THE LOGIC OF SIR WILLIAM HAMILTON: TUNNELLING THROUGH SAND TO PLACE THE KEYSTONE IN THE ARISTOTELIC ARCH

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1 BACKGROUND AND INTRODUCTION

> Every *learner* in science, is now familiar with more truths than Aristotle or Plato ever dreamt of knowing; yet, compared with the Stagirite or the Athenian, how few, even of our *masters* of modern science, rank higher than intellectual barbarians! (Hamilton, 'Philosophy of Perception', p. 40).[1]

The Enlightenment initiated the modern world as the product of a hitherto unsurpassed devotion to reason and scepticism. The scientific successes, ideals of progress, material advancement, hopes of social amelioration and freedom of the Enlightenment, were accompanied by catastrophic failures, conspicuous atrocities perpetrated in the name of reason and authority, and increasing fears of a dreadful new age of barbarism. Several Western countries incurred massive rifts, upheavals, wars, and profound societal changes that impinged upon or were feared to be the results of Enlightened thought. Following the shock-waves of the American and French revolutions, as some of the first effects of the industrial revolution were beginning to be felt, divisions at the heart of the Enlightenment between reason and scepticism resurfaced in varying guises in England, France, Germany, and Scotland. During the 18^{th} century Scotland had undergone major political changes that both weakened the country's autonomy and yet liberalised its intelligentsia in ways that helped foster that great flourishing of intellectual talent we now call the Scottish Enlightenment. This intellectual movement laid the groundwork for so many succeeding cultural and material changes across the world. A number of its leading lights were members of the University of Glasgow. It was here, in

[1] Sir Willam Hamilton, *Works of Sir William Hamilton*, with an introduction by Savina Tropea, 7 vols (Bristol: Thoemmes Press, 2001), vol. 1, *Discussions*. All references to Hamilton's articles are to this reprint edition and in this form. Page numbers correspond to, *Discussions on Philosophy and Literature, Education and University Reform*, 2nd edn (London: Brown, Green and Longmans; Edinburgh: MacLachlan and Stewart, 1853).

one of the homes in the Professors' Court of this University that William Stirling Hamilton was born on 8 March 1788.

The cultural antecedents within Hamilton's family background are interesting. In the late 17^{th} century, two ancestors were leading Covenanters; in the 18^{th}, several became somewhat distinguished academics. One of Hamilton's namesakes became the Professor of Divinity and later Principal of the University of Edinburgh — intriguingly he 'acquired a high reputation [...] for theological erudition' (Veitch, p. 5).[2] But it was in medicine that Hamilton's direct male ancestors excelled. His grandfather, Thomas Hamilton, a professor of medicine at the University of Glasgow, was fairly close to some of the more eminent medics at the University, such as William Cullen, Joseph Black, and William and John Hunter. However, Thomas was also frequently in the company of Adam Smith and James Watt, since not only were they connected through their respective roles within the University, but they were also members of the literary Anderston Club, presided over by the classical scholar and, to some extent still renowned, Professor of Mathematics, Robert Simson. Hamilton's father followed in his own father's academic footsteps but, having been Professor of Anatomy from his early 20s, he died young, aged just 31.

William Hamilton's academic lineage, the mainly Glasgow-based Enlightenment figures of his father's and grandfather's acquaintance, and the general educational ethos contributed to by a good number of the University's alumni and professors during the century of Hamilton's birth, probably played important roles in helping to mould the academic he would later become. Certainly it does seem as though Hamilton looked back into his past and may have found there sources of inspiration with regard to his somewhat pugilistic critical approach to philosophy, his legendarily extensive erudition, distinctive and in many ways exemplary pedagogical style, understanding of the nature of philosophy, and (in the works of Thomas Reid and Dugald Stewart) subject matter of extensive later study. Particularly with regard to Reid (who was the Professor of Moral Philosophy at Glasgow and is generally recognised as the founding father of the Common-Sense school), Hamilton's own development of Reidian philosophy is a significant factor that I shall briefly return to later. However, although Hamilton's intellectual inheritance from mainly Glasgow-based Enlightenment scholars must have helped shape his intellect, the educative role of his mother, Elizabeth Hamilton, should not be forgotten.

Hamilton was just 2 years old when his father died. His mother played a crucial role in his educational development. Elizabeth probably imbued in him a great keenness to excel, while balancing against this her various attempts to ensure that he did not develop too fast by, for example, returning him to school education in England following a period at Glasgow University and affording him ample leisure time during vacations to enjoy various physical pursuits and the companionship of other boys and his younger brother Thomas (who later gained some fame as a writer and the author of the novelistic account of pre-industrial Glasgow, *Cyril*

[2] John Veitch, *Memoir of Sir William Hamilton* (Edinburgh and London: Blackwood, 1869).

Thornton).[3] Hamilton's early childhood and overall educational experiences under the supervision of a strong mother were therefore, as it were, the complete obverse of the deeply unpleasant regime so notoriously inflicted by James Mill and Jeremy Bentham on John Stuart Mill, the man who would later become Hamilton's greatest antagonist (notably some nine years after his death). Mill's one-time famous attack on Hamilton in his longest philosophical work, the *Examination of the Philosophy of Sir William Hamilton*, still stands far in excess of virtually any other attempt to disparage his standing as a philosopher.[4] Since Mill, most commentators who in one way or another berate Hamilton, either merely add minor footnotes to Mill's *Examination* or uncritically accept his authority. Since Mill's *Examination* — peppered with numerous misreadings of Hamilton — is in so many ways misleading, it deserves a thorough critical reassessment which I have not judged to be appropriate or even possible within the scope of this chapter.[5]

Hamilton's principal biographer, John Veitch, claims that 'no son could cherish greater regard or a more loyal affection for a mother than he did' (Veitch, p. 12). However, Hamilton's early letters to his mother often suggest a surprisingly direct and at times high-handed manner towards her (see Veitch, pp. 25-6). Fiery, imperious, peremptory, Hamilton's style of writing in these letters possibly indicates a certain fierceness of temperament tolerated or even enjoyed by his mother. Several of his somewhat more mature letters suggest increasing tenderness towards her (see letter to his mother, dated 27 November 1807, Veitch, p. 31). By the time of her death in 1827, which profoundly affected him, Hamilton had lived with her for almost his whole life and there can be little doubt that he was deeply attached to her (Veitch, pp. 134-5). Veitch claims that Hamilton wrote to his mother 'with the familiarity of an equal in point of years, without reserve, and often strongly' but he credits her with having been conscious of 'those qualities of mind which became afterwards so remarkable' and he accords to Elizabeth the praise of resolving 'to give him every advantage of education which lay in her power' (Veitch, p. 27).

Hamilton's early school education was mainly at the Glasgow Grammar School, followed by a brief spell in the Latin and Greek classes at the University of Glasgow in 1800. He was at this time just twelve years old. Though it was not uncommon for boys to attend Scottish universities at such a young age, Hamilton was certainly

[3]Thomas Hamilton, *The Youth and Manhood of Cyril Thornton*, ed. by Maurice Lindsay (Aberdeen: The Association for Scottish Literary Studies, 1990).

[4]John Stuart Mill, *An Examination of Sir William Hamilton's Philosophy and of the Principal Philosophical Questions Discussed in his Writings* (London: Longman, Green, Longman, Roberts & Green, 1865).

[5]For one attempt to reconcile the logics of Mill and Hamilton prior to Mill's *Examination*, see, [H.L. Mansel], 'Province of logic and Recent British Logicians', *North British Review*, 33 (Nov. 1860), 401-427. There were also several attempts to defend Hamilton following Mill's attack — for example, see, [H.L. Mansel], 'Mill's Examination of Sir William Hamilton's Philosophy', *North British Review*, 43 (Sept 1865), 1-58. For a brave attempt to defend Hamilton against Mill, though not specifically addressing his logic, see Dallas Victor Lie Ouren, 'HaMILLton: Mill on Hamilton: A Re-examination of Sir William Hamilton's Philosophy' (unpublished doctoral thesis, University of Minesota, 1973).

on the younger side of the norm. But Elizabeth Hamilton decided against William continuing his studies at Glasgow in the following academic session and he and Thomas were therefore moved in 1801 to schools in England. After returning to Scotland both boys entered the University of Glasgow in 1803, where William seems to have performed well in Latin, though 'In the classes of Logic and Moral Philosophy Hamilton was greatly distinguished, having in each carried off the highest honour of the year, which was then [. . .] awarded by the votes of the members of the class' (Veitch, p. 21).

Hamilton's most notable teacher from this time was Professor George Jardine (1742-1827) in the Logic class. Jardine's teaching made a lasting impression on him (Veitch, p. 21). As his studies continued, he studied medicine from 1804, studying botany and anatomy from 1805. His medical studies continued at Glasgow throughout 1806 and, during the winter of 1806-7, at Edinburgh. However, his book purchases from around this time included a fairly broad array of philosophical, medical, and historical works (see Veitch, p. 24). Though greatly impressive by today's academic standards for undergraduates, the breadth of his reading was very much in line with the generalist nature of Scottish educational practice and was not particularly atypical of other students who would later become eminent scholars and writers. By the time of Hamilton's death he had amassed some ten thousand volumes, around eight thousand of which were purchased by the University of Glasgow where they are currently held in a special collection. Within this collection there are about one hundred and forty editions of Aristotle's works, and a good number of the works he reviewed, some of which display neat manuscript marginalia at times evincing a peculiar degree of care in, for example, comparing earlier and later editions of works by Archbishop Richard Whately — one of the Oxford logicians whom Hamilton repeatedly criticised.

If by this time Hamilton was beginning to distinguish himself as a student of marked ability at the University of Glasgow, probably the most conspicuous educational advantage Elizabeth bestowed upon her son, was her determination that he should complete his university education at Oxford. In 1807 he secured a Snell Exhibition and entered Balliol College where he continued his studies until taking his Bachelor of Arts in 1810 — he of course obtained a First. His Oxford days seem to have been highly stimulating — certainly he made many acquaintances during this period and he read voraciously. The vast extent of Hamilton's learning seems to have become somewhat legendary from around the time of his final examination at Oxford. According to one account, 'He allowed himself to be examined in more than four times the number of philosophical and didactic books ever wont to be taken up even for the highest honours [. . .]. Since that time [. . .] there has been no examination in this University which can be compared with his in respect to philosophy' (Veitch, 60). However, his first career was not in philosophy but instead law.

He became a member of the Bar in 1813 and having returned to Edinburgh he lived with his mother and his cousin, Miss Marshall, whom he later married in 1829. His wife was a devoted companion and without her hard work as an amanuensis, perhaps little of Hamilton's lectures would have survived. Though as yet we know too little about her, Lady Hamilton must have been or become through her marriage to William, one of those many women during the 19^{th} century whose knowledge of literature and philosophy far excelled the attainments accredited to them by posterity. Now that Hamilton was an advocate attempting to make a living at Edinburgh, with rather too few cases to attend to, he began to investigate his family history, though not as a light hobby but with a real purpose. Family tradition had it that William was an indirect descendant of Sir Robert Hamilton of Preston, a staunchly fierce Covenanter who died in 1701, after which time the baronetcy was not assumed by the heir and from thenceforth had lapsed into a mere family memory. After three years of research Hamilton finally presented a case to the Edinburgh Sheriff which proved that he was the heir-male to his Covenanting ancestor. Henceforth, William Stirling Hamilton became Sir William Hamilton, Baronet of Preston and Fingalton (Veitch, 69). This may seem a curious moment in Hamilton's personal history but no doubt he was motivated by several practical considerations, not least of which must have had to do with social and career advancement. From what I can gather from the occasionally sketchy accounts of his life by Veitch and Monck, Hamilton had sufficient employment as a lawyer but was only moderately successful: law was 'but a secondary pursuit for him' and instead he haunted the Advocates library with the bibliophilic zeal of an antiquarian (Veitch, p. 75).[6]

If he was less suited to the law than he might have been, his politics were also an obstruction to great material success since he was a Whig, the ruling party of the day Tory. As Veitch assesses Hamilton's politics, he was 'a man of progress' and liberal principles, though little if at all involved in party politics of any kind (Veitch, 78). Of course he knew and socialised with many of the leading Scots of the day, Sir Walter Scott, Thomas de Quincey, Francis Jeffrey, J.G. Lockhart, Macvey Napier, and many others, but he also seems to have had a fair number or European friends from Russia, France, and Germany. He visited Leipzig in 1817 and again travelled to Germany in 1820, visiting libraries in Berlin and Dresden. He was largely instrumental in the Advocates Library's purchase of an extensive collection of valuable German works. His interest in German literature and philosophy, which would later grow to unrivalled proportions among his contemporaries, dates from around this time.

Also in 1820 Hamilton applied for the Moral Philosophy professorship at Edinburgh University. Although he had strong support for this chair, not least of all from the elderly Dugald Stewart, John Wilson (better known as the famous 'Christopher North' of *Blackwood's Magazine*) secured the post due to the Town Council's patronage, though he was by no means a suitable candidate. In many ways this was quite scandalous and seems to have been entirely due to Hamilton's Whig politics (Veitch, pp. 96-103). Hamilton had to settle for a poorly paid Professorship of Civil History to which he was appointed in the following year

[6] W.H.S. Monck, *Sir William Hamilton* (London: Sampson Low, Marston, Searle & Rivington, 1881).

and he had to wait for a further 15 years before he secured a post in Philosophy. Finally, in 1836, though with only a small majority over the other candidate, he was elected to the Chair of Logic and Metaphysics at the University of Edinburgh. Veitch gives a fairly thorough account of Hamilton's appointment and the strong testimonials that supported him, but the narrowness of his majority and his failure to secure the earlier appointment of the Moral Philosophy chair indicate that University appointments in Scotland were handled in an altogether shameful manner — Hamilton was without doubt one of the most philosophically erudite and talented men in Scotland at that time but party politics, personal preferences, and unfounded doubts about his religious beliefs were allowed to prevail. In some ways little had changed since the more understandable yet equally non-academic rejection of David Hume by the University of Glasgow in the previous century, nor since the huge debacle that erupted in 1805 when John Leslie was accused of being an infidel and as a result nearly failed to secure the Mathematics Chair at Edinburgh in 1805 because he had endorsed Hume's theory of causality (see Veitch pp. 183-210).[7]

Though there is much to relate about Hamilton's life from this time on, like many scholars and dedicated teachers he led an industrious and comparatively uneventful life, though not unmarred by damaging vicissitudes, such as the deaths of his son in 1836, his brother Thomas in 1842, and a daughter in the winter of 1844-5. From the time of his appointment to the Logic and Metaphysics Chair in 1836 until around the mid 40s, Hamilton was clearly working far too hard and under a great deal of personal strain. Then, in July 1844, aged 56, he suffered a physically debilitating stroke that partially paralysed him for the rest of his life. Two years later he became embroiled in a controversy with Augustus de Morgan concerning their respective quantification systems. Academically, this is undoubtedly the most troublesome and embarrassing moment in Hamilton's career and several have agreed that he have behaved rather foolishly. According to William Kneale, Hamilton was 'a pedantic Scottish baronet' who was 'properly ridiculed by De Morgan'.[8] General opinion about the affair has been that de Morgan came out on top. [9] Possibly the intensely desperate times of the mid 40s in Britain generally, a prevailing sense of crisis in Scotland following the massive upheaval due to the disruption of the Scottish Kirk in 1843, the various bereavements Hamilton had suffered, and his loss of physical vigour may be factors that ought to be considered. Hamilton may have acted in an imperious but also a somewhat desperate manner towards de Morgan but one cannot help but wonder whether his judgment merely faltered amidst a context of great personal and social difficulties.

The details of the de Morgan controversy are tediously complex, the complexity exacerbated by Hamilton's forensic analyses of the events and his sustained sus-

[7] On Leslie see, Ralph Jessop, *Carlyle and Scottish Thought* (Basingstoke: Macmillan, 1997), pp. 36-9.

[8] William Kneale, 'Boole and the Revival of Logic', *Mind*, 57 (Apr. 1948), 149-75 (p. 152n).

[9] For example, see, William Kneale and Martha Kneale, *The Development of Logic* (Oxford: Clarendon, 1962), pp. 353-4.

picions over some years that de Morgan had plagiarised or at least was in some way trying to steal all credit from him. This issue lasted for many years well beyond Hamilton's death and although Hamilton backed down considerably from his first accusations of plagiarism in 1847, neither men really gave up. But, as de Morgan's fame rather prospered as a result of the conflict, Hamilton's prestige diminished. Peter Heath has analysed and recounted this controversy in admirable detail and therefore I shall defer to his general understanding of the whole affair, if not to his tendency to agree that de Morgan's assessment of Hamilton's quantification is right.[10] Some, though perhaps not all, of the relevant letters and other documents were collected together by Hamilton and are published within the recent Thoemmes edition of Hamilton's *Works*.[11] In very general terms, it would seem that both de Morgan and Hamilton did not fully understand each other's respective positions and arguably 'the two systems are not only distinct from, but opposed to each other.'[12] As I shall argue later in agreement with Robert Fogelin, de Morgan was mistaken concerning a fundamental point and Hamilton's system can thus be shown to be consistent and much more robust than many who subscribed to de Morgan's standpoint have assumed it to be. One of the most fruitful and important outcomes of the controversy was the effect it had on George Boole whose interest in logic and subsequent mathematization of logic was in no small part inspired by the rather public disagreement between de Morgan and Hamilton.[13] Heath insightfully remarks of the de Morgan controversy, de Morgan's notes 'contain the following, which might well serve (and was perhaps so intended) as an epigraph for the whole encounter: "Two French squadrons at B — cannonaded each other — why? Because each took the other for *Russians*. 'Then why did they fight?' Said a little girl'.[14]

Although Hamilton was not affected mentally by the stroke he suffered in 1844 and managed to continue in his Chair at Edinburgh for a good many years after, it is fair to say that he was greatly impeded by this disablement. Indeed, it is arguable that his standing as a philosopher may have suffered more from this than from Mill's *Examination*, since one of the greatest problems in studying Hamilton's works has always been simply this: he produced no *magnum opus* either in metaphysics or logic. It is reasonable, if yet somewhat whimsical, to say that, had he not been struck down in 1844 he would have at some stage during his remaining years brought his philosophical endeavours together in at least one definitive and fully mature volume. But this did not happen and after his paralysis he produced relatively little until his death at Edinburgh on 6 May 1856, aged 68.

Although Hamilton did not produce a fully definitive work on metaphysics or on

[10] Peter Heath, 'Introduction' in, Augustus de Morgan, *On the Syllogism and Other Logical Writings* (London: Routledge & Kegan Paul, 1966), vii-xxxi (pp. xii-xx).

[11] *Miscellaneous Writings and Correspondence*, *Works*, vol. 7.

[12] [H.L. Mansel], 'Recent Extensions of Formal Logic', *North British Review*, 15 (May 1851), 90-121 (p. 95n).

[13] See, Luis M. Laita, 'Influences on Boole's Logic: The Controversy between William Hamilton and Augustus De Morgan', *Annals of Science*, 36 (1979), 45-65 (p. 61; p. 65).

[14] Heath, p. xvi.

logic, he nevertheless did write rather extensively, producing material which, in the recently reprinted edition of his works by Thoemmes Press, fills seven volumes. In addition to this, he also produced an extraordinarily, many would say excessively footnoted edition of the *Works of Thomas Reid* in which he first published his fragment on Logic the 'New Analytic of Logical Forms' in 1846.[15] His footnotes in Reid's Works are at times quirky and needlessly pedantic. However, by sharp contrast with this, towards the end of his life he produced a much less cumbersome and indeed rather elegant edition of the *Works* of Dugald Stewart.[16] However, long before these scholarly works Hamilton contributed a series of substantial articles for the *Edinburgh Review* all of which, despite his failing health he managed to publish in a single volume entitled *Discussions on Philosophy and Literature, Education and University Reform* in 1852. The first of these essays, the 'Philosophy of the Unconditioned' (1829) immediately struck readers as largely unintelligible or overly philosophically sophisticated, mystical even in its complexity — but it was this article that properly launched Hamilton's career and first made him famous as the first truly eminent Scottish philosopher since Dugald Stewart who, after many years of ill health, had died in the previous year.[17]

In 'Philosophy of the Unconditioned' Hamilton reviews the work of Victor Cousin and his attempt to establish an eclectic philosophy of the Infinito-Absolute. The main thing to note here is that Hamilton constructed what he termed his Law of the Conditioned, a law prescribing the domain of positive knowledge or the realm of what may be said to be knowable — in some ways it might be regarded as a forerunner of Ayer's logical positivism, though Hamiltonian positivism is a far cry from rejecting either metaphysics or theology. A full account of this article is not appropriate here — suffice to say that Hamilton argues that all knowledge lies in a mean between two extremes of unknowables or inconditionates, which is to say all things that may be described as absolute or as infinite comprise the boundaries of knowledge and are thus strictly incomprehensible to us. The absolute and the infinite are posited as contradictories, neither of which can be positively construed to the mind but one of which, on the basis of the laws of excluded middle and non-contradiction must obtain — though which of the two obtains is incognisable. Hence, Hamilton inaugurates what I call his doctrine of *nescience*, or *learned ignorance as* the 'consummation' of knowledge ('Philosophy of the Unconditioned', p. 38). As all thought, and hence all knowledge, is conditional, of the plural, phenomenal, limited, Hamilton declares: '*To think* is *to condition*; and conditional limitation is the fundamental law of the possibility of thought' ('Philosophy of the Unconditioned', p. 14).

Educated in philosophical discourses of the much more sedate and even-tempered

[15] Thomas Reid, *The Works of Thomas Reid*, preface, notes, and supplementary dissertations by Sir William Hamilton (Edinburgh: MacLachlan, Stewart; London: Longman, Brown, Green and Longmans, 1846).

[16] *The Collected Works of Dugald Stewart*, ed. by Sir William Hamilton, 11 vols (Edinburgh: Thomas Constable, 1854-60).

[17] 'Francis Jeffrey to Macvey Napier, 23 November, 1829', in Macvey Napier [Jnr.], ed., *Selections from the Correspondence of the Late Macvey Napier* (London: Harrison, 1877), p. 68.

stateliness that typified so much earlier 18^{th} century and contemporary philosophical prose from Francis Hutchison, to Hume, Smith, Reid and Dugald Stewart, few at the time may have been able to understand what Hamilton wrote. His highly concentrated style, densely philosophical critical argumentation, frequent references to various German philosophers including Kant, Fichte, and Hegel — though all this must have dazzled and excluded many, few can have failed to catch the sense of excitement that pervaded 'Philosophy of the Unconditioned'. But this must have been more the case with his second major article 'Philosophy of Perception' (1830) which was written in an even more vigorous and racy style than his first. He now turned his attention to scepticism, principally to the scepticism of Hume, though one could be forgiven for missing this since more prominently 'Philosophy of Perception' launches an extraordinarily vitriolic attack on the man who had gained such a high reputation as a philosopher and who for many years continued to be admired by established English philosophers and writers such as John Stuart Mill and Leslie Stephen and whose job, following his death, Hamilton failed to get in 1820 — Thomas Brown. Hamilton savaged Brown for leading philosophy back into the morass of Humean scepticism and for fundamentally failing to grasp the true import of Thomas Reid's critical philosophy of Common Sense, which, according to Hamilton, had given a successful, if nonetheless relatively unsophisticated answer to Hume.

So much needs to be said about this article but I shall confine myself to just a few points: in 'Philosophy of Perception' Hamilton developed his own version of Common Sense philosophy in the form of a doctrine of perception which held that in the *act* of perception the self and the not-self were instantaneously revealed in one indivisible moment of cognition — he called this theory of perception *natural dualism* or *natural realism* and maintained that, contradistinguished from all other *representationist* theories of perception that in one way or another tended towards scepticism, *natural dualism* was a theory of immediate or *presentative* perception. Leaving aside all consideration of just how *natural dualism* was proffered by Hamilton as the best and most successful counter-argument to what some may think of as a straw man scepticism of purely theoretic indeterminacy — the absolute scepticism of Hume — I want to draw attention to Hamilton's non-Kantian notion of the relativity of knowledge by means of just one quotation:[18]

> Relatives are known only together: the science of opposites is one. Subject and object, mind and matter, are known only in correlation and contrast [...]. Every conception of self, necessarily involves a conception of not-self: every perception of what is different from me,

[18]Hamilton has often been mistakenly thought of as borrowing heavily from Kant. However, in several places he is critical of Kant and his relativity of knowledge needs to be distinguished from Kant's. On this see, Manfred Kuehn, 'Hamilton's Reading of Kant: A Chapter in the Early Scottish Reception of Kant's Thought', in George MacDonald Ross and Tony McWalter, *Kant and His Influence* (Bristol: Thoemmes Antiquarian Books, 1990), 315–347 (pp. 333–45). It should perhaps also be noted that Hamilton's notion of the relativity of knowledge is more subtle and more complex than Mill represents it as being. On this see, John Veitch, *Hamilton* (Edinburgh And London: Blackwood, 1882), pp. 201–222.

> implies a recognition of the percipient subject in contradistinction from the object perceived. [...]. In Perception, as in the other faculties, the same indivisible consciousness is conversant about both terms of the relation of knowledge. ('Philosophy of Perception', pp. 50–51).

Although the details of Hamilton's theory of the relativity of knowledge deserve separate examination, in the above we can see indications of a notion that is of critical importance to understanding Hamilton's logic, namely, the notion that subject and object exist only in correlation with one another, such that opposites, or the distinguishable terms of subject and predicate in a given proposition, may be thought of as being held together in a relationship of equation or non-equation — in this lies the germ of Hamilton's emphasis on the relativity of concepts and his quantification of the predicate.

The influence that Hamilton's two articles on the unconditioned and perception had on Victorian thought is a subject that deserves separate study. Noah Porter, professor of moral philosophy and metaphysics (1846), and later president of Yale, wrote effusively and at some length in Veitch's biography of the considerable extent of Hamilton's influence on American students (Veitch, pp. 421-8). However, although Porter thought that Hamilton had been a positive religious force in American thought, there is another side to this story. Emphasising the vastness of our ignorance in 'Philosophy of the Unconditioned', while regarding this as a prompt for our wonderment and faith, the most significant direct and lasting effect of the 'Philosophy of the Unconditioned' is perhaps best assessed in terms of the influence it had upon Henry L. Mansel, a prominent follower of Hamilton who wrote several articles defending Hamilton's logic, developed his own version it, and, more popularly, in his Bampton Lectures, gave rise to a doctrine of Christian Agnosticism.[19] But as the agnostic movement developed during the 19^{th} century, Hamilton's 'Philosophy of the Unconditioned' in comparison with its transmutation into Mansel's Christian Agnosticism can also be seen as having a profound effect on anti-Christian agnostics such as Thomas Huxley (Darwin's bulldog). As Sheridan Gilley and Ann Loades nicely put it: 'Huxley saw in Mansel the suicidally honest theologian, sitting on an inn sign and sawing it off.'[20] Hamilton's importance to the growth of agnosticism, although not widely known, has certainly been established not only by more recent scholarship but also in some of the responses to Hamilton in his own day.[21] Hence, though firstly, inspiring a new religious piety and apparent salvation from scepticism, but secondly, becoming infused into succeeding waves of religious doubt and the growth of agnostic principles

[19]For example, see Henry Longueville Mansel, *Prolegomena Logica: An Enquiry into the Psychological Character of Logical Processes* (Oxford, 1851); 'The Philosophy of the Conditioned: Sir William Hamilton and John Stuart Mill', *Contemporary Review*, 1 (1866), 31-49; 185-219; [Bampton Lectures], *The Limits of Religious Thought* (London : John Murray, 1858).

[20]Sheridan Gilley and Ann Loades, 'Thomas Henry Huxley: The War between Science and Religion', *The Journal of Religion*, 61 (1981), 285-308 (p. 297).

[21]Bernard Lightman, *The Origins of Agnosticism: Victorian Unbelief and the Limits of Knowledge* (Baltimore and London: John Hopkins University Press, 1987), p. 16; Robert Flint, *Agnosticism* (Edinburgh and London: Wm Blackwood, 1903).

more powerfully damaging to orthodox belief than even Hume's or Voltaire's more full-frontal atheistic attacks on religious belief, the full significance that Hamilton's 'Philosophy of the Unconditioned' would come to have on Victorian society, philosophy, literature, and culture, though as yet still largely unrecognised, was nothing short of immense.[22]

But Hamilton's influence can also be seen through certain of his friends or acquaintances and, of course, his students. One of his former students who was profoundly influenced by him and became the Professor of Moral Philosophy at the University of St. Andrews, James Frederick Ferrier, a philosopher whose personal and philosophical connections with Hamilton run deep and whose work is now just as undeservedly but even less well known than that of Hamilton. In literature Hamilton was a major influence on E. S. Dallas whose two volume work of literary theory, *The Gay Science,* positively teems with Hamiltonian philosophy — but Dallas has also now shrunk into the shadows and is barely known.[23] George Davie claims that Hamilton inspired a number of brilliant scholars including Ferrier and the physicist so greatly admired by Einstein, James Clerk Maxwell.[24] The Maxwell connection is particularly interesting since a recent work of literary criticism has opened up a whole new field of study — the Victorian relativity movement. Christopher Herbert argues in *Victorian Relativity* that the cultural and philosophical antecedents of Einstein's special theory of relativity are to be found in a succession of Victorian philosophers and thinkers the writings of whom have often been largely neglected for over a century. Interestingly, just as scholars who have written on agnosticism have traced its origins in the 19^{th} century back to Hamilton, Herbert also finds in Hamilton's theory of the relativity of knowledge a significant starting point for his study, seeing Hamilton at the beginning of a major Victorian re-invention of the fundamentally Protagorean relativist rejection of absolutism.[25] First suggested in 'Philosophy of the Unconditioned', developed in 'Philosophy of Perception', and, as we shall see, a key component of Hamilton's logic, Hamiltonian relativism intriguingly re-positions Hamilton as one of the 19^{th}-century's key *avante garde* thinkers.

It is too easy to see Hamilton in caricature and as of marginal importance, since this is where the later Victorians placed him and where scholars of the 20^{th} century left him. However, for all that he was hampered by a corrupt system of university patronage and indeed may not have been the most self-promoting of figures, within the context of British philosophy and literature of the 1830s, Hamilton was a

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[22]See, Ralph Jessop, 'Carlyle's Agnosticism: An Altar to the Unknown and Unknowable God', *Literature and Belief*, 25 (1&2), 381-433 (pp. 395-404).

[23]E.S. Dallas, *The Gay Science* (London: Chapman and Hall, 1866).

[24]See, George Elder Davie, *The Democratic Intellect: Scotland and Her Universities in the Nineteenth Century*, Edinburgh University Publications, History Philosophy & Economics: 12 (Edinburgh: Edinburgh University Press, 1961; repr. 1982), p. 261; John Hendry, *James Clerk Maxwell and the Theory of the Electromagnetic Field* (Bristol and Boston: Adam Hilger, 1986), p. 112.

[25]Christopher Herbert, *Victorian Relativity: Radical thought and Scientific Discovery* (Chicago and London: The University of Chicago Press, 2001), pp. 35-7.

leading and highly dynamic writer who played an important part in re-invigorating Reidian philosophy while also turning British attention arguably in a different but by no means unrelated direction towards German philosophy. It must be noted that in early 19^{th}-century Britain very few people could read German and Hamilton seems to have been one of almost a handful of intellectuals in Britain during the 1820s and 30s who had any direct knowledge of the works of Kant and other German philosophers such as Hegel. Hamilton was one of a tiny number of Germanizers who promoted the study of German philosophy and, given the rise of neo-Kantian and neo-Hegelian philosophy in Britain amongst philosophers who owed some debt to Hamilton, in this alone his influence was probably much more profound than has generally been appreciated. Another major Germanizer was Hamilton's one-time friend, the literary man of letters, Thomas Carlyle, who knew Hamilton from at least the late 1820s. Carlyle, though a literary giant of the Victorian period and even well beyond of proportions it would be difficult to exaggerate, is yet another figure who is now increasingly little known and poorly understood. Not much is positively known about their friendship but at the time of its publication in 1829 Carlyle read Hamilton's 'Philosophy of the Unconditioned' with admiration in later life he wrote a highly reverent Reminiscence of Hamilton which is published in Veitch's biography, followed by a fairly intimate letter from Carlyle written in 1834 in which he says, 'Think kindly of me; there are few in Scotland I wish it more from' (Veitch, pp. 121-127). There are undoubtedly many interesting parallels between Carlyle and Hamilton, some of which were detected in Carlyle's own lifetime by his close friend David Masson.[26] I and some others have discussed some of the interconnections between Carlyle and Hamilton elsewhere.[27]

Hamilton produced a good number of other articles for the *Edinburgh Review*, including the following selection: 'On the Revolutions of Medicine in Reference to Cullen' (1832); 'On the Study of Mathematics as an Exercise of the Mind' (1836) — controversially, though rather in keeping with his Scottish predecessors attitudes about mathematics, Hamilton did not think that mathematics was a good exercise of the mind as part of a liberal education and instead advocated philosophy while in several ways indicting the emphasis on mathematics at Cambridge; 'On the Patronage and Superintendence of Universities' (1834);[28] 'On the State of the English Universities with More Especial Reference to Oxford' (1831). Several of his articles were pointedly critical of Oxford and Cambridge and it must be said that Hamilton undoubtedly set himself up for retributive attacks from some scholars in response to his various denunciations of the established ancient universities of England. More directly germane to our subject matter and also involving his

[26]David Masson, *Recent British Philosophy: A Review with Criticisms including some Comments on Mr Mill's Answer to Sir William Hamilton*, 3rd edn (London: Macmillan, 1877),p. 69.

[27]For example see, Alex Benchimol, 'William Hamilton' in, *The Carlyle Encyclopedia*, edited by Mark Cumming (Cranbury, NJ:Farleigh Dickinson University Press, 2004), 207-9; Jessop, *Carlyle and Scottish Thought*, pp. 28-31.

[28]For example compare, Richard Olson, 'Scottish Philosophy and Mathematics 1750-1830', *Journal of the History of Ideas*, 32 (1971), 29-44 (pp. 41-4).

first of many salvos against English logicians, is his 'Logic: The Recent English Treatises on that Science' (1833).

In this article he gave yet another virtuoso performance that at once established his reputation as one of the foremost logicians of his day. But, as with some of his other slashing remarks on contemporary scholars and, as he saw it, the present philistinism of learning in Britain, in 'Logic' he was unsparing in his treatment of several Oxonian logicians. Supposedly reviewing some eight recent publications on logic, he focuses almost exclusively on Whately's *Elements of Logic*, making only brief reference to George Bentham's *Outline of a New System of Logic* (1827). As some commentators have noted, the cursory treatment of Bentham is odd since a system for quantifying the predicate is given in the work of this nephew of the more famous Jeremy Bentham.[29] Hamilton's paternity of this doctrine has often been called in doubt but since most of the specific characteristics of his quantification are largely, if not exclusively, peculiar to him, and since his own theory is so thoroughly grounded in a painstaking explication of the grounds for quantifying the predicate, I shall not engage with the tortuous historical complexities and shall instead attempt in the following sections to outline how Hamilton arrives at his quantification system and what that system itself is. Furthermore, it is likely that Hamilton was so dismissive of Bentham's *Outline* that he simply did not read all of the text and cast it aside in order that he might focus on the main logician, his principal target, Richard Whately.

Though Hamilton intimates a certain degree of respect for the natural abilities of the authors under review, he denounces their lack of genuine originality — the source of Hamilton's ire is clear and is a much repeated complaint elsewhere in his work, namely, the inadequacy of the authors' learning:

> None of them possess — not to say a superfluous erudition on their subject — even the necessary complement of information. Not one seems to have studied the logical treatises of Aristotle; all are ignorant of the Greek Commentators on the Organon, of the Scholastic, Ramist, Cartesian, Wolfian, and Kantian dialectic. ('Logic', p. 129).

And so he continues to cut and hew his way through the inadequate learning of contemporary Oxonians. But, as Whately's *Elements* stood pre-eminent, it is this work that Hamilton takes to task and he proceeds to make point after forensic point against Whately, all the time demonstrating his own vastly superior knowledge of swathes of the literature of logic and outlining several features of his own system of logic.

As part of his sustained attempt to claim priority in discovering his system of quantification, in a much later footnote in *Discussions* Hamilton asserts that on the basis of a rather vague authority — 'the tenor of the text'– the 'Logic' article shows that he 'had become aware of the error in the doctrine of Aristotle and the logicians, which maintains that *the predicate in affirmative propositions could only be formally quantified as particular'* ('Logic', p. 162n). More strongly, and with

[29] Heath, p. xvi.

much better justification, he writes in an appendix in *Discussions*, 'Touching the principle of an explicitly *Quantified Predicate*, I had by 1833 become convinced of the necessity to extend and correct the logical doctrine upon this point. In the article on Logic [...] the theory of Induction there maintained proceeds on a thoroughgoing quantification of the predicate, in affirmative propositions' (Discussions, p. 650). Certainly the 1833 'Logic' article does suggest this but it does not display the full quantification system (see 'Logic', p. 163). As I shall indicate later, although the quantification system was not fully articulated until the publication of his 'New Analytic' in 1846, a good deal of Hamilton's treatment of logic in his lectures does proceed, painstakingly towards the quantification. That he was teaching this system during the late 1830s, several years ahead of de Morgan's quite different quantification, seems to have been generally agreed and is patently evident in the wonderfully clear account given by one of his students, Thomas Spencer Baynes, whose winning essay written for a competition set by Hamilton in 1846 was later published as *An Essay on the New Analytic of Logical Forms* in 1850.[30]

In 'Logic' Hamilton very much lays out his stall. He begins with a brief definition of logic that he later elaborates:

> Nothing, we think, affords a more decisive proof of the oblique and partial spirit in which philosophy has been cultivated in Britain, for the last century and a half, than the combined perversion and neglect, which Logic — the science of the formal laws of thought — has experienced during that period. ('Logic', p. 119).

Just a few years after writing this, Hamilton commenced his Lectures on Logic as the Professor of Logic and Metaphysics at Edinburgh. These were posthumously published in four volumes, two on Metaphysics in 1859 and two on Logic in 1860. In the *Lectures on Logic*, Hamilton spends a lot more time carefully explaining his above definition of logic as 'the science of the formal laws of thought'. Though his definition of logic in the lectures is interesting and deserves discussion, I shall merely summarise some main points here.

Hamilton basically holds that Logic proper or Formal Logic is Abstract logic, dealing only with necessary inference and devoid of all adventitious or extra-logical matter or contingent considerations. However, the nature of Logic as a pure science had been greatly misunderstood in Britain: 'Bacon wholly misconceived its character in certain respects; but his errors are insignificant, when compared with the total misapprehension of its nature by Locke' (LL.I.29).[31] The British had mistakenly departed from a certain general agreement on the formal nature of

[30] Thomas Spencer Baynes, *An Essay on the New Analytic of Logical Forms, Being that which Gained the Prize Proposed by Sir William Hamilton, in the Year 1846, for the Best Exposition of the New Doctrine Propounded in his Lectures*; with an Historical Appendix (Edinburgh: Sutherland and Knox; London: Simpkin, Marshall, and Co., 1850).

[31] All references in this form are to the *Lectures on Logic*, ed. by H.L. Mansel and John Veitch (Edinburgh and London: Blackwood, 1860), vols I or II; vols 5 or 6 in the Thoemmes reprint edition of Hamilton's *Works*.

Logic among ancient and more recent German logicians. They had ignorantly or perversely deviated from centuries of collective wisdom and while German logicians from the time of Leibniz had probably done more than most to further the science, Hamilton seems to have regarded the more recent and more serious perversions and confusions in the work of recent English authors, in particular Whately, as a sort of further entrenchment of the very kind of misconstrual most likely to hinder the science (LL.I.40). It was also reprehensible that such recent British scholars should be ignorant of the German logicians: 'Great Britain is, I believe, the only country of Europe in which books are written by respectable authors upon sciences, of the progress of which, for above a century, they have never taken the trouble to inform themselves' (LL.I.33).

Distinguishing between Special or Concrete Logic (logic in its particular applications or instantiations in the several arts and sciences) and Pure or General or Abstract Logic, for Hamilton, Pure Logic has to be contradistinguished from any particular subject or discipline, the object-matter of which must necessarily be contingent due to the nature of the topics it addresses or to which Logic is being in some sense applied or put into practice (LL.I.56). However, while Special Logic is dismissed this is not to say that practical matters to do with Logic must be altogether excluded from consideration (see LL.I.61-2). He coins the term 'Modified Logic' to describe what he argues had been improperly called Applied Logic by Kant and some other German philosophers, defining Modified Logic as 'a science, which considers thought not merely as determined by its necessary and universal laws, but as contingently affected by the empirical conditions under which thought is actually exerted' (LL.I.60). Although Hamilton's treatment of Modified Logic in the second volume of his *Lectures on Logic* as the correlative second main branch of Abstract Logic, is certainly interesting, I have chosen not to discuss it but instead focus on what for Hamilton was clearly of much greater immediate importance, namely, Formal or Pure Logic. He insists that Pure Logic really comprises the whole of Abstract Logic — Modified Logic is 'a mere mixture of Logic and Psychology'; 'There is in truth only one Logic, that is, Pure or Abstract Logic', 'Modified Logic being only a scientific accident, ambiguously belonging either to Logic or to Psychology' (LL.I.63).

With such points in mind, he provisionally defines Logic as 'the Science of the Laws of Thought as Thought' (LL.I.4). Extruding the contingent, inasmuch as this is possible, Hamilton claims that Logic is only concerned with those phenomena of formal (or subjective) thought that are necessary or 'such as cannot but appear' as opposed to the contingent phenomena of thought, or 'such as may or may not appear' (LL.I.24). Hence, through his final introduction of the notion of necessary laws being the sole province of Logic, Hamilton asserts that 'Logic, therefore, is at last fully and finally defined as the science of the necessary forms of thought' (LL.I.24). Though this is Hamilton's final definition of Logic, he takes care to explain the sense or 'quality' of 'necessary' by extensively quoting Wilhelm Esser's *System der Logik*.[32] Hence, to summarize Esser's arguments as translated

[32]Hamilton translates for his students many passages from Esser. His inclusion of these nu-

by Hamilton: the necessity of a form of thought is contradistinguished from contingency by being *subjective*, which is to say that a necessary form of thought 'must be determined or necessitated by the nature of the thinking subject itself' as opposed to being determined objectively; since we are incapable of conceiving the possibility of its non-existence, such incapacity warrants the notion that the form of thought in question is *original* to, or a constitutive feature of, the thinking subject or human mind, and thus the subjective necessity of a form of thought 'must be original and not acquired'; as necessary, subjective, and original in the sense explained, the form of thought that comprises the object-matter of Logic cannot necessitate on some occasions and not on others, and thus it must also be *universal*; and finally, further enriching the sense or quality of 'necessity', according to Esser, 'if a form of thought be necessary and universal, it must be a law; for a law is that which applies to all cases without exception, and from which a deviation is ever, and everywhere, impossible, or, at least, unallowed' (LL.I.24-5). With the sense of 'necessity' within the phrase 'necessary forms of thought' hereby explained in terms of something subjectively determined by the human mind as subject and not objectively determined or extraneous to the mind, original and not acquired, universal, and a law, Hamilton gives what he regards as his 'most explicit enunciation of the object-matter of Logic' as: 'Logic is the science of the Laws of Thought as Thought, or the science of the Formal Laws of Thought, or the science of the Laws of the Form of thought; for all these are merely various expressions of the same thing.' (LL.I.25-6).

Many more recent formal logicians would generally agree that formal Logic, as contradistinguished from informal Logic and the subject matter or topics of Rhetoric, is entirely or principally concerned with deductive arguments and thus with valid inference and the necessary laws that pertain to or determine validity. But we need to take notice of the state of logic in 1830s Britain as Hamilton understood it: his contemporaries appeared to him to be woefully misguided, unscholarly in their reading of the Aristotlean tradition, and blissfully ignorant of the more recent German tradition in logic since the time of Leibniz. Hamilton's peers were, as far as he could tell, largely unaware of how the recent German writers including and after Kant had far excelled the Oxford logicians in their knowledge and understanding of the subject. Furthermore, it seems clear that part of Hamilton's crusade was against the prevailing tendency of philosophy in Britain towards the increasingly prevalent mediocrity and barbarity of his times that almost inevitably was following in the wake of and was implicitly collusive with an era of rapidly advancing materialism. Pervaded from its outset by reason and scepticism, during the early decades of the 19^{th} century the Enlightenment was evolving into new forms of a more socially pervasive utilitarian rationality and sceptical subversions of reason and faith. Acutely conscious of such trends in philosophical discourse Hamilton took considerable pains to establish his standpoint as one much more closely in line with the German approach that regarded Logic as a pure science.

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merous quotations implies the commencement of a substantial realignment in British philosophy with German logicians.

2 DOCTRINE OF CONCEPTS

> Logic [...] is exclusively conversant about thought,–about thought considered strictly as the operation of Comparison or the faculty of Relations; and thought, in this restricted signification, is the cognition of any mental object by another in which it is considered as included,–in other words, thought is the knowledge of things under conceptions. (LL.I.40).

Shortly after the above quotation Hamilton distinguishes between the act of conceiving (conception) and the thing conceived (concept) and briefly suggests a similar distinction with regard to perception, somewhat tentatively coining the term ‘percept’ some 40 years before its first recorded use in the *Oxford English Dictionary.* He takes particular care to point out that Logic is concerned with ‘thought considered as a product; that is, as a concept, a judgment, a reasoning’ (LL.I.74). The operation of Comparison, or as he later calls it, the Faculty of Comparison, produces Concepts, Judgments, and Reasonings. However, Hamilton argues that Concepts and Reasonings are modifications of Judgments, ‘for the act of judging, that is, the act of affirming or denying one thing of another in thought, is that in which the Understanding or Faculty of Comparison is essentially expressed’ (LL.I.117). This means that for Hamilton, ‘A concept is a judgment’ and as such it collects together or is ‘the result of a foregone judgment, or series of judgments, fixed and recorded in a word,–a sign’ which may be supplemented or extended by additional attributes, themselves judgments (LL.I.117). Thus, as a concept in a sense fixes a judgment or series of judgments, collecting together various attributes within a single term, so also can it be analysed into these components or amplified by the annexation of further attributes.

An important point to note here that will later be of fundamental significance to Hamilton’s quantification of the predicate, is that for Hamilton an oft-ignored and oft-violated postulate or principle of Logic is that the import of the terms used in a judgment or reasoning should be fully understood or made explicit, which is to say that ‘Logic postulates to be allowed to state explicitly in language all that is implicitly contained in thought’, that Logic demands licence to make explicit the full import of any particular concept or term, much as it attempts to do in making overt all of the steps and relations involved in a given process of reasoning or argument. Though this fundamental postulate is simple in its statement, Hamilton clearly regarded its significance as central to his project to evolve or develop Logic as a Pure science: ‘This postulate [...], though a fundamental condition of Logic, has not been consistently acted on by logicians in their development of the science; and from this omission have arisen much confusion and deficiency and error in our present system of Logic’ (LL.I.114). Interestingly, in one place he makes the postulate more precise by replacing the term ‘implicit’ with ‘efficient’ (NA.LL.II.270).[33]

[33] All references in this form are as above to the *Lectures on Logic* with ‘NA’ prefixed to indicate the ‘New Analytic of Logical Forms’ as given in vol.II.249-317.

The analysis of concepts into their often implicitly-held component attributes as also their amplification by means of adding new attributes, effectually constitutes an adherence to this postulate. Indeed, arguably, Hamilton's emphasis on the importance of being rigorously explicit elevates this postulate to the status of what one might call a meta-theoretical principle of explicitness.

With reference to the Latin '*concipere'* he explains that traditionally conception indicated 'the process of embracing or comprehending the many into the one', 'the act of comprehending or grasping up into unity the various qualities by which an object is characterised' (LL.I.120). Within our consciousness this process is typified by two cognitions, one immediate and 'only of the individual or singular', the other 'a knowledge of the common, general, or universal' (LL.I.122). Thus 'a Concept is the cognition or idea of the general character or characters, point or points, in which a plurality of objects coincide' (LL.I.122, and see, 122-3). Mirroring his notion of the formation of language, which he outlines in his *Lectures on Metaphysics*,[34] as a synthesising process that moves from chaos to the construction of general and universal terms which enable a complex and iterative relationship with the individuals or particulars into which such concepts may be analysed — a process of composition and decomposition that he occasionally describes as organic — Hamilton regards the formation of concepts as a complex process involving human agency or the *exertion* of an 'act of Comparison' upon an otherwise chaotic or confused array of presentations. Reminiscent of his highly significant relativist maxim, '*To think* is *to condition'*, Hamilton is thus acutely aware of the role of human agency or volition in the process or mental *activity* involved in the formation of concepts as unities consisting of various points drawn together by an act of comparison and the implicit intentionality within this action 'of discovering their similarities and differences' (LL.I.123). This awareness of the part played by human agency or volition, and the artificiality and partiality of concepts, becomes clearer when he invokes 'the act called *Attention'*, by means of which certain objects and qualities forming any given concept become strongly highlighted (LL.I.123). As Hamilton explains the mental operations of attention and abstraction, these two processes involved in the formation of concepts 'are, as it were, the positive and negative poles of the same act', by means of which as some objects and qualities become highlighted, others 'are thrown into obscurity' (LL.I.124, 123). He is claiming that the thinking subject's point of view or perspective, or conditioning role is crucial to the formation of concepts, and as we shall see later, point of view also plays an important role with regard to Hamilton's treatment of propositions.

What Hamilton is describing here, and in what follows this observation about the act of abstraction and attention, is the *relativity of concepts*, and thereby he begins to point up certain implicit features of concepts that emphasise their fluidity or their adaptability, as well as their inherent inadequacy or insufficiently explicit nature. For Hamilton the act of comparison combined with that of abstraction and attention *reduces* in consciousness the multiple (or really differing objects of

[34] *Lectures on Metaphysics*, II.327-332.

consciousness) into the unity that is a concept (see LL.I.124). By throwing out of view the non-resembling marks or characters or points that in reality individuate one individual from another, and by attending to the resembling points alone, we treat these *resembling* points as though they were identical and thereby synthesise them into a unity, a concept through which we think of the individuals or to which we relate these individuals. Implicitly, in opposition to mechanistically grounded (or mechanically modelled) theories of the mind, for Hamilton the formation of concepts in the mental act of conception is a single or indivisible, quasi-organic process of thought that may be analysed into components for the purpose of speaking about and better comprehending concepts (see LL.I.133). No doubt integral to or consistent with Hamilton's *natural dualism*, he regards our knowledge of what is presented and represented in consciousness — the phenomena of consciousness — as 'a direct, immediate, irrespective, determinate, individual, and adequate cognition' (LL.I.131). Such cognitions of the phenomena of consciousness, though virtually self-sufficient as cognitions, do not, for Hamilton, constitute *thought,* or so it would seem, since repeatedly he construes thought in terms of a relational or relative process in which the thinking subject is more distinctively active or operates as an agent conditioning the objects of thought. Hence, by contrast with the mere phenomena of consciousness as self-sufficient cognitions, by means of which the human mind may be thought of as, if not merely sentient, then active but unthinkingly or non-rationally so, 'A concept, on the contrary, is an indirect, mediate, relative, indeterminate, and partial cognition of any one of a number of objects, but not an actual representation either of them all, or of the whole attributes of any one object' (LL.I.131). This means that concepts are non-absolute or 'not capable of representation as absolute attributes' (LL.I.137). Earlier, in attempting to explain the inadequacy of concepts and their relativity, Hamilton pointed up the partiality of concepts, their dependence upon selective 'representation of a part only of the various attributes or characters of which an individual object is the sum' — for example, as we think *Socrates* through a particularly small range of attributes, say, *man, biped, animal*, our representation of him will be proportionately less adequate than when we think him through a greater range of attributes. Hence, a concept, construed as a collection of attributes to which the concept refers, are always one-sided or partial and to a greater or lesser extent inadequate by contrast with the plenitude of attributes that constitute any given individual.

However, this partiality or incompleteness of concepts, arising due to the combined act of abstraction and attention that conceiving necessarily involves, while it suggests the relativism of the thinking subject's conditioning influence on the objects of thought, does not seem to be the relativity that Hamilton particularly wishes to emphasise. Instead, the relativity of concepts to which he draws our attention inheres in the relational nature that a concept does not overtly make explicit but rather, in a sense, disguises by means of the fiction and even illusion of unitariness we tend to impose upon the collection of more or less resembling but nonetheless differing individual attributes that comprise any given concept.

According to Hamilton, 'A concept or notion, as the result of a comparison, necessarily expresses a relation. It is, therefore, not cognisable in itself, that is, it affords no absolute or irrespective object of knowledge, but can only be realised in consciousness by applying it, as a term of relation, to one or more of the objects, which agree in the point or points of resemblance which it expresses' (LL.I.128). Although Hamilton's account of the relativity of concepts is far from being unproblematic, he asserts that the passage just quoted resolves 'the whole mystery of Generalisation and General Terms' and thus that his notion (that a concept expresses a relation, and is not therefore an 'absolute or irrespective object of knowledge') resolves the disputes between Conceptualists and Nominalists. That concepts constitute absolutes or objects of knowledge is an 'illusion' that fundamentally arises due to our conversion of similarity into identity, and thereby, according to Hamilton, 'the real plurality of resembling qualities in nature is factitiously reduced to a unity in thought; and this unity obtains a name in which its relativity, not being expressed, is still further removed from observation' (LL.I.128).

This may sound as though Hamilton is saying that concepts are fundamentally fallacious; that they are fallaciously unities, actually pluralities. However, it needs to be noted that while he may be saying that the view that concepts are absolute objects of knowledge, or that they express unities, arises due to our conversion of similarity into identity, the illusion of concepts as objective absolutes thus generated is not due to some inherent fallaciousness within our mental faculties. Rather, this illusion is due to an insufficiently rigorous attention to the internal components, structure, and relational processes involved in any given instance of conceiving as deeply integrant to the characteristic features and nature of the product of such thought, namely, a concept. Furthermore, as he later argues, concepts, formed by comparison and hence relative or expressive of a relation, 'can only be thought of in relation to some one of the individual objects they classify', but as such 'they fall back into mere special determinations of the individual object in which they are represented. Thus it is, that the generality or universality of concepts is potential, not actual' (LL.I.134). Though Hamilton does not go on to explain that this implies that concepts have to be regarded always as in some sense provisional and thereby capable of modification, supplementation, or some other adjustment, and available for examination from differing points of view, this relativist attack on the implicit absolutism of concepts construed as non-relativistic objects of knowledge, is further pursued by Hamilton's excursus into the role of language in a way that further brings to the fore the sense in which concepts are provisional artefacts, the establishment and permanence of which is dependent on the human subject — that concepts are necessarily subjective.

He apologizes for his digression into the extra-logical domain of metaphysics, a metaphysics which he admittedly here only sketches (LL.I.131). However, it would seem that, for Hamilton, in order to analyse the products of thought (concepts) as principal components within demonstrative arguments, we need to understand the relativity of concepts by making this explicit. To do so may involve some digression into metaphysical discourse, and even the enunciation of a metaphysical standpoint consistent with or flowing out of Hamilton's *natural dualist* position, a position more or less contentious for some readers but of substantive interest in relation to the Scottish Common Sense tradition in philosophy out of which Hamilton developed the term 'natural dualism'. Clearly concerned that his constriction of Logic to the necessary laws of thought is being violated by metaphysical considerations, he also apologizes for digressing onto a similarly extra-logical consideration of language. However, just as metaphysical discourse is at the interface between Logic and the nature of the object-matter of Logic, an important part of Hamilton's description of the generation of concepts as illusorily absolute unities or objects of knowledge, though actually relative, is his eloquent and fairly extensive treatment of language. He attempts to explain how a name or term or linguistic sign somehow fixes a concept within our consciousness as though it expresses an absolute object of knowledge and not a process or relational bundle of attributes brought together by some contributing act or relationship of agency by means of which the thinking subject participates with or conditions the phenomena of consciousness to make its otherwise chaotic or confused and unfixed plenitude meaningful.

His digression onto the relationship between language and thought further deepens the relativism of his whole approach as he brings to the fore the reciprocal nature of language and thought: 'Considered in general, thought and language are reciprocally dependent; each bears all the imperfections and perfections of the other; but without language there could be no knowledge realised of the essential properties of things, and of the connection of their accidental states' (LL.I.137). He prioritises thought over speech or language but does so in such a way as to suggest that this priority really pertains to the origination of the phenomenon of language rather than being a necessary condition of all speech (see LL.I.138). Be that as it may, Hamilton's more important concern has to do with describing the reciprocal relationship between thought and language, the process of conception and the claim that, were it not for our ability to fix 'and ratify in a verbal sign' all of the constituents of a concept, it would otherwise 'fall back into the confusion and infinitude from which it has been called out' (LL.I.137). He illustrates this for his students with some nice metaphors, such as the following:

> You have all heard of the process of tunnelling, of tunnelling through a sand bank. In this operation it is impossible to succeed, unless every foot, nay almost every inch in our progress, be secured by an arch of masonry, before we attempt the excavation of another. Now, language is to the mind precisely what the arch is to the tunnel. The power of thinking and the power of excavation are not dependent on the word in the one case, on the mason-work in the other; but without these subsidiaries, neither process could be carried on beyond its rudimentary commencement (LL.I.139).

Hamilton elaborates upon the metaphor of tunnelling through a sandbank, but while his simple claim may be altogether unexceptionable (that without language thought would at best remain in the most elementary and fragile state of almost total impermanence), it is interesting to note that what his tunnelling metaphor most strongly indicates is the relationship that the thinking subject is caught within, a relationship between the ever-shifting sand of a multitudinous plenum and the crafting of signs that, unlike the relative nature of concepts with regard to their constituent parts, may seem to render in some sense or to some degree permanent for consciousness the collected attributes of any given concept, but which still leaves such concepts relative and *factitiously* unitary. But as the thinking subject's struggle to secure or make permanent through language the confusion and infinitude of the phenomena of consciousness is here highlighted, Hamilton's awareness of Logic's contest with a disintegrative atomism or absolute relativism, at once suggests his consciousness of the volitional nature of reason and the abyss of a more thoroughgoing relativism that threatens to undermine the warrantability of his project entirely. But though this exaggerates the danger of Hamilton's relativism with regard to his own project, it draws to our attention the acute sense that Hamilton has of logic's relationship through language to the surrounding and teeming chaos of the universe within which our intellects struggle to achieve order — logic, like language, is for Hamilton a process of inching forward, of a tunnelling through sand only made possible by constructing arches to hold back and make orderly the chaos that perpetually threatens to engulf us. Any elementary concepts we might have the capacity to form without the assistance of language, which Hamilton admits may be a possibility, would be 'but sparks which would twinkle only to expire', implying that without language, and we must add, without logic, whatever thought might be possible is barely worth considering (LL.I.139). It is therefore of the greatest moment for Hamilton that the logic we construct should be robust and built upon proper foundations established through the most exacting scrutiny of the work of others — this is not just a task for those who *profess* to be logicians; it is also a task involving sound architectural skills, considerable scholarship, the craftsmanship of the master builder, and a critical engagement with and demolition of the crumbling and imperfect buildings of the past.

But it all must be carried out with an acute consciousness of the materials one has to work with and the instability of the substance that only logic can hold in place. Hamilton has, more or less wittingly, but nonetheless in a most profound way, highlighted the imperfect, inchoate, factitious, anthropocentric, volitional, indeterminate, and inherently relative nature of concepts. However, in doing so he has overburdened the tenability of the fixative term or word or sign to such an extent that the permanence or ratification he seems to claim we are capable of establishing or ascribing to concepts — their 'acquired permanence' as he later describes it (LL.I.225) — begins to look questionable. That this should be the case — that Hamilton's conceptualism is in fact a dismantling of Logic's object-matter into a purely formal kind of object that describes a process only artificially or analogically rendered as a material object consisting of identifiable components — should perhaps not greatly surprise us, given that this idealism or immaterialism is so clearly congruent with his doctrine of *nescience*, and several other aspects of his philosophical position related to this doctrine.

But this is not to say that Hamilton actually undercuts the whole point of examining concepts and attempting to evolve a more complete science of the necessary laws of thought. Rather, it is to his credit that, as he persists with his enquiry, though the relativism and in general the perilously fragile condition of concepts (as factitiously wholes, and indeed thereby factitiously permanent fixtures in consciousness), is brought to his students' attention, he effectually engages in a quasi-Kantian critique of reason that at once points up the queerness, or idealism, of Logic's object-matter, while yet exploring what can be made determinate and placed under the regulation of irrefragable laws with regard to thought, an entity that he brings before us with great authenticity not as something already fixed or identical or closely analogous to material entities and physical nature (or to conceptions of physical nature that construe the material mechanistically), but rather as an object-matter evincing an indeterminacy with which it behoves exacting logical scrutiny to engage.

Having pointed up the indeterminate and relative nature of concepts he goes on to discuss the three main relations of concepts, namely, the relation they hold to their objects, to their subject, and to each other. The first relation, to their objects, is of course encapsulated by the term 'quantity' since all concepts are said to consist of a greater or lesser number of attributes (the objects of a concept). However, as is now well known but, according to Hamilton, had been largely overlooked by many contemporaneous and earlier logicians, the quantity of a concept can be distinguished into two different kinds, denominated by the terms 'extension' and 'intension' (or Hamilton's more frequently used term 'comprehension'). Although the terms 'extension' and 'intension' are well known to present-day logicians, that Hamilton regarded his contemporaries as being largely ignorant of these terms and that their importance to his own attempts to improve traditional logic is so great, provides at least two good reasons for elucidating his treatment of these terms here.

He claims that the distinction between extension and intension 'forms the very cardinal point on which the whole theory of Logic turns' (LL.I.119). He buttresses this claim by repeatedly returning to the significance of his distinction between and treatment of Extension and Intension in several later lectures, for example, when he argues that propositions can be distinguished as Intensive or Extensive depending on whether the subject or the predicate is respectively the containing whole (see, LL.I.231-3). However, as we shall see, the way in which he handles this distinction further deepens his underlying notion concerning the relativity of concepts to show, by demonstrating how extension and intension are correlatives of one another, that a relativistic analysis of concepts and arguments is possible and indeed further evolves the science of Pure Logic. However, importantly, the relativistic analysis that Hamilton's coordination of extension and intension enables, is one that is nonetheless anchored in the fundamental rule of containment, namely, the axiom which 'constitutes the one principle of all Deductive reasoning', 'that the part of a part is a part of the whole' (LL.I.119, 145, 144). It is perhaps worth pointing out that this axiom is also given in two Latin phrases by Hamilton, one of which is:

'*Prædicatum prædicati est prædicatum subjecti'*. The Editor of his lectures points out in a footnote that this is 'A translation of Aristotle's first antipredicamental rule as given in the *Categories* (see LL.I.144).

Hamilton's thorough and fairly extended explication of extension and intension in his Lecture VIII may be summarised as follows: a concept is a thought that *embraces* or, in the sense explained earlier, *brings into unity* an indefinite plurality of characters and it is also *applicable* to an indefinite plurality of objects about which it may be said, or *through* which these objects may be thought. As such, *a concept is a quantity of two different and opposed kinds*, denoted by the terms 'Intension' and 'Extension' (LL.I.140-52).

'Extension' refers to the *external* quantity of a concept, being determined by the number of objects — concepts or realities — to which the concept may be applied or which it classifies and hence *under* which these entities are said to be contained (a concept's extensive quantity is comprised of the number of objects that can be thought mediately through the concept). The extensive quantity of a concept is also referred to as its *sphere* or *breadth* and 'the parts which the total concept contains, are said to be contained *under* it, because, holding the relation to it of the particular to the general, they are subordinated or ranged under it. For example, the concepts *man, horse, dog*, &c., are contained under the more general concept *animal'* (LL.I.145). When these parts of a concept's Extension are exposed, this is called *Division*.

'Intension' refers to the *internal* quantity of a concept, being determined by the number of objects — concepts or realities — that constitute the concept and hence *in* which these entities are said to be contained (a concept's intensive quantity is the conceived sum of the attributes that constitute it, formed into a whole or unity in thought). The intensive quantity of a concept is also referred to as its *comprehension* or *depth* and 'the parts [. . .] which go to constitute the total concept, are said to be contained *in* it. For example, the concept *man* is composed of two constituent parts or attributes, that is, of two partial concepts,–*rational* and *animal*; for the characters *rational* and *animal* are only an analytical expression of the synthetic unity of the concept *man*' (LL.I.143-4). When these parts or characters of a concept's Intension are exposed, this is called *Definition*.

According to Hamilton, logicians 'have exclusively developed' the Extensive quantity of concepts. However, he asserts that the Extensive and Intensive quantities comprise 'the two great branches of reasoning' and that Intension 'is at least of equal importance' in comparison with Extension (LL.I.144-5). This claim is significant in that, placing Intension and Extension on an equal footing, as two main branches of reasoning, immediately brings the analysis of concepts under, as it were, dual aspects or two main *perspectives* from which concepts may be viewed, analysed, and through which Hamilton can further elaborate his thesis concerning the relativity of concepts by bringing any given concept's Extensive and Intensive quantities into relation with one another — which is precisely what he does at this stage in his Lectures as also in *Discussions* and his 'New Analytic of Logical Forms'.

Hamilton seems to suggest at this stage what he will later make much more explicit, that the axiom or fundamental and sole principle of all Deductive reasoning, that a part of a part is a part of the whole, originates with regard to the Intensive quantity of concepts — he certainly introduces this axiom of containment by illustrating how the Intensive quantity of a given concept, such as is signified in the term 'man', may be analysed into ever diminishing parts contained *in*, or implicit within, the concept 'man', 'till we reach attributes which, as simple, stand as a primary or ultimate element, into which the series can be resolved' (LL.I.144). If he is suggesting that the axiom of containment has been, as it were, translated for application to the Extensive quantity by the notion expressed as 'whatever is contained under the partial or more particular concept is contained under the total or more general concept', it is unclear whether he is at all troubled by the idea that the necessity of relation implicit in the axiom of containment, particularly as expressed with regard to a concept's Intension, is grounded on an analogy with physical containment that he elsewhere rejects as unwarranted — that is, with regard to the assumption that mind and body are analogically related, (LL.I.145).[35] However, he has effectually described concepts as necessarily factitious or artificial constructions, given a kind of sufficient or provisionally adequate permanence by means of language. He has also described concepts as relative continua in thought, the *analysis* of which is itself, purposive or tendentiously conducted in order that we can, as I have indicated earlier, both speak about and thereby better comprehend concepts (see LL.I.133). Thus, Hamilton can perhaps claim some licence in deploying terms such as 'contain' without issuing caveats to guard against some of the assumptions antithetical to his general philosophical standpoint of *natural dualism* that, as a term analogically related to physical containment, 'contain' itself may be said to contain or imply. However, Hamilton is relying upon a traditional technical language of containment to elucidate the Intensive and Extensive quantities which he will later claim is better replaced by the more accurate 'substantive verb, (*is, is not*)' to express the equation or affirmation or negation of identity between a given concept and the objects constitutive of it or to which it may be related (or through which other concepts or particulars may be thought) (LL.I.154). As we shall see later, he does regard the axiom or sole principle of all Deductive reasoning (that a part of a part is a part of the whole), as rather crucially originating in a thought unshakeably natural to us, namely, our knowledge of the quantity of Intension and the ways in which Intensive containment provides as it were a natural grounding for all purely logical inference — Hamilton, strenuously striving to craft Logic into a pure science, abstracted from all extra-logical matters, cannot resist bringing the laws of thought into an intimate relation with the natural, the human as irrevocably an interrelated whole consisting of the self and the not-self.

But this aside, he applies the axiom of containment to both the Intensive and the Extensive quantities of concepts. Both may be said to *contain* attributes, but they crucially differ in that, while the Intensive quantity is said to be contained *in*

[35] *Discussions*, pp. 61-2.

a concept, the Extensive is said to be contained *under* it. Thus, while both of the quantities of Extension and Intension *contain* in differing senses, and while both are determined by wholes that may be identified and thereby quantified, 'These two quantities are not convertible. On the contrary, they are in the inverse ratio of each other; the greater the depth of comprehension [Intension] of a notion the less its breadth or extension, and *vice versa*' (LL.I.142). This inverse ratio relationship between Extension and Intension is well illustrated by Hamilton as follows:

> When I take out of a concept, that is, abstract from one or more of its attributes, I diminish its comprehension [Intension]. Thus, when from the concept *man*, equivalent to *rational animal*, I abstract from the attribute or determination *rational*, I lessen its internal quantity. But by this diminution of its comprehension I give it a wider extension, for what remains is the concept *animal*, and the concept *animal* embraces under it a far greater number of objects than the concept *man* (LL.I.147).

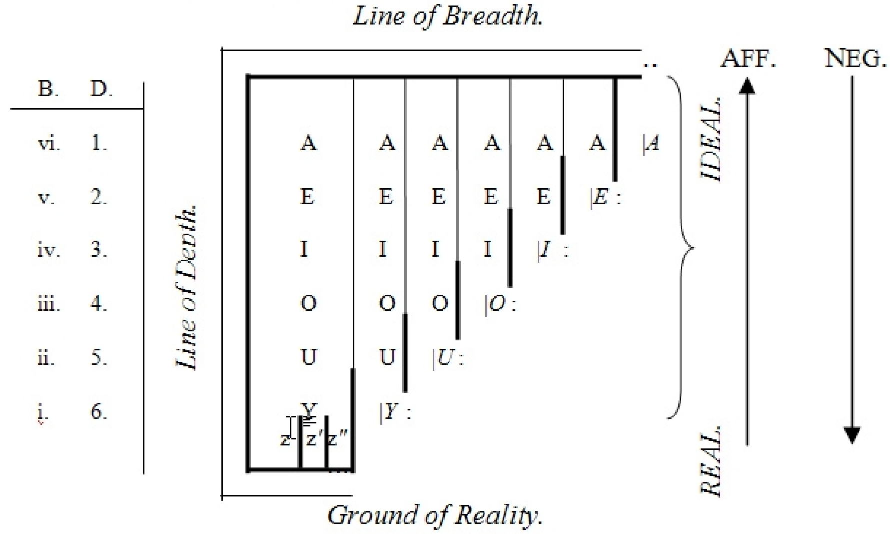
Hamilton explains the way in which this inverse ratio operates in fuller detail, pointing up as he does so that as we continue the analytic process outlined in the above quotation, the diminishment of a concept's Intensive quantity, with the correspondent amplification or expansion of its Extension, must finally result in 'that concept which all comprehension and all extension must equally contain, but in which comprehension is at its minimum, extension at its maximum,–I mean the concept of *Being* or *Existence*' (LL.I.149). And by contrast with this, when a concept's Intension is at its maximum, its Extension being then at its minimum, the concept we must end up with is that of an individual, 'the concept being a complement of the whole attributes of an individual object, which, by these attributes, it thinks and discriminates from every other' (LL.I.148).

His notion that, when Extension is at a maximum and thereby Intension at a minimum we must end with existence, raises the interesting topic of the existential import of concepts and propositions. However, leaving this aside, it should now be clear that, having described concepts as generally relative, Hamilton is laying bare the sort of things that this relativity enables with regard to how we may view and analyse concepts. In an ingenious table in his *Discussions*, which the editors of the Lectures appended to the end of Lecture VIII, Hamilton further exposes how their relational or relative nature enables us to view any given part of a concept under different relational aspects, for example: as species of a genus from one end, the genus of a species from its opposite end; how the expansion of a concept's Extension implies a coordinate reduction in its Intension; the sense in which we may say that each part 'in opposite respects, *contains* and is *contained'* (LL.I.153); and how 'the *real* identity and *rational* differences of Breadth and Depth' become exposed, such that it becomes more apparent that Extension and Intension 'though denominated *quantities*, are, in reality, one and the same quantity, viewed in counter relations and from opposite ends. Nothing is the one, which is not, *pro tanto*, the other' (LL.I.153).

When explicating these points to do with a concept's Extension and Intension in his Lectures Hamilton used 'a circular machine' which he had devised. Similar to a now-familiar child's toy consisting of a circular base with a wooden rod at its centre onto which are placed differently coloured circular discs of various circumferences, the discs of Hamilton's machine represented concepts that could be placed on the central rod representing the individual to which these concepts might refer. Each disc was placed on the rod in the correct order to illustrate several things to do with the relation of Extension to Intension, the completed device standing as an inverted cone. This 'machine' is represented and discussed by Veitch (see Veitch, pp. 250-2). As though trying to reinforce Hamilton's claim that his doctrine of the quantification of the predicate had been taught in his Lectures prior to de Morgan's quantification, the Editors' insertion of the table and the explanatory pages from his *Discussions* at the end of Lecture VIII reveal how his distinction between Extension (Breadth) and Intension (Depth), and the way in which he brings these two quantities into relation with one another, form the basis of his quantification of the predicate. That Hamilton's editors thought it necessary to do this, strongly suggests their awareness of how easy it might be not to grasp from his lectures, the extent to which Hamilton's treatment of Concepts and the two principal quantities of Extension and Intension lead to and are indeed an integral part of Hamilton's quantification of the predicate. However, the editors' insertion of this table is perhaps best understood as an attempt to bring to the fore the very sort of thing that it may be assumed Hamilton explained at this stage in his Lecture VIII by use of his 'circular machine'. I have reproduced Hamilton's table below and shall shortly provide some explanation of how it operates:

The table displays the lines of Breadth and Depth, the conceptual range as Ideal, the objective or particular range of individuals or singulars as Real, and gives lines of direction to indicate affirmation and negation (which I shall not attempt to explain). The table distinguishes between individuals or singulars (z, z', z'') and classes (A, E, I, O, U). The highest genus or widest attribute is given as A, A, A, etc., the subaltern genera and species as E, I, O, U, and the lowest species or narrowest attribute as Y. Each class is represented as 'a series of resemblances thought as one', symbolised by the same letter to denote that they are *thought* of as one, though really distinct or differing from one another in some respect(s) (this being intimated by the vertical lines separating each letter). This is in line with Hamilton's notion that a concept is relative in the sense that it brings into a unity in thought what are really discrete though resembling characters. The narrowest attribute (Y) is shown as a simple term constituted by the individuals z, z', z''. Though simple or singular in the sense that it has no extension or has a minimal extension, it is dichotomised by using a thick line '|' to denote 'not' — that is to say, if 'Y' is thinkable, as a strict logical necessity one must also be able to think 'not Y', but the narrowest attribute must otherwise be a singular attribute and not a class (as in, say, U). This is not to say that some given attribute represented by 'Y' could not be translated into 'A' in another table since any 'Y' is only the least or narrowest attribute in *relation* to U and the other letters above this, and

Table 1. Schemes of Two Quantitites



(LL.I.152; for further detail see, pp. 152-6, and *Discussions*, pp. 699-701).

the least must be in thought, if not in fact, a unity or attribute directly relatable to the individual constituents as shown beneath it. In other words, while the table posits a narrowest attribute Y, as something not consisting of any other resembling Y, closer examination of Y's Intension could involve translating it into another table of the same form in which it might be translated into A. As the thick line '|' denotes 'not' (e.g. to contradistinguish between say, 'animal' and 'not animal', the thinner lines 'merely discriminate one animal (A) , from another (A)' (LL.I.156). Hamilton's elucidation of the table more clearly begins to indicate just how his understanding of the relativity of concepts, and what a concept consists in with regard to the two quantities of Extension and Intension, combine to illustrate the necessity of quantifying the predicate: 'A is only A, not A, A, A, &c.; some Animal is not some Animal; one class of Animals is not all, every, or any other; this Animal is not that; Socrates is not Plato; z is not z'. On the other hand, E is EA; and Y is $YUOIEA$; every lower and higher letter in the series coalescing uninterruptedly into a series of reciprocal subjects and predicates, as shown by the absence of all discriminating lines. Thus Socrates (z'), is Athenian (Y), Greek (U), European (O), Man (I), Mammal (E), Animal (A)' (LL.I.155).

This needs to be further elucidated as follows: reading from left to right, the first A contains under it the concepts $EIOUY$, and the full range of all individual and hence actual Athenians, represented by 'z, z', z''.' However, the second A contains under it $EIOU$, and *not* Y ('$|Y$'), and hence neither z nor z' nor z''. The first A is a term identical *qua* term to all of the other As in the row of As, and it *resembles* (according to the thinking subject) the other As to such an extent as to be unified in thought and fixed as a unity by means of the single classificatory or general term 'A'. However, just as the first A and the second A are in all respects identical, *except* that the first A contains under it Y (and hence, in this case, z') whereas the second A does not, similarly each of the other As may be differentiated from one another and from the first A by means of reference to their respective depths or intensive quantities. This implies that each of the terms A is, as Hamilton puts it, 'only A, not A, A, A, &c. [...] one class of Animals is not all' (LL.I.155). This suggests the basis of quantification of the subject term in a simple proposition such as 'Some Animals are Mammals'. But, as we shall see shortly, it also suggests the basis of quantifying the predicate term.

Suppose that: A represents the term 'Animal'; E represents the subordinate or species 'Mammal' to A as the genus Animal; I represents the subordinate or species 'Man' to E as the genus Mammal. For ease of explanation it is necessary to follow Hamilton and ignore a reasoning that starts by quantifying the class with the universal term 'All', for it can easily be seen that were we to commence, in Breadth, with 'All A' ('all Animal') then, contained under this class are 'All E and some not E' ('all Mammal and some not Mammal', which unnecessarily complicates explication of how Hamilton's table demonstrates the need to quantify the predicate. Now, if we only attend to the range encompassed by the first five As, then this implies we must only be looking at *some* A and not all (in this case the first five). What is contained under these five As is the range of all Es (i.e. 'not E' ('$|E$')) has been excluded by abstracting or ignoring the sixth A). Translating this selection of the first five As and the range of Es listed under them, we have 'some Animals are all Mammals.' This process can of course be continued by repeating the process of removing one attribute or class of E, so that the next step in the reasoning process that examines what is contained under E, involves a further abstraction or removal of, in this case, the 4^{th} E, resulting in: 'Some E (Mammals) are all I (Man)', and so on.

If this begins to illustrate how reducing the Extension of a concept deepens or expands its Depth, and *vice versa* — the inverse ratio principle Hamilton claims to exist between Extension and Intension — the table also illustrates that, in effect, the same reasoning can be applied whether we are arguing in Depth (i.e. with regard to the Intensive quantity) or in Breadth (i.e. with regard to the Extensive quantity). This, according to Hamilton, is because, 'Though different in the order of thought, (*ratione*), the two quantities are identical in the nature of things, (*re*). Each supposes the other; and Breadth is not more to be distinguished from Depth, than the relations of the sides, from the relations of the angles, of a triangle.' (LL.I.154). The table illustrates how the same reasoning can be applied to both the Extension and Intension of a concept, so long as both the subject term and the predicate term in all of the resulting propositions are quantified consistently in accordance with the correct understanding of what the table indicates at each step in the reasoning with regard to containment (in or under). Hamilton's own illustration of this is as follows:

> In effect it is precisely the same reasoning, whether we argue in Depth — "z' is, (i.e. as subject, contains *in* it the inherent attribute), some Y; all Y is some U; all U is some O; all O is some I; all I is some E; all E is some A — therefore, z' is some A:" or whether we argue in Breadth — "Some A is, (i.e. as class, contains *under* it the subject part), all E; some E is all I; some I is all O; some O is all U; some U is all Y; some Y is z' — therefore, some A is z'." The two reasonings, internally identical, are externally the converse of each other; the premise and term, which in Breadth is major, in Depth is minor. (LL.I.154)

But, as this displays how the inverse ratio principle operates, and how so relating Extension and Intension reveals that it is only *externally* that the two quantities determine a difference in reasoning whereas, with each term consistently quantified using 'some' or 'all' as appropriate, subjects and predicates can swap position to yield formally identical conclusions ('z' is some A' and 'some A is z''), Hamilton is quick to point out in this more developed treatment of the topic explicating his 'Schemes of the Two Quantities' table that: 'In syllogisms also, where the contrast of the two quantities [Extension and Intension] is abolished, there, with the difference of figure, the differences of major and minor premise and term fall likewise.' (LL.I.154). With these few words Hamilton is touching on just how, through examining concepts Extensively and Intensively to show the inverse ratio relationship between these two quantities, the resultant quantification of all terms in a reasoning that hinges on the identification of the subject and predicate terms of a proposition, enables a significant simplification of traditional Formal Logic. However, he is also touching on a claim, the significance of which he brings to the fore much later in his Lectures, namely, that 'In fact, the *two quantities* and the *two quantifications* have by logicians been neglected *together*' (LL.I.155). As attending to the quantities of Extension and Intension reveals their '*real* identity and *rational* differences', such that it becomes more apparent that they are 'in reality, one and the same quantity', Hamilton will later show, through careful stages of his teaching of logic in the Lectures, that this identification of the two quantities is highly significant with regard to both propositions and syllogisms (LL.I.153; compare *Discussions*, pp. 701–2). However, before the full relevance of Hamilton's inverse ratio principle with regard to Extension and Intension can be illuminated with regard to propositions and syllogisms, it is necessary to outline some of the other points he makes in his Lectures concerning concepts.

If by the end of Lecture VIII it is beginning to emerge, particularly with the editors' helpful insertion of Hamilton's 'Schemes of the Two Quantities' table given above, that Hamilton is carefully working towards his quantification of the predicate, in some of the Lectures that follow, he still has much to say that will further reinforce his notion that concepts are relative, not simply in the sense that they are relational (or wholes constituted by and referring to a plurality of resembling entities, or relative with regard to their objects), but also in the sense that our construction and grasp of the depth (Intension) and breadth (Extension) of any given concept is dependent upon a subjective interrelation of clearness/obscurity and distinctness/indistinctness. This introduces the notion of the *quality* of a concept (see LL.I.157).

Thus, having elucidated the inverse ratio relation between the Extension and Intension of a concept, he goes on to consider the *subjective* relation of concepts (i.e. the relation to the subject that thinks a concept) in relation to their clearness and distinctness as similarly relative terms determining the quality of a given concept. He explains the terms 'clearness' and 'distinctness' at some length, acknowledging as he does so a considerable debt to Leibniz (see LL.I.159–65).

Hamilton points out that while intensive distinctness is at a maximum when we reach simple notions that are thereby indefinable, and while extensive distinctness is at a maximum when, to quote his translation of Esser, 'we touch on notions which, as individual, admit of no ulterior division', such distinctness is only ideal and that in fact this ideal distinctness is something we are always approaching but never in reality attaining (LL.I.170). As this ideal distinctness is regarded by Esser as an incentive to re-analyse the intension and extension of concepts, it is clear that Hamilton is suggesting to his students that this relativity and incompleteness or non-absolute condition of concepts is an important aspect of his overall approach to the study of Logic, a crucial incorporation of an awareness of the ultimate indefinability and non-absolute dimension of Logic's object matter. And in this Hamilton is arguably being entirely consistent with other aspects of his metaphysics and overall philosophical position.

In Lecture X Hamilton has more to say about the imperfection of concepts, this time returning to the problem of language. Concepts have, as it were, a propensity to be obscure and indistinct and these vices are due, partly to their very nature as wholes that bind together 'a multiplicity in unity', and partly from their dependence upon language as that which fixes concepts in consciousness (LL.I.172). He explains the problem of language by means of an illustrative analogy with methods of exchange in countries lacking an established currency. Thus, language operates much like the handing over of unquantified bags of precious metals which may or may not be closely scrutinised to see if they yield the value they purportedly signify — on most occasions the language user takes on trust that a particular term binds together what it seems to claim for itself, namely, that it does in fact represent a multitude of entities collectively amounting to a certain sum or value; but at other times, this will not be the case. This analogy of course teems with significance but there seem to be two main points that Hamilton is attempting to emphasise: firstly, 'that notions or concepts are peculiarly liable to great vagueness and ambiguity, and that their symbols are liable to be passed about without the proper kind, or the adequate amount, of thought' (LL.I.173-4); and secondly, that an important distinction, originated by Leibniz, can be made with regard to our knowledge that divides cognition into the *blind* or *symbolical* and *intuitive* (see LL.I.180-86). In short, Leibniz's notion of symbolical knowledge refers to concepts as terms taken to signify entities obscurely and imperfectly presented to the mind but which may potentially be exposed or made explicit, though we cannot think all of the ingredients that comprise the symbol or term used to refer to them. By contrast with this, intuitive knowledge is of these ingredients themselves inasmuch as this is possible. Hamilton fails to explain the significance of this distinction except to claim that thereby Leibniz and his followers in Germany superseded or overcame 'the whole controversy of Nominalism and Conceptualism,–which, in consequence of the non-establishment of this distinction, and the relative imperfection of our philosophical language, has idly agitated the Psychology of this country [Britain] and of France' (LL.I.179). However, it should be fairly obvious that by means of this distinction between symbolical and intuitive knowledge, Hamilton is providing a warrant for treating concepts in a purely formal manner in order to examine more closely the relations between concepts considered blindly or symbolically, while at the same time drawing attention to the interface between what the symbols represent with regard to the imperfect but real condition of our intuitive knowledge of the particulars to which concepts relate, and through which we both think these particulars and constitute our concepts. Once again, the relativism, imperfection, inchoateness, and mutability of concepts and the phenomenal and plural nature of our knowledge as relative, relational, provisional, and so on, is being emphasised by Hamilton, while at the same time he attempts to establish a domain or object matter of Logic at once stable, constricted, and discriminated from the material or actual, though both domains of the intuitive and symbolical are held in relation to one another as mutually informative and only theoretically discrete.

Though Hamilton is clearly aware that his various points concerning the relativity of concepts is foregrounding matter that might easily be thought of as extra-logical, having distinguished between symbolical and intuitive knowledge, in Lecture XI he appropriately turns to what he regards as 'The Relation proper' of concepts, namely, *their relation to each other* — something that can be represented symbolically and diagrammatically and which thereby establishes a set of relationships familiar to logicians. Again he discusses this in relation to the two principal quantities of Extension and Intension. Taking Extension first, he outlines five principal relations: Exclusion, Coextension, Subordination, Co-ordination, and Intersection. All of these he illustrates with simple circle diagrams, a practice used by some earlier philosophers, such as, according to Hamilton, the late 16^{th} century Christian Wiese, and of course developed later by John Venn. It is needless to reproduce Hamilton's diagrams (see LL.I.189; 256), but it is important to note that of all of these relations between concepts considered with regard to their Extension, 'those of Subordination and Co-ordination are of principal importance, as on them reposes the whole system of classification' (LL.I.189-90). He elucidates Subordination and Co-ordination with crystal clarity. However, as yet further evidence of his interest in and even fascination with relativism, he continues to draw into consideration: that there is no absolute exclusion in the relation between concepts known as Exclusion (LL.I.188); the ways in which our *perspective* on concepts results in re-describing a genus as a species and a species as a genus; how speculatively, if not practically, we may always divide a concept *ad infinitum* (LL.I.192-3); how different abstractions result in different relations of genus and species in both subordination and co-ordination; and how, admitting that there can be both a highest genus and a lowest species, neither of which are convertible into a species nor a genus respectively, within Subordination there are gradations of genus and species known as subalterns or intermediates such that, the genus lower than the highest genus, can become a species, whereas the species higher than the lowest species, can become a genus. This, as Hamilton points out all comes from Porphyry's Introduction to Aristotle's *Categories*, but it is nonetheless notable that Hamilton should emphasise the non-absolute aspects of the relations between concepts and the dependence of these various reversals of species and genus on how we regard them (LL.I.196). Point of view is indeed something he is at pains to emphasise as important to several matters, not least of which is the relation of whole to part and the distinction, which he regards as erroneous, between Logical and Metaphysical wholes, these being, contra to previous logicians, 'equally logical' (see LL.I.201-2).

Hamilton is evidently working towards some important claims to do with the whole-part relationship that will give priority to the Intensive quantity of concepts which in part relies upon his notion of *Involution* (see LL.I.202-3). Skipping over the various kinds of whole that Hamilton elucidates, the notion that a genus contains its species either potentially or actually (see LL.I.205-6), along with various other points of interest, it is in Hamilton's treatment of Comprehension or the quantity of Intension in Lecture XII that he first makes fully explicit one of his major disagreements with traditional logic, a disagreement nevertheless that he has been hinting at in one way or another from a fairly early stage. He claims that the relations of Involution and Co-ordination have been:

> altogether neglected by logicians: and, in consequence of this, they have necessarily overlooked one of the two great divisions of all reasoning [...]. In each quantity there is a deductive, and in each quantity there is an inductive, inference; and if the reasoning under either of these two quantities were to be omitted, it ought, perhaps, to have been the one which the logicians have exclusively cultivated [i.e. the deductive reasoning in Extension]. For the quantity of extension is a creation of the mind itself, and only created through, as abstracted from, the quantity of comprehension [Intension]; whereas the quantity of comprehension is at once given in the very nature of things. The former quantity is thus secondary and factitious, the latter primary and natural. (LL.I.217-8).

Now, it must be noted that by the term 'inductive' Hamilton departs from what we might call the standard treatment of or distinction between induction and deduction. Thus, he does not mean that kind of inference in which the conclusion may be said to exceed or amplify the content of the premises. He is not talking here about probable reasoning or judgements that go beyond what is delivered in the premises — his restriction of induction to a form of necessary reasoning is therefore substantially different from the empirical reasoning or induction of the physical sciences. Instead, as he makes clear in a later lecture, he is treating induction in a formal sense as differentiated from an informal or material sense, to mean a process of reasoning from *all* of the parts to the whole that these parts *entirely* constitute (see LL.I.319-26). This aside, that he is now distinguishing as he does in the above quotation between Extension and Intension may seem to be at odds with the claim he made earlier about concepts being factitiously unities, but really pluralities — now it would seem they are only partially factitious, projected, mind-dependent and mind-generated, but partially 'primary and natural'. Also, this prioritisation of Intension seems to sit uncomfortably with his attempt to hold, by means of his inverse ratio relation between the two quantities, that Extension and Intension are non-convertible but equivalent with regard to the reasoning that we apply to both quantities. Leaving aside this weaker objection, that he has now introduced a distinction between Extension and Intension that prioritises Intension as the real and primary, creates problems for Hamilton that he does not satisfactorily resolve. For example, is not the Intensive quantity also a binding together of resembling but nonetheless discrete entities held in a relationship of containing and contained, and thus is not Intension every bit as factitious as Extension? Furthermore, how can an *abstraction* from the Intensive quantity be satisfactorily described as factitious, given that it is just that, an abstraction from what is not factitious but rather 'in the very nature of things'? Perhaps, by 'factitious' Hamilton is merely asserting that, in relation to the thinking subject, Extension is a product of the mental action of abstracting from the real, Intensive quantity, and in this sense the product that is a concept's Extensive quantity may be more or less arbitrarily or selectively constructed, that Extension is both factitiously a unity, really a plurality of resembling attributes, and that its factiousness in relation to the reality of a given concept's Intension is a matter of degree. That is to say, both with regard to the degree of resemblance between a concept's attributes in Extension, and with regard to how close its Extension is to its definition, its distinctness, or the thinking subject's grasp of its Intensive quantity, the Extensive quantity may be *more or less* factitious.

Be that as it may, Hamilton does seem to be making an important distinction and claim here that may be more sympathetically understood as an attempt to ground in reality the formation of our concepts as emanating from or evolving out of a deeper or more intimately connected relationship of resemblance between the parts of a whole than is tenable of the resembling attributes in Extension. He does this by pointing up the difference between the senses of 'contain': in Extension a concept contains *under* it, its various attributes in subordination; whereas in Intension, a concept contains *in* it, its various attributes as constitutive of and as the definitional properties that make the concept what it is, just as the multitude of parts go together to constitute any given individual. To distinguish between these two different senses of 'contain', Hamilton introduces the notion of Involution mentioned above, which he explains as follows:

> In the quantity of comprehension [Intension], one notion is involved in another, when it forms a part of the sum total of characters, which together constitute the comprehension of that other; and two notions are in this quantity co-ordinated, when, whilst neither comprehends the other, both are immediately comprehended in the same lower concept. (LL.I.220).

He gives two illustrations of the Involution relationship between a concept's attributes, the second of these pointing up the inter-relatedness of concepts as involving and involved, such that we may be said to think a certain concept only in and through that of another concept:

> In this quantity [of Intension] the involving notion or whole is the more complex notion; the involved notion or part is the more simple. Thus *pigeon* as comprehending *bird*, *bird* as comprehending *feathered*, *feathered* as comprehending *warm-blooded*, *warm-blooded* as comprehending *heart with four cavities*, *heart with four cavities* as comprehending *breathing with lungs*, are severally to each other as notions involving and involved. (LL.I.223).

Suggesting as this does, a relative overlapping and even partial integration of one concept with another, such that they mutually imply and yet collectively constitute the individual or unity they define, he immediately follows this somewhat sketchy account of Involution, by differentiating between this relativist aspect (containing and contained) and the non-relative (since *not* necessarily containing and contained) co-ordination:

> Again, notions, in the whole of comprehension, are co-ordinated, when they stand together as constituting parts of the notion in which they are both immediately comprehended. Thus the characters *oviparous* [egg producing] and *warm-blooded*, *heart with four cavities*, and *breathing with lungs*, as all immediately contributing to make up the comprehension of the notion *bird*, are, in this respect, severally considered as its co-ordinate parts. These characters are not relative and co-relative,–not containing and contained. For we have oviparous animals which are not warm-blooded, and warm-blooded animals which are not oviparous. Again, it is true, I believe, that all warm-blooded animals have hearts with four cavities [. . .], and that all animals with such hearts breathe by lungs and not by gills, But then, in this case, we have no right to suppose that the first of these characters comprehends the second, and that the second comprehends the third. For we should be equally entitled to assert, that all animals breathing by lungs possessed hearts of four cavities, and that all animals with such hearts are warm-blooded. They are thus thought as mutually the conditions of each other; and whilst we may not know their reciprocal dependence, they are, however, conceived by us, as on an equal footing of co-ordination. (LL.I.223)

Hamilton hints that the significance of Involution and Co-ordination will come to the fore when he later moves on to tackle the syllogism and, seemingly regarding this as virtually a digression, his explanation of Involution in Lecture XII is unfortunately rather brief and might have been more fully elucidated, particularly given that it may be much more crucial than perhaps Hamilton realised, to how we might best understand at least one major aspect to do with the co-identity relationship within universal propositions which I shall discuss in Section IV. However, he says just about enough to suggest what one might call a grounding relativism in which his few words on this, though offered to explain the wholeness of a concept's Intensive quantity, verge on further pointing up the artificial nature of the fundamental laws of logic of identity, excluded middle, and non-contradiction. If this is less evident in the above quotation, it is more so in how he relates the involution within a concept's Intensive quantity to his non-linear notion of the formation or *evolution* of general and particular terms:

> Our notions are originally evolved out of the more complex into the more simple, and [...] the progress of science is nothing more than a progressive unfolding into distinct consciousness of the various elements comprehended in the characters, originally known to us in their vague or confused totality [the condition in which they may be said to be maximally complex].
>
> It is a famous question among philosophers,–Whether our knowledge commences with the general or with the individual,–whether children first employ common, or first employ proper, names. In this controversy, the reasoners have severally proved the opposite opinion to be untenable; but the question is at once solved, by showing that a third opinion is the true,–viz. that our knowledge commences with the confused and complex, which, as regarded in one point of view or in another, may easily be mistaken either for the individual, or for the general. [...]. It is sufficient to say in general, that all objects are presented to us in complexity; that we are at first more struck with the points of resemblance than with the points of contrast; that the earliest notions, and consequently, the earliest terms, are those that correspond to this synthesis, while the notions and the terms arising from an analysis of this synthesis into its parts, are of a subsequent formation. But though it be foreign to the province of Logic to develop the history of this procedure; yet, as this procedure is natural to the human mind, Logic must contain the form by which it is regulated. It must not only enable us to reason from the simple and general to the complex and individual; it must likewise, enable us to reverse the process, and to reason from the complex and individual to the simple and the general. And this it does by that relation of notions as containing and contained, given in the quantity of comprehension [Intension]. (LL.I.221-2).

In short, the notion of Involution takes us once again into the relativism of Hamilton's metaphysical epistemology and theory of language which he outlines more extensively in his *Lectures on Metaphysics*.[36] And, if he is, perhaps rather shakily, attempting to give priority to the reality of Intension over and against the factitiousness of Extension, he at least does so in a way that coheres interestingly with his theory of language with regard to the iterative or relative or non-linear origination of general and particular terms as a natural process of thought and indeed reasoning. It must be noted, however, that Hamilton is here attempting to foreground not the *relativity* of concepts but rather, that concepts considered Intensively enable the natural tendencies in reasoning and thought generally 'to reason from the simple and general to the complex and individual [and also] reverse the process, and [. . .] reason from the complex and individual to the simple and the general' — this, he clearly sees as a particular virtue of the Intensive quantity, namely, that, as contradistinguished from the Extensive quantity, the reversals in the direction of reasoning that we do in fact or naturally make, can only legitimately be carried out with regard to the Intensive quantity.

Although the somewhat fuzzy logic of this notion of Involution within, or characterising, the relation of the internal components of a given concept, or its Intension (as contradistinguished from Subordination in Extension), is not handled as fully as one might wish, it does seem that Hamilton is attempting to ground in an indeterminate reality of chaos or confusion, concepts as originating in, wrested out of, and partaking in a significant degree of relativism. However, I would claim that, though fluxive, quasi-organic, and arguably suggesting an indivisible and interminably iterative process, the involution relation that Hamilton seems to be suggesting inheres within a concept's Intension, warrants, or is itself peculiarly suggestive of, the distinction of constituent parts comprising wholes and the defensibility and inherent reasonableness of regarding such parts as being contained in the whole such that their relations one to the other may be understood as necessary and as such fundamental to Pure Logic as the science of the necessary laws of thought. The naturalness of this reasoning as grounded in the notion that the parts of parts *involve and are involved*, and thereby permit reversing the process of reasoning, is brought out or hinted at by Hamilton in several ways. For example, just what he seems to think of as Involution may be being suggested in what he says about *partes integrantes* with regard to Mathematical, or the Quantitative, or Integrant Whole (see LL.I.204). Perhaps the naturalness and primacy Hamilton is trying to claim of Intension, as contrasted to the factitiousness of Extension, is being indicated in the natural involutions of physical entities as we encounter them in, say, the human body, but also as may be postulated of all material phenomena as the relative, plural, and confused constituents of consciousness which, conditioned by thought, we naturally or inevitably collect together (or *grasp*) as resembling particulars. Organised into wholes, as part of the natural processes of thought and our conditioning propensity to cognise, these wholes avail, and indeed require, a corresponding analysis into their particulars and a relation of these to

[36]See, *Lectures on Metaphysics*, II.319-27.

the concept they constitute, our *act* of grasping or even perceiving them as collected together into the unity of the concept, being the synthetic reconstruction of the particulars constituting it, albeit to some degree provisionally. For Hamilton, it is as though a power of the mind — the faculty of comparison – is itself reinforced by and originates in the involution relationship as something that at once suggests the greater chaos or confused complexity into which the involutions of the parts seem to be capable of collapsing, while also suggesting the sharper discrimination into non-involved particulars that are thereby not constitutive of but which rather divide a concept into its Extensive quantity. If so, Involution at once suggests and is itself the suggestion of what a concept's Intension may be evolved into by the native processes of conception and its negative corollary, the reasoning of an analysis only prevented from disintegrating into infinite divisibility by an aptly pragmatic limitation on the extent of this science's explicitness, which Hamilton explicitly imposes (see LL.I.192-3; 210).

But in order to go at least some way towards understanding why Hamilton is now claiming that the reasoning of Intension is natural, or has a basis in reality — and indeed in a reality that he regards as otherwise confused or chaotic and theoretically if not practically open to infinite divisibility in its unconditioned state as the raw and unorganised mere phenomena of consciousness — and hence why he regards Intension as prior or superior to the quantity of Extension, we need to note at least something of the extent to which he regards the history of logic since Aristotle as flawed. We also need to note that Hamilton quite pointedly regards his prioritisation of Intension as a significant contribution towards placing the keystone in the arch of the Aristotelian logic. He claims that logicians following Aristotle rather surprisingly neglected the process of reasoning to do with Intension, even though they explicitly stated and relied upon the axiom: 'The character of the character is the character of the thing; or, The predicate of the predicate is the predicate of the subject' (LL.I.218). However, according to Hamilton, Aristotle understood the application of this axiom:

> In fact I think it even possible to show in detail, that his whole analysis of the syllogism has reference to both quantities, and that the great abstruseness of his *Prior Analytics*, the treatise in which he develops the general forms of reasoning, arises from this,–that he has endeavoured to rise to formulæ sufficiently general to express at once what was common to both kinds;–an attempt so far beyond the intelligence of subsequent logicians, that they have wholly misunderstood and perverted his doctrine. They understood this doctrine, only as applied to the reasoning in extensive quantity; and in relation to this kind of reasoning, they have certainly made palpable and easy what in Aristotle is abstract and difficult. But then they did not observe that Aristotle's doctrine applies to two species, of which they only consider one. [. . .]. This mistake,–this partial conception of the science,–is common to all logicians, ancient and modern: for in so far as I am aware, no one has observed, that of the quantities of comprehension and extension, each

> affords a reasoning proper to itself; and no one has noticed that the doctrine of Aristotle has reference indifferently to both. (LL.I.218-9).

3 DOCTRINE OF JUDGMENTS

> In the proposition *Men are animals*, we should be allowed to determine whether the term *men* means *all* or *some men*,–whether the term *animals* means *all* or *some animals*; in short, to quantify both the subject and predicate of the proposition. This postulate ['To state explicitly what is thought implicitly'] applies both to Propositions and to Syllogisms. (NA.LL.II.252)

In elucidating some aspects of Hamilton's 'Schemes of the Two Quantities' table and certain features of his doctrine of Concepts in the previous section, I have so far been indicating something of how his development and understanding of Extension and Intension underpinned his quantification of the predicate. Before examining the quantification itself, we need to look at a selection of some of the other construction work.

By 'judgment' Hamilton means 'proposition' *or* the forming of a proposition, such as '*water rusts iron',* by means of the process of judging (LL.I.227). What is relative within a judgment or proposition has at least the appearance of being much more stable than the relativity of concepts discussed at various stages above, though the very fact that Hamilton calls attention to propositions as *judgments*, keeps in focus how it is that a proposition may be regarded as at least related to some mental agency, a judgment being the product of the act of judging (akin to a concept being the product of conception). Introducing his students to the standard terms of 'subject', 'predicate', and 'copula', he points out that the subject is the determined or qualified notion, whereas the predicate is the determining or qualifying notion, the relation of determination between them being signified by the copula. He therefore defines a proposition as:

> the product of that act in which we pronounce, that, of two notions thought as subject and as predicate, the one does or does not constitute a part of the other, either in the quantity of Extension, or in the quantity of Comprehension. (LL.I.229).

Hamilton is clearly regarding subject and predicate as being held in a relation of quantity in accordance with the axiom that a part of a part is a part of the whole, but he is also, consistent with his treatment of Extension and Intension, incorporating both of these quantities, and this is how he does so:

> The first great distinction of Judgments is taken from the relation of Subject and Predicate, as reciprocally whole and part. If the Subject or determined notion be viewed as the containing whole, we have an

> Intensive or Comprehensive proposition; if the Predicate or determining notion be viewed as the containing whole, we have an Extensive proposition. (LL.I.231-2).

Hence, whether a proposition is Extensive or Intensive depends on how we view the Subject and the Predicate. Of course Hamilton acknowledges that in single propositions it is rarely clear which way the proposition is to be viewed, since it is generally unclear whether it is the Subject or the Predicate that is being regarded as the containing whole. However, according to Hamilton, 'It is only when propositions are connected together into syllogism, that it becomes evident whether the subject or the predicate be the whole in or under which the other is contained' — he thus regards the distinction of propositions into Extensive and Intensive as highly general, though of great importance, since 'it is only in subordination to this distinction that the other distinctions [he is about to introduce] are valid' (LL.I.233). The other distinctions that relate to the Extensive/Intensive distinction with regard to the subject and predicate relation may be summed up as: the distinction between Categorical (or simple propositions, in which, following the practice of Aristotle's followers, the predicate is either simply affirmed or denied of the subject, as in '*A is B*', or '*A is not B*') and Conditional propositions, Conditional propositions being further distinguished into Hypothetical ('*if A then B*'), Disjunctive ('*A or B*', or '*D is either B*, *or C*, *or A*'), and Dilemmatic or Hypothetico-disjunctive ('*If X is A*, *it is either B or C*') (see LL.I.233-242). Although it is crucial to grasp that Hamilton is showing his students how in principle the subject and predicate terms may be viewed as reciprocally related to one another in any given proposition as expressions solely concerned with a whole-part relationship, either in Extension or Intension, it is needless to elucidate this any further here. More importantly, is Hamilton's major point of difference with Aristotle and 'The doctrine of Logicians' concerning the division of propositions into four classes or species, since it is in this that Hamilton may be said to be making his most explicit statement within his Lectures so far, concerning how his system and much if not all of his previous discourse on logic has been working towards and in turn will rely upon a thoroughgoing quantification of the predicate, a quantification which notably traditional logic had failed to achieve due, not only to exclusively focusing on Extension, but also to the establishment of a class of proposition that admitted vagueness or ambiguity with regard to the subject term's quantification and thereby disabled quantification by failing to make explicit the quantity pertaining to the subject term which, according to Hamilton, is 'involved in every actual thought' though at times not in its linguistic expression (LL.I.244). But, as we shall see, for Hamilton, as this notion that quantity is either explicit or implicit enables the removal of a class of propositions in which the subject term is not quantified in expression, it also underpins the whole notion that the *predicate* term's quantity may also be made explicit.

Following Aristotle, logicians traditionally divided propositions with regard to their Extensive quantity by categorising them as: *Universal* or *General*; *Particular*; *Individual* or *Singular*; and, *Indefinite.* According to Hamilton these terms were applied with the following meanings: in Universal propositions 'the subject is taken in its whole extension'; in Particular propositions 'the subject is taken in a part, indefinitely, of its extension'; in Individual propositions 'the subject is at a minimum of extension'; and in Indefinite propositions, 'the subject is not [...] overtly declared to be either universal, particular, or individual' (LL.I.243). With regard to quantification generally, Hamilton claims that *commonly* only the Subject is regulated, whereas the predicate 'Aristotle and the logicians do not allow to be affected by quantity; at least they hold it to be always Particular in an Affirmative, and Universal in a Negative' (LL.I.244). However, he claims that this doctrine is untenable, incomplete, that it resulted in confusion, and that this confusion and incompleteness is partly due to logicians paying insufficient heed to the fundamental postulate of explicitness that I explained earlier. At this stage in his lectures, Hamilton might have simply gone straight to quantifying the predicate to show that, by making the quantification of the predicate term explicit, it is no longer requisite that we consider whether the proposition is being considered extensively or intensively. However, he is much more careful to take his students through the evolution of his logic as it involves a critique and a significant modification of traditional logic's assumption that the predicate term in an affirmative proposition is always Particular whereas in a negative proposition it is always Universal.

By contrast with the Aristotelian doctrine of the logicians, which divides propositions into the four classes or species of Universal, Particular, Individual, and Indefinite, Hamilton proposes that they should be differentiated quite differently, and he does this largely by redefining 'indefinite' and thereby eradicating as a distinct class traditional logic's Indefinite propositions, while retaining the notion of indefiniteness as describing, within any given whole, an indeterminate range of quantities sufficiently competent to be classed as the quantifier of particularity we normally express by the term 'some' — in other words Hamilton's 'indefinite' constitutes that species of judgments/propositions the logicians formerly called 'Particular'.

His redefinition of 'indefinite' seems to amount to this: for the logicians 'indefinite' meant little more than that the Extensive quantity of the subject (universal, particular, or individual) was unexpressed and thus indefinite in the sense of being unclear or inexplicit; by contrast with this, for Hamilton, 'indefinite' refers more directly to the *quantity* expressed by the terms 'some', 'many', and various other expressions so long as they designate 'some indefinite number less than the whole'. Hence, his definition of 'indefinite' refers to any quantity within a whole, ranging from (possibly) a singular to a number less than the whole to which the indefinite term refers (LL.I.246). It should perhaps be noted here that Hamilton's definition of 'indefinite' may not at first sight seem to differentiate itself sufficiently from the quantity of Individual judgments or propositions, nor from Universal propositions. As Fogelin claims, Hamilton 'held complicated views on the quantifier *some*', in which, while he sometimes referred to 'some' as meaning 'some but not all', he also used the definition 'some perhaps all', and as Fogelin argues, there seem to be good grounds for saying that Hamilton purposely incorporates both readings of 'some' (Fogelin, 153).[37] These complicated views on the meaning of Particulars, as that class of propositions which are indefinite, only come to the fore in Hamilton's 'New Analytic' (see NA.LL.II.279-80). However, that by 'some' Hamilton meant 'some but not all', and that he distinguishes between Individual and Particular Propositions does seem to be fairly clear if we confine attention solely to Lecture XIII. Not only does he give little or no indication in Lecture XIII that 'some' might mean 'some perhaps all', but he also makes it very clear that a proposition may be expressed using the indefinite article, and thereby be classed as a Particular proposition, as in one of his examples, '*An Englishman generalised the law of gravitation*'; whereas, when expressed using a proper name, it will be an Individual proposition, as in '*Newton* generalised the law of gravitation' (LL.I.247). This, however, merely illustrates what is required to transform an ordinary language statement from being a Particular proposition to an Individual one and Hamilton's explicitness principle ought to allow that all such indefinite propositions, not involving a plurality, must be expressible as individual or, where the 'some' really means 'all', as universal propositions. Though Hamilton may not have squarely tackled certain problems to do with his definition of the quantifier 'some', it is at least possible to detect in his virtual eradication of the traditional logic's class of Indefinite propositions, that he is making the treatment of the quantity of propositions more internally consistent by accommodating the indefinite within Particular propositions by removing the obstruction to them being accommodated within Particular propositions, namely, the mistaken condition of their failure to *express* whether they were universal, particular, or individual — a failure that, Hamilton might have pointed out, was entirely due to the logicians not grasping that, in an Extensive proposition, where the quantity of the subject is unexpressed it must either be a part (and hence particular) of the predicate as whole; or, in an Intensive proposition, its quantity must be capable of being expressed as the containing whole determined by the predicate as part of that whole. To re-state the quotation given earlier: 'If the Subject or determined notion be viewed as the containing whole, we have an Intensive or Comprehensive proposition; if the Predicate or determining notion be viewed as the containing whole, we have an Extensive proposition' (LL.I.231-2). By itself this will not imply what quantity ought to be appended to the subject of a given proposition where that quantity remains unknown. However, this is irrelevant as the traditional class of Indefinite propositions regards the proposition in Extension alone, and as I have claimed in support of Hamilton, viewing the traditional Indefinite proposition as Extensive, determines that the subject term must be particular in relation to the predicate term as the determining whole containing the subject term. What Hamilton has

[37] Robert J. Fogelin, 'Hamilton's Quantification of the Predicate' in, *Philosophical Interpretations* (Oxford: Oxford University Press, 1992), first published in *Philosophical Quarterly*, 26 (1976), 149-65. All references to Fogelin are to this edition. Fogelin also supplies in this edition a second article entitled 'Hamilton's Theory of Quantifying the Predicate — A Correction' (166-8), also first published in *Philosophical Quarterly*, 26.

achieved here is the removal of an anomalous class of propositions that is only possible by means of his understanding of the whole-part relationship in propositions viewed Extensively and Intensively. However, although much more might be said about all this, there is another crucial factor to removing the traditional class of problematic and confusing indefinite or indeterminate propositions, namely, his introduction of the terms 'predesignate' and 'preindesignate'.

It is easy to see how the application of these terms to the traditional class of Indefinite propositions warrants a discrimination between propositions that may thereby be deemed to be either logically adequate or inadequate. When a proposition (presumably considered as an external expression that may thus be more or less precise, as contradistinguished from a judgment, construed here as the mental process) articulates its quantity by prefixing terms such as 'all' and 'some', this is a *predesignate* proposition; whereas when a proposition does *not* articulate its quantity, the proposition is called by Hamilton, *preindesignate*. Hence, the subject and predicate terms of a proposition may also be either predesignate or preindesignate terms. But, for Hamilton, though in a proposition's external expression one or both of its terms may be preindesignate (the quantity often being 'elided in its expression'), the unexpressed quantity is *always* involved in thought (definite or indefinite 'quantity being involved in every actual thought', though not always marked by a quantifier) (NA.LL.II.250; LL.I.244). Hence, adhering to his principle of explicitness, such preindesignate terms may be translated into predesignate ones — which is to say, that unexpressed quantities may always be expressed at least in principle and indeed should be expressed in adherence to the fundamental principle of explicitness. One might therefore say that, with regard to the traditional class of Indefinite propositions, if in fact it is impossible to determine the predesignate term(s) (as quantified in thought) of a given statement purporting to be a proposition but which is somehow quite indeterminate in its meaning by means of its use of a preindesignate term(s) (hence unquantified in expression), then such a statement must be deemed to be non-propositional and inadequate for consideration within a reasoning or argument.

The differences between propositions, with regard to quantity, according to Hamilton, arise, on the one hand, 'from the necessary condition of the Internal Thought' (when we consider them specifically as Judgments), and on the other hand, 'merely from the accidental circumstances of [a proposition's] External Expression' (when we consider them as propositions) (LL.I.243). Thus, he characterises three classes of proposition as properly adequate for logical consideration: *Universal judgments or propositions*, 'in which the whole number of objects within a sphere or class are judged of,–as *All men are mortal*, or *Every man is mortal'*; *Individual judgments or propositions*, in which 'the whole of a certain sphere is judged of, but in which sphere there is found only a single object, or collection of single objects,–as *Catiline is ambitious,–The twelve apostles were inspired',* the individual(s) in question here constituting what Hamilton describes, with possible oblique reference to his notion of involution, 'determinate wholeness or totality in the form of oneness [or] indivisible unity'; and, *Particular judgments or propo-*

sitions, 'in which, among the objects within a certain sphere or class, we judge concerning some indefinite number less than the whole,–as *Some men are virtuous–Many boys are courageous–Most women are compassionate*. The indefinite plurality, within the totality, being here denoted by the words *some, many, most*' (LL.I.245-6). As he explains by means of reference to the example cited above concerning the conversion of a Particular proposition ('*An Englishman* ...') into an Individual proposition ('*Newton* ...'), although the logicians are right to treat Universals and Individuals as convertible, their correspondence one with the other is not merely due to 'the oneness of their subject', but rather: 'The whole distinction consists in this,–that, in Universals and in Individual Judgments, the number of the objects judged of is thought by us as definite; whereas, in Particular Judgments, the number of such objects is thought by us as indefinite' (LL.I.246). Hence, the major distinction between propositions in terms of quantity is between the definite (universal or individual) and the indefinite (particular). This distinction between definite and indefinite quantity, Hamilton declares most forcefully in his 'New Analytic': '*definite* and *indefinite* are the only quantities of which we ought to hear in Logic; for it is only as indefinite that particular, it is only as definite that individual and general, quantities have any (and the same) logical avail' (NA.LL.II.250).

Thus, bearing in mind that, since Universals and Individuals with regard to quantity, constitute one class of *definite* propositions, whereas Particulars constitute the only alternative quantity and thus class of *indefinite* propositions, all that needs to be added to this twofold *definite* (Universal and Individual) and *indefinite* (Particular) distinction between propositional forms to yield the traditional (yet Hamiltonized) fourfold distinction, is the standard distinction between affirmation and negation, known as the *quality* of the proposition. Hamilton briefly outlines the notion of quality, which he regards as an unfortunately ambiguous and yet generally accepted term to denote affirmation and negation (LL.I.250). Important to his quantification of the predicate, he sensibly argues against some contemporary and earlier logicians who held that affirmation and negation properly belong to the copula and not to the subject nor to the predicate terms. Drawing attention to the non-literal or non-grammatical sense in which the copula should be regarded as expressing the form of the relation between subject and predicate, he argues against certain previous and some modern logicians, that negation does not belong to the predicate term but rather to the copula, allowing him to treat subject and predicate as being held together in a reciprocal relation of whole to part, such that in a negative judgment a part is taken out of a whole, whereas in an affirmative judgment a part is put into a whole (see LL.I.251-4). All that thus belongs to the subject and to the predicate, for Hamilton, is their respective definite (Universal or Individual) or indefinite (Particular) quantities; but as this enables a distinction between propositional forms according to their quantity (definite or indefinite), traditionally given as Universal or Particular, when these are both distinguished according to their quality (affirmative or negative), this results in the traditional fourfold A, E, I, O distinction of propositional forms: A (universal affirmative); E

(universal negative); I (particular affirmative); O (particular negative).

He represents the forms A, E, I, O by using simple circle diagrams (see LL.I.255). I shall not provide these since they merely illustrate the quantity and quality aspects of the four propositional forms and since I shall shortly provide one of Hamilton's later tables from the 'New Analytic' which, by deleting one of the diagrams as redundant and adding one that expresses co-extension, incorporates these circle diagrams into a new set of four figures to which he relates both the traditional forms A, E, I, O, and his additional forms. Although it is highly likely that Hamilton pointed out to his students the quantifications of the predicate and the subject terms implied by the diagrams, for example, that the Universal Affirmative (A) in traditional logic implied an indefinite or Particular predicate, the text does not at this stage make any explicit reference either to the assumptions concerning quantification nor does it advance the thoroughgoing quantification that results in Hamilton doubling the traditional four propositional forms A, E, I, O. It must be noted, however, that much later in the Lectures he does assert that 'The nineteen useful [syllogistic] moods admitted by logicians, may [...] by the quantification of the predicate, be still further simplified, by superseding the significance of Figure' (LL.I.402). Although his quantification of the predicate does not seem to have been made fully explicit, he was clearly at the very least intimating aspects of it to his students some years before his controversy with de Morgan.

Lecture XIV is the last lecture on Hamilton's doctrine of Judgments. Before he terminates the lecture, he makes at least one significant claim worth mentioning, namely, his rejection of Modal propositions as a separate class and his argument that, for example, the modal proposition '*Alexander conquered Darius honourably*' ought to be treated as merely a complex proposition in which the mode is regarded as part of the predicate. As Hamilton points out the predicate can be more or less complex and there is no need for the Aristotelian logic's modal propositions as 'modified by the four attributions of Necessity, Impossibility, Contingence, and Possibility. [...] in regard to these, the case is precisely the same; the mode is merely a part of the predicate, and if so, nothing can be more unwarranted than on this accidental, on this extra-logical, circumstance to establish a great division of logical propositions' (LL.I.257). Once again Whately comes under fire concerning this, as also when Hamilton moves on to discuss and outline some basic points concerning the subject of the conversion of judgments or propositions, such as when the subject and predicate are transposed in a categorical proposition (see LL.I.258–9; 262–3). I shall come back to the subject of conversion briefly later, but for the time being I shall leave Hamilton's lectures and the commencement of his introduction to reasoning and the syllogism, and instead leap forward to his later work in the fragmentary but nonetheless insightful and more mature work, his 'New Analytic of Logical Forms'. For, within Lecture XIV Hamilton has arrived at a significant stage in laying the groundwork for his quantification of the predicate. Although by this stage he has yet much more to construct, it would seem that he has reached a critical point that has by now established a warrant for providing the thoroughgoing quantification that I shall illustrate in the following section.

4 QUANTIFICATION OF THE PREDICATE

> THIS NEW Analytic is intended to complete and simplify the old — to place the keystone in the Aristotelic Arch. (NA.LL.II.249)

It is abundantly clear in the 'New Analytic' that Hamilton's principle of explicitness, or rather the thoroughness of his *adherence* to this principle, is a major driving force behind his quantification of the predicate. He cites the postulate or principle as something insisted upon by Logic, but insufficiently adhered to by previous logicians, claiming on the basis of this principle, 'that, logically, we ought to take into account the *quantity*, always understood in thought, but usually, and for manifest reasons, elided in its expression, not only of the *subject*, but also of the *predicate*, of a judgment' (NA.LL.II.250; and see p. 252). Making explicit the quantities of both the subject and the predicate in a judgment or proposition facilitates regarding the subject and predicate terms as comprising 'an equation'. It is important to understand how it can be said that the whole-part relationship in Extension or Intension may be thought of as constituting either an equation or non-equation. Though in some places Hamilton does not seem to be as clear as he might have been on this point, at other places his explanation is substantially more helpful than the often rather submerged hints in the Lectures. For example, in the 'New Analytic' Hamilton's treatment of the subject of the Conversion of Categorical propositions clarifies what he thinks erroneous in traditional logic's Conversion of propositions with regard to quantity.

He regards the doctrine of Conversion as 'beset with errors' but that these errors are generated from two principal ones — Hamilton is worth quoting at length here:

> The First cardinal error is,–That the quantities are not converted with the quantified terms. For the real terms compared in the Convertend [the original proposition], and which, of course, ought to reappear without change, except of place, in the Converse [the proposition converted], are not the naked, but the quantified terms. This is evident from the following considerations:
>
> 1^o, The Terms of a Proposition are only terms as they are terms of relation; and the relation here is the relation of comparison.
>
> 2^o, As the Propositional Terms are terms of comparison, so they are only compared as Quantities,–quantities relative to each other. An Affirmative Proposition is simply the declaration of an equation, a Negative Proposition is simply the declaration of a non-equation, of its terms. To change, therefore, the quantity of either, or of both Subject and Predicate, is to change their correlation,–the point of comparison; and to exchange their quantities, if different, would be to invert the terminal interdependence, that is, to make the less the greater, and the greater the less.
>
> 3^o, The Quantity of the Proposition in Conversion remains always the same; that is, the absolute quantity of the Converse must be exactly

equal to that of the Convertend. It was only from overlooking the quantity of the predicate [. . .] that two propositions, exactly equal in quantity, in fact the same proposition, perhaps, transposed, were called the one universal, the other particular, by exclusive reference to the quantity of the subject.

4^o, Yet was it of no consequence, in a logical point of view, which of the notions collated were Subject or Predicate; and their comparison, with the consequent declaration of their mutual inclusion or exclusion, that is, of affirmation or negation, of no more real difference than the assertions,–*London is four hundred miles distant from Edinburgh,–Edinburgh is four hundred miles distant from London.* In fact, though logicians have been in use to place the subject first, the predicate last, in their examples of propositions, this is by no means the case in ordinary language, where, indeed, it is frequently even difficult to ascertain which is the determining and which the determined notion. [. . . .]

The Second cardinal error of the logicians is, the not considering that the Predicate has always a quantity in thought, as much as the Subject; although this quantity be frequently not explicitly enounced, as unnecessary in the common employment of language; for the determining notion or predicate being always thought as at least adequate to, or co-extensive with , the subject or determined notion, it is seldom necessary to express this, and language tends ever to elide what may safely be omitted. But this necessity recurs, the moment that, by conversion, the predicate becomes the subject of the proposition; and to omit its formal statement is to degrade Logic from the science of the necessities of thought, to an idle subsidiary of the ambiguities of speech. [. . . .]

1^o, That the predicate is as extensive as the subject is easily shown. Take the proposition,–*All animal is man*, or, *All animals are men.* This we are conscious is absurd, though we make the notion man or men as wide as possible; for it does not mend the matter to say,–*All animal is all man*, or, *All animals are all men.* We feel it to be equally absurd as if we said,–*All man is all animal*, or, *All men are all animals.* Here we are aware that the subject and predicate cannot be made coextensive. If we would get rid of the absurdity, we must bring the two notions into coextension, by restricting the wider. If we say,–*Man is animal*, (*Homo est animal*), we think, though we do not overtly enounce it, *All man is animal.* And what do we mean here by *animal?* We do not think,–*All*, but *Some, animal.* And then we can make this indifferently either subject or predicate. We can think,–we can say, *Some animal is man*, that is, *Some or All Man*; and, *e converso*,–*Man (some of all) is animal*, viz. *some animal.*

It thus appears that there is a necessity in all cases for thinking the

> predicate, at least, as extensive as the subject. Whether it be absolutely, that is, out of relation, more extensive, is generally of no consequence; and hence the common reticence of common language, which never expresses more than can be understood — which always, in fact, for the sake of brevity, strains at ellipsis. (NA.LL.II.257-9)

This lengthy quotation brings several things to our attention: that Hamilton regards the quantifying terms as integrant components of the 'naked' terms or concepts or judgments used in any given proposition — and hence whenever the 'naked' terms are transposed in some conversion, their clothing (the quantifier) goes with them; the terms of a proposition only exist *qua* terms as related to one another by a comparison of their respective quantities, and that their quantities will be identical or equated in an affirmative proposition, non-identical or not equated, but rather related by some confliction to do with their quantities, in a negative proposition — violation of this relation of comparison or equation/non-equation is to change without warrant the relation being asserted in their very predication; that it is mistaken to deem a proposition to be Universal or Particular solely by attending to the status of the quantification of its subject; and, Hamilton's reliance on his principle or postulate of explicitness as the fundamental principle of Logic is invoked as warranting exposure of what *must* (on pain of otherwise thinking an absurdity) be thought with regard to the predicate's quantity and thereby its relation of equation or non-equation with the quantified subject.

This last point concerning how in thought if not in linguistic expression (due to our propensity in linguistic expression to elide quantities into the 'naked' terms) strongly suggests a mental and in this sense private language of subject-predicate equation/non-equation, a mental relation of the respective quantities of both subject and predicate, that is often but not always behind the scenes and tantamount to the fundamental necessary laws of logic themselves. However, Hamilton seems to be at pains to describe these laws as in some sense natural, informing actual discourse, and in turn operating as the standard against which rhetorical utterance may be tested and to which rhetoric may be reduced without attempting to modify logic's laws to suit grammar: 'We should not do as the logicians have been wont,–introduce and deal with ['the rhetorical enouncements of common speech'] in their grammatical integrity; for this would be to swell out and deform our science with mere grammatical accidents; and to such fortuitous accrescences the formidable volume, especially of the older Logics, is mainly owing. In fact, a large proportion of the scholastic system is merely grammatical' (NA.LL.II.262). The tendency in Hamilton's logic to correct and simplify traditional logic, thus marks a major departure from regarding logic as an attempt to capture the vagaries and complexities of grammatical rules, real-life argumentation, and rhetoric in order to keep logic focused on the laws of necessary inference. However, although this approach warrants the unnatural sounding expressions that result from quantifying the predicate, as in 'All men are some mortal', Hamilton is not such a purist as to eschew utterly the notion of Pure Logic's relevance to and capacity to translate 'the rhetorical enouncements of common speech', nor is his quantification of the predicate justified solely by means of reference to a merely dogmatic assertion that quantity is 'always understood in thought' though often elided in its linguistic expression, since his arguments in support of quantifying the predicate incorporate appeals to common instances in which the quantification is made explicit: 'in fact, ordinary language quantifies the Predicate so often as this determination becomes of the smallest import' (NA.LL.II.259).

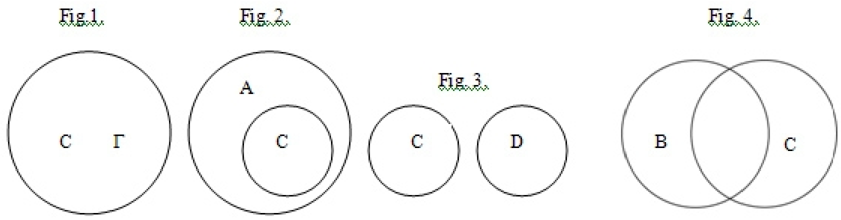
Be all that as it may, it becomes clear in the 'New Analytic' that one of Hamilton's major achievements with regard to the complex and, as he often asserts, confusing doctrines, rules, and practices of the logicians has to do with how his system effectually sweeps away various different types of conversion. With the establishment of his quantification of the predicate the only defensible type of conversion is the simple conversion he advocates. His simple conversion relies wholly upon the predicate's quantification being made as explicit as the quantification of the original subject term. Hence, simple conversion merely involves whatever transposition of terms is possible so long as the respective quantifiers of the original subject and predicate terms remain attached to these terms in order to retain in conversion any given proposition's meaning as an equation or non-equation of the quantities of the two terms in the original proposition. Over and again Hamilton emphasises the erroneousness of earlier species of conversion, both with regard to affirmative and negative propositions (see NA.LL.II.256-76). The logicians had missed 'the one straight road' of conversion, simple conversion, and Hamilton makes the ambitious but clearly defensible claim that if, by means of his quantification of the predicate, he is right in having reduced all species of conversion to the simple conversion he advocates, then 'the whole doctrine of logical Conversion is superseded as operose and imperfect, as useless and erroneous. The systems, new and old, must stand or fall with their doctrines of the Conversion of propositions' (NA.LL.II.276).

Though some elements of Hamilton's construction of his system of quantifying the predicate have been overlooked above, I think I have given an ample outline of his notions concerning the quantification and how this radically supersedes much of the traditional logic while yet incorporating and building upon at least some part of it. However, we now need to look at Hamilton's quantification procedure in some more detail and the best place to commence this is by examining the table supplied at Appendix V section (d) in the second volume of his *Lectures on Logic*, which, with only a few minor adjustments, I have replicated below as Table 2:

The only significant modification I have made to Hamilton's table as given at Table 2 above, is to adopt his later method of symbolising the Universal and Particular terms using the letters A for Universal, and I for Particular (enclosed in square brackets). This alteration should also be of some help to readers who may wish to make comparisons between Table 2 and Hamilton's more detailed 'Table of the Mutual Relations of the Eight Propositional Forms on Either System of Particularity' to which I shall refer later (see NA.LL.II.284).

Table 2 is interesting in several ways. Firstly, the various configurations of A (universal/definite) and I (particular/indefinite) under Affirmative and Negative

Table 2. Application of Doctrine of Quantified Predicate to Propositions
New Propositional Forms — Notation



Affirmative.

(1)	[A f A]	C : : Γ	All Triangle is all Trilateral [fig. 1].
(ii)	[A f I]	C : , A	All Triangle is some Figure (A) [fig. 2].
(3)	[I f A]	A , : C	Some Figure is all Triangle [fig. 2].
(iv)	[I f I]	C , , B	Some Triangle is some Equilateral (I) [fig. 4]

Negative.

(v)	[A n A]	C : : D	Any Triangle is not any Square (E) [fig. 3].
(6)	[A n I]	C : , B	Any Triangle is not some Equilateral [fig. 4].
(vii)	[I n A]	B , : C	Some Equilateral is not any Triangle (O) [fig. 4].
(8)	[I n I]	C , , B	Some Triangle is not some Equilateral [fig. 4]

Key:

1, 3, 6, 8 = Hamilton's forms

ii, iv, v, vii = Aristotelic or traditional forms. Hence, using the scholastic letters A, E, I, O: A (Universal affirmative) = ii; E (Universal negative) = v; I (Particular affirmative) = iv; O (Particular negative) = vii.

A = universal; **I** = particular; **f** = affirmation; **n** = negation; **,** = some; **:** = all.

= the affirmative copula (*is*)

= the negative copula (*is not*)

The two arrow lines above indicate which of the terms is subject and which predicate thus: where the proposition is being read in Extension, the thick end of the arrow line denotes the subject, the thin end the predicate; where the proposition is being read in Intension, subject and predicate are reversed and hence the thick end of the arrow line denotes the predicate, the thin end the subject.

To illustrate how to read Hamilton's symbolic representation of a proposition in Table 2, note that the following should be read thus:

C : , A

Extensively: *All* C is *some* A – i.e. All C *is contained under* some A

Intensively: *Some* A is *all* C – i.e. Some A *contains in it* all C.

regularly display all of the possible permutations of a thorough quantification of both subject and predicate as neatly displayed in the symbols given in square brackets such as [A f A]. Secondly, the arrow lines indicate which of the terms is the subject and which the predicate, depending on whether the proposition is to be read as Extensive or Intensive — hence, the terms may be easily transposed according to Hamilton's method of simple conversion without any alteration in the proposition's meaning using the same symbolic notation for both an Extensive and an Intensive reading (though it must be said that by this stage in Hamilton's logic, having made so much of Extension and Intension to establish the dual perspectives from which propositions may be viewed, how thus the subject and predicate terms may be transposed, and how the number of syllogistic forms can thereby be amplified, the Extension-Intension distinction seems to fall out of account as superseded by the full quantification itself). Thirdly, using Hamilton's own terminology and categorisation, the four circle diagrams need to be thought of as expressing not four but three possible principal relations of: *Toto-total Coinclusion* (fig. 1); *Toto-total Coexclusion* (fig. 3); and, brought together under one class of counter-related relations, *Incomplete Coinclusion* and *Coexclusion* (fig. 2 and 4). Fourthly, Hamilton's example of proposition (1) uses the symbol 'Γ', but since it only appears in this proposition, it is not immediately clear which of the other propositions may be said to be its contradictory or negation, though with a simple change of terms proposition (v), describing the relation of Toto-total Coexclusion (fig. 3), seems to be the most obvious contradictory of proposition (1) as describing Toto-total Coinclusion (fig. 1). These last two points require further explanation and as we shall see this will involve some discussion of a major source of difficulty and controversy concerning Hamilton's quantification.

Firstly, my third point above: in Hamilton's system the relationship between subject and predicate in each proposition needs to be thought of as a relationship of *mutuality.* This becomes much more clear when we take note of Hamilton's 'Observations on the Mutual Relation of Syllogistic Terms in Quantity and Quality' at Appendix V (e) in the *Lectures on Logic* (see NA.LL.II.285). With reference to the circle diagrams in Table 2 above, these relations can be given as follows:

1. *Toto-total Coinclusion* (fig. 1) — the relation of 'coidentity, absolute convertibility or reciprocation'.
2. *Toto-total Coexclusion* (fig. 3) — the relation of 'non-identity, absolute inconvertibility or non-reciprocation)
3. *Incomplete Coinclusion* and its counter-relation *Incomplete Coexclusion* (fig. 2 and fig.4) — the relations of 'partial identity and non-identity, relative convertibility and non-convertibility, reciprocation and non-reciprocation'. Under this counter-related pair of Incomplete Coinclusion and Coexclusion, Hamilton details all of the propositional forms he regards as intermediaries between the extreme opposites of proposition (1) Toto-total Coinclusion, and proposition (v) Toto-total Coexclusion as:

(ii) Toto-partial coinclusion (fig. 2)
(3) Parti-total coinclusion (fig. 2)
(iv) Parti-partial coinclusion (fig. 4)
(6) Toto-partial coexclusion (fig. 4)
(vii) Parti-total coexclusion (fig. 4)
(8) Parti-partial coexclusion (fig. 4)

(see NA.LL.II.285)

Though these relations and how the eight propositional forms may be said to relate to one another deserve further explanation, I shall instead draw attention to two main points here. Firstly, as Hamilton argues in his Lectures, in keeping with traditional logic a universal (A or E) may be treated as an individual or as a universal proposition — hence, the *Toto-total Coinclusion* and *Coexclusion* relations represented in Table 2's proposition (1) [fig. 1], and proposition (v) [fig. 3] respectively, may be translated in two ways as referring either to: two single/individual entities (A and B) that are (1) identical or (v) non-identical; or, two groups (collections) of things, all of which in each collection (A and B) are *sufficiently resembling to be asserted as* (1) identical or (v) non-identical — hence Fogelin seems to be right to claim that 'Hamilton develops his theory of universal propositions on an existential (rather than a Boolean) interpretation)', though arguably, as seems to be the case with his specific illustrations of propositions (1) and (v), he does also accommodate *a priori* truths constituted by wholes that may be thought of as having no existential import as physical realities, the whole and thus individual that is 'All Triangle' existing in thought, though its ontological status, as a purely ideal whole, is by no means unrelated to the whole that might be any more or less complete collection of real triangles as instantiations of the unity of thought that is 'All Triangle' (see Fogelin, p. 152). Incidentally, the complex and interesting topic of Hamilton's understanding of the existential import of propositions, which I have merely touched on here, is not perhaps as one at might first think, since Hamilton is conscious that 'the Logician has a right to suppose any material impossibility, any material falsity; he takes no account of what is objectively impossible or false, and has a right to assume what premises he please, provided they do not involve a contradiction in terms' (LL.I.322; also see, p. 338; p. 360).

The second main point I want to make here is that, though in his Lectures Hamilton seems to keep to just one sense of the quantifier 'some' it becomes clear in the 'New Analytic' that particulars, though in each and every case an *indefinite* quantity, may be indefinite in two different senses, either: as *Indefinite Definitude*, in the sense expressed by '*some, at least'* (which is to say, 'at least one, possibly more, but not all'); or, as *Definite Indefinitude*, in the sense of '*some, at most'* (which is to say, 'some, perhaps all, but not less than one') — hence, the *Incomplete Coinclusion* and *Coexclusion* relations represented in Table 2's fig. 2 and fig. 4, may also be translated in two different ways to accommodate this difference in the possible senses of the Particular quantifier 'some'. The significance of these points should become clear later, but in the meantime, I want to claim that while Hamilton made a great deal of the importance of Logic as a Pure science that must not become intermixed with any grammatical, linguistic, rhetorical, or other material concern (inasmuch as this is practically possible), as Fogelin rightly points out, 'Hamilton acknowledges both interpretations of the quantifier *some*, but only insists that each interpretation must be examined in order to capture all the everyday inference patterns a logician should study' (see Fogelin, p. 153). It does seem as though Hamilton is attempting to make his system sufficiently accommodating, such that it can encompass different readings of 'some' (or can embrace different degrees of indefiniteness), and such that his system can also be applied, not simply to both individual and universal quantities (as the only definite quantities), but also, on the one hand: to *a priori* truths or universals, the wholeness of which is as an unanalysable individual/singular, their ontological status being ideal and thus *potentially* existing only in thought, though *actually* never out of relation to, real entities; and, on the other hand and more conspicuously, his system can be applied to those universals that we might better describe as *general* terms, the *a posteriori* nature of which implies that at best they are only approximate or provisional universals which may admit of some exceptions without nevertheless losing their applicability in a syllogistic reasoning as universal terms.

Now, to come to the second main observation I want to make about Table 2 (my fourth point above): Hamilton's proposition (1) is diagrammatically represented by figure 1 in Table 2 and figures 2, 3, and 4 all look as though they illustrate relations that must stand counter to proposition (1), but since the example Hamilton gives uses a term 'Γ' that he does not replicate elsewhere in Table 2, it is not immediately clear which of the other propositions may be said to be its contradictory or negation. Figure 1 absolutely equates C with Γ and thus describes an absolute identity or Toto-total coinclusion relation between C and Γ. This is also illustrated by Hamilton's example of a possible proposition that might express this relation between the judgments or concepts C and Γ: 'All Triangle is all Trilateral'. However, surely both C and Γ collect together respectively the entire class of C (all possible shapes and sizes of triangles) and Γ (all possible Trilateral figures) and, linked by the copula, the proposition 'All C is all Γ' brings these two quantities into a relation of comparison? If so, does this mean that de Morgan's interpretation of proposition (1) is right, namely, that it expresses a complex proposition constructed by compounding 'every C is Γ' and 'every Γ is C' (de Morgan, p. 257)?

In Fogelin's first attempt to express the meaning of Hamilton's proposition (1), he accepts de Morgan's interpretation and (though shown using A and B as terms) Fogelin thus translates proposition (1) as 'All C is Γ and all Γ is C' (Fogelin, p. 151). On this reading of proposition (1), according to de Morgan, it is contradicted by one of either proposition (6) or (vii), which we may here give not as in the illustrative propositions given in Hamilton's Table 2 but, to conform to the letter symbols used in proposition (1), as follows: (6), 'some Γs are not *C*s'; (vii), 'some *C*s are not Γs' — these are, for de Morgan, the contradictories of proposition (1), 'All *C* is all Γ' (de Morgan, p. 257). To be sure, as is clearly seen simply by comparing figure 1 (illustrating proposition (1)) and figure 4 (illustrating propositions (6) and (vii)), both (6) and (vii) must stand in a negative relation to proposition (1). In a helpful but somewhat complex and possibly inaccurate table, Hamilton asserts that (6) and (vii) stand as what he calls *unilateral contraries* of (1) — this seems to suggest that he would accept that (6) and (vii) do severally contradict (1). However, as *contraries* neither (6) nor (vii) properly constitute a contradiction of (1) (see, NA.LL.II.284). Now, it might at first sight seem odd that neither Fogelin nor (in his original critique of Hamilton) de Morgan, mention proposition (v) as the contradictory of (1), since, translating this into symbols consistent with (1) and in line with de Morgan's reading, proposition (v) should be read as stating 'No C is Γ' and indeed, to be consistent with de Morgan's translation of proposition (1), he ought to have translated this as 'No C is Γ and No Γ is C'. However, de Morgan does mention proposition (v) in a later footnote as being offered as the contradictory of (1) by 'an eminent defender' (Mansel) of Hamilton's system (de Morgan, p. 258n1). But de Morgan brushes this aside as not being '*in the system'*. Quite what de Morgan means by this is rather unclear, especially since as Fogelin points out, 'de Morgan is just wrong in suggesting that the system of propositions do not pair up into proper contradictories', and he goes on to list, (1) and (8); (ii) and (vii); (3) and (6); (iv) and (v) as contradictory pairs. But still, it may seem puzzling why (1) Toto-total coinclusion and (v) Toto-total coexclusion should not be thought of as contradictories. Proposition (v)'s relation to (1), is displayed in one place by Hamilton in such a way as to suggest that (v) and (1) mutually contradict one another (see NA.LL.II.286). However, in another place Hamilton gives their relation as one of *bilateral contraries* — this is to say that both the Toto-total coinclusion of the terms in (1) as shown in figure 1 of Table 2, and the Toto-total coexclusion of the terms in (v) as shown in figure 3 of Table 2, stand *not* as contradictories of one another but both potentially false; which is of course to say, that if (1) is true though (v) must be false and *vice versa,* since (1) and (v) may both be false they are not strictly, or existentially, contradictories (see NA.LL.II.284).

Be all that as it may, as Fogelin rightly points out, the trouble lies with proposition (8) which de Morgan rejects as having no contradictory within the system. To establish against de Morgan's rejection of proposition (8) that Hamilton's system is comprehensive and not inconsistent, we need to be able to answer the question: what is the true contradictory of (8) *in the system*? This is an important question since if there is no contradictory of (8) within the system, de Morgan is right to assert that it has not been generated from any necessary laws of thought but rather by, as de Morgan so derisorily claims, on the basis of 'an arbitrary extension of the application of language' (de Morgan, p. 258n1). However, as intimated above, Fogelin ably answers de Morgan's rejection of (8) by arguing that its true contradictory is (1). Incidentally, although de Morgan can be fulsome in his praise of Hamilton — but then so can Mill — de Morgan's treatment of Hamilton's quantification scheme is at times rather scurrilously worded. It would seem that both de Morgan and Mill were, rather excessively, much given to resort to more or less veiled abusive *ad hominem* attacks on Hamilton, and I cannot help but comment here that while Hamilton's frequent denunciations of others, including de Morgan, must have to some extent provoked such responses, his achievements and reputation as a logician have most certainly suffered unduly from the cheap rhetorical tricks of opponents whose conduct ought to have been exemplarily fair, not solely as a mark of respect for Hamilton's considerable endeavours but also as a generally more virtuous way of conducting their discourse — paradoxically, it would seem that rather too often winning the argument is much more important to those who should be most concerned with striving to resolve it satisfactorily.

Fogelin's defence of Hamilton is an incisive attempt to redress the balance and although I shall not rehearse the full extent of his critical examination of Hamilton's system, to date it stands as one of the strongest defences of Hamilton's quantification of the predicate. Fogelin's defence is carried out in part by translating the controversial proposition (8), which de Morgan also claims with complete disdain was erroneously, and, so he implies, foolishly offered by one of Hamilton's defenders as the true contradictory of (1) (de Morgan, p. 259n3). As Fogelin rightly says about proposition (8), 'it is this proposition that has been the constant source of confusion' (Fogelin, 151). However, as Fogelin rightly attempts to show in his first article on Hamilton's quantification, proposition (8) is indeed the true contradictory of proposition (1). But, Fogelin's second article on Hamilton makes an important correction to his first attempt to establish that proposition (8) genuinely contradicts proposition (1), and it is therefore to this second article that I shall now refer.

According to Fogelin's reading of proposition (1) this should be interpreted as: 'Anything that is an A is identical with anything that is a B' — which 'means that there is but one thing that is an A, one thing that is a B, and these things are identical' (Fogelin, p. 167). This is to say, as Fogelin argues, that de Morgan's interpretation of proposition (1) — 'All A is B and all B is A' — is wrong. However, de Morgan was right in his interpretation of proposition (8) and hence (8) can be stated as 'Some A is not some B', which may then be translated as the contradictory of (1) as: 'Something that is an A is not identical with something that is a B'. Fogelin goes on to demonstrate just how this interpretation renders a certain syllogism (which on his previous interpretation of proposition (8) is invalid), can be shown to be valid using his second (and de Morgan's original) reading, thereby proving that Hamilton's system is 'saved from inconsistency' (Fogelin, p. 168). Although the relation of Hamilton's quantification system to syllogisms is of course important, instead of looking at this I want to enrich Fogelin's argument somewhat by considering his interpretation of proposition (1) as meaning 'that there is but one thing that is an A, one thing that is a B, and these things are identical'.

To return to Hamilton's own example, 'All Triangle is all Trilateral': as expressing the relation of Toto-total coinclusion, this expresses the nearest thing (along with Toto-total coexclusion) Hamilton would call an absolute, which is to say that the subject and predicate terms are co-inclusively related as maximally similar such that they may be said to be absolutely convertible one with the other — they are thus absolutely *reciprocal.* Whatever, if anything, *might* be said to differentiate 'all Triangle' from 'all Trilateral', such that they can be clearly if not distinctly separated into two entities (and this may merely be that the terms themselves are different signs both of which signify the same single entity), as two terms brought into a mutual relation of coidentity, this coinclusion is nonetheless one in which all material difference has been abstracted from thought, such that the co-identities are indeed absolute and the proposition that articulates this is thus effectually an implicit denial of their *non*-identity. Hence, such propositions asserting Toto-total coinclusion that involve merely a *terminal* differential assert a co-identity between two terms, and as such these propositions can be most directly contradicted by the assertion of the most minimal difference in their quantity, since with regard to their quantity alone their co-identification implies a unity or singularity — hence the assertion of parti-partial coexclusion in proposition (8) is this most direct contradiction of the unity expressed by proposition (1) in terms of two unities being co-identical, since the very ground of (1) being the expression of absolute identity is at once negated by proposition (8)'s assertion that the co-identities are co-excluding.

But, we need to keep in mind that, for Hamilton, there is no absolute *exclusion* in that relation between concepts known as Exclusion. Since he rejects absolute exclusion, but also since Hamilton so permeates his system with correlations of one sort or another, it seems fair to regard his system as ultimately one in which, as there can be no absolute exclusion, there can also be no absolute *inclusion* or perfect identity/unity. Rather, the universals of, say, the Toto-total Coinclusion proposition (1), as relating at least two terms together, are assertions of either: an *approximate* (adequate) but non-maximal co-identification; or, an *absolute* co-identification that *is* total, but *only to the extent* that the subject and predicate being equated in the proposition rely upon some merely nominal/terminal differential. For Hamilton it would seem that *some* differential is the minimum requirement for any concept, judgment, or reasoning to be *possible*, whether this differential is actual (thus rendering the universal approximate), or is so crucially dependent upon the merely terminal as to render all other distinction between them impossible. Even when proposition (1) may seem to be an affirmation or assertion of perfect identity, for (1) to exist as a proposition or material expression of the unity of thought in which A is identified with B, it must consist of at least two entities. As such, 'All A is all B' is most directly contradicted by the assertion of the sole proposition that most adequately breaches or contradicts the *relation* of coinclusion affirmed by 'All A is all B'. For (1) and (8) to be contradictories, the 'all' in (1) quantifies both subject and predicate as individuals absolutely co-identified (though the 'absolute' here, must involve *some* differential), while the 'some' in (8) must quantify the subject and predicate of (8) to be at least the individuals referred to in (1) also, which (8) must then be asserting are coexclusive to an extent beyond the necessary differential making (1) possible, such that these individuals in (8), contradicting (1), may be said to be co-exclusive. The contradictory of proposition (1), as before, is hence the assertion of Parti-partial Coexclusion in proposition (8), 'Some A is not some B', or as Fogelin interprets this, 'Something that is an A is not identical with something that is a B'.

Now, the unity of A and B's coinclusion or co-identification in thought, being expressed as a coinclusion relationship between the two terms 'A' and 'B', is the expression of a bringing into unity concepts which, as Hamilton took much trouble to explain in his Lectures, are themselves relative, since a concept consists of disparate entities thought as one by means of the degree of resemblance between their several attributes — this is why in the bulk of actual cases of proposition (1) expressing Toto-total coinclusion we need to think of their co-identity as approximate. However, the *best examples* of proposition (1) may be thought of as propositions in which the subject and predicate terms are concepts or singularities/individuals in which constituents that respectively define them, which is to say their Intensive quantities, are not mere bundles of pluralities possibly thought erroneously or merely approximately as constituting two unities. Instead, in a best example case of proposition (1) the subject and predicate terms will be constituted by attributes that so overlap or co-inform one another in meaning that they comprise what one might just as well call a true whole, a whole the parts of which are as notions involving and involved. Thus, the Intensive quantities of both the subject term and the predicate term in a best case example of proposition (1) will be involuted wholes, which are in turn related to one another by Involution to form what we might call a true whole. Such true wholes are best exemplified by *a priori* truths in which each of the terms is involved and involving in each other. Now, while this may not be fully satisfactory, some such extrapolation from at least Hamilton's notion of involution and some other elements of his work on Logic, seem to go a long way to justify Fogelin's reading of the real meaning of proposition (1), namely, that it is the assertion that there is but *one* thing that is an A, and *one* thing that is a B, *and* these things are identical. The unities co-identified in a best example case of proposition (1), as involuted wholes, themselves both involved and involving one another, bespeak the nearest true whole or unity that Hamilton can admit into his system. Hence, the only possible and most efficient and immediate contradiction of the coidentity of A and B, yet again, must be the assertion of Parti-partial coexclusion between 'A' and 'B' as expressed in proposition (8), for this is *not* to find a mere single *exception* within A or B that is not a co-identical attribute of both, but rather this is to declare the *non*-identity or co-exclusion of the two things that are A and B, and thus (8) is the contradiction of the unity that proposition (1) asserts.

However, this story, complicated enough, is not fully resolved yet. For example, it is interesting to note that Hamilton himself lists propositions (1) and (8) as *Compossible* using both senses of 'some', which is to say, that they are *not* contradictory of one another — I say 'lists' but intriguingly compossibility is expressed by means of blanks (empty spaces) in the relevant columns of Hamilton's 'Table of the Mutual Relations of the Eight Propositional Forms'. However, since a footnote mentions possible inaccuracies, I am not sure whether one can fully trust this table, nor really what its true status is, since, for one thing, it refers to 'Generals Only', which may suggest a differentiation within the universal propositions to accommodate what we might call *a posteriori* or approximate universals as contrasted with *a priori* or absolute universals (see NA.LL.II.284). However, assuming that propositions (1) and (8) *are* compossible, hence not contradictories, does this not entirely overturn Fogelin's strong defence of Hamilton's system as being consistent and, though *partially* interpreted aright by de Morgan, *crucially misinterpreted* and therefore wrongly and, one might add, unfairly dismissed by de Morgan?

First of all, a couple of simple yet weak answers to this: leaving aside the possibility that I am blundering here, Hamilton's table may simply be wrong and maybe he did intend (1) and (8) to be contradictories as Fogelin infers; and, Hamilton, de Morgan, and the seeming controversy that surrounded the meaning and place of (8) may have jointly misled Fogelin into thinking that Hamilton intended (1) and (8) to stand as contradictories. I shall take sides with Hamilton and Fogelin here: Hamilton is *not* mistaken to class (8) as compossible with (1) and Fogelin's interpretation of (8) as the contradictory of (1) does indeed not only render Hamilton's thorough quantification of the predicate for both affirmatives and negatives consistent, it also makes sense within the context of several other aspects of Hamilton's philosophical approach to logic.

However, this still leaves one problem: how can Hamilton be right to class (1) and (8) as compossible if they are to be regarded as contradictories? I think a lengthy answer could be offered, but I shall here merely intimate something of what this might be: whatever (8) is, it must be, for Hamilton, the expression of some one of two different species of Indefinitude. As such, though in one interpretation — the interpretation given by Fogelin — (8) *does* contradict (1), if (8) is to remain a Negative Particular (*Indefinite Definitude/Definite Indefinitude*) as the regular scheme of universal and particular permutations in Table 2 above determines it must be, then it cannot be typed or classified as the contradiction of (1), even though it can be *interpreted* as functioning as (1)'s contradiction. For, while it may operate in some instances and be used within a syllogism as if it is the contradictory of (1), as soon as it in fact loses its Particular status and becomes either an individual or a universal in both quantities — as soon as we *know* that its terms have to be quantified as universal/individual (hence definite), it must suddenly be transformed into proposition (v), 'Any A is not any B'/'No A is any B' — and of course, were we able to introduce only partial definitude (8) would similarly metamorphose into either (6) or (vii).

Proposition (8), though berated by de Morgan and arguably jarring with grammatical norms or certain delicacies of taste, as each and every one of Hamilton's eight propositions do, quickly makes sense when illustrated with a Venn diagram. Still, since it seems rather too easily open to misinterpretation, it is vulnerable to being regarded as a rather unnatural looking thing. Viewed in one way, it seems to have an almost spectral appearance — and yet, a logically necessary component in a schema that must be eightfold, it turns out to be a useful container within Hamilton's edifice for holding an indefinite quantity that, once grasped, functions as it needs to do within Hamilton's thorough system of quantification.

Proposition (8) may appear to be unimportant, unnatural, odd, irrelevant, even spectral in its meaning, but it nonetheless expresses a relation of partial coexclusion concerning two Particular (indefinite) quantities that can operate as the contradiction of a Toto-total coinclusion relation between the two wholes asserted as co-identical in proposition (1). But if proposition (8) strikes us as at once functionally comprehensible and yet strange, so also is proposition (1) since it seems to bespeak a unity of duality, the enouncement of an absolute or singularity only possible by comparing two things and asserting that they are one, perfectly convertible with one another but, excepting the propositional requirement of a terminal difference, in all respects the same thing. But if proposition (1) enounces a notion akin to Leibniz's principle of the indiscernibility of identicals, it is easy to regard its contradiction in (8) as being akin to a notion of infinite divisibility — some of this, is not some of that, but as, to contradict (1), this is the assertion that what is an absolute whole may be distinguished into at least two parts, what we seem to have between (1) and (8) is the clash between absolute and infinite that first brought Hamilton fame with his controversial and yet highly potent and profoundly influential Law of the Conditioned.

5 THE SYLLOGISM: SOME IMPLICATIONS

> The science now shines out in the true character of beauty,–*as One at once and Various*. Logic thus accomplishes its final destination; for as 'Thrice-greatest Hermes,' speaking in the mind of Plato, has expressed it — '*The end of Philosophy is the intuition of Unity.*' (NA.LL.II.252)

Hamilton's treatment of the syllogism deserves a whole chapter in its own right — I shall only be able to deal with it rather briefly in this section. In the Lectures Hamilton takes a considerable amount of time to explain many detailed aspects of the syllogism to his students. He distinguishes and displays four different classes of syllogism — the Categorical, Disjunctive, Hypothetical, and the Hypothetico-disjunctive (see LL.I.291-2). The categorical syllogism is also displayed in both Extensive and Intensive forms, something he will later capture in his symbolic notion as indicted in Table 2 earlier and as fully detailed in the final table of the second volume of Lectures (LL.I.295-300). Importantly, in drawing attention to the different reasonings between Extensive and Intensive syllogisms, where the copula signifies respectively 'contains under' and 'contains in', Hamilton makes the point that from what can be observed of the inverse ratio relation between Extension and Intension with regard to syllogisms, 'it is not to the mere external arrangement of the terms, but to the nature of their relation, that we must look in determining the character of the syllogism' (LL.I.300; and compare, p. 348). This is hugely important to how he will proceed to regard the syllogism within the lectures but it also bears within it the necessity of quantifying the predicate, even though at this stage he does not seem to have produced the full system of quantification as given in Table 2 earlier. With Extensive and Intensive syllogisms differentiated, Hamilton constructs three rules, in place of Whately's six, for Extensive, and three for Intension which merely invert the Extensive rules and thus are the correlatives of the rules for Extensive syllogisms (see LL.I.305-6; 315).

In all this we can see Hamilton working, as it were, from the ground up — as we shall see shortly, his ultimate position will be even more simple or general as, on the basis of his thoroughgoing quantification system, he develops a single general rule or Canon governing all valid syllogisms in both affirmative and negative moods. With regard to what is happening in the Lectures it is therefore important to remember both their instructional function and that some aspects in the Lectures are later superseded, such as, for example, his later rejection of the Rule of Reason and Consequent in favour of just three main Rules of Identity, non-Contradiction, and Excluded middle (see LL.I.290n).

In his treatment of the syllogism in the Lectures, Hamilton introduces his students to the usual suspects: the four figures, moods, the ingenious mnemonics of, for example, Barbara, Celarent, Darii, the formal fallacies, and so on, a great deal of which he lays out with painstaking detail (see LL.I.394-468). He also discusses the various forms of conversion, but since I have already touched on this subject in the previous section and how Hamilton's quantification effectually displaces other kinds of conversion with his simple conversion, it is needless to say anything more about his treatment of it in the Lectures (see LL.I.262-5; NA.LL.II.264-76). Using simple circle diagrams to illustrate the relations between the extremes or subject and predicate terms of the conclusion and the middle term, his explanations of syllogisms must have given his students an excellent grounding in the differing figures and moods. However, as so often occurs in Hamilton's expositions of the traditional logic, he marks some significant differences between his treatment and that of both his predecessors and contemporaries. One major example of this is his rejection of the fourth figure which is, of course, simply shown, in Extension, as follows:

$$\begin{array}{c} P \textit{ is } M \\ M \textit{ is } S \\ \hline S \textit{ is } P \end{array}$$

He argues that, though the fourth figure can be shown to be valid, 'the logicians, in consequence of their exclusive recognition of the reasoning in extension, were not in possession of the means of showing that this figure is a monster undeserving of toleration, far less of countenance and favour' (LL.I.424). I shall not rehearse Hamilton's arguments against the fourth figure, except to note that he shows that in this figure there is an unwarranted switch from reasoning in Extension to Intension or *vice versa* and thus it performs 'a feat about as reasonable and useful in Logic, as the jumping from one horse to another would be reasonable and useful in the race-course. Both are achievements possible; but, because possible, neither is, therefore, a legitimate exercise of skill' (LL.I.427). But, Hamilton's principal reason for rejecting the fourth figure is that it involves a mental process 'which is not overtly expressed' — in other words, when we adhere rigidly to the principle of explicitness, the fourth figure's reliance upon an intermediary conclusion becomes evident (see LL.I.427-8).

There are many other interesting features in the lectures worth mentioning, such as his treatment of the difference between Induction and Deduction, which I briefly touched on in Section 2 above. He discards what is now much more typically classed as Inductive argument, instead regarding Deduction and Induction both as formal forms of demonstrative reasoning — logicians had 'corrupted and confounded' logical deduction 'governed by the necessary laws of thought' with contingent matter and probability (LL.I.325; and see 319–26). This formal approach, according to Hamilton, is more in keeping with Aristotle's understanding of induction (see LL.I.325–6). It also has important implications for how Hamilton regards both the Sorites and Enthymeme, both of which he discusses in terms of their formal characters as syllogisms. He does of course explain that the Sorites became associated with that sophism or informal fallacy commonly referred to or illustrated by means of the examples of piling up grains of sand until what was once maintained to be a small quantity becomes large (a Progressive Sorites), and also the famous bald man example (Regressive Sorites) (see LL.I.376–8; 464–6). According to Hamilton, the Sorites only became associated with such sophistic or fallacious reasoning some time in the 15^{th} century and the failure of logicians to incorporate the Sorites as a legitimate chain-syllogism was all down to their exclusive concentration on Extension and not keeping in mind that, for Aristotle 'all our general knowledge is only an induction from an observation of particulars' (LL.I.377; 380; and see, 366–85).

His treatment of the Enthymeme is similarly interesting and informative, arguing that it is only the external form of the enthymeme that may be said to be imperfect or incomplete. As Hamilton rightly shows, an enthymeme is not merely an argument in which one of the premises is missing or suppressed; it may also be that the *conclusion* has been suppressed/omitted. But, whether the major or minor premise or the conclusion is not made explicit, this does not, for Hamilton, warrant calling an enthymeme a special or defective syllogism — it 'constitutes no special form of reasoning', nor did Aristotle maintain that it did (LL.I.387). The enthymeme illustrates an important principle that pervades so much of Hamilton's approach to logic, namely, that the mere verbal accident of elision (or in the case of the enthymeme we might say more or less deliberate omission, often serving purposes of persuasion which Hamilton somewhat oddly does not address directly) is something that the logician, in staunchly adhering to the principle of explicitness, can make explicit as something that is in thought, though not in expression (for Hamilton's treatment of the enthymeme, see LL.I.386-94). He provides some nice examples of enthymemes but I shall only quote one of these since it involves a nice quip against Hegel that suggests something of Hamilton's capacity for at least occasional touches of humour: 'There is recorded [...] a dying deliverance of the philosopher Hegel, the wit of which depends upon [its] ambiguous reasoning. 'Of all my disciples,' he said, 'one only understands my philosophy; and he does not.' But we may take this for an admission by the philosopher himself, that the doctrine of the Absolute transcends human comprehension' (LL.I.398). There is in this also more than a hint of not only Hamilton's opposition to Hegel but also to absolutism more generally.

To his credit Hamilton reminds his students of certain points he made much earlier in the lectures to do with the status of propositions with regard to their discrete components, pointing out that a syllogism is an integrated mental act and that it ought therefore to be thought of not in a merely mechanical manner:

> It is [...] altogether erroneous to maintain, as is commonly done, that a reasoning or syllogism is a mere decompound whole, made up of judgments; as a judgment is a compound whole, made up of concepts. This is a mere mechanical mode of cleaving the mental phenomena into parts; and holds the same relation to a genuine analysis of mind which the act of the butcher does to that of the anatomist. It is true, indeed, that a syllogism can be separated into three parts or propositions; and that these propositions have a certain meaning, when considered apart, and out of relation to each other. But when thus considered, they lose the whole significance which they had when united in a reasoning; for their whole significance consisted in their reciprocal relation,–in the light which they mutually reflected on each other. We can certainly hew down an animal body into parts, and consider its members apart; but these, though not absolutely void of all meaning, when viewed singly and out of relation to their whole, have lost the principal and peculiar significance which they possessed as the coefficients of a one organic and indivisible whole. It is the same with a syllogism. The parts which, in their organic union, possessed life and importance, when separated from each other, remain only enunciations of vague generalities, or of futile identities. Though, when expressed in language, it be necessary to analyse a reasoning into parts, and to state these parts one after another, it is not to be supposed that in thought one notion, one proposition, is known before or after another; for, in consciousness, the three notions and their reciprocal relations constitute only one identical and simultaneous cognition. (LL.I.275-6).

The notion of interrelation and simultaneity in the above is important to how Hamilton will go on to view the syllogism's structure as, for example, something in which it is only mere convention that always places the conclusion last, and that the relative positions of major and minor premise themselves can easily be switched around without any loss of meaning. However, I have quoted the above passage at length since it eloquently apprises us of an important general dimension of Hamilton's whole approach to Logic — for all that he is striving for a rigorously pure science free from the extra-logical and although he will later in the 'New Analytic' speak of a 'Symbolic Notation [that will display] the propositional and syllogistic forms, even with a mechanical simplicity', Hamilton constantly opposes the mechanical or rigidly structural in favour of considering the syllogism less like a dismembered material body or constructed building and more like a process — multiplex, organic, interrelated, and even fluid — and his logic needs to be seen as an attempt to capture, not only the dual perspectives afforded by Extension and Intension, but also a greater sense of the richness and complexity of formal reasoning (NA.LL.II.251).

In the previous section I referred to Hamilton's 'Observations on the Mutual Relation of Syllogistic Terms in Quantity and Quality' at Appendix V (e) in the *Lectures on Logic* (see NA.LL.II.285). In this 'Observations' section, he provides the relations between any given proposition's subject and predicate terms and, by inverting the order of the negative propositional forms given in the previous section in Table 2, he displays the best-worst quantification relationships between each of the four affirmative propositions and their corresponding negatives (see NA.LL.II.286). I have reconfigured Hamilton's presentation of these relations as follows:

Best

1.	All A is all B	Toto-total	Identity or Coinclusion
2.	All A is some B	Toto-partial	
3.	Some A is all B	Parti-total	
4.	Some A is some B	Parti-partial	
5.	Some A is not some B	Parti-partial	Non-identity or Coexclusion
6.	Some A is not all B	Parti-total	
7.	All A is not some B	Toto-partial	
8.	All A is not all B	Toto-total	

Worst

The significance of this schema becomes clear when we note how Hamilton's system of quantification brings into consideration certain aspects of syllogistic reasoning formerly ignored by Aristotelian logic. According to Hamilton:

> Former logicians knew only of two worse relations,–a particular, worse than a universal, affirmative, and a negative worse than an affirmative. As to a better and worse in negatives, they knew nothing; for as two negative premises were inadmissible, they had no occasion to determine which of two negatives was the worse or better. But in quantifying the predicate, in connecting positive and negative moods, and in generalising a one supreme canon of syllogism, we are compelled to look further, to consider the inverse procedures of affirmation and negation, and to show [...] how the latter, by reversing the former, and turning the best quantity of affirmation into the worst of negation,

> annuls all restriction, and thus apparently varies the quantity of the conclusion. (NA.LL.II.285-6).

I shall not attempt to explain this in detail. Suffice to say that the above schema of best-worst quantification relationships is used by Hamilton to construct what he calls his 'General Canon' to determine the relationship between the subject (S) and predicate (P) of the conclusion in a syllogism, on the basis of a best-worse comparison between the relationships between the subject term and middle term (M), and the middle term and predicate term of the syllogism. Hence, depending on the relationship between the subject and predicate terms of a syllogism, the relationship between S and P in the conclusion will be, for example, *totally coexclusive*, or *partially coexclusive*, or *toto-partially coexclusive*, and so on. Hamilton's 'one supreme canon of syllogism' is given as follows:

> General Canon. What worst relation of subject and predicate, subsists between either of two terms and a common third term, with which one, at least, is positively related; that relation subsists between the two terms themselves. (NA.LL.II.285)

He translates this 'General Canon' into twelve clear rules governing 36 syllogistic moods in the first three figures (the fourth figure having been rejected as pointed out above), plus 24 negative moods — these forms are detailed in the final table given at the end of the *Lectures on Logic* (see LL.II.475). However, it might be better to think of these not as rules but rather as instantiations of his General Canon since, once one has grasped how Hamilton's system operates, each of them can be translated from this Canon with relative ease (see NA.LL.II.285-9). I shall not discuss these instantiations in any detail but will instead use just one of them to illustrate how Hamilton's General Canon operates, using an example which Fogelin uses to demonstrate the correctness of his reading of propositions (1) and (8) as discussed in the previous section.

Fogelin gives the following syllogism as an example that he claims Hamilton's system allows as valid, yet which can be shown, using a Venn diagram, to be invalid if translated using his earlier translation of proposition (8), though valid using de Morgan's interpretation of this parti-partial coexclusion proposition:

Some P is not any M
Some S is some M
$\therefore$ Some S is not some P

Having already accepted Fogelin's treatment of the above syllogism which shows how it can be read as valid (and hence what proposition (8) expressing Parti-partial coexclusion means, or can be read as meaning), all I want to do here is merely illustrate how Hamilton's General Canon can be invoked to produce from the two premises the conclusion as given. I think that all this will demonstrate is that Fogelin is right that the above syllogism would be declared valid in Hamilton's system as he asserts. However, I am not entirely sure that Hamilton's table and list of interpretations of his General Canon are completely free from anomalies, and it must be said that to check this thoroughly, given the complexities of co-ordinating various mixtures of Extensive and Intensive readings with Hamilton's own symbolic representation of the negative moods, is beyond the scope of this chapter. Hence, I shall simply give Fogelin's example as unequivocally exemplifying adherence to Hamilton's General Canon, even though by treating this so simplistically I may be overlooking certain niceties and complexities concerning how it ought to be used:

Fogelin's Example:	***Relation:***	***Hamilton's rule from General Canon:***
some P is not any M	Parti-total coexclusion	VII.a: *A term parti-totally coexclusive, and a term partially coinclusive, of a third [M], are partially coexclusive of each other.* (NA.LL.II.288)
some S is some M	Parti-partial conclusion	
some S is not some P	Parti-partial coexclusion	

From what I have been so far able to ascertain, though not articulate in full, certain possible anomalies (or doubts that I still have) notwithstanding, Hamilton's General Canon does seem to work.

If, in agreement with Fogelin, I am right about this, Hamilton's system marks a most significant advancement in the treatment of the syllogism that almost entirely replaced the previous systems which Hamilton so often decried as being imperfect, flawed, confused, and misleading. No wonder that Fogelin, though acknowledging that Hamilton's achievement may not seem important 'from a modern point of view', regards his quantification system as 'a radical departure from traditional theory' (Fogelin, pp. 163–4). If Hamilton's ultimate regulation or General Canon of syllogistic forms is fully adequate, comprehensive, sufficiently general, and flexible, his treatment of the syllogism arguably marks the considerable improvement on and replacement of traditional logic's treatment of the syllogism that he himself believed it did. In place of numerous logical rules and botched attempts to bring the Aristotelic system out of the chaos in which Hamilton found it, by judiciously interpreting but not slavishly falling under the spell of Aristotle's enormous authority, Hamilton may well have developed, albeit within certain limitations, a system of formal logic that, as it culminated in a simple mechanism for making all quantities in both affirmative and negative propositions explicit, only required the capstone of his single or supreme canon of the syllogism, to warrant his grand claim that he had produced a system whose beauty resided in the very naturalness of being '*as One at once and Various.*'

6 CONCLUSION

> A mere knowledge of the rules of Rhetoric can no more enable us to compose well, than a mere knowledge of the rules of Logic can enable us to think well. (LL.I.48-9)

Augustus de Morgan deeply misunderstood the complexities that Hamilton's system could either accommodate or was pointing towards. As Fogelin claims, though de Morgan clearly did try to understand Hamilton's system 'he failed to do so and commentators since have hardly done better' (Fogelin, p. 162). According to Fogelin this has much to do with the strangeness of Hamilton's language, such as we find in his non-standard exchanges of 'all' for 'any'. True, these things do create problems, as also do Hamilton's now strange symbolism, dense tables, and the fragmentary nature of much of his work on Logic, excepting the Lectures which are abundantly clear, always informative, and even occasionally somewhat entertaining. However, I suspect that the linguistic difficulties have more to do with Hamilton's enormous ability to compress the language of his texts into an often overly taut, quasi-litigious style that is at first off-putting and certainly at times not for the faint-hearted. However, there are some more substantial reasons why de Morgan and others have floundered and in the end did Hamilton a great disservice.

In order to gain more that a foothold in Hamilton's logic it would seem that one also has to have a foothold in and possibly be to some extent persuaded by his metaphysical standpoint of *natural dualism*, for in this doctrine's opposition to absolutism — not least of all in its opposition to absolute scepticism — inheres Hamilton's relativity and the germ of what becomes a much more pervasive *correlativism* throughout so much of his writing. Hamilton's relativity places subject and predicate into simple relations of equation and non-equation, distinguishes indefinite quantities into two kinds or degrees of indefiniteness, and is founded on recognising the significance of perspective with regard to a given concept's dual quantification dimensions of extension and intension. All this seems to be an attempt to frame subject and predicate as the unity of thought from which they originate. However, the frame is really much larger, and herein lies a great part of the problem and yet potential of Hamilton's system. A part of Hamilton's relativism also brings into play not just perspective (and thereby different possible readings of propositions), but both the relations between a concept's attributes and the conditioning nature of the human subject's agency. The relations between logic (the laws that constitute the conditions of thought) and language (the terms without which we would otherwise be incapable of participating in any productive thought, argument, discourse, analysis, or articulation of logic's laws, propositional forms, and so on), and the surrounding chaos or ever-shifting sands against, and yet in relation to which, logic and language comprise our attempts to make whole, divide, and recombine this otherwise unintelligible plenum of indeterminate entities under which we are continually at risk of being submerged — as these relations form the frame or field within which Hamilton's logic is conducted, he was constructing a logical system both highly suggestive of a deeper relativism he eschewed, and yet which he virtually postulated was the condition within which logic had to operate as the very function of logic and language had to do with stabilising the multiple within unity. This, perhaps more than anything else, makes his writing complex. However, much more simply, Hamilton's enormous erudition — also a factor in the diversity he attempts to bring into order — remains an intimidating force that few, excepting some of his more devoted students and followers, have found a congenial companion.

Hamilton regarded philosophy — particularly logic and metaphysics — as the greatest gymnastic of the mind and with such a conception of his subject matter providing a large part of philosophy's *raison d'être*, he was attempting to set his students on a course of study that would challenge their intellectual abilities to their utmost through his attempts to make the study of Logic measure up to and (it must have been his hope) compete with the higher educational, intellectual, and scholarly standards that had been demanded on the Continent. The German logicians in particular, made even the Professors of Britain's one-time most prestigious university, Oxford, appear to Hamilton's eyes as little better than intellectual philistines or dilettantes. In his definition of Logic as a pure science of the necessary laws of thought, single-handedly Hamilton was laying the groundwork for improving upon and in many ways replacing the traditional Aristotelian logic that he regarded as having been sunk into a mass of confusion by centuries of almost slavish or insufficiently critical adherence to the Stagirite, coupled with numerous misinterpretations of the true nature of their subject.

Within the boldness of Hamilton's emphatic assertions and robust denunciations of the errors and confusions that he argues arose due to 'the passive sequacity of the logicians [in following] obediently in the footsteps of their great master', Aristotle, there is a genuine sense of his excitement concerning this claim that his system supersedes all previous systems new and old (NA.LL.II.262). By this stage in his work on logic it must have seemed to Hamilton as though, after many long years of arduous industry during which he had been diligently examining, summarising, and critically assessing the logics of others written mainly in Greek, Latin, English, and German, at last the numerous errors and confusions riddling centuries of Aristotelian logic could be removed. Through the earlier works of ancient and scholastic logicians, these errors and confusions had continued to sift themselves, but they also permeated the work of his Oxonian contemporaries. But now, after years of clearing away the detritus of former ages and chastising the dilettantism of more recent logicians, he could with one final push radically brush aside those aspects of the traditional logic that had significantly impeded its development or evolution into a system at once more complex and yet more orderly (see NA.LL.II.252). At last he had constructed a robust system that he could proudly advocate as the 'keystone in the Aristotelic Arch', and while this keystone had doubled the number of propositional forms, these forms constituted a structural whole based on the logic of Aristotle and thus at once the traditional logic was brought one significant step closer to the beauty and perfection which Aristotle's work seemed to promise but had not realised (see NA.LL.II.249). However, Fogelin rightly counters Hamilton's claim that his 'New Analytic' completes traditional Aristotelian logic — for, instead of merely placing the keystone in the Aristotelic arch, 'By introducing an entirely different system of classifying propositions in virtue of their potential roles in syllogisms, Hamilton made a radical departure from traditional theory' (Fogelin, p. 163). The keystone that is Hamilton's quantification of the predicate one might thus say, is far from being a pretentious claim to glory undeserved — rather, Hamilton's 'keystone' is an overly modest description for a much more radical, yet substantial and more stable construction that involved the destruction and removal of logical ruins from bygone ages, though it is plain to see that at least some of the foundations were incorporated to support Hamilton's arch. But with a final and fitting twist of irony, all too quickly the course of logic would develop during the 19^{th} century in ways that left so much of Hamilton's endeavour far behind as a curiously flawed relic.

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