Pluralisms: Logic, Truth and Domain-specificity

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Logicians and Philosophers have developed a large and rich array of logical systems – intuitionistic logic, relevance logics, free logics, a vast assortment of many-valued logics, multiple conclusion logics, extensions of classical logic of various types including temporal and modal logics and many others besides. These are naturally referred to as "logics" and can be considered and studied in the abstract, giving clear-cut notions of what follows from what according to the various different logics. Talk of "logic" (in the singular) often signals something more than a system considered in the abstract. Logic as the study of what follows from what, looks to concern more than what follows from what *relative to some chosen system*. Similarly for talk of, for example, the study of the logic of our language or of the world. Not all of the abstract systems are on a par and philosophers have often provided detailed arguments for regarding their chosen logic as of particular or unique significance, where many of the positions advocated in this way are incompatible. [1] How do we determine which of the candidate logics we should select?

It is natural to suppose that we should treat many of the abstract logical systems as just that – perhaps unsuccessful *candidates* for capturing the genuine logical consequence relation. The possibility of being a *logical pluralist* seems to allow us to reject none, or at least fewer of such candidates. The pluralist maintains that there is no unique consequence relation.

In this paper I will explore one type of logical pluralism. There are other logical pluralist positions that I will not cover here, and, in particular, I will put aside Beall and Restall's form of logical pluralism (2006). They advocate a form of pluralism based on capturing the notion of consequence with (GTT) – an argument is valid iff in every case in which the premises are all true, the conclusion is also true – and recognising multiple acceptable ways to spell out the notion of "case" in this definition. They maintain that this legitimates a wide-ranging logical pluralism, endorsing at the same time a wide range of apparently conflicting logics (including classical, intuitionistic and

[1] Uses of logical systems can go beyond capturing logical consequence relations. For example, it might be useful to employ a paraconsistent logic to work with databases that may contain inconsistent data. But that is not to say that there are true contradictions, even though it can be on the record that p and on the record that not-p. See Keefe 2014, footnote 13.

relevance logics). I rejected their position in an earlier paper (2014), arguing that they do not give us a way to endorse such a range of different logics. On the other hand, I have previously defended the viability of a much narrower form of pluralism, where, within some particular logical framework, we can endorse more than one different account of validity and logical consequence, because several alternatives may be equally good candidates for the right generalisation of the traditional classical account of validity once the non-classical framework is adopted. For example, in a many-valued setting, we can agree on the number of truth-values and the semantics of the connectives, but leave open whether to regard logical consequence as necessary preservation of some designated value – where there may be several acceptable choices for the designated value(s) – or as preservation of truth in such a way that, on one option, the conclusion must not be less true than the least true premise.[2] What options are there for a pluralism whose scope is between these widest and narrowest alternatives, or for a relatively wide scope based on a different framework from Beall and Restall's?

In this paper, I ask whether we should see different logical systems as appropriate for different domains (or perhaps in different contexts) and whether this would amount to a form of logical pluralism. One, though not the only, route to this type of position, is via pluralism about truth. Given that truth is central to validity, the commitment the typical truth pluralist has to different notions of truth for different domains may suggest differences regarding validity in those different domains. Indeed, as we'll see, the differences between the proposed multiple notions of truth are often of a type that is clearly significant in relation to logical features, such as whether or not a constructive notion of truth is at issue. I set out and assess these issues in the next section.

2. Truth Pluralism and Logical Pluralism

Crispin Wright, Michael Lynch and others defend an important form of pluralism about truth, which, very roughly, takes the following form. The conditions required for a sentence to count as true vary with the type of sentence, or the domain or region of discourse into which that sentence falls. For a sentence about concrete physical objects to be true is different from what it takes for a sentence about morality to be true, or an assertion that joke is funny, or perhaps a mathematical statement;

[2] See Keefe 2000, chapter 5, and on pluralism about supervaluationist notions of consequence, see Keefe 2001). Hjortland offers the helpful term "intra-theoretic pluralism" for this kind of pluralism. A different relatively modest logical pluralism is one that maintains that there is no uniquely correct choice of logical constants and so we can construct different equally good logics by selecting different sets from the putative logical constants.

some of these notions may involve correspondence, some may be anti-realist, with scope for a range of other differences.[3]

Truth pluralism has taken a variety of different forms. Sometimes the variation across domains in the properties in virtue of which sentences are true is combined with retaining a universal concept, or even property of truth, whereas a strong pluralist denies any common truth property across domains.[4] All versions are to be distinguished from a pluralism or relativism about *what is true*, where the same proposition may count as both true (relative to one thing) and false (relative to another).

Some pluralists about truth have maintained that logical pluralism follows, or at least is a very natural companion position.[5] In particular, a key difference between notions of truth for different domains concerns a difference over whether it is a constructive notion (e.g. superassertibility or superwarrant) or a more "objective" one. For example, you might think that "this is funny" cannot have a completely epistemically inaccessible truth-value, so you may be inclined to intuitionistic logic in that domain and, for example, deny that "p or not-p" must always be true, since (roughly) p and not-p may both be unknowable. Within certain scientific domains, by contrast, there may be no such constraints and there may instead be reasons to presuppose bivalence.

Lynch coins the phrase "Domain-specific Logical Pluralism" for the position at issue here. And Pedersen writes, "Logical pluralists maintain that the distinction between different kinds of domains is not merely verbal, but that it bears on inference. What inferences come out valid may vary from domain to domain".[6] This may be a misleading portrayal of most logical pluralists, for whom this does not capture the role of domains (see below); but it is nonetheless worth considering this domain-specific logical pluralism and some preliminary objections it faces.

One important general question is the following. If different logics govern different domains, what logic governs arguments that involve statements from more than one different domain? This question can be usefully addressed alongside consideration of the "mixed inference challenge" to pluralism about truth.[7] That challenge takes an argument with premises from different domains,

[3] See e.g. Lynch 2009, Wright 1992, 2013 and the volume of papers Pedersen and Wright 2013.
[4] E.g. Wright maintains that there is one concept but many properties and Lynch adopts a functionalism about truth whereby different concepts play the role.
[5] Pedersen (2014, p.262) maintains that "alethic pluralism argues that any alethic pluralism that accepts both realist and anti-realist conceptions of truth "brings on a commitment to logical pluralism" and Lynch writes: 'The alethic functionalist … is not required to endorse …logical pluralism. But it is likely that she will.' (2009, p. 104).
[6] Pedersen 2014, p.260.
[7] Tappolet 1997, Williamson 1994.

thus subject to different truth predicates, and considers what validity can amount to if it isn't necessary preservation of a single property of truth. Tappolet considers, for example, "Wet cats are funny; this cat is wet; therefore this cat is funny", where the first premise and conclusion are from a different domain with different standards of truth than the second premise.

One kind of response to the mixed inference challenge maintains that what matters for validity is necessary preservation of falling under the concept of truth, even if what it takes to fall under that concept is different for different types of sentence.[8] Tappolet complains, "why should we need the many truth predicates instead of the one that does the inferential job?" (2000, p.384): that one thing is necessarily preserved in valid arguments may seem to provide enough unity to undermine truth pluralism. But advocates have shown how truth pluralism nonetheless remains viable on most plausible forms of the view.

The logic governing a mixed inference, on this approach, would be the intersection of the logics governing each of the domains involved in the argument, for arguments valid in only some domains will fail to necessarily preserve some kind of truth in some other domain. This will often be the weakest of the logics for each of the two (or more) domains; for example, for an argument with realist and anti-realist domains, the logic would be intuitionistic.[9] If the picture is to be as general as possible and permit a wide range of different logics as applicable to different domains, including paraconsistent logics, it will not always be the case that the logic of one domain is contained within the other. If we had an argument spanning a range of domain representing the full diversity of consequence relations, the intersection may be nearly empty (compare Beall and Restall 2006, p.92.)[10]

It may be natural, with this approach, to assign particular significance to that logical consequence relation which is the intersection of the logics across all domains. Perhaps we should regard that as *the* logical consequence relation. That would be to abandon Domain-Specific Logical Pluralism. It would then be inaccurate to present the key consequence relation as the intersection across the specific consequence relations, since those latter will not after all be logical consequence relations. The stronger, apparently justified principles of reasoning within particular domains would then have

[8] See e.g. Wright 2013, p.133. Beall's many-valued approach (2000) – allowing different truth values, all designated values, for different predicates, and validity as preservation of designated value – arguably also falls into this category of approach.

[9] See Lynch 2008, p.137.

[10] Lynch 2008, p.139 responds to the objection that the first approach he considers – the logic of a mixed inference is the logic governing the weakest element of it – requires "the assumption that the logics in question can be ordered, in the sense that the stronger logics are extensions of the weaker logics" by maintaining, without further explanation that this is not "as unreasonable constraint on those logics that apply only to specific domains of inquiry". But he also considers the above objection of taking the logic to be the *intersection* of the logics governing the elements of the inference.

to count as non-logical. Lynch says, "... in domains which, according to this suggestion, nonetheless *appear* classical … bivalence must be true for some *non-logical* reason. And one might wonder what that reason might be." Suppose, then, we resist this option and uphold domain-specific logical pluralism and the different logics for different domains; there will potentially be other different logics for different mixed inferences, depending on what overlapping logics are at issue and mixed inferences in which it is the very weakest (intersection) logic at play will be rare.

In the next section, keeping Domain-specific Logical Pluralism on the table, we consider a different approach.

3. Cotnoir's algebraic approach

I turn next to a different, interesting approach, offered in Cotnoir 2013. He seeks to tackle the problem of mixed inferences while avoiding the charge of being committed to a single (possibly disjunctive) notion of truth as what is preserved in common to all the domains. He focuses on "Strong Pluralism" – pluralism about truth which is not committed to a unique property of truth, a view which he endorses and attributes to Wright.[11] The basic idea is to model the language and logic with a more complex range of truth-values than those employed in a bivalent – or even typical multi-valent – system. His values are n-tuples, where there is a place in the n-tuple for each domain. We can then represent a sentence as true in one particular domain but not others, by giving it a value of 1 in the place corresponding to the domain in which it is true, but not in the other places. Cotnoir uses this idea to go on to define an algebraic notion of validity.

Cotnoir's "admissible valuations" are the valuations satisfying certain definitions of the connectives that extend the classical definitions. In brief (for the model he largely focuses on): values in each place are either 0 or 1, and the constraints imposed by the definitions of the connectives mirror at each place of the n-tuple the classical rules for the connective, so, for example, the negation of a sentence "flips" between 0 and 1 at each place of the n-tuple and each place of the n-tuple for (A ∨ B) takes 1 at the relevant place of the n-tuples if it is 1 at that place for A or for B (or for both). He then introduces a notion of ordering, ≤, between n-tupled values, where (using v(A) for the value of A), v(A) ≤ v(B) iff v(A ∨ B) = v(B). There is a top value – with 1 in every place of the n-tuple – and a bottom value – 0 in every place; there are lots of values between which have different mixtures of 0s and 1s. These values aren't well-ordered; for example, simplifying to n=2 with v(A) = <0,1> and v(B) =

[11] Note that this denial of a unique property is compatible with accepting a unified *concept* of truth. On some conceptions of properties (e.g. concept nominalism), that would surely be enough for commitment to a single general property of truth, in which case the distinction between weak and strong pluralism would collapse.

<1,0>, we would have neither v(A) ≤ v(B) nor v(B) ≤ v(A) since v(AvB) = <1,1> which is not equal to v(A) or v(B). The logical consequence relation is then defined as follows: A is the consequence of some premises iff v(the conjunction of the premises) ≤ v(conclusion).

Cotnoir goes on to show how the logic resulting from this is classical because the framework is a Boolean Algebra.[12] The details of this argument do not matter for our purposes. We can grant that the framework he describes, given the assumptions he makes, does indeed deliver classical logic. Running a simplified illustration will show how some classically valid inferences come out valid, but not necessarily in the expected way. If v(A) = <0,1> and v(B)= <1,0>, v(A&B)=<0,0>. A,B trivially entails A&B because whether the entailment holds turns on whether the conclusion is at least as true as the conjunction of the premises, when in this case the conclusion is identical to the conjunction of the premises. So the entailment holds even though the premises are each true in some way and the conclusion has the very bottom value.

In accommodating inferences involving several domains, Cotnoir has provided a framework that has a universal logic. But this does not sit well with the position considered above, according to which logical pluralism is a natural partner for pluralism about truth given the different logical behaviour central to the differences in the truth properties (e.g. whether it is a constructive notion of truth). Cotnoir seeks to accommodate logical pluralism by generalising the framework in certain ways to be described. I shall argue that these fail and that they illustrate a more general problem in relation to pluralisms about logic and truth.

Cotnoir anticipates the following important objection to his semantic values, as used to capture the domain-specific properties of truth. On his account, a sentence, S, falling squarely into domain i and true according to the corresponding notion of truth will take 1 in the i-th place of its semantic value and zeros in all the other places while its negation will take 0 in the i-th place and 1 in all others. This fails to capture the idea that S and its negation are squarely within that one domain and thus only truth-apt in the corresponding sense. As Cotnoir summarises, “why think that the negation of a true descriptive proposition must be morally true, mathematically true etc.…?” (Cotnoir 2013, p.574.) In response, he suggests introducing a third value, ½, reflecting the status of a sentence in a domain for which it is not truth-apt. S will then take value ½ in all places except i, as will its negation. Cotnoir goes on to describe the resulting many-valued logic, which turns out to be both paracomplete and paraconsistent. For example, the Law of Excluded Middle fails, as he illustrates with an example of a

[12] Along the way to showing this, he shows that his set of values with conjunction and disjunction forms a lattice with the top value taking 1 in every place and the bottom value taking 0 in every place.

sentence taking ½ in all places of its n-tuple value, where its negation will thus take the same value, as will the disjunction of these two sentences.[13]

This development of the framework does not have the advantages advertised, however. Most importantly for our purposes, it merely shifts the universal logic from classical logic to this paraconsistent and paracomplete alternative, for this is the logic that determines whether an argument is valid or sentence is logically true – even if the premises and conclusions are from within a single domain rather than being a mixed inference. Cotnoir appears to deny this, however, and says, "One can, if one wishes, insist that propositions in some domains are always classically evaluated while allowing for non-classical domains of discourse. One merely stipulates that, at some coordinates in the n-tuples, components are always only selected from 1 and 0." (p.14). But that doesn't make for Bivalence within the domain in the desired sense that all sentences from that domain are true or false; rather all sentences *at all* are true or false in that domain. As he earlier asks, "why think that the negation of a true descriptive proposition must be morally true, mathematically true etc.?" It should be compatible with taking mathematical truth to be bivalent to deny that either "Jack is funny" or "Jack is not funny" is mathematically true. This seems to make a mockery of the idea of mathematical truth – limited to the mathematical domain – within a pluralism about truth.

Cotnoir also claims to provide a framework that can accommodate domains in which truth is constructive and intuitionistic logic holds,[14] but, I will argue, the same problem threatens. The original Boolean algebra delivers classical logic and so leaves no room for a constructive notion of truth, and thus fails to capture the notion of truth that Wright – whose position he seeks to stay close to – defends for a domain such as humour and treats as key to his central notion of superassertibility. Cotnoir's response to this kind of problem is to develop a variant on the framework described above which validates intuitionistic logic. Whereas the original classical version was based on a Boolean algebra, an intuitionistic version is based on a Heyting Algebra. The resulting consequence relation, $|=_I$, is intuitionistic.

[13] This illustration is an odd case of a sentence that doesn't fall into any domain. The Law of Excluded Middle also fails for more typical sentences within some domain, as the first disjunct, and thus the second disjunct too, will typically take value ½ in some of the places and thus the disjunction will not take the top value (1 in all places). The only cases where it will take the top value is when the disjuncts have 1 or 0 in each place, which will be the rare cases of sentences simultaneously in all domains, e.g. perhaps itself a disjunction with a disjunct from each domain.

[14] "In this section, I extend the algebraic account of validity to non-classical domains, showing how the account can handle domains for which paracomplete, paraconsistent, and intuitionistic logic seem most appropriate" (p.13)

The mechanics of the system and the construction of semantic values is somewhat different in this case from the previous versions. In the classical version, the n-tuple consists of 1s and 0s, where the third possibility of ½ is added in the many-valued variant. The talk of the Boolean Algebra is then introduced at the level of the behaviour of the resulting many-place values. This is how Cotnoir gets quickly to the conclusion that classical logic (captured by that Boolean Algebra) holds for mixed inference, i.e. allowing for the full range of n-tuples that could be assigned. In the intuitionistic alternative, we are told that the ith place of the n-tuple are members of the ith Heyting Algebra, where for some domains, this could be the special case of a Boolean Algebra. This suggests an approach to the logic of a single domain that is determined not by the n-tuple values of the corresponding sentences, but only the component corresponding to that domain. I will argue that this approach is problematic.

Note, first, a complication. The Heyting Algebra framework encompasses Boolean Algebras and thus Classical Logic as a special case. Standard many-valued domains do not fit this framework, so to also make room for a many-valued domain, a more general, weaker type of algebra is needed. I will not go into the details here.

To assess the viability of the suggested solution to allowing a classical domain alongside intuitionistic ones (and other domains, when further generalised), it will be useful to ask the following question: what would it be for an argument to be strictly within one domain alone? There are general issues for pluralism about truth arising from the specification of domains, and I won't enter into those here. A natural informal reply is that the sentences within domain i are truth-apt within that domain only, so only receive a value on that co-ordinate. But that won't fit within Cotnoir's framework, where each sentence has an n-tupled value, so must have *some* value in place k even if it isn't truth-apt within that domain (where, on the original version it would be 0 and it would be ½ on the many-valued version). Cotnoir says, "As in the semantics prior, 1 in the *i*th place represents that the proposition is in domain *i* and has the property $true_i$. Likewise for 0 in the *i*th place and $false_i$. To see why this won't do, suppose m is a domain respecting classical logic, then H_m, the corresponding Heyting Algebra will be a Boolean Algebra with only the values 0 and 1. But in that case, every sentence will take 0 or 1 in the mth place and, by this definition, count as in the domain.[15] E.g. if m is the mathematical domain, moral statements would not be $true_m$ and so would take 0 in the mth value and thus count as part of that domain after all. This illustrates a general problem regarding

[15] Might it be represented instead by alternative Boolean Algebra, so that the logic is still classical, even though Bivalence does not hold? If Bivalence is a logical feature of a domain (as Lynch argues, 2008) then surely only the two-valued Boolean Algebra will be able to capture the domain.

assignment of any kind of semantic value in relation to a particular truth predicate when the sentence is not truth-apt in that respect.

Perhaps we could take a different approach to understanding what it is to be in a particular domain, where this cannot be read off from the semantic value assigned. If we had a classification of sentences – maybe just atomic sentences – into domains that was independent of the assignment of the semantic values, then we could allow sentences that were assigned 1 or 0 in the mth-placed value, while not being part of the m-th domain. Such a position muddies the waters with regards to the previously fairly appealing idea of being truth_m-apt (i.e. apt for propositions within the mth domain). For a sentence from outside domain m could be true_m (taking value 1 in the mth place). Being true_m, it may naturally be counted as truth_m-apt despite the pluralist's typical presentation of truth_m-aptness as reserved for sentences in domain m. If, on the other hand, it counts as neither true_m nor truth_m-apt, despite receiving value 1 at the corresponding place of the semantic value, then that renders those values mystifying, standing in, as they do, sometimes for truth of the appropriate kind and sometimes for something else entirely. But to focus specifically on validity within a particular domain, an argument would then count as valid within that domain – valid_m – iff necessarily if the premises and conclusion are all within the domain and the premises are true, so is the conclusion. If we preserved Cotnoir's framework in conjunction with this, the very same argument could count as valid_m but not valid_{SP}, i.e. according to the Strong Pluralist's notion of validity used for mixed inferences. Whether an argument is valid_m would depend on its content not just its form; we cannot straightforwardly say that not-not-p so p is valid_m as it is only valid_m if p is from the right domain. This consequence and the general structure outlined here is not dependent on the Cotnoir framework, so I will return to it below.

To summarise. I have found Cotnoir's framework to be unsuccessful and suggest one moral is that you shouldn't try to capture something not being truth-apt with a semantic value at all. The algebraic approach requires us to make sense of question whether a truth predicate applies outside its domain: some value has to be assigned at that place. This does not respect the spirit of truth pluralism, a flaw that will be shared by other algebraic approach that model validity by considering behaviour over all domains in this way. I have examined Cotnoir's position because it provides an unusual attempt to model the logic associated with pluralism about truth in such a way as to accommodate arguments that are not purely within one domain and also to allow for different logics for different domains. I turn next to some more general problems for "Domain-Specific Logical Pluralism", whether or not that is seen as arising from pluralism about truth.

4. Domains and their logics

As we have seen, "Domain-specific Logical Pluralism" (DLP) is the kind of logical pluralism that we might expect to be combined with pluralism about truth given the key role for different domains for truth pluralism. This form of logical pluralism could also be defended without commitment to pluralism about truth, however. For you could think that different logics govern different domains even though it is the same, univocal truth property which is involved: in different domains, different rules for necessary truth-preservation may apply. This kind of position might be appealing when we consider the range of different phenomena that have prompted philosophers to seek non-classical logics, and we reflect on the difficulties of trying to confront them all at once. Consider, for example, future contingents, vagueness, discourse involving empty names, quantum behaviour and statements about what is funny: these have all been prompts for adopting a non-classical logic of some type. The debates on these individual issues are messy enough on their own and it looks desirable to avoid having to accommodate all these phenomena at once within the single all-encompassing logic. DLP seems to offer us a way to carve off the different problems and treat them individually. This could also vindicate the common practice of ignoring some problematic phenomena – e.g. ignoring vagueness – for if we isolate off a particular domain, we don't need to worry about the logic specific to that domain when considering arguments outside it.

The idea of different logics for different domains is often ignored in debates about logical pluralism that aren't linked in to pluralism about truth.[16] Field, for example, argues that the most interesting versions of logical pluralism will maintain that several "all-purpose logics" can all be right and, as such, don't really disagree, despite appearances. The idea of an "all-purpose logic", as encapsulated in its name, is a logic applicable across all domains and all contexts and in the light of all messy phenomena. As Field emphasises, a defender of a particular all-purpose logic may have some role for other logics without them capturing logical consequence, perhaps because extra premises hold. For example, he explains how Putnam's quantum logic could fit the bill and yet in dealing with macroscopic objects, classical logic still be appropriately used because it holds to such a high degree of approximation in such contexts.[17] I agree that this kind of usability of a logic in a given domain –

[16] Domain-specific logical pluralism is not the kind of position Beall and Restall are interested in, for example. They would, I think, regard it as a form of relativism, since what is valid is relative to the domain/context. Beall and Restall's logical pluralism turns on the different interpretations of case in the defining principle of logical consequence, (GTT) "An argument is valid_x if and only if, in every case_x in which the premises are true, so is the conclusion." Pedersen (2014) seeks to adopt their framework by explaining how different notions of case could be appropriate to different domains. But (GTT) commands a universality in its biconditional, and limiting the notion of case to a single domain does not provide a "precisification" of "case" in the manner required for Beall and Restall's framework.

[17] Field 2009, p.344.

an enhanced consequence relation – can be explained in ways that are compatible with commitment to a logical monism involving a different (all-purpose) logic: that two different logic "apply" in this kind of way in two different domains does not demonstrate the rightness or viability of DLP and we need this to be reflected in the characterisation of DLP. But, even if a logical pluralism advocating several different all-purpose logics would be particularly striking, Domain-Specific Logical Pluralism could still be a very interesting position. If the one and only logic for some domain is different from that of another domain (so that in each case it is not merely that a different logic can be used because it makes for a good approximation or because we can reasonably make certain additional assumptions), that will show that there can be no all-purpose-logic, which would be a very significant conclusion worthy of the title of "Logical Pluralism".

A bundle of central problems for domain-specific logical pluralism surround the individuation of domains which I will not discuss in any detail here.[18] There are general problems in marking out domains. Most arguments concerning moral matters will involve factual (non-moral) claims: should those claims thereby fall into the moral domain after all? It leaves us with even less grasp of the idea of such a domain if "this cat is wet" in Tappolet's example above is counted as falling in the moral domain. The alternative of regarding those arguments as cross-domain and thus not necessarily subject to the logical laws of morality threatens to leave very few arguments within the moral domain.

Whereas domains such as mathematics, morality and perhaps humour might be thought to be individuated by their subject-matter, logically challenging phenomena such as vagueness, future contingents and empty names surely aren't unified by their subject-matter. Vague language can be about anything, for example, so the phenomenon and its logical repercussions cuts across several domains. Moreover, staying outside the domain of vagueness would need to be a matter of more than just avoiding sentences that are actually borderline – e.g. sticking to premises and conclusions that happen to be definitely true or false – as assessment of validity requires consideration of all possible valuations and a clear non-borderline sentence is often *possibly* vague, thereby impacting on the logic of the argument. So, we should not assume that the subject-matter of an argument can always determine the right logic, as phenomena such as vagueness can also impact on the argument while cutting across subject-matters. And the logical relevance of a phenomenon such as vagueness is even harder to isolate (in the absence of a specific subject-matter), since the possible vagueness of a non-borderline sentence is still relevant to the assessment of validity.

[18] Issues about domains are addressed in Lynch 2009 and Wyatt 2013, among many other places.

Even if we found a way to isolate sentences within particular domains, we would again face the problem that typical arguments involving those sentences also involve sentences from outside those domains. Might there then be another logic governing cross-domain arguments? We have seen above that treatments of "mixed inferences" can offer a logic that differs from the logics of other domains.[19] Should we then call this a new domain – the domain of mixed inferences – allowing that sentences are in this domain as well as in their more exclusive domain? After all, the idea of DLP was that different logics could be upheld because they were limited to different domains, so a mixed-inference logic suggests a mixed-inference domain. If so, that would be an extreme example of a domain with no unity of subject-matter. If that domain contains all sentences – since all sentences could be part of some mixed inference in combination with sentences from any other domain – then it looks to yield a universal logic. Other logics will then just look like the kind of enhanced consequence relations described above.[20] In short, we need a logic of mixed inferences since arguments needn't remain within a single pure domain. But such a logic threatens to undermine the pluralism of the other so-called consequence relations since it has a universality.

We can frame this as we did earlier (section 3). Take an argument, A, B so C, whose premises and conclusion fall within domain m. We then assess it as valid_m since it is validated by the enhanced rules specific to that domain. But the argument is also in the broader domain, SP, and that same argument is not valid_{SP}, since not validated by the weaker rules.[21] If we regard both as genuine logical consequence relations, we have a logical pluralism beyond Domain-Specific Logical Pluralism, since the very same argument is both valid and not valid. The earlier discussion suggests the natural move of identifying genuine validity with the weaker consequence relation. Lynch, however, suggests that we should regard A, B so C as valid simpliciter because there is a legitimate notion of validity with respect to which it is valid. If truth_m is the notion of truth at issue in relation to A, B and C, why not take that argument to be valid simpliciter if it necessarily preserves that property of truth? I respond that the notion of truth relevant to assessing the argument may not be settled relative to the narrowest interpretation of the argument itself. Even if all the premises and conclusions are from within a particular domain, we may want to consider whether adding an extra

[19] The arguments below do not require that the logic of mixed inferences is distinct from all of the logics of the pure domains – it could, rather, be the weakest, e.g. intuitionistic logic if the domain-specific options are just classical logic and intuitionistic logic. The same questions arise as to the status of the stronger logics.
[20] Perhaps, then, we should deny that there is any mixed *domain* – the domain-specific logical pluralist could consider the logic governing mixed inferences (the intersection of the logics of the pure domains) without requiring a corresponding *domain*. Different logics may then result from different domains either by being logics of those different domains or resulting from the interaction of different domains in a more complex way. Whether there is a mixed *domain* or not, the argument regarding the universality of the logic of mixed domains still holds.
[21] Or, if SP is not strictly a *domain*, then the assessment via the most general logic still applies.

premise to that argument would preserve validity, and since the extra premise could be from outside the domain, we should think of a broader notion of validity as key. Moreover, if the argument is of a *form* that is not generally necessarily truth-preserving, then this may be enough to prevent it qualifying as a valid argument.

There will be principles that hold in a domain – e.g. bivalence – that look logical but must be taken to hold for some other reason. Lynch objects that bivalence would then hold in some realm for some *non-logical* reason and it isn't clear what kind of reason that could be. But there can be a range of very general features of a domain that needn't be logical. They can be metaphysical or, for example, reflect the fact that there is no scope of vagueness within that domain. Not all arguments that necessarily preserve truth can count as genuine logical consequence relations. "Jon is a bachelor, therefore Jon is unmarried" necessarily preserves truth but there is no space to reflect this in the true logical consequence relation. It is typically recognised that the logical consequence relation must be *formal* and an argument such as this is surely not valid "in virtue of form", but rather because of the special meaning of the terms "bachelor" and "unmarried". Even if integral to the notion of truth relevant to the domain, principles that hold in a domain in virtue of features specific to that domain threaten to fail tests of formality.[22]

Finally, I turn to a related, more general, simple worry about Domain Specific Logical Pluralism. One of the central features of logic is often taken to be its topic-neutrality. A logic that is confined to a single domain cannot have that neutrality. Although that objection may seem simplistic (can't we just reject that characterisation of logic as wrong?), it ties in with the worries about pinning down domains. For an argument to be valid, it must be the case that on no interpretation of the non-logical vocabulary are the premises all true and the conclusion false. Within DLP, there must be a restriction to interpretations within the domain; but if the domain is not clearly demarcated, the test cannot be adequately applied. Suppose, for example, that we hope to put aside vagueness and focus on the "precise" domain. Any sentence could be vague, at least as far as its structure is concerned. So, even if the premises are precise, the consideration of all possible interpretations will include some vague ones. We thus would need to limit the interpretations quantified over for the test to the precise ones. But then we can only reach a conclusion about what follows from what on the assumption of necessary precision of the relevant elements of the argument. Since this is clearly a false assumption, the suggestion that this captures the genuine consequence relation for that

[22] Beall and Restall (2006), for example, identify the settled core features of logical consequence relations as necessity, normativity and formality: to qualify as one of the genuine consequence relations of their logical pluralism, a relation must have these features. Although details of Beall and Restall's "settled core" are controversial, a requirement of formality is widely accepted and the domain-specific relations in question meet none of the candidate more detailed specifications of the formality criterion.

domain looks questionable. When the domains are demarcated by subject-matter (e.g. morality or humour), then the limitation to interpretations within the domain may be less problematic than the previous case. But this will still only do if we can isolate the sentences in that domain in such a way as to encompass sentences involved in typical arguments concerning that subject-matter as indicated above, this is often not easy. Topic-neutrality allows for the kind of generality required.

I have argued that the claim that the right logic varies across different domains is not sustainable, whether in a version stemming from pluralism about truth or not. It is still open to the logical pluralist to reject topic neutrality as a requirement on logic,[23] but I put aside that option and turn, in the next section, to consider a model whereby different rules of reasoning are permitted in different domains or contexts, where this can be superimposed over a universal (and topic-neutral) logic.

5. Relative validity, contexts and domains

This section considers a treatment of arguments from different domains that adopts the suggestion that different rules of reasoning are called upon and justified in different contexts. Timothy Smiley offers an account of *relative validity* which tells an appealing story about formalising arguments with suppressed *rules*, according to which arguments are modelled as valid relative to rules that are justified in the context.[24] I will suggest that this account may be of use in tackling issues considered above.

Just as there are contexts when a reasoner is justified in assuming something which can act as a suppressed premise of their argument, so a reasoner can sometimes be justified in using a specific rule which – though not strictly valid – needs no further justification in the context. Smiley emphasises that a formalisation of someone's everyday argument as involving a suppressed rule can be better than one diagnosing a suppressed premise. The push to do the latter instead can have the artificiality of traditional logician's attempt to regard arguments as disguised syllogisms. The contextually-justified rule can, he maintains, be a casual ad hoc one specific to the particular circumstances (e.g. "it's Tuesday, so this is Paris" – justified by a timetable) or it can be a formal system of axioms or rules. We can consider the rule employed as part of system R, which is reasonably assumed in the context, and endorse the argument, in the context, because it is *valid relative to system R*. If R contains a rule that is not itself necessarily truth-preserving, then an argument can be valid relative to R, without being strictly valid.

[23] See e.g. Shapiro 2014 for a view taking this line, focusing on a range of fruitful mathematical theories that use different logics (e.g. intuitionistic analysis).
[24] Smiley 1995. See also Keefe 2010 for further discussion of Smiley's account of relative validity.

The key notions are spelled out as follows. The premises of an argument imply the conclusion by R – i.e. the argument is valid relative to R – iff there is no way to falsify that inference without falsifying R. An inference counts as being falsified if there is an assignment of Ts and Fs to sentences such that the premises are assigned T and the conclusion is assigned F. A rule is falsified if some instance of it is falsified. When considering what can be falsified, "the only restriction [on the assignment of Ts and Fs] is that all occurrences of the same sentence should be assigned the same truth-value" (Smiley 1995, p.730).

The strategies for assessing an argument explicitly stated as merely "A, so B" are then the *Suppressed premise strategy* (SPS) and the *Suppressed rule strategy* (SRS). SPS requires finding a missing premise, P, which is true, needs no further justification in the circumstances and is such that "A, P; so B" is valid. SRS involves finding a rule, R, which is truth-preserving, needs no further justification in the circumstances and which results in "A; so B", being valid by R. (It is worth noting that system R includes rules *and axioms*, so SPS is a special case of SRS.) As an example, take "Alan is taller than Benji, Benji is taller than Carmine; so Alan is taller than Carmine". SRS detects rule "from X is taller than Y and Y is taller than Z, infer X is taller than Z".SPS needs to call upon, e.g., "for all X, Y and Z, if X is taller than Y and Y is taller than Z, then X is taller than Z" and use the rule of universal instantiation and modus ponens. Smiley claims that "the rule strategy thus takes the argument as it comes" (p.731) and can result in a formalisation that may seem closer to the original intentions of the subject.

Here are three ways that relative validity and formal validity can come apart.

1) Argument is relatively valid but falls short of being absolutely valid.

The argument uses a rule which is justified in the circumstances, but not absolutely valid. Smiley's case involving "it's Tuesday, so this is Paris" is an example of this that is dependent on a casual rule justified in quite specific circumstances. As we will explore below, this category of arguments can include cases where rules are justified given the domain over which we are operating.

2) Argument is relatively valid and absolutely valid (i.e. necessarily truth-preserving), but not formally valid (by the chosen system).

The "taller than" case above is an example of this.

3) The argument involves a notion of relative validity that is stronger than the notion of validity assumed by the classical logician.

This is a possibility given the definition of falsifying a rule – restricted only by the requirement that all occurrences of the same sentence are assigned the same truth-value – such that for some

instance of the rule, the premises are true and the conclusion false. This allows the falsification of, for example, “A; so A or B”, and so there could be systems relative to which that rule is not valid. So, for example, we can consider validity relative to a relevance logic and there may be contexts in which these stricter conditions are justified.

Should we take this to show that the genuine notion of logical consequence is weaker than classical logic and, indeed, is a minimal consequence relation, weaker than most standard alternatives? The natural approach here is to identify this with the consequence relation that endorses only the minimal logic common to all contexts (compare the discussion above).

To settle the correct consequence relation, however, we would need to determine what rules *are* common to all contexts. Someone convinced that Classical Logic is the true logical consequence relation might argue, for example, that Reductio Ad Absurdum *is* always justified, even if it is disputed. Conversational participants may together have some control over what rules are added because justified in the context – e.g. the timetable rule – without being able to exclude others. Arguably, however, there are contexts in which classical logic is too strong – it is said, for example, that reasoning about an inconsistent database requires rules of a relevance logic that block explosion, so that it is not the case that the recording of contradictory facts on the database warrants inferring any conclusion at all. Or take another application of a relevant logic: Lewis argues that a paraconsistent logic is a suitable “logic for equivocators”.[25] If we cannot rule out that A is ambiguous across the premises not-A and A or B, then we should not infer B; rather the rules of a paraconsistent logic are better placed to guide our reasoning if we cannot be sure we are avoiding equivocation. The classicist can respond by explaining how these cases do not show that there is any argument genuinely of the classical form in question (e.g. disjunctive syllogism or reductio) that fails to be valid. For example, the database argument really needs its premises and conclusion appended with “according to the database”, so that what looks like P and not-P are actually the non-contradictory “according to the database P” and “according to the database not-P”. Similarly, corresponding to the potentially equivocating English argument, there are several different propositional contents (corresponding to the various disambiguations) and none of those invalidate disjunctive syllogism.

The alternative position which does identify logical consequence with the very weakest common core of all candidate logics, may seem truer to the spirit of the approach to relative validity as explained above, for example, in the attempt to “take the argument as it comes”. I will not here try to settle the choice between these options (and others) – it would require tackling much-disputed

[25] Lewis 1998.

questions about the essential nature of logical consequence. I have instead illustrated how Smiley's account of relative validity may allow for a flexibility of theoretical position with which it is combined, as well as flexibility for the everyday reasoner in what counts as valid relative to the context they are in. We may thus see as independent the question of what the true logical consequence relation is, or, indeed, what the true logical consequence relations (plural) are, since logical pluralism may also remain a live option. For the approach may be compatible with a form of logical pluralism that is not domain-specific: if there is no uniquely correct logical consequence relation, we may consider this story about relative validity superimposed over different logics, where the latter variation is not a matter of variation between contexts or over different domains.

Let us return to our earlier issues of truth pluralism and domain-specific logical pluralism in the light of this framework. The truth-pluralist emphasised different features of truth in different domains, where many of these appeared logical in character. For example, in a domain where a constructive notion of truth is needed, the rule of Reductio Ad Absurdum is not justified, whereas in other domains it is justified. In those latter domains, we can take the justified system R to be one that includes the additional rule of Reductio and arguments can be justifiably formalised as involving that rule and thus as valid relative to R. We could then accommodate the difference in appropriate reasoning in different contexts. The contexts here can be determined by the domains under consideration. A context purely dealing with mathematics justifies rules applicable to that domain, whereas a context concerned with attributions of humour does not. A mixed inference will, in effect, combine both domains, thus undermining the justification of any rules that are only valid in one of them.

The truth pluralist talks of different *domains*, whereas this approach to relative validity focuses on different *contexts*. But we can see the latter as more general by considering the domains relevant to a context. A single domain, say mathematics, may be at issue in some context. This can mean that the evaluation of the same argument (same premises, conclusions and relevant domains), can be relatively valid in one context and not in another – it is not dictated purely by the domain in which the premises and conclusion are situated, if statements from other domains may be relevant. The focus on contexts rather than domains is also more suitable for dealing with phenomena such as vagueness and the thought that in many contexts it is appropriate to ignore vagueness and reason with rules that are legitimate in the absence of vagueness.

We can then allow that the same (weak) underlying logic governs all domains, strictly speaking, while granting that additional rules are warranted within a specific domain. Even if we maintain that these rules do not count as logical, since not reflective of absolute validity, they share many features

of logical rules and play a comparable role. They appear a long way down the continuum from the ad hoc timetable rule "it's Tuesday so this is Paris". In reasoning and in evaluating arguments it is often relative validity that is paramount, and that is most appropriate for formalising arguments.

Regarding pluralism about truth specifically: the role of the minimal core logic here mirrors the approach to the mixed inference challenge that declares an argument valid if it preserves falling under the concept of truth (allowing for different properties of truth for different domains). The truth pluralist would thus then need a response to the standard objection that truth should then be identified with that property that is necessarily preserved in validity, but if such a response can be given the framework of relative validity could offer an additional way to approach the variety of arguments involving the variety of types of truth.

The above context-based framework can thus be employed by the domain-specific logical pluralist, but that framework also allows for logical pluralism that does not involve several domains. Different contexts may demand rules of classical logic, where others only justify intuitionistic rules, even when the same domain (e.g. mathematics) is at issue.[26]

According to Lynch, the claim that "there is more than one logic governing our reasoning" is the key claim of logical pluralism (2008, p.132). The framework of relative validity allows us to accommodate the idea here without commitment to multiple logical consequence relations. For principles and rules can govern our *reasoning* without their reflecting logical consequence: other informal rules may be crucial to our reasoning in certain contexts for pragmatic reasons and stories about how to reason will involve more than just principles of logic.

References

Beall, JC (2000). "On Mixed Inferences and Pluralism about Truth Predicates", *Philosophical Quarterly,* 50: 380–82.

Beall, JC and Restall, G. (2006). *Logical Pluralism*. Oxford: Clarendon Press.

Caret, C. (2017). "The Collapse of Logical Pluralism has been Greatly Exaggerated", *Erkenntnis* 82: 739–60.

Cotnoir, A. J. (2013) "Validity for Strong Pluralists", *Philosophy and Phenomenological Research* 86: 563–79.

Field, H. (2009) "Pluralism in Logic", *Review of Symbolic Logic* 2:342–59.

[26] For other recent papers connecting logical pluralism with ideas about context, see Caret 2017 and Simard Smith 2018.

Hjortland O.T. (2012) “Logical Pluralism, Meaning-Variance, and Verbal Disputes”, *Australasian Journal of Philosophy* 91: 355–73.

Keefe R. (2000) *Theories of Vagueness*. Cambridge: Cambridge University Press.

(2001) “Supervaluationism and Validity”, *Philosophical Topics* 28: 93-105.

(2010) “Relative Validity and Vagueness”, *The Force of Argument: Essays in Honor of Timothy Smiley*. Routledge.

(2014) “What Logical Pluralism cannot be”, *Synthese* 191: 1375-1390.

Lewis, D.K. (1982) “Logic for Equivocators”, *Noûs* 16: 431–41.

Lynch, M. P. (2008) “Alethic Pluralism, Logical Consequence and the Universality of Reason”, *Midwest Studies in Philosophy* 32: 122–140.

(2009) *Truth as One and Many*. Oxford University Press.

Pedersen, N.J.L.L. and Wright C.D (2013) *Truth and Pluralism*. Oxford University Press

Pedersen, N.J.L.L. (2014) “Pluralism × 3: Truth, Logic, Metaphysics”, *Erkenntnis* 79: 259–277.

Smiley, T. J. (1995) “ A Tale of Two Tortoises”, *Mind* 104: 725–736.

Shapiro, S. (2014) *Varieties of Logic*. Oxford: Oxford University Press.

Smith, P. Simard. (2018) “Assessment Context-Sensitive Logical Claims”, *Inquiry*

Tappolet, C. (1997) ‘‘Mixed Inferences: A Problem for Pluralism about Truth Predicates’’, *Analysis* 57: 209–10;

(2000) *“*Truth Pluralism and Many-valued Logics: A Reply to Beall”, *Philosophical Quarterly*50: 382–85.

Williamson, T. (1994) “A critical study of *Truth and Objectivity*”. *International Journal of Philosophical Studies*, 30:130–144.

Wright, C.J.G. (1992) *Truth and Objectivity*. Harvard University Press.

(2013) “A Plurality of Pluralisms” in *Truth and Pluralism* eds N.J.L.L. Pedersen and C.D. Wright, Oxford University Press.

Wyatt, J. (2013) “Domains, Plural Truth and Mixed Atomic Propositions”, *Philosophical Studies* 166: 225–36