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# Modified Collatz conjecture or *(3a + 1) + (3b + 1)I* Conjecture for Neutrosophic Numbers $\langle Z \cup I \rangle$

**W.B. Vasantha Kandasamy[1], K. Ilanthenral[2], and Florentin Smarandache[3]**

[1] Department of Mathematics, Indian Institute of Technology (Madras), Chennai, 600 036, India. E-mail: vasantha@iitm.ac.in
[2] School of Computer Science and Engg.,VIT University, Vellore, 632 014, India. E-mail: ilanthenral@gmail.com
[3] Department of Mathematics, University of New Mexico, USA. E-mail: smarand@unm.edu

**Abstract**: In this paper, a modified form of Collatz conjecture for neutrosophic numbers $\langle Z \cup I \rangle$ is defined. We see for any $n \in \langle Z \cup I \rangle$ the related sequence using the formula $(3a + 1) + (3b + 1)I$ converges to any one of the 55 elements mentioned in this paper. Using the akin formula of Collatz conjecture viz. $(3a - 1) + (3b - 1)I$ the neutrosophic numbers converges to any one of the 55 elements mentioned with appropriate modifications. Thus, it is conjectured that every $n \in \langle Z \cup I \rangle$ has a finite sequence which converges to any one of the 55 elements.

**Keywords:** Collatz Conjecture, Modified Collatz Conjecture, Neutrosophic Numbers.

## 1 Introduction

The Collatz conjecture was proposed by Lothar Collatz in 1937. Till date this conjecture remains open. The 3n – 1 conjecture was proposed by authors [9]. Later in [9] the 3n ± p conjecture; a generalization of Collatz Conjecture was proposed in 2016 [9].

However, to the best of authors knowledge, no one has studied the Collatz Conjecture in the context of neutrosophic numbers $\langle Z \cup I \rangle = \{a + bI / a, b \in Z; I^2 = I\}$ where I is the neutrosophic element or indeterminancy introduced by [7]. Several properties about neutrosophic numbers have been studied. In this paper, authors for the first time study Collatz Conjecture for neutrosophic numbers. This paper is organized into three sections.

Section one is introductory. Section two defines / describes Collatz conjecture for neutrosophic numbers. Final section gives conclusions based on this study. Extensive study of Collatz conjecture by researchers can be found in [1-6]. Collatz conjecture or 3n + 1 conjecture can be described as for any positive integer n perform the following operations.

If n is even divide by 2 and get $\frac{n}{2}$ if $\frac{n}{2}$ is even divide by 2 and proceed till $\frac{n}{2^t}$ is odd.

If n is odd multiply n by 3 and add 1 to it and find 3n + 1. Repeat the process (which has been called Half of Triple Plus One or HTPO) indefinitely. The conjecture puts forth the following hypothesis; whatever positive number one starts with one will always eventually reach 1 after a finite number of steps.

Let n = 3, the related sequence is 3n + 1, 10, 5, 16, 8, 4, 2, 1.

Let n = 11, the related sequence is 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1.

Let n = 15, the related sequence is 15, 46, 23, 70, 35, 106, 53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2, 1.

In simple notation of mod 2 this conjecture can be viewed as

$$f(n) = \begin{cases} n/2 & \text{if } n \equiv 0 \pmod 2 \\ 3n+1 & \text{if } n \equiv 1 \pmod 2 \end{cases}.$$

The total stopping time for very large numbers have been calculated. The 3n – 1 conjecture is a kin to Collatz conjecture.

Take any positive integer n. If n is even divide by 2 and get $\frac{n}{2}$ if $\frac{n}{2}$ is odd multiply it by 3 and subtract 1 to i.e. 3n – 1, repeat this process indefinitely, [9] calls this method as Half Or Triple Minus One (HOTMO).

The conjecture state for all positive n, the number will converge to 1 or 5 or 17.

In other words, the 3n – 1 conjecture can be described as follows.

$$f(n) = \begin{cases} \frac{n}{2} & \text{if } n \equiv 0 \pmod 2 \\ 3n-1 & \text{if } n \equiv 1 \pmod 2 \end{cases}$$

Let n = 3, 3n – 1 = 8, 4, 2, 1.
Let n = 28, 14, 7, 20, 10, 5.
n = 17, 50, 25, 74, 37, 110, 55, 164, 82, 41, 122, 61, 182, 91, 272, 136, 68, 34, 17.

Several interesting features about the 3n – 1 conjecture is derived and described explicitly in [9].

It is pertinent to keep on record in the Coltaz conjecture 3n + 1 if n is taken as a negative number than using 3n + 1 for negative values sequence terminate only at – 1 or – 5 or – 17. Further the 3n – 1 conjecture for any negative n, the sequence ends only in – 1.

Thus, for using 3n + 1 any integer positive or negative the sequence terminates at any one of the values {–17, –5, –1, 0, 1} and using 3n – 1 the sequence for any integer n positive or negative terminates at any one of the values {–1, 0, 1, 5, 17}.

## 2 Collatz Conjecture for the neutrosophic numbers ⟨Z ∪ I⟩

In this section, we introduce the modified form of Collatz conjecture in case of neutrosophic numbers ⟨Z ∪ I⟩ = {a + bI / a, b ∈ Z and $I^2 = I$} where I is the neutrosophic element or the indeterminancy introduced by [7]. For more info, please refer to [7].

Now, we will see how elements of ⟨Z ∪ I⟩ behave when we try to apply the modified form of Collatz conjecture.

The modified formula for Collatz conjecture for neutrosophic numbers n = a + bI is (3a + 1) + (3b + 1)I; if a = 0 then 3bI + I = (3b + 1)I is taken if b = 0 then 3a + 1 term is taken, however iteration is taken the same number of times for a and bI in n = a + bI.

If n ∈ ⟨Z ∪ I⟩ is of the form n = a, a ∈ Z then Collatz conjecture is the same, when n = aI, a ∈ I, $I^2 = I$ then also the Collatz conjecture takes the value I; for we say aI is even if a is even and aI is odd is a is odd.

For 3I, 9I, 27I, 15I, 45I, 19I, 35I, 47I, 105I, 101I, 125I are all odd neutrosophic numbers.

Now 12I, 16I, 248I, 256I etc. are even neutrosophic numbers.

The working is instead of adding 1 after multiplying with 3 we add I after multiplying with 3.

For instance consider n = 12I, the sequence for n = 12I is as follows:

12I, 6I, 3I, 3 × 3I + I = 10I, 5I, 16I, 8I, 4I, 2I, I.

So the element n = 12I has a sequence which terminates at I.

Consider n = 256I, the sequence is 256I, 128I, 64I, 32I, 16I, 8I, 4I, 2I, I so converges to I.

Take n = 31I, 31I is odd so the sequence for n = 31I is

31I, 94I, 47I, 142I, 71I, 214I, 107I, 322I, 161I, 484I, 242I, 121I, 364I, 182I, 91I, 274I, 137I, 412I, 206I, 103I, 310I, 155I, 466I, 233I, 700I, 350I, 175I, 526I, 263I, 790I, 385I, 1156I, 578I, 289I, 868I, 434I, 217I, 652I, 326I, 163I, 490I, 245I, 736I, 368I, 184I, 92I, 46I, 23I, 70I, 35I, 106I, 53I, 160I, 80I, 40I, 20I, 10I, 5I, 16I, 8I, 4I, 2I, I.

Let n = 45I the sequence is 45I, 136I, 68I, 34I, 17I, 52I, 26I, 13I, 40I, 20I, 10I, 5I, 16I, 8I, 4I, 2I, I.

So if n ∈ Z then as usual by the Collatz conjecture the sequence converges to 1. If n ∈ ZI then by applying the Collatz conjecture it converges to I. Now if x ∈ ⟨Z ∪ I⟩ that is x = a + bI how does x converge.

We will illustrate this by an example.

Now if x = a + bI, a, b ∈ Z \ {0}; is it even or odd? We cannot define or put the element x to be odd or to be even. Thus to apply Collatz conjecture one is forced to define in a very different way. We apply the Collatz conjecture separately for a and for bI, but maintain the number of iterations to be the same as for that of a + bI. We will illustrate this situation by some examples.

Consider n = 3I + 14 ∈ ⟨Z ∪ I⟩. n is neither odd nor even. We use (3a + 1) + (3b + 1)I formula in the following way

3I + 14, 10I + 7, 5I + 22, 16I + 11, 8I + 34, 4I + 17, 2I + 52, I + 26, 4I + 13, 2I + 40, I + 20, 4I + 10, 2I + 5, I + 16, 4I + 8, 2I + 4, I + 2, 4I + 1, 2I + 4, I + 2, 4I +1, I + 4, I + 2.

So the sequence terminates at I + 2.

Consider n = 3I – 14 ∈ ⟨Z ∪ I⟩, n is neither even nor odd.

The sequence for this n is as follows.

3I – 14, 10I – 7, 5I – 20, 16I – 10, 8I – 5, 4I – 14, 2I – 7, I – 20, 4I – 10, 2I – 5, I – 14, 4I – 7, 2I – 20, I – 10, 4I – 5, 2I – 14, I – 7, 4I – 20, 2I – 10, I – 5, 4I – 14, 2I – 7, I – 20, 4I – 10, 2I – 5, ... , I – 5.

So for n = 3I – 14 the sequence converges to 2I – 5.

Consider n = – 5I – 34; – 5I – 34, –14I –17, –7I – 50, –20I –25, –10I –74, –5I –37, –14I, –110, –7I –55, –20I – 164, –10I – 82, –5I –41, –14I –122, –7I –61, –20I –182, –10I –91, –5I – 272, –14I –136, –7I – 68, –20I – 34, –10I –17, –5I – 50, –14I –25, –7I – 74, –20I –37, –10I –110, –5I –55, –14I –164, –7I –82, –20I –41, –10I –122, –5I –61, –14I –182, –7I –91, –20I –272, –10I –136, –5I –68, –14I –34, –7I –17, –20I –50, –10I –25, –5I –74, –14I –37, –7I –110, –20I –55, –10I –164, –5I –82, –14I –41, –7I –122, –20I –61, –10I –182, –5I –91, –14I –272, –7I –136, –20I –68, –10I –34, –5I – 17. (1)

n = – 5I – 34, converges to –5I – 17.

Let n = –10I –17, –5I – 50, –14I –25, –7I –74, –20I – 37, –10I –110, –5I – 55, –14I – 164, –7I –82, –20I – 41, –10I – 122, –5I –61, –14I –182, –7I –91, –20I –272, –10I – 136, –5I – 68, –14I – 34, –7I –17, – 20I – 50, –10I –25, –5I –74, –14I –37, –7I – 110, –20I – 55, –10I – 164, –5I – 82, –14I – 41, 7I – 122, –20I – 61, –10I – 182, –5I – 91, –14I – 272, –7I –136, –20I –68, –10I – 34, –5I – 17.

Thus, by using the modified form of Collatz conjecture for neutrosophic numbers ⟨Z ∪ I⟩ we get the following collection A of numbers as the limits of finite sequences after performing the above discussed operations using the modified formula 3(a + bI) + 1 + I or (3a + 1) + (3b + 1)I; a,

$b \in Z \setminus \{0\}$ if a = 0 then (3b + 1)I formula and if b = 0 then 3a + 1 formula is used.

A = {1, –1, 0, I, –I, 1 + I, –I + 1, –1 + I, –1 – I, –17, –5, –17I, –5I, 1 + 2I, 1 – 2I, –1 – 2I, –1 + 2I, 2 – I, 2 + I, –2 – I, –2 + I, –5 + I, –5 + 2I, –5 – 17I, –5 – I, –5 – 2I, –51 + 1, –5I + 2, –5I – 2, –5I – 1, –5I – 17, –17 – I, –17 + I, –17I + 1, –17I – 1, –17 – 2I, –17 + 2I, –17I + 2, –17I – 2, 1 + 4I, 4I + 1, 4 – I, 4I – 1, –34 –5I, –17I –10, – 17 – 10I, –34I – 5, –17 – 20I, –17I – 20, – 68I – 5, – 68 – 5I, – 5I + 4, – 5 + 4I, –17 + 4I, –17I +4}.

Thus, the modified 3n + 1 Collatz conjecture for neutrosophic numbers $\langle Z \cup I\rangle$ is (3a + 1) + (3b + 1) I for n = a + bI $\in \langle Z \cup I\rangle$, a, b $\in Z \setminus \{0\}$.

If a = 0 then we use the formula (3b + 1)I and if b = 0 then use the classical Collatz conjecture formula 3a + 1. It is conjectured that using (3a + 1) + (3b + 1)I where a, b $\in Z \setminus \{0\}$ or 3a + 1 if b = 0 or (3b + 1)I if a = 0, formula every n $\in \langle Z \cup I\rangle$ ends after a finite number of iterations to one and only one of the 55 elements from the set A given above. Prove or disprove.

Now the 3n – 1 conjecture for neutrosophic numbers $\langle Z \cup I\rangle$ reads as (3a – 1) + (3bI – I) where n = a + bI; a, b $\in Z \setminus \{0\}$; if a = 0 then (3b – 1)I = 3bI – I is used instead of 3n – 1 or (3a – 1) + (3b – 1) I.

If b = 0 then 3a – 1 that is formula 3n – 1 is used.

Now every n $\in \langle Z \cup I\rangle$ the sequence converges to using the modified 3n – 1 Collatz conjecture (3a – 1) + (3b – 1)I to one of the elements in the set B; where

B = {1, 0, –1, I, 5I, 5, 17, 17I, –I, 1 + 2I, 1 – 2I, –1 + 2I, –1 –2I, 1 + I, I – 2, I + 2, –I –2, –I + 2, I – 1, –I – 1, 5 + I, 5 – I, 5 – 2I, 5 + 2I, –I + 1, 5 + 17I, 17 – I, 17 + I, 17 – 2I, 17 + 2I, 17 + 5I, 5I – 1, 5I – 2, 5I + 1, 5I + 2, 17I – 1, 17I – 2, 17I + 1, 17I + 2, 17 + 10I, 17I + 10, 34 + 5I, 34I + 5, 17 + 20I, 20 + 17I, 68 + 5I, 68I + 5, 5I – 4, 5 – 4I, 17 – 4I, 17I – 4, –4I + 1, –4I – 1, –4 + I, –4 – I }.

We will just illustrate how the (3a – 1) + (3b – 1)I formula functions on $\langle Z \cup I\rangle$.

Consider 12 + 17I $\in \langle Z \cup I\rangle$ the sequence attached to it is 12 + 17I, 6 + 50I, 3 + 25I, 8 + 74I, 4 + 37I, 2 + 110I, 1 + 55I, 2 + 164I, 1 + 82I, 2 + 41I, 1 + 122I, 2 + 61I, 1 + 182I, 2 + 91I, 1 + 272I, 2 + 136I, 1 + 68I, 2 + 34I, 1 + 17I, 2 + 50I, 1 + 25I, 2 + 74I, 1 + 37I, 2 + 110I, 1 + 55I, 2 + 164I, 1 + 82I, 2 + 41I, 1 + 122I, 2 + 61I, 1 + 182I, 2 + 91I, 1 + 272I, 2 + 136I, 1 + 68I, 2 + 34I, 1 + 17I.

The sequence associated with 12 + 17I terminates at 1 + 17I.

Thus, it is conjectured that every n $\in \langle Z \cup I\rangle$ using the modified Collatz conjecture (3a – 1) + (3b – 1)I; a, b $\in Z \setminus \{0\}$ or 3a – 1 if b = 0 or (3b + 1)I if a = 0, has a finite sequence which terminates at only one of the elements from the set B.

## 3 Conclusions

In this paper, the modified form of 3n ± 1 Collatz conjecture for neutrosophic numbers $\langle Z \cup I\rangle$ is defined and described. It is defined analogously as (3a ± 1) + (3b ± 1) I where a + bI $\in \langle Z \cup I\rangle$ with a ≠ 0 and b ≠ 0.

If a = 0 the formula reduces to (3b ± 1)I and if b = 0 the formula reduces to (3a ± 1).

It is conjectured every n $\in \langle Z \cup I\rangle$ using the modified form of Collatz conjecture has a finite sequence which terminates at one and only element from the set A or B according as (3a + 1) + (3b + 1)I formula is used or (3a – 1) + (3b – 1)I formula is used respectively. Thus, when a neutrosophic number is used from $\langle Z \cup I\rangle$ the number of values to which the sequence terminates after a finite number of steps is increased from 5 in case of 3n ±1 Collatz conjecture to 55 when using (3a ± 1) + (3b ± 1)I the modified Collatz conjecture.

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