In Defence of Dimensions

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Caspar Jacobs*

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Abstract

The distinction between dimensions and units in physics is commonplace. But are dimensions a feature of reality? The most widely-held view is that they are no more than a tool for keeping track of the values of quantities under a change of units. This anti-realist position is supported by an argument from underdetermination: one can assign dimensions to quantities in many different ways, all of which are empirically equivalent. In contrast, I defend a form of dimensional realism, on which some assignments of dimensions to quantities better describe reality than others. The argument I provide is a form of inference to the best explanation. In particular, the technique of dimensional analysis is explanatory, but it is only successful for certain systems of dimensions. Since these dimensional systems support scientific explanations, we have reason to believe that they are real.

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1 Introduction

The world was not made of mathematics. In it we find not abstract numbers, but concrete quantities such as mass, charge, velocity, blood pressure, productivity, and happiness. What distinguishes these quantities from pure numbers is that they have a dimension. Dimensional quantities measure the

*Institute for Philosophy, Leiden University, Nonnensteeg 1–3, 2311 BE Leiden, Netherlands, c.a.jacobs@phil.leidenuniv.nl

amount of some 'stuff' in the world. For example, mass is a quantity whose dimensions are primitive: we simply say that mass has 'dimensions of mass' (M). This near-truism relays important information, for it means that mass quantities measure the amount of some physical stuff of a particular system, rather than some merely abstract feature of it. Other quantities have complex dimensions: velocity, for instance, has dimensions of length over time (L/T).[1] This means that velocity measures the amount of some stuff that is, in a particular way, related to distance and duration. Any investigation of our concrete, palpable, physical world must start with dimensional quantities.

Yet, the very notion of dimensionality is little discussed by either physicsts or philosophers.[2] The implicit—but dominant—take on dimensions is anti-realist: they are seen as no more than a bookkeeping device for keeping track of changes of units of measurement. The main purpose of dimensions, namely their use in the technique of dimensional analysis discussed below, is likewise considered no more than a helpful tool to simplify certain calculations. Langhaar (1951, 51), in his textbook on dimensional analysis, makes this consensus explicit: 'The general conclusion that emerges from the discussions is that the concept of dimensions is of little importance to philosophy. On the other hand, dimensions serve a mathematical purpose. *They are a code for telling us how the numerical value of a quantity changes when the basic units of measurement are subjected to prescribed changes.* This is the only characteristic of dimensions to which we need ascribe significance in the development of dimensional analysis.' In other words, dimensions do not animate the lifeless world of mathematics; they are just another way to churn out numbers.

The observation underlying this anti-realism is that the dimensions of quantities are underdetermined. I just said that velocity has dimensions of L/T, but there are in fact self-consistent dimensional systems in which these and other quantities are assigned different dimensions than the ones they standardly have. These systems are empirically equivalent: there is no experiment that could rule out one or rule in another. The anti-realist case against dimensions is thus a traditional form of underdetermination of theory by experiment. The conclusion looms that any particular assignment of dimensions is a matter of conventional choice. This is indeed the stance

[1] I prefer the term 'complex quantity' to the more common 'derived quantity', because the latter seems to imply that such a quantity is less fundamental.

[2] The work of Susan Sterrett (2009, 2019, 2021) is an exception, as are recent papers by Lange (2009), Skow (2017) and Jalloh (forthcoming).

of the Bureau International des Poids et Mesures in the 9^{th} edition of the SI brochure (2019, 24): 'Physical quantities can be organized in a system of dimensions, where the system used is decided by convention.'

The aim of this paper is to offer a defence of realism about dimensions. The realist rejoinder to underdetermination is an inference to the best explanation: some dimensional systems are more explanatory than others. Specifically, the technique of dimensional analysis offers explanations of the functional dependence between quantities; but only if particular dimensional systems are used. The best explanation for the explanatory success of those systems is that they more closely track the truth about the dimensionality of the quantities involved. The exact form of realism is thereby left open—I will not further discuss the 'metaphysics' of dimensions here.

Of course, many an anti-realist is not moved by this type of abductive inference. But I do not set out to convince the committed sceptic. My more modest aim is to convince those already sympathetic to some form of realism, that their realism should also encompass dimensions. There are matters of fact about dimensions that we can come to know, albeit indirectly. Moreover, dimensional analysis can help explain the way in which physical quantities hang together. Dimensions are thus of significant importance to philosophy.

2 The Function of Dimensions

First, an abbreviated account of the formal machinery of dimensions:[3] let a *system of dimensions* consist of

(a) an ordered n-tuple $\mathcal{D}$ of *base dimensions*, denoted by Roman capitals such as M, L and T; and

(b) a *dimension function* $[\cdot] : \mathcal{Q} \to \mathbb{Z}^n$ from the set $\mathcal{Q}$ of quantities into n-tuples of integers, which are called the 'ddimensional exponents' of the quantity in question.

The n^{th} item of $[Q]$ indicates how many of the n^{th} base dimension Q has. For a concrete example, suppose that the base dimensions are mass, length and time: $\mathcal{D} = \langle \mathrm{M}, \mathrm{L}, \mathrm{T} \rangle$. Then $[m] = \langle 1, 0, 0 \rangle$ since mass (m) has dimensions of mass; and $[v] = \langle 0, 1, -1 \rangle$, since velocity ($v$) has dimensions of length over

[3] For various more complete axiomatisations of dimensional systems, see Guggenheim (1942); Whitney (1968); Bunge (1971); Sharlow (2009); Raposo (2018); Zapata-Carratala (2022). For more on the history of dimensions, see De Clark (2017) and Mitchell (2017).

time. I will use the 'mechanical' dimensions M, L and T to illustrate certain claims, but the argument of this paper applies equally to the wider class of dimensions that includes temperature of electric current.

It is often more convenient to write out the dimensions of a quantity as a product of powers of base dimensions, such that $[v] = \mathrm{L}^1\mathrm{T}^{-1}$ or even $[v] = \mathrm{L}/\mathrm{T}$. The assumption that the dimensions of any quantity are a product of powers of the base dimensions is known as 'Bridgman's lemma'. I will continue to write out the dimensions of quantities in this way in the rest of the paper.

That was a formal account of dimensions. But what are dimensions? The International Vocabulary of Metrology (VIM, 2008) issued by the Joint Committee for Guides in Metrology defines the dimension of a quantity as follows:

> expression of the dependence of a quantity on the base quantities of a system of quantities as a product of powers of factors corresponding to the base quantities, omitting any numerical factor.

This definition does not get us very far. Firstly, it is silent on the dimensions of the 'base quantities' themselves. What does it mean to say that $[m] = \mathrm{M}$? The VIM definition at most entails that mass depends on itself. Secondly, as pointed out by Grozier (2020), the definition does not define what is meant by a 'correspondence' between dimensional factors and base quantities. Finally, it is left completely open what 'dependence' here means—we will see below that what it means depends on whether one is a realist or an anti-realist.

It is more helpful to proceed with how dimensions are used in science. What is the function of dimensions? I will not give an explicit definition of dimensions in this paper, but an implicit definition in terms of their functional role. This will suffice to characterise dimensions for our purposes. I will emphasise three functions in particular: the relation between dimensions and units; the behaviour of complex quantities under changes of unit; and the so-called 'quantity calculus'. These functions are uncontested between realists and anti-realists.

Firstly, to every base dimension is associated a class of units of measurement. The dimension M, for instance, is associated to a class of units that includes the pound, the gram and the kilogram; the dimension L is associated to a class of units that includes the inch, the metre and the kilometre. Therefore, the fact that mass has dimensions of mass already tells us that it can be measured in pounds or grams, but not in inches. The necessity of a choice of unit is one of the ways in which dimensional quantities differ from pure numbers.

Secondly, the units for quantities with complex dimensions are derived from those with basic dimensions. Thus, the possible units for velocity, which has dimensions of L/T, are the km/h, km/s, m/s, etc. This is one way in which to understand the 'dependence' of the VIM definition. This dependence becomes relevant when one changes from one system of units to another. Suppose one were to use seconds instead of hours as the unit of time. Every time-quantity is then multiplied by 3600, since there are 3600 seconds in an hour. But one must also change the numerical value of velocity, since the units of velocity depend on those of time. From $[v] = \mathrm{L}/\mathrm{T}$ it follows that one must divide every velocity quantity by the conversion factor between the old and the new units of time. So, 1 km/h becomes 1/3600 km/s. The dimension symbols 'L' and 'T' here represent change-ratios: ratios between units. The dimensions of a quantity then determine the factor by which its value will change as a function of these ratios. Skow (2017) calls this the 'dimension function' of a quantity. If Langhaar's anti-realism is correct, this function is the 'only characteristic of dimensions to which we need ascribe significance'.

Finally, it is possible to carry out arithmetic with dimensional quantities, but only if certain rules are followed (the 'quantity calculus').[4] These are as follows:

1. It is possible to add or subtract quantities with the same dimensions only;
2. It is possible to multiply and divide quantities of any dimension with each other; the dimension of a product of powers is $[Q_1^{p_1} Q_2^{p_2} ... Q_n^{p_n}] = [Q_1]^{p_1} [Q_2]^{p_2} ... [Q_n]^{p_n}$;
3. Any other operation (sin, exp, etc.) applied to a dimensional quantity is undefined.

There is more to say about the justification of these rules—why couldn't I add the mass of the Eiffel Tower to the distance between London and Edinburgh?—but for my purposes here it suffices to note that they are widely accepted within the physics community.

So far, I have covered common ground between realist and anti-realist accounts of dimensions. But there is a difference between the function of

[4] For a history of the quantity calculus, see de Boer (1995).

dimensions, and whatever actually realises that function. It is here that realism and anti-realism come apart.

3 Dimensional (Anti-)Realism

The anti-realist account of dimensions was already hinted at in the introduction: it holds that dimensions are no more than a useful tool for keeping track of unit changes. The base dimensions just are change-ratios, and the 'dependence' in the VIM definition solely concerns the numerical dependence of the change in value of a complex quantity on these ratios. This account comes close to Fourier's conception of the dimension of a complex quantity as a 'conversion factor' (De Clark, 2017). The operationalism of Percy Bridgman committed him to a similarly deflationary view. The anti-realist claims that a quantity's dimensions does not track reality. This leaves room for the peaceful coexistence of incompatible dimensional systems, which assign different dimensions to the same quantities.

The existence of incompatible dimensional systems is one of the main motivators for anti-realism, so I will now discuss it in more detail. The point is that there are many different ways to assign dimensions to quantities, all of which are both consistent with the relevant dynamical equations and compatible with the same empirical evidence.[5] It follows that the dimensions of quantities are underdetermined. I identify three ways in which they are underdetermined: by the choice of base dimensions; by the freedom to eliminate dimensions; and by the freedom to increase the number of dimensions.

With respect to the first point, we have already seen that it is customary to distinguish between base and complex quantities. But there is no reason one must adopt M, L and T as base dimensions. It is also possible, for example, to let the set of base dimensions consist of M, T and V—$\mathcal{D} = \langle \mathrm{M}, \mathrm{T}, \mathrm{V} \rangle$—where the latter is a dimension of velocity: $[v] = \mathrm{V}$. It would then follow from $v = x/t$ that $[x] = \mathrm{VT}$, rather than L. As the term 'base dimension' suggests, this is akin to a different choice of basis for a linear space.[6] The new system of dimensions seems to express equally well the way in which distance, duration and velocity depend on each other. It is, moreover, empirically equivalent to the system on which M, L and T are the base dimensions. After all, equations such as $v = x/t$ carry a theory's

[5] This claim is found *inter alia* in Bridgman (1931); Dingle (1942); Langhaar (1951); Ellis (1964); Pankhurst (1964).

[6] I thank an anonymous reviewer for pointing out the terminological resemblance.

empirical content, and such equations are preserved across different choices of base dimensions.

The second element of underdetermination consists of the freedom to eliminate dimensions. To stick with velocity as a simple example, one could decide to eliminate the dimension L so that $\mathcal{D} = \langle \mathrm{M}, \mathrm{T} \rangle$, and set $[x] = \mathrm{T}$. This means that distances are measured in units of time. For example, one could choose units in which one light-year is equal to one year. It follows that $[v] = [x]/[t] = \mathrm{T}/\mathrm{T} = 1$: velocity becomes 'dimensionless'. Unlike in the previous case, this system of dimensions may seem decidedly worse. Previously, $[v] = \mathrm{L}/\mathrm{T}$ expressed the way in which velocity depends on both distance and duration, whereas $[v] = 1$ seems uninformative of the relation between velocity, length and time. However, recall that for the anti-realist the notion of 'dependence' is merely mathematical. The relation $[v] = 1$ only expresses the fact that in this reduced system of dimensions the value for v does not vary when one chooses a different unit for time: a particle that moves one light-year per year has the same velocity as one that moves one light-second per second. It is the equation $v = x/t$ that expresses the physical relation between velocity, length and time, but that equation itself remains the same. Therefore, these dimensional systems are again empirically equivalent.

Finally, it is also possible to increase the number of dimensions. For example, one could introduce a base dimension $[v] = \mathrm{V}$ for velocity in addition to M, L and T, so that $\mathcal{D} = \langle \mathrm{M}, \mathrm{L}, \mathrm{T}, \mathrm{V} \rangle$. This doesn't quite work, because now $[v] = \mathrm{V} \neq \mathrm{L}/\mathrm{T} = [x]/[t]$. But this is easily fixed by the introduction of a novel proportionality constant, c, with dimension $[c] = \mathrm{VT}/\mathrm{L}$. One can then set $v = cx/t$, so that $[v] = \mathrm{V} = \mathrm{V}\frac{\mathrm{LT}}{\mathrm{LT}} = [c][x]/[t]$. The notion of a dimensional constant is familiar: the gravitational constant G and the spring constant k likewise carry dimensions. For an anti-realist, the only role played by such constants is to fix the conversion factor between quantities of different dimensions. In this case, c converts values for distance and duration into a value for velocity. One may object as before that $[v] = \mathrm{V}$ is uninformative of the dependence of velocity on length and time, but as we saw above the anti-realist does not require the dimensions of v to provide any such information. In addition to the choice of base dimensions, then, the number of base dimensions is also underdetermined by the empirical evidence.

The core thesis of anti-realism about dimensions is that all of these systems of dimensions describe reality equally well. There is no fact of the matter as to whether $[v] = \mathrm{V}$ or $[v] = \mathrm{L}/\mathrm{T}$; there are just different ways of keeping track of changes of units. This position is similar to conventionalism about spacetime geometry. Just as spacetime conventionalism says that there simply is no matter of fact as to the true geometry of spacetime, so dimensional conventionalism says that there is no matter of fact as to the true dimensions of quantities.[7] Another author of a textbook on dimensional analysis, Ipsen (1960, 44), explicitly calls dimensions 'conventional'.

Perhaps there are reasons to reject some non-standard systems, for example because they are in tension with further theoretical commitments. In this vein, Jalloh (ms.) argues that although operationalism considers the number of dimensions conventional, it holds that there is a privileged choice of base dimensions in terms of our fundamental measurement procedures. This would attenuate the anti-realist's conventionalism, but not overcome it. Likewise, the geometric conventionalist may hold that on any adequate account, space is three-dimensional (in the sense, unrelated to the dimensions discussed in this paper, that at least three coordinates are required to specify any point on it), but deny that there are any further facts about the geometry. In either case, there remains a significant element of convention.

For the realist, on the other hand, dimensions are something more than just a code. The dependence relation of the VIM definition is metaphysically weighty. If $[v] = \mathrm{L/T}$, for example, then it is in the nature of velocity to be related to space and time in a particular way. This was the notion of dimensions held in most of the 19th-century before Fourier (De Clark, 2017), as well as that of Richard Tolman (1917).

What is this 'something more'? I am not aware of any comprehensive realist account of dimensions. Perhaps dimensions are second-order properties of quantities, which in turn stand in second-order relations to each other. Or perhaps the function of dimensions is to classify quantities into natural kinds, such as the kind of mass quantities and the kind of length quantities. On this view, dimensions make for similarity and dissimilarity between quantities.[8] Yet another possibility is what Skow (2017) calls 'Construction', namely the view that the values of complex quantities are 'constructed from' the values of some simple quantities. Thus, velocity values are constructed from distances and durations. This of course requires a privileged set of fundamental quantities. Skow raises an issue for Construction, namely that it presupposes—falsely, Skow believes—that the units of these fundamental quantities uniquely determine the correct units of the complex quantities.

[7] For a recent revaluation of geometric conventionalism, see Dürr and Read (2023).

[8] However, note that although identity of dimensions is often seen as a necessary condition for identity of kind, it is not a sufficient condition: heat capacity and entropy have the same dimensions, but are of a different kind.

The realist may need to parry this objection.

I will not consider these various possibilities here. Instead, I define dimensional realism more broadly as any view on which some select class of systems of dimensions more closely represents reality than others. The claim is not just that some systems of dimensions are better than others; anti-realism may concur that some systems are simpler or more fruitful. The claim is rather that some systems of dimensions are closer to the truth. This form of realism is not committed to the claim that there is a unique system in which quantities have their 'true' dimensions. Some aspects of the choice of dimensions may remain conventional even for the realist, if that conventionalism is limited in scope.

There isn't so much a sharp divide between realism and anti-realism, then, as a spectrum. On one far end, the full-blown anti-realist espouses wholesale conventionalism: no system of dimensions, however contrived, is worse than any other. On the other extreme, the committed realist claims that there is a unique true system of dimensions. I believe that the intermediate position I will defend in this paper is closer to realism than to anti-realism. On this position, the second and third aspects of underdetermination described above are not purely conventional. There are matters of fact about the dimensional relations between quantities, such as whether a pair of quantities have the same dimensions or not. This form of realism is sufficiently robust to rebuke the accusation of conventionalism.

4 Primer on Dimensional Analysis

The argument from underdetermination is predicated on the claim that it makes no difference which system of dimensions one employs. This is true insofar as empirical adequacy is concerned: the predictions of a theory remain the same whatever the dimensions of the quantities involved are. But the realist's rejoinder to underdetermination has typically focused on explanatory differences. While there are many empirically equivalent theories, some of them offer better explanations of the observed phenomena than others. They deserve our realist commitment on that basis. The anti-realist has often rejected this form of argument by assimilating explanatory strength to the pragmatic virtues (van Fraassen, 1980). It is not my intention to rehearse that debate here. I will assume the validity of abductive inferences. The more specific question then is whether this type of inference can be applied successfully to dimensions: are some dimensional systmems more explanatory than others? I answer in the affirmative. In particular, I believe that the technique of dimensional analysis is explanatory, but only once certain dimensions are adopted. By an inference to the best explanation, I conclude that those dimensions better describe reality.

I will elaborate on this argument in the next section, but first I must introduce dimensional analysis itself. In a nutshell, dimensional analysis is a technique to determine the functional relation between some quantities from just their dimensions—without any derivation from dynamical laws. Suppose one wanted to know the relation between the period P of a mass m attached to a spring with spring constant k suspended in a gravitational field of strength g. The quantities involved are standardly assigned dimensions as follows: $[P] = \mathrm{T}$; $[m] = \mathrm{M}$; $[k] = \mathrm{M}/\mathrm{T}^2$; and $g = \mathrm{L}/\mathrm{T}^2$. The crucial insight then is that there is a unique product of powers of m, k and g in dimensions of T only: $\sqrt{m/k}$. To see this, first note that g is the only quantity with dimensions of L. Therefore, no product that features g can have dimensions of T only, so g does not feature in the sought-after expression. The only other quantity with dimensions of T is k, but because it also has dimensions of M one has to divide k by m. This yields the expression k/m, which has dimensions $[k]/[m] = 1/\mathrm{T}^2$. From there one can immediately infer that $P = \alpha\sqrt{m/k}$, where α is a dimensionless constant.

This may not seem like the most exciting result. Yet the importance of dimensional analysis lies in the fact that one can find this expression without deriving it from the equations of motions. Although far from difficult, it is a little more involved to derive the period directly from Hooke's law, $ma = -kx$. For more complex systems such a derivation may prove intractable.

Dimensional analysis does require antecedent knowledge of the relevant laws. Those laws tell us which quantities are involved in the first place. We know that P may depend on m because the relevant force law contains m. Moreover, the dimensions of complex quantities themselves are ultimately derived from the laws in which they occur. Given the dimensions of m, a and x, for instance, it follows from Hooke's law that $[k] = [m][a]/[x] = \mathrm{M}/\mathrm{T}^2$. Campbell (1924) and Ellis (1964) criticise dimensional analysis on this basis. If we already know the relevant equations, why not just solve them directly? Of course, dimensional analysis may sometimes be simpler to carry out, but a derivation from dynamical laws seems more fundamental—hence more explanatory. Whilst I do not dispute that knowledge of the laws is normally required to carry out dimensional analysis, I will argue in the next section that this does not mean that the latter technique is less explanatory. Indeed, dimensional analysis is often more explanatory than a 'differential analysis' of the equations of motion.

Before that, however, I first present a more formal account of dimensional analysis. The theorem that powers dimensional analysis is known as the 'Buckingham Π-Theorem':

Theorem 1 (Buckingham Π-Theorem)**.** *Any law* $f(Q_1, ..., Q_N) = 0$ *in terms of* N *quantities* Q *of* n *base dimensions can be rewritten as a law* $f(\Pi_1, ..., \Pi_m) = 0$ *in terms of* $m \coloneqq N - n$ *dimensionless quantities* Π*.*

The Π-terms are products of powers of the Q-terms.

For an example of the Π-theorem in action, consider once more the period of a spring. There are four quantities (P, m, k and g) in three dimensions (L, T, M), so the Π-theorem entails that there is one dimensionless Π-term. This term is our kP^2/m, which is unique up to a dimensionless proportionality factor. Therefore, the law that relates these quantities is of the form $f(kP^2/m) = 0$, from which it follows that P is proportional to $\sqrt{m/k}$.

The Buckingham Π-theorem relies on three assumptions (Jalloh, forthcoming):

1. Any law that relates the quantities Q is expressible as a function $f(Q_1, ..., Q_n) = 0$;
2. The dimension of any quantity is a product of powers of the base dimensions (Bridgman's lemma);
3. The law f is dimensionally homogeneous: each term has the same dimensions.[9]

I will adopt these assumptions in what follows.

However, is dimensional realism really entitled to these assumptions, in particular dimensional homogeneity? Typical justifications of dimensional homogeneity are based on an anti-realist account of dimensions. The main motivation for homogeneity is that it ensures that an equation remains valid under an arbitrary change of units. As Maxwell (1873, 1-2) wrote: 'The formulae at which we arrive must be such that a person of any nation, by substituting for the different symbols the numerical values of the quantities as measured by his own national units, would arrive at a true result.' The unit-independence of the laws is a reasonable requirement: the validity of the laws should not hinge on an arbitrary choice of units!

Ths justification assumes the viewpoint of dimensions as a bookkeeping device. If, as dimensional realism asserts, dimensions are (for example)

[9] Lange (2009) calls this condition 'dimensional consistency', whereas he defines dimensional homogeneity as the weaker condition that an equation holds for any choice of unit. I will not follow Lange in his non-standard choice of terminology.

real properties of quantities, one may wonder what such properties have to do with unit changes. Fortunately, dimensional realism can offer a deeper justification of homogeneity. For a dimensional realist, equations satisfy dimensional homogeneity because the dimensions of a quantity determine its kind, and only quantities of the same kind can have the same value. If an equation is inhomogeneous it equates quantities of different kinds, so it is trivially false. This justification of dimensional homogeneity can be traced back to Lodge (1888, 47): 'I have recently been laying stress on the fact that the fundamental equations of mechanics and physics express relations among quantities and are independent of the mode of measurement of such quantities; much as one may say that two lines are equal without enquiring whether they are going to be measured in feet or metres; and indeed, even though one may be measured in feet and the other in metres.' Because I do not wish to consider in detail the metaphysical nature of dimensions, I will not say more on this issue here. However those details turn out, it seems clear that dimensional homogeneity is also justified—perhaps even more so—on a realist conception of dimensions.

Therefore, the realist can rely on the Π -theorem just as much as the anti-realist. This means that the path is free to appeal to dimensional analysis in my defence of dimensional realism.

5 How Dimensions Explain

I now return to the realist's rejoinder to the anti-realist's argument from underdetermination in the form of an inference to the best explanation. The inference runs as follows:

1. Dimensional analysis, when successful, is explanatory;
2. Dimensional analysis is successful only when quantities are assigned particular dimensions;
3. If dimensional analysis is explanatory only when quantities are assigned particular dimensions, then those are the quantities' true dimensions;
4. Therefore, those are the quantities' true dimensions.

I will comment on each of the premises in turn.

5.1 Dimensional analysis, when successful, is explanatory

The first premise says that dimensional analysis not only delivers true results, but is also explanatory. In the previous section, I mentioned one reason to believe that this premise is false: for dimensional analysis to work, knowledge of the fundamental laws is necessary. With that knowledge one could just as well derive the sought-after expression directly by solving a set of differential equations. The putative explanation from dimensional analysis is 'screened off' by this more fundamental differential analysis of the system's dynamics.

Lange (2009) has argued, to my mind convincingly, that this objection is unsuccessful. He shows that dimensional analysis is often more explanatory than direct derivation. I have little to add to his results, so I will simply summarise them here.

Lange identifies at least four senses in which dimensional analysis is often more explanatory than differential analysis:

1. *Dimensional analysis abstracts away from explanatorily redundant features.* It is often the case that a differential analysis contains more detail than needed. For example, it is possible to derive from the law of universal gravitation that the period of a planet's elliptical orbit of radius r around a massive star is inversely proportional to $r^{3/2}$. But this would hold even if the law of universal gravitation were different, for example if the value of G were different or if F_g were proportional to M^2 instead of M. To the extent that dimensional analysis is insensitive to such details it offers a better explanation.

2. *Dimensional analysis explains distinct features of the same system in distinct ways.* The explanation offered by differential analysis is often the same even for distinct explananda. For example, suppose one wants to know both (i) why T is proportional to $\sqrt{m/k}$, and (ii) why it does not depend on g. In order to explain these features by differential analysis, one simply derives the equation $P = \alpha\sqrt{m/k}$ from the equations of motion: both (i) and (ii) follow immediately. Dimensional analysis, on the other hand, offers distinct explanations: (i) is explained by the fact that m must be divided by k to cancel out the mass dimension M, whereas (ii) is explained by the fact that g is the only one of these quantities that has dimensions in L. Dimensional explanations are better because they are more fine-grained.

3. *Dimensional analysis explains similar features of different system in similar ways.* Different systems may involve similar dimensional de-

pendencies; dimensional analysis allows one to unify these features in a way that differential analysis does not. For example, the expressions for the wave speed of a (longitudinal) sound wave and of a (transversal) wave in a piece of rope are physically similar, despite the fact that the dynamical equations governing these systems are quite different. Dimensional analysis offers a more unified explanation of these expressions.

4. *Dimensional analysis explains distinct features of different systems in distinct ways.* Conversely, similar systems may involve different dimensional dependencies; dimensional analysis allows one to track these differences. For example, the expressions for the period of a pendulum and that of a spring look similar, but the former depends on g whereas the latter does not. Differential analysis cannot explain this dissimilarity beyond the brute fact that one eventually arrives at different expressions. But dimensional analysis can locate the difference: the period of a pendulum, unlike that of a spring, may depend on the length l, so here it is not the case that g is the only quantity with dimensions in L. Again, dimensional explanations are often more precise than brute calculations.

Based on these results I take the first premise to be firmly established.

5.2 Dimension analysis is successful only when quantities are assigned particular dimensions

Lange does not pay particular attention to the second premise, since it is not his aim to defend dimensional realism (in fact, he seems to operate with a broadly anti-realist conception). But the effectiveness of dimensional analysis depends on the choice of dimensions. The realist can use this fact to privilege a select class of dimensional systems.

The crucial fact is that dimensional analysis is most effective when m, that is, the number N of quantities involved minus the number n of base dimensions, is minimised. This follows directly from the Π-theorem, which states that any law can be re-expressed as a function of m dimensionless quantities. The smaller m, the fewer terms are involved in this function, and so the more determinate it becomes. In the case where $m = 1$ there is a unique dimensionless product Π, such that $f(\Pi) = 0$. From the dimensional homogeneity of f it then follows that Π is equal to some dimensionless constant, c. In that case the relation between the original quantities is known uniquely up to a proportionality factor. Therefore, dimensional analysis is most 'powerful' when m is minimised.[10]

I will now show that m is minimised, and hence the power of dimensional analysis is maximised, only for certain dimensional systems. In particular, m is determined by the number of base dimensions. I will use the period of the spring as a case study throughout the remainder of this section. When that example was presented in the previous section, there were four quantities (P, m, k and g) in three dimensions (M, L and T). So, $N = 4$, $n = 3$, and $m = N - n = 1$. Therefore, there is a unique dimensionless product of the relevant quantities, kP^2/m, which means that their relation is fully determined up to a proportionality constant. For different choices of dimensional system, dimensional anlysis fails to yield such a specific result.

Consider first the elimination of a dimension. Concretely, suppose one were to eliminate the mass dimension by setting $[m] = \mathrm{L}$. It would then follow from the dimensional homogeneity of the equations of motion that $[g] = [k] = \mathrm{L}/\mathrm{T}^2$. In this system of dimensions there no longer is a unique dimensionless product of the quantities involved. For there are now four quantities ($N = 4$) but only two dimensions ($n = 2$), so the Π-theorem entails that there are $m = 2$ dimensionless Π-terms. These are kP^2/m and gP^2/m. Therefore, dimensional analysis at most yields the result that either $T = \alpha\sqrt{k/m} * f(gP^2/m)$ or $T = \alpha\sqrt{g/m} * f(kP^2/m)$, where α is a dimensionless constant and f is an arbitrary function. This is less informative than the previous result in two ways: firstly, there are two distinct expressions to choose from; and secondly, the form of f is left completely open. The correct result is only recovered if f is identically set to 1 in the first expression. It is therefore clear that the elimination of a dimension reduces the explanatory power of dimensional analysis.

Now consider the introduction of a new dimension. Here, one can show the inverse: the introduction of a further dimension can increase the power of dimensional analysis. For example, suppose one had started out with L and T as the only two base dimensions. We have just seen that in that case dimensional analysis does not yield a unique expression for the period, because $m = 2$. The introduction of a base dimension M for mass would decrease m to 1 to yield a unique dimensionless product, namely kP^2/m—this is just the result derived in the previous section. Just as the elimination of dimensions can reduce the power of dimensional analysis, then, the intro-

[10] What if $m = 0$? Then there is no functional dependence at all between the quantities involved. Since $[x] = \mathrm{L}$ and $[t] = \mathrm{L}$, for example, there simply is no functional dependence of x on t only. I am only interested in cases where $m > 0$.

duction of a new dimension can increase it.

But it is not always better to introduce a dimension. Consider a dimensional system in which force, F, has a base dimension $[F] = \mathrm{F}$ in addition to M, L and T. This would violate the dimensional homogeneity of Newton's second law, $F = -ma$, since $[m][a] = \mathrm{ML}/\mathrm{T}^2$. This is easily fixed: introduce a novel dimensional constant h with dimensions $[h] = \mathrm{FT}^2/\mathrm{ML}^2$, and modify the equation of motion to read $F = hma$. Then dimensional homogeneity is restored. This constant may seem mysterious: what does it represent? But it does no more than codify the proportionality between F and ma, in the same way that k codifies the proportionality between F and x. There is now an additional dimension, F, so $n = 4$; but there is also an additional quantity, h, so $N = 5$. The difference m therefore remains the same, and so the Π-theorem dictates that there is a unique dimensionless product of T, k, m, g and h. This is kP^2/hm, from which it follows that T is proportional to $\sqrt{hk/hm}$. This is essentially the same result as before, except for the presence of the novel constant. The addition of a further dimension has therefore not increased the explanatory power of dimensional analysis. Conversely, it would not decrease the power of dimensional analysis to eliminate the F-dimension.

The broader conclusion is this: if the introduction of a dimension also involves the introduction of a novel constant, or if the elimination of a dimension also involves the elimination of a constant, then dimensional analysis remains equally explanatory. In such cases, the change in n is compensated for by an equal change in N, so m is kept constant.

It may thus seem that there is no overall preference for a system with four rather than five dimensions. This would pose a problem for realism, which aims to remove elements of conventionality. The presence of fewer dimensional constants, however, is preferable on broadly Occamian principles. The literature is divided on the nature of constants of nature—but whether they are physical quantities, aspects of the laws, properties of interactions or kinematical structures, they would seem to come at some ontological cost.[11] Even a deflationary account of constants must consider laws without them as syntactically simpler. Bridgman, for instance, held that 'the dimensional constants are to be regarded as an evil, to be tolerated only if they make possible more information about the physical variables' (1931, 49). If it makes no difference to dimensional analysis whether there is an additional constant, the correct choice is to opt for one fewer constant and hence one

[11] For example, on the account offered in Jacobs (2022) such constants represent fundamental cross-value space structure.

fewer dimension.

I therefore posit this maxim: first minimise the difference m between the number N of quantities and the number n of dimensions; then minimise the number n of dimensions itself. If an additional dimension decreases m, the explanatory power of dimensional analysis is enhanced; if it leaves m the same, Occam's razor disfavours the additional constant. The 'Goldilocks' assignment of dimensions is thus the one that is most parsimonious of the ones that are maximally explanatory. Dingle (1942, 338) already expressed a similar insight: 'The net result of these considerations, then, is that if we wish to give the greatest scope of usefulness to the principle of dimensional homogeneity, we must choose sufficient fundamental magnitudes to prevent two or more magnitudes from having the same dimensions, but not so many that indeterminable 'constants of nature' are forced into our equations.' Yet Dingle does not draw the conclusion that dimensions correspond to an element of reality. I argue that the optimal system of dimensions deserves our realist commitment.

I do not claim, however, that there is a unique optimal system. Although the above maxim fixes the number of dimensions, it does not fix the choice of base dimensions. For example, a system of dimensions in which K is a base dimension instead of M is equally explanatory. I will comment on the significance of this at the end of this section, but first let me briefly discuss the final premise.

5.3 If dimensional analysis is explanatory only when quantities are assigned particular dimensions, then those are the quantities' true dimensions;

The final premise is simply a statement of the validity of inference to the best explanation. If the 'Goldilocks' dimensions were not the true dimensions of the quantities in question, the explanatory power of dimensional analysis when exactly these dimensions are adopted would seem miraculous.

But isn't the explanatory power of dimensional analysis also miraculous if dimensional realism is correct? How does a dimensional system fix the lawlike relation between a set of quantities? In a traditional inference to the best explanation, the explanation is often causal. Thus, we believe that alpha particles exist because their existence provides the best causal explanation of the observed track in the cloud chamber. Dimensions do not seem to possess any causal powers. So, how can dimensions explain?

I intend to leave this question open, because the answer will ultimately depend on a broader metaphysical account of dimensions. For my argument's validity, it is enough to have established that dimensional analysis is explanatory one way or another. But let me offer a sketch of a possible answer. We saw earlier that dimensional homogeneity is a core assumption in Buckingham's Π-theorem. The realist justifies this assumption by thinking of quantities with different dimensions as being of different kinds. This makes homogeneity a physical principle, rather than a merely formal requirement. Dimensional homogeneity acts as a constraint on the way in which quantities can hang together: it is impossible for a mass quantity to be equal to a length quantity in somewhat the same way that it is impossible for a square peg to fit into a round hole. In the words of Susan Sterrett (2021, 674): '[t[he fact that physical equations (equations of physics) must satisfy the requirement of dimensional homogeneity, provides an explanation of how mathematical equations as used in physics can be informative about the world.'[12] This is how the dimensions of quantities explain why they can only depend on each other in a particular way.

The staunch anti-realist may not have much faith in this style of explanation, but it is not my hope to convince the sceptic. For a realist, the argument presented here establishes the conclusion that some dimensional systems better describe reality than others, namely those that first minimise m and then minimise n. Although the choice of dimensional system is underdetermined by the data, it is constrained by a theory's explanatory commitments.

I have not yet discussed the third element of conventionality: the choice of base dimensions. That is because this choice has no effect on the power of dimensional analysis. In the example of the spring, one could as easily have chosen L, T and F as base dimensions, where F is a dimension of force. Then $[P] = \mathrm{T}$, $[k] = \mathrm{F/L}$, $[m] = \mathrm{FT^2/L}$, and $[g] = \mathrm{L/T^2}$. There are still four quantities in three dimensions, $m = 1$, and the only dimensionless product is kP^2/m as before. This is no surprise: a change in base dimensions affects neither the number of dimensions n nor the number of quantities N, so m is left the same. Therefore, the inference to the best explanation does not

[12] Sterrett thinks of dimensional homogeneity as akin to a grammatical rule, so that an inhomogeneous equation is ill-formed. This interpretation does not lend itself as well to realism—and seems inconsistent with her claim above—since grammatical constraints are formal rather than physical. I do not believe it is correct to think of homogeneity as grammatical: a law such as $F = m$ is well-formed, but false. Lange (2009, §8) likens the principle of dimensional homogeneity to 'meta-laws' such as symmetry principles. It has the status of a general law that explains the more specific first-order laws. This analysis is more amenable to realism, but it carries metaphysical commitments that not all realists may want to accept.

yield a unique set of true dimensions for a set of quantities.

This is not a problem for realism as I have defined it, which does not require a unique dimension for any quantity but only a restricted class of privileged dimensional systems. The latter suffices to reject anti-realism.

Nevertheless, this intermediate position may not seem to deliver on the promise of dimensional realism, namely that the dimensions of a quantity reflect their nature. I concur: dimensional analysis does not reveal the nature but the dimensional structure of quantities. As such, the realism I defend is best understood as a version of structural realism.[13] The structure in question consists of the dimensional relations between quantities. By this, I mean relations of the form $[F] = [m][a]$ or $[P] = [\sqrt{m/k}]$, that is, relations that say that the dimension of some quantity is a particular function of the dimensions of certain other quantities. On the one hand, these relations are preserved under changes of base dimensions. For example, whether I choose force or mass as a base dimension, it remains the case that $[F] = [m][a]$. On the other hand, they are not preserved under a change in the number of base dimensions. For example, $[g] = [k]$ is false in a system with three base dimensions, but true in a system with only two. The claim that there is a fact of the matter as to the dimensional relations between quantities certainly goes beyond the conventionalism of Bridgman, Langhaar and the SI.

Importantly, only the dimensional relations are relevant for dimensional analysis. The principle of dimensional homogeneity, for example, only requires identity of dimensions; it does not matter which dimensions the terms in an equation have. The key result, that there are m dimensionless Π-terms, is likewise insensitive to a change of dimensional basis.

Like with structural realism in philosophy of science, there are two varieties of structural dimensional realism: epistemic and ontic. The first position says that while quantities have a unique set of true dimensions, we cannot know what they are. All that is accessible to us are their dimensional relations. The latter position, meanwhile, says that the dimensional relations are all there is. There just is no fact of the matter whether $[v] = \mathrm{L}/\mathrm{T}$ or V; the relation $[v] = [x]/[t]$ expresses all there is to know about the dimensions of v, x and t. The former position is perhaps easier to make sense of, because it is not committed to 'dimensional relations without dimensions'. But the latter position is more parsimonious, since the dimensional rela-

[13] Johnson's (1997, Ch. 4) view is an early precursor of dimensional structuralism. Jalloh (ms.) defends a similar position, which he calls 'functionalism'. A functionalist analysis is one way to obtain a form of structuralism, but it is not the only one; I therefore prefer 'structuralism' as a more neutral term.

tions are the working posits of dimensional analysis whereas the dimensions themselves are idle.

The argument that I have presented, however, is neutral between the difference between ontic and epistemic structural realism. I therefore leave a discussion of their respective merits to future work.

6 Dimensions and Fundamental Laws

In the previous section I established that certain dimensional system increase the explanatory power of dimensional analysis and therefore deserve our realist commitment. However, this only works if it is the same system (up to a choice of base dimensions) in each instance that affords successful dimensional analyses. If it were the case that different dimensional analyses require different, incompatible dimensional systems, then an inference to the best explanation yields inconsistent results. In this section, I discuss a putative case of such non-uniqueness and show that it does not threaten dimensional realism. The key point is that in order to find out the most fundamental dimensional system, one has to consider the most fundamental applications of the laws.

The most famous example of non-uniqueness is the 'Rayleigh-Riabouchinsky paradox'.[14] In 1915, Lord Rayleigh showed that dimensional analysis could yield the equation for the heat transfer from a stream of fluid to a solid rod. In his analysis, Rayleigh assumes that heat and energy have different dimensions. Riabouchinsky pointed out in a response that kinetic theory allows one to equate their dimensions. But it turns out that this reduces the number of dimensions without a reduction in the number of quantities, so dimensional analysis yields a less determinate result. The maxim from the previous section would therefore rule that heat and energy have different dimensions. This may not seem problematic, except for the fact that other applications of dimensional analysis are most successful when the dimensions of heat and energy are equal. If incompatible dimensional systems are equally explanatory, then an inference to the best explanation cannot favour one over the other.

In order to tackle this problem I will consider a slightly simpler example that does not concern heat, namely whether force is a base dimension in addition to M, L and T. We have seen in the previous section that an additional base dimension F for force necessitates the introduction of an additional constant, leading to an overall less parsimonious system. It would

[14] For more on this paradox, see Jalloh (ms.) and references therein.

thus seem that it is inadvisable to include force as a base dimension. Bridgman (1931), however, discusses a case in which dimensional analysis becomes more explanatory when a dimension of force is added. The explanandum of Bridgman's example is the expression for the velocity v of a small massive droplet falling under the influence of gravity in a viscous liquid. The relevant quantities are six in number: the velocity v; the diameter of the droplet, D; the densities d_1 and d_2 of both the droplet and the liquid; the viscosity of the liquid, f; and the gravitational field, g. If one chooses M, L and T as base dimensions, such that $[f] = \mathrm{M/LT}$, then there are $6 - 3 = 3$ dimensionless quantities. In that case dimensional analysis does not yield any useful results. If, on the other hand, one includes F as a base dimension for force, such that $[f] = \mathrm{FT/L^2}$, then there are only $6 - 4 = 2$ dimensionless quantities: vf/gd_1D^2 and d_1/d_2. Since v only occurs in the first expression one can infer that $v = gd_1D^2/f \times \theta(d_1/d_2)$, where θ is an arbitrary function. This is not far off from the correct result, namely that $v = gD^2(d_1 - d_2)/f$.[15] Unlike in the case of the spring, here a system with an additional base dimension for force is more explanatory. By an inference to the best explanation, we should conclude that force is a base dimension after all.

The problem of underdetermination seems to have returned in full force. There is a way out: the former example is, in a sense to be made precise, more fundamental than the latter. Bridgman already notes that the analysis of the droplet is essentially a problem in statics. The droplet does not accelerate, because at every point in time the gravitational force on the droplet is equal to the upwards pressure of the liquid. It follows that the second law $F = ma$ plays no role in the derivation of the droplet's velocity. On the other hand, the spring-load does accelerate, so it is a dynamical problem that requires the second law to derive the period. Dynamics includes statics as a special case, namely when all net forces are zero. Because dynamics is more general than statics, it is also more fundamental.[16]

Lange uses this fact to explain the Rayleigh-Riabouchinsky paradox. Lange's point is that the choice of F as an additional base dimension does not ignore the information that $[F] = [ma]$, but rather reflects our knowledge that the dependence of F on m and a is irrelevant to the problem at hand. It is possible to put this in modal terms: the expression for v would remain the same even if force were not proportional to mass times accelera-

[15] Indeed, if one were to consider only the difference in densities $(d_1 - d_2)$, dimensional analysis would yield exactly this expression!

[16] See Hunt *et al.* (2023) on the claim that more general theories are more fundamental.

tion, for instance if F were proportional to m^2a instead. Since the relation between the dimensions of F and those of m and a are irrelevant, we may assume that force has a base dimension unrelated to M, L and T. The same is not the case for the period P of a pendulum, since the derivation of P from the equations of motion relies on the second law. Likewise, distinct dimensions for heat and energy do not reflect our ignorance of kinetic theory, but rather our knowledge that in Rayleigh's example no thermal heat is converted into mechanical energy. Even if heat and energy were not interconvertible, Rayleigh's expression for heat transfer would have remained the same.

I agree with Lange that this correctly explains why it is legitimate to use base dimensions for heat or force in certain cases. But this does not answer the non-uniqueness objection, and in fact may seem to make it worse: if different choices for the number of dimensions are justified in different cases, how can one of those choices more closely match reality? To solve this puzzle, I return to the claim that dynamics is a more fundamental branch of mechanics than statics. In order to find out the fundamental system of dimensions, one should look at the most fundamental applications of dimensional analysis. This means that one should prefer the dimensional system that best explains the period of the spring over one that best explains the velocity of the droplet. To put this point differently, the latter explanation would have worked even if the world's laws were different from what they actually are. If we are interested in the actual dimensions of quantities, we should consider the actual laws that relate those quantities. If the second law were false, force may have had its own base dimension. But we wish to know the actual relation between the dimension of F and those of m and a, so we must consider the second law. After all, it is the second law that determines how F, m and a are related in the actual world. The dimensional analysis of the period does rely on the second law, and it tells us that $[F] = [m][a]$.

The uniqueness objection is therefore answered by the claim that in order to discover the fundamental dimensional relations between quantities, one should consider the most fundamental dimensional explanations. As far as I am aware, those explanations always presuppose the same dimensional relations. Of course, I have not proven this: it remains possible that there are pairs of equally fundamental dimensional analyses that nevertheless use dimensional systems that differ over more than just the choice of base dimensions.[17] If Lange's explanation of the Rayleigh-Riabouchinsky paradox

[17] One possible counterexample concerns directions in space. There are some applications

is correct, however, then there is no reason to expect such cases to occur, for Rayleigh-Riabouchinsky-style paradoxes occur exactly when a dimensional analysis leaves out some fundamental law.

7 Conclusion

I have defended the claim that the dimensions of quantities—or, more precisely, the relations between the dimensions of quantities—track a real feature of the natural world. Although it is possible to assign the same set of quantities different dimensions, some of those assignments are more explanatory than others. The best explanation for this fact is that those dimensional systems better describe reality.

But dimensional analysis does not uniquely determine a dimensional system. In particular, it leaves open the choice of base dimensions. Therefore, my account is neutral as to which individual dimensions are real. It does not say, for example, that L, M and T are more fundamental than F. It does, however, say that there are exactly three mechanical dimensions, and that $[F] = [m][a]$—contrary to consensus conventionalism.

I have defined dimensional realism in broad brushstrokes: it simply consists of the claim that some dimensional systems better describe reality than others. This leaves it almost completely open what dimensions are. I hope that my defence of dimensions will spur on philosophers to anwer this question. In addition to the intrinsic interest in what kind of entities dimensional realism commits one too, the answer to this question also bears on the choice between epistemic and ontic structural dimensional realism. For example, if dimensions are conveived of as higher-order properties of quantities, then it would seem that each quantity must ultimately possess a unique true dimension—a 'nature' that we cannot know. Skow's (2017) account of dimensions in terms of a 'definitional connection' explicitly assumes a privileged choice of base dimensions. Other accounts are better suited to ontic structuralism. In this vein, Jalloh (ms.) has recently developed a functionalist account of dimensions, on which dimensions are relationally-defined nomic roles.

Finally, dimensional realism provides a sense in which reality is truly made of physical stuff: of masses, charges, velocities, and so forth. Dimen-

of dimensional analysis on which it is advantageous to assign distinct dimensions to distances in orthogonal directions. If those applications are fundamental, we should conclude that there are in fact three distinct length-dimensions. I lack the space to discuss this example in more detail.

sions are an indication that these quantities are distinct from pure numbers. The dependence of the effectiveness of dimensional analysis on the choice of dimensional system indicates that it matters which quantities are made of which kind of stuff. It will not do to throw every quantity on the same heap—assign every quantity the same base dimension, or none at all—because then dimensional analysis is silent on their mutual relations. This realisation should serve as an antidote to recent proposals in favour of a fully dimensionless physics (Whyte, 1954; Duff, 2015; Barbour, 2021).[18] If dimensional realism is correct, such a physics is all but physical.

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[18] This is not to say that only a dimensional realist can opppose dimensionless physics. Grozier (2020), for instance, shows that Duff incorrectly applies dimensional analysis.

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