The Confirmation of Singular Causal Statements by Carnap's Inductive Logic

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Abstract
The aim of this paper is to apply inductive logic to the field that, presumably, Carnap never expected: *legal causation*. Legal causation is expressible in the form of singular causal statements; but it is distinguished from the customary concept of scientific causation, because it is *subjective*. We try to express this subjectivity within the system of inductive logic. Further, by semantic complement, we compensate a defect found in our application, to be concrete, the impossibility of two-place predicates (for causal relationship) in inductive logic.

Keywords: Carnap, inductive logic

1 Problem of legal causation

What we call "singular causal statements" in this paper are the statements of the following kind:

(1) The X's parking on C Street caused the later traffic jam.

Hart and Honoré called this type of causation "legal causation" since it sometimes develops into a legal dispute (Hart & Honoré, 1985). We might as well call it "common-sense causation," considering its wider applications (cf. Hart & Honoré, 1985, p. 9). But in this paper, we uniformly call it "legal causation."

Legal causation is differentiated from scientific causation in that there are no support, such as *general laws* in Hempel and Oppenheim's schema[1]. Thus, here arises a problem: How can we *justify* legal causation?

[1] Cf. Hempel and Oppenheim (1948, especially pp. 138–140). Herein, Hempel and Oppenheim referred to Carnap's inductive logic as well (Hempel & Oppen-

The aim of this paper is to apply Carnap's inductive logic to this justification problem. Of course, I know that this is not in accord with its public image. Besides, legal causation is seemingly outside of Carnap's interest.

It was, rather, the theorists of *probabilistic causality* who treated this problem. For example, Patrick Suppes took up the statement very similar to (1), and showed interest in Hart and Honoré's work as well (Suppes, 1970, pp. 7–8). Nevertheless, I prefer Carnap's logic to probabilistic causality[2]. Why? To begin with, let me state the reasons.

2 Why I do not favor probabilistic causality

In appearance, probabilistic causality is suitable to analyze legal causation. But, in my view, it is still defective for the analysis.

Firstly, following probabilistic causality, we must reduce causal relationship to *conditional probability*, so that cause and effect are split into the two arguments of the probability function: P(*effect*, *cause*). Herein, the connection of cause and effect is considered in a mathematical way. But, I think, we intuit something real in causal connection, and it is not reducible to any mathematical relations. For instance, if you heard a dog barking when you tapped a desk, you will perhaps think: Your tapping the desk caused the barking. In this instance, we may say, you directly intuit the connection of the two events; and this intuition is concerned with something real, not reducible to mathematical relations. We must preserve this character of causation, but regrettably, probabilistic causality disregards it[3].

Secondly, most theories of probabilistic causality are practically comparative[4]. All they can do is showing the comparison like

heim, 1948, pp. 167f.). However, their reference were exclusively made for their theory of *systematic power*, that is, the power of the deductive systematization of a universal statement $\mathcal{T}$ over the data $\mathcal{K}$ in question. But it has little to do with our present interest.

[2]In this paper, the word "probabilistic causality" means "*a theory* of probabilistic causality" as well.

[3]As we shall see, in our application of inductive logic, this character is preserved. See (5)–(ii) below.

[4]According to Salmon, there were at least three theories of probabilistic causality so far (Salmon, 1980, p. 50): Good's *theory of causal network* (Salmon, 1980, pp. 51f.), Reichenbach's *causal theory of time* (Salmon, 1980, pp. 56f.), and Sup-

$P(A_t, B_{t'}) \geq P(A_t)$. This analysis is, however, suitable for heuristics rather than for justification. Taking up Suppes' theory, for example, it narrows the class of *prima facie* causes (Suppes, 1970, p. 12) down to the *genuine* cause (Suppes, 1970, p. 24), *screening off spurious* causes (Suppes, 1970, p. 21, p. 24). This is an approach of heuristics. But we need a theory of justification now.

Even if probabilistic causality is regarded as a theory of justification, it will certainly not meet our requirements. Suppose, for example, we calculate the conditional probability of not being attacked, given that an individual was inoculated (cf. Suppes, 1970, pp. 12f.). For this calculation, probabilistic causality presupposes, in advance, the data that 749 people were not attacked within a total of 818 people; by comparison, 276 not attacked within 279 inoculated. And on the basis of this data, the calculation is made this way: $P(\textit{not attacked, the total}) = \frac{749}{818} \leq P(\textit{not attacked, inoculated}) = \frac{276}{279}$. This is how probabilistic causality concludes that inoculation is a (prima facie) cause of not being attacked.

However, our present object of study, legal causation, lacks very much this kind of objective data. Rather, the data used in it is, in most cases, *subjective*. And this subjectivity keeps away probabilistic causality from the analysis of legal causation.

3 Subjectivity of legal causation

But, why is legal causation so subjective? Where does the subjectivity come from? To clarify these points, let us consider the following scenario imaginable on the preceding example (=1).

(2) A policeman was searching for the cause of the traffic jam that occurred on C Street, which he thought was the cause of the traffic accident investigated. The accident happened at the moment when a certain driver spun into the opposite lane to avoid the traffic jam. He remembers that X had parked his car on this narrow street when he patrolled. A few weeks later, he judged: the cause of the traffic jam was the X's parking.[5]

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pes' *probabilistic theory of causality* (Salmon, 1980, pp. 59f.). Reichenbach's and Suppes' were altogether comparative (Salmon, 1980, p. 60). Still, Good tried to make a quantitative theory; but it was ignored because of its "forbidding" style.

[5]There is a possibility to treat this problem in terms of *abnormality* (Hart &

Can we think this judgement objective? Presumably, the policeman observed C Street for a long time, and then, found that even short-term parking, such as X's, sufficiently caused a traffic jam. Again, based on this observation, he was convinced: the cause of the traffic jam was the X's parking (=1). Against this judgement, however, X can protest that on another street, such as A Street, even long-term parking rarely causes a traffic jam. In any case, we can say, the justification of legal causation is *subjective.*

This subjectivity of legal causation originates from its *context.* According to Hart and Honoré, the context of legal causation is much different from that of scientific causation (Hart & Honoré, 1985, p. 24). To take an example,

(3) The sudden increase of traffic on C Street caused the later traffic jam.

We can classify this statement into scientific causation[6].

In some cases, we can be content with (3) as an explanation for the traffic jam. But in other cases, we cannot; we are tempted to ask an additional cause. This is because we cannot help attributing the harmful traffic jam to somebody else. Hart and Honoré called the contexts of this latter kind *attributive contexts* (Hart & Honoré, 1985, p. 24). And it was strictly distinguished from the former cases called *explanatory contexts* (Hart & Honoré, 1985, p. 24). It is this difference of contexts that differentiates legal causation from scientific causation, and makes legal causation *subjective.*

4 Similarity

In my opinion, inductive logic is suitable to express this subjectivity. Surely Carnap refuses this application. However, we can find a passage where he came close to our problem:

(4) Suppose X owns a house whose value is $10,000, and he wonders whether to insure his house against fire. He will then

Honoré, 1985, pp. 37–40). But I leave this possibility aside in this paper.

[6] Define "traffic jam" as "vehicles slowing down to a specific speed." Then, "the increase of traffic" prevents vehicles from passing an intersection with a traffic light smoothly once. This suffices to cause a slowdown of vehicles, and so a traffic jam. A general law is of course available in this argument.

> make his decision in view of the probability that his house will burn down during the next year. But, how can he predict it? He predicts it with regard to his knowledge *e* that contains information of previous experiences concerning *similar* houses.[7]

Here, Carnap admits that the *evidence* of inductive inference is gathered in terms of *similarity*[8]. This is true of (2) as well. In that situation, the policeman gathered the evidence in terms of similarity; concretely, he gathered the evidential events (parking) that all, similarly, occurred on C Street.

However, this choice is arbitrary. The policeman was certainly in a position to choose the street other than C. In fact, X can protest, against the policeman's judgement, that on another street, such as A, even long-term parking rarely causes a traffic jam.

The policeman may respond, against this objection of X's, that his evidence are all similar to the original case, the X's parking. But, against this response, X can further protest that the policeman's cognition of similarity is, after all, subjective.

5 Inductive logic

In my opinion, this subjectivity with a lax criterion of similarity is well expressed in inductive logic—this is the original aim of our inquiry. Let us then *design* a formal language, which constitutes, in a sense, a base of the following arguments[9].

(5) The Design of Language $\mathfrak{L}_3^2$

- (i) "ε_1," "ε_2," and "ε_3" are the individual constants in $\mathfrak{L}_3^2$.
- (ii) "__ is parking" and "__ causes a traffic jam" are the predicate constants in $\mathfrak{L}_3^2$.

[7](Cf. Carnap, 1962, p. 256, p. 263). The sentences are modified by Kaneko.

[8]Similarity is dealt with in the argument of the *inference by analogy* as well (Carnap, 1962, pp. 207–208, pp. 567–571). But the similarity concept in our argument is about individuals. In contrast, the similarity in the inference by analogy is about properties.

[9]Carnap called this step a "classification of the signs" (Carnap, 1942, p. 24; Carnap, 1962, p. 65). Again, I want the readers to take note that "$\mathfrak{L}_N^\pi$" below means the language with N individual constants and π predicates (Carnap, 1962, p. 123).

$\mathfrak{L}_3^2$ is a language of first-order logic including Davidson's logic of event (Davidson, 1967). Its informal explanation is as follows. "ε_3" is considered to be the X's parking on C Street. "ε_1" and "ε_2" are the evidential events gathered by the policeman. We can regard "ε_1" as the Y's parking on C street, and "ε_2" as the Z's parking on C street, for example. With this evidence, we can formulate the justification of the policeman's judgement (=1) as follows:

(6) $\mathfrak{c}^*((\varepsilon_3$ causes a traffic jam$), \{(\varepsilon_1$ is parking$) \wedge (\varepsilon_1$ causes a traffic jam$)\} \wedge \{(\varepsilon_2$ is parking$) \wedge (\varepsilon_2$ causes a traffic jam$)\} \wedge (\varepsilon_3$ is parking$))$

Here, "$\mathfrak{c}^*$" is a probability function peculiar to Carnap, which is called a $\mathfrak{c}$-*function*[10]. Its second argument, "$\{(\varepsilon_1$ is parking$) \wedge (\varepsilon_1$ causes a traffic jam$)\} \wedge \ldots \wedge (\varepsilon_3$ is parking$))$," expresses evidence, which is called an *individual distribution* (cf. (13) below).

How to assign concrete values to this formula is the core of the present study. Carnap's answer is this[11]:

(7) Let "s_{M}" be the number of individual constants of which molecular predicate M is predicated in evidence e. Further let "w_{M}" be the width of M, and "s" the number of individual constants observed up to then, "κ" the number of Q-predicates in $\mathfrak{L}_N^{\pi}$. Then, the probability that M is predicated of the

[10] Strictly speaking, $\mathfrak{c}^*$ is no more than one option among many $\mathfrak{c}$-functions. In *Foundations*, Carnap narrowed all of possible $\mathfrak{c}$-functions down to this one (Carnap, 1962); its definition is (7) below.

[11] We think of this definition of $\mathfrak{c}^*$ as an expression of the subjectivity of inductive logic. But some might object that the subjectivity of inductive logic is adequately expressed in λ-system:

(†) $\mathfrak{c}_{\lambda}(\mathrm{M}(\varepsilon_{s+1}), i) = \frac{s_{\mathrm{M}} + \frac{w_{\mathrm{M}}}{\kappa}\lambda}{s+\lambda}$ (Carnap, 1951, p. 33)

My answer to this objection is as follows. It is true that each person freely chooses λ's argument in (†), and we may perhaps attribute the subjectivity of inductive logic to that choice. However, the choice of λ is merely a choice of the inferential system (Laplace's system, Reichenbach's system, etc.); after the choice, however, everything works *objectively.* But our problem is why our inductive reasoning is *subjective* even after the choice. And my solution to this problem is: "Because Carnap's inductive logic is based on his possible world semantics. Considered in terms of this semantics, his inductive logic is likely regarded as the reflection of a personal view of the world." This opinion of mine is derivable only from *Foundation.* This is why now we stick to Carnap's former system, although the detailed argument is put off to another paper (Kaneko, 2010).

next individual constants ε_{s+1} iscalculated with the following formula:

$$\mathfrak{c}^*(\mathrm{M}(\varepsilon_{s+1}), e) = \frac{s_{\mathrm{M}} + w_{\mathrm{M}}}{s + \kappa} \qquad \text{(Carnap, 1962, p. 568)}$$

We must follow up the unfamiliar words herein. For the explanation, we may as well divide the formula into two components: the logical factor $\frac{w_{\mathrm{M}}}{\kappa}$ and the empirical factor $\frac{s_{\mathrm{M}}}{s}$ (Carnap, 1962, p. 568). Firstly, the explanation of "κ" in $\frac{w_{\mathrm{M}}}{\kappa}$ is provided.

(8) Only for abbreviation, we write "$\mathrm{P}_1 \wedge \mathrm{P}_2(e_1)$"[12] instead of "$\mathrm{P}_1(e_1) \wedge \mathrm{P}_2(e_1)$," for example, and call it a *molecular predicate expression.* Moreover, we can give the name "$\mathrm{M}(e_1)$" to "$\mathrm{P}_1 \wedge \mathrm{P}_2(e_1)$," for example, and call it a *molecular predicate.* (Carnap, 1962, pp. 104–105)

Here, "P_1" and "P_2" are *primitive monadic predictates*, such as "__ is parking" and "__ causes a traffic jam" in $\mathfrak{L}_3^2$. In $\mathfrak{L}_3^2$, we can form four molecular predicates:

(9) $\forall e[\mathrm{Q}_1(e) \longleftrightarrow (e \text{ is parking}) \wedge (\text{e causes a traffic jam})]$
$\forall e[\mathrm{Q}_2(e) \longleftrightarrow (e \text{ is parking}) \wedge \neg(e \text{ causes a traffic jam})]$
$\forall e[\mathrm{Q}_3(e) \longleftrightarrow \neg(e \text{ is parking}) \wedge (e \text{ causes a traffic jam})]$
$\forall e[\mathrm{Q}_4(e) \longleftrightarrow \neg(e \text{ is parking}) \wedge \neg(e \text{ causes a traffic jam})]$

These four molecular predicates "Q_1"∼"Q_4" are called *Q-predicates.* Their formal definition is as follows:

(10) The molecular predicates defined in the following way are called *Q-predicates.*

$$\forall e[\mathrm{Q}_i(e) \longleftrightarrow (\neg)\mathrm{P}_1(e) \wedge \ldots \wedge (\neg)\mathrm{P}_\pi(e)]$$
(Carnap, 1962, p. 125)

"P_1"∼"P_π" are π primitive predicates in $\mathfrak{L}_{\mathrm{N}}^{\pi}$. "$(\neg)$" stands for affirmation or negation. In general, there are 2^π Q-predicates in $\mathfrak{L}_{\mathrm{N}}^{\pi}$ (Carnap, 1962, p. 125). "κ" in $\frac{\mathrm{w_M}}{\kappa}$ expresses this number, 2^π.

Next, we take up the numerator of $\frac{\mathrm{w_M}}{\kappa}$, that is, "$\mathrm{w_M}$."

[12] This is also expressible as "$\lambda e[\mathrm{P}_1(e) \wedge \mathrm{P}_2(e)]$," using Church's lambda operator (Carnap, 1956, p. 3, p. 14).

(11) Any formula $\mathfrak{M}$(e) in $\mathfrak{L}_{\mathrm{N}}^{\pi}$ is expressed by a disjunction of Q-predicates as follows:

$$\forall e[\mathfrak{M}(e) \longleftrightarrow \mathrm{Q}_{i1}(e) \vee \mathrm{Q}_{i2}(e) \vee \ldots \vee \mathrm{Q}_{iw}(e)] \quad \text{(Carnap, 1962)}^{13}$$

By this theorem, we can substitute "$\mathrm{Q}_1(\varepsilon_3) \vee \mathrm{Q}_2(\varepsilon_3)$" for "($\varepsilon_3$ is parking)," for example. The number of Q-predicates which we substitute for formula $\mathfrak{M}$ is called the *width* of $\mathfrak{M}$ (Carnap, 1962, p. 127). It is marked with the second subscript "w" of the last disjunct in (11). "w_{M}" expresses it.

In this way, the logical factor is explained. Let us then proceed to the other factor, that is, the empirical factor.

(12) If molecular predicates $\mathrm{M}_1, \ldots, \mathrm{M}_{\mathrm{p}}$ fulfill the following conditions, then they are called *forming a division* (Carnap, 1962, pp. 107–108).

(i) $\models$ [14]$\forall e[\mathrm{M}_1(e) \vee \ldots \vee \mathrm{M}_{\mathrm{p}}(e)]$ (exhaustiveness)

(ii) For any M_i, M_j $(1 \leq i, j \leq p)$, $\models \forall e \neg[\mathrm{M}_i(e) \wedge \mathrm{M}_j(e)]$ (exclusiveness)

(iii) For no M_i $(1 \leq i \leq p)$, $\models \neg \exists e\, \mathrm{M}_i(e)$ (M_i is not logically empty)

(13) The conjunction, in the following way, stating, over s individual constants and p molecular predicates forming a division, which predicate is predicated of which individual constant is called an *individual distribution.*

$$e_k = \ulcorner \mathrm{M}_{k1}(\varepsilon_{j1}) \wedge \mathrm{M}_{k2}(\varepsilon_{j2}) \wedge \ldots \wedge \mathrm{M}_{ks}(\varepsilon_{js}) \urcorner \quad \text{(Carnap, 1962, p. 111)}^{15}$$

In $\mathfrak{L}_3^2$, one of the four Q-predicates in (9) occupies each position of "M_{k1}"∼"M_{ks}."

"s" in the empirical factor expresses the number of individual constants in this individual distribution, and "s_{M}" expresses the number of individual constants in s that exemplify M, the predicate in question, which is one of "M_{k1}"∼"M_{ks}."

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[13] The proof was made in (Kaneko, 2010, (18)).

[14] "$\models$" means "logically true" though Carnap used "$\vdash$."

[15] "$\ulcorner\ \urcorner$" is Quine's quasi-quotes. But I place legibility prior to strictness in this paper.

6 Subjectivity

Now that we obtained the minimum knowledge of inductive logic, we can proceed to the calculation of (6), that is, the confirmation of legal causation.

(14) $\mathfrak{c}^*((\varepsilon_3 \text{ causes a traffic jam}), \mathrm{Q}_1(\varepsilon_1) \wedge \mathrm{Q}_1(\varepsilon_2) \wedge (\varepsilon_3 \text{ is parking}))$

from (9)

$$= \frac{\mathfrak{c}^*(\mathrm{Q}_1(\varepsilon_3), \mathrm{Q}_1(\varepsilon_1) \wedge \mathrm{Q}_1(\varepsilon_2))}{\mathfrak{c}^*(\mathrm{Q}_1(\varepsilon_3) \vee \mathrm{Q}_2(\varepsilon_3), \mathrm{Q}_1(\varepsilon_1) \wedge \mathrm{Q}_1(\varepsilon_2))}$$

from def. of conditional probability and (11)

$$= \frac{\frac{2+1}{2+4}}{\frac{2+2}{2+4}}$$

from (7); note that both $\mathrm{Q}_1(\varepsilon_1)$ and $\mathrm{Q}_1(\varepsilon_2)$ exemplifies $\mathrm{Q}_1(e) \vee \mathrm{Q}_2(e)$

$$= \frac{3}{4}$$

In this way, we can trace the process of the policeman's judgement (=1). But I do not mean this is the actual process. My emphasis is, rather, on another point; that is, the policeman's conception over the evidence directly affected his reasoning. In other words, (14) is no more than the result of (5)[16]. To see this, let us consider another formation of language. Suppose, for example, X conceived the following formation in order to object against the policeman's judgement:

(15) The Design of Language $\mathfrak{L}^2_{3\mathrm{X}}$

- (i) "ε_0," "ε_2," and "ε_3" are the individual constants in $\mathfrak{L}^2_{3\mathrm{X}}$.
- (ii) "__ is parking" and "__ causes a traffic jam" are the predicate constants in $\mathfrak{L}^2_{3\mathrm{X}}$.

Here, "ε_2" and "ε_3" are the same as in $\mathfrak{L}^2_3$. (X concedes in this respect.) But X removes "ε_1," and instead, puts "ε_0," which means the W's parking on A Street, for example. Thereby, he protests that ε_0 did not cause any traffic jam at all.

In this language, the probability of (6) is calculated as follows:

[16] Carnap also admitted that the design of language played an important role in inductive logic (Carnap, 1962, p. 54).

(16) $\mathfrak{c}^*((\varepsilon_3 \text{ causes a traffic jam }), \mathrm{Q}_2(\varepsilon_0) \wedge \mathrm{Q}_1(\varepsilon_2) \wedge (\varepsilon_3 \text{ is parking})) = \dfrac{1}{2}$

This is how the probability of the policeman's judgement is lowered below $\frac{3}{4}$ (=14).

This comparison of $\mathfrak{L}^2_{3\mathrm{X}}$ with $\mathfrak{L}^2_3$ shows how influential the design of a language is in inductive logic. And the design is due to the person who wants to or refuses to confirm the legal causation in question. In this very respect, the subjectivity of inductive logic is brought to light.

7 The first criticism: on my subjective interpretation of inductive logic

In this way, Carnap's inductive logic gives a good framework to legal causation, which was advanced at the beginning of this paper. Let us then review this conclusion in the rest; that is, we scrutinize it from other viewpoints, especially from those of critics.

Firstly, let us take up our subjective interpretation of inductive logic. Some experts may say: "Carnap's inductive logic is concerned with the objective confirmation procedure in natural science. So your interpretation is besides the mark." However, it is relatively easy to respond to this criticism. As stated in Section 3, we have already entered an unexplored field, namely legal causation. And it is much different from the customary field of scientific causation. Thus, we may say, we have dealt with a completely new problem that Carnap never expected[17].

On the other hand, some experts on philosophy of probability may ask about the relationship between our subjective interpretation of inductive logic and Ramsey's *subjective theory.* As for this question, we can refer to Carnap's treatment of Ramsey's theory (Carnap, 1962, pp. 45–47). Therein, Carnap reduced Ramsey's subjective theory to his *logical theory.* Ramsey, in turn, admitted that probability theory is, in general, a branch of logic (Ramsey, 1926, p. 82 etc.).

[17]It is true that Canap had some ethical perspectives in his application of inductive logic to decision theory (Carnap, 1962, p. vii, pp. 252–279; Carnap, 1971); and, previously, I also followed this line (Kaneko, 2011). But now I came to think Hart and Honoré's distinction—between scientific causation and legal causation, or between explanatory context and attributive context—is more crucial for our argument.

Later, Carnap characterized his inductive logic as the pure and theoretical part of *normative decision theory* (Carnap, 1971, p. 26). Thereby, he regarded the agents following inductive logic as a kind of rational robot (Carnap, 1971, p. 17, p. 26). But, according to our analysis, even such agents cannot be perfect robots because, as we saw in Section 6, the source of their inference, the design of language, is far from mechanical objective procedures.

8 The second criticism: on my treatment of causal relation

The second criticism is against our awkward formulation of causation. In $\mathfrak{L}_3^2$ (=5), we formulated causation in the following way:

(17) (ε_3 causes a traffic jam)

This formula is composed of one individual constant "ε_3" and a one-place predicate "__ causes a traffic jam." However, causation is nothing but causal *relationship*; so its formulation must be made with a two-place predicate like "__ causes __." Nevertheless, we have hitherto persisted in the one-place predicate.

To tell the truth, Carnap admitted two-place predicates in his system (Carnap, 1962, p. 114). But the problem is that he did not develop this idea any further[18].

In my opinion, it is impossible to develop the language with two-place predicates in inductive logic. One of the reasons is that Carnap confined his arguments to the language only with one-place predicates (Carnap, 1962, pp. 123f.). Therefore, all items, such as Q-predicate (cf. 10), were defined only by one-place predicates.

The theory that lies behind inductive logic is *combinatorics* (Carnap, 1962, pp. 156f.; Carnap, 1966, p. 23). If two-place predicates are introduced to inductive logic, the number of items, such as Q-predicate, will be extravagantly large[19].

[18] In "Meaning Postulates," Carnap touched on this problem once again. But it seems to me that he did not make any significant progress (Carnap, 1956, pp. 226–229).

[19] We can see this complexity even on one-place predicates. See (Carnap, 1962, p. 139)

However, this reason is not decisive. The true reason was the lack of theory. Carnap complained that there was no "theory for relations" in inductive logic, stating in parallel to the history of deductive logic (Carnap, 1962, pp. 123–124). Although he showed an optimistic attitude to this problem (Carnap, 1966, p. 33), such a theory for relations has not been developed yet. In my opinion, it is not necessary to invent such a theory for inductive logic. Instead, the lack of theory can be complemented by *semantics*. Let me elaborate on this idea below.

9 Carnap's semantics

As we shall see in the next section, however, our semantic complement is a kind of *model-theoretic semantics*. But the relationship between Carnap's system and model-theoretic semantics is not so clear. We must hence clarify the relationship between these two theories in advance.

Regarding this problem, two points are to be noted. Firstly, Carnap presumably did not know such model-theoretic semantics as we know today. Secondly, inductive logic is also classified into Carnap's semantics. Let us begin with this second point.

Carnap's semantics has two faces. One is the face obedient to Tarski's tradition: from the definition of truth (Tarski, 1933) to a theory of meaning (a truth-conditional theory of meaning)[20]. This face appears in Carnap's earlier studies of semantics (e.g. Carnap, 1942, pp. v–55).

The other face is L-semantics. This is the field for the explication of logical concepts, such as logical truth, logical consequence, and so on. The noteworthy here is the introduction of *state-descriptions*, Carnap's peculiar notion of *possible worlds*[21]. Based on this notion,

[20] But we must note: Tarski's concept of semantics is somewhat different from a theory of meaning (Tarski, 1944, p. 345). So we must take Davidson's program into consideration when we think about "Tarski's tradition" mentioned above (Davidson, 1962, p. 23).

[21] Let me define this notion for the subsequent arguments.

(†) The conjunctions introduced, as follows, by predicating one Q-predicate of each individual constants in $\mathfrak{L}_N^\pi$ are called state-descriptions:

$$\mathfrak{Z}_\mathrm{i} = \ulcorner \mathrm{Q}_{i1}(\varepsilon_1) \wedge \mathrm{Q}_{i2}(\varepsilon_2) \wedge \ldots \wedge \mathrm{Q}_{ic}(\varepsilon_N) \qquad \text{(Carnap, 1962, p. 116)}$$

Carnap thought, we could explicate the concept of probability as well (Carnap, 1942, pp. 96–97; Carnap, 1956, p. iii). That is why our present object of study, inductive logic, is classified into L-semantics[22].

In contrast, our semantic complement in the next section is a part of Tarski's tradition. To complicate matters further, it is stated in the form of *model-theoretic semantics*. It is true Tarski opened up the modern semantics[23]; but, even so, it still seems difficult to imagine model-theoretic semantics from Carnap's peculiar style.

As for this problem, Hintikka daringly severed the connection between these two theories (Hintikka, 1973). Presumably influenced by Church's criticism (Church, 1943), Carnap moved on to the study of intensional logic in *Meaning and Necessity*. Therein, according to Hintikka, model-theoretic semantics was closest at hand to Carnap's thought (Hintikka, 1973, p. 375)[24]. Nevertheless, Carnap did not lay his hand on it. This is because the model-theoretic semantics expected of him was Kripke-style possible world semantics (Hintikka, 1973, p. 374 etc.); Carnap adhered to his syntactic formulations of possible worlds—state-descriptions, so that he did not come up with Kripke-style semantics. That was why he failed in developing his theory to model-theoretic semantics (Hintikka, 1973, pp. 374–375, pp. 377–378)[25].

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[22] Actually, Carnap regarded the $\mathfrak{c}$-function as a semantical function (Carnap, 1962, p. 164, p. 283, p. 522).

[23] As for the relationship between Tarski's argument and model-theoretic semantics, we can learn a lot from Raatikainen (2008). Therein, he indicated two points that differentiate Tarski's argument from model-theoretic semantics on the two parts of model-theoretic semantics: $\mathcal{M} = \langle \mathcal{D}, \mathcal{I} \rangle$. Regarding $\mathcal{D}$, he pointed out: Tarski's approach is possibly differentiated from the customary concept of domain in model-theoretic semantics, since Tarski seemingly considered only one fixed domain referred by an infinite sequence of objects (Raatikainen, 2008, p. 109). Regarding $\mathcal{I}$, he pointed out: $\mathcal{I}$ is possibly in contradiction with Tarski's commitment that he never presupposes any semantical concepts, since we can regard $\mathcal{I}$ as a semantical concept of designation (Raatikainen, 2008, pp. 112–113). But at the same time, in his article, a relief measure is also provided to reconcile these two theories (Raatikainen, 2008, p. 109, pp. 112–113).

[24] For example, like model-theoretic semantics, Carnap adopted *classes* as semantic values of predicate constants (Carnap, 1956, p. 19, p. 83). This idea is not found in his earlier system (Carnap, 1956, p. 166; Carnap, 1942, p. 18).

[25] Let us take an *individual concepts* as an example (Carnap, 1956, p. 41). This intensional object is considered to be a function that assigns one object in the discourse of universe $\mathcal{D}$ to each individual expression $\mathfrak{A}_i$ (e.g. "WBA boxing champion") with regard to a possible world w_j: $\mathit{Intension}(\mathfrak{A}_i, w_j) \in \mathcal{D}$ (Hintikka, 1973,

10 Semantic complement

Nevertheless, it was the same idea, state-descriptions, that enabled Carnap to complete his inductive logic. Should we then abandon model-theoretic semantics? My answer is, "No." If only L-semantics is narrowed down (not to include model-theoretic semantics in it), we may allocate model-theoretic semantics to the other face of Carnap's semantics: a truth conditional theory of meaning. This is how we may classify the following argument into the first face of Carnap's semantics, which gives a meaning to each expression of $\mathfrak{L}_3^2$ in the manner differentiated from inductive logic.

Now then, let us embark on the semantic complement[26]. Therein, we aim at proving the following conditionalized T-sentence in context γ and in model $\mathcal{M}$:

(18) ((19) and (20)) $\Longrightarrow$ [{(17) is true in $\mathcal{M}$ in γ} $\longleftrightarrow$ (21)]

(17) is the sentence in question. (19), (20) and (21) are as follows:

(19) $\exists ! e[(e \text{ is a traffic jam of C street}) \wedge (\mathrm{T}(e) \subseteq \text{D-Term}(\gamma)) \wedge (\mathrm{T}(e) < \langle now \rangle(\gamma)]$

(20) $\exists ! e[\exists ! x\{(e \text{ is parking of } x \text{ by X}) \wedge (x \text{ is a car})\} \wedge (e \text{ occurs on C street}) \wedge (\mathrm{T}(e) \subseteq \text{D-Term}(\gamma)) \wedge (\mathrm{T}(e) < \langle now \rangle(\gamma))]$

(21) $\imath e[\exists ! x\{(e \text{ is parking of } x \text{ by X}) \wedge (x \text{ is a car})\} \wedge (e \text{ occurs on C street}) \wedge (\mathrm{T}(e) \subseteq \text{D-Term}(\gamma)) \wedge (\mathrm{T}(e) < \langle now \rangle(\gamma))]$

causes $\imath e[(e \text{ is a traffic jam of C street}) \wedge (\mathrm{T}(e) \subseteq \text{D-Term}(\gamma)) \wedge (\mathrm{T}(e) < \langle now \rangle(\gamma))]$

Here, "$\imath$" is the iota operator, "T" expresses a function that assigns each event the time when it happens. "D-Term" expresses a function that assigns each context a *discourse term*[27]."γ" is an individual constant that designates the context in situation (2). "$\langle now \rangle$" is Kaplan's

p. 376). To a certain extent, Carnap had this idea (Carnap, 1956, p. 181). Nevertheless, it was not developed; as just stated, his syntactical notion of possible worlds prevented him from introducing the primitive concept of w_j.

[26] It was already examined twice: in (Kaneko, 2009) and in (Kaneko, 2011).

[27] Tense expression is always concerned with a specific length of time. Suppose, for example, you ask, "Did he lock the door?" Then, it is not likely that you intended to ask whether he had ever locked the door. Like this example, when we use tense expression, we are supposed to have a specific length of time in mind. This is nothing but the discourse term stated in the text. See (Iida, 2002, pp. 338–339).

character (Kaplan, 1989, pp. 505f., p. 548).“$(t_1 < t_2)$” means that t_1 is earlier than t_2. It is to be noted that these are not Carnap’s devices.

Let me explain (18) further. It says: “Provided that X parked his car on C Street in fact (=20) and there was a traffic jam on the street in fact (=19), then the expression ‘(ε_3 causes a traffic jam)’ (=17) actually means, ‘the X’s parking caused the traffic jam’ (=21=1).” Recall that (17) was composed of a one-place predicate “__ causes a traffic jam.” On the contrary, (21) is composed of a two-place predicate “__ causes __.” Hence, we can say, (17) actually means causal relationship represented by (21) if (18) is proved. This is the role of truth condition (18), which follows Tarski’s tradition.

Hereafter, we make use of model-theoretic semantics to prove (18). Concretely, we introduce an intended model of $\mathfrak{L}_3^2$: $\mathcal{M} = \langle \mathcal{D}, \mathcal{I} \rangle$. The statements below are parts of this model necessary for our proof:

(22) $\mathcal{I}(\text{“}\varepsilon_3\text{”}) = \imath e[\exists! x\{(e \text{ is parking of } x \text{ by X}) \wedge (x \text{ is a car})\} \wedge (e \text{ occurs on C street}) \wedge (\mathrm{T}(e) \subseteq \text{D-Term}(\gamma)) \wedge (\mathrm{T}(e) < \langle now \rangle(\gamma))]$

(23) $\mathcal{I}(\text{“\_\_ causes a traffic jam”}) = \{ e_1 \mid \forall y[(e_1 \text{ occurs on } y) \rightarrow \exists! e_2((e_2 \text{ is a traffic jam of } y) \wedge (\mathrm{T}(e_1) < \mathrm{T}(e_2)) \wedge (\mathrm{T}(e_1) \subseteq \text{D-Term}(\gamma)) \wedge (\mathrm{T}(e_2) \subseteq \text{D-Term}(\gamma)) \wedge (\mathrm{T}(e_1) < \langle now \rangle(\gamma)) \wedge (\mathrm{T}(e_2) < \langle now \rangle(\gamma)) \wedge (e_1 \text{ causes } e_2))] \} = \{\mathcal{I}(\text{“}\varepsilon_1\text{”}), \mathcal{I}(\text{“}\varepsilon_2\text{”}), \mathcal{I}(\text{“}\varepsilon_3\text{”})\}$

(24) $\mathcal{I}(\text{“\_\_ is parking”}) = \{ e \mid \exists s \exists x(e \text{ is parking of } x \text{ by } s) \} = \{\mathcal{I}(\text{“}\varepsilon_1\text{”}), \mathcal{I}(\text{“}\varepsilon_2\text{”}), \mathcal{I}(\text{“}\varepsilon_3\text{”})\}$

By the way, earlier, we made sure: the present semantics is classified into the first face of Carnap’s semantics. Indeed, these statements are translatable into the language of that semantics under the name of “extensional neutral language M_e” (Cf. Carnap, 1956, pp. 168f.). Let us see the translation as well:

(25) “ε_3” designates the X’s parking on C street.

(26) “__ causes a traffic jam” designates a cause of a traffic jam.

(27) “__ is parking” designates parking.

(25) corresponds to (22), (26) to (23), and (27) to (24), respectively.

The translation of (25) to (22) is not so problematic. If only we adopt the first-order language as M_e, we can somehow translate ordinary expression (25) into (22). Then, however, two points are to be noted: First, interpretation function $\mathcal{I}$ in (22) is the translation of the designation in (25)[28]. Second, the definite description appearing in (22) is not the expression of an individual concept (cf. note 25).

In contrast, the translation of (26) to (23) is more problematic. This is because the *class expressions*—besides, two—appear on the right side of (23). We can, however, make use of Carnap's notion of a *neutral entity* in this case (Carnap, 1956, pp. 153f.).

Let us regard "a cause of a traffic jam" in (26) as such a neutral entity. On the one hand, it is supposed to have an *intensional property*, which is expressed as the connotation on the first right side of (23). On the other hand, it is also supposed to have an extension, which is expressed as the extensional class expression on the second right side of (23).

In this way, we can interpret the two class expressions on the right side of (23) as the two aspects of one and the same neutral entity in (26). The same explanation is true of the translation of (27) to (24) as well. This is how we may say: the present model-theoretic semantics (semantic complement) is classified into the first face of Carnap's semantics.

Let us then return to the proof of (18). In this proof, firstly, we premise (19) and (20). These are factual statements. But premising factual statements in semantics is not question-begging. We can include empirical information in semantics, which was already shown in the extensional class expression in (23) and (24)[29].

On these premises (19) and (20), it suffices for the proof of (18) only to deduce its consequential part: the biconditional " '(ε_3 causes a traffic jam)' is true in $\mathcal{M}$ in $\gamma \longleftrightarrow$ (21)."

For the proof of this biconditional, we can refer to the empirical information stated in (23): $\mathcal{I}$("ε_1") $\in \mathcal{I}$("__ causes a traffic jam"). With this factual information, and from the customary definition of truth in model-theoretic semantics[30], we obtain the following:

(28) "(ε_1 causes a traffic jam)" is true in $\mathcal{M}$ in γ

Based on this fact, we can move on to the proof of $\Longleftarrow$ in the bicon-

[28] This is recognizable from the past controversy on Kripke semantics of the first-order modal logic.

[29] Cf. (Carnap, 1956, p. 70, pp. 163–164) and (Carnap, 1962, pp.126–127).

[30] "(ε_3 causes a traffic jam)" is true in $\mathcal{M}$ in $\gamma \longleftrightarrow \mathcal{I}$("$\varepsilon_1$") $\in \mathcal{I}$("__ causes a traffic jam")

ditional (in short, (28) $\longleftrightarrow$ (21)). It is clear from the definition of material conditional that the problem for the proof of $\Longleftarrow$ is whether we can obtain (28) on the assumption of (21). But we have already obtained (28) above. So $\Longleftarrow$ holds.

The direction $\Longrightarrow$ is more problematic. On the assumption of (28), can we obtain (21)? By reference to (22)∼(24), firstly, we obtain the following statement from (28), based on the customary definition of truth (cf. note 30):

(29) $(\iota e[\ldots 22 \ldots]$ occurs on C street$) \rightarrow \exists ! e_2\{(e_2$ is a traffic jam of C street$) \wedge (\mathrm{T}(\iota e[\ldots 22 \ldots]) < \mathrm{T}(e_2)) \wedge (\mathrm{T}(\iota e[\ldots 22 \ldots]) \subseteq \text{D-Term}(\gamma)) \wedge (\mathrm{T}(e_2) \subseteq \text{D-Term}(\gamma)) \wedge (\mathrm{T}(\iota e[\ldots 22 \ldots]) < \langle now\rangle(\gamma)) \wedge (\mathrm{T}(e_2) < \langle now\rangle(\gamma)) \wedge (\iota e[\ldots 22 \ldots]$ causes $e_2))\}$[31]

Here we focus on the following theorem:

(30) $\exists ! e A(e) \longleftrightarrow A(\iota e A(e))$[32]

We apply this theorem to (20) above; and from Conjunction Elimination, we obtain the following:

(31) $(\iota e[\ldots 22 \ldots]$ occurs on C Street$)$

From Modus Ponens pertaining to (31) and (29), we obtain the following:

(32) $\exists ! e_2\{(e_2$ is a traffic jam of C street$) \wedge (\mathrm{T}(\iota e[\ldots 22 \ldots]) < \mathrm{T}(e_2)) \wedge (\mathrm{T}(\iota e[\ldots 22 \ldots]) \subseteq \text{D-Term}(\gamma)) \wedge (\mathrm{T}(e_2) \subseteq \text{D-Term}(\gamma)) \wedge (\mathrm{T}(\iota e[\ldots 22 \ldots]) < \langle now\rangle(\gamma)) \wedge (\mathrm{T}(e_2) < \langle now\rangle(\gamma)) \wedge (\iota e[\ldots 22 \ldots]$ causes $e_2))\}$

Here, further, we focus on the following theorem:

(33) $(\exists ! e A(e) \wedge \exists ! e[A(e) \wedge B(e)]) \rightarrow \iota e A(e) = \iota e[A(e) \wedge B(e)]$[33]

We apply this theorem to (19) and (32); thereby, we obtain the following identity:

(34) $\iota e[\ldots 19 \ldots] = \iota e[\ldots 32 \ldots]$

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[31] "$[\ldots 22 \ldots]$" expresses the counterpart of (22). The same is true of the similar expressions below.

[32] The proof was made in (Kaneko, 2009, p. 50).

[33] The proof was made in (Kaneko, 2009, p. 51).

Again, we apply (30) to (32); and from Conjunction Elimination, we obtain this:

(35) $(\imath e[\ldots 22\ldots]$ causes $\imath e[\ldots 32\ldots])$

Finally, by the rule of substitution of identical things[34] with (35) and (34), we obtain (21) above. This is how $\Longrightarrow$ holds.

In this way, we could prove (18) in the intended model and in the proper context. Based on this, we may say, our awkward formulation (17) surely expresses the causal relationship between the X's parking and the traffic jam, which is nothing but the legal causation questioned at the beginning of this paper.

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[34] For any α and β, if $\alpha = \beta$ and $A(\alpha)$, we may infer $A(\beta)$.

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