

The authors venture to define the notion of linguistic geometry in this book. It is pertinent to keep on the record that linguistic geometry differs from classical geometry. Many fundamental concepts and notions of classical geometry are not valid or extendable in the case of linguistic geometry. We discuss in depth the linguistic variables, graphs and geometry essential for social information networks. And then develop the new notion of linguistic geometry and its related properties. The concept of linguistic social information geometric networks is also developed.

LINGUISTIC GEOMETRY AND ITS APPLICATIONS

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# Linguistic Geometry and its Applications

**W. B. Vasantha Kandasamy  
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Florentin Smarandache**

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## PREFACE

For the first time, the authors venture to define the notion of linguistic geometry in this book. It is pertinent to keep in the record that linguistic geometry differs from classical geometry. Many basic or fundamental concepts and notions of classical geometry are not true or extendable in the case of linguistic geometry. Hence, for simple illustration, facts like two distinct points in classical geometry always define a line passing through them; this is generally not true in linguistic geometry. Suppose we have two linguistic points as tall and light we cannot connect them, or technically, there is no line between them. However, let's take, for instance, two linguistic points, tall and very short, associated with the linguistic variable height of a person. We have a directed line joining from the linguistic point very short to the linguistic point tall. In this case, it is important to note that the direction is essential when the linguistic variable is a person's height. The other way line, from tall to very short, has no meaning. So in linguistic geometry, in general, we may not have a linguistic line; granted, we have a line, but we may not have it in both directions; the line may be directed. The linguistic distance is very far. So, the linguistic line directed or otherwise exists if and only if they are comparable. Hence the very concept of extending the line infinitely does not exist. Likewise, we cannot say as in classical geometry; three non-collinear points determine the plane in linguistic geometry. Further, we do not have the notion of the linguistic area of well-defined figures like a triangle, quadrilateral or any polygon as in the case of classical geometry. The best part of linguistic geometry is that we can define the new notion of linguistic

social information geometric networks analogous to social information networks. This will be a boon to non-mathematics researchers in socio-sciences in other fields where natural languages can replace mathematics.

This book has three chapters. The first chapter discusses in depth the linguistic variables, linguistic graphs and linguistic geometry essential for social information networks. The second chapter discusses and develops the new notion of linguistic geometry and its related properties. The final chapter defines for the first time the concept of linguistic social information geometric networks.

Every chapter has suggested problems which will familiarise the readers with these concepts.

We acknowledge Dr K.Kandasamy with gratitude for his continuous support.

W.B.VASANTHA KANDASAMY  
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## Chapter One

### **LINGUISTIC VARIABLES, LINGUISTIC GEOMETRY, GRAPHS AND THEIR PROPERTIES**

In this chapter, we mainly describe and develop three concepts introduction to linguistic variables, linguistic terms and related concepts. Secondly, we introduce the concept of classical geometry and, finally, the basic concept of linguistic social networks.

Here we define or represent a linguistic variable only by a linguistic (ling) set of words or terms. We do not visualize a linguistic variable to be sentences. Further, we do not associate the notion of fuzzy membership with the ling variable [29-31].

These are developed to an extent by Zimmerman [32-34]. However, for us to analyze social problems, we by no means need to use the concept of distance or numerical values. It is sufficient if we can solve these problems linguistically. For us to study social problems, it is enough if we study this linguistic mathematics which is very sensitive to the problems more approachable by non-mathematicians like socio-scientists. Researchers over three decades in mathematical modelling and



solving real-world and social issues using mathematics realized the need to expand the spectrum of research to non-mathematicians (socio-scientists, psychologists etc.). In solving specific real-world problems, the tools of linguistic mathematics and linguistic models may be helpful and easy to adapt.

Several concepts in geometry or analytical geometry cannot be generally taken verbatim in linguistic theory.

Further, ling theory is more language-based terminology that can be given geometrical and algebraic structures, though not in the case of the classical sense. We can build all forms of structures with some limitations (we use by default of notation the terminology ling instead linguistic).

We do not claim that ling mathematics can be a complete alternative to classical or existing mathematics. Our only claim is that ling mathematics can give rise to ling models, which can be very powerful in tackling social problems. Social problems are more easily representable as linguistic models as most of the social issues cannot be measured with real numbers or fuzzy values.

For the suffering of a person when she lost her close relative can never find a value numerically or as fuzzy membership in  $[0, 1]$ . But however it can find a ling term which can fully capture her sufferings. Thus ling mathematics will certainly be the best tool to study and analyze social issues and some social problems.

However, to tackle any sort of problem, we need the concept of distance.

We can depend on two ways of measuring the ling distance; that is, either by using ling geometry or by using ling analytical geometry. However, to use the latter method, we need to convert the problem or the ling terms into a ling coordinate plane or ling analytical plane, but converting the ling set or into a ling coordinate plane, we basically need to have the ling set to be a ling continuum or at least a totally ordered ling set. But getting at all times a totally ordered ling set is not feasible; for it highly depends on the ling variable under consideration, so using ling coordinate geometry has some limitations.

But if we try to replace it with ling geometry, we will be able to define a ling distance provided the two ling terms or ling points under consideration are comparable.

We have discussed the fundamental way of how ling geometry is introduced; it is described and developed in the following chapter.

Now we still have limitations for all the time the ling variables may not be comparable also, limitations of developing ling geometry on par with classical Euclidean geometry is nearly impossible. However, we can develop some basic properties of classical geometry in the case of ling geometry, but not several concepts like area, circle etc, which may not be possible.

Now we proceed to develop the notion of the ling variable and its related ling set/term. We give examples of a few of the ling variables. Age of people, height of people, weight of people, character of persons, performance of students in the class room, performance of the workers, employer in a

company, performance of a teacher, quality product made by an industry, speed of car, quality of fruit, yield of a plantation, customers view on a product, service at a hotel, performance index of a school / college etc, etc can be visualized as ling variables.

When we say a ling set/term associated with the ling variable we only describe it by ling words and not as described and defined by Zadeh or Zimmerman who these as view the ling set/term as sentences and a fuzzy membership associated with them. This is the main feature where we differ from the linguistic term/set used by [24-6]. By a ling set or ling words or ling term it is always associated with a ling variable.

We will first provide some examples of them to make our basic definition clear.

**Example 1.1.** Consider the ling variable age of people. The ling set/term associated with the ling variable  $V$  is given by [youngest, oldest].

We see youngest is the superlative to describe the just born and the oldest is the greatest value viz 100 years which is take as the span of life of a person in general.

Thus terms like very young, young, just young, very very young, just youth, youth so on, middle aged, just middle aged, so on; old, just old, very old, very very old, ..., oldest will be in the ling continuum [youngest, oldest].

The following observations are mandatory.

- i) Age of a person directly depends on time. As time goes on age of a person increases steadily.

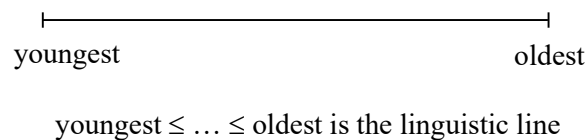
Secondly the process is continuous in no way it stops with progressing time, unless otherwise the person dies and after which the ling subinterval of the ling continuum stops for that particular person

For instance [youngest, middle age], where it so happens that person dies in his middle age. However no person can ever skip the term youngest for he starts at the youngest at his birth and each day he / she grows older and older and may attain the oldest or terminate at any other linguistic term in the ling continuum [youngest, oldest].

Now this is clearly a continuous process as time passes and as time never stops.

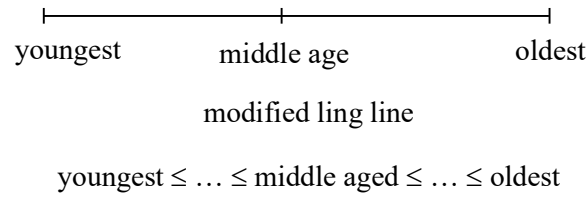
Thus this ling continuum [youngest, oldest] is a totally ordered set. Youngest is the least ling term and oldest is the greatest ling term.

This ling continuum can be given a ling line representation given by the following figure.



**Figure 1.1**

However we can also have the notion of modified ling line which is given by the following figure 1.2.



**Figure 1.2**

Here we visualize the modified ling line as a ling line analogous to real line where middle age is the ling mid point of the ling line and the ling terms on the left of the ling point middle age are the negation of the ling terms on the right or vice versa.

Thus the negation of youngest is oldest and that of oldest is youngest.

The negation of young is old and that of old is young

$$\neg \text{young} = \text{old}$$

$$\neg \text{old} = \text{young}$$

$$\neg \text{young, means negation of young;}$$

That is  $\neg \text{young} = \text{negation of young.}$

Thus modified ling line happens to be like the logical line where every ling term on the ling line has a negative one, however the line mid term is the middle most term which is analogous to 0 of the real line.

Now we observe the following points regarding these ling continuum.

- i) It (ling continuum) is a totally ordered set and it is a continuum.
- ii) Viewing this ling line as a modified ling line we see it has a mid point analogous to 0 in the classical real line where the negative of any ling term on this ling continuum is nothing but the logical negation of that ling term. However, 0 has no negation. It is like the neutral terms.
- iii) The ling line is not only continuous it is an increasing one. For tallest is the greatest element and shortest is the least term.

Now any part of the ling continuum or ling interval say [old, oldest] = P is called as the ling subinterval or ling subcontinuum of the ling continuum [youngest, oldest].

Clearly;  $\text{old} \leq \dots \leq \text{oldest}$ .

This ling subinterval or ling subcontinuum is also increasing and is a continuous one.

In fact every ling subcontinuum of the ling continuum [youngest, oldest] is also a continuous one and is an increasing one.

Now we see the cardinality of each of its ling subcontinuum is also of infinite cardinality as that of the cardinality of [youngest, oldest].

Now consider the finite subset  $P$  of the ling continuum [youngest, oldest], given by

$$P = \{\text{youngest, old, young, just old, just young, middle aged, very old, very young, just middle aged}\} \subseteq [\text{youngest, oldest}].$$

We see the ling set  $P$  is a totally ordered set for

$$\begin{aligned} \text{young} \leq \text{very young} \leq \text{just young} \leq \text{young} \leq \text{just middle aged} \\ \leq \text{middle aged} \leq \text{just old} \leq \text{old} \leq \text{oldest}. \end{aligned}$$

However we cannot say  $P$  is a continuous ling set what we can say is only a totally ordered set.

Next we consider yet another ling variable height of people in the following example.

**Example 1.2.** Take the ling variable height of people. The ling set associated with this ling variable is the ling continuum [shortest, tallest]. Growth in height occurs continuously, till a stipulated period of time.

When we say continuous it does not mean increasing as time goes on the person slowly grows say at a maximum of 7 feet. However we can say it is increasing in some period of life it remains constant and then it increase and remain the same; that is why we can only say continuous and not increasing.

The person's height at birth is denoted by shortest and that of the height in the middle age attains the maximum height or the greatest height the person attains.

The difference between the ling variable age and the ling variable height is that a person ceases to grow after middle age

but the age does not stop to increase till his / her death. This is yet another difference between the linguistic variables; age and height.

However both the ling variables are associated with ling sets / terms which are a ling continuum?

We have given the difference between the two ling variables height and the age.

So in case of the ling variables even if they are ling sets / terms which are ling continuous still they may not in general enjoy the same properties. They may enjoy different properties.

We will list out the difference and similarities of these two ling variables.

<b>Ling variable - age</b>	<b>Ling variable - height</b>
1. ling set is a continuum [youngest, oldest]	ling set is a ling continuum [shortest, tallest].
2. It is a continuous one	Clearly this is also a continuous one.
3. Increases steadily till the end	Increases or is a constant at some time and after middle age it is constant so is not increasing steadily.
4. Every finite ling subset is totally ordered	Every finite ling subset is totally ordered or is a constant.

Now we will describe yet another ling variable by an example.



**Example 1.3.** Consider the ling variable V, colour of the eyes of internationals. The ling set associated with this ling variable V is given by S; where

$S = \{\text{black, brown, dark brown, light brown, green, blue, hazel}\}$  is the ling set associated with the ling variable V.

We make the following observations

- i) S is only a finite ordered set
- ii) S is not a totally ordered.

We can compare only the 3 ling terms, light brown, brown and dark brown.

$\text{light brown} \leq \text{brown} \leq \text{dark brown}$ . However it cannot be compared for one person as the colour of his / her eyes remains the same no change takes place. So this comparison is only mathematical it cannot yield to any form of comparison of persons; so we conclude linguistically invalid comparisons for the given ling variable V.

Apart from this we will not be able to compare any or every pair of distinct ling terms in S.

Thus S can be a partially ordered set not ling. Partially ordered set for the ling variable V. So we see ling variables are of different types will give way to different types of ling sets / continuum.

Infact what is important some of the ling variables given here are time dependent. For the ling variable age it is fully time

dependent but the ling variable height is time dependent only for a period.

However the ling variable colour of eyes of internationals is time independent. For the colour of the eyes has no relevance to the time at all.

We provide yet another example.

**Example 1.4.** Consider the ling variable performance of students in the classroom. In general the ling variable  $V$ , students performance has the ling set / continuum associated with it is  $[\text{worst}, \text{best}] = S$ . This is also a ling continuum.

One of the important observations is that this ling variable is not time dependent for if we take the general performance of students in a class room. This cannot be evaluated as marks as they are not writing their exams. The performance of them in the classroom and assessed by their class teacher. However for a class of 30 students we may get a maximum of 30 values if the 30 of them is evaluated on a particular day and they are distinctly different.

However this is different from the same ling variable performance of one student continuously on all working days for an academic year.

Now we have only a finite set of ling terms / points associated with the ling variable performance of a student in a classroom for an academic year.

Suppose the ling terms associated with it are {just fair, bad, very bad, just bad, fair, just good, good, very good, very fair, very very fair, very very bad, very very good}.

The expert feels that his performance in an academic year happens to fall under these 13 ling terms. His performance is fluctuating from very very bad to very very good. From this ling set one can conclude he is not a consistent performer.

His performance fluctuates very badly and his performance in the classroom may heavily depend on his fluctuating moods, teacher is not in a position to put him as a good or a bad or fairly performed student. As from the ling set associated with him in one can only conclude his performance is a highly fluctuating one.

However we wish to record the same set cannot be used for some other student. His / her ling set of terms would be very different.

Thus neither is the ling set contribute to a continuous ling set nor is it increasing it is only highly fluctuating. This ling variable and the ling set associated with it is different from the example 1.3 in which we have studied the ling variable age, height and colour of the eyes of internationals.

We provide yet another example of a ling variable which is different from all these four different ling variable given in examples 1.1 to 1.4.

**Example 1.5.** Let us consider the ling variable speed of a car on road. The speed of the car on road can start from nil speed gradually increase to very fast and then reduce to slow, very slow and finally nil speed if signal falls and this pattern can repeat itself several times if it is a long distance travel.

Let S be the ling continuum associated with the ling variable speed of a car. Now before we give the ling set we make the following assumptions the car start from nil speed slowly increases and becomes fast then becomes less fast as it approaches the signal if the signal falls then it becomes nil speed once again is very slow as it picks up becomes fast, very fast and again slows it to fast, to just fast and then just slow, slow and comes to nil speed if the signal falls.

For this ling variable we have the following curve if we take the car's total long journey from start to destination given by the following figure.

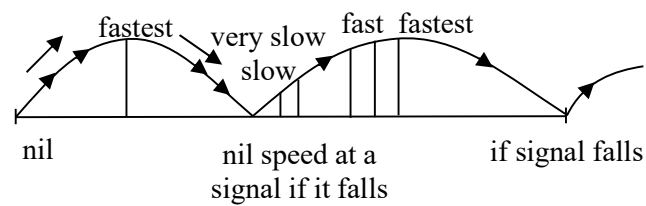


Figure 1.3

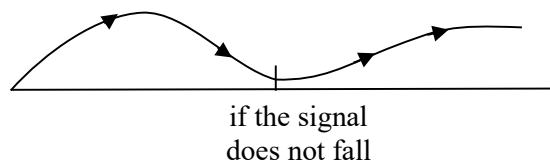


Figure 1.4

This pattern will repeat depending upon the number of signals the car faces from start to destination or some other need to apply break.

So we can represent the ling set associated with the ling variable speed of car from start to destination as follows.

$$S = [\text{nil, fastest}] \cup [\text{fastest, nil}] \cup [\text{nil, fastest}] \cup [\text{fastest, very slow}] \cup [\text{very slow, fastest}] \dots \cup [\text{fastest, nil}].$$

So this is a repeating pattern of increasing and slowing decreasing to nil than increasing to decreasing, very slow or nil and so on.

This ling set associated with the ling variable speed of a car on road happens to be very different from all other four ling variables given in examples 1.1 to 1.4.

**Example 1.6.** Next we study the ling variable quality of mango fruits. The ling set associated with the ling variable, the quality of mango fruit is given by

$$S = \{\text{good, bad, ripe, very ripe, unripe, big, small, very small medium size, just big, sour, just medium size, insipid, bitter very tasty, sweet, very sweet and so on}\}.$$

Clearly we cannot order this ling set for it is not comparable so is not a totally ordered set. However it is partially ordered set for we have

$$\text{very small} \leq \text{small} \leq \text{just medium size} \leq \text{medium size} \leq \text{just big} \leq \text{big sweet} \leq \text{very sweet and unripe} \leq \text{ripe} \leq \text{very ripe}.$$

From the above relations we can easily understand the ling set S is only a partially ordered set and is not a totally ordered set.

This example 1.6 is very different from examples 1.1 to 1.5. All are different in their own way. Thus the ling sets associated with the ling variables in general are distinctly different.

Now we give yet another example of a ling variable different from these.

**Example 1.7.** Consider the ling variable friendships of 10 people. The ling set S is as follows.

$S = \{\text{same profession, neighbourers, studying in same college, researchers on same topic, playing for same team, and so on}\}.$

However we see S has meaning for we cannot give any relations. We have to take in two first to show friendship linguistic dyads or threes to get linguistic triads and so on.

This ling set S is meaningless. So a general friendship ling set S is in no way helpful to study the problem.

This is yet another example which is distinctly different from the other examples 1.1 to 1.6 given earlier.

Though friendship is a ling variable V, still a study of them using a ling set associated with V is not possible for us to build ling set to describe; then we need more information about the friendship.

For instance take even in a classroom of 70 students pick up 5 of them, there may not be even a pair of friends. So far us to study friendship some more relevant information is needed. However in chapter two using ling geometry we have developed

ling dyad, ling triads and in fact ling quadrilaterals to study the friendship of four persons where additional information is supplied.

Now having seen some examples of ling variables and ling sets associated with it and their usefulness and their limitations we next proceed onto discuss about ling sets associated with some ling variables with finite sets / collection.

**Example 1.8.** Suppose there are 18 persons in group of very different age groups in an ice cream shop; the owner of the shop wants to approximately assess their age and interest in enjoying the ice cream as a study to know which age groups enjoy ice cream. He does not want to ask their age and get information for roughly he wants their age.

The best approach is to fix their age linguistically like young, old, middle age and so on.

He puts them (the 18 of them) in the following ling set S which is associated with the ling variable age.

$S = \{\text{young (3), youth (4), just young (2), just middle age (1), very young (2), just old (1), adult (5)}\}$ ; where the number in the bracket denotes the number of persons in that age group.

Now the ling set S associated with this ling variable age group is a finite totally ordered set.

$$\begin{aligned} \text{very young} \leq \text{just young} \leq \text{young} \leq \text{adult} \leq \text{youth} \leq \text{just} \\ \text{middle age} \leq \text{just old.} \end{aligned}$$

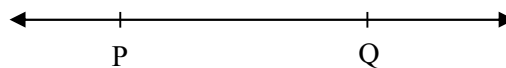
We see adult come to shop followed by youth, young, just young and very young, ling set become handy in these situations also. This example 1.8 is very different from other examples.

Having seen ling variables and ling sets we now proceed onto give a brief introduction to geometry.

Geometry is a branch of mathematics which measures the points, lines, planes, etc. However geometry is mostly taught only at the school level.

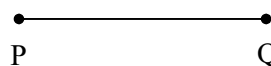
A point is described in classical geometry with its specific location in space.

A line in coordinate geometry is determined by two distinct points, a line is fixed by two distinct points and the line extends to infinity in both directions.



**Figure 1.5**

The line segment PQ is a part of a line between two points. P and Q are referred to as end points

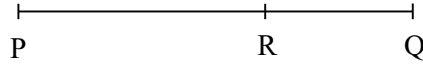


**Figure 1.6**



We represent the line segment by PQ.

If R is point that lies between P and Q on PQ then  $PR + RQ = PQ$ .



**Figure 1.7**

Angles: angle is the union of two rays where the ray PQ, or line PQ starts at an end point and extends to infinity in the other direction. Any two points can determine a straight line. We cannot use the other concepts of geometry to develop ling geometry as it cannot be achieved.

This is the limitations of ling geometry.

However we wish to record some of the important deviations of ling planes and planes we use in classical geometry.

We know in the classical or Euclid geometry one can have a point but it cannot fix the plane. Similarly, two points in classical geometry can only uniquely fix a straight line extending to infinite on both sides given three points we see that fixed uniquely a plane.

Keeping this in mind we will describe the ling geometry, ling point, linguistic plane and the ling line.

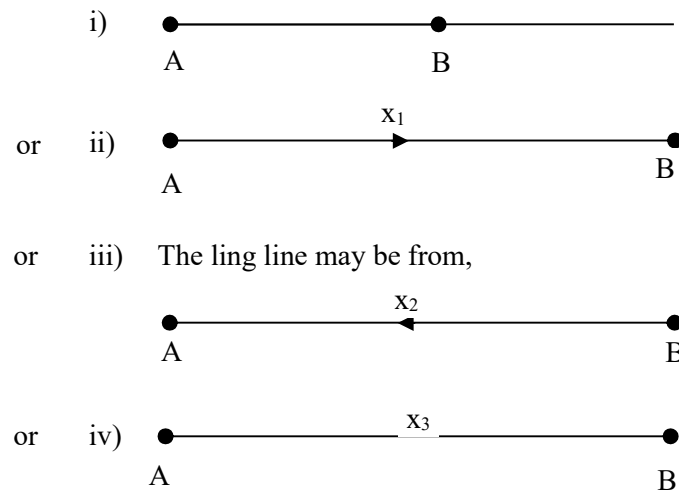
First of all for us to place a ling point we need the following.

- i) The ling variable V.
- ii) The ling set S associated with the ling variable V S may be finite or infinite.
- iii) The ling plane L which is associated with the ling variable V and the associated ling set S.
- iv) The ling term / point in this S will find its place only in this ling plane L. So to every ling set we have a ling plane. This the ling plane is highly dependent on the ling set associated with the ling variable.
- v) Given any two ling points / terms in S we cannot say the ling line will exist. For the ling line to exist the two given ling points must be comparable. Further as in case of classical geometry we do not say the line can be extended infinitely on both sides.

Infact such a concept is not possible in ling geometry. Given any two ling points  $A, B \in S$ .

- i) The line from A to B may exist or may not exist. Will exist if and only if A and B comparable.
- ii) The ling line AB is not infinitely extendable.
- iii) The ling line AB can be viewed in classical geometry only as a line segment. But we call it as a ling line connection AB.

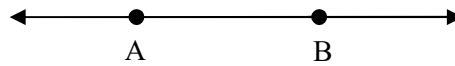
The ling line from A to B may be of four types



**Figures 1.8**

All these find its place only on the ling plane determined by the ling variable V and the ling plane associated with the ling set S of the ling variable V.

This is the main difference between the points and line classical geometry where



**Figure 1.9**

$\overline{AB}$  is the line segment and in classical geometry the two points A and B has an infinite line extending in both directions.

However in case of ling line, they exist only depending on the ling plane associated with it and for any two ling points we may not have a ling line associated with it. The ling lines are always finite, never infinite and they are labeled as far, near, just far or just near and so on.

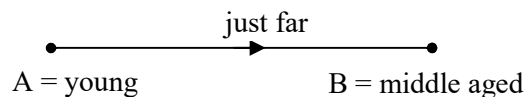
To this effect we will give some examples.

**Example 1.8.** Suppose we have the ling variable  $V$ , age of people and let  $S = [\text{youngest, oldest}]$  be the ling set associated with the ling variable  $V$ .

Let  $L$  be the ling plane which describes the age of people.

Let us consider the ling points;  $A = \text{young}$  and  $B = \text{middle aged} \in S$ .

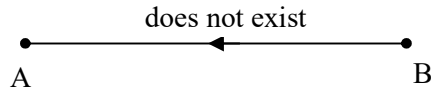
We can describe these ling points  $A$  and  $B$  in the ling plane  $L$  in a very unique way which is given in the following figure.



**Figure 1.10**

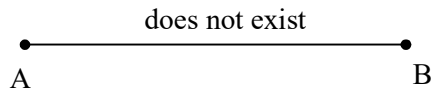
We see the ling line is only from  $A$  to  $B$  that is the young  $A$  can reach the middle age  $B$  and the ling distance is just far.

However the line  $BA$  cannot exist as no middle aged person can reach to young age. So  $BA$  does not exist.



**Figure 1.11**

Thus we also cannot put as we have only A to B and B to A does not exist for  $A \leftrightarrow B$  to exist both A to B and B to A must exist.



**Figure 1.12**

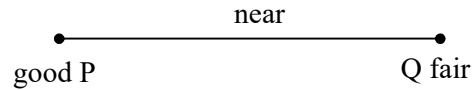
Now consider the ling variable V, be the performance of students in a classroom.

Consider the ling set S associated with the ling variable V and let L be the ling plane associated with the ling variable V and the ling set S.

Now we take the ling points  $P = \text{good}$  and  $Q = \text{fair} \in S$ .

We find the ling distance between P and Q. Is PQ a directed one or an undirected ling line?

It can be only an undirected one for we can reach from P to Q or Q to P as a student good at studies may go to fair at studies and equivalently a person who is fair at studies may become good at studies in due course of time.



**Figure 1.13**

So no direction is needed.

So every distinct pair in  $S$  we have no direction as performance of a student in the class room is taken only for a period of time and a ling time exist.

Now we come to yet another example. Consider the ling variable  $V$  colour of the eyes of people internationally.

Let  $S$  be the ling set associated with  $V$ ,

$S = \{\text{green, blue, brown, black, light brown, dark brown}\}$ .

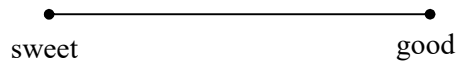
Let  $L$  be the ling plane associated with  $V$  for the ling. set  $S$ .

We see for no pair of elements in  $S$  we can have a distance defined for no pair is comparable. We cannot say dark brown can move to light brown or brown as they are fixed concepts relating the colour of each persons eye.

Now consider another type of ling variable  $V$  which gives the quality of fruits. Let  $S$  be the ling set associated with  $V$ ;

$S = \{\text{good, bad, sour, sweet, tasty, small, big, very big, and so on}\}$ . Let  $L$  be the ling plane.

We cannot match tasty to sour for they are different and cannot match each other. However we can associate that is sweet to good and vice versa.



**Figure 1.14**

We have discussed more in depth these ling concepts in the following chapter II.

We have given a brief introduction here mainly for emphasis and prepare the reader what sort of study is carried out about ling geometry.

However at this stage we will not be in a position to carry out the concepts of circle, etc. in the ling plane. Ling geometry has lot of limitations also.

Further the main advantage of using ling geometry is that as applications we can go to ling graphs which will be used in social information network.

Now we introduce some simple and relevant concepts on social information network which can help in building the notion of ling social information network that basically uses ling geometry.

One is very well aware of the fact that graphs play a vital role in the social network analysis as a way of representing social relations and quantifying important social structure properties. Anthropology uses the concept of graph theory so is

social psychology. The graphs helps the socio scientist to discover patterns.

Graphs have been used to represent both directional and non directional relations.

Directed graphs or digraphs can be used to represent directional relations.

In the graphs  $G$  used for social information networks the graphs  $G$  has set of nodes and set of lines. We usually avoid reflexive ties or loops in these graphs. Mostly the graphs which are consider are simple graphs. It may be directional or non directional.

A graph with only one node is trivial. A graph with  $n$  nodes and no lines is empty.

We can just recall 7 families A, B, C, D, E, F and G in a grated community we list out.

A is a neighbor of B, C and E, F and G in a grated community we list out.

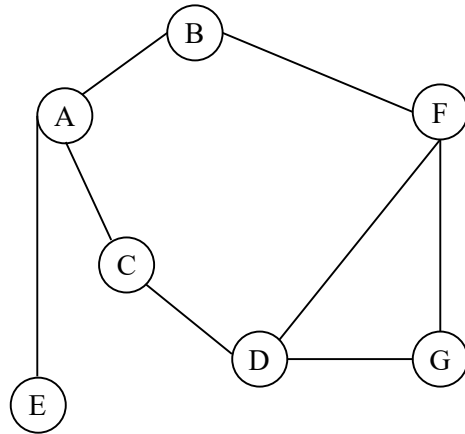
A is a neighbor of B, C and E. D is a neighbor of F, G and C. F is a neighbor of D, G and B.

D is a neighbor of F, G and C and

F is a neighbor of B, D and G.

We represent this by a graph.

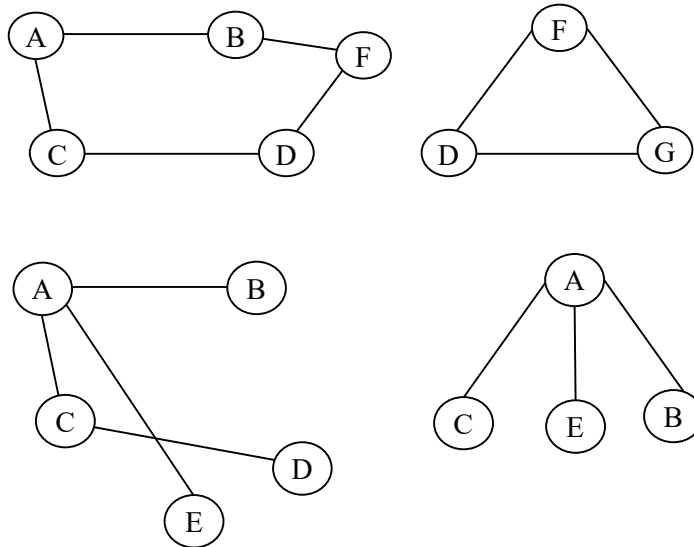


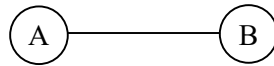


**Figure 1.15**

The following observation is vital. In case of graphs associated with social network we may not have an associated edge weight.

We can discuss about the subgraph of these graphs





**Figure 1.16**

and so on.

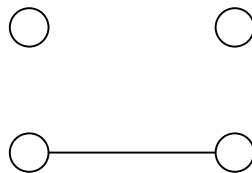
Now a dyad represents a pair of actors and the possible tie between them.

We have two types of dyads for undirected graphs.

A dyad representing a pair of actors and the possible tie between them, is a (node-generated) subgraph consisting of a pair of nodes and the possible line between the nodes.

There are only two types of dyadic states in an undirected graph.

- i) Undirected relation represented as a graph that is actors in the dyad have a tie present or they do not.



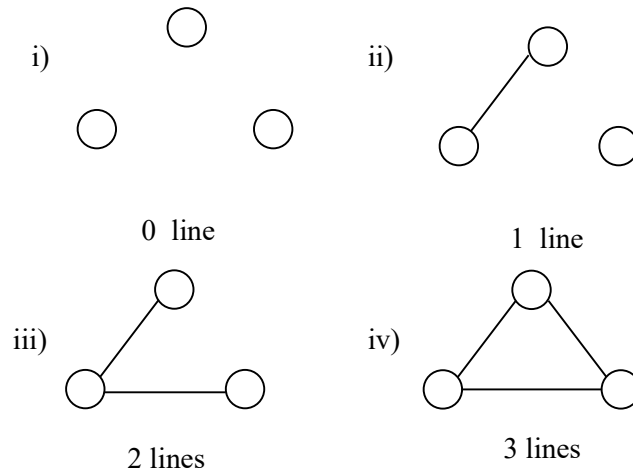
**Figure 1.17**

Triadic analysis is also based on subgraphs where the number of nodes is three.

A triad is a subgraph consisting of three nodes and the possible lines among them.

A triad may be in one of the four possible states depending on whether zero, one, two or three lines are present among the three nodes in the triad.

The four possible triadic states are shown in case of undirected triads in the following figures.



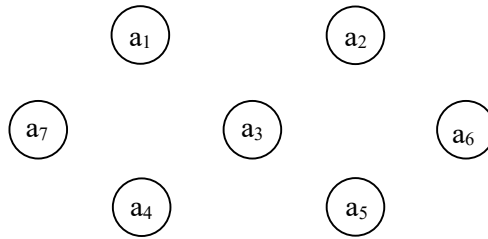
**Figures 1.18**

Four possible triadic states in an undirected graph.

However the mean nodal degree is  $\bar{d} = \frac{2L}{g}$  (g number of nodes in the graph G and L number of lines for the graph with these g nodes).

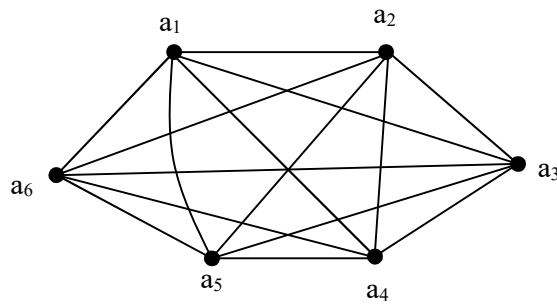
$$\text{The variance of the degrees } S_D^2 = \frac{\sum (d(n_i) - \bar{d})^2}{g}.$$

We give examples of empty, complete and intermediate graphs in the following.



**Figure 1.19**

Empty graph with 7 nodes it has no lines that is why empty  $L = 0$  and  $n = 7$  ( $L$  - lines  $n$  - nodes).



**Figure 1.20**

The graph is a complete is one which has  $\frac{6 \times 5}{2} = 15$  possible lines the density is 1. Here  $g = 6$  and  $L = 15$ . Next we describe an intermediate graph.

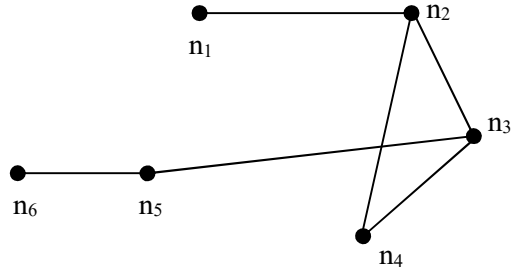


Figure 1.21

These graphs are neither empty nor complete.

For the following graph we give the degree of the nodes.

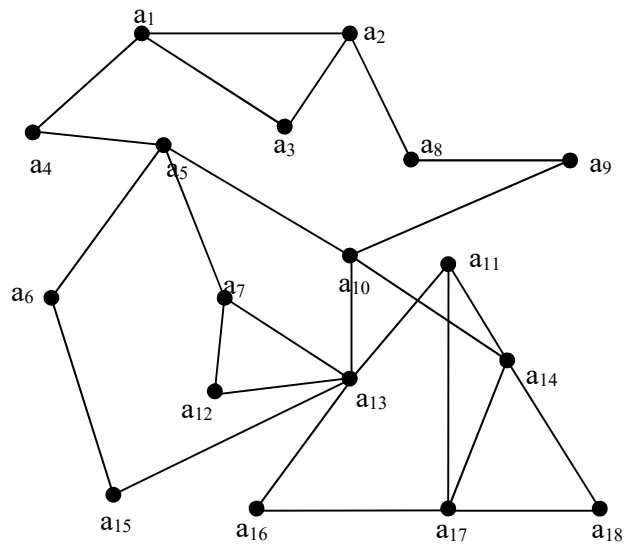
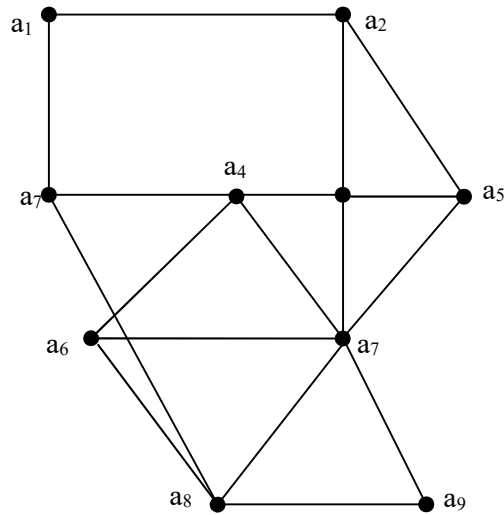


Figure 1.22

<b>node = <math>a_i</math></b>	<b>degree = <math>d(a_i)</math></b>
$a_1$	3
$a_2$	3
$a_3$	2
$a_4$	2
$a_5$	4
$a_6$	2
$a_7$	3
$a_8$	2
$a_9$	2
$a_{10}$	4
$a_{11}$	3
$a_{12}$	2
$a_{13}$	6
$a_{14}$	4
$a_{15}$	2
$a_{16}$	2
$a_{17}$	4
$a_{18}$	2

Next we give an example of connected and disconnected graphs by the following figures 1.23 and 1.24.

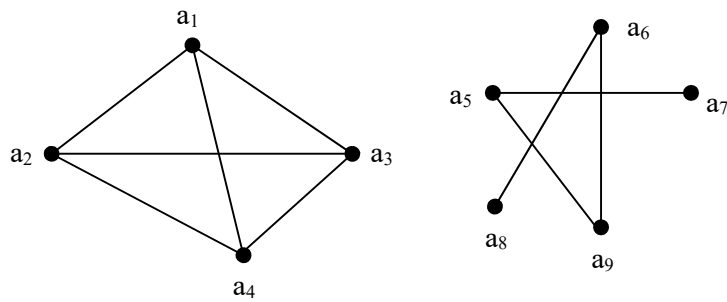
However the reader is requested to refer [27] for concepts like cycles, walk, closed walk, tours etc.



**Figure 1.23**

This is a connected graph.

Now we provide an example of a disconnected graph.

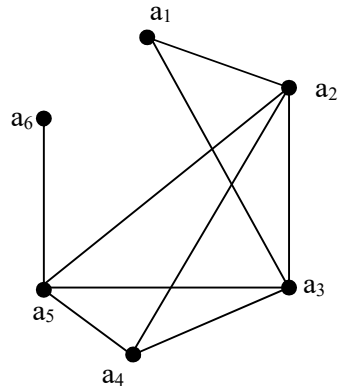


**Figure 1.24**

The connected subgraphs of a graph are known as the components of the graph.

The above graph has two components.

Now we give an example of a graph and give its geodesics and diameter.



**Figure 1.25**

**Geodesic distances**

$d(a_1, a_2) = 1$	$d(a_2, a_6) = 2$
$d(a_1, a_3) = 1$	$d(a_3, a_4) = 1$
$d(a_1, a_4) = 2$	$d(a_3, a_5) = 1$
$d(a_1, a_5) = 2$	$d(a_3, a_6) = 2$
$d(a_1, a_6) = 3$	$d(a_4, a_5) = 1$
$d(a_2, a_3) = 1$	$d(a_4, a_6) = 2$
$d(a_2, a_4) = 1$	$d(a_5, a_6) = 1$
$d(a_2, a_5) = 1$	

Diameter of the graph

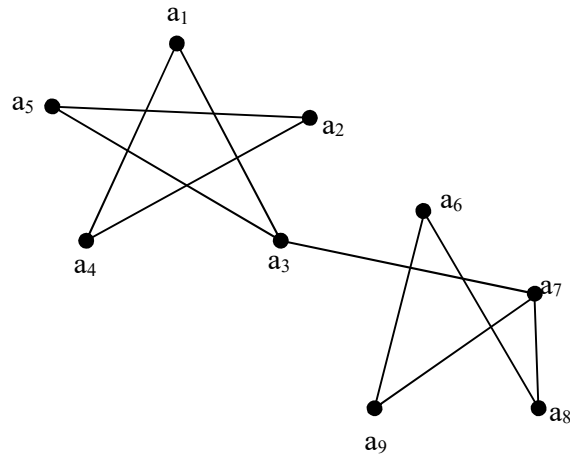
$$\begin{aligned}
 &= \max d(a_i, a_j) \\
 &= d(1, 6) = 3 \quad 1 \leq i, j \leq 6.
 \end{aligned}$$



Likewise we can also find the diameter of the subgraph.

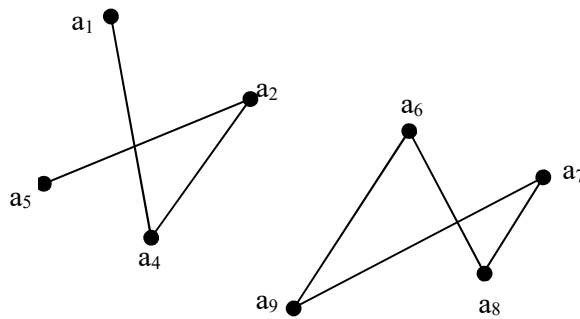
The connectivity of a graph is a function of whether a graph remains connected when the nodes or lines are deleted.

A cut point - A node  $a_i$  is a cut point if the number of components in the graph that contains  $a_i$  is fewer than the number of components.



**Figure 1.26**

$a_3$  node is the cut point or node cut.



**Figure 1.27**

Example of a cut point in a graph

Next we proceed onto describe the bridge.

A bridge is a line that is critical or crucial to the connectedness of graph. The removal of a bridge leaves more components than when the bridge is included.

We will illustrate this situation by an example.

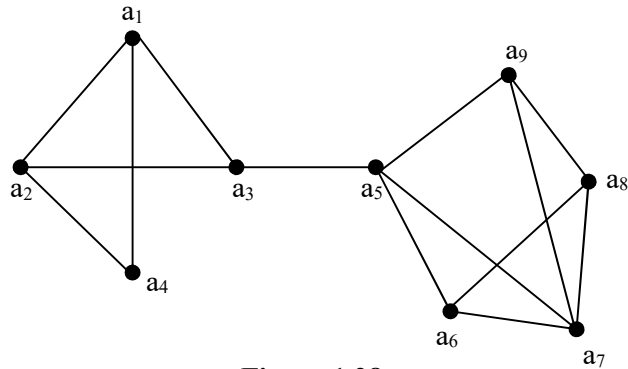


Figure 1.28

We find the graph without the  $(a_3, a_5)$  is given in figure;

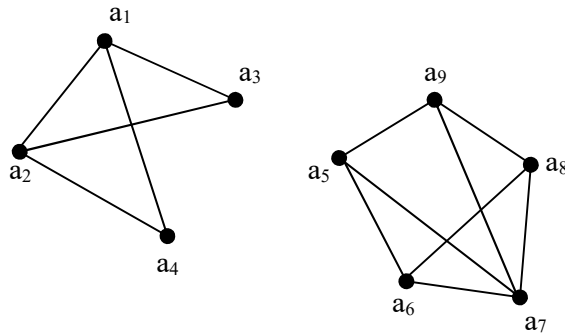


Figure 1.29

We define the notion of  $n$  node connected.

The value  $m$  is the minimum number of nodes that must be removed to make the graph disconnected. Thus removing any number of nodes less than  $m$  does not make the graph disconnected. For any value  $n$  less than  $m$ , the graph is said to be  $n$ -node connected.

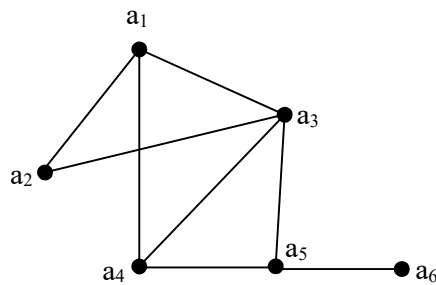
Clearly a complete graph has no cut point.

The reader is expected to familiarize with the concepts of cyclic and acyclic graphs.

Also they are supposed to know the concept of a complement of a graph  $G$ .

However we just provide examples of trees and forest. They can also be mentioned as cyclic graphs and acyclic graphs.

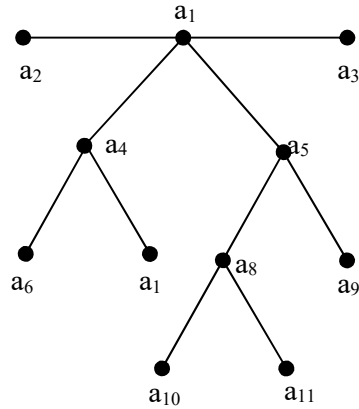
The following figure gives a cyclic graph.



**Figure 1.30**

We prove examples of trees and forests or a cyclic graph.

A connected graph and is a cyclic is called a tree.



**Tree Figure 1.31**

Trees are simple graphs since they contain the minimum number of lines that is essential to be connected and they do not contain cycles.

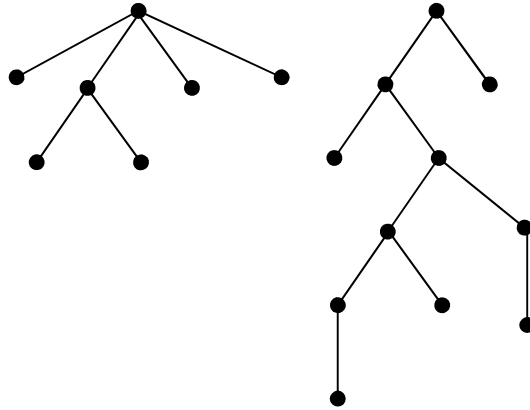
In trees every line in the graph is a bridge.

The removal of any line can make the graph disconnected.

A forest is a graph that is disconnected (has more than one component) and contains no cycles is a forest.

In forest each component is a tree.

The following figure gives an example of a forest.



**Forest Figure 1.32**

Next we proceed onto give examples of bipartite and complete bipartite graphs.

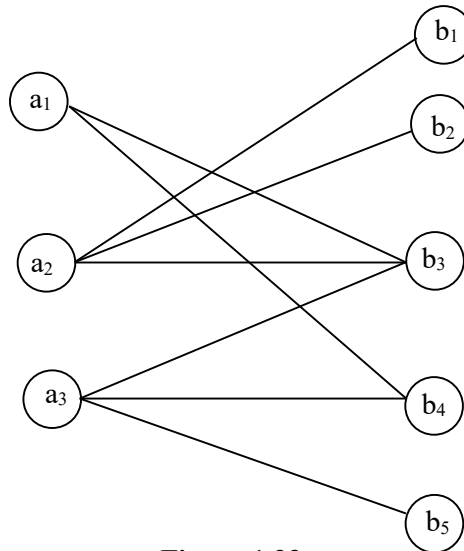
If the nodes of the graph can be partitioned into two subsets  $P_1$  and  $P_2$  that every line in  $L$  is an unordered pair of nodes in which one node is in  $P_1$  and the other nodes is in  $P_2$ .

Then we define such graphs as bipartite graphs.

We provide an example of the bipartite graph in figure 1.33 where clearly  $P_1$  and  $P_2$  are disjoint.

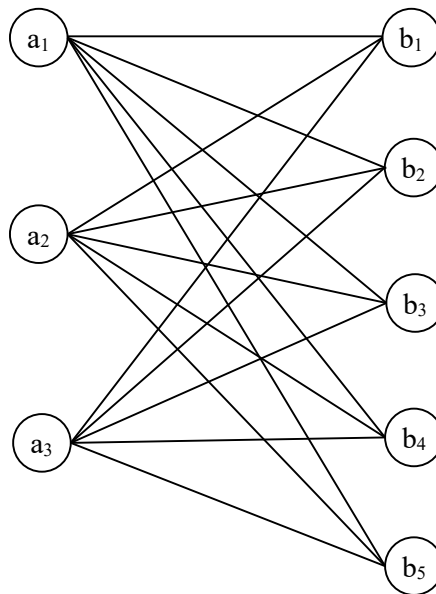
$$P_1 = \{a_1, a_2, a_3\} \quad \text{and}$$

$$P_2 = \{b_1, b_2, b_3, b_4, b_5\}.$$



**Figure 1.33**

We provide an example of a complete bipartite graph in the figure 1.34.

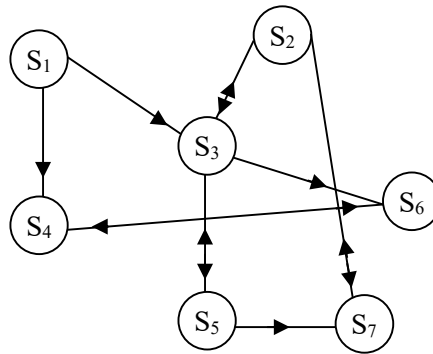


**Figure 1.34**

Now we proceed onto describe the notion of digraph or directed graph.

We give the friendship of 7 children studying in the same class.

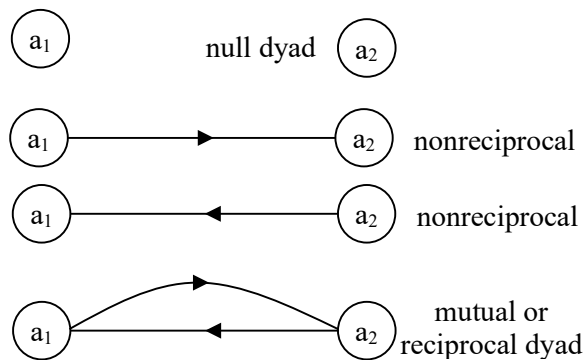
We denote the students by the nodes  $\{S_1, S_2, \dots, S_7\}$



**Figure 1.35**

Their friendship is not mutual only one sided or non reciprocating.

Thus in case of directed dyads or dyads in directed graphs we have 4 possible states of dyads given in the follows.



**Figure 1.36**

The number of dyads in the graph 1.35 is

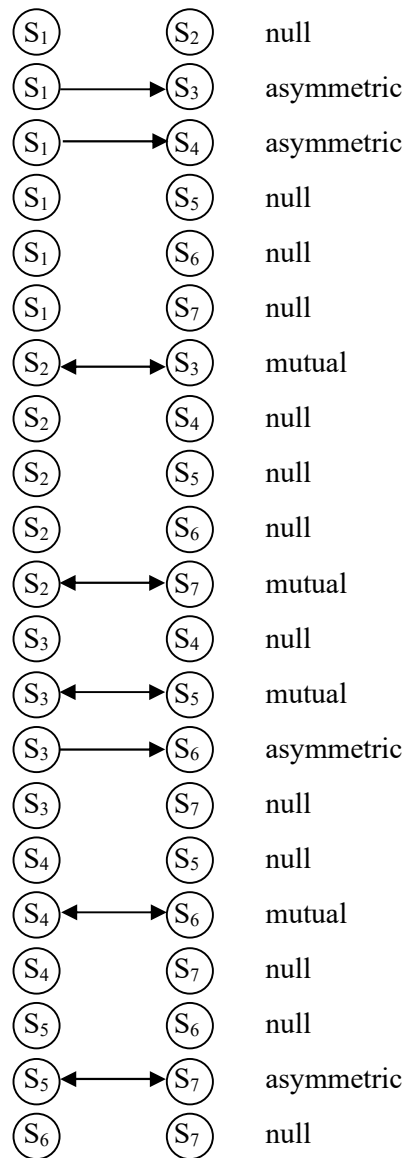


Figure 1.37



Unlike undirected graphs in case of directed graphs we study nodal in degrees and out degrees.

In a directed graph a node can be either adjacent to or adjacent from another node depending on the direction of the arc.

The in degree of a node  $d_I(S_1) = 0$  is the number of nodes adjacent to  $S_1$ .

The out degree of a node is the number of nodes adjacent from  $n_i$

Out degree is the number of arcs originating from  $S_1$

Out degree of  $S_1$  is  $d_0(S_1) = 2$ . Here  $d_0(S_i)$  denotes the out degree and  $d_I(S_i)$  denotes the in degree of the node  $S_i$

**Out degrees of nodes in G    In degrees of nodes in G**

$d_0(S_1) = 2$	$d_I(S_1) = 0$
$d_0(S_2) = 2$	$d_I(S_2) = 2$
$d_0(S_3) = 3$	$d_I(S_3) = 4$
$d_0(S_4) = 0$	$d_I(S_4) = 2$
$d_0(S_5) = 2$	$d_I(S_5) = 1$
$d_0(S_6) = 1$	$d_I(S_6) = 2$
$d_0(S_7) = 2$	$d_I(S_7) = 1$

All the formulas of classical graph theory can be used in case of ling graphs also.

We just bring out the classification of these nodes of directed graphs.

Recall  $d_i(a_i)$  denotes the in degree of the node  $a_i$  of the directed graph  $G$ .

Thus  $d_o(a_i)$  denotes the out degree of the node  $a_i$  of the directed graph  $G$ .

- We say
- i) Isolate if  $d_i(S_i) = d_o(S_i) = 0$ .
  - ii) Transmitter if  $d_i(S_i) = 0$  and  $d_o(S_i) > 0$ .
  - iii) Receiver if  $d_i(S_i) > 0$  and  $d_o(S_i) = 0$ .
  - iv) Carrier or ordinary if  $d_i(S_i) > 0$  and  $d_o(S_i) > 0$ .

The distinction between a carrier and an ordinary node is that although both kinds have both positive in degree and positive out degree and the carrier has both out degree and in degree equal to 1 whereas ordinary has both in degree and out degree greater than 1.

That is

Carrier;  $d_i(S_i) = d_o(S_i) = 1$

Ordinary;  $d_i(S_i) > 1$  and  $d_o(S_i) > 1$

These concepts help in the topology of describing the roles or positions or actors in social networks.

The formulas vary appropriately, interested readers can refer [27]. The concept of walk, trail etc; can be had from [27] for directed graphs.

We briefly mention the concept of reachability and connectivity in directed graphs.

In a graph a pair of nodes is reachable if there is a path between them.

However, in order to define reachability in a directed graph we must consider directed paths. Specifically if there is a directed path from  $S_i$  to  $S_j$  then the node  $S_j$  is reachable from  $S_i$ . To make this book a self contained one we however recall the definition of directed path.

A directed walk is a sequence of alternating nodes and arcs so that each arc has its origin at the previous node and its terminus at the subsequent node.

More simply in a directed walk all arcs are pointing in the same direction. The length of a directed walk is the number of instances of arcs in it (an arc is counted each time it occurs in the walk).

Thus a directed path or simply a path in a directed graph consists of all arcs pointing in the same directions. The length of the path is the number of arcs in it.

In a semi walk the direction of arcs is irrelevant.

A semi path joining  $S_i$  and  $S_j$  is a sequence of distinct nodes where all successive pairs of nodes are connected by an arc from the first to the second or by an arc from the second to the first for all successive pairs of nodes.

In a semi path the direction of the arcs is irrelevant.

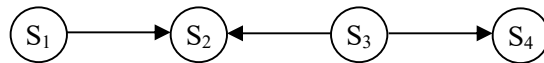
The length of a semi path is the number of arcs in it.

Every path is a semi path but every semi path need not be path.

A pair of nodes may be

- i) Weakly connected
- ii) Unilaterally connected
- iii) Strongly connected and
- iv) Recursively connected.

The pair of nodes  $S_i$  and  $S_j$  are weakly connected if they are joined by a semi path  $S_1$  to  $S_4$  is weakly connected.

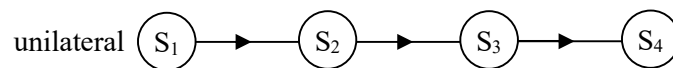


**Figure 1.38**

A pair of nodes  $S_i$  to  $S_j$  is unilaterally connected if they are joined by a path from  $S_i$  to  $S_j$  or a path from  $S_j$  to  $S_i$ .

We illustrate this situation by the following figure.

The path connecting  $S_1$  to  $S_4$  is unilateral.

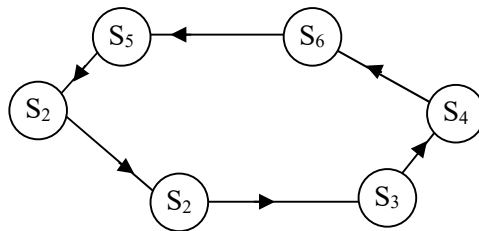


**Figure 1.39**

A pair of nodes  $S_i$  and  $S_j$  are strongly connected if all the pairs, of nodes are strongly connected.

That is the pair of nodes  $S_i$  and  $S_j$  strongly connected if there is a path from  $S_i$  to  $S_j$  and a path from  $S_j$  to  $S_i$ , the path from  $S_i$  to  $S_j$  may contain different nodes and arcs than the path from  $S_j$  to  $S_i$ .

We will illustrate this situation by one example.

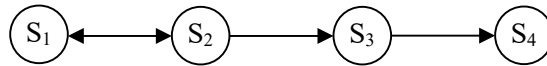


**Figure 1.40**

Next we proceed onto define recursively connected nodes.

A pair of nodes  $S_i$  and  $S_j$  are recursively connected if they are strongly connected and the path from  $S_i$  to  $S_j$  uses the same nodes and arcs as the path from  $S_j$  to  $S_i$  in the reverse order.

We will illustrate this by an example.



**Figure 1.41**

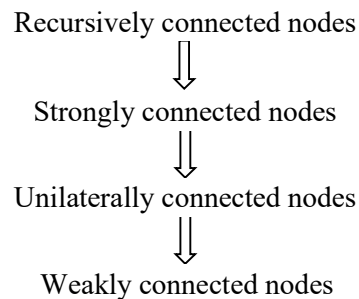
Now we make the following observations.

All recursively connected nodes are always strongly connected nodes however a strongly connected one need not in general be recursively connected. Evident from the figure 1.36 to 1.39 given.

Consider a strongly connected nodes they are clearly unilaterally connected nodes however in general a unilaterally connected nodes in general is not strongly connected nodes. This is also evident from the given figures in 1.36 to 1.39.

Now every unilaterally connected nodes are weakly connected nodes. However weakly connected nodes in general are not unilaterally connected. Evident from the figures 1.36 to 1.39.

Thus we see



Now we can extended this notion to the directed graphs in general. We only talked about the connectivity in a pair of nodes, now we describe for directed graphs.

A directed graph is weakly connected if all pairs of nodes are weakly connected.

Unilaterally connected directed graph is a graph in which every pair of nodes are unilaterally connected.

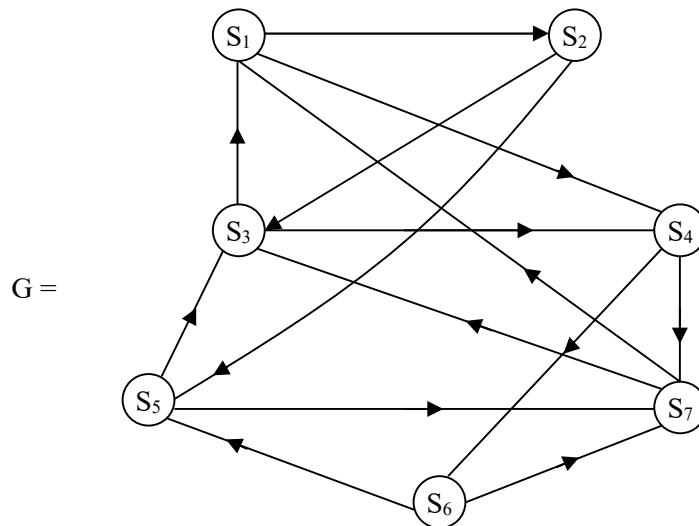
A directed graph is strongly connected if all pairs of nodes strongly connected.

A directed graph is recursively connected if all pair of nodes are necessarily connected.

Recall (geodesic) distance between a pair of nodes in a graph is the length of a shortest path between the two nodes and is the basis for defining the diameter of a graph.

Now we give an example of a directed graph and find the geodiscs and the diameter of a directed graph.

Let  $G$  be a directed graph given by the following figure 1.42.



**Figure 1.42**

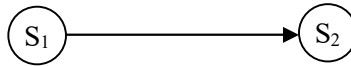
**Geodiscs distances**

$d(S_1, S_2) = 1$	$d(S_2, S_1) = 2$
$d(S_1, S_3) = 2$	$d(S_3, S_1) = 1$
$d(S_1, S_4) = 1$	$d(S_4, S_1) = 3$
$d(S_1, S_5) = 2$	$d(S_5, S_1) = 2$
$d(S_1, S_6) = 2$	$d(S_6, S_1) = 3$
$d(S_1, S_7) = 2$	$d(S_7, S_1) = 1$
$d(S_2, S_3) = 1$	$d(S_3, S_2) = 2$
$d(S_2, S_4) = 2$	$d(S_4, S_2) = 4$
$d(S_2, S_5) = 1$	$d(S_5, S_2) = 3$
$d(S_2, S_6) = 3$	$d(S_6, S_2) = 4$
$d(S_2, S_7) = 3$	$d(S_7, S_2) = 3$
$d(S_3, S_4) = 1$	$d(S_4, S_3) = 2$
$d(S_3, S_5) = 3$	$d(S_5, S_3) = 1$
$d(S_3, S_6) = 2$	$d(S_6, S_3) = 2$
$d(S_3, S_7) = 2$	$d(S_7, S_3) = 1$
$d(S_4, S_5) = 2$	$d(S_5, S_4) = 2$
$d(S_4, S_6) = 1$	$d(S_6, S_4) = 3$
$d(S_4, S_7) = 1$	$d(S_7, S_4) = 4$
$d(S_5, S_6) = 3$	$d(S_6, S_5) = 1$
$d(S_5, S_7) = 1$	$d(S_7, S_5) = 4$
$d(S_6, S_7) = 1$	$d(S_7, S_6) = 3$

The diameter is 4.

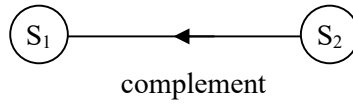


We now list out some special type of directed graphs.



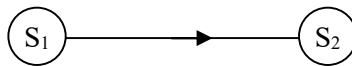
**Figure 1.43**

a directed dyad its complement is given in Figure 1.44



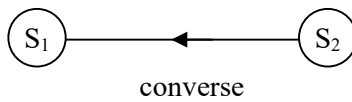
**Figure 1.44**

The converse of figure 1.45 is



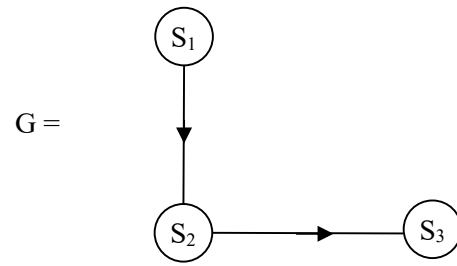
**Figure 1.45**

Given in figure 1.46



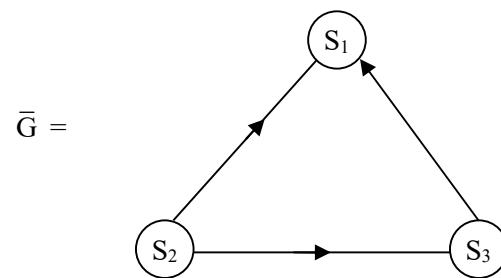
**Figure 1.46**

The complement  $\bar{G}$  (or  $G^c$ ) of the directed graph  $G$ ;



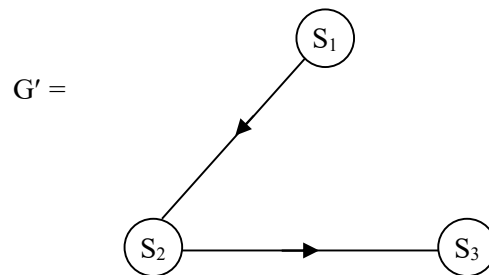
**Figure 1.47**

$\bar{G}$  is the complement of  $G$  given in figure 1.48



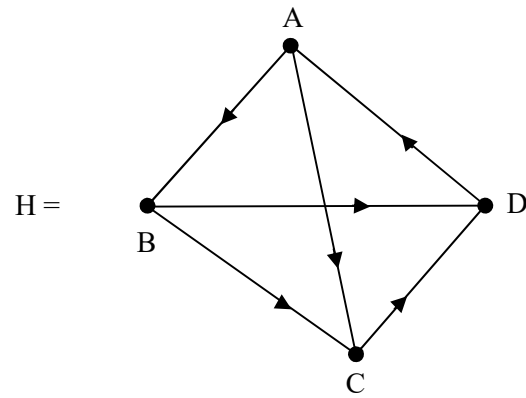
**Figure 1.48**

The converse  $G'$  of  $G$  is as follows:

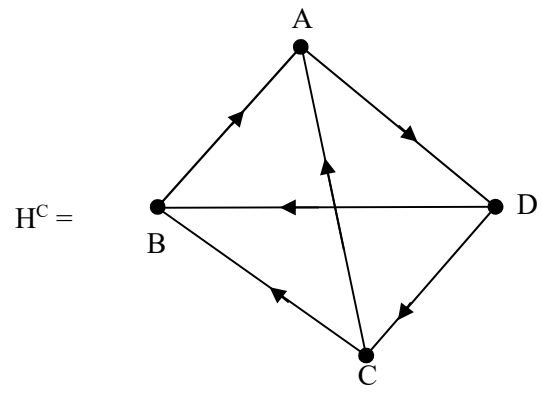


**Figure 1.49**

We give the complement of this complete digraph H.

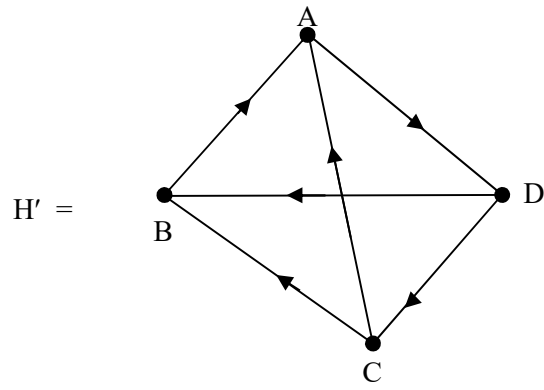


**Figure 1.50**



**Figure 1.51**

The converse of H is given by

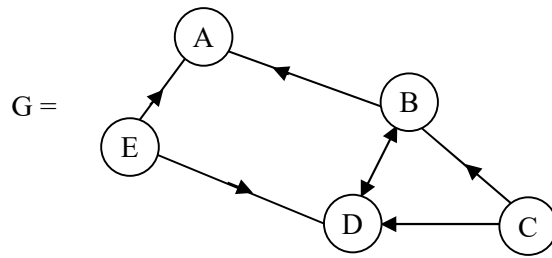


**Figure 1.52**

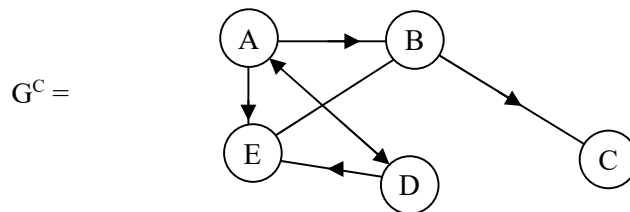
We see  $H^C = H'$  are identical.

We provide a simple exercise of finding all those directed graphs  $G$  whose converse is identical with the complement of  $G$ .

Consider the directed graph  $G$ .



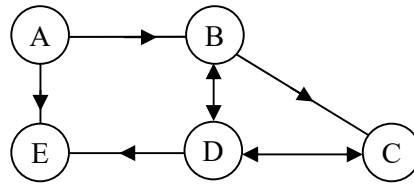
**Figure 1.53**



**Figure 1.54**

The complement of  $G$  is given in figure 1.54.

The converse  $G'$ , of  $G$ , is given by the following figure.



**Figure 1.55**

Clearly  $G^C \neq G'$ ; that is converse and complement of  $G$  are distinct.

The converse of a directed graph might be helpful in thinking about relations that have opposites.

For instance the converse of a directed graph representing a dominance relation ( $S_i$  wins over  $S_j$ ) would represent a submissive relation ( $S_j$  loses to  $S_i$ ).

On the other hand complement of a directed graph might be used to represent the absence of a tie or as not relation. For instance if  $S_i$  chooses  $S_j$  as a friend in the complement  $S_i$  does not choose  $S_j$  as a friend.

Now we briefly discuss about the valued graphs and directed valued graphs or valued directed graphs.

In a valued graph or a valued directed graph each line carries a value. In case of linguistic valued graph the values will be linguistic terms / words from the appropriate ling. set.

A valued graph consists of set of nodes

$\{S_1, S_2, \dots, S_y\} = N$  a set of lines

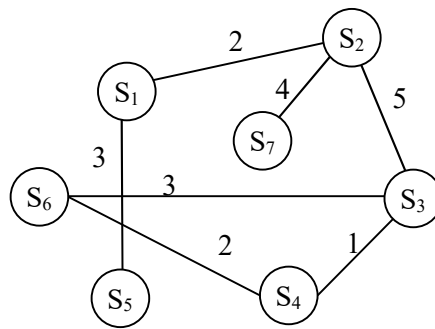
$M = \{l_1, l_2, \dots, l_L\}$  and a set of values

$V = \{v_1, v_2, \dots, v_L\}$  attached to lines  $l_i$ .

We can also denote a valued graph by  $G_V(N, M, V)$ . Valued graphs are also known as weighted graphs or weighted digraphs.

We will be calling in chapter III our valued linguistic graphs as linguistic weighted graphs.

We will illustrate by example the notion of valued graphs and directed valued graphs.



**Figure 1.56**

$$\langle S_1, S_2 \rangle = l_1 \quad v_1 = 2$$

$$\langle S_1, S_5 \rangle = l_2 \quad v_2 = 3$$

$$\langle S_2, S_3 \rangle = l_3 \quad v_3 = 5$$

$$\langle S_2, S_7 \rangle = l_4 \quad v_4 = 4$$

$$\langle S_3, S_6 \rangle = l_5 \quad v_5 = 3$$

$$\langle S_3, S_4 \rangle = l_6 \quad v_6 = 1$$

$$\langle S_4, S_6 \rangle = l_7 \quad v_7 = 2$$

It is to be noted here  $\langle S_i, S_j \rangle = \langle S_j, S_i \rangle$  as the graph is undirected.

Now we provide an example of a valued directed graph.

Let

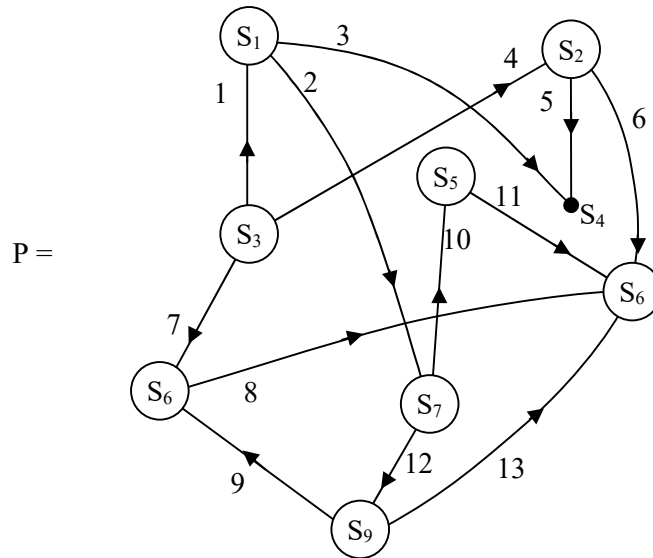
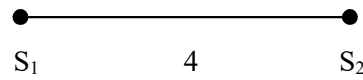


Figure 1.57

$l_1 = \langle S_1, S_4 \rangle$	$v_1 = 3$
$l_2 = \langle S_1, S_7 \rangle$	$v_2 = 2$
$l_3 = \langle S_2, S_4 \rangle$	$v_3 = 5$
$l_4 = \langle S_2, S_8 \rangle$	$v_4 = 6$
$l_5 = \langle S_3, S_1 \rangle$	$v_5 = 1$
$l_6 = \langle S_3, S_2 \rangle$	$v_6 = 4$
$l_7 = \langle S_3, S_6 \rangle$	$v_7 = 7$
$l_8 = \langle S_5, S_8 \rangle$	$v_8 = 4$
$l_9 = \langle S_6, S_8 \rangle$	$v_9 = 8$
$l_{10} = \langle S_7, S_5 \rangle$	$v_{10} = 10$
$l_{11} = \langle S_7, S_9 \rangle$	$v_{11} = 12$
$l_{12} = \langle S_9, S_6 \rangle$	$v_{12} = 12$
$l_{13} = \langle S_9, S_8 \rangle$	$v_{13} = 13$

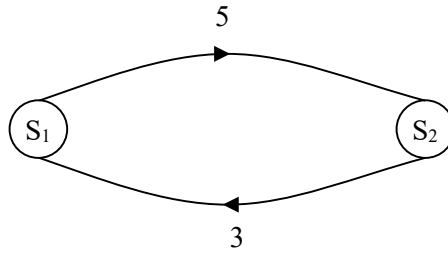
Dyads in valued graphs. A dyad in a valued graph has a line between nodes with a specific strength or value.



**Figure 1.58**

A dyad in a valued directed graph has arcs between the nodes the value  $v_1 = 5$  and  $v_2 = 3$  where  $l_{12} = \langle S_1, S_2 \rangle; v_1 = 5$   
 $l_{21} = \langle S_2, S_1 \rangle; v_2 = 3.$

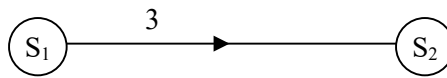




**Figure 1.59**

These values may be measuring the strength.

It may also happen



**Figure 1.60**

arc  $\langle S_2, S_2 \rangle$ ;  $v_1 = 3$



**Figure 1.61**

$l = \langle S_2, S_1 \rangle$       $v_2 = 5$ .

Such study is also interesting.

The value of a path (semi path) is equal to the smallest value attached to any line.

The value of a path is thus the weakest link in the path.

This idea makes most sense if larger value indicate stronger ties.

In general paths that include those arcs or lines with large values will have higher path values, whereas paths that include lines with small values will have lower path values.

Since all values in a path at level  $C$  are greater than or equal to  $C$ . This concept is used in social networks in the study of cohesive subgroups for valued graphs.

The valued graphs also play a role in the reachability of a pair of nodes.

For one can generalize reachability for a pair of nodes of strengths of reachability in a valued graph.

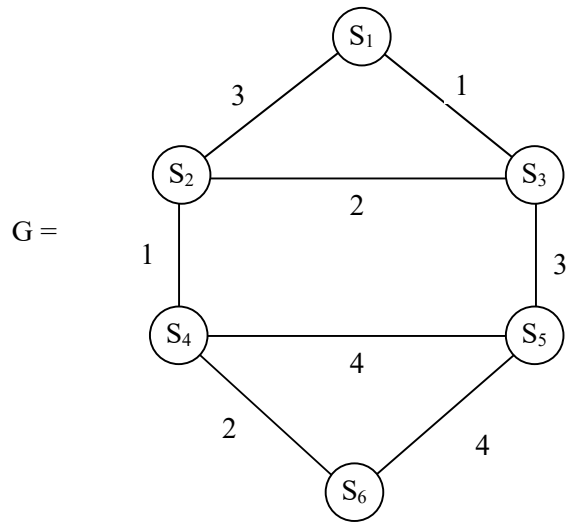
In a valued graph two nodes are reachable at level  $C$  if there is a path at level  $C$  between them. In other words we can say if two nodes are reachable at level  $C$  then there is atleast one path between them that includes no line with a value less than  $C$ . If two nodes are reachable at level  $C$  then they are reachable at any value less than  $C$ .

Now we show the path length in case of valued graphs is associated with cost.

If the values of a graph corresponds to the cost associated between two nodes  $S_i$  and  $S_j$ , then if we define the length of the path as the sum of the values of the lines in it.

We will describe a path in a valued graph by an example.

Let  $G$  be a valued graph given by the following figure.



**Figure 1.62**

- $(S_1, S_2) = l_1 \quad v_1 = 3$
- $(S_1, S_3) = l_2 \quad v_2 = 1$
- $(S_2, S_3) = l_3 \quad v_3 = 2$
- $(S_2, S_4) = l_4 \quad v_4 = 1$
- $(S_3, S_5) = l_5 \quad v_5 = 3$
- $(S_4, S_5) = l_6 \quad v_6 = 4$
- $(S_4, S_6) = l_7 \quad v_7 = 2$
- $(S_5, S_6) = l_8 \quad v_8 = 4$

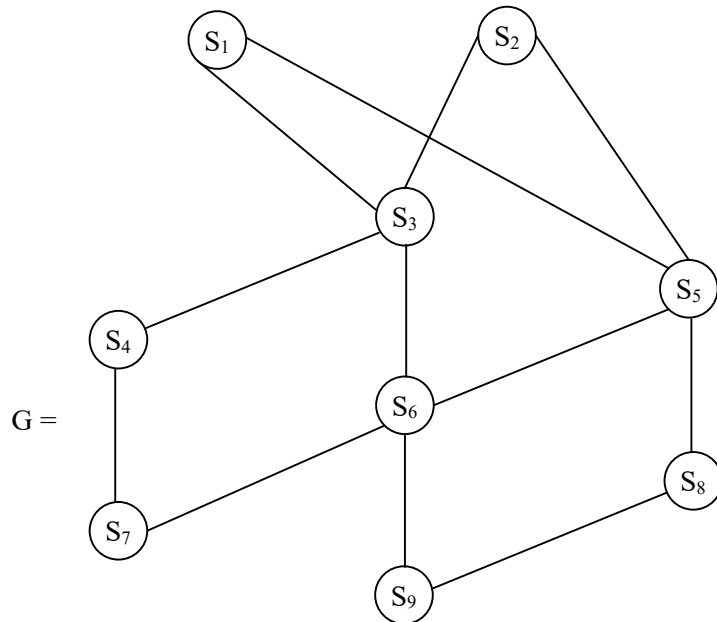
Path	Length	Value
$S_1 S_2 S_4 S_6$	6	1
$S_1 S_3 S_5 S_6$	12	2

$S_1 S_2 S_4 S_5 S_6$	12	1
$S_1 S_3 S_2 S_4 S_6$	6	1
$S_1 S_3 S_2 S_4 S_5 S_6$	12	1
$S_1 S_2 S_3 S_5 S_6$	12	2

We just give a brief account of matrices for graphs.

The adjacency matrix associated with the social network or graph is defined as the socio matrix.

Consider the graph  $G$  given by the following figure 1.63.



**Figure 1.63**

The table associated with the graph  $G$  is as follows.

	S <sub>1</sub>	S <sub>2</sub>	S <sub>3</sub>	S <sub>4</sub>	S <sub>5</sub>	S <sub>6</sub>	S <sub>7</sub>	S <sub>8</sub>	S <sub>9</sub>
S <sub>1</sub>	-	0	1	0	1	0	0	0	0
S <sub>2</sub>	0	-	1	0	1	0	0	0	0
S <sub>3</sub>	1	1	-	1	0	1	0	0	0
S <sub>4</sub>	0	0	1	-	0	0	1	0	0
S <sub>5</sub>	1	1	0	0	-	1	0	1	0
S <sub>6</sub>	0	0	1	0	1	-	1	0	1
S <sub>7</sub>	0	0	0	1	0	1	-	0	0
S <sub>8</sub>	0	0	0	0	1	0	0	-	1
S <sub>9</sub>	0	0	0	0	0	1	0	1	-

The socio matrix M associated with the graph G is as follows.

$$M = \begin{matrix} & \begin{matrix} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 & S_8 & S_9 \end{matrix} \\ \begin{matrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \\ S_8 \\ S_9 \end{matrix} & \begin{bmatrix} - & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & - & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & - & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & - & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & - & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & - & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & - & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & - & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & - \end{bmatrix} \end{matrix}$$

Now the socio matrix M of the graph G is a  $9 \times 9$  matrix which is symmetric with '-' filled along the diagonal which implies  $l(S_i, S_i) = ii$  is undefined as we have not allowed loops in the graph G and  $x_{ij} = x_{ji}; 1 \leq i, j \leq 9; i \neq j$ .

The incidence matrix is a matrix that associates the nodes with lines.

$$l_1 = (S_1, S_3) = (S_3, S_1)$$

$$l_2 = (S_1, S_5) = (S_5, S_1)$$

$$l_3 = (S_2, S_3) = (S_3, S_2)$$

$$l_4 = (S_2, S_5) = (S_5, S_2)$$

$$l_5 = (S_3, S_4) = (S_4, S_3)$$

$$l_6 = (S_3, S_6) = (S_6, S_3)$$

$$l_7 = (S_4, S_7) = (S_7, S_4)$$

$$l_8 = (S_5, S_6) = (S_6, S_5)$$

$$l_9 = (S_5, S_8) = (S_8, S_5)$$

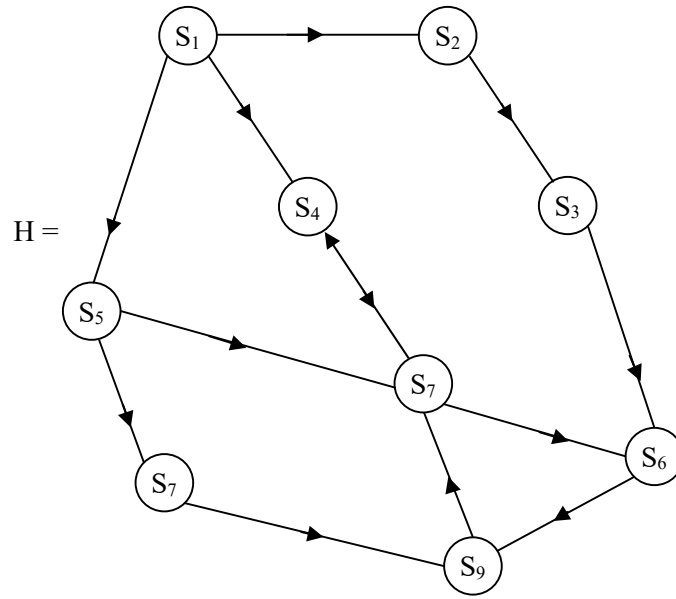
$$l_{10} = (S_6, S_7) = (S_7, S_6)$$

$$l_{11} = (S_8, S_9) = (S_9, S_8)$$

	$l_1$	$l_2$	$l_3$	$l_4$	$l_5$	$l_6$	$l_7$	$l_8$	$l_9$	$l_{10}$	$l_{11}$
$S_1$	1	1	0	0	0	0	0	0	0	0	0
$S_2$	0	0	1	1	0	0	0	0	0	0	0
$S_3$	1	0	1	0	1	1	0	0	0	0	0
$S_4$	0	0	0	0	1	0	1	0	0	0	0
$S_5$	1	0	1	1	0	0	0	1	1	0	0
$S_6$	0	0	0	0	0	1	0	1	0	1	0
$S_7$	0	0	0	0	0	0	1	0	0	1	1
$S_8$	0	0	0	0	0	0	0	0	1	0	0
$S_9$	0	0	0	0	0	0	0	0	0	0	1

This table gives the incidence matrix of the graph G.

Now we give the directed graph H and its adjacency matrix N.



**Figure 1.64**

The adjacency matrix (social matrix) of the directed graph H be N.

$$N = \begin{matrix} & \begin{matrix} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 & S_8 & S_9 \end{matrix} \\ \begin{matrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \\ S_8 \\ S_9 \end{matrix} & \begin{bmatrix} - & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & - & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & - & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & - & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & - & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & - & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & - & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & - & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & - \end{bmatrix} \end{matrix} .$$

On similar lines one can get the incidence matrix of directed graphs. We have given a brief description of ling variables, ling terms / ling continuum. We have also recall some basic needed in this book on geometry. Finally we have developed the concept of graphs that will be used in social networks. We suggest a few problems for the reader. The \* (starred) problems are difficult to solve.

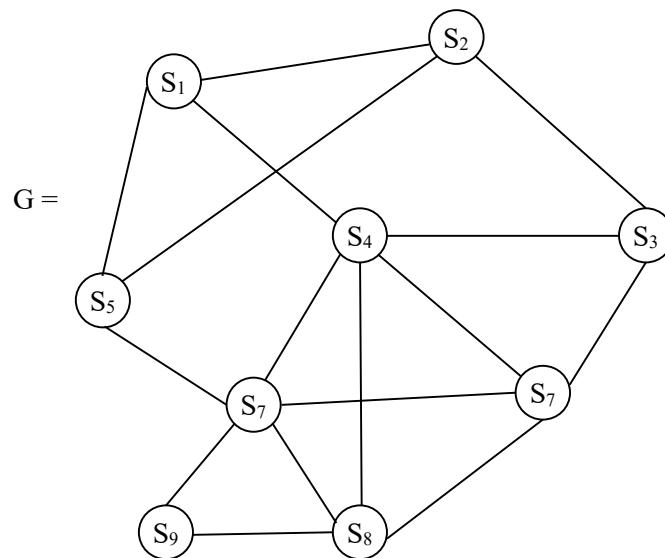
### **SUGGESTED PROBLEMS**

1. Give a ling variable whose associated ling set is only of finite cardinality.
2. Describe the concept of totally ordered set and partially order set and illustrate them by examples.
3. Can we say all ling sets associated with all ling variables are totally ordered sets? Justify your claim.
4. Give an example of a ling variable whose associated ling set is totally orderable.
5. Give an example of a ling variable whose ling set is of infinite order and not totally orderable.
6. Suppose one takes the ling variable to be the quality of mango fruits,  $V$ , with the associated set  $S$  of the ling variable  $V$  be totally ordered.
  - a) What is the cardinality of the ling set  $S$ ?
  - b) Can you substantiate by an example the study of ling sets of ling variables can perform better than fuzzy methods? Substantiate your claim.



7. Describe the ling set associated with the ling variable employee satisfaction at workplace.
8. Is speed of a car on road a ling variable V?
  - a) How is this ling variable V different from the ling variable age of people?
  - b) How is this ling variable V different from the ling variable eyes of internationals?
  - c) Is this the same like the ling variable weather report of a month?
9. Describe the ling set associated with the ling variable customer service at a restaurant.
10. Can friendship between group of people be treated as a ling variable? Justify your claim.
11. Can you describe the ling line L for any ling variable? Substantiate your claim.
12. Describe the ling geometric plane associated with the ling variable age of people.
13. Can the ling variable colour of eyes of cats give way for a ling set? Justify your claim.
14. How is this ling variable given in problem 13, is different from the ling variable performance of workers in a company? List out the difference and similarities if any.
15. Can a ling distance be defined on a ling set associated with a ling variable? Substantiate your claim.

16. Can all ling variables have a ling geometric plane associated with it? Justify your claim.
17. What are dyads? How many types of dyads can be defined in general?
18. What is a triad? How many types of triads exist?
19. Give an example of a directed graph which is strongly connected.
20. Give an example of a directed graph  $G$  which has for every node the in degree is equal to out degree.
21. For the graph  $G$  describe in figure 1.65 Calculate the diameter of  $G$ .

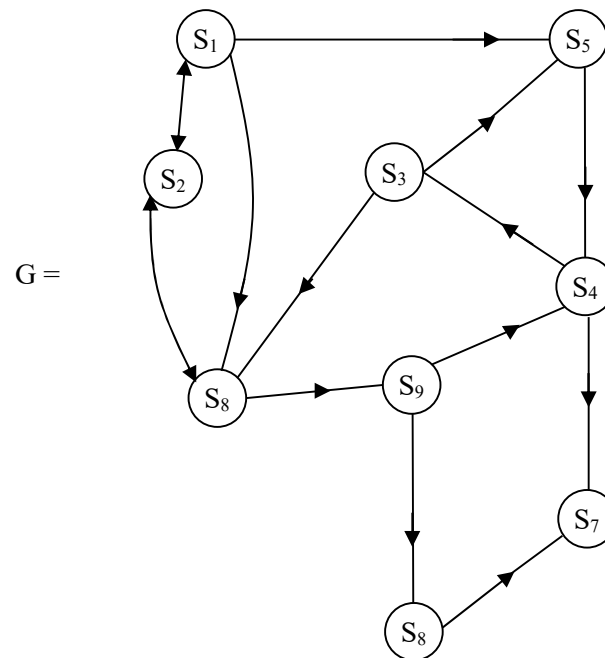


**Figure 1.65**

- i) How many complete triads are in  $G$ ?

- ii) Does  $G$  contain a complete subgraph of order four.
- iii) Can the complement of  $G$  be defined?
- iv) Does  $G$  have the converse?
- v) Find the path from  $S_2$  to  $S_9$ .
- vi) Find the longest walk that exist in  $G$ ?

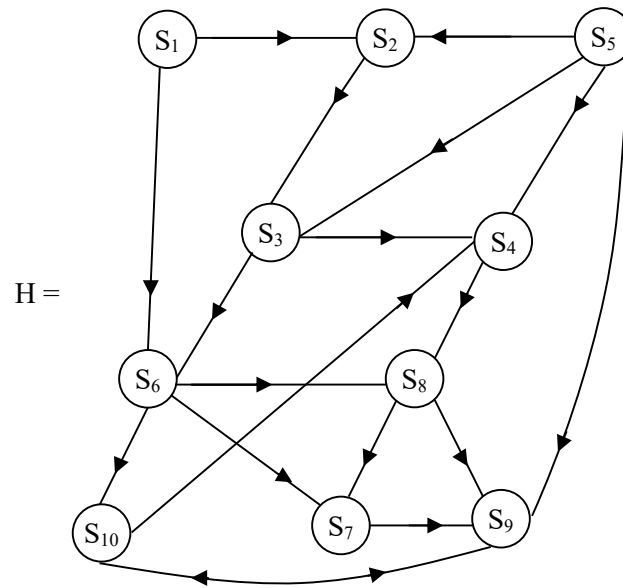
22. Let  $G$  be the directed graph given by the following figure 1.66.



**Figure 1.66**

- a) What is the diameter of  $G$ ?

- b) Is  $G$  a strongly connected graph?
  - c) Is  $G$  a weakly connected graph?
  - d) Determine the path from  $S_1$  to  $S_8$ .
  - e) Does there exist a semi path from  $S_3$  to  $S_9$ ?
  - f) Does this graph  $G$  have a cut point?
  - g) Does this graph  $G$  have a bridge?
  - h) Find the complement  $G^C$  of  $G$ .
  - i) Find the converse  $G'$  of  $G$ .
  - j) Will  $G^C = G'$ ? Justify your claim.
  - k) Does  $G$  contain a pair of points which is recursively connected?
23. Does there exist a graph such that  $G = G^C$ ?
24. Does there exist a directed graph  $G$  such that  $G = G'$ ?  
(That is the graph  $G$  identical with the converse)
25. Does there exist a graph  $G$  for which  $G = G' = G^C$ ? That is the graph  $G$  is identical with  $G$ 's complements and  $G$ 's converse.
26. Define a bipartite graph.
27. Show by examples in general all graphs are not bipartite.
28. For the directed graph  $H$  given in the following figure.



**Figure 1.67**

- a) Find the indegrees and outdegrees of the digraph H.
  - b) Find all the geodesic distances and the diameter of H.
  - c) Is H weakly connected or strong by connected or recursively connected? Justify by proof.
29. For the following directed graphs  $G_1$ ,  $G_2$ ,  $G_3$  and  $G_4$  find  $G_i'$  and  $G_i^c$ ;  $1 \leq i \leq 4$ .

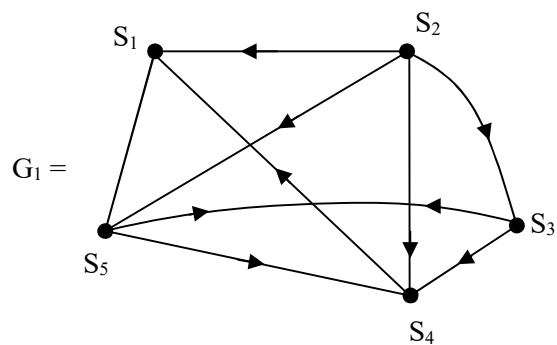


Figure 1.68

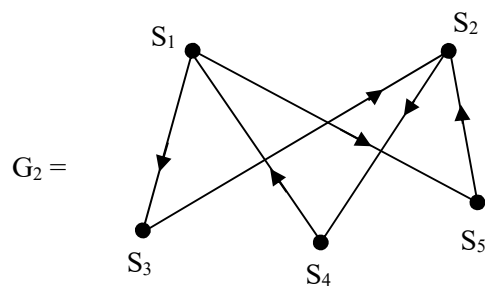


Figure 1.69

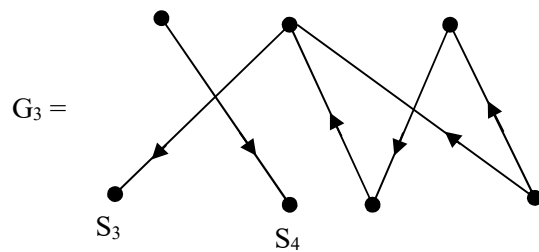
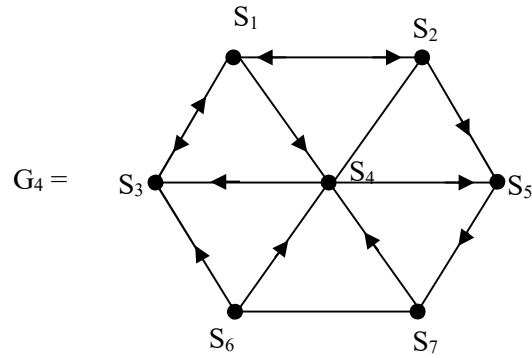


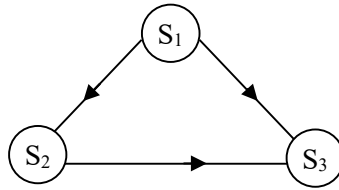
Figure 1.70



**Figure 1.71**

- a) Do we have  $G_i$  such that  $G_i = G_i^c$ ;  $1 \leq i \leq 4$ ?
  - b) Can we have  $G_i$  such that  $G_i^c = G'_i$ ;  $1 \leq i \leq 4$ ?
  - c) Distinguish each  $G_i$  for its speciality and distinctness ( $1 \leq i \leq 4$ ).
  - d) Can the complement of a bipartite graph be a bipartite graph?
30. What will be the complement of a complete directed graph? Will it be complete?
  31. What is the converse of the complete directed graph? Will it be complete?
  32. Obtain any special and distinct properties enjoyed by complete directed graph.

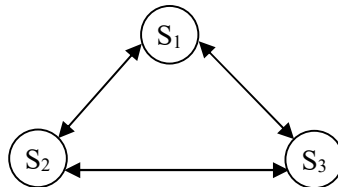
33. Let  $G =$



**Figure 1.72**

i) Find  $G'$  and  $G^c$ ?

34. Let  $H =$



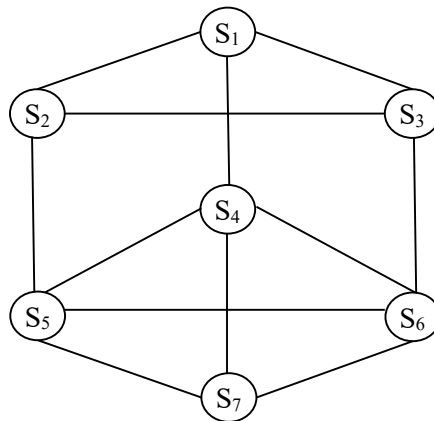
**Figure 1.73**

be a complete directed graph. Find  $H'$  and  $H^c$ .

35. Compare  $H$  and  $G$  given in problems 34 and 33 respectively.

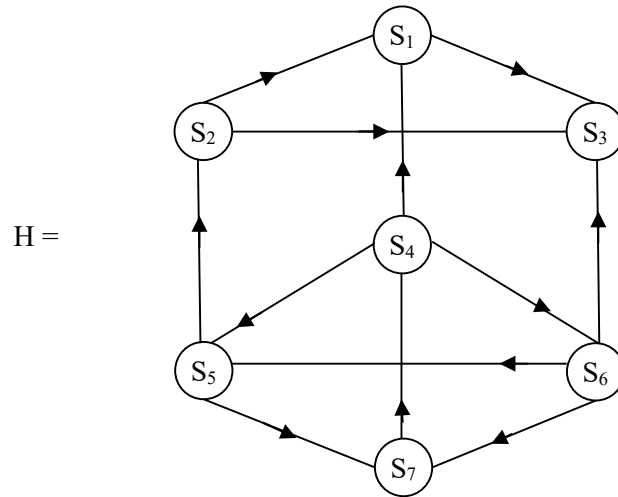
36. Calculate the diameter of the following two graphs and compare their diameter.

$G =$



**Figure 1.74**



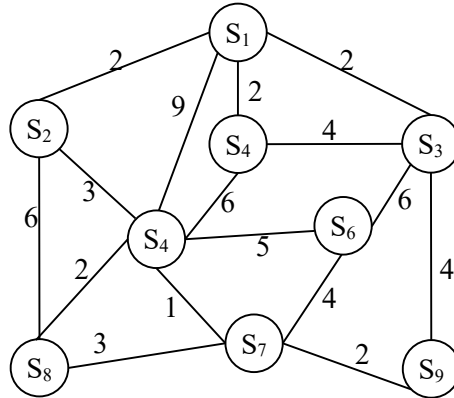


**Figure 1.75**

- a) Which of the graphs have a bigger diameter?
  - b) Define a path from  $S_1$  to  $S_7$  in both G and H.
  - c) Are they strongly connected or weakly connected?
  - d) Find the complements of G and H.
  - e) Find the converse of G and H.
  - f) What are main differences between the graphs G and H?
37. For the graphs G and H given in problem 36 find the socio matrix and the incidence matrix.
38. a) What is a weighted graph?
- b) How is it different from the usual graphs?

c) What are the advantages of using weighted graphs?

39. Consider the valued graph  $G$  given by the following figure.



**Figure 1.76**

Calculate the path values from  $S_i$  to  $S_j$ . Calculate the length and value  $i \neq j; 1 \leq i, j \leq 9$ .

40. For the graph  $G$  in figure 1.76 in problem 39.

- a) Find the adjacency matrix of  $G$ .
- b) Is  $G$  in problem 39 connected or not?
- c) Does  $G$  have cut point? Justify your claim.

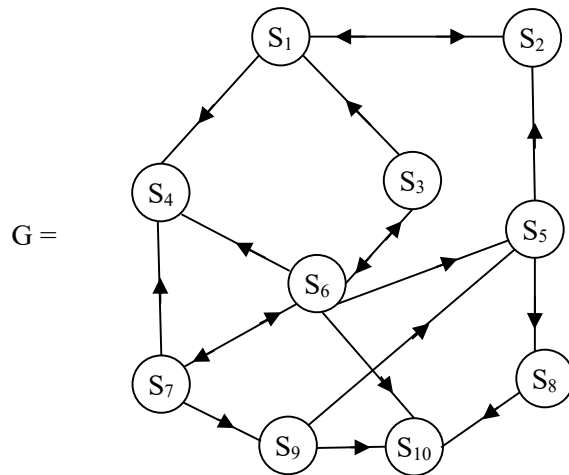
41. There are 10 students  $S_1, S_2, \dots, S_{10}$  from a same apartment.

$S_1$  is a neighbor of  $S_2$  and  $S_4$

$S_4$  is a neighbor of  $S_5$  and  $S_6$ .

$S_7$  is a neighbor of  $S_3$  and  $S_8$ .

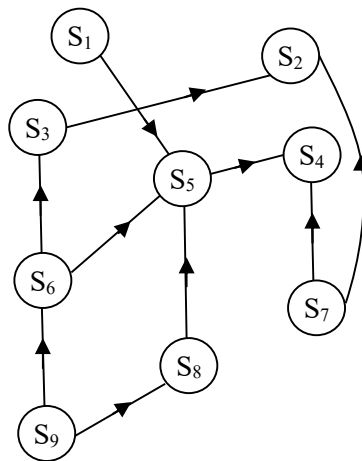
- a) Find the neighbour graph  $G$  of these 10 students.
  - b) Is it a directed graph?
  - c) Is the graph  $G$  connected or not?
  - d) How many lines are there in  $G$ ?
  - e) Find all the subgraphs of  $G$ .
42. Give an example of cyclic graph, tree and forest.
43. Can forest have complements? Justify your claim.
44. For the following directed graph  $G$ .



**Figure 1.77**

- a) Find all out degrees of  $G$ .
- b) Find all in degrees of  $G$ .

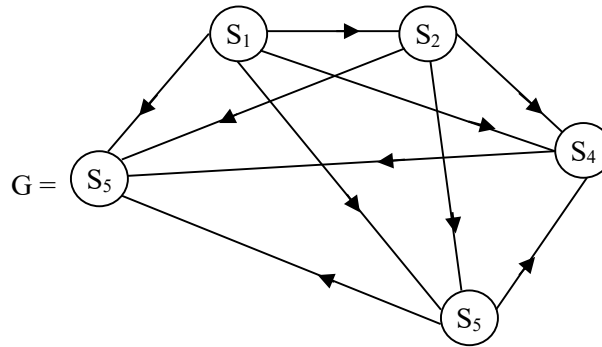
- c) Find  $G^C$  and  $G'$  of  $G$ .
  - d) Find the path  $S_1$  to  $S_{10}$ .
  - e) Is it weakly connected?
  - f) Can the complement of  $G$  be recursively connected?
45. Give an example of a unilaterally connected graph.
  46. Give an example of a weakly connected graph which is not unilaterally connected.
  47. Give an example of a unilaterally connected graph which is not strongly connected.
  48. Give an example of a strongly connected graph which is not recursively connected.
  49. Give an example of a valued directed graph.
  50. What is reachability in a valued graph?
  51. Give an example of a directed graph which has none of its nodes to be carrier or ordinary.
  52. For the directed graph given by the following figure 1.78.



**Figure 1.78**

- i) How many nodes are isolate?
- ii) How many are transmitter?
- iii) How many are receiver?
- iv) How many are carrier or ordinary?

53. For the complete directed graph given by the follow figure 1.79



**Figure 1.79**

- a) Can this directed graph G have isolate nodes? Justify your claim?
  - b) Can a complete directed graph have nodes which are isolate?
  - c) Can G have transmitter node?
  - d) Describe all receiver nodes if any in G.
  - e) Can this G have carrier or ordinary nodes in G?
54. Define density of a directed graph and find the density of the directed graph given in figure 1.79 given in problem 53.

55. Give a directed walk of the graph G given in problem 53.
56. Define a directed triad. Find a directed triad in the graph G given in problem 53.
57. Define the notion of matrix permutation.
- a) Give an example of matrix permutation of a socio matrix.
- b) If X gives a socio matrix give X permuted.

	s <sub>1</sub>	s <sub>2</sub>	s <sub>3</sub>	s <sub>4</sub>	s <sub>5</sub>	s <sub>6</sub>
s <sub>1</sub>	-	1	0	1	0	0
s <sub>2</sub>	1	-	0	0	1	1
s <sub>3</sub>	1	0	-	1	0	1
s <sub>4</sub>	0	1	1	0	-	0
s <sub>5</sub>	0	0	1	1	1	-

- c) Where can one use the concept of matrix permutation?
58. Define connectivity of a graph.
59. Give an example of a 2-node connected graph.
60. Give an example of a 5-node connected graph.
61. Define line connectivity or edge connectivity of a graph.
62. Describe point connectivity or node connectivity of a graph.

63. For the following graph G

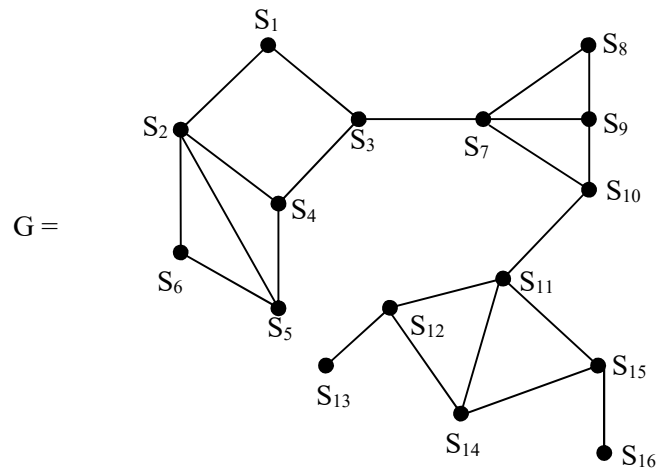


Figure 1.80

- Which of the lines are bridges of this graph G.
- Does this graph contain a node cut or cut point?
- For the tree T given in figure

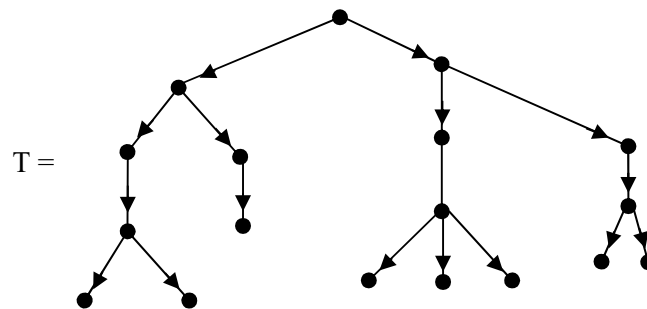
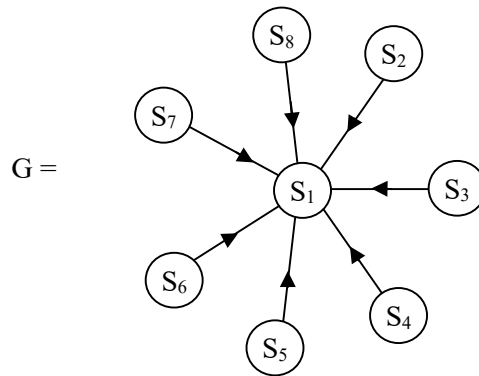


Figure 1.81

Does  $T^c$  and  $T'$ , the complement and converse exist?  
Justify your claim.

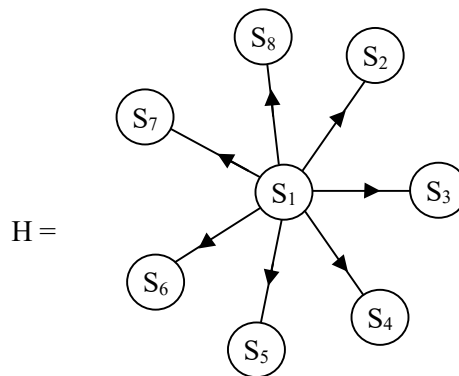
65. Let  $G$  be a star graph.



**Figure 1.82**

- a) Does  $G$  have  $G^C$  and  $G'$ ? Justify your claim.
- b) What are the values of  $d_0(S_i)$  and  $d_1(S_i)$ ?
- c) Are the nodes isolate or transmitter or receiver or carrier or ordinary? Substantiate your claim!

66. Suppose  $H$  is the star graph given by the following figure.

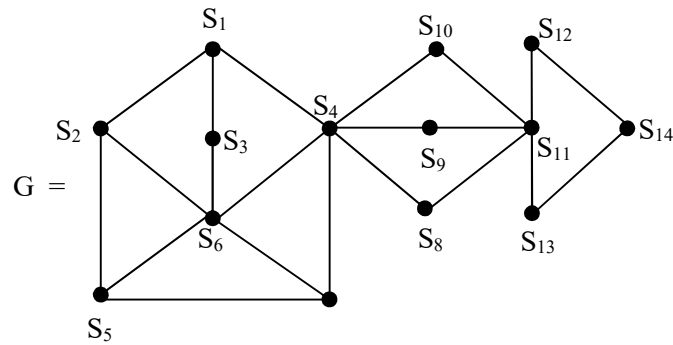


**Figure 1.83**



Study questions (a), (b) and (c) of problem 65 for this H.

67. Give an example of a graph which comprise of a 2-node cut.
68. Give an example of a graph which comprise a 3-node cut.
69. Let  $G$  be a graph.

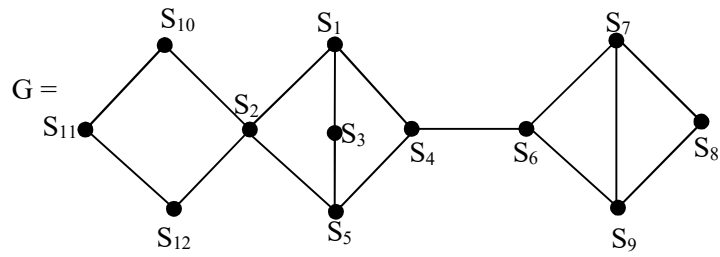


**Figure 1.84**

be a graph.

- a) Does  $G$  have 2 cut node? How many nodes must be removed to make  $G$  and is connected are?
70. If the node  $S_4$  is removed, does the graph  $G$  given in problem 69 in figure 1.84 become disconnected?
71. If the node  $S_{11}$  is removed in the graph  $G$  given in problem 69 in figure 1.84 will  $G$  become disconnected.
72. Give an example of graph which becomes disconnected by removing an edge or a node.

73. Let  $G$  be a graph given by the following figure 1.85.



**Figure 1.85**

- i) Which is the bridge of the graph  $G$ ?
- ii) Which is the cut node of  $G$ ?
- iii) What is the length of the path from  $S_{11}$  to  $S_8$ ?

## Chapter Two

### FUNDAMENTALS OF LINGUISTIC GEOMETRY

In this chapter, we introduce the basic concepts of linguistic geometry. We have given the basics of classical geometry and basics of linguistic graphs in Chapter I. However, for us to define linguistic geometry we need to define only a linguistic plane.

When we say a linguistic plane it is not unique. The ling plane depends on the ling variable or variables under consideration.

We can also have the notion of semilinguistic planes.

Thus one has to keep in mind the following facts.

The ling planes in general are not unique; they depend on the ling variables under consideration, however, they are just like planes used in classical geometry.

There is yet another type of ling plane which we call as semi ling planes which are different from general ling planes.

These semi ling planes are also many and they also depend on the ling and real linguistic variables we consider.

Given any two linguistic points we cannot always connect them by a linguistic line.

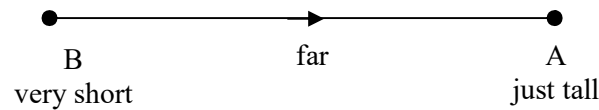
Sometimes the ling lines may be directed or sometimes undirected and at times cannot find a ling line between them. We use ling in the place of linguistic in this book.

Before we proceed onto define these concepts abstractly we will provide some examples of them.

**Example 2.1.** Let us consider the ling variable height of people. The ling continuum associated with this ling variable height is given by [shortest, tallest] = S.

Consider any two ling terms or points in S, say  
A = (just tall) and B = (very short).

In the ling plane which is associated with height we see the ling distance between A and B is far



**Figure 2.1**

We observe the following facts.

- i) very short < just tall so these two points are orderable, that is when we find the distance

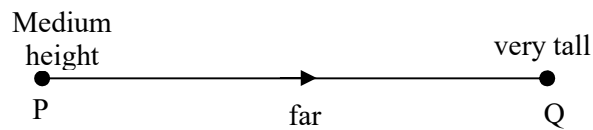
between them. This distance we call as ling distance. Since the ling variable pertains to height of persons, clearly it cannot become less after some time for any person. It can only become greater. For we see the less height person there is a chance to become tall.

Hence we have to say the distance from B to A is ‘far’.

Further we cannot measure the distance from A to B for it is undefined, for the reasons given above.

Thus ling geometrical the distance from B to A is far and the ling distance from A to B cannot be defined. This is limitations of ling geometry. So only meaningful distance can be measured. Further we see unlike classical geometry between two linguistic points at times we have associate a distance and direction, only B to A is defined and A to has no meaning.

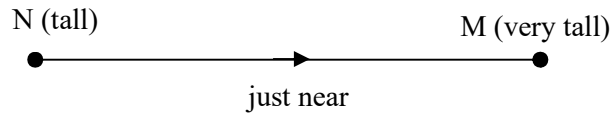
Consider the two ling variables in S, P = medium height and Q = very tall. The ling distance between P and Q is as follows.



**Figure 2.2**

The ling distance from the point P to Q is ‘far’. However we cannot define ling distance from Q to P as a very tall person cannot become medium height. So PQ is defined QP remains undefined.

Consider the two ling points M (very tall) and N (tall)  $\in$  S. We see the ling distance is defined only from N to M and from M to N the distance is undefined.

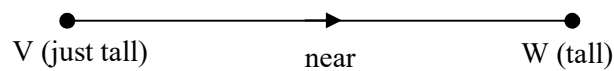


**Figure 2.3**

Consider the two ling points W = (tall) and V (just tall).

The linguistic distance from V to W is defined, however the ling distance from W to V is undefined.

The ling distance from V to W is given by the following figure 2.4.



**Figure 2.4**

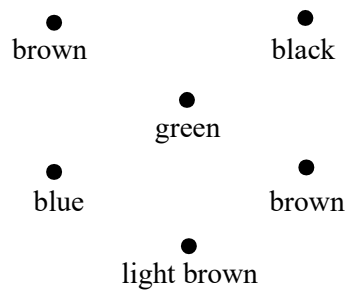
In fact the ling distance from V to W is near.

Further we wish to state that if the ling set associated with some ling variable is a ling continuum then the distance between every pair of ling point / terms is defined with one and only one direction and not both ways, that is it is defined from less ling value to more ling value the other way from more ling value to less ling value cannot be defined or is undefined.

Now consider the ling variable colour of the eyes of people internationally. The ling set associated with this ling variable is given by

$$S = \{\text{green, bluish, hazel, brown, dark brown, blacks, light brown}\}.$$

Now taking the plane giving the colour of eyes we see no distance concept can be defined on S; for these ling terms / points do not have a ling line connected with them as the ling points colour of eyes of people cannot change; the colour is fixed at birth so finding ling distance for any pair is impossible as they do not exist.



**Figure 2.5**

They are infact a set of scattered points.

So all ling variables do not in general give way to ling sets which has a ling distance defined between any two points in them.

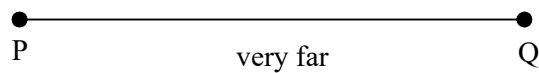
One mentioned above can only give scattered ling points in the ling plane of colour of eyes of different nationalities. So we have seen there are ling variables whose associated ling sets

do not give even a ling pair of points which have ling distance defined between them.

Consider the following example which enjoys a special type of ling distance.

**Example 2.2.** Let us consider the ling variable performance aspects of students in the classroom. The ling set which is a ling continuum associated with this ling variable is [worst, best] = V. How to find the ling distance between any two points in the set V.

Consider the two ling points P = (good) and Q = (very bad) in V. The ling distance between P and Q is very far.

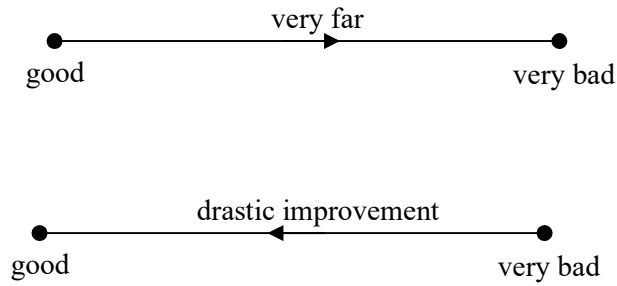


**Figure 2.6**

Clearly we do not give any directed line in this case as it can be P to Q or Q to P. Hence in this ling set we do not give any direction. However we can define linguistic distance in another way, where the measurement of distance is not in terms of far or near but the distance is improvement, little improvement, drastic improvement, deterioration and so on. If the performance of a student changes from good “very bad” we measure it as deterioration.

If the very bad performed student changes to good we can say or measure the distance as drastic improvement. This is represented by the following figures 2.7.



**Figure 2.7**

We see the distance measurement in this case is linguistic words. The words may be of different shades not very much orderable or comparable as elements from a continuum.

However it is clear from the figure 2.7 the direction is mandatory but in case of figure 2.7 the direction is not necessary or irrelevant, for we check the nearness or far away from the two ling words, time can do both P to Q or Q to P.

As the ling concept very bad student can in due course of time may become good and it is also possible a good student can become very bad also.

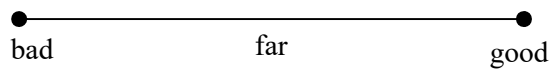
That is why we do not give any direction in case of ling points in the ling set continuum

[worst, best].

Hence this is an example where direction between any two ling points / terms in V are always possible as the ling set is a ling continuum, every pair is comparable as V is a totally ordered set.

The main difference between this V given in the above example and S given in example 2.1 are different for the ling continuum S is also only a totally ordered set but in S only every pair is ordered and there is direction as it pertain to height. The ling term height cannot be increasing and decreasing it can only increase.

Hence it is always a ling directed straight line whereas the ling continuum V, every pair has ling straight line but no direction need be put as these ling concepts are such that from one to another and vice versa is always possible. For instance in due course of time a good student (performance wise in studies) can become bad student (performance wise in studies) and vice versa. That is there is all possibilities a bad students in studies in due course of time may become good so we only can say.



**Figure 2.8**

No direction is put. We have given two types for both of them the ling set is a ling continuum which is totally ordered. But they behave differently. The ling variables under consideration of these ling sets play a vital role in their behavior.

Finally we see we can also the distance in terms different set of ling. terms improved performance, not satisfactory, no significant change, slight deterioration and so on.

However we now still consider yet another example.

**Example 2.3.** Let us consider a basket full mangoes both ripe, partially ripe or unripe of various sizes. We want to study the ling set associated with the ling variable quality of these mangoes.

The ling set associated with these ling variable be

$$W = \{\text{big, very big, small, medium size, ripe, very ripe, unripe, raw, just ripe}\}.$$

Now consider the ling plane associated with this. We analyse if these ling terms in  $W$  can be ordered. The answer is they can only be partially ordered.

For we cannot relate ripeness with size for we can have small size mangoes which is well ripe and also tasty. There can be big mangoes which are ripe or unripe or just ripe.

Now ripe cannot be compared with size. We have

$$\text{unripe} \leq \text{just ripe} \leq \text{ripe}$$

$$\text{small} \leq \text{big} \leq \text{very big so on.}$$

Thus  $W$  can only be a partially ordered set.

Now as the mangoes are not on the trees only in a basket already plucked we cannot compare the size connecting say as it can be small variety and big variety.



**Figure 2.9**

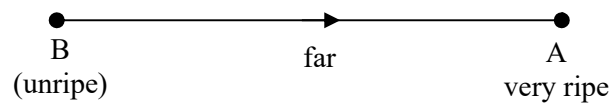
We cannot find a ling line connecting them for no relation exist for small to big and big can never become small.

However if we have A (very ripe) and B (unripe) in the ling set W. Both comparable for a unripe mango can always become ripe or very ripe in due course of time.

However a very ripe mango can never become unripe.

So we can have a ling line which has direction.

That is



**Figure 2.10**

and the direction is B to A and its ling distance is far.

This example the ling set W is different from the ling continuum V and S used in other examples.

Thus we have by no means a fixed way of finding ling distances it may be directed or undefined or at times undirected. We have given examples of all the three situations.

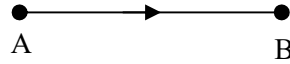
These ling distance concept is used in social information networks using ling geometry.

If A and B are two persons, the following relations can occur.



**Figure 2.11**

A does not know B and B does not know A and they are not friends

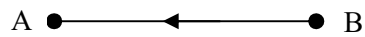


**Figure 2.12**

A is friend of B but B knows A but is not a friend of A so the ling line is from A to B and not from B to A.

Consider the A and B, A knows B and A is not a friend of B but B is a friend of A.

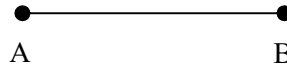
So



**Figure 2.13**

The ling line is directed and is from B to A, different from all the two types mentioned.

Now it so happen A is a friend of B as well as B is a friend of A. Thus we have



**Figure 2.14**

This ling line is not directed so is different from the other two cases.

Thus this sort ling lines will be very helpful in studying social information networks as well.

We can define the four figures as ling dyad (linguistic dyad) which forms the basis or foundation of social information networks, in this case linguistic social information networks.

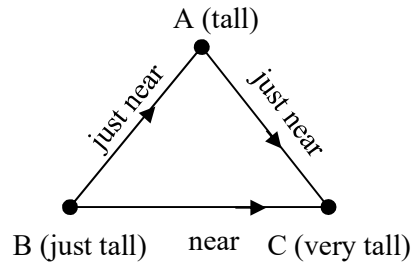
Next we define ling triangles in ling planes, which is the ling geometric triangles. Unlike these ordinary triangles given in classical geometry we may not be able to give all properties in the ling geometry of triangles. Whatever is possible alone we will give here.

Consider the example 2.1 where  $S$  given is a totally ordered set in fact  $S = [\text{shortest}, \text{tallest}]$  is a ling continuum associated with the ling variable height of persons.

Let us consider 3 linguistic terms in  $S$  given by

$A = \text{tall}$ ,  $B = \text{just tall}$  and  $C = \text{very tall}$ .

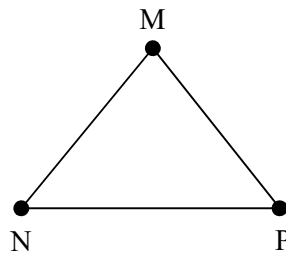
Now we give the ling triangle associated with them.



**Figure 2.15**

We know the ling triangle is such that the edges are directed and instead just of the length we can say ling lengths.

Now when we have triangle in classical geometry.



**Figure 2.16**

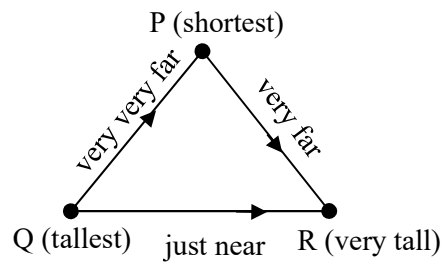
We say since the side  $MN = MP$  we call it as the isosceles triangle.

Consider the ling triangle ABC given in Figure.

The ling distance  $BA = AC = \text{just near}$ .

However  $BC = \text{near}$ . Now we call this ling triangle as linguistically isosceles triangle.

We will give different types of ling triangles taking ling terms from S.



**Figure 2.17**

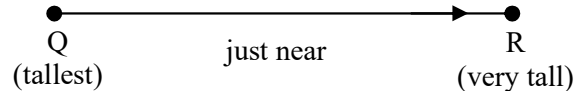
The ling terms shortest, very tall, tallest are in S.

Clearly P(shortest) to become Q tallest will be very very far from  $P \rightarrow Q$  however Q to P has no meaning or is undefined for a tallest man/woman cannot become shortest by any means. Consider the ling terms P(shortest) to R(very tall). The ling distance is defined only from P to R that is  $P \rightarrow R$ . We see the distance from P to R is very far for the shortest to become very tall.

We cannot define any ling line or ling distance from R to P for no person who is very tall can ever become shortest.

Consider the ling terms Q(tallest) and R(very tall) in the ling continuum. We cannot get a ling line or define a ling distance from R to Q; we have ling distance only from Q to R given by

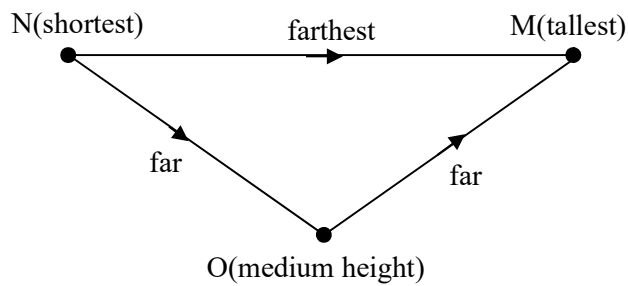




**Figure 2.18**

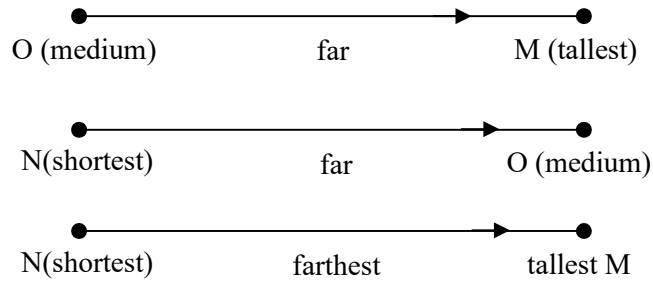
Clearly the ling triangle ABC is different from the ling triangle PQR. ABC is an isosceles ling triangle where as PQR is not so.

Now consider the ling terms M (tallest), N(shortest) and O(medium height) from S. The ling triangle formed by MNO is given in the following.



**Figure 2.19**

We see the ling triangle with ling distance is as follows



**Figure 2.20**

Now we see this is a complete linguistic triad. Triads play a major role in the study of social network. Now we see these ling geometrical triangles given in figures 2.17 and 2.19 are ling triads which will throw light in the study and research in social linguistic information networks.

In view of this we have the following theorem.

**Theorem 2.1.** *Let  $L$  be a ling variable which increases with time.  $S = [a_1, a_n]$  be a ling continuum ( $a_1 \leq a_2 \dots \leq a_n$ ) associated with  $L$ .*

- i)  *$S$  is a totally ordered set.*
- ii) *For every distinct pair of ling terms in  $S$  there exists a unique directed ling line which measures the distance.*

Proof. Follows from the simple fact if  $a_i \in S$ ,  $a_{i+1} \in S$  is reach after a time period so  $a_i \rightarrow a_{i+1}$  only exist and  $a_{i+1}$  to  $a_i$  cannot be defined or is undefined  $1 \leq i \leq \infty$ ; since the ling variable is time dependent.

**Example 2.4.** Let  $L$  be a ling variable age of people. Clearly  $L$  is time dependent for as time increases age also increases for some period of time.

$S = [\text{youngest, oldest}]$  be the ling continuum associated with the linguistic variable  $L$ , age of people.

The ling set  $S$  satisfies the conditions of theorem 2.1.

**Example 2.5.** Let  $L$  be a ling variable, growth of plants in terms of height. Clearly  $L$  is time dependent. Let  $M = [\text{stunted, very tall}]$  be the ling continuum associated with the ling variable  $L$ .

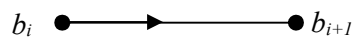
$M$  satisfies the conditions of theorem 2.1.

Interested readers can get more examples of these situation of this type.

Now we give the following result which relaxes the condition. The ling line is not directed.

**Theorem 2.2.** *Let  $L$  be a ling variable which is time independent (that is occurrence of the ling variable has no relevance to time. Though changes take place with time that is it does increase with time or decrease with time). Let  $T$  be the ling set/continuum  $T = [b_1, b_n]$  associated with the ling variable  $L$  ( $b_1 < b_n$ )*

- i)  $T$  is a totally ordered set.
- ii) For every pair  $b_i, b_{i+1}$  of distinct ling terms in  $T$  we have



**Figure 2.21**

though  $b_i < b_{i+1}$  still we have the possibility  $b_i \rightarrow b_{i+1}$  as well as  $b_{i+1} \rightarrow b_i$  that is

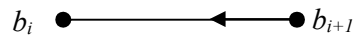


Figure 2.22

Proof. Follows from the fact as T is not time dependent we say occurrence of  $b_i$  and  $b_{i+1}$  occurs with change of time but can be increasing or decreasing. However because T is a ling continuum we have

$$b_1 \leq b_2 \leq \dots \leq b_i \leq b_{i+1} < \dots \leq b_n$$

for any  $i, j \in N = \{1, 2, \dots, \infty\}$ .

We give examples of this situation.

**Example 2.6.** Consider the linguistic variable L performance of students performance in the studies in the class room.

Clearly  $B = [\text{worst}, \text{best}]$  is the ling continuum / set associated with the ling variable L.

We clearly see

$$\text{worst} \leq \dots \leq \text{bad} \leq \dots \leq \text{far} \leq \dots \leq \text{good} \leq \dots \leq \text{best}$$

Now if we take good and bad  $\in B$  we see  $\text{good} \leftrightarrow \text{bad}$  for

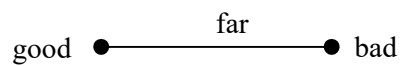


Figure 2.23

for a good student can become bad or a bad student can become good so no direction need be given but the measurement or change takes place with times for it can be changing from good to bad or bad to good.

Hence this stands as an example of theorem 2.2.

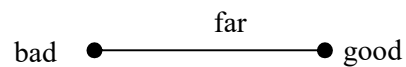
We give yet another example.

**Example 2.7.** Let us consider the performance aspects of a worker in a factory as the ling variable L.

Let the ling set associated with the ling variable L is given by  $S = [\text{very bad}, \text{very good}]$ .

This is a ling continuum so totally orderable. This ling set and the ling variable is independent of time.

Thus if good and bad  $\in S$  we have the ling distance is given by



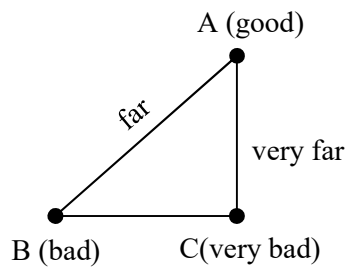
**Figure 2.24**

For a worker can become from bad to good or change from good to bad both can happen with change of time that is in due course of time. Thus this is also an example of theorem 2.2.

Now having seen examples of both types we will discuss about analogous to triads the notion of ling triads used in ling. Geometry in social information networks.

Consider the triads or ling triangles with ling elements / terms from S. Take the ling set  $\{\text{good, bad, very bad}\} \in S$ .

The ling triangle ABC given by this set is as follows.



**Figure 2.25**

This ABC is a complete triad.

**Theorem 2.3.** *Let L and S be as in theorem 2.1. Then every triangle in S gives way to a ling geometrical triangle which is a directed linguistic triad.*

Proof. From theorem 2.1 we know every pair contributes to a directed ling line or a ling line which is directed. So every triple in S will give way to a directed triad as every of ling pair is a ling line which is a directed dyad.

**Theorem 2.4.** Consider the ling variable  $L$  and the ling set  $T$  be as in theorem 2.2.

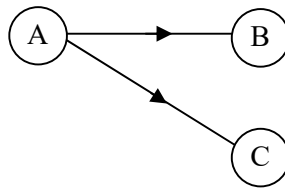
Every set of distinct triples gives way to a undirected ling triad or a ling geometric triangle which is not directed.

Proof. Follows from the fact on  $T$  every distinct pair of ling terms gives way to undirected ling dyad and as  $T$  is a ling continuum every triple gives way to a ling undirected triad.

All these concepts will be useful to ling geometrical social information network.

Now we give a few illustrations of ling social linguistic geometrical information network.

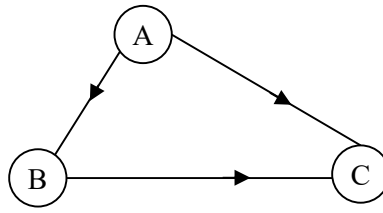
**Example 2.8.** Consider  $A, B, C$ , three persons. Suppose  $A$  is a friend of  $B$  and  $C$  and  $B$  and  $C$  are not friends given in figure 2.26.



**Figure 2.26**

This is an incomplete directed ling geometrical triad.

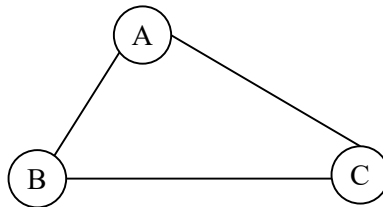
If  $A$  is a friend of  $B$  and  $B$  is a friend of  $C$  and  $C$  is a friend of  $A$  then we have the following triad.



**Figure 2.27**

This is a complete geometrical ling directed triad.

Now consider the set of 3 people ABC where A is a friend of B and B is a friend of A. Similarly B is a friend of C and C is a friend of B and A is a friend of C and C is a friend of A.



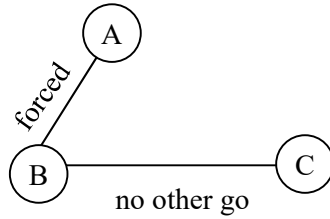
**Figure 2.28**

ABC is a complete ling geometrical undirected triad, that is it is not a directed one.

Now how do ling triads forms ling triads can be of any form complete directed or undirected but the ling distance is the ling line with some ling term between two values.



Even if friendship is strong, weak, motivated, no other go, forced, and so on.



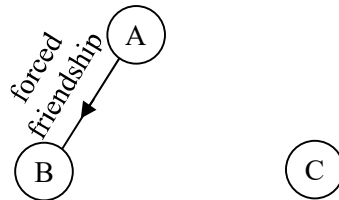
**Figure 2.29**

So we can study the ling variable friendship. We can have several types of friendships. Let S be the ling set associated with the ling variable friendship.

Consider the ling plane people and friendship.

We can connect some, cannot connect some linguistically. For if A, B, C are 3 people. A is a friend of B by force C is not a friend of A and B.

The related forbidden ling triad is as follows.



**Figure 2.30**

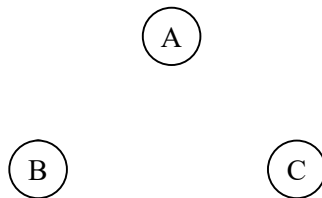
The term forced friendship is used when in working place they do not find any friend but however they need some one in that group to help them in their absence or other type of official functioning.

We see in general or classical forbidden triad we may not have the concept of weightages that too ling terms used for weights for the edges. This is the main difference and it is also better in expressing their friendship. So in study of social information networks we can use as applications the concept of ling geometrical triads, etc.

Thus we can have the following types of ling triads. We have to keep in mind in case of ling triad every edge or the distance between two ling points will always be weighted or marked with the ling distance.

We give the ling triads associated with the ling variable of friendship. The friendship can be strong, loyal, very strong, reciprocating, one sided, forced, weak, based on profession, based on bad habits, based on work, based on playing games living in the same place, etc etc.

Let A, B and C be 3 persons we list out all possible types of ling triads.

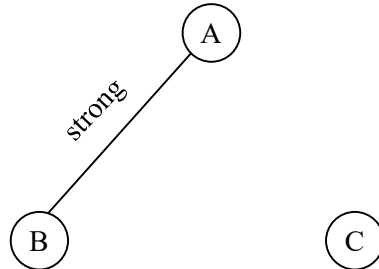


**Figure 2.31**

This figure shows no friendship exists between the three people.

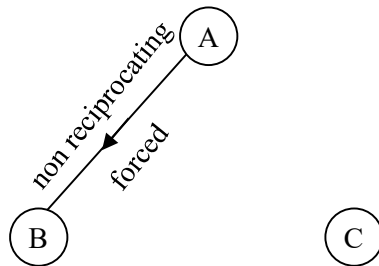
We just mention any one of types of friendship.

Next, consider the following figure;



**Figure 2.32**

C is not a friend of A and B, however A and B are friend whose bonding is both reciprocating as well as strong.



**Figure 2.33**

C is not a friend of A and B but the friendship of A with B is non reciprocal also only a forced one.

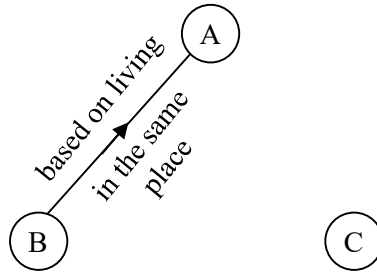


Figure 2.34

A is in no way friend of B but B prefers to be a friend of A as both belong to or live as neighbours and travel together.

Consider the friendship of the 3 persons A, B and C.

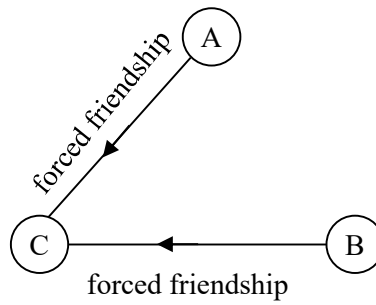
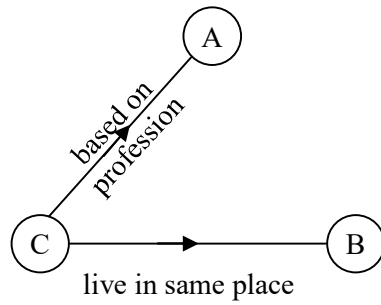


Figure 2.35

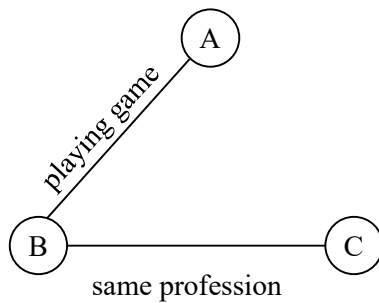
Both A and B are forced friends of C as they all work in the same place and C happens to be their boss.



**Figure 2.36**

Consider the 3 persons A, B and C. C has a type of friendship to A and B but A and B are not friends. C is a friend of A based on the profession both do, however A being in a powerful level does not care for the friendship of B but C has to depend on B for his profession so he is a friend.

Further C is very lovely so he keep or leans on B as a friend as both live as neighbours. Further B does not reciprocate it. However A and B do not know each other.

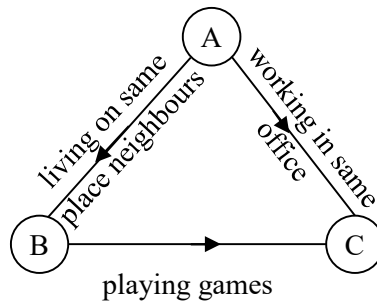


**Figure 2.37**

Consider the case given in figure 2.37. A, B, C are three persons where A and B enjoy reciprocating friendship based on playing games B and C have the same profession which is also a reciprocating friendship.

Consider 3 persons A, B and C given by figure 2.38.

A and C do not know each other.



**Figure 2.38**

A is only a friend of both B and C however both B and C do not reciprocate the friendship with A.

For A and B are neighbours and A as a neighbor needs B's help but B does not care A to reciprocate.

Similarly A and C work in the same office. Once again A wants to keep C as his friend however C does not reciprocate his friendship to A.

Further B is a friend of C as they play games together however C is not a friend of B and the friendship is non reciprocating.

The researcher is forced to study only these type of friendships. For it reflects on the nature and personality of all the three of them which is distinctly different from the classical social information network triads.

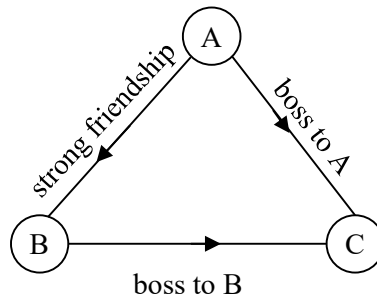
From this ling triad one can make a observation that A is not a very independent strong personality. He wants some one in his work place to be his friends. Similarly he wants some one to be friend in his living place.

This shows the character of A either he should be a friendly one or he should be suffering some sort of loneliness or isolation wants to be with same one to share or fall back in time of need or should be a warm personality in general.

Consider the nature of B; clearly B is not the same type of character as that of A, that is why he is not reciprocating the friendship of A however he must be a jovial or a happy person he wants some one to play games with him.

Consider C; C is very independent does not care for friendship that is why no reciprocating but however both of them (A and B) are attracted towards C and want to be friends with him. Further C is not a very warm person to reciprocate it back.

Now consider the persons A, B and C.



**Figure 2.39**

Consider the figure 2.39. We see A and B enjoy strong bond of friendship, that is reciprocating healthy friendship.

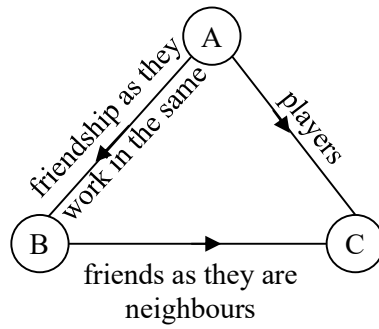
However both A and B are only friend of C for they want to maintain a cordial relationship with their boss.

However C does not reciprocate the friendship to both B and A.

The experts analyse this situation in the following way. If C tries to reciprocate his friendship with both A and B he may lose his identity as they are his subordinates or it may be he wants to maintain a respectful distance from all his subordinates.

Consider the persons A, B and C. Their friendship bond is given in the following figure 2.40.





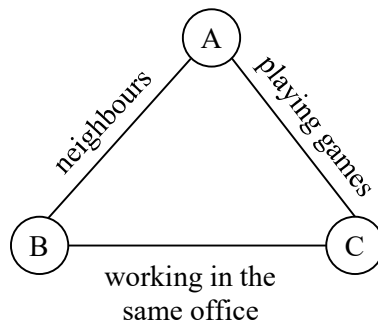
**Figure 2.40**

Consider the ling triad ABC. We see A and B enjoy a reciprocating friendship based on the fact they work in the same office.

Similarly B and C enjoy a reciprocating friendship as they are neighbors.

But A is a friend of C but C does not reciprocate it as they are both players of the same team.

Consider finally the 3 persons A, B and C. The ling triad relating them is as follows.



**Figure 2.41**

The ling triad associated with A, B and C is a reciprocating friendship. For no friendship mentioned in one way or non reciprocating very balanced perfect ling triad.

Using ling geometry is social information networks will be a boon to socio scientist. We have only so far discussed about only ling triads of all types however the edges are weighted with ling. terms based on which the friendship woroks.

Next we will introduce the concept of ling lines important property viz. intersecting ling lines.

Before we proceed to discuss and develop those notions present in classical geometry in case of ling geometry we face a lot of limitations.

1. Given a point in classical geometry we say we can have infinite number of lines passing through that point each of infinite length extending both sides

Clearly this concept has no relevance in case of ling geometry. Basically for us to have a ling line

- i) we need two distinct ling points.
- ii) we need those two points to be related or comparable otherwise no ling line can join them.
- iii) Suppose we have ling set P associated with ling variable yield of plants (paddy). We see

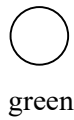
$P = \{\text{good, yield, no yield, poor yield, very good yield, very bad yield, bad yield and so on}\}$

The growth and appearance can be stunted, tall, short, very short, very tall, tallest and so on.

$\{\text{green, brown, yellow or dark green and so on}\}$ .

Consider the ling plane associated with the ling variable yield of plants.

Consider the ling point green in the ling plane.



**Figure 2.42**

We as in case of the usual classical geometry cannot say there are infinitely many lines passing through the given point green in the linguistic plane of plants growth.

We have no ling line passing through the ling point green in the ling plane relating growth and yield of plants.

**Theorem 2.5.** *Let  $L$  be a ling plane associated with same ling variable  $V$ . Let  $S$  be the ling set associated with the ling variable  $V$ .*

*Let  $x \in P$  be any ling term or point in  $L$ .*

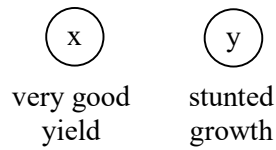
*There exist no ling line passing through the ling point  $x$ .*

Proof. We have given examples and shown for us to have a ling line we need two distinct ling points which are related. Then only we can have a ling line with specified ling (distance) term associated with it.

Now consider two ling points / terms

$x =$  stunted and  $y =$  very good yield

in the ling plane associated with the yield of plants given by the following figure 2.43.



**Figure 2.43**

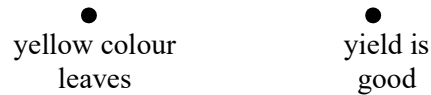
We see the two ling terms are not related or comparable. For the ling term stunted growth or stunted has no relevance with the ling term very good yield, for we know if the plant has a stunted growth then it cannot have any yield more so a very good yield.

From this we make a conclusion like our classical geometry which states that given two distinct points in a plane we have a unique line passing through it. This is never true in our linguistic points in a plane.

Thus we see this is in general flouted in case of ling planes. For by no means we can ever have a ling line passing through the ling points stunted and very good yield.

Now consider the two ling terms / points a, b, c in the ling set P, which are associated with the ling plane yield of plants where a = green and b = good yield.

Clearly when we have the plants to be green in colour we are sure the yield will be good or very good. Or we can still say the ling distance in this case short or near described by the following figure 2.44.



**Figure 2.44**

Clearly these two ling terms / points cannot be related in any way.

Thus we see as in case of classical geometry we cannot in general say if we have two distinct ling points in a related ling plane there is a unique line passing through them.

**Theorem 2.6.** *Let L and S be as in theorem 2.1. We have pairs of distinct ling points in general such that no line can pass through them.*

Proof follows from counter examples.

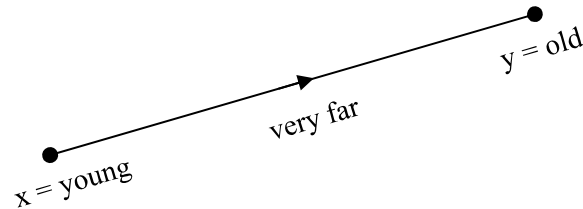
This sort of theorems were proved once before but however to emphasize these ling properties we repeat only this will make one understand the ling geometry is basically different from the classical geometry.

Also we will give yet another example where in the ling plane for a specific ling variable such that for every pair of distinct ling points there is always a directed ling line connecting them.

**Example 2.9.** Let us consider the ling variable age of people. Let  $L$  be the ling plane associated with ling variable  $V$ . Suppose  $S = [\text{youngest, oldest}]$  is the ling continuum / set associated with the ling variable  $V$ .

Consider  $x = \text{young}$  and  $y = \text{old}$  two ling points / terms in the ling set  $S$ .

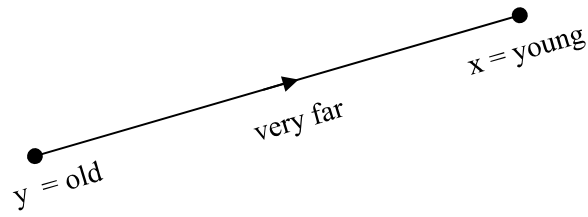
Now the position of these points in the ling plane  $L$  is given by the following figure.



**Figure 2.45**

The line connecting  $x$  with  $y$  is ling which is described by the ling term “very far”.

Further the ling line  $xy$  is directed from  $x$  to  $y$ . We cannot say  $xy$  can be just represented by an undirected line or by a line from  $y$  to  $x$  given in the following figure 2.46 in the linguistic plane.

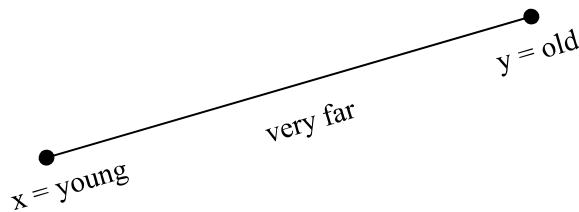


**Figure 2.46**

For it is naturally wrong that a person becomes from old to young which is an absurdity.

As ling concepts are very much related to ling logic this is not agreeable as it is wrong.

Further if one tries to represent in the ling plane L as follows.



**Figure 2.47**

This is also absolutely wrong for direction of the ling line or ling distance is mandatory as in classical geometry we only give the distance as  $xy$  (or  $PQ$ ) which is never infinite.

Keeping all these in mind we have mentioned about them by several examples.

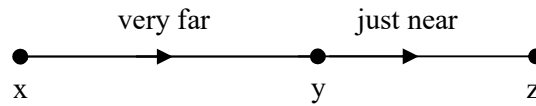
Finally, we cannot always say three given ling points determine a ling plane uniquely or three ling terms are collinear and so on. We provide more examples.

**Example 2.10.** Consider the ling variable V height of people and the ling plane associated with it. Let the ling set associated with this ling variable be [shortest, tallest] = S.

Consider any 3 points

$x = \text{short}$ ,  $y = \text{just tall}$  and  $z = \text{very tall} \in S$ .

We have following representation of them in the ling plane.



**Figure 2.48**

In fact we cannot make a ling triangle using them in general. This three ling terms only give rise to a ling directed line.

So if the ling set is a ling continuum we have only ling straight line for any number of ling points belong to the ling continuum.

If the ling points are not comparable then we can have three ling points to form a ling triangle.

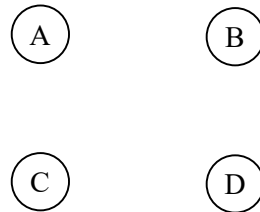
We have discussed about ling geometrical properties of ling points, ling straight lines and ling triangles.



Next we proceed onto describe ling quadrilateral and ling polygon on ling geometric planes.

Consider the ling geometry of four ling points measuring or relating the friendship of four persons. Let A, B, C and D be four ling points in this special ling plane.

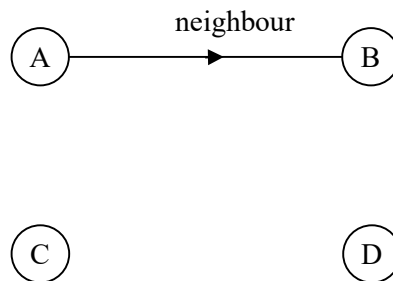
We study all possible ling relations.



**Figure 2.49**

The four people A, B, C and D are not related.

Consider the four people A, B, C and D in the ling plane of friendship.



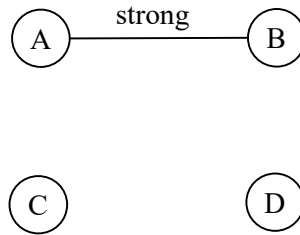
**Figure 2.50**

We see of the four persons in the ling plane of the ling variable friendship.

We see C and D are not friends of A and B, however A is a friend of B but B is not a friend of A.

Since A is a neighbor of B. However B does not reciprocate the friendship.

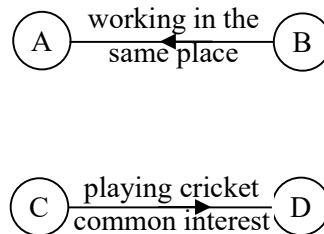
Consider A, B, C and D four persons.



**Figure 2.51**

We see A is a friend of B and B is a friend of A. It is a strong friendship and an instant of reciprocating friendship.

For A, B, C and D four persons.

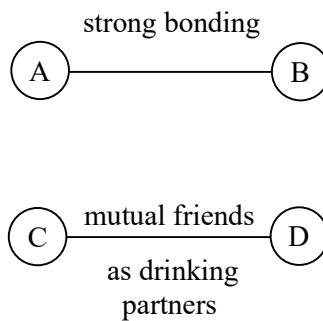


**Figure 2.52**

We see the above figure shows the friendship of B with A which is not reciprocative but is due to working in the (same) common place.

Similarly C is a friend of D but D is not a friend of D this friendship is also not reciprocating, the friendship due to playing the same game.

Consider the four persons A, B, C and D.



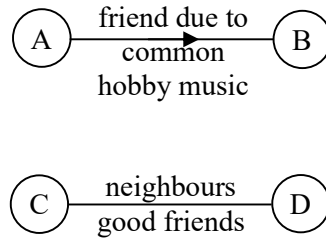
**Figure 2.53**

Unlike the friendship given in figure 2.52.

This friendship of A with B and C with D are reciprocating.

Now consider the four persons A, B, C and D.

The friendship can also be of the form

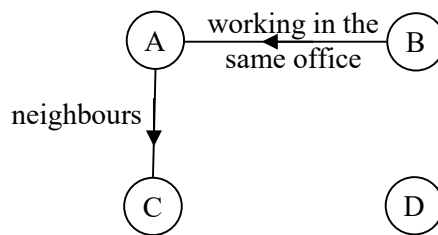


**Figure 2.54**

A is a friend of B taking B for company to music concerts but B never wishes to reciprocate, C and D hold a strong friendship bond as neighbor they are very good friends.

This figure 2.54 is different from the other figures.

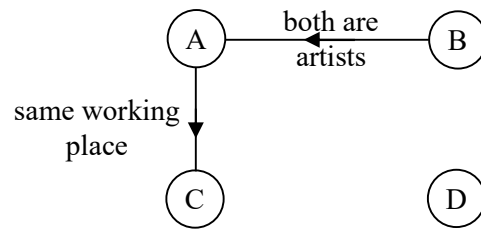
Consider the four persons A, B, C and D. Their friendship level is described by the following figure 2.55.



**Figure 2.55**

B is a friend of A but A does not reciprocate his friendship. However B keeps A as a friend to get some help when needed in office from B. A is a friend of C but C does not reciprocate. A maintains a friendship as C is his only neighbor.

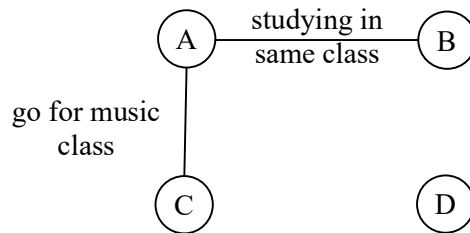
Now consider the friendship of A, B, C and D given by the following figure 2.56.



**Figure 2.56**

Clearly A is a friend of B and B is a friend of A both are artists from the same troop. However A is a friend of C he maintains a friendship with C as C his immediate boss. But C does not reciprocate the same.

Take the four persons A, B, C and D.



**Figure 2.57**

A is a friend of B and B is a friend of A both are classmates. Also A is a friend of C for they learn music from a

same teachers. The friendship is reciprocating. However B and C are not friends of A, and D is not a friend of A, B and C.

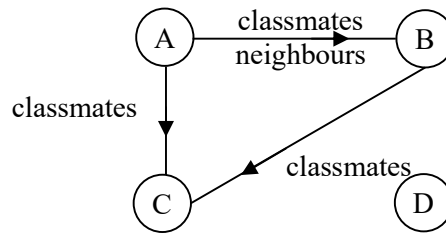


Figure 2.58

A, B, C and D all study in the same school and same class. However, A is a friend of both B and C, and A is the class topper, but both C and B do not reciprocate. Also, B is a friend of C and C does not reciprocate. Finally, D is not a friend of A, B and C.

Consider four persons A, B, C and D. Their ling diagram Figure 2.59 of friendship is given in the following.

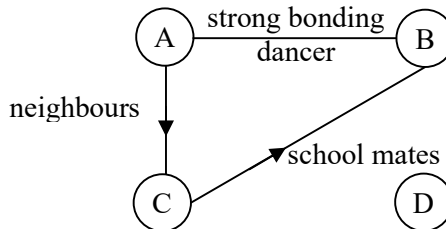
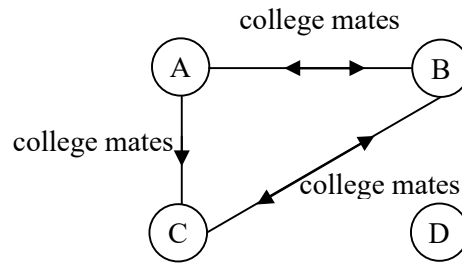


Figure 2.59

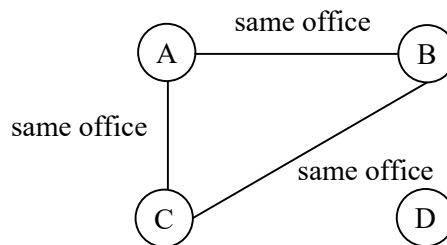
A and B share a strong bondage of friendship as both are dancers. However A and C are neighbours so A is a friend of C but C does not reciprocate. But C is a friend of B as both are classmates but B does not reciprocate the friendship to C. This is different type of friendship different from figure 2.58.



**Figure 2.60**

C, A, B and D study in the same college in the same batch. D is not a friend of A, B and C. However A and B are close friends and they reciprocate. Similarly B and C are also friends who reciprocate. However A is a friend of C but C does not reciprocate to A.

A, B, C and D work in the same office. We see



**Figure 2.61**

We see A is a friend of both B and C and they reciprocate B and C hold a reciprocating friendship. However D is not a friend of any of the 3 of their colleagues.

Consider the ling friendship of A, B, C and D given by the following figure 2.62.

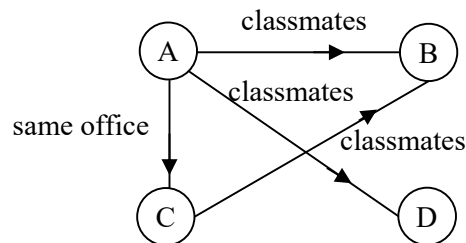


Figure 2.62

A, B, C and D are studying in the same college A is a friend of B, C and D, however none of them reciprocate it to A. C is a friend of B but B is not a friend of C.

Now consider A, B, C and D working as teachers in the same college.

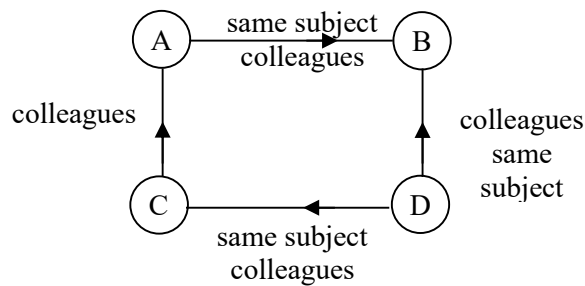


Figure 2.63

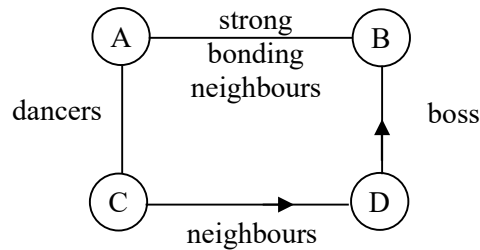


A is a friend of B and B does not reciprocate to A. C is a friend of A but A does not reciprocate.

D is a friend of C and B both C and B do not reciprocate to D.

This is called as ling ring in social information network. Their friendship is based on common subject. Thus they share their friendship as subject is same. However C is a friend of A as he is his neighbours cadre. But C does not reciprocate the friendship.

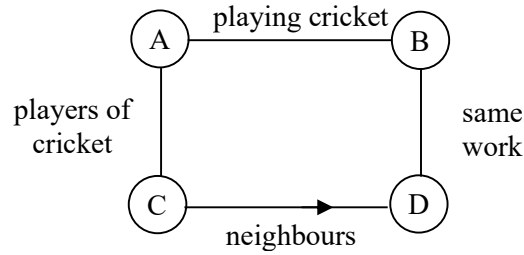
C is a friend of D as C is a neighbor, but D does not reciprocate. D is a friend of B as B is his boss. B does not reciprocate the friendship to D.



**Figure 2.64**

We see A and B are neighbours and they are good friends D is a friend of B as B is his boss however non reciprocating. C is a friend of D as is neighbor of C. A and C are dancers and are very good friends.

Now we give the friendship bond of A, B, C and D given by the following figure.



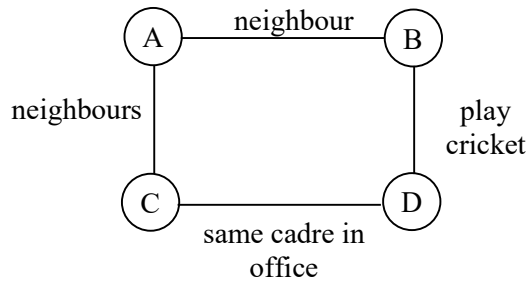
**Figure 2.65**

A, B and C belong to one group of cricket players.

A and B hold a strong bond of friendship A and C hold a strong bond of friendship.

However B and C are not friends B and D work in the same place C and D are neighbours. C is a friend of D and D is not a friend of C. ABCD is a ling ring.

Consider A, B, C and D as the four persons. All four work in the same place and the four of them live in the same quarters and are friends.

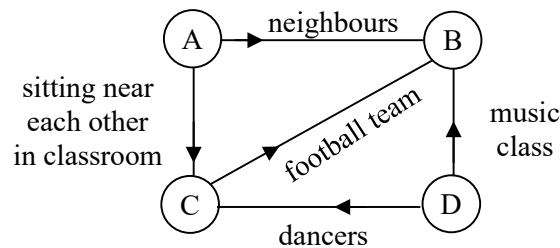


**Figure 2.66**

A and B are neighbours and they work and stay in the same place. Similarly A and C are neighbours. C and D belong to same cadre so they are friends. B and D are very good friends as they are players of the company, clearly ABCD is strong ling ring which can be used in as a social information linguistic network.

Thus we can have several ling quadrilaterals connecting four friends, complete or otherwise.

We will provide one or two such ling quadrilaterals in the following depicting the friendship of four persons A, B, C, D. These four students study in the same classroom in a school. The ling quadrilateral measuring their friendship and the type of friendship is depicted below.



**Figure 2.67**

We see A is a friend of B as he is also the neighbor, but B does not reciprocate, A is a friend of C for they both should sit in the same bench in the classroom. But C does not reciprocate his friendship. From this we follow A is a friendly type want some company be it in his school surroundings or his home surroundings.

C is a football player and B belong to the same team. C is a friend of B but B does not reciprocate the friendship.

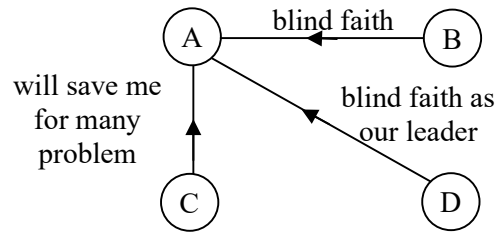
C and D go for dance classes together and D is a friend of C but C does not reciprocate D friendship D is a friend of B as both B and D go to music class after school. However B does not reciprocate the friendship of D. Now if we observe the nature of the student D he wants some friends while travelling and in the extra classes where they learn. This is a bit different from the student D. However B happens to be very different from the rest of the three persons for B is not reciprocating the friendship of any one of them as seen from figure 2.67.

If we analyze the personality of C, we observe he is more interested in sports and wants to maintain a friend from his football team.

Not interested even the classmate who should always sit with him in class or he is interested in his friend who travels with him for the dance class thus C's preference is distinct and different from other.

So if we use the concept of linguistic geometry or to be more precise the ling graphs for ling geometric social information network one can derive the relations more appropriately than other social information networks.

Let us take four persons not educated but living together as fishermen from the same place.



**Figure 2.68**

When the friendship graph or the ling quadrilateral was drawn it happened to be star graph and ego centric on A. A was their leader and they (B, C and D) are least bothered about anybody's friendship. They go for fishing with him and have a blind faith whatever be the demanding situation while in the sea A will take care of them.

This sort of ling graph forced us to study star ling geometric figures or graphs.

One is very familiar with the concept of star graphs or equivalently ego centric graphs.

Now in case of ling plane we call them as ling star graph of ling ego centric graph.

Once again we wish to say that when we say a ling plane we do not associate with the coordinate ling plane discussed in ling analytical geometry. For in these geometric planes we do not have the concept or notion of vertical and horizontal axis. They are just like planes discussed as in classical geometry that is why we call them as ling geometric planes. Ling geometric planes are different from ling analytical planes.

A point in ling geometric plane can be

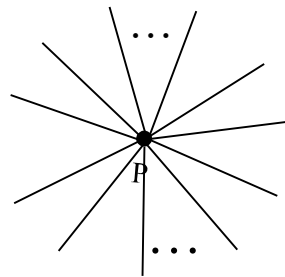
• good or • bad or • fair • or • short or • tall or • young or • old  
or • medium height and so on

whereas in case of ling analytical planes we have (x,y) where both x and y are ling terms say for instance

(old, tall) or (young, medium height) or (good, devoted) and so on. So in case ling analytical geometry every point in that plane is a pair of ling terms whereas in ling geometry every point is only a ling point as in case of classical geometry.

However other properties of classical geometry are not true in case of ling geometry.

- i) A point in classical geometry has infinite number of lines passing through it



**Figure 2.69**

If P is a point in the plane in classical geometry there are infinite number of lines passing through it figure 2.69 depicts it. However in case of linguistic geometry if L is a ling plane

associated with some ling variable  $V$  and  $S$  is a ling set associated with it and if  $\text{good} \in S$  then the representation of the ling point  $\text{good}$  is just a point given by Figure 2.70 in the ling plane.

●  $\text{good}$

**Figure 2.70**

No line passes through it intentionally we go on repeating these in this book for not only the concepts are new and they are very different from the classical geometry.

So this should be first registered in the mind set of people or reader / research who wants to study or work with ling geometry.

So a ling point in a plane is a ling point with no line passing through it.

Next consider a ling plane  $L$  associated with the ling variable  $V$ . Let  $S$  be the associated ling set of the ling variable  $V$ .

Let  $P$  and  $Q$  be two ling points in  $S$ .

●                      ●  
P                              Q

**Figure 2.71**

The ling point P and Q do not have a ling distance in general unless P and Q are comparable.

This is a very important factor as we see in case of classical geometry in their plane given any two points in that plane their exist a line connecting them uniquely.

In Classical Geometry plane

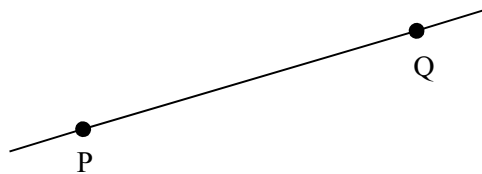


Figure 2.72

P and Q are two distinct points in that geometric plane. Then there always exist a unique line passing through P and Q. The distance between P and Q is denoted by PQ. It is to be noted we do not have any direction associated with it naturally or unless we specify it.

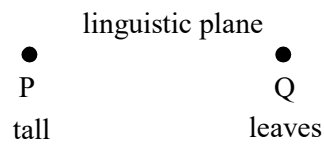
But if we take the ling plane P and the ling variable V as yield of plants. Associated with V is the ling set given by

{tall, short, stunted, medium height, very short, green, dark green, yellow, brown, leaves with brown good yield, bad yield, medium yield very bad yield} = S.

Consider the ling geometric plane L. Let P = tall and Q = leaves with brown dots be two distinct ling terms in S.



We see

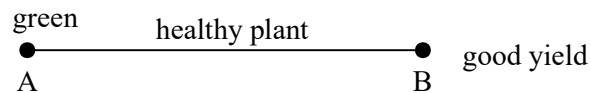


**Figure 2.73**

We cannot find any connection or a ling line between P and Q. They cannot be related in any way. So the classical geometry property given two point on the plane there exists a unique line connecting them is flouted in case of ling geometry for given any two ling points on the ling plane in general there does not exist a ling line or ling distance defined between them. The point P and Q given in that ling plane in figure 2.73 is an evidence or an illustration of the same.

Consider A and B two ling points in S.

A = green colour of plants and B = good yield



**Figure 2.74**

It is to be noted that the ling distance exist and no ling direction exist for green colour of plant and good yield results in healthy plant as the conclusion so they are related.

Now we give yet another example where every pair of ling points have a ling distance associated with them.

Consider the ling plane L take the ling variable V age of people.

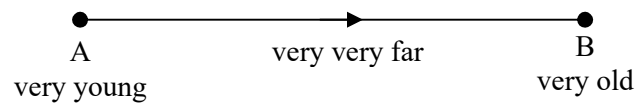
Let  $S = [\text{youngest, oldest}]$  be the ling continuum or set associated with V.

Clearly S is a totally ordered set. Every pair of distinct points are connected by a ling length and the ling distance between them is finite varying from [nearest, farthest] = S.

Every pair of distinct ling points in S has a ling distance associated with it on the ling plane L related with the ling set S.

Suppose we consider the two ling points A = (very young) and B = (very old) in the ling set S.

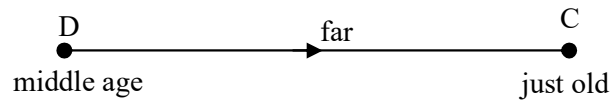
Their position and distance on the ling plane L is as follows.



**Figure 2.75**

Consider the ling points D = middle aged and just old = C in S.

The ling distance between them in the ling plane L is as follows.

**Figure 2.76**

An important thing to be observed that the ling distance is fixed and ling line has a direction associated with it. We cannot find the distance from B to A for it is impossible for a very old person to become very young.

So the only relevant distance is from very young to very old, it is infact directed.

Similarly there is no distance from just old to middle age for a old just person can never become middle aged.

Only the ling distance from middle aged to just old can occur.

This is also a major difference from the ling geometry and classical geometry.

Now having seen both directed and undirected ling lines and also the non existence of ling lines we now proceed onto discuss about ling star graphs.

Recall we have already described about the ling triad and ling dyads.

Now we describe ling star graphs. They are also directed ones in most cases.

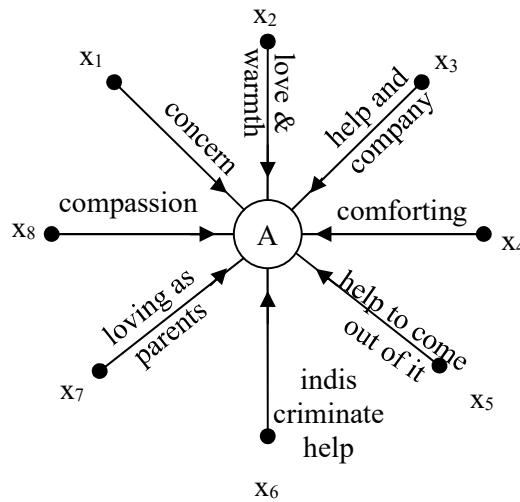
We first illustrate this situation by some examples.

**Example 2.10.** Consider a famous leader A. He is famous for he serves for the poor, needy, those who are sick, old and so on.

Now we take the ling variables connecting the leader A with

$x_1 = \text{poor}$ ,  $x_2 = \text{needy}$ ,  $x_3 = \text{old}$ ,  $x_4 = \text{sick}$ ,  $x_5 = \text{in problem}$ ,  $x_6 = \text{handicapped}$ ,  $x_7 = \text{a orphan}$ ,  $x_8 = \text{a destitute}$ .

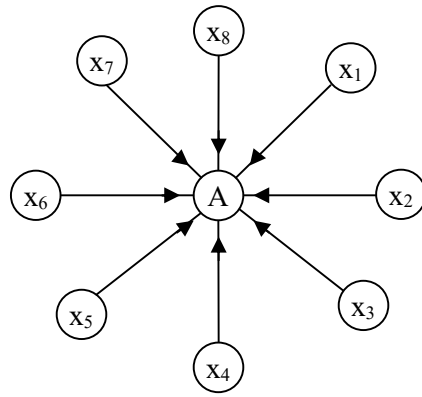
We will give ling star graph and the ling distance connection  $x_i$  with A.



**Figure 2.77**

None of the people in the set  $\{x_1, x_2, \dots, x_8\}$  know each other or any one of them. The reason for being attracted towards A is given as the ling distance.

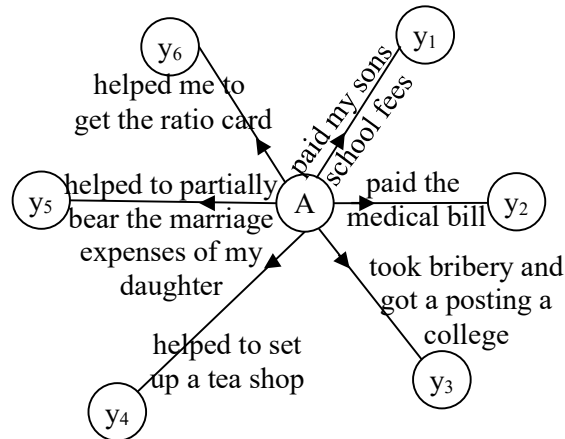
Now each has cause to like him and opt him as their leader in admiration. This is a ego centric linguistic star graph. In general classical star graph no such ling distance is possible only the direction will be provided. That is the classical star graph looks as follows.



**Figure 2.78**

Thus we claim the ling star graph can measure the cause of attraction towards the leader A in a very sensitive way of course linguistically than the usual or classical star graphs used in social information networks.

Now consider the opposite of this there is a political leader P; he attracts people by money getting them job etc. He goes behind them. We give the ling distance of P to the set of people  $\{y_1, y_2, \dots, y_6\}$  by the following figure 2.79.



**Figure 2.79**

We see the six people have given how P the political leader has voluntarily helped those people  $y_1, y_2, \dots, y_6$  in various ways.

They all know his service to them was with anticipation of votes from those 6 families. So none of them are attracted towards him as a political leader; infact it is he who has volunteered to help them in turn they have promised their vote to him.

Thus this is a ling star graph which is entirely different from the ling star graph given in figure 2.79 in which A is their leader.

It is infact ego centric and people go to him and in return he expects nothing from him (A). Clearly it is ego centric and A is distinctly different from P.

Now we have seen ling star graphs of two types. Interested reader can develop more in this direction.

We suggest in the following problems for the reader to solve to become more familiarized with these view notions. Starred problem are very difficult to solve.

### **PROBLEMS**

1. Prove in a ling plane associated with a ling variable, any point on the ling plane has no ling line passing through it.
2. Prove or disprove a ling plane associated with a ling variable whose related ling set is not a totally ordered set; the ling distance does not exist in general for every pair of points in that ling plane.

Illustrate your claim by an example.

3. Given  $L$  is a ling plane associated with the ling variable  $V$  denseness of plants in a forest area.
  - i) What will be the form of the ling set  $S$  associated with  $V$ ?
  - ii) Is  $S$  a ling continuum?
  - iii) Is  $S$  a totally ordered set?
  - iv) Is  $S$  only a partially ordered set?
  - v) Is  $S$  finite or of infinite order?
  - vi) For every pair of distinct ling term in  $S$ , does there exists a ling distance? Prove your claim.
  - vii) What type of ling graph one can get?
  - viii) Will the ling distance using  $S$  will be directed or otherwise?
  - ix) How many ling triads we can get?
4. What are the special features associated with the ling distances of the ling set  $S$ ?
5. How is ling geometry different from the classical geometry?
6. Can one say ling geometry will play a better role in ling social information network? Justify your claim by examples!

- 7\*. Apply ling geometry to real world problems.
- 8\*. Prove or disprove using ling star graphs in the appropriate ego centric models is better than classical star graphs.
9. Can we claim ling social geometric information network models will be more sensitive than classical social information network?
10. How many types of ling triads can be formed in studying ling friendship model?
11. Give an example of a ling variable where its related linguistic set has a ling plane in which there is no pair of distinct ling points which has a ling distance defined.
12. Give an example of a ling variable  $V$  whose ling set  $S$  is such that its resulting ling plane has no ling direction defined for any pair of distinct ling terms in  $S$ .
13. Give an example of a ling variable  $V$  for which if  $S$  is the ling set and  $L$  is its ling plane such that  $S$  has a pair of ling points which has ling distance defined and some points in  $S$  which has no distance defined.
14. How many ling geometric quadrilaterals can be got which studies the ling variable friendship?
- 15\*. Suppose one is interested in studying ling geometry then is there any means by which the concept of ling perpendicular can be defined?
16. Prove or disprove there does not exist a ling triangle which is equilateral. Substantiate your claim.



17. Obtain 5 different examples of ling set  $S$  and their associated ling variable  $V$  and their related ling plane  $L$  such that we can have ling geometric isosceles triangles.
18. Suppose  $L$  is the ling plane associated with the ling variable  $V$ , the performance of students in the classroom. Let  $S = [\text{worst, best}]$  be the ling continuum associated with the ling variable  $V$ .
  - a) Prove the ling quadruple
 
$$\{\text{good, just good, bad, just bad}\}$$
 form a ling rectangle.
  - b) Does there exist a ling quadruple which can form a ling square? Justify your claim!
- 19\*. Can we have the concept of ling circle in a ling plane for any ling set associate with some ling variable  $V$ ? Justify or substantiate your claim!
- 20\*. Does there exist a ling variable for which there is a ling set which has ling geometrical polygon? Illustrate this by an example.
21. Do we have an example of a ling plane whose ling variable can have a ling set which can yield a ling circle or any other form of ling curves?
22. Give example of ego centric ling star graph for friendship.

23. What is the ling variable that will give ling star graph away from the central element?
24. Can we have the concept of parallel ling lines? Justify your claim.
- 25\*. Can we have the concept of perpendicular ling lines? Substantiate your claim.
26. Prove the concept of medium in a ling triangle is always possible!
27. What is the difference between ling geometry and ling analytical geometry? Justify all your claims.
28. Obtain an equivalence of pythagorian property of right analysed triangles in case of ling triangles.
29. Suppose we have a ling variable V students performance in the class room. If  $S = [\text{worst, best}]$  as a ling set / continuum associated with the variable V.
  - a) Prove or disprove the ling triple  $\{\text{bad, fair, good}\}$  forms triangle.

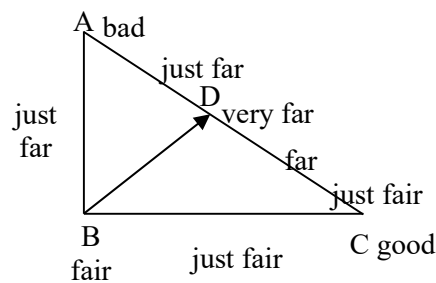


Figure 2.80

(D is the mid point of AC ling distance of AC is very far ling distance AB is just far and that of BC is just far ling dist (AB) together with ling dist (BC) is ling distance AC. Evident from the figure).

This is true because ABC the ling triangle is a ling isosceles).

b)\* Can this concept lead to the ratio and proportion of ling lines?

30. Prove for V and S as in problem 30 if

A (good) and B (best)  $\in$  S..

i) Find the ling distance between A and B.

ii) If very good a ling point D on the ling line AB prove  $AD + DB = AB$ .

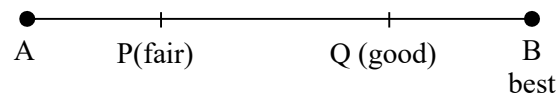
iii)\* Can this concept lead to ling ratio of dividing ling line? Justify your claim.

31\*. If the ling set S is not totally ordered; can problem 30 be true? Substantiate your claim.

32\*. Prove the concept said in problem 30 (ii) is not possible in case of classical geometry.

33. The concept of ratio and proportion in ling geometry would be different or same as in case of classical geometry? Justify your claim.

34. Let



**Figure 2.81**

be a ling line.

a) If P is a ling point on AB. Can  $AP + PB = AB$ ?

b) If Q(good) is a ling point on AB.

Will  $AQ + QB = AB$ ?

35\*. If linguistic variable L under consideration is indeterminacy.

i) Find the ling. set S associated with L.

ii) Will ling distance between any point always exists?

iii) What is the ling. distance measurement used in S?

## Chapter Three

### LINGUISTIC SOCIAL INFORMATION GEOMETRIC NETWORKS

In this chapter we develop the concept of linguistic social information networks. We have discussed the about linguistic geometry but however we have got very meager form of defining linguistically geometrically concepts. They are discussed both in chapter I and II.

We in this chapter all these concepts of linguistic social networks or to be more precise linguistic social information geometric network. Linguistic social networks play a role two ways i) when the nodes of the network are from linguistic set we define them and (ii) if the nodes are linguistic sets and weights of the edges are linguistic terms which measures the distance. It is in this type the ling geometry places a role. For us the linguistic distances that is when edges of the networks are weighted with linguistic values we call them as linguistic valued graphs. The linguistic graphs can be directed or undirected. We will discuss both types as well as both types of linguistic social

geometric networks. We see geometric for the ling distance concept is inculcated in these linguistic social networks.

In the first type the linguistic nodes alone are used and when we say an edge exist we say yes and when there is no edge we say no and for the diagonal we say not defined or undefined or nd or ad.

Now we will only be discussing about linguistic social networks. For the study of social networks analysis geometric linguistic graph theory plays the vital role. When we say geometric linguistic graphs they are nothing but geometric linguistic figures which enjoy directions and some linguistic weights associated with them. This was elaborately discussed in chapter II of this book. We with default of notation call this as linguistic graph also. So is in the study of linguistic social networks linguistic graphs plays the principal role.

In linguistic graph theory provides a appropriate language which can be used to label and denote many social structural properties. This ling language also provides us the basic concepts that allows us to refer quite precisely those ling property.

Ling graph theory also provides us mathematical operations and ideas with which we measure the properties both in a quantified and in a ling way which is purely on logic and not on numbers or fuzzy numbers.

This ling mathematical geometrical graph theory helps us to derive results about ling graphs from which we can arrive the testable statements.

However the classical graph theory has limitations but the use of ling graphs when used as networks can yield better methodology and results.

But in case of ling graph theory almost all properties of classical graph theory can be used, however it is more powerful as the terms can give more than sheer numbers as ling terms which is more meaningful and appropriate ling graph theory cannot only give the representation of the social network but more so better describable of the same.

Ling network consists of linguistically describe actors with also linguistically describable ties between them, but classical networks have limitations.

Ling model gives the most simple representation of a situation that contains some representations of the elements and it represents linguistically the situation. So when ling graph is used for the ling social network, ling points (called ling nodes) are used to represent actors linguistically and ling lines or ling ties connecting these ling points / terms. In this way ling graph represents is a ling model of a ling social network in a way that a model train ling set (ling nodes) linguistically by using ling tie / lines.

This linguistic social networks can also formally represent social relations and linguistically quantifying the vital social properties linguistically. The main positive or advantageous quality of social information networks is that a number is replaced by a qualifying ling. term which is better in understanding the ties or relations.

Ling graph theory can be better replacement of graph in studying social structural properties of anthropology, social psychology, organizational research, sentiment analysis, in business, customer care, environmental changes etc etc.

These ling graphs will also powerfully do pattern recognition. We will be replacing the classical matrices by ling matrices as an alternative way to represent and integrate the ling network data.

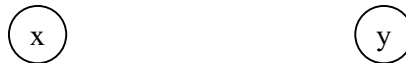
However the ling matrices will exactly represent the information of a ling graph and the former is more powerful for calculation and computer analysis. However one should efficiently restructure the computer analysis pages (GRADAD, UCINET, STRUCTURE, SNAPS NEGOPY) into ling formulated theory / structures.

However all examples / illustrations used are not data collected from real-world social problems they are only hypothetical linguistic data only for demonstration purposes.

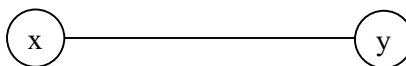
We first describe properties of ling graphs, where a line between two ling nodes is nondirectional.

Now given a pair of ling points from a ling set S.

Let  $x, y \in S$



**Figure 3.1a**



**Figure 3.1b**



The two possibilities in the non directional case are 3.1a and 3.1b.

We give a brief description of both the figures 3.1(a) and 3.1(b) for two different ling sets.

**Example 3.1.** Let  $L$  be the ling variable associated with the colour of the eyes of internationals.

Let  $S$  be the ling set associated with this ling variable  $L$   
 $S = \{\text{green, blue, brown, light brown, dark brown}\}.$

Clearly no pair of points in  $S$  are comparable so we have no line connecting any pair in  $S$ .



**Figure 3.2**

No ling line exists between these two ling nodes.



**Figure 3.3**

No ling line exist between dark brown or brown.

Consider the pair of ling variables blue and light brown in  $S$ . We get the following figure 3.4.



**Figure 3.4**

None of the ling pairs are comparable as colour of the eyes do not change and once one gets the colour of eyes it remains the same unless in very extraordinary cases the eye colour changes due to some eye problems, so a ling line undefined or does not exist.

**Example 3.2.** Let  $V$  be a ling variable  $S$  age of a person be a ling set associated with the ling variable age.

- i) Can be have a ling pair in  $S$  for which are distinct such that the ling line between them is undefined
- ii) Can we have a ling pair of distinct elements in  $S$  such that they have unidirectional ling line defined between them

Now before we answer these two questions we make the following observations.

- i) The ling variable age of a person is such that it is ever increasing with time.
- ii) It is not possible to reverse the ling terms for old can never become or old can never become middle age. Only old can become very old.

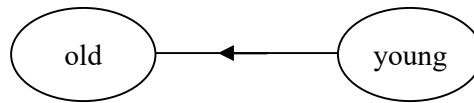
The youth cannot become the youngest (born baby's age) and so on. So the ling set  $S$  associated with age is a ling

continuum [youngest, oldest]; it is in the increasing order and it is a totally ordered set.

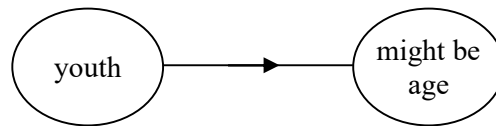
So given young and old we cannot have a unidirectional line connecting them.

Neither we have a distinct ling pair which is such that the ling line is undefined.

For the ling set  $S$  is in the increasing order and it can only be a directed line.



**Figure 3.5**



**Figure 3.6**

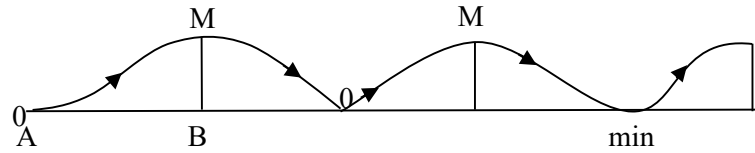
So the ling line connecting any distinct pair of elements is always a directed ling line.

This is a very special case.

Next we give an example of a undirected ling line.

**Example 3.3** Let us consider the ling variable  $V$ ; speed of the car on road which is travelling a long distance. Let  $S$  be the ling set associated with ling variable  $V$ .

Now the car starts from rest and increases its speed reaches a maximum then slows down at the speed breaker if the signal falls the speed falls to zero then gradually it starts and increases in speed and the pattern repeats which is described by the following figure 3.7.



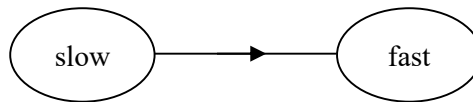
**Figure 3.7**

If we observe the figure the speed increases attains a max then decreases to zero or min near signal.

Let us consider the ling variable in the interval  $[A, B]$ .

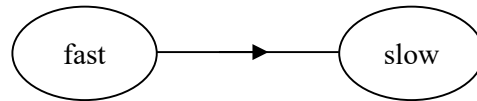
Let  $\text{slow}, \text{fast} \in [A, B]$ ;

Now as  $\text{slow}, \text{fast} \in [A, B]$  we can say



**Figure 3.8**

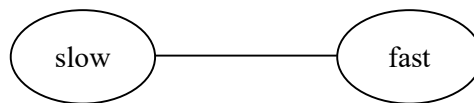
If however if the two ling terms  $\text{slow}, \text{fast} \in [B, C]$  then we see the ling line connecting it is directional given by



**Figure 3.9**

However if no mention is made about the interval just if it says slow,  $\text{fast} \in S$  then we cannot assign any directional ling line connecting slow to fast.

To best way to represent this by the ling line is



**Figure 3.10**

Thus we have only four types and the nature of undefined or directional or unidirectional.

All these four concepts highly depend on the ling variable and its related ling set plays a vital role.

So we have two types of graphs directed graphs and undirected graphs.

In ling graphs nodes are ling nodes or ling terms or ling concepts etc. The line lines describes the ling ties which may or may not exist or ling relations between two ling variables.

To have a ling graph we need the following entities.

- i) A ling variable  $V$ .
- ii) A ling set  $S$  associate with the ling variable  $V$ .
- iii) The ling graph  $G$  consists of two sets of information the set of long nodes  $\{s_1, s_2, \dots, s_n\}$  from  $S$  and a set of ling lines (provided it should defined related to  $S$  and  $V$ )  $\{l_1, \dots, l_t\}$  gives the linguistic distance between distinct pair of ling concepts (or nodes or terms). There are  $n$  ling nodes and  $t$  number of ling lines.

We say there is no order in the ling nodes  $(s_i, s_j)$  if  $l_t$  is a ling line which is undirected or is nondirectional that is a dichotomous relation.

We say  $s_i$  and  $s_j$  the ling nodes are adjacent if there is ling line  $l_p = (s_i, s_j)$  and  $l_p \in \{l_1, \dots, l_t\}$ .

Thus a ling node is incident with a ling line and a ling line is incident with the ling node if the ling node is an unordered pair of ling nodes defining the ling line.

Thus each ling line is incident with two ling nodes in the unordered pair (or non directional ling line) that define the ling line.

Thus a line graph that contains only one ling node is trivial and all other ling graphs are non trivial.

A ling graph that contains  $n$  ling nodes and no ling line is defined as empty linguistic graph.

Clearly trivial and empty line graphs do not have any big role to play in the new notion of ling graphs.

So in ling social networks these ling graphs correspond to a ling network consisting of only one ling term (trivial graphs).

The ling network consisting of more than one ling term / actor but no ling ties or ling lines between ling terms / nodes is a ling empty graph.

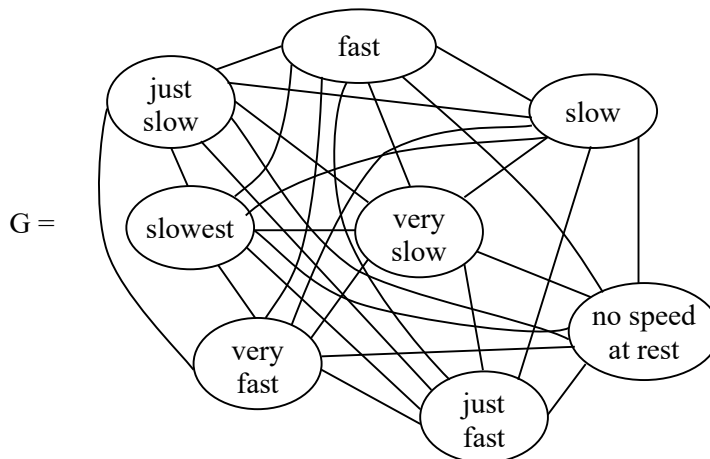
We will use the term actors / ling actors more specifically and ling ties / ling lines to describe a ling social network which speaks of ling distances so to be more appropriate we should call them as ling geometrical network or graph.

We will describe this situation by an example.

**Example 3.4.** Suppose we are measuring the speed of the car on road which is taken as the ling variable  $V$ . Let  $S$  be the ling set associated with  $V$ .

Take the ling nodes / terms from  $S$  which are

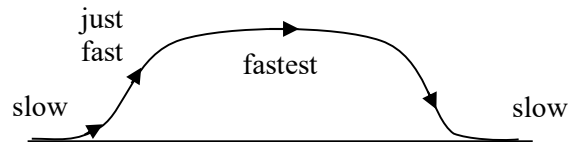
{fast, slow, just slow, slowest, not running (zero speed) medium speed, very fast, just fast, very slow}.



**Figure 3.11**

We see  $G$  is an undirected ling graph which is complete for every ling node is adjacent with every other ling node.

The ling variable speed has no direction for it is stationary and reaches fast or fastest then slow down or comes to rest.



**Figure 3.12**

This is an extreme situation.

We give the reverse of this situation.

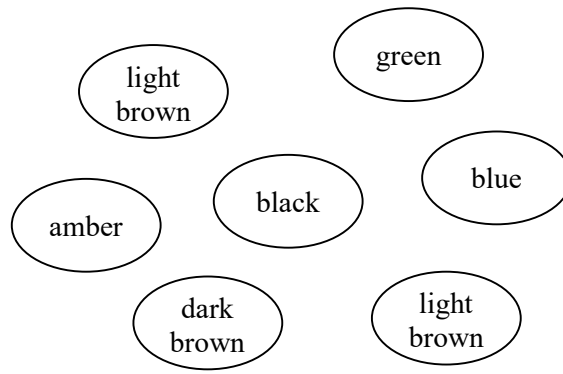
**Example 3.5.** Let  $V$  be the ling variable associated with the colour of the eyes of internationals.

Let  $S$  be the ling set associated with the ling variable  $V$ .

$S = \{\text{amber, dull yellow, brown, dark brown, black, blue, green, light brown}\}$ .

Taking these colours as the ling nodes / terms we get the following empty ling graph.





**Figure 3.13**

Consider yet another example of directed ling graph.

Recall as in classical sense a ling directed graph  $G$  is one where the pair of ling nodes are ordered pair  $(n_i, n_j) \neq (n_j, n_i)$  in general  $i \neq j$ .

That is the ling line  $n_i$  to  $n_j$  may exist and  $n_j$  to  $n_i$  may not exist or is undefined.

**Example 3.6.** Consider the age of people as the ling variable  $V$  and let  $S$  be the ling set associated with the ling variable  $V$ .

In this case  $S$  is a ling continuum given by

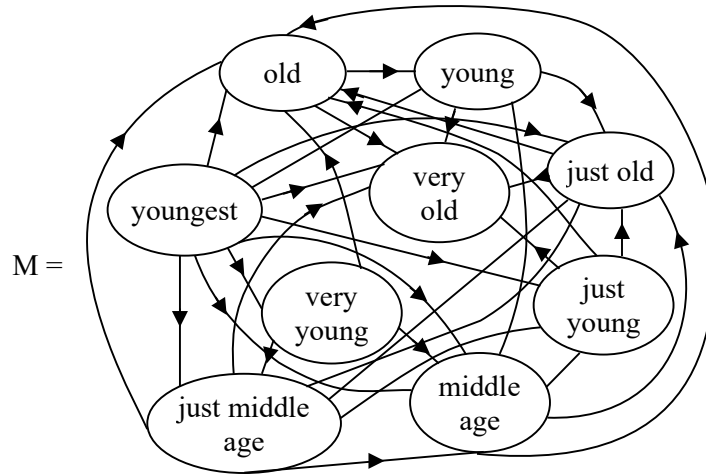
[youngest, oldest].

That is  $S$  is a totally ordered by set and is in the increasing order.

Consider a set of ling nodes

$P = \{\text{old, young, very old, just old, youngest, very young, just young, middle age, just middle aged}\} \in S = [\text{youngest, oldest}]$ .

The ling graph associated with P is as follows.



**Figure 3.14**

Clearly the ling graph M is directed and it is also complete for every ling node is adjacent with every other ling node.

We see that the ling set which forms the vertices of the ling graph takes its values from a totally ordered set.

Further we the additional condition reverse ling ties are undefined. For a old person can become older, very old and oldest.

However a old person cannot become middle aged or young or so on. Thus this is a very explicit example of directed ling graphs. For young cannot become very young or youngest.

However in case of the ling variable speed of the car on road, we see the fastest can become slow at nearing signals and a slowest at signal can reach the fastest at a point and remain for same time in the same speed. However the ling set associated with the ling variable speed of the car on road is a totally ordered set but is not an increasing set all the time it can increase or decrease.

Thus this is an excellent example of totally ordered set which is not increasing at all time, it can be decreasing or remain constant at time and can also be increasing, however the ling variable age of people is associated with a ling set which is steadily increasing with time, it cannot be increasing or remaining constant at any time.

Now as in case of classical graph theory in case of ling graphs theory, we can define the notion of ling subgraphs, ling dyads and ling triads.

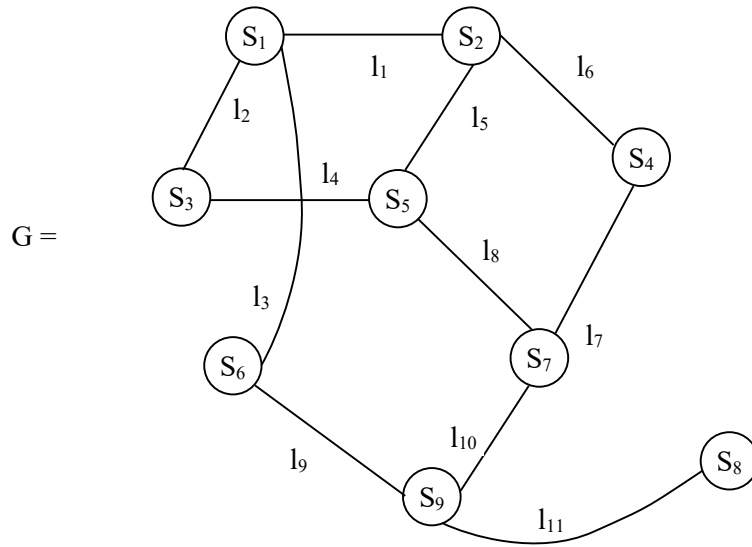
A ling graph  $G_s$  is said to be a ling subgraph of the ling graph  $G$  if the set of ling nodes of  $G_s$  is a ling subset of the set of ling nodes of  $G$  and the set of ling lines of  $G_s$  is also a subset of the set of the ling line of the ling graph  $G$ .

We will first illustrate this situation by some examples.

**Example 3.7.** Let  $G$  be a ling graph with ling nodes,

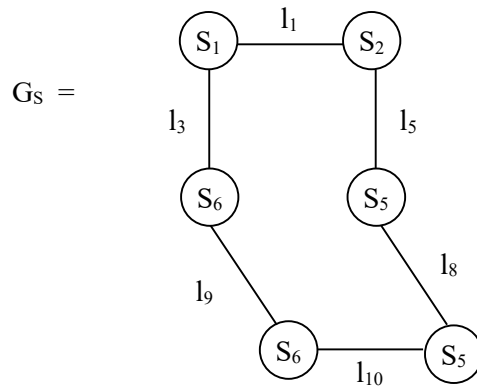
$S = \{S_1, S_2, \dots, S_9\}$  and let

$L = \{l_1, l_2, \dots, l_{11}\}$  be the set of ling lines associated with the ling graph  $G$  given by the following figure 3.15.



**Figure 3.15**

Let  $Q = \{S_1, S_2, S_5, S_9, S_6, S_7\}$  and  $M = \{l_1, l_3, l_5, l_8, l_9, l_{10}\}$  gives a ling subgraph  $G_S$  of  $G$  given by the following figure 3.16.



**Figure 3.16**

Clearly  $Q \subseteq P$  and  $M \subseteq L$  and the ling subgraph  $G_s$  has  $Q$  as its ling nodes and  $M$  as its ling lines.

Now consider the pair of ling subsets of  $P$ ,  $T = \{S_2, S_3\} \in P$  clearly  $T$  is a proper ling subset of  $P$ .

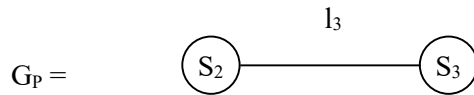
The ling subgraph associated with  $T$  is



**Figure 3.17**

which is a empty ling subgraph of  $G$ .

Consider the set  $W = \{S_1, S_6\} \subseteq P$ . Clearly  $\{l_3\}$  is the ling line connecting or relating  $S_1$  with  $S_6$ .



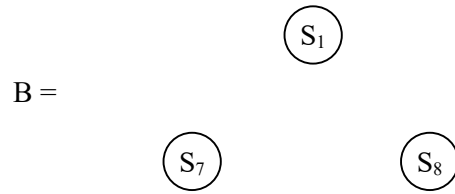
**Figure 3.18**

$G_p$  is a ling subgraph of the ling graph  $G$  which is a ling dyad.

Consider the ling subset  $N = \{S_1, S_7, S_8\} \subseteq P$ .

We see there is no ling lines connecting ling nodes of  $N$ .

The ling subgraph  $B$  of  $G$  is given by the following figure 3.19.

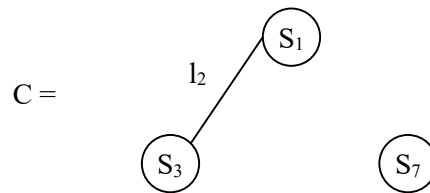


**Figure 3.19**

Clearly B is a ling empty subgraph which is ling empty triad of  $G_S$ .

Consider  $A = \{S_1, S_3, S_7\} \subseteq P$ , be the ling subset of P. The ling lines associated with A is given by  $\{l_2\}$ .

The ling subgraph C of G is given by the following figure 3.20.

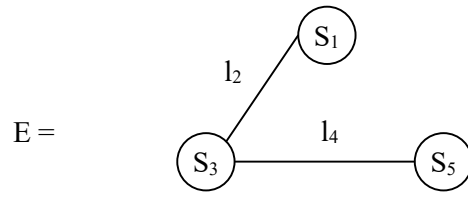


**Figure 3.20**

Clearly C is a ling triad which is not complete.

Consider the ling subset  $D = \{S_1, S_3, S_5\} \subseteq P$ . The ling lines associated with the ling subset D is  $\{l_2, l_4\}$ .

Let E be the ling subgraph using the ling subset D and the ling line  $\{l_2, l_4\}$ . E is given by the following figure 3.21.



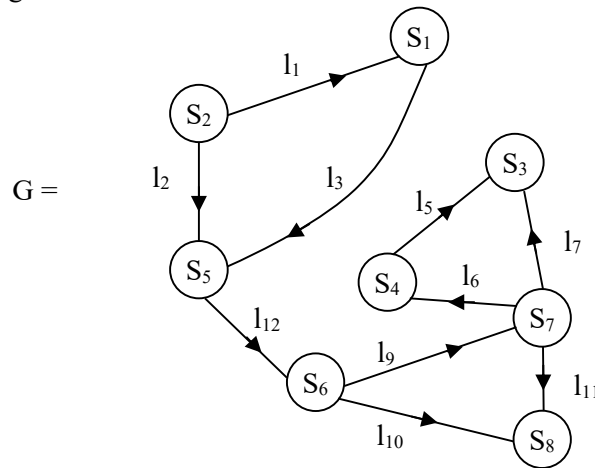
**Figure 3.21**

We see E is a ling subgraph of the ling graph G which is an incomplete ling triad.

Now in this ling graph G we have no ling triad which is complete as a ling subgraph.

Given yet another example of a directed ling graph.

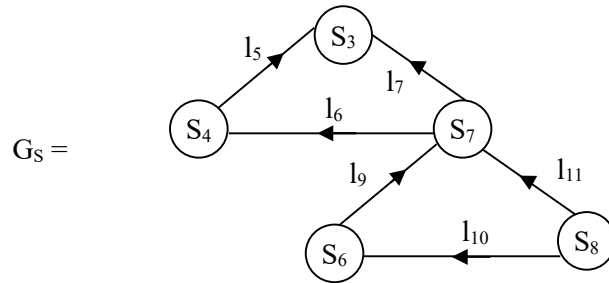
**Example 3.8.** Let G be a directed ling graph with ling node set  $W = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8\}$  and the ling lines set  $P = \{l_1, l_2, l_3, l_4, l_5, l_7, l_8, l_9, l_{10}, l_{11}, l_{12}\}$  given by the following figure 3.22.



**Figure 3.22**

Let  $G_S$  be the ling subgraph of the ling graph  $G$ , with ling node subset  $V = \{S_3, S_4, S_7, S_6, S_8\} \subseteq W$  and ling line subset

$M = \{l_5, l_7, l_6, l_9, l_{11}, l_{10}\} \subseteq P$  given by the following figure 3.23.



**Figure 3.23**

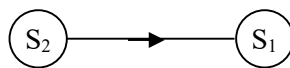
Clearly  $G_S$  is a linguistic directed subgraph of  $G$ .

Consider the pair of two linguistic terms  $S_1$  and  $S_3$ . We see they are just a ling empty dyad.



**Figure 3.24**

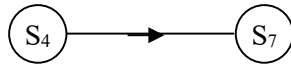
Now this pair  $S_2$  and  $S_1$  in  $W$  is a ling directed dyad.



**Figure 3.25**



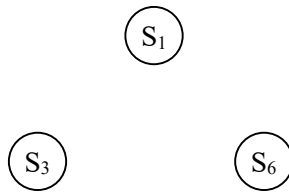
The pair of ling terms  $S_4$  and  $S_7$  is again a ling directed dyad given by



**Figure 3.26**

This is again a directed ling dyad.

Consider the ling subset  $\{S_1, S_3, S_6\} \in W$ . We see this is again a ling empty graph given by the figure 3.27.

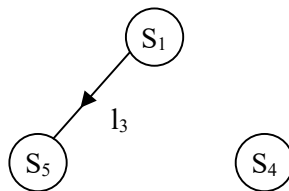


**Figure 3.27**

This is a empty ling triad. Consider the ling subset

$$\{S_1, S_5, S_4\} \in W.$$

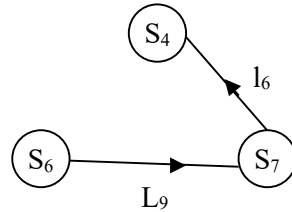
The ling subgraph given by this ling subset.



**Figure 3.28**

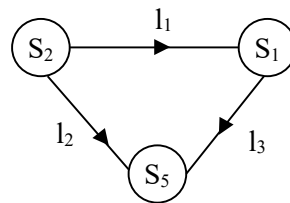
Clearly this is a ling triad which is not complete.

Now the ling subset of  $W$  given by  $\{S_4, S_7, S_6\} \in W$  gives way to a incomplete ling triad given by the following figure 3.29.



**Figure 3.29**

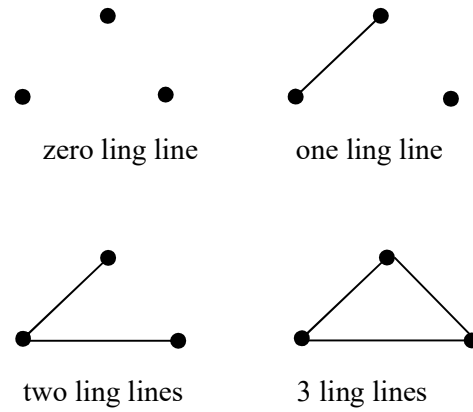
Finally take the ling subset  $\{S_1, S_2, S_5\} \in W$ . We see this ling subset gives a complete directed triad given by the following figure 3.30.



**Figure 3.30**

Now having seen examples of ling subgraphs which are either ling directed triads or ling directed dyads.

We also in case of linguistic undirected triads we have four possibilities.



**Figure 3.31**

The ling degree of a ling-node denoted by  $d(n_i)$  is the number of ling lines that are incident with it.

Thus degree of the ling node is the number of ling nodes adjacent to it.

A ling node with ling degree equal to 0 is called a linguistic isolate.

Now we will illustrate the ling degrees of the ling nodes of a ling graph.

Consider the ling variable weather report of a month. Let  $S$  be the ling set associated with this ling variable.

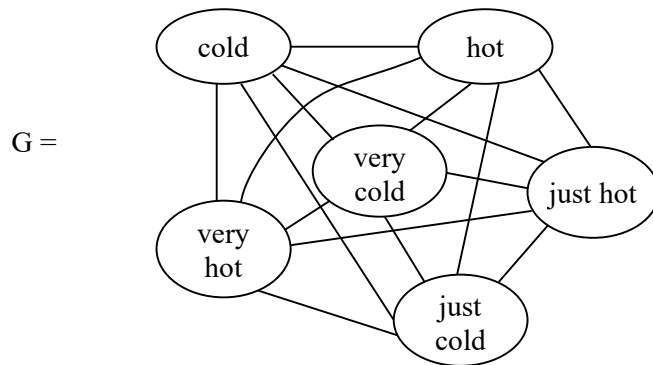
$S = \{\text{cold, very cold, hot, just cold, very hot, just hot, medium (pleasant weather) just medium, very very cold}\}$ .

The weather can vary from hot to cold or cold to hot. So it is meaning less to associate direction with it, so any network or graph is not directed.

Let us take

$$P = \{\text{cold, hot, very hot, just hot, very cold, just cold}\} \subseteq S.$$

The undirected ling graph LG associated with the ling set P is given by following figure 3.32.



**Figure 3.32**

$$\begin{aligned} \text{We see } d_L(\text{cold}) &= d_L(\text{hot}) = d_L(\text{very cold}) = \\ & d_L(\text{just hot}) = d_L(\text{very hot}) = d_L(\text{just cold}) = 5. \end{aligned}$$

It is very clear that the every node in G is 5.  $d_L$  denotes it is a ling-degree.

Further when the degree of every node is of the same degree then the ling graph or for that matter any graph is complete.

We give yet another example of a ling graph which is not directed.

**Example 3.9.** Consider the ling variable yield of paddy in a particular field for one season. The yield depends on growth of the plant and the amount of green (chlorophyll) present in the leaves (we ignore other factors).

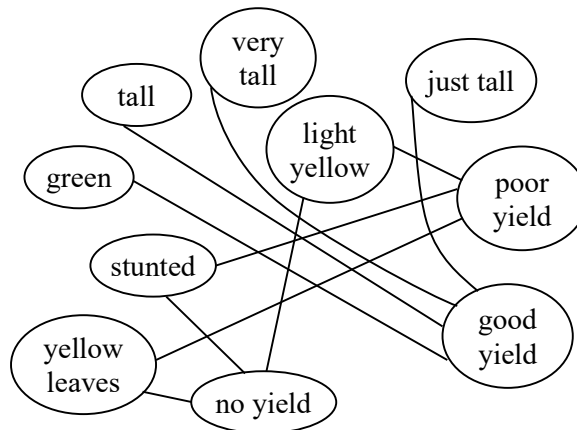
Let  $S$  be the ling set associated with this ling variable yield.

$S = \{\text{tall, just tall, very tall, medium height, short, very short, just short, green, light yellow, yellow, brown, brown dots on leaves, good yield, medium yield, no yield, just good yield, poor yield, stunted growth, poorest yield}\}$ .

Now take a ling subset  $M \subseteq S$  where

$M = \{\text{green, light yellow stunted, tall, very tall, good yield, no yield, poor yield, just tall, yellow leaves}\}$ .

The ling graph  $G$  associated with the ling set  $M$  yields a non directed ling graph.



**Figure 3.33**

$d_L(\text{tall}) = 1, d_L(\text{very tall}) = 1, d_L(\text{just tall}) = 1,$

$d_L(\text{light yellow}) = 2, d_L(\text{green}) = 1, d_L(\text{stunted}) = 2,$

$d_L(\text{poor yield}) = 3, d_L(\text{yellow leaves}) = 2,$

$d_L(\text{no yield}) = 3, d_L(\text{good, yield}) = 4.$

This is the way the deg of nodes are determined in case of linguistic non directed graph.  $d_L$  ling degree of the ling node of the undirected ling graph.

Now we find the degree of nodes directed ling graphs for the following example.

**Example 3.10.** Let us consider the ling variable height of people. Let S denote the ling set associated with the ling variable height.

$S = [\text{shortest, tallest}]$  is a ling continuum.

This S is a totally ordered.

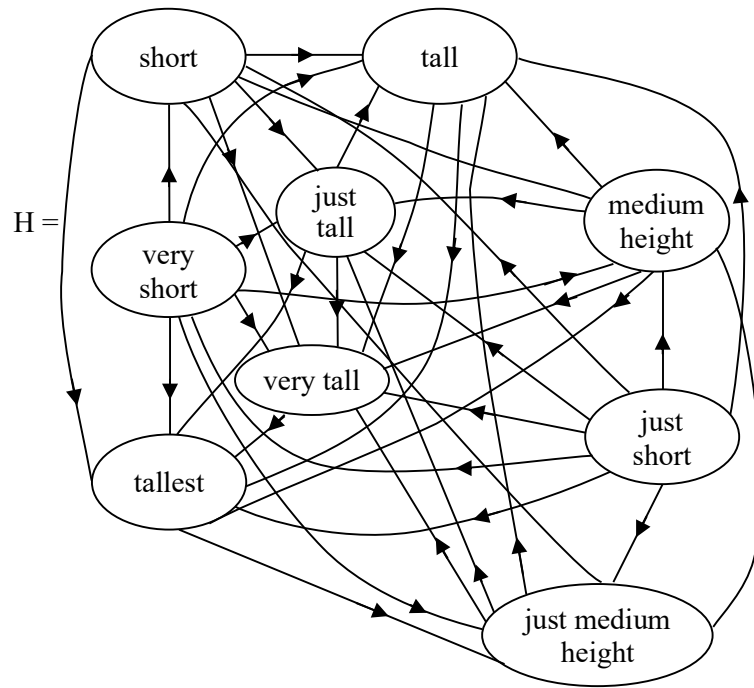
We should be careful to note a person is tall he/she cannot become short.

So we have total ordering and it is also an increasing one.

In such cases we have only a directed ling graph.

Take  $N = \{\text{short, tall, just tall, very short, medium height, very tall, just short, tallest, just medium height}\} \subseteq S.$

The directed ling graph H is as follows



**Figure 3.34**

Now because  $H$  is directed ling graph we have the concept of in degree denoted by  $d_L^0$  and out degree denoted by  $d_L^1$ .

$$d_L^1(\text{short}) = 2$$

$$d_L^0(\text{short}) = 6$$

$$d_L^1(\text{very short}) = 0$$

$$d_L^0(\text{very short}) = 8$$

$$d_L^1(\text{just short}) = 1$$

$$d_L^0(\text{just short}) = 7$$

$$d_L^1(\text{just medium height}) = 3$$

$$d_L^0(\text{just medium height}) = 5$$

$d_L^1$ (medium height) = 4	$d_L^0$ (medium height) = 4
$d_L^1$ (just all) = 5	$d_L^0$ (just tall) = 3
$d_L^1$ (tall) = 6	$d_L^0$ (tall) = 2
$d_L^1$ (very tall) = 7	$d_L^0$ (very tall) = 1
$d_L^1$ (tallest) = 8	$d_L^0$ (tallest) = 0

- i) H is a complete directed ling graph with  $d_L^1 + d_L^0 = 8$  as H has 9 ling nodes
- ii)  $d_L^1$  (tallest) = 8 which is the greatest term in the ling continuum and  $d_L^0$  (tallest) = 0
- iii)  $d_L^1$  (very short) = 0 as it is the minimal term in the ling subset N of S so  $d_L^0$  (very short) = 8
- iv) Medium height is the most middle term of the continuum as well as the in ling sub set N so  $d_L^1$  (medium height) = 4 =  $d_L^0$  (medium height).

Thus N has both in degree = out degree = 4.

The mean degree in case of non directed ling graph is

$$d_L^1 = \frac{\sum d_L(n_i)}{g} = \frac{2L}{g}$$

(L is the number of lines and g is the number of ling nodes in the ling graph).



Further the variance of the ling degree is the same as that of usual graphs given by  $S_{LD}^2 = \frac{(\sum d_L(n_i) - \bar{d}_L)^2}{g}$ .

We will verify these results in due course of time. Now density of a ling graph denoted by

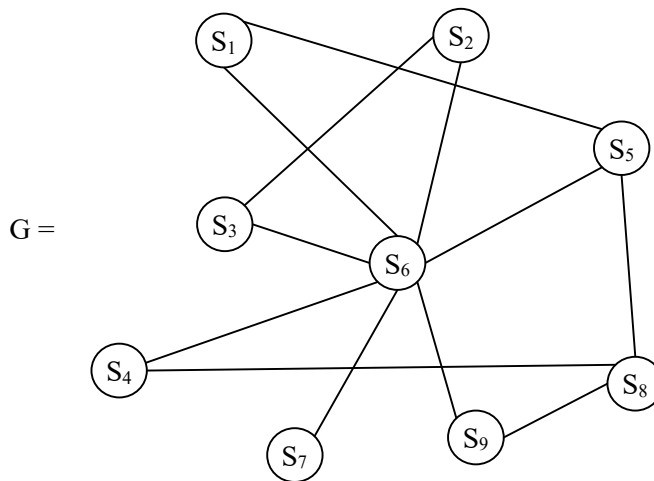
$$\Delta_L = \frac{L}{g(g-D/2)} = \frac{2L}{g(g-1)}$$

$$= \frac{\bar{d}}{g-1}.$$

The results also hold good in case of ling subgraphs of a ling graph.

Next we will give examples of a connected ling graph and a disconnected ling graph.

**Example 3.11.** Let  $G$  be a ling graph given by the following figure 3.35.



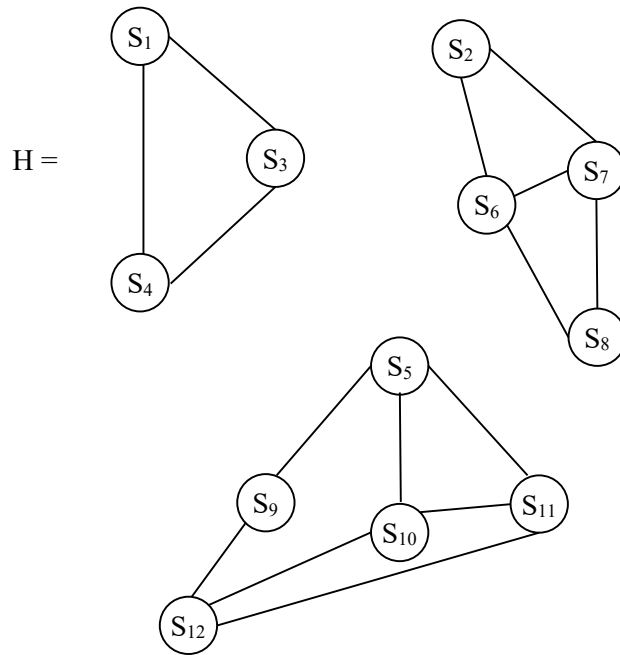
**Figure 3.35**

Clearly  $G$  is a connected ling graph where  $S_i$  are ling nodes ( $1 \leq i \leq 9$ ).  $G$  is connected as there is a path between every pair of nodes in this ling graph  $G$ .

That is we say  $G$  is connected if all pairs of nodes are reachable.

Now we give an example of a ling graph which is disconnected by the following figure.

Let  $S = \{S_1, S_2, \dots, S_{12}\}$  be a ling set and  $H$  be the ling graph associated with  $S$  given by the following figure 3.36.



**Figure 3.36**

Clearly  $H$  is a disconnected ling graph as there are pair of ling nodes in  $H$  which have no path between them.

For instance take  $S_1$  and  $S_2$  there is no path between  $S_1$  and  $S_2$  of  $H$ .

Consider  $S_5$  and  $S_8$  there is no path connecting  $S_5$  and  $S_8$ .

Consider  $S_3$  and  $S_{12}$  of  $H$ , clearly there is no path connecting  $S_3$  and  $S_{12}$  in  $H$ .

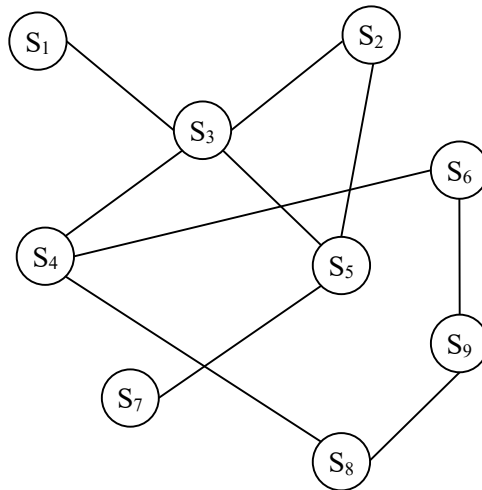
Thus  $H$  is a disconnected graph and there are 3 components related with it.

As in case of usual or classical graphs or ling graphs we have the same formula to be true in case of diameter and geodesic distances.

We will illustrate this case by the following example.

**Example 3.12.** Let  $G$  be a ling graph given by the following figure.

Let  $S = \{S_1, S_2, \dots, S_9\}$  be the ling nodes of  $G$ .



**Figure 3.37**

## Geodesic distances

$d_L(S_1, S_2) = 2$	$d_L(S_2, S_6) = 3$
$d_L(S_1, S_3) = 1$	$d_L(S_2, S_7) = 2$
$d_L(S_1, S_4) = 2$	$d_L(S_2, S_8) = 3$
$d_L(S_1, S_5) = 2$	$d_L(S_2, S_9) = 4$
$d_L(S_1, S_6) = 3$	$d_L(S_3, S_4) = 1$
$d_L(S_1, S_7) = 3$	$d_L(S_3, S_5) = 1$
$d_L(S_1, S_8) = 3$	$d_L(S_3, S_6) = 2$
$d_L(S_1, S_9) = 4$	$d_L(S_3, S_7) = 2$
$d_L(S_2, S_3) = 1$	$d_L(S_3, S_8) = 2$
$d_L(S_2, S_4) = 2$	$d_L(S_3, S_9) = 3$
$d_L(S_2, S_5) = 1$	$d_L(S_4, S_5) = 2$
$d_L(S_4, S_6) = 1$	$d_L(S_4, S_8) = 1$
$d_L(S_4, S_7) = 3$	$d_L(S_4, S_9) = 2$
$d_L(S_5, S_6) = 3$	$d_L(S_5, S_7) = 1$
$d_L(S_6, S_7) = 4$	$d_L(S_6, S_8) = 2$
$d_L(S_6, S_9) = 1$	$d_L(S_7, S_8) = 4$
$d_L(S_5, S_7) = 5$	$d_L(S_8, S_9) = 1$

diameter of the ling graph  $G = \max \{d_L(i, j)\} = d_L(S_7, S_9) = 5$ .

Next we discuss about the connectivity of ling graphs.

Connectivity as in case of usual graph is a function of whether a ling graph remains connected when nodes/or lines are deleted.

To this end we will just recall the definition of cut points in a ling graph.

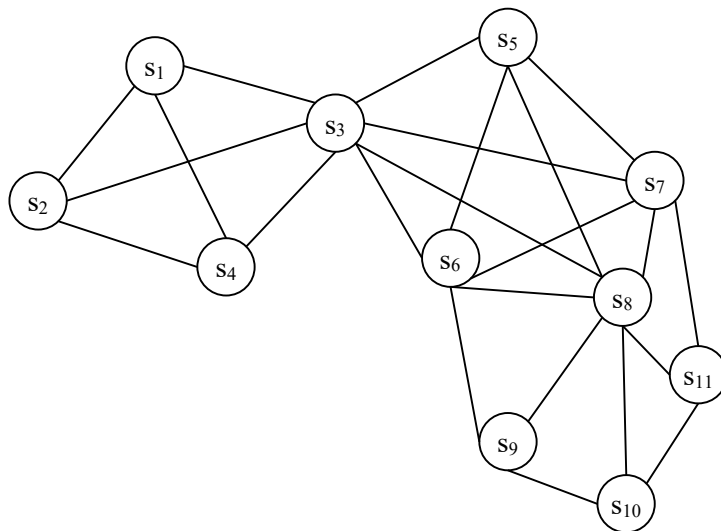
A ling node  $S_i$  is a cut point if the number of components in the ling graph that contains  $S_i$  is fewer than the number of components in the ling subgraph that results from deleting  $s_i$  from the ling graph.

The concept of a cut point can be extended from a single node to a set of nodes to keep the ling graph connected.

A cut point is a 1-node cut.

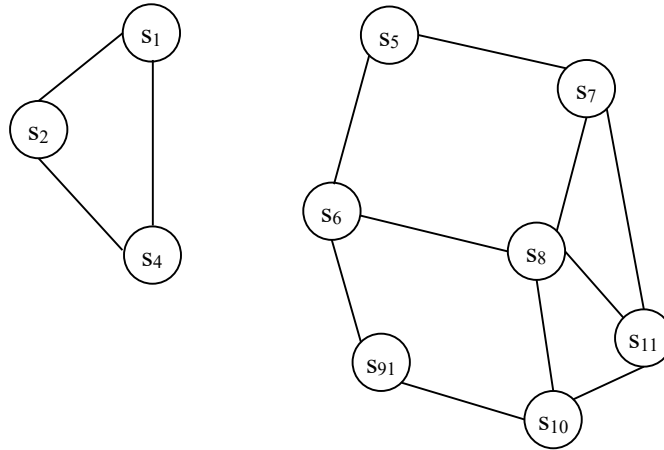
We will first illustrate this situation by an example.

**Example 3.13.** Let  $S = \{s_1, s_2, \dots, s_{11}\}$  be a set of ling points / terms given by the following figure.



**Figure 3.38**

Clearly the node  $s_3$  is a node cut or cut point



**Figure 3.39**

The ling graph without the ling node  $s_3$ .

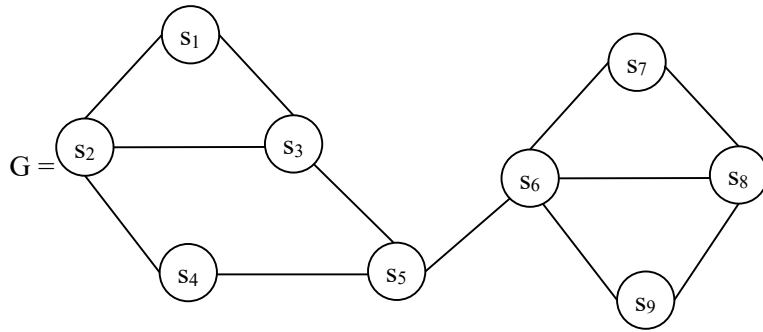
This is an example of cut point in a ling graph.

The notion of bridge in a ling graph. A bridge is a line that the ling graph containing the line has fewer components than the ling subgraph that is obtained after the line is removed.

The removal of a bridge leaves more components than when the bridge is included.

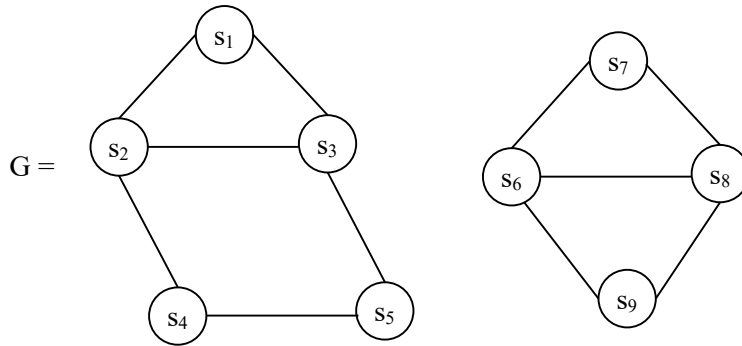
We will illustrate this situation in case of ling graphs by an example.

**Example 3.14.** Let  $S = \{s_1, s_2, s_3, \dots, s_9\}$  be the ling set, given by the ling graph.



**Figure 3.40**

Clearly  $l(s_5, s_6)$  is the bridge. So if the bridge  $l(s_5, s_6)$  is removed we get into two components.



**Figure 3.41**

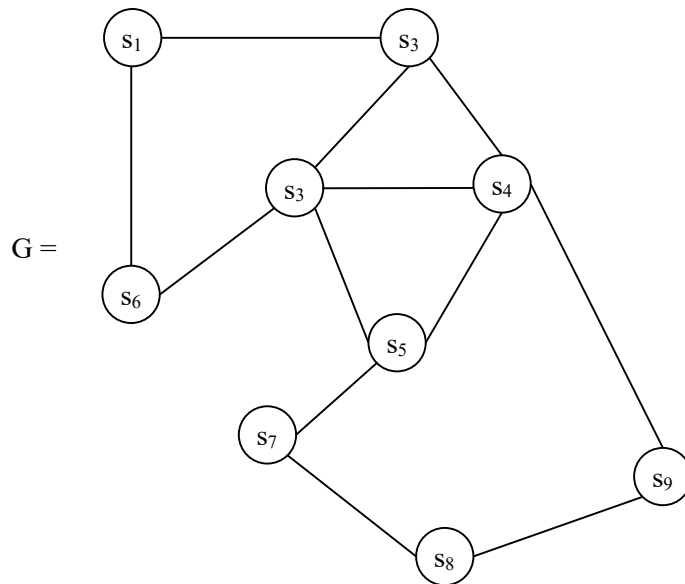
Removal of the bridge  $l(s_5, s_6)$  we get two components ling subgraphs of  $G$ .

We can as in case of classical graph theory develop these notions to ling graph theory the concept of  $l$ -line cut and  $k$ -node cut  $k$  and  $l$  greater than one line-connectivity and edge connectivity of ling graphs.

We have ling trees ling forest and ling cyclic graphs as in case of classical graphs.

We will illustrate them by same examples.

**Example 3.15.** Suppose  $S = \{s_1, s_2, s_3, \dots, s_9\}$  be a ling set. Using the ling set  $S$  we get the ling cyclic graph  $G$  which is as follows.



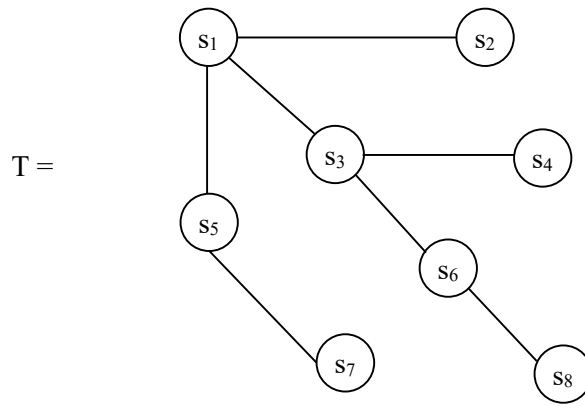
**Figure 3.42**

Clearly  $G$  is a ling cyclic graph.

Consider the ling set  $M = \{s_1, s_2, s_3, s_4, s_5, s_7, s_6, s_8\}$ .

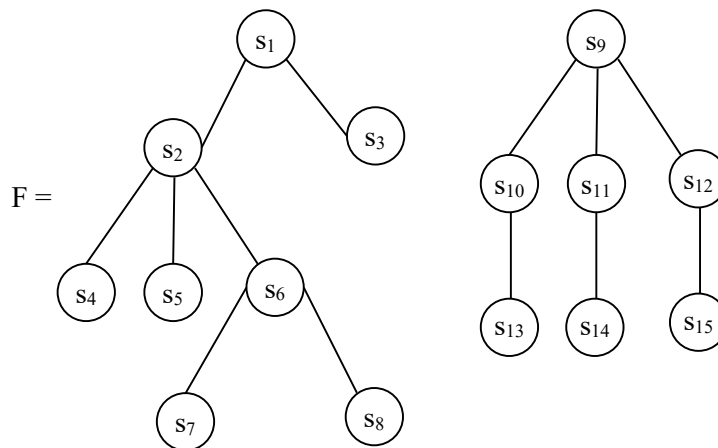
The ling tree  $T$  associated with the ling set  $M$  is as follows.





**Figure 3.43**

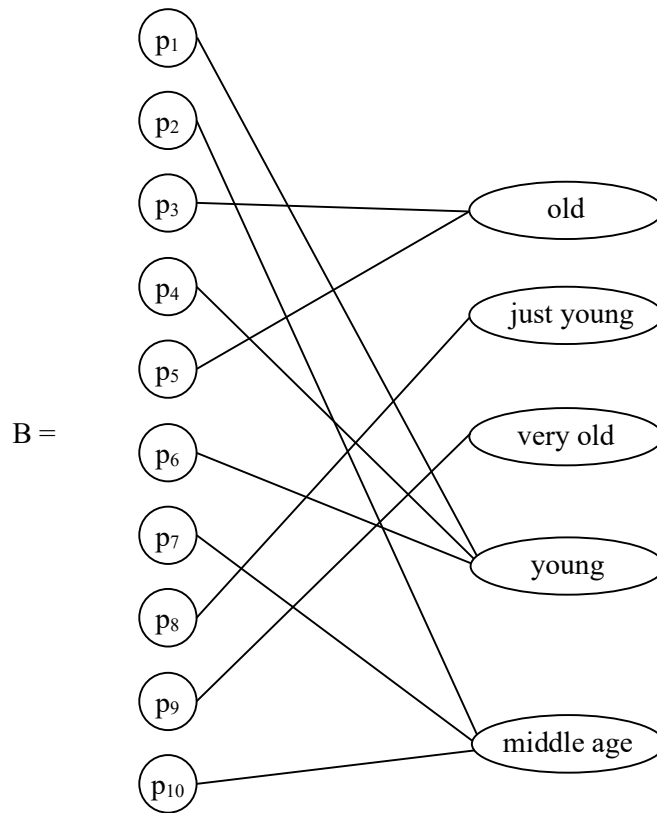
Now we give an example of a ling forest F using the ling set  $S = \{s_1, s_2, \dots, s_{15}\}$ .



**Figure 3.44**

Now we provide examples of ling bipartite graphs and ling complete bipartite graph.

**Example 3.16.** Consider the ling variable age of 10 persons  $\{p_1, \dots, p_{10}\}$  and age falls under  $\{\text{old, just young, young, middle aged, very old}\}$ . We have the following bipartite ling graph.

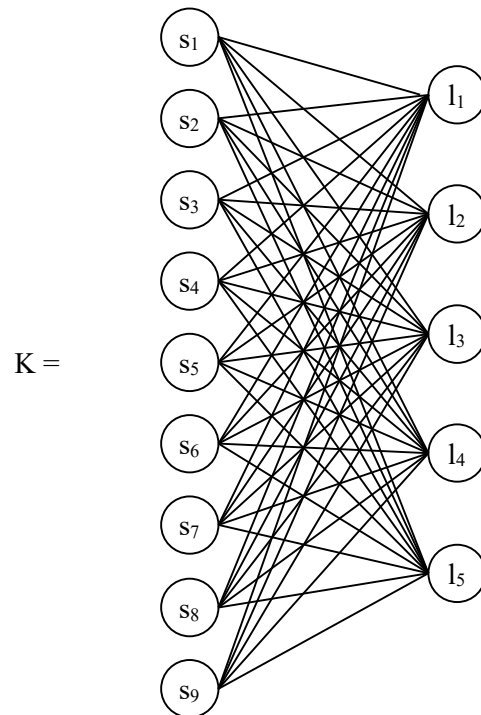


**Figure 3.45**

Clearly B is only a ling bipartite graph which is not a complete ling bipartite graph.

**Example 3.17.** Let  $V = \{s_1, s_2, s_3, s_4, \dots, s_9\}$  and

$W = \{l_1, l_2, \dots, l_5\}$  be two ling sets. Now let  $K$  be the bipartite graph using the ling sets  $V$  and  $W$  which is given by the following figure 3.46.



**Figure 3.47**

Clearly  $K$  is a complete ling bipartite graph.

In case of ling directed graphs all properties that are true for classical directed graphs are also true ling graphs.

The concept isolate, transmitter, receiver and carrier or ordinary can also defined exactly in a similar way in case of ling graphs.

The concept of reachability and connectivity digraphs are also true in case of ling directed graphs.

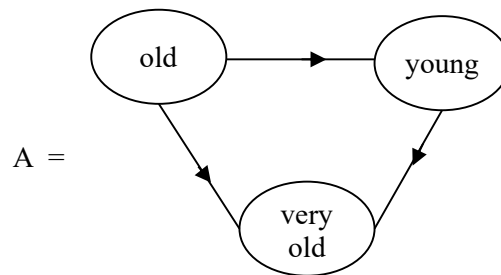
However the concept of complement and converse of a directed graph is not true in case of ling direct graph for meaning would be changed.

We will show this by examples.

Let us consider the ling variable age of people.

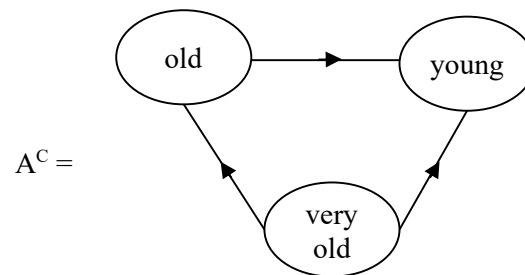
Let  $S = [\text{youngest, oldest}]$  be the ling set associated with the ling variable age of people. Clearly  $S$  is a ling continuum.

Let



**Figure 3.48**

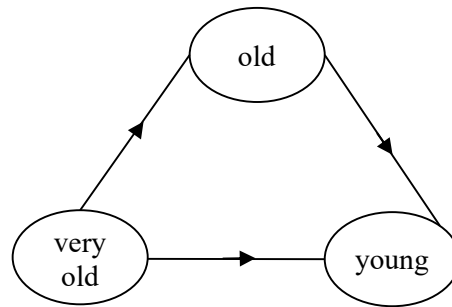
The ling complement of  $A$  denoted by  $A^C$  is as follows.



**Figure 3.49**

Clearly  $A^C$  the ling complement is not a ling triad as the ling lines are absurd very old person cannot become old or a very old person cannot become young and old person cannot become young. So  $A^C$  does not exist.

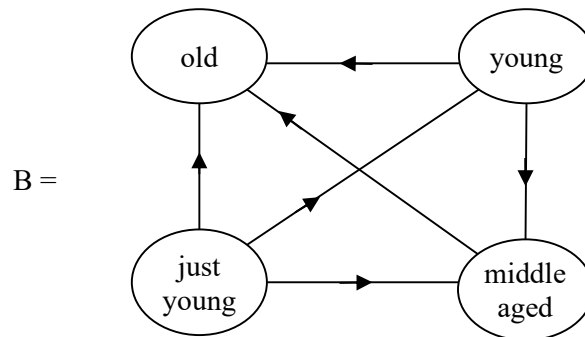
Consider the ling converse  $A'$  of  $A$  given by the following figure 3.50.



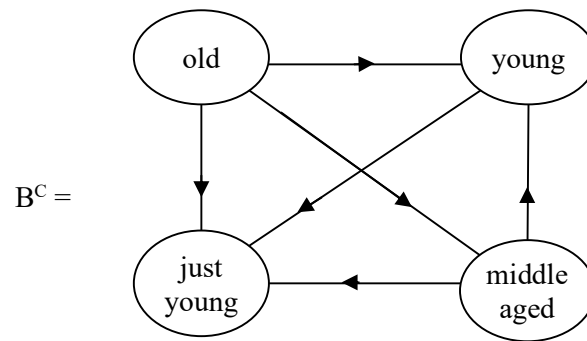
**Figure 3.50**

which is absurd and cannot be defined.

Consider



**Figure 3.51**



**Figure 3.52**

$B^C$  does not exist so  $B^C$  is not a ling graph. For middle aged to just young one cannot go back so on for all nodes in figure 3.52.

Similarly  $B'$  the converse of this ling graph does not exist.

We wish to keep on record that for ling graphs  $G$  its complement and converse cannot be meaningful.

Now we go for the interesting concept of ling valued graphs and ling valued directed graphs of the ling graphs the edges are given ling terms of weights.

We call such ling graphs as ling valued graphs.

We will first give an example of the same.

**Example 3.18.** Let us consider the ling variable age  $V$ , let  $S$  be the ling set associated with it.  $S = [\text{youngest}, \text{oldest}]$  be the ling continuum associated with ling variable age  $V$ .

$$P = \{\text{old, young, just young, very old, just old, very young, middle aged}\} \subseteq S.$$

The ling graph using P with ling edge values is as follows.

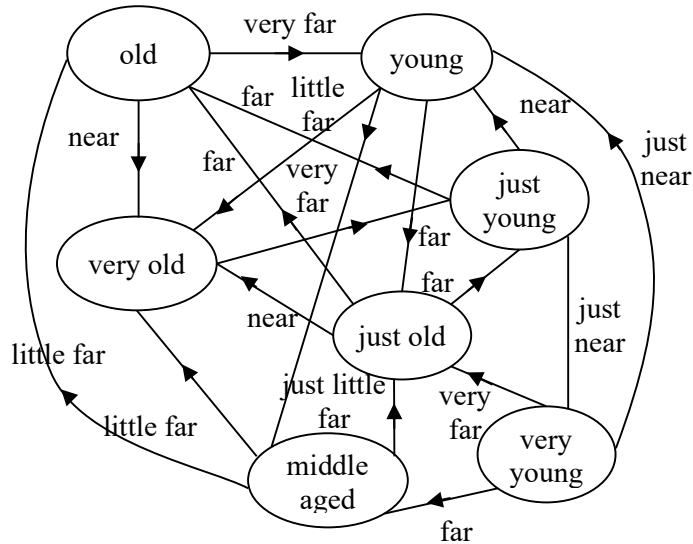


Figure 3.53

Next we will describe the format of ling socio matrix by some examples.

**Example 3.19.** Consider the ling variable V, the speed of the car on road and let S be the ling set associated with V.

$$P = \{\text{slow, fast, just fast, very slow, very fast, normal speed, just slow, at rest}\}$$

be the subset of S. Now we give the ling graph G using P as the ling node set by the following figure 3.54.

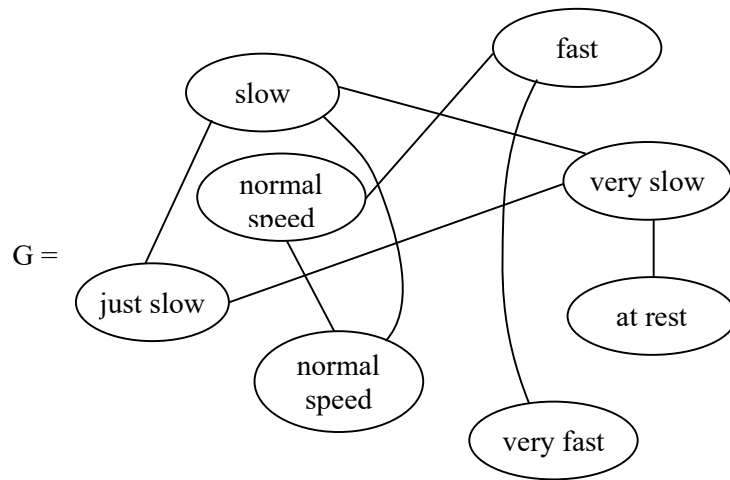


Figure 3.54

Now let  $M_L$  be the ling socio matrix associated with G.

	slow	fast	just fast	just slow
slow	-	ud	ud	d
fast	ud	-	d	ud
just fast	ud	d	-	ud
just slow	d	ud	ud	-
veryslow	d	ud	ud	d
at rest	ud	ud	ud	ud
normalspeed	d	ud	d	ud
very fast	ud	d	ud	ud



veryslow	at rest	normalspeed	very fast
d	ud	d	ud
ud	ud	ud	d
ud	ud	d	ud
d	ud	ud	ud
–	d	ud	ud
d	–	ud	ud
ud	ud	–	ud
ud	ud	ud	–

Clearly the ling socio matrix uses the ling term d (defined) if there is a relation tie between the ling nodes otherwise we put the ling term ud (undefined).

However as we assume the ling graph is simple that is no loops all the diagonal elements as in case of socio matrix is filled with ‘-’ which implies undefined.

Next we give an example of a ling directed graph H and obtain its ling socio matrix.

**Example 3.20.** Let S be the ling set associated with the ling variable age of people.

Let  $T = \{\text{old, young, very old, just old, middle aged, just young, very young}\}$

be the ling subset of S.

Let H be the ling directed graph by taking the ling nodes as the set T.

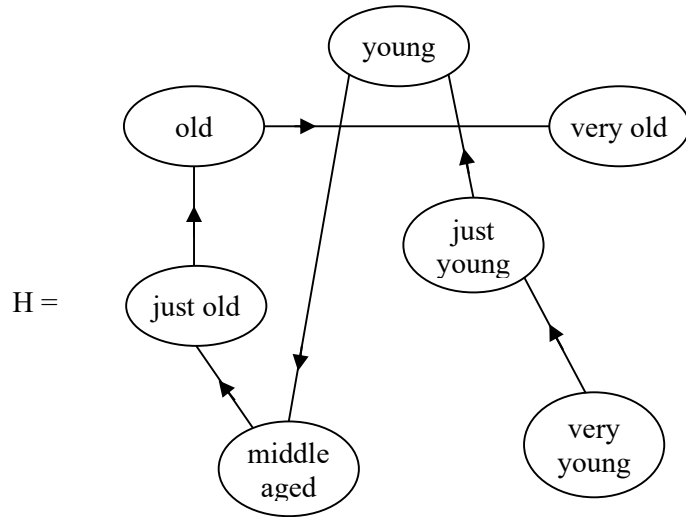


Figure 3.55

The ling socio matrix  $N_L$  associated with the ling directed graph H is as follows.

	very young	just young	young
very young	-	d	d
just young	ud	-	d
young	ud	ud	-
$N_L =$ middle aged	ud	ud	ud
just old	ud	ud	ud
old	ud	ud	ud
very old	ud	ud	ud

	middle aged	just old	old	very old
	ud	ud	ud	ud
	ud	ud	ud	ud
	d	ud	ud	ud
	–	d	ud	ud
	ud	–	d	ud
	ud	ud	–	d
	ud	ud	ud	–

ud - undefined

d - defined

- - no loops

We make the following observations.

- i) In case of ling non directed graphs the ling socio matrix is symmetric with entries from {–, ud, d / - no loop, un - undefined, d - defined}

They are square matrices of order  $t \times t$  if the ling non directed graph has t-ling nodes.

- ii) In case of ling directed graphs w see the ling socio matrix is a square matrix which is not symmetric but has its entries from {–, ud, d / - no loop, ud - undefined, d - defined}

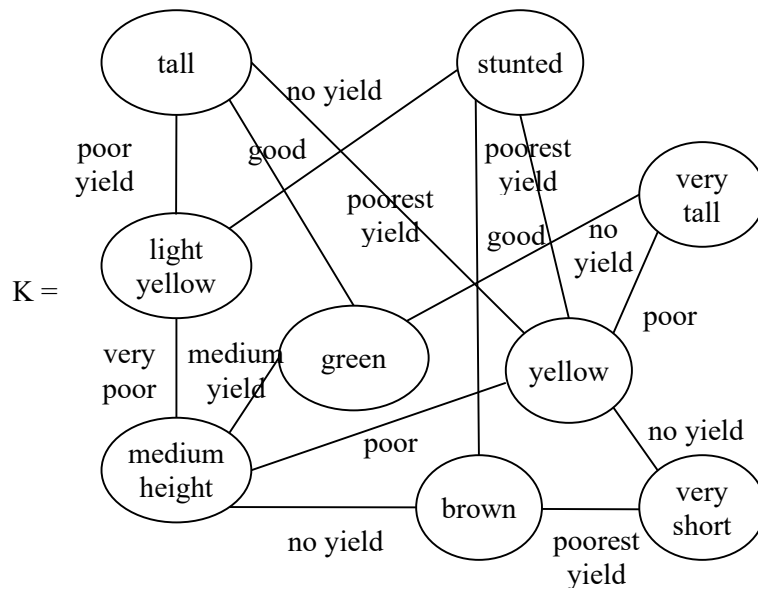
Now we have given illustration of both.

We will now discuss about ling valued graphs and ling valued directed graphs and find its ling socio matrix by examples.

**Example 3.21.** Let us consider the ling variable V yield of paddy crop. The ling set associated with this ling variable V is given by

$S = \{\text{tall, stunted growth, medium height, short, very short, green, yellow, very tall, brown dots on leaves, light yellow}\}.$

Let K be the ling graph associated with the ling variable yield given by following figure.



**Figure 3.56**

We now give the ling socio matrix  $P_L$  associated with ling valued non directed graph K.

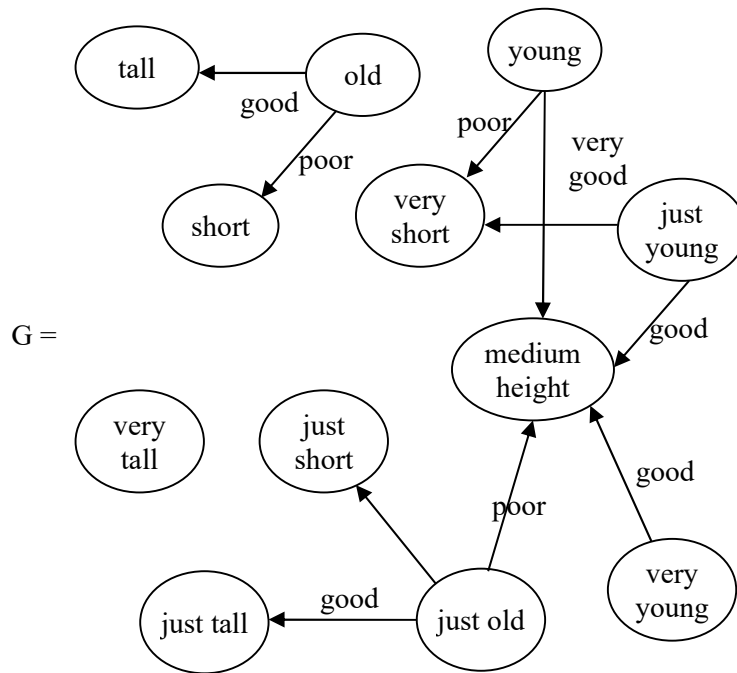
		tall	stunted	very tall	light yellow
	tall	–	ud	ud	poor yield
	stunted	ud	–	ud	no yield
	very tall	ud	ud	–	ud
	light yellow	poor yield	no yield	ud	–
P <sub>L</sub> =	green	good	ud	good	ud
	yellow	poor yield	no yield	poor	ud
	medium ht	ud	ud	ud	very poor yield
	brown	ud	poorest yield	ud	ud
	very short	ud	ud	ud	ud

	green	yellow	medium ht	brown	very short
	good	poor yield	ud	ud	ud
	ud	no yield	ud	poorest yield	ud
	good	poor	ud	ud	ud
	u	ud	very poor yield	ud	ud
	–	ud	medium yield	ud	ud
	ud	–	poor	ud	ud
	medium yield	poor	–	no yield	ud
	ud	ud	no yield	–	poor yield
	ud	ud	ud	poor yield	–

We see ling – valued socio matrix for this valued ling graph K is a square symmetric matrix with entries from the set  $\{-, ud, d / - \text{ no loop, ud-undefined, d - defined}\}$ .

Now we give an example of a valued ling directed graph.

**Example 3.22.** Let us consider the valued ling directed graph, age and height of people. The ties of relation measures the growth of people is good or bad or so on. So the line value is growth associating the ling values height and age of people.



**Figure 3.57**

The ling nodes of the valued directed graph G is as follows  $\{\text{tall, old, young, short, very short, medium height, very tall, just short, just tall, very young, just young and just old}\}$ .

We give the ling valued socio matrix  $R_L$  associated with G.

		tall	old	young	short	very short
$R_L =$	tall	–	ud	ud	ud	ud
	old	good	–	ud	poor	ud
	young	ud	ud	–	ud	poor
	short	ud	ud	ud	–	ud
	very short	ud	ud	ud	ud	–
	just young	ud	ud	ud	ud	poor
	medium ht	ud	ud	ud	ud	ud
	very tall	ud	ud	ud	ud	ud
	just short	ud	ud	ud	ud	ud
	just tall	ud	ud	ud	ud	ud
	just old	ud	ud	ud	ud	ud
	very young	ud	ud	ud	ud	ud

just young	medium ht	very tall	just short	just tall	just old	very young
ud	ud	ud	ud	ud	ud	ud
ud	ud	ud	ud	ud	ud	ud
ud	ud	ud	ud	ud	ud	ud
ud	ud	ud	ud	ud	ud	ud
ud	–	good	ud	ud	ud	ud
ud	–	ud	ud	ud	ud	ud
u	ud	–	ud	ud	ud	ud
ud	ud	ud	–	ud	ud	ud
ud	ud	ud	ud	–	ud	ud
ud	ud	ud	poor	good	–	ud
ud	good	ud	ud	ud	ud	ud

Clearly the entries of the ling value are from the set

$$\{-, ud, good, poor, very\ good\}$$

where – no loop, ud – undefined.

Now having seen examples of ling valued directed graphs and ling valued non directed graphs we proceed onto describe the notion of ling valued bipartite graph and its associated ling socio matrix by some examples.

**Example 3.23.** Consider the two ling variables student performance and teachers ability to teach.

The two sets of ling nodes are given by

$$D = \{good, bad, very\ bad, very\ good, fair, very\ poor, poor\}$$

where D denotes the ling set performance of students in the class room.

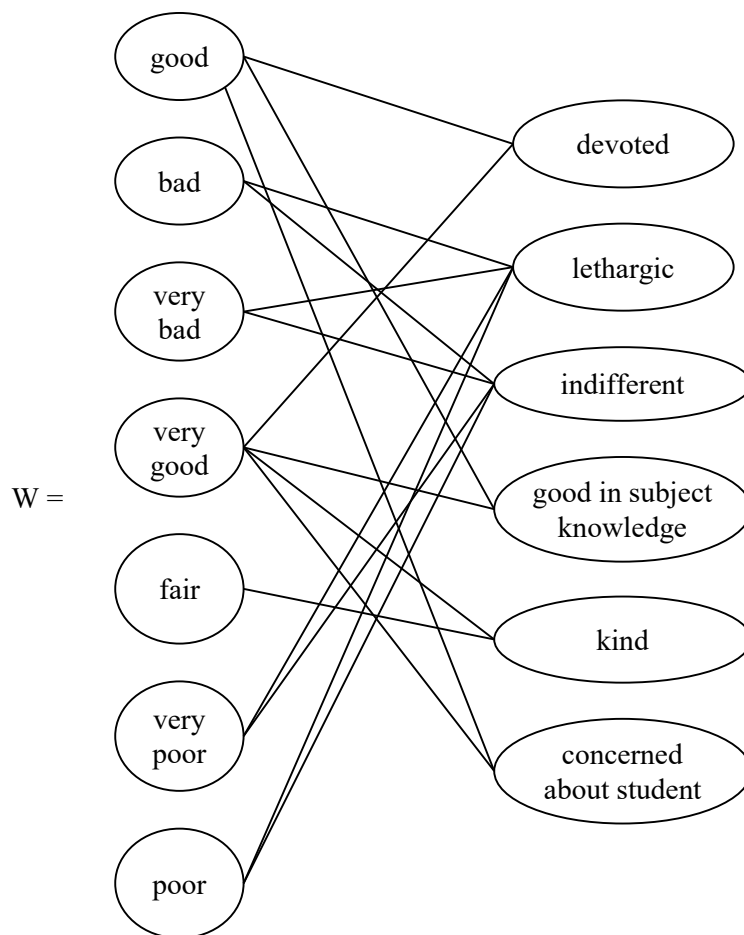
Let

$$R = \{devoted, lethargic, indifferent, good\ subject\ knowledge, \\ kind, concerned\ about\ students\}$$

be the ling set associated with the devolution of teachers and their attitude.

Now we give the ling bipartite graph W given by the following figure.





**Figure 3.58**

The ling socio matrix  $P_L$  associated with  $W$  is as follows.

		devoted	lethargic	indifferent
$P_L =$	good	d	ud	ud
	bad	ud	d	d
	very bad	ud	d	d
	very good	d	ud	ud
	fair	ud	ud	ud
	verypoor	ud	d	d
	poor	ud	d	d

	good in subject knowledge	kind	concerned about students
	d	ud	d
	ud	ud	ud
	ud	ud	ud
	d	d	d
	ud	d	ud
	ud	ud	ud
	ud	ud	ud

$\left. \vphantom{\begin{matrix} d \\ ud \\ ud \\ d \\ ud \\ ud \\ ud \end{matrix}} \right]_{7 \times 6}$

We see  $P_L$  is a  $7 \times 6$  ling socio matrix with entries from the ling set {ud, d, ud - defined, d - defined}

One can easily see these ling socio matrix associated with bipartite ling graphs.

They are not square ling matrices so they cannot be symmetric.

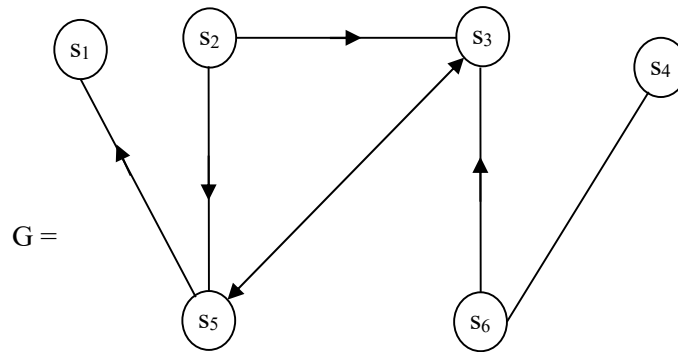
Further these ling socio matrices associated with ling bipartite graphs do not contain loops.

In the following we suggest a few problems to the reader. Starred problems are difficult to solve

By working with these problem can become familiar with these new notions.

### PROBLEMS

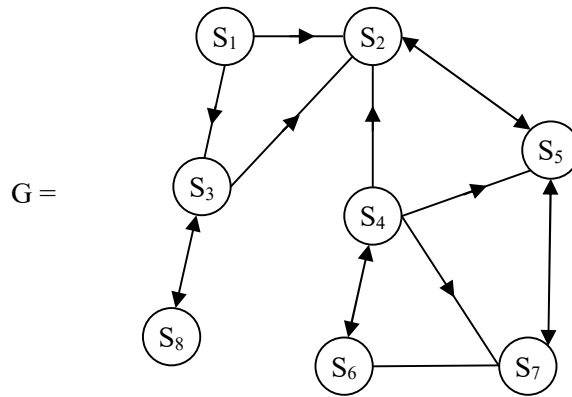
1. Bring out the common features and differences between classical graphs and ling graphs.
2. Define all forms ling dyads and illustrate by examples.
3. How many forms of ling triads can be defined? Illustrate them by an example.
4. Give an example of a ling graph describing friendship between four persons.
5. Give an example of a ling empty graph how is it different from the ling trivial graph.
6. Draw the ling graph  $G$  using age of people as ling variables and its associated ling set with 10 ling nodes.
  - i) Will  $G$  be a ling directed graph?
  - ii) Find the ling socio matrix  $M$  of  $G$ .
  - iii) Will  $M$  be symmetric or non symmetric matrix?
  - iv) Can  $G^C$  and  $G'$  have any sense associated with them?
7. Let  $G$  be a ling graph given by the following figure 3.59.



**Figure 3.59**

- a) Find the ling complement graph  $G^C$  of  $G$ .
  - b) Find the converse  $G'$  of  $G$ .
  - c) Obtain the ling socio matrices of  $G$ ,  $G^C$  and  $G'$  and compare them.
8. Let  $V$  be the ling variable associated with age of people  $S$  be the ling associated with  $V$ . Let  $G$  be the ling graph with ling vertex set
- $$P = \{\text{old, just old, young, very young, middle aged, just young, very old, oldest}\} \subseteq S.$$
- i) Is  $G$  a ling directed graph?
  - ii) Find all ling subgraphs of  $G$ .
  - iii) Does  $G$  contain complete triads?
  - iv) Does  $G^C$  and  $G'$  have any meaning? Justify your claim.

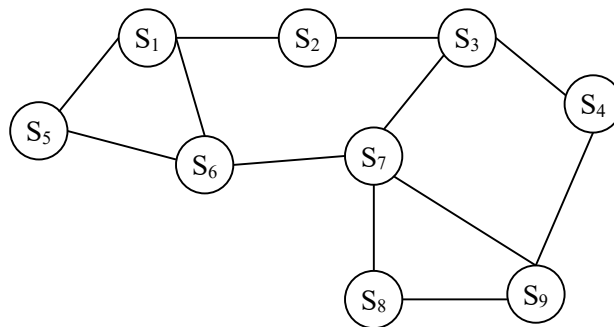
- v) Give the ling socio matrix associated with G.
9. Let G be a ling graph given by the following figure 3.60.



**Figure 3.60**

- i) Find all ling triads of G.
- ii) Give the ling socio matrix  $M_L$  associated with G.
- iii) Can  $M_L$  be a symmetric ling matrix? Justify your claim.
- iv) Does G have cut points?
- v) Does G have a bridge?
- vi) Find complement  $G^C$  of G.
- vii) Find the converse  $G'$  of G.

10. Give an example of a ling graph  $G$  with 3 cut points.
  
11. Give an example of a line graph  $G$  such that  
 $G = \text{complement of } G; G^C = \text{converse of } G; G'$
  
12. Give an example of a ling graph  $G$  for which  $G^C$  and  $G'$  does not exist.
  
13. Give an example of a ling graph  $G$  for which only  $G^C$  exist and  $G'$  the converse does not exist.
  
14. Give an example of a ling graph  $G$  for which only  $G'$  exist and  $G^C$  does not exist.
  
15. For the ling graph  $G$  with ling node set  $S = \{S_1, S_2, \dots, S_9\}$  is given in following figure 3.61.



**Figure 3.61**

- i) Find the ling mean degree  $\bar{d}$  of  $G$

- ii) Find the ling variance of degrees  $S_D^2$
  - iii) Find the ling density of the graph G.
16. What is a cut point in a ling graph?
- a) Give an example of a ling graph with 1-cut point.
  - b) Give an example of 3-cut point ling graph.
17. Give an example of a connected graph.
18. Give an example of a bridge in graph.
19. Give examples of
- i) Cyclic ling graph.
  - ii) ling tree
  - iii) Ling forest
20. What is ling geodiscs in a ling graph?
21. Define ling - diameter of a ling graph G.
22. Give a real world problem where ling graphs yield a better result than classical graph (graphs as networks).
23. Let S be a ling set associated with the ling - graph G.

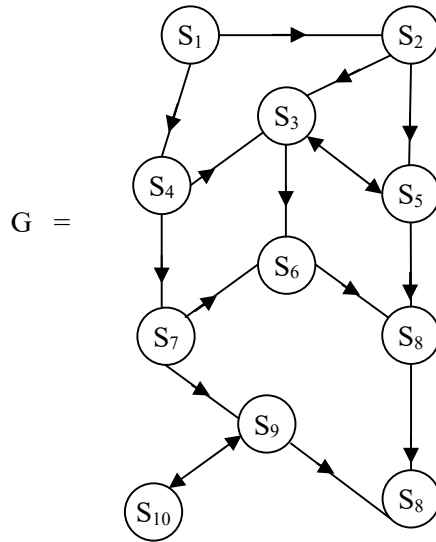


Figure 3.62

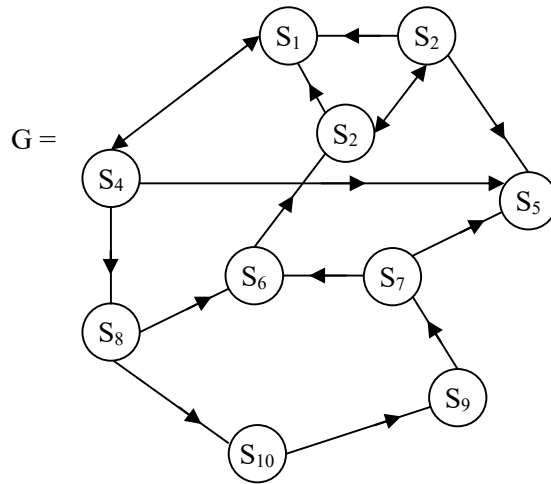
- i) Find the ling mean  $\bar{d}$  of G.
- ii) Find the ling density of the ling graph G
- iii) Find  $S_D^2$  the variance of the ling-graph G.
- iv) Find  $M_L$  the ling socio matrix of G.
- v) What is the cut point of G so that G becomes disconnected?
- vi) Can  $G^C$  be defined for this G?
- vii) How many complete ling triads does G contain?



- viii) Find the ling path  $S_1$  to  $S_{11}$
  - ix) Define ling walk from  $S_3$  to  $S_{10}$ .
  - x) Define geodiscs in  $G$ .
  - xi) List out the ling in degree  $d_L^I$  and ling out degree  $d_L^O$  of  $G$ .
  - xii) What is ling diameter of  $G$ ?
  - xiii) What is ling isolate of  $G$ ?
  - xiv) Determine the ling transmitter of  $G$ .
  - xv) Is  $G$  carrier or ordinary?
  - xvi) What is the ling receiver of  $G$ ?
  - xvii) Is the ling semi path from  $S_3$  to  $S_{11}$  same as the ling path from  $S_3$  to  $S_{11}$ .
  - xviii) Does  $G$  have a ling bridge? Justify
  - xix) What is the node cut in  $G$ ?
  - xx) Obtain any other nice properties associated with ling directed graphs  $G$ .
24. Give an example of a friendship ling graph  $G$ .
- a) Will  $G$  be directed? Justify your claim.
  - b) For this  $G$  find the ling variance of out degrees and ling variance of in degrees and show how

these measures quantify how unequal the friendship in that ling network.

25. Give an example of a ling complete bipartite graph  $G$ .
  - a) Find the ling socio matrix  $M$  of  $G$ .
  - b) Can the ling socio matrix  $M$  of  $G$  be symmetric? Justify your claim.
  - c) Obtain all special features enjoyed by ling bipartite complete graphs.
26. Give an example of a weakly connected ling graph which is not unilaterally connected.
27. Provide an example of a unilaterally connected ling graph which is not strongly connected.
28. Give an example of a strongly connected ling graph which is not recursively connected.
29. Give an example of a ling valued directed graph.
30. What is reachability in a valued ling graph?  
Give example of a ling graph in which every pair of ling nodes are reachable.
31. Give an example of a disconnected ling graphs.
32. Are ling forests disconnected ling graphs?
33. Let  $G$  be a ling graph given by the following figure 3.63.



**Figure 3.63**

- i) Find a directed walk from
  - a)  $S_1$  to  $S_{10}$
  - b)  $S_3$  to  $S_9$
  - c)  $S_2$  to  $S_8$
  
- ii) Find the directed path from
  - a)  $S_6$  to  $S_2$
  - b)  $S_2$  to  $S_9$
  - c)  $S_4$  to  $S_9$
  - d)  $S_3$  to  $S_9$
  
- iii) Find all directed triads.
  
- iv) Find  $G^C$  and  $G'$  of  $G$ .

By an example prove that every ling path is a ling semipath but not every ling semipath is a ling path.

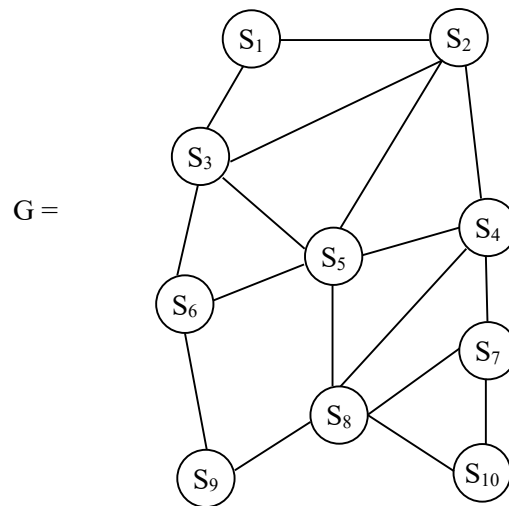
34. What is ling semicycle how is it different from ling cycle and ling semiwalk?

35. What is ling path length?

Give an example to show the ling path length and the purpose of it.

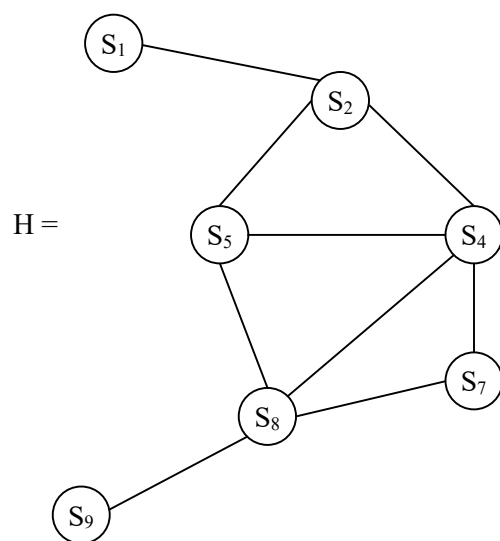
36. How is ling valued graphs different from valued graphs? Justify by an examples.

37. Give an example of a ling valued graph and describe the special features associated with it. Let  $G$  be a ling graph given by the following figure 3.64.



**Figure 3.64**

i) For the ling subgraph H of G



**Figure 3.65**

Calculate  $\Delta$  and  $\Delta_s$ .

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