

Russell, Wittgenstein and Synthesis in Thought

Summary

Wittgenstein held that Russell's multiple relation theory of judgment fails to explain an atomic judgment's representation of entities as combined. He demonstrated this failure as follows. Under the multiple relation theory, an atomic judgment is a complex whose relating relation is *judgment*, the universal, and whose terms include the entities the judgment represents as combined. Taking such a complex we may arrive through the substitution of constituents at a complex whose relating relation is again *judgment* but whose terms do not include entities which are logically suited for combination. This second judgment complex will not represent any of its terms as combined, for entities that are logically uncombinable are unrepresentable as combined. Russell's theory does not, however, explain how the original judgment differs from the complex arrived at by the substitution of constituents such that the former but not the latter represents certain of its terms as combined.

1 Complexes, Substitutability and Judgment

1.1

The multiple relation theory of judgment sits within the context of Russell's general theory of complexes. Let's begin with a brief discussion of this general theory, as held in and around 1913, before turning our attention to judgment.

'A complex', Russell writes, is 'anything which has *constituents*' (Russell 1984: 79). Such constituents are said variously to figure, occur, appear or enter in the complex which they constitute. Further, Russell talks of *modes* of occurring (figuring etc.) in complexes. A constituent of a complex does not merely occur in the complex: it occurs there *in a particular way*. So far this remains somewhat abstract; Russell goes on, however, to provide some examples of modes of appearance. Most centrally, Russell discusses the modes of appearance found within members of a particular family of complexes – the family, that is, of atomic complexes. A Russellian atomic complex may have any number of constituents. Such a complex of $n+1$ constituents consists of n entities appearing as self-standing units and a single entity appearing in such a way as to go between and relate together the n self-standers. The appearance as self-standing unit Russell calls appearance as term; to go between and relate together n entities appearing as term is to appear as n -relating relation.¹

The idea of a mode of appearance in complexes is of central importance to Russell's metaphysics. Indeed, Russell types his entities by reference to such modes. A *term*, for instance, is defined as an entity which can appear in a complex as term. An *n-ary relation* is an entity which can appear as n -relating relation. A unary relation Russell calls a *predicate*, relations of all orders are called *universals*, and terms which are not universals are called *particulars*. So for example we find:

An entity which can occur as “precedes” occurs in “A precedes B” will be called a *relation*. (Russell 1984: 80)

A particular is defined as an entity which can only enter into complexes as the subject of a predicate or as one of the terms of a relation, never itself as a predicate or relation. (Russell 1984: 55-6)

The idea here is quite general: an entity’s ‘logical type’ is a matter of the ways in which it can and cannot appear within complexes.

This idea provokes an obvious question. To say that an entity *e* can appear in complexes in mode *m* is to say that a complex is possible in which *e* appears in *m*. Trivially, then, there is no complex possible in which Socrates is related to Plato by something that is not a dual relation – by Aristotle, say. Such an ‘impossible complex’ would ‘go against’ the nature of Aristotle as a particular. But is it the case that so long as the elements of a complex are not there playing roles that ‘go against’ their type, the complex will be possible? A dual relation is an entity such that there are two terms it can relate, but are we further to take it that a dual relation is capable of relating any two terms? A ready response to this question begins with the thought that so long as the types in view are defined in terms *only* of modes of complex appearance (as they indeed are), and not in terms, say, of specific entities or ranges of entities, then the only thing that ‘goes against’ an entity’s type will be its appearing in a mode which is given as impossible for that type. If, on the other hand, a mode is given as possible for its type, an entity’s appearing in that mode will ‘be in line with’ its type no matter what other entities are in play in the complex. Next, we may take it in the context of Russell’s theorising that an entity has a ‘fully determinate type’ – a type, that is, such that for any mode, the mode will either ‘go against’ or ‘be in line with’ it. (The type ‘term’ is not a fully determinate type: some terms can appear as dual relating relation, others cannot.) This is a straightforward assumption to make: it is very hard to see what Russell might be doing here if he were to hold that only certain abilities and inabilities to appear in complexes in a certain mode go to characterise types of the species he has in view. Finally – and here is the key step – we can note that ‘being in line with’ and ‘going against’ (determinate) entity types *make in themselves* for a sense of possibility and impossibility: they make for *possibility/impossibility so far as the entity types are concerned*. If *a* and *b* are terms and *r* is a dual relation then *so far as the types of the three entities are concerned* it is possible for *r* to relate *a* and *b*.

The weight of this perspective on complex possibility may be appreciated by emphasising that the types under discussion here are theoretically basic for Russell. An entity’s ability to figure in a certain way in complexes is not explicable by reference to some further, more primitive feature of that entity. Possibility so far as such entity types are concerned is thus possibility so far as basic natures of entities are concerned. Above, we called these basic natures *logical*; such possibility, such ‘being in line with’ and ‘going against’ basic natures, can correspondingly be called *logical* possibility. Adopting this terminology, then, it will be logically possible for *being next to* to relate Socrates and Plato, because *being next to* is a dual relation and Socrates and Plato are terms. Similarly, so long as *mortality* is a term – which it is for Russell – then it will be logically possible for Socrates to be next to *mortality*.² What is logically impossible is for Socrates to be in between Plato – that is, for there to be an atomic complex in which Socrates and Plato are related by *being in between* – for *being in between* is not a dual relation. Similarly, it is logically impossible for Socrates and Plato to combine as subject and predicate respectively: Plato is a particular and so not a predicate.

It will be useful for us to recast this thought. Along with the idea of entities appearing in complexes in a variety of modes, Russell talks also of a constituent's *position* within a complex, of the *substitution* of entities within complexes, and of a complex's *form*:

A complex has a property which we may call its "form", and the constituents must have what we call determinate "position" in this form. (Russell 1984: 81)

[T]wo complexes have the same form if the one becomes the other when the constituents of the other are successively substituted for the constituents of the one. This might be thought to constitute a definition of "having the same form", but in fact it does not; for it is necessary that the substituted term should be in the same position in the new complex as the old term occupied in the old complex, and the sameness of position thus involved cannot be explained without the notion of form. (Russell 1984: 113)

A complex has a form – a structure, as Russell sometimes calls it (Russell 1984: 114) – within which its constituents occupy various positions. With this idea goes an idea of the substitution of entities. We may substitute a constituent of a complex for another entity to obtain a new complex differing from the old in that the position occupied in the old complex by the entity substituted out is occupied in the new complex by the entity substituted in. These ideas connect relatively straightforwardly for Russell to the notion discussed above of an entity's appearing in a complex in a certain mode, for forms are identified with – or at least put into immediate correspondence to – patterns together of modes of appearance. There is, for example, such a thing as 'the form of all subject-predicate complexes' (Russell 1984: 114), and again as 'the form of dual complexes' (Russell 1984: 115). There is, this is to say, a subject-predicate form shared by all complexes in which one entity appears as term and a second as unary relating relation. This form has two positions: the subject position and the predicate position. The subject position is a term position, a position such that an entity occupying it appears in the complex as term; the predicate position is a unary relating relation position, a position such that an entity occupying it appears in the complex as unary relating relation.

This expanded terminology provides for a recasting of the above thought that, under an obvious notion of logical possibility, a complex in which the constituents' modes of appearance are 'in line with' their basic natures as complex constituents is a logically possible complex. In terms now of substitution this point may be made as the following principle (S) of substitutability:

- (S) If there is a logically possible complex A in which entity e1 appears in a mode m (e.g. as term, as dual relating relation) and a logically possible complex B in which a second entity e2 appears in that same mode m, then there is a logically possible complex A' which is the result of substituting e2 in for e1 in any position in which it occurs in A in mode m.

Someone who resists the thought as originally cast commits to the contradiction of entities whose basic natures as complex constituents both suit them and do not suit them to play certain roles together in a complex. Alternatively, now, it may be remarked that if a complex position is, say, a term position – if it is a position such that something occupying it appears in the complex as term – then an entity whose fundamental nature as a possible complex constituent suits it to appear in complexes as term is an entity whose fundamental nature suits it to appear in that position. Principle (S) is implicit in Russell's basic thinking about the nature of complexes and their constituents.

1.2

Let's move on from the general theory of complexes to Russell's theory of judgment. The first thing to note in making this move is Russell's identification of complexes with facts.³ The atomic complex 'Socrates' love for Plato' – a complex in which Socrates and Plato appear as term and *love* appears as relating relation – is the atomic fact that Socrates loves Plato. It is, this is to say, what whose existence provides the truth condition of a judgment that Socrates loves Plato.⁴ So Russell wrote in 1910:

If A loves B, there is a such a complex object as 'A's love for B', and vice versa; thus the existence of this complex object gives the condition for the truth of the judgment 'A loves B'. (Russell 1966: 157)

As for the judgment itself, Russell famously proposes that S's judgment that A loves B is an atomic complex whose terms are S, A, *love* and B, and whose relating relation is *judgment*, the universal. More generally, Russell makes the following claim:

(J) A judgment by S whose truth condition is the existence of an atomic complex c is itself an atomic complex whose terms include S and the constituents of c and whose relating relation is *judgment*.

Russell calls atomic complexes of more than two terms *multiple relation complexes*. Russell's claim (J) is the heart of what is therefore known as his *multiple relation theory of judgment*.⁵

(J) is motivated and developed by Russell in a number of interesting ways. For the most part, however, we shall not explore such matters. Our focus will rather be on Wittgenstein's criticism of the multiple relation theory. The most straightforward formulations of this criticism are found in the 1913 *Notes on Logic* where Wittgenstein writes:

Every right theory of judgment must make it impossible for me to judge that this table penholders the book. Russell's theory does not satisfy this requirement. (Wittgenstein 1979: 103)

And again:

A proper theory of judgment must make it impossible to judge nonsense. (Wittgenstein 1979: 95)

What Wittgenstein has in mind here by a nonsense judgment, I shall suggest, is a judgment whose truth condition involves an entity's appearing (or not appearing) in a manner which goes against its basic logical nature. In Russell's terms, and scoped to his atomic judgments, a nonsense judgment is a judgment whose truth condition is the existence of a 'logically impossible atomic complex' – an atomic complex, that is, in which an entity figures in a way which 'goes against' its basic nature as discussed above. An example of such a complex would be that in which a table is related to a book not by the dual relation *bigger than* but by a penholder, or that in which Desdemona is related to Cassio not by the relation *love* but by the particular Iago. Wittgenstein complains that Russell's theory is consistent with the existence of such nonsense judgments.

1.3

It is not immediately clear either how Russell's theory is to be consistent with the possibility of nonsense judgment or why Wittgenstein found this consistency objectionable. There is, however, something we can note right away which appears to be very much in the area of Wittgenstein's complaint. According to (J), Othello's judgment that Desdemona loves Cassio is a complex one of whose constituents is *love*, the universal. Indeed, *love* figures in the complex that is Othello's judgment as term. Given (S), then, *love* may here be substituted for any other term, including the particular Iago: there is a logically possible complex which is the result of substituting Iago for *love* in Othello's judgment. Similarly, there is a logically possible complex which is the result of substituting a penholder for *bigger than* in a judgment that a certain table is bigger than a certain book. Leaving to one side for now the surrounding content of Wittgenstein's objection, The proposal I want to make in this section is that Wittgenstein brought the principle (S) of substitutability to bear in this way on Russell's thesis (J).

Before pursuing this idea, we should note that Russell's commitment to principle (S) is not something he explicitly recognises at the time of Wittgenstein's criticism. Indeed, despite writing in 1903 that 'it is characteristic of the terms of a proposition [a complex] that any one of them may be replaced by any other entity [term] without our ceasing to have a proposition' (Russell 1992b: 45), Russell spends considerable effort in his 1913 work *Theory of Knowledge* theorising about complexes in ways which contravene precisely this (narrowed) version of (S).⁶ My expanded suggestion will thus be as follows. In *Theory of Knowledge* – the work under way when Wittgenstein first made his criticism – Russell was under the impression that he had the right to disallow certain substitutions that (S) would allow. More particularly he was under the impression that he could disallow certain unwelcome looking substitutions in his multiple relation judgment complexes. In his objection to the multiple relation theory, however, Wittgenstein pressed upon Russell his commitment to (S). The criticism was thus doubly painful for Russell: undermined was not only his theory of judgment but also much of his more general theorising at the time about complexes. The principle support for this suggestion, beside the consideration of section 1.1 above that it is incoherent of Russell to deny (S), derives from Russell's reaction to Wittgenstein's attack.

It is notable that from the time of the impact of Wittgenstein's criticism Russell drops all theorising at odds with the principle of substitutability.⁷ This is, however, only suggestive. What provides the strongest evidence for my proposal is Russell's discussion of judgment in his 1918 lectures *The Philosophy of Logical Atomism*. He writes there:

'Othello believes that Desdemona loves Cassio.' There you have a false belief. You have this odd state of affairs that the verb 'loves' occurs in that proposition and seems to occur as relating Desdemona to Cassio whereas in fact it does not do so, but yet it does occur as a verb, it does occur in the sort of way that a verb should do. I mean that when A believes that B loves C, you have to have a verb in the place where 'loves' occurs. You cannot put a substantive in its place. Therefore it is clear that the subordinate verb (i.e., the verb other than believing) is functioning as a verb, and seems to be relating two terms, but as a matter of fact does not when the judgment happens to be false. (Russell 1968: 225)

In Othello's belief that Desdemona loves Cassio *love* occurs not as term but in a manner proper to verbs (relations). This is given in the fact that 'you have to have a verb in the place where 'loves' occurs. You cannot put a substantive in its place.' What can be replaced in a complex only by a relation, Russell asserts in 1918,

must be appearing there in a manner proper to relations only. *Love* can be replaced in A's belief that B loves C only by a relation and so it must appear there not as term but in a manner proper to relations.

So whilst Russell engaged in theorising opposed to (S) in 1913, by 1918 the principle is deployed at the forefront of his reasoning. And with this new endorsement of (S) comes a repudiation of the multiple relation theory of judgment:

[A] main thing one wants to notice in this matter [of judgment is] ... the impossibility of putting the subordinate verb on a level with its terms as an object term in the belief. That is a point in which I think that the theory of judgment which I set forth once in print some years ago was a little unduly simple, because I did then treat the object verb as if one could put it as just an object like the terms, as if one could put 'loves' on a level with Desdemona and Cassio as a term for the relation 'believe'. (Russell 1968: 226)

Unwilling to accept the result of substituting Iago for *love* in Othello's belief that Desdemona loves Cassio, Russell rejects his earlier theory in which *love* appears in Othello's belief as term on a level with Desdemona and Cassio. His multiple relation thesis (J) is thrown out.

The question now opens up for Russell: how then does *love* occur in Othello's belief? It must occur *as a relation* – in a manner proper to those entities capable of appearing as relating relations – but the only manner proper to relations in the theory of atomic complexes is that of relating relation. And *love* certainly can't occur there in that manner, as actually relating Desdemona to Cassio, for as Russell notes the possibility of false judgment would then be foreclosed. What we must have here, therefore, is a way of appearing in complexes that is neither as term nor as relating relation. We must have here, that is to say, complexes of a form quite different from that of any atomic complex:

You cannot get in space any occurrence which is logically of the same form as belief. When I say 'logically of the same form' I mean that one can be obtained from the other by replacing the constituents of the one by the new terms. If I say 'Desdemona loves Cassio' that is of the same form as 'A is to the right of B'. Those are of the same form, and I say that nothing that occurs in space is of the same form as belief. I have got on here to a new sort of thing, a new beast for our zoo, not another member of our former species but a new species. The discovery of this fact is due to Mr. Wittgenstein. (Russell 1968: 225-6)

It can be shown* that a judgment, and generally all thought whose expressions involves *propositions*, must be a fact of a different logical form from any of the series: subject-predicate facts, dual relations, triple relations, etc. In this way, a difficult and interesting problem of pure logic arises, namely the problem of enlarging the inventory of logical forms so as to include those forms appropriate to the facts of epistemology.

* As I have come to know through unpublished work of my friend Mr. Ludwig Wittgenstein. (Russell 1984: 46)⁸

This task of enlarging the inventory of logical forms, of giving a theory of the relevant types of non-atomic complexes, was not, however, pursued.

1.4

Let's take stock. We have suggested that in his attack on Russell's theory of judgment, Wittgenstein pressed upon Russell his obligation to the principle (S) of substitutability. As a consequence of this obligation, Wittgenstein pointed out, there is the possibility for Russell of substituting Iago for *love* in Othello's belief. We

have also given an explanation of Russell's devastation at Wittgenstein's attack. This was due both to Russell's recognition of his commitment to the principle of substitutability, and so of the invalidity of a good part of his thinking at the time about complexes, and also to the loss of his multiple relation theory. Russell takes it that you have to have a dual relation in the place where *love* occurs in 'A believes B loves C', and so, given the principle of substitutability, that his proposal (J) must be rejected. It remains to be answered, however, *why* you must have a dual relation where *love* occurs in 'A believes B loves C'. Why not keep the multiple relation theory and accept the logical possibility of substituting Iago for *love* in Othello's jealous judgment? To provide an answer to this question will be the major task of the remainder of this paper. Our focus in this task will be more on Wittgenstein than on Russell: rather than investigating why Russell set himself against the possibility of substituting Iago for *love* in Othello's judgment, we shall be concerned principally to see what role was played by that possibility in Wittgenstein's rejection of Russell's theory.⁹

2 Synthesis in Thought

2.1

Russell developed his basic multiple relation judgment idea in a number of ways. The most striking of these is made in the 1913 manuscript *Theory of Knowledge*:

Thus, if we call the subject *S*, and the relating relation (of which "understanding" is presupposed by all the others) *U*, and the objects *x*, *R*, *y* (taking the case of a proposition asserting a dual relation for the sake of illustration), and γ the form of dual complexes, the total complex which occurs when the subject has the relation *U* to the objects in question may be symbolised by $U(S, x, R, y, \gamma)$. (Russell 1984: 115)

Under earlier versions of the multiple relation theory the terms of a multiple relation judgment were exhausted by the judgment's subject and the constituents of the complex whose existence is its truth condition. In 1913, however, Russell includes also the form of this latter complex. The aim of this section is to explain why he does this. Russell holds for quite general reasons that forms are terms, but why should multiple relation judgment complexes contain forms appearing as term?

Russell accounts for his new proposal as follows:

Suppose we wish to understand "*A* and *B* are similar". It is essential that our thought should, as is said, "unite" or "synthesize" the two terms and the relation; but we cannot *actually* "unite" them, since either *A* and *B* are similar, in which case they are already united, or they are dissimilar, in which case no amount of thinking can force them to become united. The process of "uniting" which we *can* effect in thought is the process of bringing them into relation with the general form of dual complexes. (Russell 1984: 116)

The introduction of form terms to multiple relation judgments is to account for an atomic thought's 'uniting' or 'synthesizing' the constituents of the complex whose existence is its truth condition. Russell believes there to be a certain synthesis in thought, and his account of this phenomenon is that a thought's terms include a form.

Looking to understand this, a first thing to note is that a complex's form is talked of in *Theory of Knowledge* as the 'mode of combination' of its constituents (see, e.g., Russell 1984: 98). This makes good sense. For a complex to be a subject-predicate complex – for it to be of the subject-predicate form – is for it to

have two constituents one of which appears as term and the other as predicate (as unary relating relation), and considered together, these ways of appearing are a way of combining. Taking on board this perspective on forms, then, we can see Russell's suggestion to be that combining things in thought is a matter of having them together in thought with a mode of combination. But of what notion of 'combining in thought' is this intended as an account?

First, we should be clear to separate the current issue of unity in thought from the general issue of the unity of a complex. It is an ongoing issue in Russell how a complex is to be distinguished from the mere collection of its constituents. But Russell is not worried here about how the complex that is Othello's judgment that Desdemona loves Cassio is to be distinguished from the collection of its constituents – from the collection, that is, of *judgment*, Othello, Desdemona, *love* and Cassio. The unity in question regarding S's judgment is rather a unity *within* the judgment, and it is a unity not of all five but of just three of its constituents: Desdemona, *love* and Cassio.¹⁰ Next, we are told that the phenomenon in view is not that of an actual synthesis. If Desdemona, *love* and Cassio are not actually united into the complex 'Desdemona loving Cassio' then they do not become so by Othello's believing that Desdemona loves Cassio. Othello can believe that Desdemona loves Cassio even if Desdemona does not in fact love Cassio. A further handle on the matter may be gained by asking what kind of mental act would *not* involve a synthesis of the relevant kind. Here the suggestion might be of acts of thinking of objects one by one.¹¹ But still, what are these contrast cases contrasting with? A few sentences further on in his manuscript Russell writes:

[W]hen we are concerned with a proposition which may be false, and where, therefore, the actual complex is not given, we have only, as it were, the "idea" or "suggestion" of the terms being united in such a complex; and this, evidently, requires that the general form of the merely supposed complex should be given. (Russell 1984: 116)

In thought, entities are not actually united and so an actual complex is not given; what we have rather is the 'idea' or 'suggestion' of entities being united. In a belief that A and B are similar we have the suggestion or idea that A, B and *similarity* are combined into the complex 'A similar to B'.

Taking the above points together, Russell's notion of 'synthesis in thought' may, I suggest, be clarified as concerning *representation*. Consider a belief that A is red. For Russell this belief contains the entities A and *redness*. More than this, though, there is in the belief the suggestion of these two entities being united into the complex 'A is red'. This is not to say, of course, that within the belief A and *redness* are *actually* united together into the complex 'A is red' – that would rule out the possibility of falsity. What it means, rather, is that within the belief A and *redness* are *represented* as united together into the complex 'A is red'. So Peter Hylton writes:

The judgement must represent the objects as combined in the same way that they are actually combined in the corresponding fact (if there is one). If the judgment does this simply by bringing it about that the objects are so combined, then this no longer appears to be uniting them in thought only. ... How can we give a meaning to 'unite in thought' which keeps this notion clearly distinct from uniting in reality? Russell's answer is that the judgement represents the constituents as combined in the right way not by so combining them but by including 'the way they are to be combined' as a further entity, the logical form, which the judging mind combines with the others. (Hylton 1990: 345-6)

The suggestion is as follows. A judgment that certain entities are combined in a certain way – a judgment whose truth condition is that certain entities combine in a certain way – is a judgment that represents those entities as so combined. (So much is, I take it, pleonastic: to have the truth condition that certain entities combine in a certain way *is* to represent those entities as so combined.) This representation/having a truth condition is something Russell is concerned in *Theory of Knowledge* to explain. More, given that he is committed to an atomic judgment that p being a multiple relation complex whose existence is sufficient for its being judged that p – he is committed, that is, to its being internal to a multiple relation judgment complex that it represents what it does –, Russell is concerned to explain such a judgment’s representation of entities as combined by reference to an internal feature of the complex. The account Russell offers in 1913 of such representation is that a judgment with the truth condition that certain entities combine in a certain mode contains amongst its terms not only those entities but also that mode of combination.

2.2

We might worry here that simply including the way things are represented as standing amongst the constituents of a judgment does not obviously do the required work – it is not obvious how this explains the fact that the judgment represents things as standing together in the included way. A thought’s synthesising certain things might seem rather different from its containing both those things and a mode of synthesis. Leaving such concerns to one side, however, let’s turn instead to the context of Russell’s 1913 proposal.

Russell’s discussion in *Theory of Knowledge* of the inclusion of form terms within multiple relation judgments begins with a repudiation of an earlier version of the multiple relation theory as given in his 1912 *The Problems of Philosophy*:

What is the proof that we must understand the “form” before we can understand the proposition? I formerly held that the objects alone sufficed, and that the “sense” of the relation of understanding would put them in the right order; this, however, no longer seems to me to be the case. Suppose we wish to understand “*A* and *B* are similar”. It is essential that our thought should, as is said, “unite” or “synthesize” the two terms and the relation... (Russell 1984: 116)

In *The Problems of Philosophy* Russell had written the following:

It will be observed that the relation of judgment has what is called a ‘sense’ or ‘direction’. We may say, metaphorically, that it puts its objects in a certain *order*. (Russell 1998: 73)

When an act of believing occurs, there is a complex, in which ‘believing’ is the uniting relation, and subject and objects are arranged in a certain order by the ‘sense’ of the relation of believing. ... When the belief is *true*, there is another complex unity, in which the relation which was one of the objects of the belief relates the other objects. Thus, e.g., if Othello believes *truly* that Desdemona loves Cassio, then there is a complex unity ‘Desdemona’s love for Cassio’, which is composed exclusively of the *objects* of the belief in the same order as they had in the belief... . (Russell 1998: 74)

Russell’s proposal in 1912 is that the judgment’s relating relation makes it the case that the judgment represents certain of its terms as combined in a certain way. A judgment’s relating relation puts its terms into a certain semantic order. This proposal was never elaborated: at no point did Russell say *how* the relating relation effects the semantic ordering. Rather than asking what Russell might have said here, however, I should like to examine

the chronology of events immediately surrounding Russell's expression on 24 or 25 May 1913 of his change of mind.

On 20th May 1913, whilst Russell was in the midst of writing part I of *Theory of Knowledge*, Wittgenstein came to Russell 'with a refutation of the theory of judgment which I [Russell] used to hold'.¹² Three or four days later, on 23rd or 24th May, Russell moved on in his work to part II chapter I of *Theory of Knowledge*, that part in which judgment is first broached.¹³ There Russell remarks the need to explain a synthesis in judgment, suggests that his earlier theory of *The Problems of Philosophy* failed to meet this need, and proposes instead that judgments contain forms as one of their terms. The suggestion is thus available that Russell's rejection in *Theory of Knowledge* of the 1912 version of his theory of judgment was prompted by Wittgenstein. Indeed, this suggestion would seem most probably true. Wittgenstein's refutation of 20th May would, we may suppose, be of the theory of judgment of *The Problems of Philosophy*: there was no version intervening between that version and the as yet uncomposed version of *Theory of Knowledge*, and Wittgenstein would most likely address himself to Russell's most recent account. If this is granted, however, then to deny that the two refutations are the same, or at least closely related, would mean supposing a considerable coincidence of timing between Wittgenstein giving his refutation to Russell and Russell three or four days later penning for the first time a quite different objection to the same theory. Russell nowhere gives or discusses any objection to his theory of *The Problems of Philosophy* other than that written so shortly after the encounter with Wittgenstein.

Moving the chronology on, Wittgenstein visits Russell again on 26th May:

I [Russell] showed him a crucial part of what I had been writing. He said it was all wrong, not realizing the difficulties – that he had tried my view and knew it wouldn't work. I couldn't understand his objection – in fact he was very inarticulate – but I feel in my bones that he must be right, and that he has seen something I have missed. (Russell 1992a: 459)

Following this meeting, in the week leading to 18th June 1913, Wittgenstein writes to Russell saying 'I can now express my objection to your theory of judgment exactly' (Wittgenstein 1995: 29). Wittgenstein then makes for the first time his complaint that Russell's theory does not rule out the possibility of nonsense judgment.¹⁴

Considering these later events, we may infer from Wittgenstein's letter that the 'crucial part' of his manuscript shown by Russell to Wittgenstein on 26th May included Russell's *Theory of Knowledge* version of the multiple relation theory of judgment, written only a day or two before that meeting. This inference fits happily with the suggestion made just above that Russell's 1913 rejection of the theory of judgment of *The Problems of Philosophy* derived at least partially from Wittgenstein: Russell is showing Wittgenstein how he proposes to deal with the difficulty Wittgenstein had raised six days earlier for his 1912 version of the multiple relation theory. Indeed, a nicely cohering picture of the whole episode may now be drawn. On 20th May Wittgenstein puts to Russell that there is a synthesis in thought which is not successfully accounted for by his 1912 theory. Russell agrees and on arriving on 23rd or 24th May in his work at judgment rejects that theory and pens a new version in which a form is added to a judgment's terms in order to effect the required synthesis. On 26th May Russell shows Wittgenstein this new idea. Wittgenstein objects inarticulately, telling Russell that the suggestion 'wouldn't work', that adding a form to the terms of a multiple relation judgment won't make for the judgment's synthesis. Later, Wittgenstein finds himself able to make his objection more clearly, and complains of Russell's theory that it does not rule out nonsense judgment.

Plausible as it may be, this history of the engagement between 20th May and 18th June is of course somewhat speculative: it is supported in part by only circumstantial evidence. Its purpose, however, is not to carry any great weight of exegesis. What is hoped for rather is a clue as to where and how we might look to connect together what may have so far appeared rather disparate lines of concern with Russell's theory, namely the concern with a synthesis in thought and Wittgenstein's nonsense judgment criticism. And what is suggested by the history is that the two are tightly linked. Our story finds Wittgenstein presenting his nonsense judgment objection to Russell precisely as the objection that Russell's *Theory of Knowledge* version of his theory of judgment still does not account for the synthesis in thought. I want to suggest that this is indeed what Wittgenstein is doing. Wittgenstein's claim that Russell's multiple relation theory is consistent with the existence of nonsense judgment was intended as a demonstration that that theory, even in its 1913 version, does not account for thought's synthesis.

2.3

To see how this demonstration works, let's first consider (again) that the species of nonsense in view is that of a judgment whose truth condition involves an entity playing a combinatorial role to which it is logically unsuited. It is a premise of Wittgenstein's criticism of Russell that such judgment is impossible. There is no such thing as judging that Socrates and Plato are related by what is not a dual relation – by Aristotle, say, or by *being in between*. There is no such thing as representing Desdemona and Cassio as combined as subject and predicate respectively. Representability entails logical possibility. This is a significant premise which we do not, unfortunately, have space here to explore. As a minimal contextualising gesture, however, consider that the equivalent for Frege of representing Desdemona and Cassio as combined as subject and predicate would be to represent Cassio as a concept mapping Desdemona to the True. To do this one would need something like a 'concept expression referring to the object Cassio'. Any such suggestion would, of course, be anathema, and even contradictory, to Frege. Similarly, the suggestion is anathema to Wittgenstein that an entity might be represented as combining in a manner to which it is logically unsuited.

Next, let's turn to the possibility of unwelcome substitutions in Russellian multiple relation judgments. Consider a judgment by Aristotle that Socrates is human. The truth condition of this judgment, for Russell, is that Socrates and *humanity* are combined as subject and predicate: the judgment represents those two entities as so combined. Now under the multiple relation theory Aristotle's judgment is a multiple relation complex whose relating relation is *judgment* and whose terms are Aristotle, Socrates and *humanity* (and perhaps also the dual complex form). In appearing as term, however, *humanity* is substitutable in this multiple relation complex for any other term. (The possibility of such substitutions is, recall, something to which Russell's basic ideas about complexes and their constituents commit him – even if this is something Russell does not recognise at times in his 1913 writings. Part of Russell's devastation at Wittgenstein's attack stemmed from his coming to see that his commitment to principle (S) renders much of his theorising at the time about complexes invalid.)¹⁵ Let's then consider substituting *humanity* in Aristotle's multiple relation judgment for the term Plato. What can we say about the complex that results from this substitution? Well one thing we can say about it is that it is not a nonsense judgment representing Socrates and Plato as combined as subject and predicate: there is no such possible judgment. But this raises a challenge for Russell. How does Aristotle's judgment that Socrates is

human differ from the result of substituting *humanity* there for Plato such that the former represents Socrates and *humanity* as combined as subject and predicate where the latter does not represent Socrates and Plato as so combined? How, that is to say, does Russell rule out the nonsense judgment that Socrates and Plato are combined as subject and predicate?

This challenge is urgent, for if it is not met then Russell's attempted explanations of a judgment's representational synthesis will be seen to fail. If it is not explained what the difference is between the two judgment complexes such that the one but not the other represents certain of its terms as combined – if nothing in Russell's theory rules out the nonsense judgment that Socrates and Plato are combined as subject and predicate – then the representation by Aristotle's judgment that Socrates is human will not have been explained. Transparently, however, nothing we have seen of Russell's theorising meets this challenge. The 1912 idea that the sense of the judgment relating relation will put the terms into the relevant semantic order is by itself inadequate: nothing Russell says makes for an explanation of how it is that when Plato is substituted for *humanity* in Aristotle's judgment the sense of the relating relation doesn't order Socrates and Plato just as before it ordered Socrates and *humanity*. And the 1913 proposal that a judgment's representation of entities as combined is explained by its containing a form term is seen straightaway not to work: if Aristotle's judgment contains a form term then so too will the result of substituting *humanity* in that judgment for Plato. As developed, then, the multiple relation theory of judgment can be seen not to explain the representation of an atomic judgment. Russell's theory of judgment fails to be a theory of judging that something is the case. This, I suggest, is the intended force of Wittgenstein's criticism.^{16, 17}

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¹ 'An atomic complex is one [which] ... can be analyzed into certain terms related by a single relation' (Russell 1984: 176). More, '[t]he way in which a relating relation occurs in an atomic complex is quite different from the way in which its terms occur' (Russell 1984: 90). For an account in Russell of these different modes of appearance see, e.g., Russell 1998: 74.

² Of course, one might want to work with a second species of possibility under which it is impossible for Socrates to be next to *mortality*. The point emphasised here is simply that the structure of Russell's theorising is such that there is a key sense of possibility in which Socrates *can* be next to *mortality*. The complex in which Socrates is related to *mortality* by *being next to* is a complex in which each constituent appears in a manner in keeping with its basic, logical nature.

³ See, e.g., Russell 1907: 45. Due perhaps to the influence of Wittgenstein, Russell is by 1913 not wholly confident about the identification, writing that 'there is certainly a one-one correspondence of complexes and facts, and for our present purposes we shall assume that they are identical' (Russell 1984: 80).

⁴ See Russell 1984: 130.

⁵ See, e.g., Russell 1966: 155.

⁶ See, for example, the discussion of 'heterogeneous complexes' in part II chapter II of Russell 1984.

⁷ Consider especially here the first six chapters of Russell 1984 (as prepared in 1914-15 for publication), Russell 1949 and Russell 1968.

⁸ This citation is from *Theory of Knowledge* as it was prepared for publication in October 1914. (The editor of *Theory of Knowledge* note that it was unlikely that this passage was written before October 1913 (see Russell 1984: xxxvii).)

⁹ Nicholas Griffin has given an answer to the question of what Wittgenstein found objectionable with Russell's theory which can at best, I think, be plausible only as (part of) an answer to the question of why Russell was

devastated at Wittgenstein's attack. Griffin links Wittgenstein's criticism to Russell's theory of types, writing that '[w]hat Wittgenstein was able to show was that Russell's multiple-relation theory of judgment was inconsistent with his theory of types, the lynchpin of the logic of *Principia*' (Russell 1992a: 461). As Michael Potter points out, however, the intricate and technical nature of the demonstration makes its attribution to Wittgenstein implausible (Potter 2009: 129-130). Wittgenstein was neither competent with the formal details of Russell's theory of types nor much concerned at any point in his career to offer complex technical arguments of the kind Griffin suggests. To these remarks I would add that Wittgenstein's criticism of Russell's theory is repeated in the *Tractatus* (Wittgenstein 1961: 5.5422), and the suggestion that Wittgenstein was concerned as late as 1918 to argue against Russell on the basis of certain technical details of his theory of types is wholly implausible.

¹⁰ Russell's more or less unwavering account from 1903 to 1918 of the unity of an atomic complex is that it is the product of the complex's relating relation playing a binding role within the complex. To occur as relating relation is to go between and bind together the entities appearing as term. In the case of a judgment complex, then, the complex is unified in virtue of the copulative appearance of the universal *judgment*.

¹¹ Stuart Candlish imagines a 'mind leap[ing] from A to love to B, or group[ing] them in that order' and claims that 'this is not a judgment that A loves B' (Candlish 1996: 117). David Pears agrees, adding further that such mental leapings are distinguished from judgments precisely by the absence from the former of the synthesis with which Russell is concerned in *Theory of Knowledge*. There is, Pears writes, a 'general difference between combining the constituents [of a complex] in thought and thinking of them one by one' (Pears 1989: 174).

¹² Letter from Russell to Morrell #782, postmarked 21 May 1913, cited at Russell 1984: xxvii.

¹³ See for this chronology the editor's introduction to *Theory of Knowledge* (Russell 1984: xxvii).

¹⁴ What Wittgenstein writes at this point is:

I can now express my objection to your theory of judgment exactly: I believe it is obvious that, from the prop '*A* judges that (say) *a* is in the Rel *R* to *b*', if correctly analysed, the prop '*aRb*∨~*aRb*' must follow directly *without the use of any other premiss*. This condition is not fulfilled by your theory. (Wittgenstein 1995: 29)

Deciphering, note that Russell at the time used the proposition 'p or not p' to mean 'p is significant' (see Russell and Whitehead 1935: 171). Wittgenstein is thus complaining that it does not follow from Russell's theory of the nature of a judgment that what is judged makes sense. The theory does not rule out nonsense judgment.

¹⁵ The suggestion this paper makes for Wittgenstein's criticism has been reacted to more than once with the claim that if this is Wittgenstein's point it falls flat because Russell held in 1913 that a universal occurring as term cannot (always) be substituted for a particular. The purpose of much of sections 1.1 and 1.3 above was to suggest that whilst Russell may indeed have held this, his basic theorising about the nature of complexes and their constituents commits him to the contrary. In a similar vein, it has been objected that this paper does not address the details of the final version of the multiple relation theory of judgment Russell lays out later on in *Theory of Knowledge*, a version of the theory in which Russell can perhaps be read as at least trying to meet Wittgenstein's concerns as he then understood them. These details depend, however, upon an implicit rejection of principle (S), and so cannot be seen as an adequate response to Wittgenstein. I mentioned above that from the time of the impact of Wittgenstein's criticism Russell abandoned all work inconsistent with (S). Whilst the criticism certainly had impact of a kind the first time it was made (Russell felt straightaway that Wittgenstein, though inarticulate, must have seen something he had missed), it was probably not until the two met again face to face on June 18th that Russell came to see (S) as the basis of Wittgenstein's point. On June 19th Russell wrote to Morrell that 'yesterday' he had felt 'ready for suicide'. 'All that has gone wrong with me lately comes from Wittgenstein's attack on my work – I have only just realised this' (Russell to Morrell, #811 pmk. June 20th 1913, cited at Russell 1984: xix). Work on *Theory of Knowledge* ceased at this point.

¹⁶ David Pears has presented an interpretation of Wittgenstein's critique which compares interestingly to that given in this paper. Pears starts from what has been a central concern of this section: that of 'the general difference between combining the constituents [of a complex] in thought and thinking of them one by one' (Pears 1989: 174). Instead, however, of tackling questions as to the nature of this difference and of how it is to be given theoretical account, Pears raises the question of how the subject knows that the way entities are combined in his thought is a real possibility for those things. He writes:

[I]f we turn now to the second thing that Russell's theory was designed to explain, the subject's knowledge that *aRb* really is a possibility, we have independent evidence that an objection to this part of the theory was among the criticisms made by Wittgenstein at that meeting [of 26 May 1913]. (Pears 1989: 178-9)

At this time, Pears says, Wittgenstein made the point that:

if acquaintance with the constituents of a proposition is going to explain how the subject knows that he has put them together in a way that makes sense, then acquaintance must be intensional. For example, he must be acquainted with *a* and *b* as objects of the right kind to combine with *R* to produce *aRb*. (Pears 1989: 179)

It has, however, been argued earlier in Pears' paper that 'acquaintance is an extensional relation not involving any knowledge of truths about its object' (Pears 1989: 172). Thus Russell's theory fails to explain how the subject 'knows that what he thinks is a real possibility' (Pears 1989: 180). Pears continues:

When Wittgenstein's point is put like this, it is a point against Russell's 1910 theory of judgment [in which the only acquaintances needed by someone who understands the proposition *aRb* are with *a*, *R* and *b*], and of course it does make an impact on that theory. However, it is clear that Wittgenstein actually made the point against the 1913 theory, which required acquaintance with the form of dyadic relations as well as acquaintance with the three constituents. It follows that Wittgenstein must have argued that, even when Russell had brought in acquaintance with the form, he had not made any progress toward a solution of the problem. For it remained unexplained how the subject knows that the constituents can be combined within this form. In short, if the form is treated as an object of acquaintance, it recreates the problem it was designed to solve. (Pears 1989: 179)

Thus Pears, like me, finds Wittgenstein arguing that the introduction by Russell of form terms into his multiple relation judgments (and the addition therewith of a requirement on a judger of acquaintance with forms) fails to serve the purpose for which it was intended. Where I read that purpose to be a synthesis in thought, however, Pears does not find Russell to be after an account of this synthesis and reads the inclusion of form terms as intended rather to explain the subject's knowledge that the (given) synthesis is indeed a genuine possibility.

Russell certainly considered acquaintance by the subject with the objects of a judgment (including, in 1913, a form) a precondition on the judgment's existence. This issues from his general position that '[a]ll cognitive relations – attention, sensation, memory, imagination, believing, disbelieving etc. – presuppose acquaintance' (Russell 1984: 5). It is not obvious, however, that in the case of a propositional attitude these prior acquaintances were supposed to explain, or constitute, knowledge by the subject that the cognition is genuinely propositional (and not 'nonsensical'). Indeed, it is not obvious what Russell's interest should be in such knowledge: might a subject not make a judgment in its absence? What does seem clear, by contrast, is that the primary purpose of Russell's 1913 inclusion of form terms in judgment was to explain not a knowledge by the subject that the thought's synthesis is a genuine possibility but rather the synthesis itself:

The process of "uniting" which we *can* effect in thought is the process of bringing them [*A*, *B* and *similarity*] into relation with the general form of dual complexes. (Russell 1984: 116)

It is frustrating that Pears fails to explore this key remark.

¹⁷ A recent commentator whose interpretation comes close to mine is Graham Stevens, who writes:

If the subordinate relation is to be treated as a term on a par with the elements it is intended to relate, then there will be no logical difference between the relation and its referents and relata. In other words, in place of a judgment-complex of the form $J\{S, x, R, y\}$ we are in fact left with an analysis which, at best, will be of the form $J\{S, x, y, z\}$. A theory of judgment which fails to explain the difference between a judgment like 'Othello believes that Desdemona loves Cassio' and one like 'Othello believes that Love desdemonas Cassio' is simply one which fails to account for the difference between judgements and non-judgements; in other words, it simply fails to provide an explanation of judgement at all. (Stevens 2005: 96)

And:

Wittgenstein's target ... was the failure of the [multiple relation] theory to account for the division of propositional content into parts which reflect and preserve its unity and hence debar nonsensical pseudo-judgments such as 'this table penholders the book'. (Stevens 2005: 105)

Stevens' principal interest in the episode of Wittgenstein's criticism is, however, to explain its impact on Russell; he is much less occupied with Wittgenstein's half of the transaction. The Wittgensteinian sounding notion of representing entities as combined – of representing that something is the case – is not brought into clear view. Further, he locates Russell's commitment to the possibility of unwelcome substitutions in judgment complexes as the consequence not of his basic ideas of complexes and their constituents, but rather of a thesis that the constituents of a judgment must stand on an equal ontological footing if orders of propositions are to be generated at the epistemological rather than the ontological level (see, e.g., Stevens 2005: 103).

Christopher Pincock argues against Stevens as follows:

Stevens's idea is that when a relation is not a relating relation for a complex, Russell must treat it as on a par with terms that are not relations. As the multiple-relation theory requires invoking relations that are not relating relations, the theory cannot make the required distinctions. For example, if Russell allows $U(o, d, L, c)$, then he must also allow $U(o, L, d, c)$, that is, Othello understands that love desdemonas Cassio.

Now, on the interpretation of the theory that I have developed, the complexes that Stevens says Russell must countenance are ruled out as logically impossible. No commitments that Russell defends in *Theory of Knowledge* require what Stevens requires. For example, Russell's theory of complexes denies that if there is a complex where A is a part of α , then there is a possible complex in which α is a part of A . So even things are heterogeneous in some complexes. When it comes to the network of relations involved in the full analysis of an understanding complex, the same restrictions will be in place. We can block Stevens's objection, then, by emphasizing Russell's theory of complexes and the restrictions that it imposes. (Pincock 2008: 121-2)

By 'Russell's theory of complexes', however, Pincock is referring not to Russell's ideas of complexes as discussed in section 1.1 above, but rather to higher level discussions of complexes which are given up from the time of – and, I suggest, as a result of – the impact of Wittgenstein's criticism. To repeat, Russell does not, given his basic ideas of complexes and their constituents, have the right to theorise as he does in opposition to principle (S). (It may be, of course, that given his understanding of Russell's commitment to unwelcome substitutions Stevens cannot so easily defend himself against Pincock.)