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Leibniz-Clarke correspondence, Brain in a vat, Five-minute hypothesis, McTaggart's paradox, etc. are clarified in quantum language [Revised version]

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Abstract
Recently we proposed "quantum language" (or, "the linguistic Copenhagen interpretation of quantum mechanics"), which was not only characterized as the metaphysical and linguistic turn of quantum mechanics but also the linguistic turn of Descartes=Kant epistemology. We believe that quantum language is the language to describe science, which is the final goal of dualistic idealism. Hence there is a reason to want to clarify, from the quantum linguistic point of view, the following problems: "brain in a vat argument", "the Cogito proposition", "five-minute hypothesis", "only the present exists", "Copernican revolution", "McTaggart's paradox", and so on. In this paper, these will be discussed and clarified in quantum language. That is, these are not in quantum language. Also we emphasize that Leibniz's relationalism in Leibniz-Clarke correspondence is regarded as one of the most important parts of the linguistic Copenhagen interpretation of quantum mechanics. This paper is the revised version of ref. [18] (Open Journal of Philosophy, 2018 Vol.8, No.5, 466-480).

Key phrases: Dualism, Idealism, Quantum Language, Linguistic Copenhagen Interpretation,

1 Review: Quantum language (= Measurement theory);

Following refs. [6, 7, 8, 14], we shall review quantum language (i.e., the linguistic Copenhagen interpretation of quantum mechanics, or measurement theory), which has the following form:

$$\underset{\text{(= measurement theory)}}{\boxed{\boxed{\text{Quantum language}}}} = \underset{\text{(Axiom 1)}}{\boxed{\text{measurement}}} + \underset{\text{(Axiom 2)}}{\boxed{\text{causality}}} + \underset{\text{(how to use Axioms 1 and 2)}}{\boxed{\text{linguistic (Copenhagen) interpretation}}} \tag{1}$$

We believe, from the scientific point of view, that quantum language is the only successful dualistic idealism throughout all philosophical history. In this paper we assume that "idealism"="metaphysics"= "a discipline that cannot be verified by experiment". Mathematics is of course successful metaphysics, but it is not dualistic.

1.1 Mathematical Preparations

Now we briefly introduce quantum language (= measurement theory) as follows.
Consider an operator algebra $B(H)$ (i.e., an operator algebra composed of all bounded linear operators on a Hilbert space H with the norm $\|F\|_{B(H)} = \sup_{\|u\|_H=1} \|Fu\|_H$), and consider the pair $[\mathcal{A}, \overline{\mathcal{A}}]_{B(H)}$ (or, the triplet $[\mathcal{A} \subseteq \overline{\mathcal{A}} \subseteq B(H)]$), called a *basic structure*. Here, $\mathcal{A}(\subseteq B(H))$ is a C^*-algebra, and $\overline{\mathcal{A}}$ ($\mathcal{A} \subseteq \overline{\mathcal{A}} \subseteq B(H)$) is a particular C^*-algebra (called a W^*-algebra) such that $\overline{\mathcal{A}}$ is the weak closure of $\mathcal{A}$ in $B(H)$.
The measurement theory (=quantum language= the linguistic Copenhagen interpretation) is classified as follows.

$$\text{(A)} \quad \underset{\text{(= quantum language)}}{\text{measurement theory}} = \begin{cases} (\text{A}_1)\text{: quantum system theory} & (\text{when } \mathcal{A} = \mathcal{C}(H)) \\ \\ (\text{A}_2)\text{: classical system theory} & (\text{when } \mathcal{A} = C_0(\Omega)) \end{cases}$$

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That is, when $\mathcal{A} = \mathcal{C}(H)$, the C^*-algebra composed of all compact operators on a Hilbert space H, the (A_1) is called quantum measurement theory (or, quantum system theory), which can be regarded as the linguistic aspect of quantum mechanics. Also, when $\mathcal{A}$ is commutative (that is, when $\mathcal{A}$ is characterized by $C_0(\Omega)$, the C^*-algebra composed of all continuous complex-valued functions vanishing at infinity on a locally compact Hausdorff space Ω (*cf.* refs. [25, 27])), the (A_2) is called classical measurement theory.
Also, note that, when $\mathcal{A} = \mathcal{C}(H)$, i.e., quantum cases,

(i) $\mathcal{A}^* = Tr(H)$ (=trace class), $\overline{\mathcal{A}} = B(H)$, $\overline{\mathcal{A}}_* = Tr(H)$ (i.e., pre-dual space), thus, ${}_{Tr(H)}\big(\rho, T\big)_{B(H)} = \mathrm{Tr}_H(\rho T)$ $(\rho \in Tr(H), T \in B(H))$.

Also, when $\mathcal{A} = C_0(\Omega)$, i.e., classical cases,

(ii) $\mathcal{A}^* =$"the space of all signed measures on Ω", $\overline{\mathcal{A}} = L^\infty(\Omega,\nu)(\subseteq B(L^2(\Omega,\nu)))$, $\overline{\mathcal{A}}_* = L^1(\Omega,\nu)$, where ν is some measure on Ω, thus, ${}_{L^1(\Omega,\nu)}\big(\rho, T\big)_{L^\infty(\Omega,\nu)} = \int_\Omega \rho(\omega)T(\omega)\nu(d\omega)$ $(\rho \in L^1(\Omega,\nu), T \in L^\infty(\Omega,\nu))$ (*cf.* ref. [25]).

Let $\mathcal{A}(\subseteq B(H))$ be a C^*-algebra, and let $\mathcal{A}^*$ be the dual Banach space of $\mathcal{A}$. That is, $\mathcal{A}^* = \{\rho \mid \rho$ is a continuous linear functional on $\mathcal{A}$ $\}$, and the norm $\|\rho\|_{\mathcal{A}^*}$ is defined by $\sup\{|\rho(F)| \mid F \in \mathcal{A}$ such that $\|F\|_{\mathcal{A}}(= \|F\|_{B(H)}) \le 1\}$. Define the *mixed state* ρ $(\in \mathcal{A}^*)$ such that $\|\rho\|_{\mathcal{A}^*} = 1$ and $\rho(F) \ge 0$ for all $F \in \mathcal{A}$ such that $F \ge 0$. And define the mixed state space $\mathfrak{S}^m(\mathcal{A}^*)$ such that

$$\mathfrak{S}^m(\mathcal{A}^*)=\{\rho \in \mathcal{A}^* \mid \rho \text{ is a mixed state}\}.$$

A mixed state $\rho(\in \mathfrak{S}^m(\mathcal{A}^*))$ is called a *pure state* if it satisfies that "$\rho = \theta\rho_1 + (1-\theta)\rho_2$ for some $\rho_1, \rho_2 \in \mathfrak{S}^m(\mathcal{A}^*)$ and $0 < \theta < 1$" implies "$\rho = \rho_1 = \rho_2$". Put

$$\mathfrak{S}^p(\mathcal{A}^*)=\{\rho \in \mathfrak{S}^m(\mathcal{A}^*) \mid \rho \text{ is a pure state}\},$$

which is called a *state space*. It is well known (*cf.* ref. [25]) that $\mathfrak{S}^p(\mathcal{C}(H)^*) = \{|u\rangle\langle u|$ (i.e., the Dirac notation) $\mid \|u\|_H = 1\}$, and $\mathfrak{S}^p(C_0(\Omega)^*) = \{\delta_{\omega_0} \mid \delta_{\omega_0}$ is a point measure at $\omega_0 \in \Omega\}$, where $\int_\Omega f(\omega)\delta_{\omega_0}(d\omega) = f(\omega_0)$ $(\forall f \in C_0(\Omega))$. The latter implies that $\mathfrak{S}^p(C_0(\Omega)^*)$ can be also identified with Ω (called a *spectrum space* or simply *spectrum*) such as

$$\underset{\text{(state space)}}{\mathfrak{S}^p(C_0(\Omega)^*)} \ni \delta_\omega \leftrightarrow \omega \in \underset{\text{(spectrum)}}{\Omega} \tag{1}$$

For instance, in the above (ii) we must clarify the meaning of the "value" of $F(\omega_0)$ for $F \in L^\infty(\Omega,\nu)$ and $\omega_0 \in \Omega$. An element $F(\in \overline{\mathcal{A}})$ is said to be *essentially continuous at* $\rho_0(\in \mathfrak{S}^p(\mathcal{A}^*))$, if there uniquely exists a complex number α such that

- if ρ $(\in \overline{\mathcal{A}}_*$, $\|\rho\|_{\overline{\mathcal{A}}_*} = 1)$ converges to $\rho_0(\in \mathfrak{S}^p(\mathcal{A}^*))$ in the sense of weak* topology of $\mathcal{A}^*$, that is,

$$\rho(G) \longrightarrow \rho_0(G) \quad (\forall G \in \mathcal{A}(\subseteq \overline{\mathcal{A}})), \tag{2}$$

then $\rho(F)$ converges to α.

And the value of $\rho_0(F)$ is defined by the α.

Definition 1. [Observable] According to the noted idea (*cf.* ref. [3]), an *observable* $\mathsf{O}:=(X, \mathcal{F}, F)$ in $\overline{\mathcal{A}}$ is defined as follows:

(i) [σ-field] X is a set, $\mathcal{F}(\subseteq 2^X$, the power set of $X)$ is a σ-field of X, that is, "$\Xi_1, \Xi_2, ... \in \mathcal{F} \Rightarrow \cup_{n=1}^\infty \Xi_n \in \mathcal{F}$", "$\Xi \in \mathcal{F} \Rightarrow X \setminus \Xi \in \mathcal{F}$".

(ii) [Countable additivity] F is a mapping from $\mathcal{F}$ to $\overline{\mathcal{A}}$ satisfying: (a): for every $\Xi \in \mathcal{F}$, $F(\Xi)$ is a non-negative element in $\overline{\mathcal{A}}$ such that $0 \le F(\Xi) \le I$, (b): $F(\emptyset) = 0$ and $F(X) = I$, where 0 and I is the 0-element and the identity in $\overline{\mathcal{A}}$ respectively. (c): for any countable decomposition $\{\Xi_1, \Xi_2, \ldots, \Xi_n, ...\}$ of Ξ $\big($i.e., $\Xi, \Xi_n \in \mathcal{F}$ $(n = 1, 2, 3, ...)$, $\cup_{n=1}^\infty \Xi_n = \Xi$, $\Xi_i \cap \Xi_j = \emptyset$ $(i \ne j)\big)$, it holds that $F(\Xi) = \sum_{n=1}^\infty F(\Xi_n)$ in the sense of weak* topology in $\overline{\mathcal{A}}$.

1.2 Axiom 1 [Measurement] and Axiom 2 [Causality]

Measurement theory (A) is composed of two axioms (i.e., Axioms 1 and 2) as follows. With any *system* S, a basic structure $[\mathcal{A}, \overline{\mathcal{A}}]_{B(H)}$ can be associated in which the measurement theory (A) of that system can be formulated. A *state* of the system S is represented by an element $\rho(\in \mathfrak{S}^p(\mathcal{A}^*))$ and an *observable* is represented by an observable $\mathsf{O} := (X, \mathcal{F}, F)$ in $\overline{\mathcal{A}}$. Also, the *measurement of the observable* O *for the system* S *with the state* ρ is denoted by $\mathsf{M}_{\overline{\mathcal{A}}}(\mathsf{O}, S_{[\rho]})$ $\big($ or more precisely, $\mathsf{M}_{\overline{\mathcal{A}}}(\mathsf{O} := (X, \mathcal{F}, F),\ S_{[\rho]})$ $\big)$. An observer can obtain a measured value x $(\in X)$ by the measurement $\mathsf{M}_{\overline{\mathcal{A}}}(\mathsf{O}, S_{[\rho]})$.

The Axiom 1 presented below is a kind of mathematical generalization of Born's probabilistic interpretation of quantum mechanics. And thus, it is a statement without reality.

Now we can present Axiom 1 in the W^*-algebraic formulation as follows.

Axiom 1 [Measurement]. *The probability that a measured value* x $(\in X)$ *obtained by the measurement* $\mathsf{M}_{\overline{\mathcal{A}}}(\mathsf{O} := (X, \mathcal{F}, F),\ S_{[\rho]})$ *belongs to a set* $\Xi(\in \mathcal{F})$ *is given by* $\rho(F(\Xi))$ *if* $F(\Xi)$ *is essentially continuous at* $\rho(\in \mathfrak{S}^p(\mathcal{A}^*))$.

Next, we explain Axiom 2. Let $[\mathcal{A}_1, \overline{\mathcal{A}}_1]_{B(H_1)}$ and $[\mathcal{A}_2, \overline{\mathcal{A}}_2]_{B(H_2)}$ be basic structures. A continuous linear operator $\Phi_{1,2} : \overline{\mathcal{A}}_2$ (with weak* topology) $\to \overline{\mathcal{A}}_1$(with weak* topology) is called a *Markov operator,* if it satisfies that (i): $\Phi_{1,2}(F_2) \geq 0$ for any non-negative element F_2 in $\overline{\mathcal{A}}_2$, (ii): $\Phi_{1,2}(I_2) = I_1$, where I_k is the identity in $\overline{\mathcal{A}}_k$, $(k = 1, 2)$. In addition to the above (i) and (ii), we assume that $\Phi_{1,2}(\mathcal{A}_2) \subseteq \mathcal{A}_1$ and $\sup\{\|\Phi_{1,2}(F_2)\|_{\mathcal{A}_1} \mid F_2 \in \mathcal{A}_2 \text{ such that } \|F_2\|_{\mathcal{A}_2} \leq 1\} = 1$.

It is clear that the dual operator $\Phi^*_{1,2} : \mathcal{A}^*_1 \to \mathcal{A}^*_2$ satisfies that $\Phi^*_{1,2}(\mathfrak{S}^m(\mathcal{A}^*_1)) \subseteq \mathfrak{S}^m(\mathcal{A}^*_2)$. If it holds that $\Phi^*_{1,2}(\mathfrak{S}^p(\mathcal{A}^*_1)) \subseteq \mathfrak{S}^p(\mathcal{A}^*_2)$, the $\Phi_{1,2}$ is said to be deterministic. If it is not deterministic, it is said to be non-deterministic. Also note that, for any observable $\mathsf{O}_2 := (X, \mathcal{F}, F_2)$ in $\overline{\mathcal{A}}_2$, the $(X, \mathcal{F},\ \Phi_{1,2}F_2)$ is an observable in $\overline{\mathcal{A}}_1$.

Definition 2. [Sequential causal operator; Heisenberg picture of causality] Let $(T, \leq)$ be a tree like semi-ordered set such that "$t_1 \leq t_3$ and $t_2 \leq t_3$" implies "$t_1 \leq t_2$ or $t_2 \leq t_1$". The family $\{\Phi_{t_1,t_2} : \overline{\mathcal{A}}_{t_2} \to \overline{\mathcal{A}}_{t_1}\}_{(t_1,t_2)\in T^2_{\leqq}}$ is called a **sequential causal operator**, if it satisfies that

(i) For each t $(\in T)$, a basic structure $[\mathcal{A}_t \subseteq \overline{\mathcal{A}}_t \subseteq B(H_t)]$ is determined.

(ii) For each $(t_1, t_2) \in T^2_{\leqq}$, a causal operator $\Phi_{t_1,t_2} : \overline{\mathcal{A}}_{t_2} \to \overline{\mathcal{A}}_{t_1}$ is defined such as $\Phi_{t_1,t_2}\Phi_{t_2,t_3} = \Phi_{t_1,t_3}$ $(\forall(t_1, t_2), \forall(t_2, t_3) \in T^2_{\leqq})$. Here, $\Phi_{t,t} : \overline{\mathcal{A}}_t \to \overline{\mathcal{A}}_t$ is the identity operator.

Now we can propose Axiom 2 (i.e., causality). (For details, see ref. [14].)

Axiom 2[Causality]; For each $t(\in T$="tree like semi-ordered set")), consider the basic structure:

$$[\mathcal{A}_t \subseteq \overline{\mathcal{A}}_t \subseteq B(H_t)]$$

Then, the **chain of causalities** is represented by a **sequential causal operator** $\{\Phi_{t_1,t_2} : \overline{\mathcal{A}}_{t_2} \to \overline{\mathcal{A}}_{t_1}\}_{(t_1,t_2)\in T^2_{\leqq}}$.

1.3 The linguistic Copenhagen interpretation (= the manual to use Axioms 1 and 2)

Since so-called Copenhagen interpretation is not firm (*cf.* ref. [5]), we propose the linguistic Copenhagen interpretation in what follows. In the above, Axioms 1 and 2 are kinds of spells, (i.e., incantation, magic words, metaphysical statements), and thus, it is nonsense to verify them experimentally. Therefore, what we should do is not "to understand" but "to use". After learning Axioms 1 and 2 by rote, we have to improve how to use them through trial and error.

We can do well even if we do not know the linguistic Copenhagen interpretation (= the manual to use Axioms 1 and 2). However, it is better to know the linguistic Copenhagen interpretation , if we would like to make progress quantum language early. I believe that the linguistic Copenhagen interpretation is the true Copenhagen interpretation (*cf.* ref. [5]).

In Figure 1 (mentioned later), I remark:

(B$_1$) ⓧ: it suffices to understand that "interfere" is, for example, "apply light".
ⓨ: perceive the reaction.

That is, "measurement" is characterized as the interaction between "observer" and "measuring object (= matter)". However,

(B$_2$) in measurement theory (=quantum language), "interaction" must not be emphasized.

Therefore, in order to avoid confusion, it might better to omit the interaction "ⓧ and ⓨ" in Figure 1.
After all, we think that:

(B$_3$) it is clear that there is no measured value without observer (i.e., "I", "mind"). Thus, we consider that measurement theory is composed of three key-words:

$$\underset{\text{(I, observer, mind)}}{\boxed{\text{measured value}}}, \qquad \underset{\text{(body(= sensory organ), thermometer, eye, ear, compass (e.g., polar star))}}{\boxed{\text{observable (= measuring instrument)}}}, \qquad \underset{\text{(matter)}}{\boxed{\text{state}}}, \tag{2}$$

The essence of the manual is as follows:

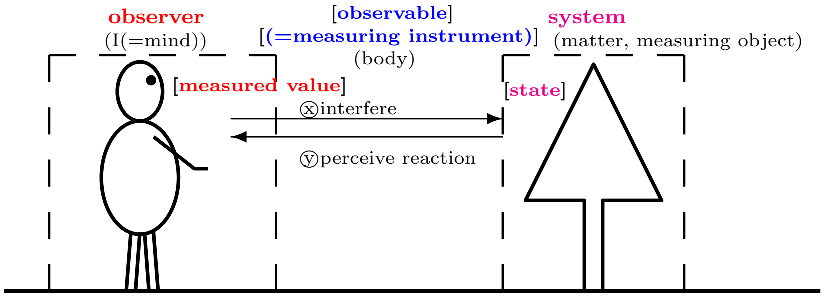


Figure 1: [Descartes Figure]: Image of "measurement(=ⓧ+ⓨ)" in mind-matter dualism

The linguistic Copenhagen interpretation says that

(C$_1$) **Only one measurement is permitted. Thus, Axiom 1 can be used only once.** And therefore, the state after a measurement is meaningless since it can not be measured any longer. Thus, the collapse of the wavefunction is prohibited (*cf.* ref. [13]; projection postulate). We are not concerned with anything after measurement. Strictly speaking, the phrase "*after the measurement*" should not be used. Also, the causality should be assumed only in the side of system, however, a state never moves. Thus, the Heisenberg picture should be adopted, and thus, the Schrödinger picture should be prohibited.

(C$_2$) "Observer"(="I") and "system" are completely separated. Hence, the measurement $\mathsf{M}_{\overline{\mathcal{A}}}(\mathsf{O} := (X, \mathcal{F}, F), S_{[\rho]})$ does not depend on the choice of observers. That is, any proposition (except Axiom 1) in quantum language is not related to "observer"(="I"), therefore, there is no "observer's space and time" in quantum language. And thus, it does not have tense (i.e., past, present, future).

(C$_3$) there is no probability without measurements (Bertrand's paradox in Section 9.12 of ref.[14]))

(C$_4$) Leibniz's relationalism concerning space-time. See Section 4 later.

and so on. We consider that the above (C$_1$) is closely related to Parmenides' saying (born around BC. 515 in ancient Greek)[There are no "plurality",but only "one"] and Kolmogorov's extension theorem (*cf.* [20]). For details, see ref. [14].

Remark 1 "Who measured?" is not essential. An observer may be satisfactory for anyone, if "observer" and "system" are completely separated. For example consider the following cases:

($\sharp_1$) Jack measures Tom's body temperature.

($\sharp_2$) A doctor measures Tom's body temperature.

($\sharp_3$) Tom's body temperature is measured.

($\sharp_4$) An observer measures Tom's body temperature

($\sharp_5$) I measure Tom's body temperature.

($\sharp_6$) Tom measures Tom's body temperature.

($\sharp_7$) I measure my body temperature (when I am Tom)

The above are all the same. See the above ($\sharp_6$) and ($\sharp_7$), which may be misleading, since (C_2) says that "observer" and "system" are completely separated. The meaning od "separation" will be clarified in Section 2.1; Brain in a Vat. Also, identification of "observer" and "I" in (C_2) may be misleading. Thus, we may say that

($\flat$) any statement in quantum language should be expressed without using "I" if it is possible.

In this sense, quantum language is quite different from Descartes philosophy.

Remark 2 Experiment verification must be possible also for any statement in quantum language. For example, "Apple falls down a tree" can carry out experiment verification. Thus, this is a statement in quantum language. On the other hand, the statement "Now I am here" can not carry out experiment verification. Thus, this is not a statement in quantum language.

1.4 The history of world description

[The location of quantum language in the history of world-description (*cf.* refs.[8, 14])]

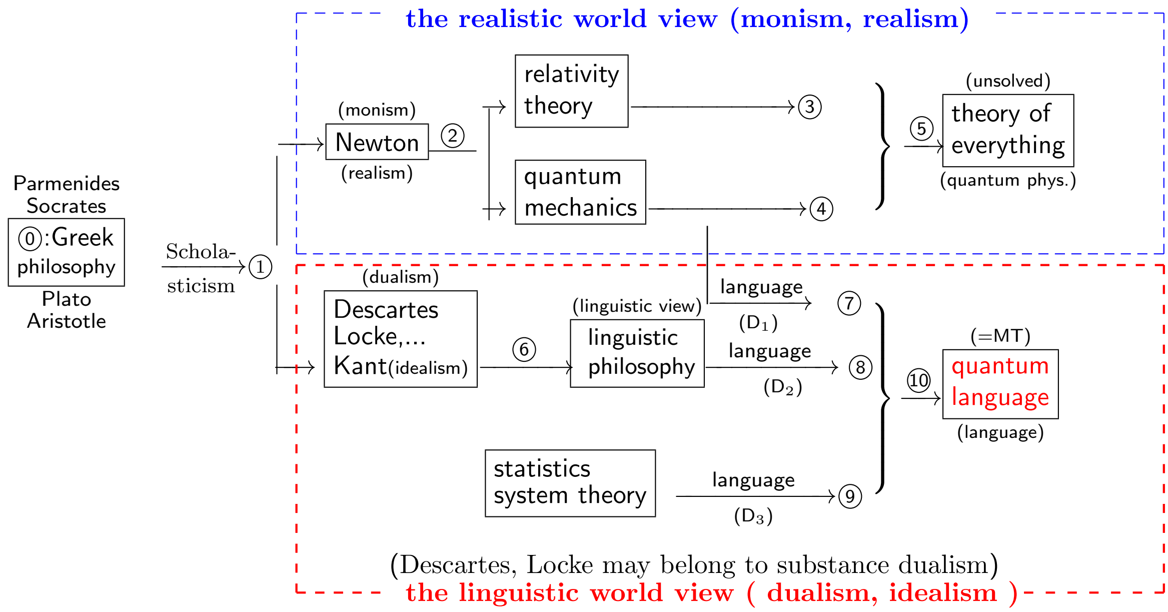


Figure 2: The history of the world-description

In refs. [14, 15], I asserted that the following four are equivalent:

(D_0) to propose quantum language (*cf.* 10 in Figure 2)

(D_1) to clarify so-called Copenhagen interpretation of quantum mechanics (*cf.* 7 in Figure 2)

(D_2) to find the final goal of the dualistic idealism (*cf.* 8 in Figure 2)

(D_3) to reconstruct statistics in the dualistic idealism (*cf.* 9 in Figure 2)

1.5 The Copernican Revolution

In Figure 2, "$\xrightarrow[\text{(D}_1\text{)}]{language;⑦}$" should be called "the linguistic CR(=Copernican Revolution)" in the sense below:

$$
\begin{array}{ccc}
\text{(substance dualism)} & & \text{a priori + a posteriori} \\
\boxed{\text{Descartes(dualism)}} & \xrightarrow[\text{recognitive CR}]{\text{idealism}} & \boxed{\text{Kant (dualism)}} \\
\text{(the world is previous, recognition is later)} & & \text{(recognition is previous, the world is later)} \\
 & & \Big\downarrow \text{linguistic turn} \\
\text{(realism)} & & \text{Axioms+Copenhagen interpretation} \\
\boxed{\text{quantum mechanics(dualism)}} & \xrightarrow[\text{linguistic CR}]{\text{idealism}(\approx\text{language})} & \boxed{\text{quantum language (dualism)}} \\
\text{(the world is previous, language is later)} & & \text{(language is previous, the world is later)}
\end{array}
$$

Kant's Copernican revolution (i.e., the above recognitive CR (*cf.* ref. [19])) should be praised as the discovery of "idealism", though the true discovery may be due to the above linguistic CR.

1.6 Philosophy made progress

In the above Figure 2, let us focus on the history of the dualistic idealism in the linguistic world view such as

$$
\boxed{\text{Plato}} \longrightarrow \boxed{\text{Descartes}} \longrightarrow \boxed{\text{Kant}} \longrightarrow \boxed{\text{Wittgenstein}} \tag{3}
$$

Note that physics obviously made progress in Figure 2, on the other hand, the (3)'s progress is not clear. In ref. [15], we asserted that, if "(philosophical) progress" is defined by "approaching quantum language", then

(E) the (3) does not only imply time series but also progress, that is,

$$
\underset{\text{starting point}}{\boxed{\text{Plato}}} \xrightarrow[\text{progress}]{} \overset{\text{dualism}}{\boxed{\text{Descartes}}} \xrightarrow[\text{progress}]{} \underset{\text{idealism}}{\overset{\text{dualism}}{\boxed{\text{Kant}}}} \xrightarrow[\text{progress}]{} \underset{\text{language}}{\boxed{\text{Wittgenstein}}} \xrightarrow[\text{progress}]{} \underset{\text{idealism}(\approx\text{language})}{\overset{\text{dualism}}{\boxed{\text{Quantum language}}}}
$$
$$
\text{(if "progress" is defined by "approaching quantum language")} \tag{4}
$$

Here,

- Plato: the founder
- Descartes: the discoverer of dualism (though the true scientific discovery is due to N. Bohr (*cf.* [2])). Also, Berkeley's saying: "To be is to be perceived" is essential to idealism (cf. ref. [15]).
- Kant: the discoverer of idealism (in the sense of the above Section 1.5)
- Wittgenstein: he emphasized the importance of language.

This is natural since we assume [(D_2); quantum language is the final goal of the dualistic idealism]. That is, we consider that the (4) is the history which gropes after the language in which science is written. Also, for the linguistic approach to the mind-body problem, see ref. [16].

Remark 3 As mentioned in ref. [15], we do not agree with the following "progress";

$$
\underset{\text{starting point}}{\boxed{\text{Plato}}} \xrightarrow[\text{progress}]{} \overset{\text{dualism}}{\boxed{\text{Descartes}}} \xrightarrow[\text{progress}]{} \underset{\text{idealism}}{\overset{\text{dualism}}{\boxed{\text{Kant}}}} \xrightarrow[\text{progress}]{} \boxed{\text{Husserl}} \xrightarrow[\text{progress}]{} \boxed{\text{brain science}}
$$

That is because we think that

- philosophy should be metaphysics, and thus it isn't in the immature state of the science.

1.7 Quantum language is the language to describe science

Also, since the (D) says that

$$\underset{(⑨ \text{ in Figure 2})}{\text{“statistics”}} \bigcup \underset{(⑦ \text{ in Figure 2})}{\text{“quantum information theory”}} \bigcup \underset{(⑧ \text{ in Figure 2})}{\text{“dualistic idealism”}} \subset \text{“quantum language”}$$

it is natural to assume that

(F) quantum language is the language to describe science, that is,

$$\text{proposition in quantum language} \Longleftrightarrow \text{scientific proposition (=experiment verifiable proposition)}$$

which is the most important assertion of quantum language. Also, we assume that this (i.e., to make the language to describe science) is the true purpose of the philosophy of science.
Remark 4 Note that the theory of relativity cannot be described by quantum language. However, we want to assert the (F). We think that the theory of relativity (and more, the theory of everything) is too special, an exception.

2 What we cannot speak about in quantum language

In this section we clarifies the following well-known philosophical statements:

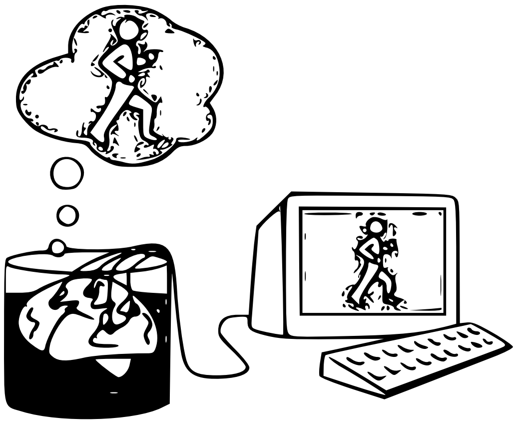
(G) “brain in a vat problem”, “the Cogito proposition”, “five-minute hypothesis”, “only the present exists”, “McTaggart’s paradox” and so on.

which are “what we cannot speak about in quantum language”, that is, non-scientific propositions.

2.1 Brain in a vat argument

Suppose (*cf.* ref. [23]);

(H_1) a mad scientist has removed your brain, and placed it into a vat of liquid to keep it alive and active. The scientist has also connected your brain to a powerful computer, which sends neurological signals to the brain in the way the brain normally receives them. Thus, the computer is able to send your brain data to fool you into believing that you are still walking around in your body.



Then, you may say;

(H_2) “Am I a brain in a vat?” Or, “Can I check whether I am a brain vat or not?”

Note that the question (H_2) is related to “I”. Or, precisely, “observer”=“I”, “system (=measuring object)”=“I”, thus, “observer” and “system” are not separated. Thus, the linguistic Copenhagen interpretation (C_2) says that this (H_2) is not a statement in quantum language. Thus, the (H_2) is not scientific, that is, there is no experiment to verify the statement (H_2).
Remark 5 Since we receive several questions for the above argument in ref. [18], we add Remarks 1 and 2 and the following. If you are Tom, (H_2) is the same as

(H_3) “Can Tom himself check whether Tom is a brain vat or not?”

Here, “observer”=“Tom”, “system (=measuring object)”=“Tom”, thus, “observer” and “system” are not separated. Thus, this is not a statement in quantum language. This should be compared to the following.

(H_4) “Can Jack check whether Tom is a brain vat or not?”

which is the statement in quantum language.

2.2 The Cogito proposition

It is well known that Descartes proposed the Cogito proposition "I think, therefore I am", as the first principle of philosophy since he believed that this proposition has no room for doubt. That is, Descartes think that

(I_1) I confirm "I think, therefore I am"

However, this is doubtful. Note that the proposition (I_1) is related to "I". Or, precisely, "observer"="I", "system (=measuring object)"="I", thus, "observer" and "system" are not separated. Thus, the linguistic Copenhagen interpretation (C_2) says that this (I) is not a statement in quantum language. Thus, the (I_1) is not scientific, that is, there is no experiment to verify the statement (I_1).
Remark 6 Since we receive several questions for the above argument in ref. [18], we add the following. As brain death determination,

(I_2) A doctor confirms "Tom thinks, therefore Tom is alive"

In this case, we see that "observer"="doctor", "system (=measuring object)"="Tom". Hence "Tom thinks, therefore Tom is alive" is the proposition in quantum language. For the more precise argument, see Section 8.4 [Cogito – I think, therefore I am] in ref.[14].

2.3 What is "I"?

Descartes proclaimed that he discovered "I". Then, we have the natural question:

What is "I(discovered by Descartes)"?

If (E) is true (i.e., $\boxed{\text{Descartes}} \xrightarrow[\text{progress}]{} \boxed{\text{Quantum language}}$), this question can be answered as follows. In quantum language, several words ("I"(="observer"), "observable", "matter", "measurement", etc.) are undefined such as point, line, plane etc. in Hilbert's geometry (i.e., *The Foundations of Geometry* (1899)). D. Hilbert said that

- The elements, such as point, line, plane, and others, could be substituted by tables, chairs, glasses of beer and other such objects.

For example, the readers should note that the term "measurement" is used trickily in the quantum linguistic answer of Monty-Hall problem (*cf.* ref. [10]).

2.4 Five-minute hypothesis

The five-minute hypothesis, proposed by B. Russell (*cf.* ref. [24]), is as follows.

(J_1) The universe was created five minutes ago. Or equivalently, the universe was created ten years ago.

Now we show that this (J_1) is not the statement in quantum language as follows (i.e., The first answer (i) and the second answer (ii))

The first answer (i): Note that this hypothesis (J_1) is related to "tense". Thus, the linguistic Copenhagen interpretation (C_2) says that this (J_1) is not a statement in quantum language. Thus, the (J_1) is not scientific, that is, there is no experiment to verify the statement (J_1).

The second answer (ii): There may be another understanding as follows. If we consider that ["observer"∈"the universe"], the proposition (J_1) cannot be described in quantum language. That is because the linguistic Copenhagen interpretation (C_2) says that "observer" (="I") and "measuring object"(="the universe") have to be completely separated. (Also, see Remark 7 (b) later.)

Some may want to relate this hypothesis to skepticism (*cf.* ref. [24]), However we do not think that this direction is productive.
Remark 7(a): Also, the above (J_1) should be compared to the following (J_2)

(J_2) The universe was created in A.D. 2008. (Or equivalently, now is A.D. 2018, and the universe was created ten years ago.)

This (J_2) can be denied by experiment, that is, it is different from the fact. Thus, this is a proposition in quantum language.
(b): If the (J_2) is a proposition in quantum language, the hypothesis [“observer”$\in$“the universe”] in the second answer (ii) may be doubtful. If so, we may not understand the meaning of [“observer”$\in$“the universe”] completely. Thus, I may not be confident in the second answer (ii).

2.5 Only the present exists

It is well known that St. Augustinus (AD.354-AD.430) said that

- *the past does not exist because of its being already gone, that the future does not exist because of its not coming yet, and that the present really exists.*

Here, consider

(K) “Only the present exists”

Note that this proposition (K) is related to “tense”. Thus, the linguistic Copenhagen interpretation (C_2) says that this (K) is not a statement in quantum language. Thus, the (K) is not scientific, that is, there is no experiment to verify the (K).

2.6 McTaggart’s paradox

In ref. [21], McTaggart asserted “the Unreality of Time” as follows.

The sketch of McTaggart’s proof

(L_1) Assume that there are two kinds of times. i.e., “observer’s time (A-series)” and “objective time (B-series)”. (Note that this assumption is against the linguistic Copenhagen interpretation (C_2).)

(L_2) $\cdots\cdots$

(L_3) After all, the contradiction is obtained

Therefore, by the reduction to the absurd, we get;

(L_4) A-series does not exist (in science).

About this proof, there are various opinions also among philosophers. Although I can not understand the above part (L_2) (since the properties of A-series are not clear), I agree to him if his assertion is (L_4) (*cf.* ref. [8]). That is, I agree that McTaggart noticed first that observer’s time is not scientific.

2.7 Is “What we cannot speak about we must pass over in silence” true?

It should be noted that “what we cannot speak about” depends on language. As mentioned in the above, the Cogito proposition “I think, therefore I am” is “what we cannot speak about in quantum language”. However, thanks to Descartes said “I think, therefore I am”, dualism was developed. This fact may imply that

(M) “What we cannot speak about we must pass over in silence” is not true.

However, we think that Descartes’ success is accidental luck. Or, we may consider that the true discoverer of dualism is N. Bohr, the leader of the Copenhagen school (*cf.* [2]). Since Wittgenstein (*cf.* ref. [26]) said “The limits of my language mean the limits of my world.”, he had should propose “my language”. We are sure that it will fall into a play on words by the argument without “my language”.

3 What is space-time in quantum language ?

The problems ("What is space?" and "What is time?") are the most important in modern science as well as the traditional philosophies. In this section, reviewing ref. [14], we give the quantum linguistic answer to these problems. As seen later, the answer is similar to Leibniz's relationalism concerning space-time. In this sense, we consider that Leibniz is one of the founders of the linguistic Copenhagen interpretation.

3.1 How to describe "space" in quantum language

In what follows, let us explain "space" in quantum language.

For example, consider the simplest case, that is,

(N) "space"$=\mathbb{R}_q$(line (=one dimensional space))

Since classical system and quantum system must be considered, we see

$$
\text{(O)} \begin{cases} (\mathrm{O}_1)\text{: a classical particle in the one dimensional space } \mathbb{R}_q \\ \\ (\mathrm{O}_2)\text{: a quantum particle in the one dimensional space } \mathbb{R}_q \end{cases}
$$

In the classical case, we start from the following state:

$$
(q,p) = (\text{"position"}, \text{"momentum"}) \in \mathbb{R}_q \times \mathbb{R}_p
$$

Thus, we have the classical basic structure:

(P_1) $[C_0(\mathbb{R}_q \times \mathbb{R}_p) \subseteq L^\infty(\mathbb{R}_q \times \mathbb{R}_p) \subseteq B(L^2(\mathbb{R}_q \times \mathbb{R}_p)]$

Also, concerning quantum system, we have the quantum basic structure:

(P_2) $[\mathcal{C}(L^2(\mathbb{R}_q)) \subseteq B(L^2(\mathbb{R}_q)) \subseteq B(L^2(\mathbb{R}_q))]$

Summing up, we have the basic structure

$$
\text{(P)}\ [\mathcal{A} \subseteq \overline{\mathcal{A}} \subseteq B(H)] \begin{cases} (\mathrm{P}_1)\text{: classical } [C_0(\mathbb{R}_q \times \mathbb{R}_p) \subseteq L^\infty(\mathbb{R}_q \times \mathbb{R}_p) \subseteq B(L^2(\mathbb{R}_q \times \mathbb{R}_p))] \\ \\ (\mathrm{P}_2)\text{: quantum } [\mathcal{C}(L^2(\mathbb{R}_q)) \subseteq B(L^2(\mathbb{R}_q)) \subseteq B(L^2(\mathbb{R}_q))] \end{cases}
$$

Since we always start from a basic structure in quantum language, we consider that

$$
\begin{aligned}
&\text{How to describe "space" in quantum language} \\
\Leftrightarrow\ &\text{How to describe [(O):space] by [(P):basic structure]}
\end{aligned} \tag{3}
$$

This is done in the following steps.

Assertion 3. [**The linguistic Copenhagen interpretation concerning space**]

How to describe "space" in quantum language

(Q_1) Begin with the basic structure:

$$
[\mathcal{A} \subseteq \overline{\mathcal{A}} \subseteq B(H)]
$$

(Q_2) Next, consider a certain commutative C^*-algebra $\mathcal{A}_0(= C_0(\Omega))$ such that

$$
\mathcal{A}_0 \subseteq \overline{\mathcal{A}}
$$

(Q_3) Lastly, the spectrum Ω ($\approx \mathfrak{S}^p(\mathcal{A}_0^*)$) is used to represent "space".

For example,

(R$_1$) in the classical case (P$_1$):

$$[C_0(\mathbb{R}_q \times \mathbb{R}_p) \subseteq L^\infty(\mathbb{R}_q \times \mathbb{R}_p) \subseteq B(L^2(\mathbb{R}_q \times \mathbb{R}_p))]$$

we have the commutative $C_0(\mathbb{R}_q)$ such that

$$C_0(\mathbb{R}_q) \subseteq L^\infty(\mathbb{R}_q \times \mathbb{R}_p)$$

And thus, we get the space $\mathbb{R}_q$ (i.e., the spectrum) as mentioned in (O)

(R$_2$) in the quantum case (P$_2$):

$$[\mathcal{C}(L^2(\mathbb{R}_q)) \subseteq B(L^2(\mathbb{R}_q)) \subseteq B(L^2(\mathbb{R}_q))]$$

we have the commutative $C_0(\mathbb{R}_q)$ such that

$$C_0(\mathbb{R}_q) \subseteq B(L^2(\mathbb{R}_q))$$

And thus, we get the space $\mathbb{R}_q$ (i.e., the spectrum) as mentioned in (O)

3.2 How to describe "time" in quantum language

In what follows, let us explain "time" in quantum language.

This is easily done in the following steps.

Assertion 4. [**The linguistic Copenhagen interpretation concerning time**]

How to describe "time" in quantum language

(S$_1$) Let T be a linear tree like semi-ordered set. Usually it suffices to consider that $T = \mathbb{R}$, or $T = \mathbb{Z}$(the set of all integer).

(S$_2$) For each $t(\in T$="tree like semi-ordered set")), consider the basic structure:

$$[\mathcal{A}_t \subseteq \overline{\mathcal{A}}_t \subseteq B(H_t)]$$

And consider a **chain of causalities** which is represented by a **sequential causal operator** $\{\Phi_{t_1,t_2} : \overline{\mathcal{A}}_{t_2} \to \overline{\mathcal{A}}_{t_1}\}_{(t_1,t_2)\in T^2_{\leqq}}$.

(S$_3$) Then, the T can be used to represent "time".

For the details, see ref. [14].

4 Leibniz-Clarke Correspondence

The above argument urges us to recall Leibniz-Clarke Correspondence (1715–1716: *cf.* ref. [1]), which is important to know both Leibniz's and Clarke's ($\approx$ Newton's) ideas concerning space and time.

(T) [The realistic space-time]

Newton's absolutism says that the space-time should be regarded as a receptacle of a "thing." Therefore, even if "thing" does not exits, the space-time exists.

On the other hand,

(U) [The metaphysical space-time]

Leibniz's relationalism says that

(U_1) Space is a kind of state of "thing".

(U_2) Time is an order of occurring in succession which changes one after another.

Therefore, I regard this correspondence as

$$\underset{\text{(realistic view)}}{\boxed{\text{Newton } (\approx \text{Clarke})}} \quad \underset{\text{v.s.}}{\longleftrightarrow} \quad \underset{\text{(linguistic view)}}{\boxed{\text{Leibniz}}}$$

It should be noted that

(V_1) Newton proposed the scientific language called Newtonian mechanics, in which his absolutism (T) was explained. Therefore it is understandable.

On the other hand,

(V_2) Leibniz could not propose a scientific language. Hence, Leibniz's relationalism (U) is incomprehensible and literary.

However, I believe that Leibniz's relationalism (U) can be formulated as mentioned in Section 3 in quantum language. Therefore, we assert that Leibniz's relationalism (U) [= Assertions 3 and 4] should be regarded as one of the most important parts of the linguistic Copenhagen interpretation of quantum mechanics (see (C_4) mentioned in Section 1.3).

Remark 8 Note that the great disputes in the history of the world view are always formed as follows:

$$\underset{\text{(monistic realism)}}{\overset{\text{Einstein,...}}{\boxed{\text{realistic world view}}}} \quad \underset{\text{v.s.}}{\longleftrightarrow} \quad \underset{\text{(dualistic idealism)}}{\overset{\text{Bohr,...}}{\boxed{\text{linguistic world view}}}}$$

For example,

Table 1 : The realistic world view vs the linguistic world view

Dispute ＼ R vs. L	R: the realistic world view	L: the linguistic world view
Greek philosophy	Aristotle	Plato
Problem of universals	Nominalisme(William of Ockham)	Realismus(Anselmus)
Space·times	Clarke(Newton)	Leibniz
Quantum mechanics	Einstein (*cf.* [4])	Bohr (*cf.* [2])

(*cf.* Note10.7 in Chapter 10 (Section 10.7) of ref. [14]).

5 Conclusion

Dr. Hawking said in his best seller book [A Brief History of Time: From the Big Bang to Black Holes, Bantam, Boston, 1990]:

- Philosophers reduced the scope of their inquiries so much that Wittgenstein the most famous philosopher this century, said "The sole remaining task for philosophy is the analysis of language." What a comedown from the great tradition of philosophy from Aristotle to Kant!

We think that this is not only his opinion but also most scientists' opinion. And moreover, we mostly agree with him. However, we believe that, if "the analysis of language" was rewritten to "the creation of language", then Dr. Hawking would not have been critical to philosophy. That is because the task of phycisists is just the creation of language, i.e., the language called Newtonian mechanics, the language called the theory of relativity, etc.

Also, since Wittgenstein (*cf.* ref. [26]) said "The limits of my language mean the limits of my world.", he had should propose "my language". We are sure that the argument without "my language" will fall into a play on words.

In this paper, we introduced quantum language, and in the framework of quantum language, we discussed the followings:

(W) "brain in a vat argument", "the Cogito proposition", "five-minute hypothesis", "only the present exists", "McTaggart's paradox", and so on.

And we showed that the above propositions in (W) are not in quantum language, that is, these are not scientific. Or equivalently, we have no experiment to verify the above propositions in (W).

Also we emphasize that Leibniz's relationalism in Leibniz-Clarke correspondence is clarified in quantum language, and it should be regarded as one of the most important parts of the linguistic Copenhagen interpretation of quantum mechanics.

Since I received several questions for ref. [18], I prepare this paper as the revised version of ref. [18]. I think everyone can understand proof easily in this revised paper.

I hope that my assertions will be examined from various points of view[2] .

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