neg A))}\,(+\to\text{I.}^{**})^2$$

Proof of (CNI*).

$$
\begin{prooftree}
\AxiomC{$[+A]^1$}
\noLine
\UnaryInfC{$\vdots$}
\noLine
\UnaryInfC{$\bot$}
\AxiomC{$[\ominus\neg(A \to A)]^2$}
\RightLabel{(Repetition)}
\BinaryInfC{$\bot$}
\RightLabel{$(\mathrm{SR}_2)^2$}
\UnaryInfC{$+\neg(A \to A)$}
\AxiomC{$[\ominus\neg A]^3$}
\RightLabel{$(\ominus\neg\mathrm{E.})$}
\UnaryInfC{$\oplus A$}
\RightLabel{$(\mathrm{Weak\ I.})^1$}
\BinaryInfC{$\oplus\neg(A \to A)$}
\RightLabel{$(\oplus\neg\mathrm{E.})$}
\UnaryInfC{$\ominus(A \to A)$}
\AxiomC{$[+A]^4$}
\RightLabel{(Repetition)}
\UnaryInfC{$+A$}
\RightLabel{$(+\!\to\mathrm{I.}^{**})^4$}
\UnaryInfC{$+(A \to A)$}
\RightLabel{(Rejection)}
\BinaryInfC{$\bot$}
\RightLabel{$(\mathrm{SR}_2)^3$}
\UnaryInfC{$+\neg A$}
\end{prooftree}
$$

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