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The limits of classical mereology:

Mixed fusions and the failures of mereological hybridism

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Abstract

In this thesis I argue against unrestricted mereological hybridism, the view that there are absolutely no constraints on wholes having parts from many different logical or ontological categories, an exemplar of which I take to be 'mixed fusions'. These are composite entities which have parts from at least two different categories – the membered (as in classes) and the non-membered (as in individuals). As a result, mixed fusions can also be understood to represent a variety of cross-category summation such as the abstract with the concrete, the physical with the non-physical, and the possible with the impossible, just to name a few.

Proposed by David Lewis (1991) alongside his defence of classical mereology (the major theory of parthood which permits such transcategorial composites through its principle of unrestricted composition) it is my contention that mixed fusions are an under-examined consequence of indiscriminate mereological fusion which harbour a multitude of complications. In my attempt to discern their substantive character, throughout this thesis I make a case study of mixed fusions and uncover several problematic consequences which I think follow from their most plausible assessment.

These include: (1) that mixed fusions' probable membership relations may lead to dubious foundational loops in the mereological Universe, or (2) otherwise that mixed fusions oblige an implausible ontological priority of the mereological Universe as a whole; (3) that mixed fusions contradict the reductive account of set theory they are proposed within, by plausibly being seen to have the same members as their class parts, and (4) that mixed fusions therefore confound a mereological thesis of Composition as Identity, which some (including Lewis) use to support classical mereology – a consequence which is potentially self-defeating; (5) that mixed fusions as sums of abstract and concrete entities both subvert Lewis's (1986) system of modal realism, while (6) also undermining less expansive theories of possible worlds; and finally, (7) that even where some of the foregoing is resisted, it remains implausible that mixed fusions are ontologically innocent, because their supposed distinction from their parts in this case ensures that they need to be counted as additional entities in one's ontology.

To be clear, I do not advance a theory of mereological hybrid nihilism in the sense of denying *all* cases of transcategorial composition. (I only cover a few select instances of mereological hybridism via mixed fusions after all.) Rather, I deny that mereological hybridism is plausible in full generality, by demonstrating that any cases of it are at least limited by the constraints that I identify. This in turn vindicates a call for a restriction on parthood theories and

composition principles which allow certain types of categorially mixed entities – including restricting classical mereology with its principle of unrestricted composition.

Although theories of parthood like the standard classical mereology are not ordinarily developed for the sake of mereological hybrids like mixed fusions, these and other transcategorial composites are still among the logical consequences of such parthood systems operating with sufficient generality. The significance of my thesis, then, comes from showcasing how some of these kinds of entities do not conform to the systems in which they are included as required, and hence I argue for the rejection of unrestricted mereological hybridism as well as any mereological principles which support it.

Declaration by author

This thesis is composed of my original work, and contains no material previously published or written by another person except where due reference has been made in the text. I have clearly stated the contribution by others to jointly-authored works that I have included in my thesis.

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No publications included.

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Statement of parts of the thesis submitted to qualify for the award of another degree

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Research involving human or animal subjects

No animal or human subjects were involved in this research.

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abstract–concrete distinction, grounding, mereology, ontological innocence, possible worlds, property inheritance, set theory, transcategorial sums, the universe, unrestricted composition

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For My Mum

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List of abbreviations used in the thesis

CAI	Composition as Identity
GEM	General Extensional Mereology
MK	Morse–Kelley set theory
MKU	Morse–Kelley set theory, with individuals
NBG	Neumann-Bernays-Gödel set theory
NBGU	Neumann-Bernays-Gödel set theory, with individuals
OI	Ontological innocence
ZFC	Zermelo–Fraenkel set theory with the axiom of choice
ZFCU	Zermelo–Fraenkel set theory with the axiom of choice, with individuals

The limits of classical mereology: Mixed fusions and the failures of mereological hybridism

Chapter 0

Introducing mereological hybridism

0.0 Introduction to this whole story

Mereological hybridism is the view that some composite wholes have parts from multiple different logical or ontological categories. A mereology – or, a theory of parthood and parthood relations – is characterised by a description of relations believed to hold between parts and wholes, and also between parts of wholes. Mereologies have been developed in response to long-standing philosophical problems concerning the possible part-whole configuration of existents, as well as questions and issues surrounding the identity of objects and the structure of physical reality, among other things. Some contemporary metaphysics has similarly brought into focus an understanding of existents as belonging to special categories, either for the sake of regimenting the conceptualisation of existence at large, or because of the belief that such categories identify natural divisions in reality, and so facilitate the recognition of entities in their proper place. Mereological hybridism occurs when the applications of mereological theories operate in such a manner as to transcend category divisions, natural or otherwise, and results in wholes or sums with parts from multiple different categories. In this thesis I argue that mereological hybridism fails in a wide variety of contexts.

Consider the following example. Within the context of the classic Platonic dual ontological division into the abstract (such as numbers) and the concrete (such as material beings), a mereology may permit the formation of wholes with parts from either side of this divide. One such theory which allows for these fusions is *classical mereology*, the standard and canonical theory of parthood, which also goes by the name of *General Extensional Mereology* (hereafter 'GEM'). Reasonable questions arise when considering such transcategorial sums – are these composites themselves abstract or concrete, or something else? If such sums are meant to be either abstract or concrete, natural concerns arise over how to justify that something abstract has concrete parts (and vice versa). Or, if these sums are neither abstract nor concrete, then what are these different kinds of thing, what category do they belong to, and how does this accord with our prior conception of the abstract and the concrete as categories?

What is at stake in answering these sorts of questions in cases like these is whether the alleged frameworks and contexts which engender them are worth supporting, or if these systems ought to be reconsidered. For instance, if none of the three options in the above case of abstract and concrete hybrid fusions are plausible, then it is reasonable to suspect that there is something wrong with cross-category summation on this occasion. Indeed, on my view, the prospects of widespread mereological hybridism are dim. For the sake of answering such questions, in this thesis I examine a range of instances of mereological hybridism that all arise from a single type of claimed existent –

one which is easily recognised as able to encompass a large variety of different categories – a type of entity which David Lewis (1991) proposes and defends: 'mixed fusions'. I argue that these instances expose inconsistencies, incompatibilities, and even contradictions with mereological hybridism, and claim that the best response to these problems is to restrict composition principles which allow them.

According to Lewis, mixed fusions are the mereological sum of at least one 'class' and one 'individual'. The foremost difference between classes and individuals that make mixed fusions mereological hybrids on my view concerns the sorts of membership relations that these kinds of entities are involved in. Following Lewis's initial definitions in this regard, classes have one main feature, and are simply entities that have members (1991, 4), such as the class {Magpie, Possum} with Lewis's two cats as members. Individuals, however, are said to have two core traits, being defined by Lewis as entities which do not have any members but which are yet themselves members (1991, 4). Maintaining the usual understanding that all ordinary space-time creatures are memberless, the cats Magpie and Possum are expected to be individuals here, insofar as they are members of some class, as in the previous example. Mixed fusions, then, would be given in cases such as where the individual Magpie and the class {Magpie, Possum} together compose a whole, and their mereological hybridism would be given in their being fusions of both membered and non-membered things.

Under Lewis's system of classes, individuals and part-whole theory, the mereological hybridisation which occurs in such cases is universal. Therein Lewis attests that mixed fusions exist due to a principle of mereological composition which allows for such distinct types of entity to form a composite whole. This is a principle found in GEM – Lewis's mereology of choice – of *unrestricted composition*. In essence, unrestricted composition asserts that any plurality of entities composes a mereological unit, a whole, and they do so regardless of what kinds of entities these may be or to what categories they belong. As Lewis puts it, unrestricted composition entails that "whenever there are some things, no matter how many or how unrelated or how disparate in character they may be, they have a mereological fusion. [...] That means that if I accept individuals and I accept classes, I have to accept mereological fusions of individuals and classes." (1991, 7) This universality of composition clearly also indicates the availability of other instances of cross-category summation.

What makes mixed fusions such a good case study in this regard then is that, depending on how we understand the nature of classes and of certain kinds of individuals, mixed fusions are also able to represent many further examples of mereological hybridism besides their primary case of logical division along the lines of membered and non-membered things. For instance, because on

the standard view classes are abstracta, and abstracta are usually conceived to be non-physical, non-extended, non-spatial, as well as being arguably necessary entities, fusions of these with individuals which are either concrete (like Magpie on the ordinary conception), or physical, or extended, or spatial, or contingent (all of which concreta variously can be on standard accounts) undoubtedly leads to instances of mereological hybridism, insofar as it is the case that the assorted contrastive divisions mentioned each count as categories in their own right. There is also the example of mixed fusions being transcategorial sums of possibilia and impossibilia following Lewis's (1986) account of possible worlds as concrete maximal sums, whereby the mereological whole which any possible world comprises only has parts which are possible too. As Lewis too regards classes as abstracta (Lewis 1986, 87 and 94, 1991, 142; cf., Nolan 2002, 21 and 155), his position that no possible worlds have abstracta for parts entails that classes are thereby impossible entities (merely in the sense of not being a part of any possible worlds), which would again lead to mixed fusions being mereological hybrids, and in this instance, of possibilia and impossibilia.

As there are many complexities and intricacies surrounding this variety of potential divisions of reality into categories, in terms of how justifiable each of them are in their own right, or for the overall notion of categories altogether, along with the fact that some candidate individuals may themselves be abstracta (e.g., propositions, fictional characters, gods), among others, my objections to widespread mereological hybridism are somewhat conditional on the circumstances holding that the various potential forms of mereological hybridism would require – or at the least, I demonstrate inconsistency and incoherency for a variety of mereological pairings based on logical and conceptual grounds. Which is to say that even if it is not the case that there really are both abstracta and concreta, as one example, I at least show how certain conceptions of these as categories logically precludes there being mereological fusions of things from each of them.

In this respect, my overarching claim in this thesis is that unrestricted composition is incompatible with many standard views of particular logical and ontological category divisions, and that, under such conditions, unrestricted composition is to be rejected. In turn, this amounts to my rejection of GEM, insofar as it is taken literally to be an absolutely general and topic neutral theory and expected to be able to apply to entities from any category whatsoever with certain combinations of categories in unison. On my view, the best answers to the questions above work against the plausibility of mereological hybridism, and that thereby any sort of theories or principles which tend to support the view in specific instances, such as GEM or unrestricted composition, also face objections based upon these type of consequences they accrue. Because I think that mereological hybridism fails in many cases, and that mixed fusions suitably provide a representative example for each of these instances of failure, I make my arguments against mereological hybridism by putting

mixed fusions front and centre, although I expect that my criticisms can relevantly generalise. With mixed fusions as my main entity under scrutiny, we need a thorough account of them so as to provide a basis for their critical examination.

I also think that Lewis's discounting of much attention to mixed fusions means that he has overlooked their problematic status and their consequences which I believe undermine his reductive project and system of principles – to the point that a reduction of set theory to the theory of singleton functions, GEM and plural quantification cannot be achieved. Moreover, I think that mixed fusions serve as a good example of why unrestricted composition would fail in domains which include sets or classes, and that thereby mixed fusions stand as an exemplary threat to GEM being an absolutely general theory. This is also why I think that mixed fusions serve as an exemplary instance of the failures of mereological hybridism, in that their potential for instances of cross-category fusing overturns the frameworks which they are supposed to inhabit.

0.1 Chapter overviews

To support these claims against mereological hybridism and GEM in general, I make several arguments across each of my chapters, covering some, but not all, of the categorial divisions already mentioned, and using Lewis's mixed fusions as the main example.

0.1.1 Chapter 1

As mixed fusions serve as a case study for all of the problems that I examine with mereological hybridism, Chapter 1 is entirely exegetical. There I start with an explanation of mixed fusions following their proposed properties and relations, both implicit and explicit, from Lewis's (1991) system. In addition I present much of the conceptual apparatus required to follow the rest of the thesis.

0.1.2 Chapter 2

In Chapter 2 I argue for the correction of some flaws that I perceive in Lewis's account of mixed fusions, alongside offering some clarifying terminology to also be of assistance. Beyond that I discuss a preliminary case of mereological hybridism under the distinction of things being members of classes or not – rather than having members or not – and argue that a prohibition Lewis makes on mixed fusions themselves being members of classes is unjustifiably strong. I claim that if we follow the consequences of Lewis's framework explicitly, there is no good reason why mixed fusions in general cannot be members.

However, I also show that this being corrected leads to a highly unfavourable – but not necessarily paradoxical – instance of a foundational or grounding loop for the mereological

Universe. This is the entity which is supposed to have all other things as its parts, according to GEM and other mereologies which have such a Universe. The foundation-looping consequence is a result which, while not ultimately thwarting of mereological hybridism in this instance, is still indicative of some of its potential, and very unattractive, consequences. This looping result is also relevant for the debate in Chapter 4.

0.1.3 Chapter 3

In Chapter 3 I examine a somewhat related case of mereological hybridism as that from Chapter 2, which this time involves the counterpart to something *being* a member, that of something *having* members or not, and the categorial divide between those things which have members, and those things which do not. Here I argue that, even though mixed fusions are mereological sums of something membered and something memberless by definition, mixed fusions should not themselves have members – a claim stronger than what Lewis makes (which is just that they do not have members), and for weaker reasons than mine. Keeping Lewis's framework of standard set and class theory in mind, I make my case by showing that if mixed fusions are like classes and membered that this either leads to contradictions of the standard membership theory, or otherwise implausible revisions to what the part-whole structure of sets or classes could even be.

In addition to this, I also show that the *Composition as Identity* (CAI) thesis that Lewis (1991) famously defends – which basically asserts that wholes are 'nothing over and above' their parts – fails in the case of mixed fusions. This is because if mixed fusions are identical to their parts, as per CAI, then they have the same members that their parts do, even as they have different parts from them for being their fusion, which leads to the same contradictions of the standard membership theory as before. I argue that Lewis's position on mixed fusions having members is implicitly inconsistent because he explicitly says mixed fusions don't have members, although CAI implies otherwise, and Lewis endorses both the claim about mixed fusions and the thesis about composition. Accordingly, I show that Lewis has to give up CAI on pain of contradiction, which means he also loses a thesis he takes to support GEM, which makes my case against GEM in its facilitation of mereological hybridism in this instance also enhanced by CAI's failure.

0.1.4 Chapter 4

In Chapter 4 I debate recent work by Trogdon and Cowling (2019), one of the only other works which is strictly focussed on the nature of mixed fusions. The debate here is over the grounding status of mixed fusions, and specifically over whether mixed fusions are better understood as being grounded by their proper parts, or alternatively that they are grounders of their proper parts. Where Trogdon and Cowling argue that mixed fusions are grounded by their proper parts, I show that their presentation either leads to contradictions or results in their premises better supporting the

conclusion that they explicitly argue against, that of mixed fusions grounding their own proper parts.

Although I am not specifically interested in making a case myself for priority monism – the view that there is a unique mereological Universe which is fundamental, and where any entity numerically distinct from the mereological Universe is grounded by it – I argue that this is the consequence of Trogdon and Cowling's premises, as the mereological Universe under Lewis's system is a mixed fusion. Therefore, the relation to mereological hybridism in this instance involves both the potentially abstracta/concreta fusion instantiated by mixed fusions, as well as how their parts' distinct membership relations operate.

Further to this, I show that by recalling the result obtained in Chapter 2, that mixed fusions being members leads to grounding loops involving the mereological Universe, that this new consequence from Trogdon and Cowling's premises serves to remove the loops but only at the cost of overdetermining the grounding relations. Therefore I argue that instead of facing a problematic trilemma of choosing between either unsupported proper part grounding, unfavourable grounding loops, or unacceptable grounding overdetermination, that we are better off rejecting GEM and its attendant mereological hybridism in such contexts, to avoid mixed fusions and these associated problems with their grounding.

0.1.5 Chapter 5

In Chapter 5 I examine how the claim to mixed fusions could be affected by the classic categorial distinction between abstract and concrete entities (Katz 1998). Most commonly classes are taken to be abstract entities (Linnebo 2018), where abstracta are routinely depicted in terms of the features they lack (e.g., non-spatiotemporal, non-mental, non-changing, non-causal, inter alia) (Kaufman 2002). Concrete entities, on the other hand, are commonly taken to have at least some of the positive features which abstracta do not have, with ordinary space-time creatures like cats, cars and caves being some examples of concreta. A natural question arises in the case of mixed fusions, and serves for a key inquiry into the prospects of the plausibility of unrestricted mereological hybridism – if mixed fusions have at least one part which is an abstractum (the class) and one which is a concretum (which individuals that are ordinary things should be), which of these kinds of entity is the mixed fusion?

I argue that on one plausible construal of the abstract–concrete distinction that mixed fusions can be of neither kind in these instances, and should therefore be rejected – which is a de facto rejection of unrestricted composition and GEM.

Throughout the rest of the chapter I consider Lewis's options for resisting this consequence of the abstract–concrete distinction so described, drawing from his (1986) account of making the distinction, and also examining how both his system of concrete modal realism and conception of classes as entities which exist outside possible worlds (1986; 1991) are impacted. In the end I argue that the most credible option in terms of claims that Lewis makes in defence of his systems actually still leads to significant problems that justify a restriction on composition after all. Specifically in his claims to trans-world fusion which involve parts from different possible worlds, but also in his claims to supra-world summation which involve parts from both outside and inside possible worlds.

However, I do not end up with a conclusion that shows *all* abstracta/concreta fusions to be objectionable. Instead, my claims to reject mereological hybridism and to restrict composition in the case of abstract and concrete sums is when these are meant to also be supra- and trans-world fusions, as they can be within systems like Lewis's.

0.1.6 Chapter 6

Chapter 6 has similar themes to Chapter 5, yet covers a debate about the prospects of transcategorial sums in general, rather than just the restricted case of only considering abstracta and concreta. More specifically, I criticise a debate between Simons (2003) and Varzi (2006) which is over the referent of the term 'the Universe', as well as investigate the prospects of transcategorial sums in more general terms – sums which, of course, exhibit mereological hybridism. I also show how Smid (2015) confounds the issue by offering that sums may be of multiple or no categories, options which I reject, as do Varzi and Simons, because categories as presented are comprehensive across all things, and mutually exclusive. In closing I argue that only some but not all transcategorial fusing may be acceptable.

Consequently I argue that because we can have cases both where transcategorial fusion fails and succeeds that we are unable to assume in advance that it will necessarily fail or succeed, and are thereby compelled to refrain from assuming in advance that any case of mereological hybridism is acceptable. I therefore argue that we must do the work to show category compatibility, and consequently further claim that GEM is an unacceptable system for its imprudent enabling of mereological hybridism. In the end I thereby side with Simons in rejecting the mereological Universe as a transcategorial sum, but for stronger reasons than he gives specifically on this angle.

0.1.7 Chapter 7

In Chapter 7 I examine Lewis's argument for the 'ontological innocence' of GEM, and conclude that it cannot be innocent in the case of mixed fusions, as these would in fact be additional entities to accept in our ontology. In his defence of GEM Lewis famously argues for its ontological

innocence in the sense that when it comes to our ontological commitment to the number of entities that there are, that we do not incur numerically more commitments by accepting fusions of things which we are already committed to. As Lewis puts it "given a prior commitment to cats, say, a commitment to cat-fusions is not a *further* commitment." (1991, 81)

Recalling, however, that Lewis thinks that mixed fusions are distinctly *unlike* any of the other entities in existence (i.e., they are neither individuals nor classes) makes them a problem. This is because, as Varzi explains, the idea that mereology is innocent is based on the assumption that there are *not* irreducible composites which exemplify properties and relations that are not reducible to or grounded on the properties and relations exemplified by the things that compose them (2014, 63). With mixed fusions on Lewis's account being unable to be reduced to even just their class parts *qualitatively*, then, I argue that each of them would indeed be an additional ontological cost and thereby *not* innocent, as they must therefore altogether be counted as both a separate entity in kind *and* in number.

In reaching this conclusion I further argue that a mereological Universe which contains mixed fusions, as in Lewis (1991), is decidedly non-innocent ontologically, and should thereby be rejected. This, in turn, amounts to another rejection of mereological hybridism in the case of mixed fusions.

0.1.8 Chapter 8

Finally, in Chapter 8 I review the results of the preceding chapters and establish my multiple arguments against mixed fusions as a cumulative result against general mereological hybridism, and thereby against GEM due to unrestricted composition.

Although GEM serves as a major instigator of mereological hybridism, I further claim that the cases I make against mereological hybridism involving mixed fusions should be able to generalise. This is to the effect that any claimed instances of composition across categories like the ones I discuss should be seen as too problematic to maintain, and that composition is indeed to be restricted to avoid such cases, whether or not the mereological system is classical.

I also suggest avenues for future work that follow on from the conclusions reached herein.

Chapter 1

What are these things called Mixed Fusions?

1.0 Introduction

David Lewis's (1991) aim in *Parts of Classes* is to provide a better understanding of the ontology of the mathematical theories of sets and classes (1991, 6). There Lewis takes seriously the prospects of ontological universality, the idea that we can provide a comprehensive and all inclusive account of the things that exist, and regiment them under the conditions of a single system. One of Lewis's key proposals for achieving this better understanding is to defend a reduction of class theory (or set theory, depending on how these terms are defined) primarily to the theory of singleton functions, a reduction which is meant to clarify the ontological basis of the more general membership theory.

Lewis facilitates this reduction by enlisting GEM and the logical resources of plural quantification (Lewis 1998, 203; Forrest 1991, 495), two theories which he takes to be absolutely general or topic-neutral in the sense that they apply to entities of any kind or from any domain whatsoever (1991, 62). In conjunction with these two theories, Lewis also proposes a bespoke package of theses for the mereological structure of classes and their parthood involvement with other entities, a package which is headed by his work's *Main Thesis*, that the parts of a class are all and only its subclasses (1991, 7). On Lewis's overall account it is proposed that the general case of membership from standard class theory reduces to the special case of membership in singletons (the unit classes which only have one member each) following the mereological framework provided, which entails that the key mereological components of classes with multiple members are the singletons (1991, 54).

The single, comprehensive system which Lewis depicts from this combination of theses and theories is one which is also meant to simultaneously encompass both the mereological and membership aspects of everything else that exists. This includes the world of ordinary objects, such as cats and buildings and sandwiches – things which are usually understood to be strictly different from classes in not having members but also usually themselves expected to be members, which under these specifications regarding membership are technically termed 'individuals' – as well as whatever else it is that may exist. With this differentiation in mind, and by following GEM's principle of 'unrestricted composition' which in essence asserts that any plurality of entities composes a mereological unit, a whole, Lewis very much anticipates the existence of something else, and things which are admittedly unfamiliar, namely composite entities which have at least one part which is an individual and at least one part which is a class – what Lewis calls 'mixed fusions'.

Lewis's claim to the existence of mixed fusions exemplifies an acceptance of 'mereological hybridism', which I have characterised as any view which claims the existence of composite wholes with parts from multiple different categories, either logical or ontological. What primarily makes

mixed fusions mereological hybrids on my view concerns the sorts of membership relations that their parts of classes and individuals are involved in. Following Lewis's initial definitions that classes are entities which have members and that individuals are entities which do not have any members but which are yet themselves members, I see the primary mereological hybridism of mixed fusions as stemming from their being fusions of both membered and non-membered things. Yet it is rather plausible that mixed fusions also exemplify other instances of mereological hybridism for the fact that classes are usually understood to be abstract entities, in at least the sense of not being in space and time, and that there are many potential individuals which are concrete entities, in the sense of being in space and time, provided such things do not have any members and are also eligible to be members, classifications which Lewis more or less also adheres to himself.

However, Lewis makes no defence specifically of his system's mereological hybridist consequences, even as he admits that unrestricted composition coupled with the full generality of GEM leads to plenty of other kinds of unheard-of things which are mereological sums of multiple entities that we have heard of, such as his infamous 'trout-turkeys', whole creatures said to be composed of the front half of a trout plus the back half of a turkey, which altogether exist as spatially scattered sums of halves of those two ordinary animals (1991, 7). Instead, we get offered strategies for coping with them – strategies such as to restrict our quantifiers so as to avoid their discussion, among others (1991, 80) – strategies that do nothing to address any of the natural questions which arise when faced with such unfamiliar beings as mixed fusions clearly portend, questions such as: what kind or category might these entities belong to? How might these newly recognised entities fit within our current scheme or prior beliefs about (certain) categories? Is it really plausible that there are such other logical or ontological divisions? These sorts of questions take on an extra degree of significance in the context of certain categories which are understood to be both exclusive and exhaustive, such as with how the abstract/concrete distinction is traditionally understood (cf., Kaufman 2002), and I discuss them considerably in Chapters 5 and 6.

On my view, the best answers to these questions work against the plausibility of mereological hybridism, and that thereby any sort of theories or principles which tend to support the view in specific instances, such as GEM or unrestricted composition, also face objections based upon these type of consequences that they accrue. Because I think that mereological hybridism fails in many cases, and that mixed fusions suitably provide a representative example for each of these instances of failure, I make my arguments against mereological hybridism by putting mixed fusions front and centre, although I expect that my criticisms can relevantly generalise. With mixed fusions as my main entity under scrutiny, we need a thorough account of them so as to provide a basis for their critical examination.

The natural starting point for our account of mixed fusions is to begin with Lewis's introduction of them, to then continue with his overall portrayal of their features and attributes, as well as some of the relations that he thinks that they are in, both implicit and explicit for all. However, as I have found several problems within Lewis's account of what mixed fusions are like and how they are connected to other entities, I will be following up this initial presentation with a critical evaluation of the proposals, and argue for certain clarifications and specifications to be adopted instead. Additionally, besides an account of mixed fusions from Lewis's original perspective, we also need the details of the context in which Lewis first introduces them, such as the class theory, the mereology, and the package of theses which Lewis advances. Beyond this we also need some indication of how mixed fusions are supposed to be representative of mereological hybridist entities. Therefore, the extent of this chapter is to provide all of the core details about mixed fusions that are relevant for the thesis ahead, starting from the source and going on to their clarification, correction and qualification, in effect setting us up for the remainder of the work.

Before we start looking at mixed fusions proper we take an overview of the theories operating within Lewis's system, as well as some of the terminology used throughout.

1.1 Lewis's selected theories and terminology

First it should be noted that Lewis uses admittedly idiosyncratic terminology when it comes to set and class theory (1991, 4). While Lewis explicitly restricts the term 'class' to entities which have members, he restricts the term 'set' to any class which is itself also a member of a class, as well as uses it for the null set, which has no members. This means that for Lewis there is no 'null class', such that even though the null set *is* a subset of every class, it is *not* a subclass of any class, and thereby – following his Main Thesis that the parts of a class are all and only its subclasses – is not a part of any class. For Lewis the classes which are not members of any classes are the so-called proper classes (with the reasons given for them not being members discussed in later chapters). Note too that Lewis sometimes uses the terms 'class' and 'set' interchangeably, with the context in which they are deployed providing clarity for their meaning (something which I may do as well).

To be explicit, the way Lewis's system of sets and classes handles proper classes makes it a version of what is often known as Morse–Kelley set theory (MK) (Burgess 2015, 460), which is a proper extension of the canonical set theory of Zermelo–Fraenkel with the axiom of choice (ZFC). MK is a proper extension of ZFC in that MK contains all of ZFC's theorems, yet can also prove more theorems than ZFC (see Mendelson 2015, 396 fn for a formal definition of 'proper extension', and 2015, 4.6.1 for MK). Furthermore, because Lewis's system also has resources to quantify over proper classes via plural quantification (1991, 62-71, following Boolos 1984), which are resources

beyond those found in MK itself, it is also arguably a more powerful system than even MK (Nolan 2002, 193). As both ZFC and MK can also be developed so as to include individuals (or 'urelemente' in the German, thereafter initialising the systems as ZFCU and MKU) Lewis also proceeds with this development for the sake of having a system which is anticipated to be able to capture all existents and not just ones with members.

Because Lewis reserves the term individual for anything that is a member of a class but does not itself have members, he also calls the null set an individual, and defines 'urelement' as any individual other than the null set. The following is a diagram to help show how Lewis uses the terms, with the more general higher terms connected to two more specific lower ones each.

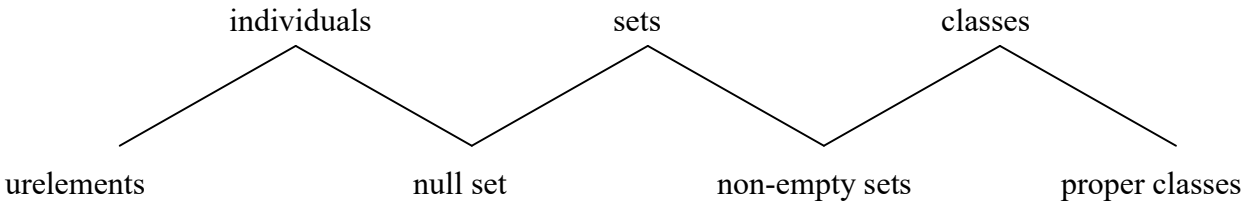

Figure 1.1 A linking diagram of Lewis's terminology.

Besides the class theory and the admittedly idiosyncratic terminology, Lewis's system also makes use of GEM. This is broadly accounted for in the next subsection, alongside a small explanation of Lewis's Main Thesis and some of its consequences.

1.1.1 Lewis's Main Thesis and GEM

In *Parts of Classes* Lewis's Main Thesis is that the parts of a class are all and only its subclasses. For an example, consider the set of three elements a, b and c, i.e., {a, b, c}. According to the Main Thesis the parts of {a, b, c} are seven: the plural class {a, b, c} which is a subclass of itself; its proper subclasses {a, b}, {a, c} and {b, c}; and the unit classes, or singletons, {a}, {b} and {c} which are also proper subclasses of the classes they share members with. Similarly to the subclass/proper subclass distinction, the term 'part' here permits two readings, either proper part or improper part.

While a proper part of an entity *E* is a part of *E* that is not *E* itself, the improper part of an entity *E* is *E* itself. For instance, of a brick wall under an ordinary conception, a proper part of the brick wall is one of its bricks, just as a proper part of one of the bricks is one of its own molecules.

However, the improper part of a brick wall here is the brick wall itself, and the improper part of a brick is the brick itself, and so on.[1]

One consequence of Lewis's Main Thesis is that singletons are mereological 'atoms' (1991, 15), a name for entities defined as having no proper parts, and hence singletons such as {a}, {b} and {c} are atoms according to Lewis's system. Using a mereological model we can see an instance of the structure of the parthood relations that Lewis claims that classes have, with proper parthood relations going from the over elements to the under elements.

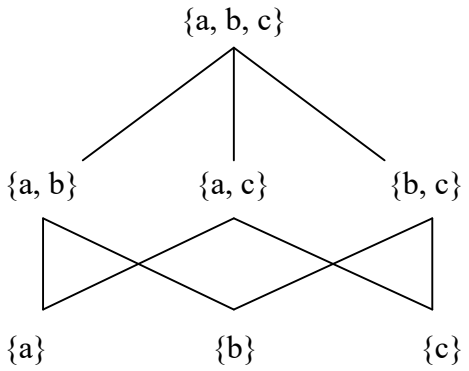

Figure 1.2 A Lewis system model of class parthood.

This model also reflects the classical theory of parthood that Lewis is committed to. Informally, GEM states, among other things, that (1) all things are mereological, in the sense that everything has parts, and at the very least, each has itself as a part; (2) any plurality of things mereologically combine to form a whole, including the plurality of all things, if there is such a plurality; and (3) every whole is mereologically unique, such that any two entities cannot share all of the same parts and not be the same thing. In short, GEM asserts that (1) parthood is reflexive; and that mereological composition is both (2) unrestricted, and (3) unique.[2]

What GEM makes no claims to, however, is what it is specifically that exists. Instead, it puts forth a number of rules and conditions for what the mereological relations any existent is bound to or is involved in, such that GEM entails ontological commitments only conditional on what it is that exists. Although GEM can be adapted to more limited domains, it can be understood to be topic-neutral or absolutely general in the sense that it applies to entities from any ontological category

[1] For deniers of bricks and brick-walls, under an anti-GEM view called mereological nihilism, see van Inwagen (1990) (as an instance of the denial of composition in the case of artefacts), and Rosen and Dorr (2002), and Unger (1979) (for denials of composition in general). See Kearns (2011) for a GEM violating example of an extraordinary wall which has itself as a proper part, due to time-travel.

[2] There are other rules in the system and many ways to formulate GEM. For extensive overviews see Varzi (2016), Simons (1987) and Hovda (2009).

whatsoever (cf., Ryle 1958, and Uzquiano 2006). This absolute generality is how Lewis understands the scope of GEM (1991, 62 and 75), and goes towards explaining why he seeks to apply it to domains which include sets and classes. Because Lewis thinks that both classes and individuals exist and that GEM covers the mereological structure of all existents, he applies it to each of them and all things together accordingly.

That being said, the Main Thesis is not a claim about the parts of classes that Lewis always believed. As Lewis admits, in his earlier thinking he did not consider classes to be mereological entities at all. "I used to think [that] the part-whole relation applies only to individuals, and not sets. [...] I might at least have granted that the part-whole relation applies to classes in a trivial way: even if a class has no proper parts, as I then thought, at least it should have itself as an improper part." (1991, 3 and fn 2). Here Lewis is following the consequence of GEM's claims to absolute generality and of the reflexivity of parthood to the effect that any class is at the least single parted, i.e., at least a mereological atom. The Main Thesis goes much further than this, however, and says that the mereological structure of a domain which consists of only all classes mimics the mereological structure of an atomistic version of GEM, a claim which GEM itself takes no stance on. To justify the Main Thesis Lewis offers defences of his First and Second theses, of which the Main Thesis is a conjunction (1991, 6). We will consider these supporting theses later.

For now we start to look at Lewis's presentation of mixed fusions.

1.2 Ignored but undenied

As mentioned, Lewis thinks that mixed fusions and other sorts of heterogeneous fusions like trout-turkeys exist because he accepts both GEM's principle of unrestricted composition and the existence of things which are their parts. In recognising that the existence of such entities may be unpalatable for some, one of Lewis's strategies for minimising our concern over such unfamiliar beings is to advise that if we wish to ignore them we may do so, but only in cases where we are restricting the range of our quantifiers, either implicitly or explicitly (1991, 80). In this way, Lewis is happy to maintain that these kinds of things exist even as he acknowledges both that we are not accustomed to speak or think about such things, and also that mixed fusions and others may be different from ordinary, much-heard-of things – such as by their lack of cohesion and causal integration (1991, 80). However, Lewis attests that it is in the cases where we may wish to speak literally of everything, or otherwise leave our quantifiers sufficiently wide open, in which we must affirm the existence of such creatures.

Although Lewis does not cover mixed fusions in that much detail, asserting that they and other heterogeneous fusions resist concise classification (1991, 8), over the course of *Parts of Classes* Lewis indeed opts not to ignore mixed fusions absolutely, both referring to and making claims about them in roughly ten places throughout the work's almost 150 pages. There Lewis builds up a small, explicit account of what he thinks mixed fusions are like. For instance, we learn that, according to Lewis, mixed fusions cannot themselves have members (and are thereby unlike classes) (1991, 7-8 and 16-17), yet are also not themselves members of classes (and are thereby unlike individuals) (1991, 8, 15, 21, 103 and 118); that an entity of which all others are supposed to be part – the mereological Universe – is a mixed fusion (which Lewis calls "Reality") (1991, 8); and that mixed fusions are 'ontologically innocent', because a prior commitment to individuals and classes is expected to ensure that their recognition as existents does not increase our ontological commitments in terms of the number of entities that we accept – something which Lewis argues for as being the case with every composite whole (1991, 81-87).

Beyond Lewis's explicit statements about mixed fusions, however, are the implicit connotations and consequences for them when framed in the surrounding context of class theory, GEM, and Lewis's package of theses about the parts of classes, as well as some of the evaluative claims Lewis puts forth, or his other proclaimed beliefs and assumptions. Here we can see implied either from Lewis's framework or particular statements that mixed fusions are grounded in their proper parts and their class parts' members; that mixed fusions as sums of concreta and abstracta are themselves abstracta, and also that Lewis finds no problems with mixed fusions and other transcategorial sums in terms of a variety of other instances of mereological hybridism; that mixed fusions are peculiarly identical to their parts; and that mixed fusions are, in some cases, irreducible composites. Notably, some of these implicit consequences are contradictory, either with each other or with some of the explicit ones, and are some of what motivates my criticism in this thesis of the prospects of mixed fusions being plausible existents. Overall, although Lewis characterises mixed fusions to a degree and does not deny their existence, he does think that they can be left out of the domains of all but our most unrestricted quantifying, and thereby be mostly ignored. (1991, 8 and 80)

As both the explicit and implicit characterisations Lewis makes of mixed fusions have consequences for the later chapters, some of the more involved details on these fronts will be revealed closer to the moment they are needed and addressed properly there, with more of the explicit details given to be the focus of this chapter in the presentation ahead. On this note, let's begin to examine Lewis's account of mixed fusions.

1.3 Salt beef sandwiches: The beginning of mixed fusions

Lewis first introduces mixed fusions in the context of justifying and explaining one from his package of theses about the parts of classes and their relation to other entities, namely his *Division Thesis*, that Reality divides exhaustively into individuals and classes. On Lewis's account, 'Reality' is the name he gives to the absolute whole or mereological sum of everything that exists. When taking GEM as an absolutely general theory, the notion that there is an entity of which all others are parts is a theorem (Bohn 2009), and this totality entity is a type that I will refer to in general as a 'mereological Universe'. The distinction to keep in mind here is that Lewis's proposed mereological Universe which he calls Reality is one which is explicitly said to consist of individuals and classes as well as mereological fusions thereof. However, such an absolute composite as Lewis's Reality is not the only way to conceive of a mereological Universe if, for example, one denies the existence of classes, which would preclude there being mixed fusions altogether. To get a feel for how Lewis conceives of mixed fusions, and the general attitude he has towards them, it is instructive to quote his qualification of the Division Thesis in full.

> Roughly speaking, the Division Thesis says that there is nothing else except individuals and classes. But that is not exactly right. If we thought that Reality divided exhaustively into animal, vegetable, and mineral, that would not mean that there was no such thing as a salt beef sandwich. The sandwich is no counterexample, because the sandwich itself divides: the beef is animal, the bread is vegetable, and the salt is mineral. Likewise, the Division Thesis permits there to be a mixed thing which is neither an individual nor a class, so long as it divides exhaustively into individuals and classes. I accept a principle of *Unrestricted Composition*: whenever there are some things, no matter how many or how unrelated or how disparate in character they may be, they have a mereological fusion. [...] That means that if I accept individuals and I accept classes, I have to accept mereological fusions of individuals and classes. Like the mereological fusion of the front half of a trout plus the back half of a turkey, which is neither fish nor fowl, these things can be mostly ignored. They can be left out of the domains of all but our most unrestricted quantifying. They resist concise classification: all we can say is that the salt beef sandwich is part animal, part vegetable, part mineral; the trout-turkey is part fish and part fowl; and the mereological fusion of Possum [one of Lewis's cats and presumably non-membered] plus the class of all cat-whiskers is part individual and part class. Likewise Reality itself – the mereological fusion of everything – is mixed. It is neither individual nor class, but it divides exhaustively into

> individuals and classes. Indeed, it divides into one part which is the most inclusive individual and another which is the most inclusive class. (1991, 7-8)

Lewis's presentation and discussion here is very informative, on several fronts. On one, it introduces us to the notion of mereological decomposition which is complementary to that of mereological composition, in that if something is composed of parts, it can also be decomposed, or 'divided', into them. For Lewis, then, some of the plausibility of the Division Thesis can be seen to rest on the expectation that there are two basic entity types, which together exhaustively "mark the line that matters: the line between the membered and the memberless." (1991, 4) This dividing line between classes and non-classes is also meant to suggest that the things which make up Reality, as its proper parts, are themselves exhaustively decomposable into either things which are membered, or things which are not. (For my take on the best way to mark this division, see section 2.2.1.)

On another front, this discussion further introduces us to Lewis's classical expectation that parthood is to be understood as topic-neutral or absolutely general, in that composition relations are supposed to hold between things "no matter how many or how unrelated or how disparate in character they may be", as well as the fact that the existence of a mereological fusion of everything, a mereological Universe, implies that all things altogether are in a composition relation of composing this whole. On a further front, this discussion also introduces us to the fact that there are other customarily overlooked creatures which are compatible with unrestricted composition, such as the trout-turkey, besides just those which are the mixed fusions. We are also shown that unrestricted composition is also compatible with ordinary things like sandwiches too, and is not solely in the business of introducing us to ontological exotica.

Finally, the discussion shows that Lewis thinks that Reality is a mixed fusion – the biggest mixed fusion of them all! – and that he does not expect it to be a member of any classes, because it "is neither individual nor class". In fact, Lewis expects that all mixed fusions are non-members, which is something to be elaborated upon below, and also contested in Chapter 2. This also means that Lewis expects that Reality does not have any members, in it not being a class either, which is something Lewis also expects of all mixed fusions, that these are memberless. This understanding is mentioned below and to be discussed further in Chapter 3, where I deny that Lewis is able to maintain this claim about mixed fusions while still being consistent with his thesis of CAI.

1.4 What else does Lewis say about mixed fusions?

Although Lewis, as quoted earlier, does say that mixed fusions can be mostly ignored, he makes several other mentions of them throughout *Parts of Classes*, each of which helps to build a picture

of what he thinks they are like and how they fit into the world amongst everything else. We will now go through most of the things Lewis explicitly says about mixed fusions, and the claims he makes about them, before we go on to describe his other theses. For starters, when it comes to axiomatising set theory with 'singleton' as the only set-theoretical primitive – as Lewis proposes in his reduction of set theory – we can consider the prospects of what the candidate entities which fall under the broad categorisation of classes and individuals are or could be, besides those kinds of entities construed via their membership relations. For instance, Lewis says we should include certain unofficial axioms, such as those which embody "those of our customary metaphysical opinions about classes that deserve credence. [Including] for instance, an axiom stating that ordinary things – cats, etc. – are individuals, rather than classes or mixed fusions." (1991, 46)

On the notion of whether mixed fusions can *be* members Lewis says, "the 'salt beef sandwiches': the mixed fusions, ignored but undenied ... have no singletons and are not members of anything." (1991, 21) On the prospects of whether mixed fusions can have members, Lewis says "a mixed fusion, part individual and part class [does] have singletons as parts, yet probably we would not want to say that it has members." (1991, 16) The last explicit, albeit indirect, claim that Lewis makes about mixed fusions is that they are *ontologically innocent*, in the sense that a commitment to their existence is not to count as an additional commitment beyond that already made to classes and individuals. As this is a claim that Lewis defends about composite entities in general (1991, 81-87), it also applies to mixed fusions.[3]

From here we can update from our previous linking diagram of the terminology that Lewis uses to an Euler diagram which includes mixed fusions, as well as indicates the various membership properties and relations each kind of entity is involved in.

[3] I challenge Lewis's claims to the ontological innocence of GEM in Chapter 7.

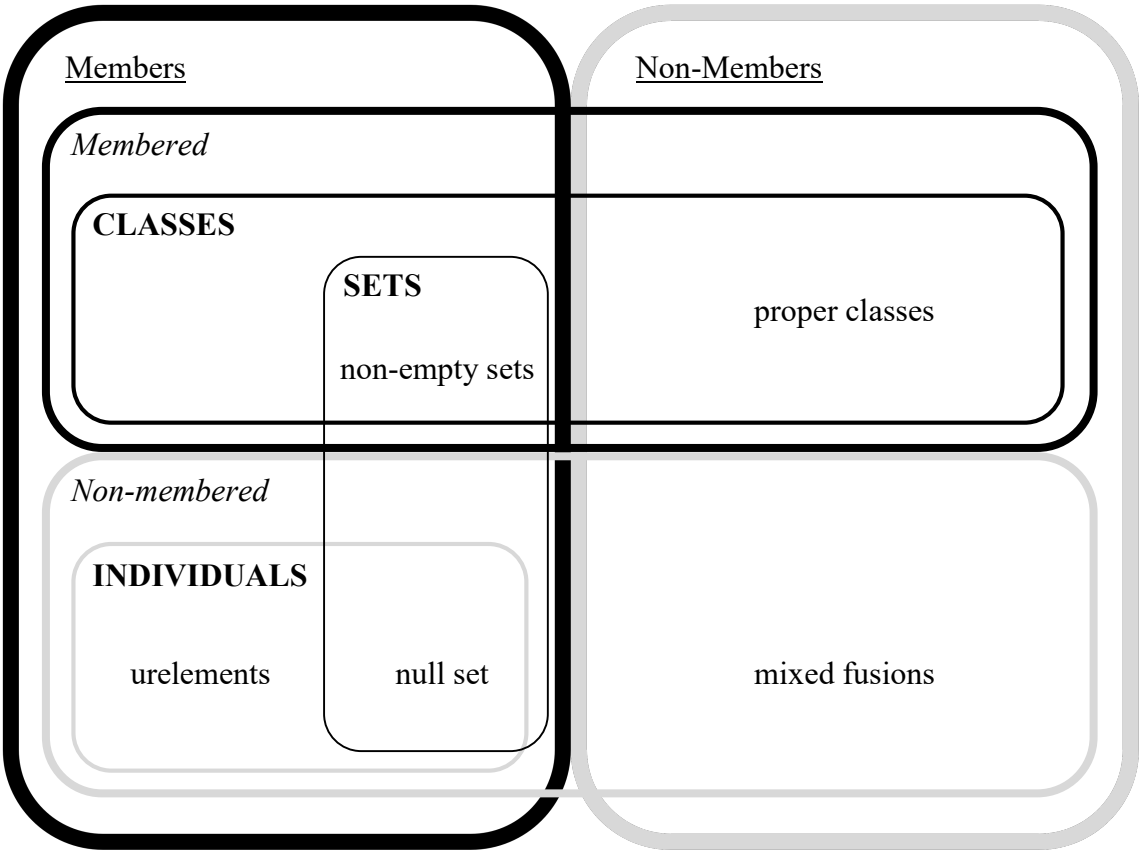

Figure 1.3 An Euler diagram of Lewis's terminology, with membership relations indicated.

Alternately, we can show this arrangement in a table.

Lewis's terminology	individual	set	class	is a member	has members
urelements	✔			✔	
null set	✔	✔		✔	
non-empty sets		✔	✔	✔	✔
proper classes			✔		✔
mixed fusions					

Table 1.1 Lewis's terminology, with membership relations indicated.

As is clearer from the table, mixed fusions are conveyed mostly negatively by Lewis. What we have so far that is positive however are some of their direct implications. For example, we know that Lewis thinks that mixed fusions are transcategorially valid, in that their existence is not denied even as they have parts which at least cut across the logical division of the membered and the non-membered. We also know that Lewis thinks that mixed fusions are ontologically innocent, as noted before. So we can summarise Lewis's explicit portrayal of mixed fusions in the following table.[4]

[4] With numbers indicated for my thesis chapters where this portrayal is primarily discussed.

	Mixed fusions	Lewis's direct views
2	Are members?	None of them
3	Have members?	None of them
6	Are transcategorially valid?	Yes
7	Are ontologically innocent?	Yes

Table 1.2 Lewis's direct characterisation of mixed fusions.

With regards to most of these explicit characterisations of mixed fusions, I think Lewis is wrong. Additionally, I also think that Lewis's system of claims around membership, composition, identity and possible worlds, among other things, actually serves to indirectly amount to a different understanding of mixed fusions altogether. My allegations here are to be uncovered and justified as we proceed through the chapters.

For now we look towards the rest of Lewis's theses to complete the overview of his reductive system.

1.5 Completing Lewis's package of theses

In support of his Main Thesis Lewis provides five others, one of the most important being the

First Thesis One class is a part of another iff the first is a subclass of the second. (1991, 4)

To understand the First Thesis, note that on standard class theory, the notion of a subclass of a class refers either to a class itself or to a class which it includes in terms of having all of its members, yet is still distinct from it in terms of its other members. This means that subclasses come in two varieties, proper and improper, where a proper subclass of a class C is a subclass of C that is not C itself and the improper subclass of any class is just that class itself. For an example of a proper subclass, consider the three membered class {Leśniewski, Tarski, Goodman}, which has as one of its proper subclasses the two membered class {Leśniewski, Goodman}. Outside of Lewis's mereological comparisons and claims, the structure of class and subclass relations are already well known as being akin to how the parthood relation is characterised by GEM where the parts of anything are either proper or improper, and the proper parts of anything are mereologically distinct from that thing while the thing's improper part is just the thing itself.

For our example, then, under the First Thesis, the class {Leśniewski, Goodman} is a proper part and subclass of {Leśniewski, Tarski, Goodman}, as are each of the singletons {Leśniewski}, {Tarski}, {Goodman}, as well as its other proper subclasses. By following the First Thesis, and Lewis's specification that the null set is not a null class, we get the understanding that any class is

both a subclass and part of itself, as an improper subclass and improper part of itself, and that any proper subclass of a class is also a proper part of that class.[5]

Lewis offers three reasons to believe in his First Thesis, while also claiming that it is far too evident to need an argument (1991, 7). The first reason is that it appears to conform to common speech, in that it seems natural to say, for instance, that the class of frogs is part of the class of animals (1991, 5). Secondly, there appear to be no formal obstacles to it being true, because of the formal analogy between GEM's part-to-whole relation and the class theory's (specifically non-empty) subclass-to-class relation, a formal analogy which has long been understood (Oliver 1994). Lewis's claim here is that for classes their subclasses just are their parts, we know it, and we speak accordingly (1991, 5).[6]

Finally, Lewis submits that there is a certain utility that comes from accepting the First Thesis which far surpasses any misgivings one could have towards it, with the remainder of *Parts of Classes* supposed to stand testament to its acceptance. Here Lewis sees the accepting of the First Thesis as being the principal step towards separating the ontological "innocence" of mereological fusion from the ontological "extravagance" of singleton formation, because the making many things into one via mereological fusion appears to Lewis to be an ordinary phenomenon – as we seem to not really have more than we started with – contrasted with the making many things out of one, via singleton formation – as we end up with far more than we may ever have imagined, with the infinitude of sets (1991, 6).

With regards to the second reason, for Lewis the Main Thesis (with its support in the First Thesis) is supposed to demonstrate that the formal similarity between the subclass-to-class relation and the part-to-whole relation is sufficient to show that the non empty subclass-to-class relation is, in fact, a species of the part-to-whole relation. This formal similarity can be conveyed easily with the help of some simple models, one of which we encountered earlier:

[5] What we don't get is the reverse, that any proper part of a class is a subclass. That's what the Second Thesis (to be introduced) is for.

[6] For strong criticism of Lewis's defence of the First Thesis, see Oliver (1994).

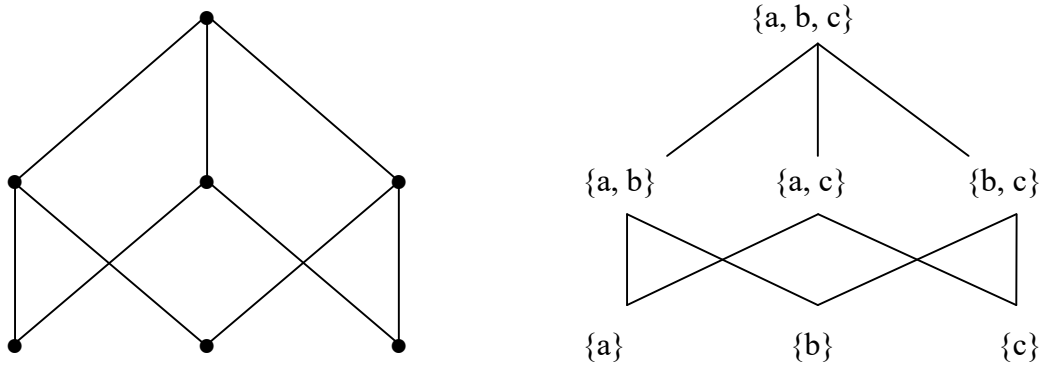

Figure 1.4 (Left) A generic, atomic, GEM parthood model. (Right) A Lewis system of class parthood model.

On the left side is a generic model of the GEM part structure which holds in the case of three mereological atoms. To read the models, lines go up the page to represent relations of proper parthood between elements.[7] On the right side as before is a model of the GEM part structure which holds among a three membered class and its subclasses, in the case that Lewis's account of the parts of classes is correct. On this account the unit classes, or singletons, of set theory are all mereological atoms, following the Main Thesis.

A requisite terminological clarification that Lewis makes here involves specifying that the null set – which is the only set which has no members – is a set *only*, and is not at all a class (1991, 4). This is because the null set, on standard set theory, is a (proper) subset of every set, and thereby a (proper) subclass of every class. Yet because there is no parallel null part of every entity under GEM, completely following the formal similarity between the theories would lead to severe GEM deviations. Where usually the null set would be shown on a model as being below the unit classes as the zero-element of a Boolean algebra (see Varzi 2016), this is a model for Lewis specifically of *non-empty classes*, none of which have the null set as a part, even as all of them have it as a proper subset. As Lewis insists on the terminological distinction between sets and classes, singletons are understood as mereological atoms from having only one non-empty subclass, themselves, by having only one member. In light of all of the qualifications given, Lewis also considers the null set to be an individual, because it is ordinarily able to be a member of sets and classes in the respective theories – although regardless of whether these theories are extended to also include urelements as individuals – and by definition it is also memberless as well.

What the First Thesis does not tell us, however, is whether or not classes have any other parts *besides* their subclasses, hence Lewis offers his

[7] Note that the left can also be a model of a mereological Universe which comprises only three atoms, whereby the topmost overlying element is the mereological Universe itself, and everything else is a part of it.

Second Thesis No class has any part that is not a class. (1991, 6)

In conjunction with the First Thesis, the Second Thesis is stated to serve as the basis for the Main Thesis, and is treated to an argument in the form of a *reductio* proof, mostly because it seems to Lewis "far less evident" than the First Thesis (1991, 7). (Due to space limitations I don't provide the proof in this thesis. It can be found in Lewis 1991, 9-10. See Nolan 2002 and Tennant 2013 for good discussions and analyses of it.) To defend the Second Thesis Lewis makes use of the First Thesis along with the Division Thesis and the remaining two of the

Priority Thesis No class is part of any individual. (1991, 7)

Fusion Thesis Any fusion of individuals is itself an individual. (1991, 7)

In support of these two auxiliary theses, Lewis says

> The Priority Thesis and the Fusion Thesis reflect our vague notion that somehow the individuals are 'basic' and 'self-contained' and that the classes are somehow a 'superstructure'; 'first' we have individuals and the classes come 'later'. (In some sense. But it's not that God made the individuals on the first day and the classes not until the second.) Indeed, these two theses may be all the sense that we can extract from that notion. We don't know what classes are made of – that's what we want to figure out. But we do know what individuals are made of: they're made of various smaller individuals, and nothing else. (1991, 9)

Here we can see that, even as Lewis supplies the theses, he does not provide much of a defence of them overall. This is regrettable because these theses are all tendered as supporting the cornerstone of an important understanding of the mereology in set theory, the Main Thesis, yet none of them receive much justification, which means that it does not either. To be honest, I am surprised that the Main Thesis does not receive adequate support from Lewis, who thinks that the supporting theses are either self-evident or otherwise only offers them as reflecting vague notions that we may or may not have. That the theses which are supposed to justify the Main Thesis either have no argument behind them, or when they do have an argument like in the proof of the Second Thesis still rely on three other barely supported theses to succeed, makes it very difficult to allow Lewis his premises on the grounds he expects alone. This is not to say that they may not be plausible independently of Lewis's presentation, but it does mean that one has to put a bit of work and faith in to go all the way with Lewis from the outset.

Despite these misgivings about the lack of support the theses get, we can go along with them for the sake of argument and treat them as proposals to be reviewed, especially if other claims are said to depend on them. What is most important are the background conditions within which these theses are proposed, namely the mereology which infuses the whole project. This is because – on the understanding that relations of parthood and composition are indeed completely general and operate in the way advised by GEM, and provided that both membered and non-membered things exist – it will be the case that there are fusions of classes with non-classes, as well of fusions of non-classes with non-classes, and of classes with classes, and any class will at the very least have itself as an improper part. It would do us well to know if we can say more than this for the sake of our understanding of the mereology, the set theory, and the variety of existents, as Lewis's proposed package of theses tries to do. Which makes Lewis's system a useful starting point for considering the same sorts of questions as he does, and something that we can make the most of in starting us off, even if it is more of a suggestive proposal altogether than something which is properly secured.

1.5.1 Lewis's implicit theses, and Reality

In addition to the preceding it may be worth making explicit some of the implicit theses which Lewis is committed to alongside those which have been explicitly stated, as the above. The first is an implicit commitment of Lewis's regarding the type of entity formed when it is composed only of classes, namely the

Class Fusion Thesis Any fusion of classes is itself a class.[8]

The Class Fusion Thesis is an implicit commitment for Lewis following the conjunction of the Priority Thesis, that no class is part of any individual, and the fact that mixed fusions necessarily have both at least one individual and one class as proper parts. In this case, Lewis expects that the only things which have class parts are either classes or mixed fusions, and because mixed fusions are not classes for Lewis, any fusion of classes is expected to itself be a class.

Lewis's commitment to the Class Fusion Thesis also further clarifies why MKU is his adopted set theory. This is because MKU permits proper classes (and also individuals), which standard set theory does not allow (although it can also be extended to include individuals.) The clarification arrives because Lewis thinks that all proper classes are fusions of a large number of singletons – which are, by the Main Thesis, all mereological atoms (as noted earlier) – to the point that the fusion of *all* singletons is a proper class, and a proper class which is the most inclusive class (1991, 8). After characterising Reality as a mixed fusion that is the mereological fusion of everything, recall that Lewis states that it "divides into one part which is the most inclusive

[8] Tennant phrases this as Lewis's "tacit assumption that *the fusion of any class with a class is a class*." (2013, 726)

individual and another which is the most inclusive class" (1991, 8). A simplified part model of Reality, as such (yet where none of the entities are presumed to be mereological atoms), can be given in the following:

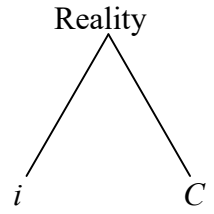

Figure 1.5 A simplified parthood model of Reality.

Lewis claims that proper classes such as *C* are in our ontology "willy-nilly, be they useful or be they useless. We do not go out of our way to posit them; rather we can't keep them away, given our Main Thesis and Unrestricted Composition." (1991, 19) I believe it is the implicit commitment to the Class Fusion Thesis which also helps to manifest such proper classes on Lewis's view.

The second implicit thesis which Lewis holds is what I call the

Mixed Fusion Thesis Any fusion of both a class and an individual is neither of these.

This thesis is first implied by how Lewis describes the Division Thesis as permitting there to be a "mixed thing which is neither an individual nor a class" (1991, 7), as well as in how Lewis characterises mixed fusions as being memberless, which excludes them from being classes, as well as being non-members, which excludes them from being individuals.

I treat these two additional assumptions of Lewis's like they are theses in their own right because of the significant role they play in Lewis's attitudes towards how he believes fusion amongst various entities plays out, which is also something to be revealed in more detail as we continue in later chapters. A further point to note with them is that even without their explicit statement as theses by Lewis himself, they are employed just as often as, if not more than, the other theses which Lewis provides, and also receive just as little support as they do.

1.5.2 A list of Lewis's theses

To close this section, I list Lewis's implicit and explicit theses as a revised package of eight, as advised from the foregoing. Of the original six, five maintain their name, yet I rename the Fusion Thesis to the *Individual Fusion Thesis* to maintain greater clarity in its distinction to the additional two of the Class Fusion and Mixed Fusion theses.

Main Thesis The parts of a class are all and only its subclasses.

First Thesis	One class is a part of another iff the first is a subclass of the second.
Second Thesis	No class has any part that is not a class.
Division Thesis	Reality divides exhaustively into individuals and classes.
Priority Thesis	No class is part of any individual.
Individual Fusion Thesis	Any fusion of individuals is itself an individual.
**Class Fusion Thesis*	Any fusion of classes is itself a class.
**Mixed Fusion Thesis*	Any fusion of both a class and an individual is neither of these.

1.6 Conclusion, and summary of the task ahead

So, what are these things called mixed fusions? We have seen that they are posited by Lewis because of accepting the existence of classes, via the acceptance of set theory, along with accepting the existence of non-classes, via more or less ordinary things, coupled with a composition principle which allows for their formation, such as that of unrestricted composition from GEM, which asserts that any pluralities form composites.

In addition to the motivation for thinking that they exist we have seen that, according to Lewis, mixed fusions are neither membered nor members – that they are neither class nor individual – and that they are a sort of transcategorial sum at the least in being fusions of both membered and non-membered things. In terms of Lewis's explicit characterisation of mixed fusions, that's about all we have, which leaves us with the implications of his statements and views to characterise them further. So, besides the more explicit statements on the nature of mixed fusions which Lewis supplies, I myself submit an early statement of several more characterisations which are only implied, leaving my fuller accounts of what I think are some of the implications for Lewis's conception of mixed fusions for later chapters, with these characterisations to be examined further on.

They include that mixed fusions are grounded in their proper parts (relevant for Chapters 2 and 4), as I will argue is evidenced by Lewis's Priority Thesis that no class is part of any individual, along with his claim that in some sense we have individuals 'first' and the classes 'later' (1991, 9). They also include that mixed fusions can be mereological sums of an abstract part (as classes may be abstracta) and a concrete part (as ordinary things are commonly taken to be concreta) (discussed in Chapter 5 and 6); as well as that mixed fusions are acceptable transcategorial sums (discussed in Chapter 6), being exemplars of such cross-category fusing as that of both abstract and concrete

entities, of both membered and non-membered entities, and also of both possible and impossible-entities (the latter because Lewis does believe in mereological fusions both across possible worlds and outside them). Finally, they include that mixed fusions are irreducible to their proper parts (discussed in Chapter 7), with this being so in the case of them not being classes – at least (and potentially more) depending on their transcategorial instances.

As my aim in this thesis is to have convinced you that mixed fusions should be denied it is fair to expect that I think Lewis is either wrong with regards to the matters above (and many others about mixed fusions), or that I think his presentation leads to problems internally. Indeed, I think that Lewis's discounting of much attention to mixed fusions means that he has overlooked their problematic status and their consequences which I believe undermine his reductive project and system of principles – to the point that a reduction of set theory to the theory of singleton functions, GEM and plural quantification cannot be achieved. Moreover, I think that mixed fusions serve as a good example of why unrestricted composition would fail in domains which include sets or classes, and that thereby mixed fusions stand as an exemplary threat to GEM being an absolutely general theory. In line with this is why I also think that mixed fusions serve as an exemplary instance of the failures of mereological hybridism, in that their potential for instances of cross-category fusing overturns the frameworks which they are supposed to inhabit.

As Lewis's defence of the acceptability of mixed fusions is weak and also undermined by his other express commitments, or so I claim and will later defend, in this chapter I sought first to clarify Lewis's portrayal of mixed fusions, as well as give an account of his system of views, to have a basis of detail from which to later show why mixed fusions should be denied, and also thereby how mereological hybridism should be rejected. Besides establishing the basis of this account, in this chapter we also gained a lot of the concepts and terminology that will be useful for the examination of mixed fusions as transcategorial sums ahead.

Even though Lewis suggests that we can mostly ignore mixed fusions, in the sense that they exist but we don't pay them any attention, I think that we are compelled to completely ignore them in the end, in virtue of their proposed existence bringing troubles which are reason enough for us to reject them after all. Besides this, I also think that how Lewis portrays mixed fusions is inconsistent with how they should fit his explicit package of theses and principles, to the point that some of the things he asserts about them in particular directly contradict some of his other views, such as the thesis of CAI, and the ontological innocence of composites. I think it is by thoroughly considering how mixed fusions are supposed to fit within the framework that Lewis offers that we are able to determine just how problematic a proposed entity type they are. My engagement with each of

Lewis's characterisations of mixed fusions forms the basis of my thesis chapters, and both my disputes and agreements with them can be summarised in the following table:

	Mixed fusions	**Lewis's views**	**My views**
2	Would be members?	None of them	Many, but not all, of them
3	Would have members?	None of them	All of them
4	Would be grounders *of* or grounded *by* their proper parts?	Grounded *by* their class parts' members, if not by all their proper parts	Grounders *of* their proper parts, if grounded at all
5	Would be abstract (A), concrete (C), or something else?	Equivocal on whether A if A+C fusions or C if A+C fusions; presumably A if A+A and C if C+C	C if A+C fusions; A if A+A; C if C+C
6	Would be transcategorially valid?	Yes	Not absolutely
7a	Would be irreducible to their proper parts?	Yes	Yes (if 3 is resisted)
7b	Would be ontologically innocent?	Yes	No (if 3 is resisted)

Table 1.3 The differences between Lewis's and my characterisations of mixed fusions.

Overall, the task of my thesis is to not ignore mixed fusions in Lewis's manner, i.e., by accepting their existence yet forgoing much attention to them. Instead, I put them under great scrutiny for the sake of having a clearer understanding of what these entities that Lewis proposes are both supposed to be like and would be like, if they existed. From doing so, I go on to demonstrate the implausibility of the existence of mixed fusions, and thereby reveal both the futility of unrestricted mereological hybridism and the failure of the composition principles and mereological systems which allow it.

Chapter 2

Mixed fusions and being a member

2.0 Introduction

In the last chapter we heard of Lewis's explicit positive presentation of mixed fusions as being ontologically innocent, non-membered non-members. Following this characterisation, each mixed fusion's membership status gives us a first pass at explaining the ways in which they can be understood to be mereological hybrids, because the primary and guaranteed representations they can instantiate are those involving their proper parts having members or not. For instance, because mixed fusions are necessarily sums of at least one class, which has members, and at least one individual, which does not have members, mixed fusions stand as a clear case of mereological hybridism in that they are fusions of entities from the distinct, mutually exclusive categories of the membered and the non-membered (consequences of which I will examine in Chapter 3). Yet this may not be the only way in which a mixed fusion could be perceived as demonstrating mereological hybridism along membership lines, as it is the case that on the other side of anything *having* members is the obvious consequence that some things are *being* members. While it is the case for individuals that each of these are members of classes, as defined, this is not the way things are for all classes under the Morse-Kelly class theory which Lewis adopts within his system. Here, some classes are ineligible to be members, for certain technical reasons involving their infinite membership size, the so-called 'proper classes'.

At first glance the existence of proper classes in this system appears like it could token another instance of mereological hybridism worth questioning – e.g., why does a fusion of something which is a member (an individual) and something which is not a member (a proper class) form an entity which is not a member (a mixed fusion)? Why are mixed fusions like proper classes in this regard? Similarly, if we reflect on the situation of mixed fusions which are composed of non-empty sets, or non-proper classes, and individuals we have a comparable question – why does a fusion of multiple things which are all members form an entity which is not a member? Even as this latter example does not properly represent a case of mereological hybridism along the lines of the fact that the two types of entities are members, and so do not belong to different categories in this particular way, the similar issue of why mixed fusions are unlike all of their proper parts in this instance does arise. So why can't mixed fusions be members of classes?

The reason Lewis gives for mixed fusions not being members is not related to the parts they are said to have, however, but rather because Lewis is unwilling to expand the scope of the membership function to include them, reportedly for the sake of avoiding paradox.

> If we accept the mixed fusions of individuals and classes, must we also posit some previously ignored classes that have these mixed fusions as members?

> No; we can hold the mixed fusions to be ineligible for membership. Mixed fusions are forced upon us by the principle of Unrestricted Composition. Classes containing them are not likewise forced upon us by a corresponding principle of unrestricted class-formation. That principle is doomed in any case: we dare not say that whenever there are some things, there is a class of them, because there can be no class of all non-self-members. Nor are classes containing mixed fusions forced upon us in any other way. Let us indulge our offhand reluctance to believe in them. (1991, 8)

Considering that, following the common definitions of individuals or urelements given within a standard set theory, it is usually sufficient that if some *x has* no members that that *x is* a member, can there really be, as Lewis suggests, such things as non-membered non-members – what I term *anti-members* for ease of exposition – within such a system of classes? If there can be, are mixed fusions plausibly some of them, as well as the only ones?

As it turns out, yes, anti-members are compatible with class theory after all, but, as I will argue, not in the way that Lewis attests. As it stands, Lewis's restriction on membership for *all* mixed fusions is unjustified, for being unnecessarily strong, and there is no good reason for *every* mixed fusion to be a non-member. However, following results from Nolan (2002) and Schwarz (2005), there are things which have a claim to be anti-members within Lewis's system which are not themselves mixed fusions, and this result in turn can be used to show that at least not every mixed fusion can be a member after all.

Besides articulating my disagreements with Lewis on these matters and correcting his system to follow his obligations, I propose to do two further things in this chapter. The first is to clarify the logical space under consideration, as I do not believe it has received such a treatment before. The second is to discuss and show an example of a mixed fusion being a member that, once his account is corrected, Lewis is obliged to accept in his system, which, although it is not paradoxical, is still a problem for some of his espoused views, and also serves to make his system unpalatable overall. The end result is to show that Lewis's system is not nearly as mathematically harmonious as he intends it to be, and that it also has some metaphysical consequences which further make it unfavourable, both of which are clearly problems for accepting the combination of systems which Lewis advocates.

2.1 Being a member

What does it take for something to *be* a member of a set or a class? What, if anything, determines any given entity's membership eligibility? Although these questions are simple to ask, they can be difficult to answer, because they appear to involve reasons both intrinsic and extrinsic to any candidate entities under consideration. For example, whether something is a member or not depends to some degree both on the theory of membership in play and how the system in operation is said to mandate such belonging, as well as on certain aspects of any candidate entity themselves. This much will be made clear.

From the outset of this sort of question, what is most apparent is that for anything to be a member in the manner of sets or classes there needs to be a system of sets or classes in play. Otherwise there are no things which are members, and there are no things with members – i.e., everything is an anti-member under such conditions. Provided that there is such a system in play though, the question of what it takes for something to be a member becomes whether the membership function within the system is such that the prospective member is eligible to be one. So what might determine eligibility?

2.1.1 ZFC

Within the canonical system of set theory, Zermelo-Fraenkel set theory with the axiom of Choice (ZFC), there is no restriction on the field of members. Here, everything is eligible to be a member. In this case, membership is a system wide affair, where the entities said to be within the scope of the set theory ensure that all entities in the domain are eligible to be members – regardless of anything to do with the entities in particular. What this means is that *all* things are members therein. However, as ZFC is a so-called 'pure' set theory which only considers membership relations for sets alone, the notion of "all things" within such a system actually only includes things which are sets. This includes the null set as well, which is also known as the empty set and sometimes represented with '∅', itself the only thing that has no members yet is a member in this system. We can table the membership division in ZFC as follows:

ZFC member types	Is a member
Has members	(i) non-empty sets
Does not have members	(ii) the null set

Table 2.1 ZFC member types.

2.1.2 ZFCU

All is not lost, however, as ZFC can be augmented to include other non-membered members beyond the null set. Although these things are recognised to be member-wise identical to the null

set, for being empty of members themselves, they are still usually defined in distinction to it. So on this augmentation there are the sets and the non-sets – the former consisting of the non-empty sets and the empty one; the latter consisting of whatever it is that we are including that is not a set. Such an extension is called ZFCU, with the 'U' standing for 'urelemente',[1] a common name for non-membered members. Other common names for such entities within the mathematical literature are 'atom' and 'individual', but here I will stick to using urelement and individual as required, as atom already has too many other connotations.

Why augment ZFC in this way? As Mendelson explains

> [Certain set and class theories] do not allow for objects that are not sets or classes. This is all well and good for mathematicians, since only sets or classes seem to be needed for dealing with mathematical concepts and problems. However, if set theory is to be a part of a more inclusive theory having to do with the natural or social sciences, we must permit reference to things like electrons, molecules, people, companies, etc., and to sets and classes that contain such things. Things that are not sets or classes are sometimes called *urelements*." (2015, 303)

This broader application of set theory is useful for many things in philosophy. It also makes a very plausible assumption – that ordinary things, such as people and cats and natural science objects are not themselves sets, and therefore do not have members of their own. In one respect these sorts of things are permitted to be members by the very intent to include them in the set theory, but in another respect they are permitted to be members on similar grounds for why the null set is a member – they do not seem to conflict with the set theory by arriving at any paradoxes resulting from their having improprietous instances of membership. In essence, as the null set is not known to lead to any contradictions within ZFC, the presumption is that an augmentation of ZFC which merely enlarges the number of member-wise identical elements is not going to lead to any contradictions either. On the face of it, this is quite reasonable. Similarly to before, we can table the membership division in ZFCU as follows:

ZFCU member types	Is a member
Has members	(i) non-empty sets
Does not have members	(ii) the null set; urelements

Table 2.2 ZFCU member types.

[1] German for "original elements" (Potter 2004, 24).

2.1.3 Proper class theories

Some limitations of ZFC led set theorists to develop alternative yet closely related versions of axiomatic set theories which are still consistent with the core of ZFC, yet which enlarge the domain of existents to also include proper classes. For instance, by the inclusion of proper classes the set theory of von Neumann-Bernays-Gödel (NBG) was able to be developed to have only finitely many axioms, a feature which Zermelo-Fraenkel set theory lacked (Szudzik 2019). The set theory of Morse-Kelley (MK) includes proper classes as well, yet cannot be finitely axiomatised (Potter 2004, 312-3). It further differs from NBG though, mainly by the range it allows for certain bound variables, such that these may range over all classes and not just over all sets. This means it is an impredicative theory in terms of how it comprehends classes. With the introduction of proper classes within these theories, which can be adapted to also include urelements, their membership divisions are tabled as follows:

NBGU/MKU member types	Is a member	Is not a member
Has members	(i) non-proper classes (i.e., non-empty sets)	(iii) proper classes
Does not have members	(ii) the null set; individuals	*none defined*

Table 2.3 NBGU/MKU member types.

Some of the motivations for not extending membership to certain classes involve either the kinds of members they are expected to have, or otherwise the amount of members they are said to have – with the sense that any class's "size" is related to its number of members. On the former motivation, for example, the class of ordinals cannot belong to a class of higher ordinality without contradiction (Nolan 2002, 174). On the latter motivation, when it comes to cases of infinite membership, any infinitely membered class is always going to have strictly more non-singleton subclasses than singleton subclasses, as implied by Cantor's theorem, which leads to there being infinities of continually greater cardinality. As both NBG and MK can include a limiting principle such that there are a maximum number of members possible that any given class may have, this entails that not everything can be a member, as such classes will strictly have more subclasses than they have members. If membership is given iteratively, such as starting at the null set and working through the singletons, then rising to two-membered classes, then three, and so on ad infinitum, the consequence is that the greatly membered classes will not be able to be members, because the available singletons have, as it were, 'run out' before reaching the top of that subclass hierarchy.

This maximum number of members notion comes from a principle of the 'limitation of size' which serves to maintain a conception of so-called large classes, classes which Maddy (1983) describes as "'too big'" to be members "in the sense that their members occur arbitrarily high up in

the iterative hierarchy, ruling out the possibility of a stage after which they have all been formed." (1983, 121) What the principle provides is literally a size limit on the classes, by stating that some things comprise a set if and only if there is no 1-1 map from the entire universe into them (Uzquiano 2006). In the case that there is a 1-1 map, this principle conditions the distinction between set sized classes and the proper classes uniformly, as "not only does it say that the proper classes are larger than all [other classes], also it says that all proper classes are the same size – namely, the largest size." (Lewis 1991, 98)

While Lewis's system accords, mostly, with MKU in how it handles proper classes and membership, there are some additional reasons Lewis's system includes proper classes which are different from the motivations just given. As explained in Chapter 1, it is Lewis's commitment to the Class Fusion Thesis, that any fusion of classes is itself a class, which also results in his theory being one involving proper classes. This is because this thesis entails that any fusion of only singletons is a class, which also clearly includes that the fusion of all and only singletons is a class. As all of the singletons account for all of the members, a class containing all of the members must be the largest class possible. Let's call it C. With self-membership prohibited on the standard set theory of ZFC, and accordingly its class theoretic extensions, C cannot have a singleton, and thereby be a member, without contradicting the rule against self-membership. So the system excludes C from being a member while maintaining that C has members, making it a proper class as defined.

Although C is what Lewis (1991, 8) refers to as "the most inclusive class", it is not the only proper class in his system. This is because C has *n* many atomic parts in the singletons, following Lewis's Main Thesis, yet strictly has more than *n* many non-singleton subclasses, by Cantor's Theorem, for the same reasons as indicated above. Therefore, in this case, because there will be a strictly greater infinity of non-singleton proper subclasses of C than that of the number of singletons altogether it means that a great majority of C's non-singleton proper subclasses will themselves be proper classes (cf., Lewis 1991, 27).

Lewis's incorporation of proper classes into his system is further entailed by his discussion of restricting singleton formation for some classes, as a means to avoid a paradox, such as in the case of the class of all sets that are non-self-members. As Lewis says "[it] had better not be a set, on pain of Russell's paradox" (1991, 18), which is the case whereby a class is apparently self-membered only under the condition that it is not self-membered. Lewis's assessment of the situation here is that "we *can* fuse all the singletons of sets that are non-self-members, thereby obtaining a proper class of sets, but this proper class does not in turn have a singleton." (1991, 19)

2.2 Lewis's membership restriction

Even though Lewis's system parallels MKU, and thereby standard set theory with urelements, in most of its class-theoretic aspects, it has the unique feature of also including entities previously unrecognised by all of the other set or class theories – viz., non-membered non-members, or what I have dubbed 'anti-members'. On Lewis's view, the only candidate anti-members are the mixed fusions, which he explains as existing solely because of their key types of parts and a suitable principle of composition.

Lewis's member type divisions	Is a member	Is not a member
Has members	(i) non-proper classes (i.e., non-empty sets)	(iii) proper classes
Does not have members	(ii) individuals (includes the null set)	(iv) mixed fusions

Table 2.4 Lewis's (1991) member types.

The distinction Lewis's account of member types brings to the table is its inclusion of things which none of the other membership theories account for. However, Lewis is not explicit on this point, that he is introducing us to something unfamiliar to all prior theories of sets and classes – something which does not sit well with his stated aim of adhering to ordinary mathematical practice (e.g., 1991, vii and 58-59). Lewis is, however, explicit about why he thinks that mixed fusions are not members. As quoted in the introduction to this chapter, he is concerned that a principle of unrestricted class-formation, which would permit mixed fusions as members, would result in paradox (1991, 8).

Looking carefully at what Lewis says about restricting mixed fusion membership, while he is correct to say that classes containing mixed fusions are not forced upon us by a principle of unrestricted class-formation – as such a principle would lead to paradox – I think he does go too far here in suggesting that there is not *any* other way classes containing mixed fusions could enter into our consideration. What about a principle of restricted class-formation which also permits mixed fusions to be members so long as none of them are problematic?

Amongst discussion of his expansive regimentation of Lewis's proof for the Second Thesis using a natural-deduction framework, Tennant criticises Lewis on this same point, and instead endorses a very liberal membership principle which aims only to avoid paradoxes stemming from too large classes (2013, 733), which in turn means that he accepts mixed fusions being members in general. Eschenbach and Heydrich also comment that Lewis's absolute restriction does not seem necessary, when they say "mixed sums [i.e., our mixed fusions] are, according to Lewis, excluded from set membership, hence from the domain of the singleton function. This feature, however, does

[not] seem very essential for the account. The singleton function has to be partial anyway, since otherwise, Russell's paradox threatens." (1995, 735)

As we can see, the notion that the singleton function needs to be partial – as in restricted – in some ways for the sake of avoiding paradoxes is agreed upon, and the key differences are in the claims towards just how much restriction is required. The tension here between Tennant's more liberal view of class formation and Lewis's highly restrictive one is a tension between what is necessary and what is sufficient – with the key target on both accounts to be a view of class formation which avoids set-theoretical paradoxes.

While I think Lewis is right to restrict class formation to avoid paradox, his restriction of all mixed fusions from being members is unnecessary, and thereby unjustified, as it is inappropriately strong for the sake of achieving the end of avoiding paradox. Tennant's view, while not strictly explicit about the potential paradoxes which may lurk, is more justified in terms of allowing for mixed fusions as members, provided we take it more generally in being flexible to the point of permitting mixed fusions as members so long as no paradoxes arise.

Furthermore, while it is true that in the class theory adopted there is no principle of unrestricted class-formation, the last clause from Lewis above is unnecessarily strong: we may, after all, be obliged to accept classes containing mixed fusions, and expressly under a restricted version of the membership function. Tennant states, and I agree, that it is "puzzling" that Lewis seeks to restrict class-formation to exclude mixed fusions (2013, 733).

If I can hazard a guess, I imagine it follows somewhat from Lewis's notion regarding individuals being 'vaguely prior' to classes (1991, 9), such that if mixed fusions were also able to be members 'after' they formed mereologically from the 'initial' individuals and classes, and were at least members if not also membered, then a new round of class-formation would ensue, disrupting this 'vague' priority.[2] As it stands, this is a very weak reason to exclude mixed fusions from being members in general, because it is only a sentiment.

After all of these considerations, I don't believe Lewis is entitled to deny that *all* mixed fusions are members of classes without further argument.

2.2.1 The line that matters

When Lewis offers his Division Thesis, that Reality divides exhaustively into individuals and classes, he is trying to "mark the line that matters: the line between the membered and the

[2] Curiously, Tennant does not follow through on any implications of what mixed fusions being members of classes might mean for the rest of Lewis's program, though, as I do in this chapter.

memberless." (1991, 4) While the idea behind the Division Thesis gets it right that such a division as that between the membered and the non-membered is to be jointly exhaustive, the meaning which Lewis gives to each of the key terms in the thesis entails that what he says is either not correct or is otherwise insufficiently general. Due to some unforeseen, and unintended, consequences of how Lewis's Division Thesis and his surrounding package of theses do not adequately capture the full variety of possible entities when it comes to membership relations (as discussed in Nolan 2002, Appendix 2 and Tennant 2013, 725-6) I believe it is worth clarifying the possible membership distinctions once and for all.

To explain where the problems are, we start by recalling that what Lewis means by 'Reality' is the classical mereological Universe. This is a problem for how the Division Thesis is stated if it is to be acceptably general because the existence of a mereological Universe, classical or otherwise, is a substantial thesis and its existence under several guises is contested (see my Chapter 6 for discussion). So I offer that rather than accepting Lewis's meaning of the classical mereological Universe by 'Reality' here that we are better off meaning the totality of beings/entities in a way that is suitable to cover either a plurality of things *or* a singular entity, to avoid the unnecessary excluding of restricted and non-compositional views.

Next, because what Lewis means by 'Reality' is the classical mereological Universe, what Lewis mostly intends by 'division' is GEM style mereological decomposition. We know this because Lewis says "the Division Thesis permits there to be a mixed thing which is neither an individual nor a class, so long as it divides exhaustively into individuals and classes." (1991, 7) Here 'mixed things' meaning "mixed fusions of individuals and classes." (1991, 8)

With the conditions of GEM the 'division' is clearly mereological here. However, as we can neither presume that composition takes place (cf., restrictions of composition in e.g., Markosian 2008, and nihilism about composition in e.g., Rosen and Dorr 2002), nor even that everything is mereological (cf., Uzquiano 2006; Cowling 2014), the notion of something mereologically dividing exhaustively into individuals and classes is not guaranteed. This is because it may be the case that some things either do not mereologically divide (if not everything is mereological) or will not properly be composites of two supposedly mutually exclusive and jointly exhaustive types of things. These are some of the reasons why the Division Thesis looks to be insufficiently general to me.

So, without the conditions of GEM, I think the best way to look at the type of division which takes place is to consider one which is logical rather than mereological – as in the logical sorting of entities into kinds or categories. This way of understanding the Division Thesis should

still fit Lewis's intentions following his remark about "the line that matters", as well as cover his own proposed entities appropriately, yet it also looks to be sufficiently general to be able to hold in other contexts without making any presumptions about the mereological status of existents.

Therefore, without making any prior assumptions as to what kinds of member involved entities there are or can be, we can start by simply naming the potential member types in a noncommittal manner, both to avoid some of the loaded expectations that may come along from doing it in other ways, and for further clarity. Hence we can discern a generic logical member type division.

Generic logical member types	Is a member	Is not a member
Has members	(i) membered members	(iii) membered non-members
Does not have members	(ii) non-membered members	(iv) non-membered non-members

Table 2.5 The four generic logical divisions of the membered and the non-membered.

Following Lewis's assessment that mixed fusions are not members (1991, 8, 15, and 21) and his avoiding saying that they have members (1991, 16, 17, and 18) what the logical division in the table makes explicit is that while Lewis is also more generally committed to anti-members with mixed fusions being the only candidate of these he accepts, the logical space neither enforces nor ensures mixed fusions to be the only non-membered non-members.

From here, as it stands, what is expressed with the Division Thesis is clearly not correct after the more general qualification that the division be logical and that the totality involved may be plural. This is because by the terms in the thesis it neglects to account for any anti-members at all, which neither individuals nor classes are. As for these, although on Lewis's explicit view the only anti-members are mixed fusions, which leads him to later interpret the Division Thesis as saying "in other words, everything is an individual, a class, or a mixed fusion of individual and class" (1991, 99), in an appendix, Nolan shows how following Lewis's several theses and definitions does not strictly preclude there being anti-members which are mereological fusions of only classes and which are not themselves classes or individuals (2002, 195-200).

Additionally, Nolan (2002, Chapter 7; 2004) shows that there are available modifications to Lewis's system which should be acceptable for the fact that they reduce a significant ontological cost of there being impossible objects which have all of their parts outside of possible worlds – the singletons and all of their collective fusions. Nolan does this by using structuralist mathematical tools which Lewis (1998) himself later fully endorses, and also by asserting proper class many individual entities across appropriately infinitely many possible worlds. This is in contrast to how earlier Lewis (1991) supplies his ontology with sufficient objects for mathematics in asserting

proper class many singleton entities outside of any possible worlds. However, in the process of Nolan's modifications, Lewis's Individual Fusion Thesis (mentioned earlier in Chapter 1) is contravened.

This is because what Lewis expects of every fusion of only individuals is that these are something which both has no-members yet is a member, i.e., which is another individual, yet under the modifications which Nolan makes it becomes the case that there are large fusions of individuals which are not themselves individuals (nor classes).[3] Although losing the Individual Fusion Thesis in this manner seems a fair trade off for eliminating sets as impossibilia, it does introduce another kind of anti-member into the picture, this time ones without any class parts at all.

From these considerations I believe Nolan is still warranted in avoiding the Division Thesis as is, and also because the purpose Lewis takes it to serve, as evidenced by his reinterpretation of it, could only be satisfied by bringing in other principles which Nolan takes to have little intuitive support (2002, 199), such as the Class Fusion Thesis met in Chapter 1. For our purposes avoiding the Division Thesis as is is also beneficial, both because it misses the mark by sorting things into only individuals and classes and also by being insufficiently general to include anti-members, as there possibly may be. So I propose to try and clear up once and for all how a Division Thesis could be formulated which at least satisfies the original sentiment that the line that matters is the line between the membered and the memberless. As a first pass we have the

Logical Division Thesis Each thing is either membered or non-membered.

Clearly this at least gives us the logical disjunction we are after – things either have members, or they do not – and it makes no commitment to a mereological Universe, any sorts of composites, or even if there is more than one thing. Where set and class theorists posit that there are both things with members and things without,[4] anyone who instead rejects sets and classes (or similarly membered things) will posit that there are only things without members, which is still acceptable via the logical disjunction.

Although the mutual exclusivity and joint exhaustiveness of this Logical Division Thesis is sufficient, one thing I find lacking with this definition is that one of the kinds is only offered as the contrary to the other. Not that I think negatively defining something is inappropriate, after all abstracta are commonly defined negatively in contrast to concreta, by their purported comparative

[3] The proof for this is mentioned further in section 2.4 ahead.

[4] Understood in the formal manner that set/class membership works within those theories. A plurality, for instance, can be said to have members, as can categories too, without this meaning the same thing as it does for sets and classes. (Simons 1982)

lacks (Rosen 2017; and see Chapter 5), and I have no great problem with that. Rather it is that I think the kinds of things which are non-membered are better off with a name to convey their kind, rather than just having a descriptor, and especially when there are a multitude of properties which such entities may have. And this can't just be 'individual' because that specifically designates things which have no members but which *can* be them. I think we need a term that stands for something that doesn't have any members, yet isn't meant to specify whether something *is* a member or not.

So, while Lewis was reluctant to annoy us by imposing novel coinages of terms (1991, 4), I'm optimistically willing to offer another neologism, both because, unlike Lewis, I don't believe I can hijack any words already in common use in this context, and also to help us to avoid spelling out each time the properties such entities do or do not have (much how 'anti-member' serves to condense the notion of a 'non-membered non-member'). So my proposal here is meant, again, for the sake of benefitting our discussions by making our communications, hopefully, clearer.

The idea which I want to follow is, I think, somewhat analogous to that with the word 'abstracta'. This is because, even though abstracta can be defined in terms of their characteristics negatively, and in opposition to concreta, say, they still have a name of their type which is not just a negated name. I think this can be done with non-membered things too in a way that, like 'individual', still suggests the individuality and singularity of a non-membered thing as an existential unit. One of the problems with Lewis's Division Thesis, I believe, is that 'individual' for him means non-membered member, yet, as I will argue in the next section, plenty of mixed fusions should also be treated as non-membered members – just not all of them. Besides this, there is the implicit tension remaining that there is another aspect beyond just having members or not which comes from the introduction of sets and classes into one's ontology. That is of course whether or not something *is* a member. As the logical division table above shows we have four kinds of things, yet all of which still follow from the basic division of whether something is membered or not.

So what I propose is that we call anything which is non-membered a *singulate*. We can also describe anything non-membered as 'singulate', with the contrary being that anything non-singulate is membered. Some of what this name is meant to convey is that, for example, while with any non-empty set there is always at least two things (the set and its at least 1 member) that with a singulate there is always at least one thing (the singulate itself), and maybe at most one thing (again, the singulate itself).

Depending on the membership theory, some singulates will be members, and some singulates may be anti-members. The null set, for instance, is a singulate which is *always* a member.

This terminology can also capture Nolan's large fusions of individuals which are not individuals, and the possible class fusions he mentions which are not classes – which are each singulates that are anti-members – as well as capture any mixed-fusion (anti-member or not) and any individual (these can remain as originally conceived). With classes just being either proper classes or non-empty sets following Lewis's terminological distinction, we get a two-way division across the memberless and the membered of: *singulates* and *classes*; and when including the additional scope of something being a member or not we get a four-way division of: *individuals* and *anti-members* (the singulates); and *sets* and *proper classes* (the classes).

Now that we have this terminological addition at hand, the best version of the Division Thesis which strictly goes along the lines of the logical division between the membered and the memberless, while also giving each entity a specific name identifying (to a degree) their own kind, is the

Membered Division Thesis Each thing is either a singulate or a class.

Following this thesis we have it that reality, as one or many things (i.e., not Lewis's Reality necessarily), divides exhaustively into singulates and classes, where singulates are non-membered things and classes are membered things. The division is not mereological, but rather logical, in that entities are categorised or sorted one way or the other. Whether all singulates are members, or whether some are anti-members; and whether there are singulates with class parts, or classes with singulate parts is to be determined by the constraints of the different theories of membership.

Finally, in addition to whether entities *have* members or not, it may be useful to bring in some terminology to disclose whether they *are* members or not (hence my 'to a degree' qualification above). For this purpose I adopt the terms *held* and *free*, where anything which is a member of any class is 'held', and anything which is not a member of any class is 'free'. This allows that if certain 'large' entities (such as proper class-sized classes) are still able to be accommodated as members of classes for some reason (see Nolan 2002, 173-4),[5] we can still distinguish between those membered things which are not members, and those membered things which are members, without needing to indicate their size, or the amount of members that they have. In turn this gives us the

Members Division Thesis Each thing is either held or free.

[5] Although I acknowledge that this is a strategy which could be taken up, in this thesis I follow Lewis's usage, leaving proper classes unable to be members.

Finally, this brings us to a conjoined division thesis applicable in all ways via the four-way logical division, the

Membership Division Thesis Each thing is either a singulate or a class, and is either held or free.

Additionally this means we also have: *individuals* and *the null set* and *held classes* and *non-empty sets* (the held); and *anti-members* and *free classes* (the free). We can mark this all again in another table which includes the names I've proposed alongside their logical descriptions.

Membership Division names	Is a member (held)	Is not a member (free)
Has members (classes)	(i) sets (non-empty); held classes	(iii) free classes
Does not have members (singulates)	(ii) individuals; the null set	(iv) anti-members

Table 2.6 Different proposed names/descriptors for the four possible kinds of membered and non-membered entities.

Altogether the Membership Division Thesis satisfies to capture what is needed in terms of marking the line that matters, the line between the membered and the memberless, yet also includes how things can be divided along another line in virtue of that primary one.

2.3 Are mixed fusions members?

Now we can circle back to some questions that are more central to the questions of the chapter – what is it that permits or precludes some things from being members? Can mixed fusions be members? If not, why not? The notion from Tennant (2013) earlier gives us a hint towards the least restrictive direction that we can take, and it is that we ensure that paradox is avoided in the formation of sets and classes. While this is an ordinary desideratum on any account, it arises forcefully in these contexts because these are the contexts which historically have introduced us to the very types of paradox that we aim most to avoid. Lewis's caution to avoid a paradoxical "class of all non-self-members" (1991, 8) by restricting class-formation and his ensuring that "the class of all sets that are non-self-members had better not be a set, on pain of Russell's paradox" (1991, 18) are both instances of acting towards this aim. Our key concern here is determining exactly what we could let in before it is too much and the system breaks down.

Recalling our Tables 2.5 and 2.6, our question is what can fit into which of the logical divisions? On standard set theory, and the well known extensions of it that we have seen in NBG and MK, it is usually the case that if any non-membered entities are introduced in addition to the

null set, when extending these to NBGU or MKU for instance, that all of these non-membered entities will also themselves be members of at least some of the sets within the theory.

2.3.1 Things without members are usually themselves members

On the textbook definition of including non-membered entities into a class theory, Mendelson (2015) explains that, ordinarily, membership systems are

> designed to handle classes, not concrete individuals. The reason for this is that mathematics has no need for objects such as cows and molecules; all mathematical objects and relations can be formulated in terms of classes alone. If nonclasses are required for applications to other sciences, then the [membership system] can be modified slightly so as to apply to both classes and nonclasses alike. (2015, 233)

To be clear, what Mendelson means by "concrete individuals" is objects that are not classes (2015, 233). He also restricts this understanding specifically to be with regards to things of at least the "natural or social sciences [like] electrons, molecules, people, companies, etc." (2015, 303) Yet we can clarify this notion further to be more inclusive if we want, to also include potentially abstract individuals, or individuals that do not fit well under either division of abstract or concrete, if such there be. (In Chapter 5 I consider some of the conceptual background for entities such as these.) As Lewis himself reminds us,

> Remember what an individual is: not necessarily a commonplace individual like Magpie or Possum, or a quark, or a space-time point, but anything whatever that has no members but is a member. If you believe in some remarkable non-classes – universals, tropes, abstract simple states of affairs, God, or what you will – it makes no difference. They're still individuals, however remarkable, so long as they're members of classes and not themselves classes. (1991, 8-9)

With mixed fusions being introduced as non-membered non-members by Lewis, something that is 'not a class' as Mendelson describes them now means more than just individuals/urelements, however. Clearly the issue of member eligibility comes to the fore here. We can see this especially in generalising the following quote, that "If there were [objects that are not classes], then the definition of equality would have to be changed, since all such [objects that are not classes] have the same members (namely, none at all)." (Mendelson 2015, 233) The equality being expressed here is strictly membership equality, as standard class theories all contain an axiom of Extensionality, the principle that no two classes have the same members.

Understood more broadly, this notion is usually applied to entities altogether, such that when individuals are introduced into membership systems, the definition of equality via members needs to be modified so as not to identify the null set with every individual – which would violate Extensionality in there being multiple distinct things which are identical along the lines of their lack of members. (Mendelson 2015, 304 explains how to define equality in a different way for classes, the null set and individuals.) As Mendelson sums it up "the domain of any [standard class theoretic membership system] will be divided into two disjoint parts[6] consisting of the classes and the urelements, and the classes are divided into sets and proper classes." (2015, 304)

This characterisation of the standard domain exposes how unusual Lewis's proposal is when it comes to standard class membership accounts. What the explanations above also show is that it is usually sufficient that if some entity does not have members, then it is eligible to be a member. As we ourselves are going along with Lewis's depiction of mixed fusions as non-membered (1991, 16) (explained in Chapter 3), we can see that this lack of members for them is the main reason that Lewis's restriction on being members for mixed fusions is unjustified. The differences arise here precisely because of GEM revealing other entities being in the domain of all existents, which then need to be compared against those already included and considered. So, although it is tempting to think that all non-membered things can themselves be members, with the introduction of GEM into the framework, it turns out that not all non-membered things can be members after all, for reasons similar to the restriction of membership for large classes. As we will see further in the next sub-section, it is in this way that Lewis's account properly goes beyond the standard accounts of class membership systems in entailing the existence of anti-members, instead of merely stipulating them.

2.3.2 Justified mixed fusion membership restriction

Although we answer the question of whether any mixed fusions can be members positively, this answer requires nuanced qualification. The further point of interest beyond the absolute question is that of *which* mixed fusions can be members? Could it be all of them? Only some of them? Perhaps just the 'small' sized ones?[7] From all I have said, it is clear that if there are some paradoxically problematic instances of mixed fusions being members that these should be able to end up in the

[6] Interestingly, Mendelson's account parallels that of Lewis's depiction of Reality dividing into the most inclusive class and individual as demonstrated earlier, except Mendelson's makes no mention of mixed fusions, of course. While I cite Mendelson (2015), the 1st edition of this work was published in (1966), so could have had some influence on Lewis's (1991) conception, although this work is not cited therein.

[7] What is meant by 'small' in this sort of context follows both the relative distinctions of size (1991, 88-91), as well as three hypotheses about the size of Reality (1991, 93-4) that Lewis makes. Roughly, all the things which don't have as many members or parts as the most inclusive class in Lewis's system are small, including many mixed fusions, all of the singletons and sets, and, on Lewis's (1986) view, any given concrete world. Everything else is 'large'.

division (iv) as anti-members, and that this is a justified restriction on membership in these instances, even as the standard conception of membership in normal applications of set/class theories do not even consider anti-members as an option.

The first case to be covered in this second stage of questions involves that of the opposite situation to Lewis's absolute restriction on class-formation. Can there be *no* restriction on class-formation for mixed fusions? The short answer is no, especially in the current context of proper class inclusion, because of a cardinality argument against it. As Wolfgang Schwarz explains,

> there are strictly *more* mixed fusions and proper classes than there are singletons: by [classical] mereology, there is no one-one correspondence between the fusions of any atoms and the atoms themselves. Hence there are more fusions of singletons than there are singletons. Since the singleton relation is a one-one map from the sets into the singletons, it follows that there can be no such map for the remaining fusions of singletons, the proper classes. As for every proper class, its fusion with any individual is a mixed fusion, the mixed fusions also outnumber the singletons. (2005, 14)[8]

The fact that there are more fusions of singletons than there are singletons, and also more proper classes than there are singletons, and that each of these fusions of singletons can form a further fusion with any memberless non-member understood as one of Lewis's individuals, say, shows that there are more mixed fusions than singletons too. Further to this, the proper classes make up the bulk of all of the classes that there are – to the point that there are more proper classes than there are sets even (Bremer 2010, 33). This all means that not all mixed fusions can have singletons, as in be a member of a singleton, and thereby any other classes, because there are just not enough singletons to "go around", as it were.

Now we are at a point somewhere between where some mixed fusions can be, but not all mixed fusions can be members of classes. Presumably any small-sized mixed fusion under these conditions can be a member of a class ('small-sized' here in terms of the number of parts such a mixed fusion has), because it has only really ever been that it is the large-sized entities which invite trouble in these circumstances. Although we have considered some large-sized mixed fusions glancingly, the only large-sized mixed fusion that I expect we might have any specific interest in is what Lewis calls Reality, because it is the mereological Universe, the sum of all things. Does Reality have a singleton? Is Reality a member of any classes?

[8] Nolan (2002, 166-67) also has a cardinality argument against all non-membered things being members in a similar context, which leads to him rejecting Lewis's Fusion Thesis, that all fusions of individuals are themselves individuals.

2.4 The mereological Universe as a member?

Before we answer that specific question, we look at a similar case arising in a smaller domain consisting of sets and individuals and their fusions, but no proper classes. In this case Uzquiano (2006) derives a paradox from the mereological sum, *U*, of all there is in a set sized domain having a singleton (in this context *U* is a smaller domain counterpart to Lewis's Reality, and an unacknowledged mixed fusion). In this case, we are introduced to a type of entity which has its own singleton as a proper part, viz. *U* as the fusion of all things (in its domain), which includes its own singleton. This leads to there being two different types of singletons – the ones which are parts of their members, and the ones which are not. This latter type of singleton is of the type Uzquiano calls the *R*s:

> 1. For every o, o is one of the *R*s iff there is some x such that $o = \{x\}$ and o is not part of x (2006, 145)

Continuing, from the principle of unrestricted composition we have some entity, *r*, which is the mereological sum of the *R*s. We also have, as a consequence of a set-based version of Lewis's Main Thesis (the parts of a set are all and only its non-empty subsets) the notion of *Unique Decomposition*, whereby a singleton is part of a sum of singletons if and only if it is one of them. The paradox which Uzquiano derives then arises when we ask whether $\{r\}$, which exists in accordance with unrestricted membership in the original context, is part of *r*?

> Since *r* is a mereological sum of some singletons, the *R*s, by Unique Decomposition, we have:
>
> 2. $\{r\}$ is part of r iff $\{r\}$ is one of the *R*s.
>
> 3. $\{r\}$ is part of r iff there is some x such that $\{r\} = \{x\}$ and $\{r\}$ is not part of x,
>
> By extensionality, we conclude:
>
> 4. $\{r\}$ is part of r iff $\{r\}$ is not part of r,
>
> which leads to a contradiction. (Uzquiano 2006, 145)

The contradiction is avoided in this smaller domain as one would expect – by denying that every entity has a singleton, and viz. by denying an earlier assumption that every entity is a member (2006, 141). In Uzquiano's example this means denying that *U* specifically is a member, because *U* as a member is the entity which introduces things having their own singleton as a proper part. Can we derive the same paradox in the case of proper classes and Lewis's Reality?

As in the first case, if Reality were to have its own singleton {R} as a proper part there would clearly be at least two types of singletons – ones which are "N"s, in not being part of their sole member (which {R} is not), and ones which are "B"s, in being part of their sole member (which {R} is). Similarly again, following the principle of unrestricted composition under GEM there would be a unique fusion of the singletons which are Ns, a fusion which we can call F. As understood, F is the fusion of all singletons which are not part of their sole member. If F would have a singleton, {F}, the question of whether {F} is a part of F arises and invites the paradox in this context. Is {F} a part of F? Is it one of the Ns? If the answer to the question of parthood is no, then {F} is supposed to be a part of F, because it is a singleton which *is not* a part of its member. It is one of the Ns. If the answer to the question of parthood is yes, however, then {F} is not supposed to be a part of F, because it is a singleton which *is* a part of its member. It is not one of the Ns. That is, for {F} to be a part of the thing which is the fusion of the Fs is for that thing not to be the fusion of the Ns. This leads to the contradiction that F is the fusion of the Ns iff F is not the fusion of the Ns.

To frame this argument slightly more formally, I model it after Uzquiano (2006). The problem I am showing is that under the condition where all of the singletons which are not proper parts of their members, the Ns, have a fusion F; and F is a member of {F} and:

1) {F} is a proper part of F iff {F} is one of the Ns;

then

2) {F} is a proper part of F iff $\exists x$ ({F} = {x} $\wedge$ {F} is not a proper part of x);

which is to say

3) {F} is a proper part of F iff {F} is not a proper part of F;

which is a contradiction.

Although at first this looks like the ultimate mixed fusion of Reality being a member leads to contradiction, we need to go back a little and ask, why think that F would have a singleton after all? If we have been maintaining that membership for proper classes is generally prohibited, then, in fact, membership for F would be prohibited too. This is because – following Lewis's Class Fusion Thesis and the amount of singletons it has as parts – F is a proper class. To see this, notice that if F has every singleton that is not a proper part of its sole member as part, and thereby every single thing which is a member of such singletons as its members, then it will be a large class. Since

singletons are themselves mereological atoms according to Lewis, any singleton of a singleton will count as a member of F.

So on the ordinary condition that every singleton has a singleton, F then has at least as many members as there are singletons altogether. Now, because any proper class has proper class many members, for it to be the case that F has that many members it needs to have proper class many singletons which are some of its subclasses, which it does. Accordingly, if F has at least as many members as there are singletons altogether, then F has at least as many members as any proper class. Given the limitation of size rule which Lewis adheres to, F thereby counts as a proper class in being the same size as any proper class, which means that it will not itself have a singleton under these conditions.[9]

Even if we forego Lewis's Class Fusion Thesis, however, we get a similar result with the fusion of proper class many atoms, of lacking a singleton (and being a member at all) whichever kind of atoms these may be, whether individual or class themselves. This is for the same reasons as before, where there will be strictly more fusions of the atoms than there are atoms altogether, necessitating that there are large fusions which miss out on being members because there are simply not enough singletons around to permit them to be. As Nolan (2002) (paraphrased)[10] explains

> Most of the fusions of proper-class many [atoms] will need to be denied the status of [members] themselves, and once singletons are denied to these entities (or most of these entities), the other fusions of [atoms] can all be admitted to be [members] without causing any trouble. This seems to be the obvious approach to take in the light of this result, and it ought not to be too surprising that the counter-intuitive results of this theory happen in connection to things having proper-class many parts: the paradoxes of set (and class) theory have been known (or suspected) to stem from some postulated things being "too big" since the time of Cantor. (2002, 167)

Although Nolan proves that this result applies to fusions of atomic individuals (2002, 166), I expect the result is able to generalise to mixed fusions as well, even with atomic singleton parts. This is because the argument is to do with the significant differences between the amount of atoms that there are, and their fusions, rather than the types of entities under consideration.

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[9] My thanks to Gabriel Uzquiano for advice on this point.

[10] This is paraphrased because Nolan is specifically talking about fusions of atoms that are individuals, and whether such fusions have the status of individuals themselves, as per Lewis's Fusion Thesis.

This restriction on membership is clearly the sort of restriction that it is justified to assert, in the sense that so long as the fusion F of all those singletons is not itself a member, a type of paradox will not arise. However, at this point, while it is fair to say that Lewis would undoubtedly endorse a restriction on mixed fusions being members to avoid Cantorian paradoxes, his lack of a defence for why small-sized mixed fusions could not be members shows why his absolute restriction is too strong and unjustified. With this opening into at least some mixed fusions being able to be members it becomes clear that any mixed fusion being a member or not is less to do with the thing being a mixed fusion, and more to do with its number of parts and the membership system which contains it.

While the avoided paradox means that Reality having its own singleton does not lead to a paradox in this regard (unlike in Uzquiano's more restricted example system which follows ZFC, whereby everything is a member), the understanding that so-called large fusions of parts generally are not themselves members due to an insufficient number of singletons also suggests that Reality is unable to be a member purely for the fact that it has the most number of parts possible, in literally having all the parts. However, as the unfounded contradiction to be derived from Reality having a singleton above leaves it open that Reality *can* have one, and thereby be a member of its singleton and other classes, it is worth exploring what else may be seen to happen in the case that Reality is a member. This is especially considering Lewis's own take on properties, whereby for a thing to have properties is for it to be a member of a class (Rosen 2015, Nolan 2002, McElhoes 2016). Reality is a special mixed fusion because it is the sum of all things in this context, and the notion that it has no properties is absurd. (This is especially considering Lewis's CAI claims, as discussed further in section 2.5.3 and also in Chapter 3, which should give Reality – and any mixed fusion really – its parts' properties in at least some manner, for the identity claim to make sense.)

To be clear, although most large-parted entities cannot be members due to the lack of singletons, this does not mean that all of them cannot be members. In the case of classes, while it looks to be reason enough to restrict class-formation when some classes could lead to certain paradoxes if they themselves were members, such as in the case of the class of ordinals, or in our example of fusion F above, this kind of restriction usually only applies because of the members of the classes involved and from what classes they are, and not specifically due to their amount of parts as classes. Similarly, although there will be many mixed fusions which are just like Reality in being proper class sized, in terms of parts, it is unclear if any one of these in particular cannot be members of classes for any other reason.

This means that instead of a restriction on membership being in general for all large-parted entities, the restriction on membership for large things is here mostly only a totality restriction. This

is in the sense that definitely not all large things (and of distinct types) can be members, and more because there are strictly not enough singletons for each of them to be members, and less because of their contents as entities. This is especially clear in the cases of large things which are not also classes, because without members they do not appear to have any kinds of contents as such to make their being members lead into paradox. So, while the great majority of large things will be non-members (cf., Lewis 1991, 27; Nolan 2002, 167) and regardless of their type (as in class or not) we are unlikely to know in advance *all* of which particular entities will lead to paradoxical problems in their own way, beyond that which arises due to a collective overload or those involving particular instances of contradiction already encountered, like with fusion F.

This all seems to leave us with at least two basic ways to go about advancing a restriction on membership – fully fledged like Lewis, and denying multiple types of entities from being members, with the guarantee that all of the problematic ones should be avoided; or strictly case by case, never expecting all or even a majority of cases to go through, but leaving it open that any potential or proposed case is worth looking at for the sake of plausible inclusion, rather than adverting to an outright ban from the outset. This is what might be considered a genuinely principled restriction on membership for certain types of things. I therefore end this section with the conclusion that, although not all mixed fusions can be members, many could be, including the mixed fusion which is Reality. As such, it is worth considering what being a member might entail for such a specific entity.

2.5 Problems with Reality as a member

Recall Lewis's statement, after characterising Reality as a mixed fusion, that it "divides into one part which is the most inclusive individual and another which is the most inclusive class" (1991, 8). Also recall the simplified part model of Reality in Figure 1.5 with C as the most inclusive class. The mixed fusion which is Reality, on that model, has some interesting yet problematic properties when it is treated as a member. Yet I don't think these are paradoxical properties, so let's examine why. As before, let's call Reality's singleton $\{R\}$. Considering that Reality has a singleton $\{R\}$ and C is both the mereological fusion of all singletons *and* a proper class with all individuals as its members, the singleton $\{R\}$ is both a part and member of C, as well as a proper part of Reality.

Meanwhile Reality is also a member of C, among infinitely many other classes, which are *all* also proper parts of Reality. While a thing having its own singleton as a proper part (and in general being a member of any of its proper parts) is highly unusual there is nothing clearly in the *letter* of either the mereological or class theories adopted which prohibits such occurrence. The problem we are going to pursue is that it seems like there should be an explicit prohibition in *spirit*

– after all the two theories of GEM and MKU do not exhibit these sorts of loop structures independently, so why should we accept their combination leading to such structures?

2.5.1 Well-founding against loopwholes

To properly illustrate the issue, we acknowledge a core class theoretical principle of 'Foundation',[11] which Lewis follows in his account and supplies as the principle that "No class intersects each of its own members." (1991, 103) As is understood, Foundation prohibits (1) membership loops and (2) infinitely descending chains of membership, such as:[12]

(1) $\ldots \in y \in x \in z \in y \in x$ (x has member y has member z has member x has member y...)

(2) $\ldots \in x3 \in x2 \in x1 \in x0$ ($x0$ has member $x1$ has member $x2$ has member $x3$...)

In other words, Foundation conditions classes to be *well-founded*. The importance of the well-founding of sets (and classes, as they arise) for standard membership theories is a common refrain amongst membership theorists. For instance, according to Maddy (1988, 484) supporters of the iterative conception of sets (such as Boolos 1971, 498 and Shoenfield 1977, 327) "often see [Foundation] as built into the very idea of the stages." Similarly, Aczel (1988, xvii-xviii) writes that "For the iterative conception only well-founded sets exist and [Foundation] and the other axioms of ZFC [from which the MKU ones are derived] are true when interpreted in the iterative universe of pure sets." These attitudes towards Foundation and its role in the well-founding of orthodoxically membered entities are also reflected in Adrian Mathias's mature view, that "set theory itself is ultimately the study of well-foundedness." (Kanamori 2016)

In a similar vein, one of the core mereological principles from GEM is 'Antisymmetry', which conditions mereological entities to be well-founded against certain loops by saying that if some things x and y are each part of each other, that they are the same thing. Antisymmetry prohibits (3) proper part loops, such as:[13]

(3) $\ldots < j < k < l < j < k$ (k has proper part j has proper part l has proper part k...)

An interesting difference between standard variations of membership theories and GEM is that instances of infinitely descending relations of proper parthood are not specifically excluded under GEM, even while loops are (Varzi 2016, 3.4). These instances arise in cases where there are things whose parts all have further proper parts – what Lewis named 'gunk' (1991, 20). While gunk is

[11] Although Lewis uses the German term 'Fundierung'. Across the literature Foundation is also known as 'Regularity'.

[12] '∈' is a common symbol for the 'is a member of' relation.

[13] For the sake of this illustration '<' represents the 'is a proper part of' relation.

non-well-founded (Cotnoir and Bacon 2012), and its existence is accepted as an epistemic possibility by Lewis (1991, 20-21; cf., Hall 2010), the system in *Parts of Classes* is flexible with regards to whether everything is either composed of just mereological atoms, or a mixture of both gunk and atoms.[14]

For classical mereologists, Antisymmetry is advanced on the grounds that it and other core mereological principles of transitivity and reflexivity[15] are "partly constitutive of the meaning of 'part'" (Simons 1987, 1.1.1). Although Varzi says that this "simply amounts to regarding identity as a limit (improper) case of parthood" (2016, 2.1), which, for some, engenders the "sense in which the antisymmetry axiom just is an extensionality principle for *improper* parthood" (Cotnoir 2010, 401) it *is* the case that Weak Supplementation, another GEM principle, entails "Antisymmetry so long as parthood is transitive and reflexive" (Varzi 2016, 3.1). While these statements suggest that Antisymmetry may merely be classical mereological orthodoxy, it is orthodoxy which Lewis subscribes to, and which mereological extensionalist commitments entail.

In all, the picture we receive from Reality being an individual, as within Lewis's system in this case, is that Reality itself is 'double' non-well-founded via cross systems through an infinite series of loops, as in:

(4) $\ldots \in \{R\} <$ Reality $\in \{R\} <$ Reality (Reality has proper part $\{R\}$ has member Reality...)

This type of double non-well-founding is also rather widespread in this case because it arises in every instance of a class which has $\{R\}$ as a subset. As far as I know, this phenomenon of double non-well-founding in itself has not received any attention in the literature. So what are we to make of such a novel yet apparently widespread occurrence wherein the mereological Universe is a rampant 'loopwhole'?

Before we set off I reiterate that because neither the frameworks of MKU nor GEM explicitly prohibit such an occasion by design, that there is no immediate formal objection to Reality having such a structure (as shown in the discussion earlier surrounding the fusion 'F' of all singletons which are not proper parts of their own members.) Yet clearly the spirit of each theory is violated, as both of them have restrictions on their respective relations which prohibit these being in loops. The conceptual issue for me here is that if the merging of the two systems brings non-well-

[14] Recall that as singletons are mereological atoms on Lewis's view, such atoms must at least exist, even if there is also gunk elsewhere.

[15] i.e., the principles "If x is part of some part of y, then x is part of y" (Lewis 1991, 74) and "everything is part of itself" (Varzi 2016, 2.1), respectively.

founding with it, the phenomena is normalized. So why not go for a genuinely non-well-founded system from the outset?

It is also worth considering what this looping and double non-well-founding issue could entail in the context of certain relations being treated as 'prior' to others, and to consider what this means for the prospects of combining such general theories as GEM and MKU.

2.5.2 Member priority and part priority

The contemporary debate puts two positions on priority relations involving parthood, known as *priority pluralism* and *priority monism*, in opposition. Roughly, priority pluralism takes the view that wholes ontologically depend on their parts, whereas priority monism takes the converse view that parts ontologically depend on their wholes. The notion of ontological dependence in use within the debates has recently centred mostly around either what is called metaphysical grounding or notions of fundamentality, able to be broadly construed as notions of "in virtue of", or of "priority".[16] So it may be said that a priority pluralist takes the view that composite entities exist in virtue of their proper parts, which makes these parts prior to, or grounding of, the whole, whereas it may be said that a priority monist takes the view that proper parts exist in virtue of composite entities which have them, making composites prior to, or grounding of, the proper parts.

Pre-theoretical intuitions about how parthood operates may incline one towards a part-to-whole direction of priority – drawing from such everyday cases as the building of a brick wall, perhaps, where the individual bricks as proper parts at least 'come first' as entities before the whole of the wall is completed. Yet Schaffer, a proponent of priority monism,[17] discusses how 'common sense' may also recognise whole priority in cases of esteemed 'genuine unity', even as it designates proper part priority in cases of 'mere heaps' (2014, 3.2.1). However, such talk of mixed priority with some wholes being posterior and some being prior is not reflected in the debate proper, as far as I can tell, and it is usually seen as a one or the other affair. To be sure, there is nothing in the system of GEM itself which guarantees an overall direction, as Bader (2013) makes clear, "Formal mereological systems are neutral regarding questions of priority. They characterise parthood structure but are silent on questions of ontological priority. Accordingly the idea that wholes are prior to their parts is not in conflict with [GEM]." (2013, 14) Even so, as Cotnoir (2013) explains, it is the default view that proper parts are prior to their wholes, which he frames as "Necessarily, the existence of parts is metaphysically prior to the existence of the wholes they compose." (2013, 67)

[16] See, for instance, Audi (2012).

[17] Schaffer's (2010; 2014) preferred view is a priority concrete monism, the view that exactly one fundamental concrete object exists, and is characterised as targeting concrete objects and counting by basic tokens, to get the universe as One.

Priority relations involving members are less debated, however, and the usual view is that members are prior to their classes, in the sense that the classes depend on their members for their existence. Lewis's commitment to the priority of members to their classes can be found in his acceptance of the standard iterative conception of set, which makes a non-membered member (the null set) the foundational element of all classes. This is what Lewis calls "the orthodox iterative conception of set" (1991, 61), and is expressed using similar language by Boolos, who states that it is an "important characteristic [of] sets ... that, in a sense [the] elements of a set are "prior to" it." (1971, 215-216).[18]

This conception is also indirectly expressed in Lewis's Priority Thesis, that no class is part of any individual, a thesis which is to be contravened in the case that any mixed fusions are individuals. Similarly, Lewis's (Individual) Fusion Thesis, that any fusion of individuals is itself an individual, does not stand in the case that there are proper class many individuals (Nolan 2002). With all small mixed fusions able to be individuals, there will be proper class many of them, because one individual which mereologically fuses with each singleton, of which there are proper class many, means there are proper class many mixed fusions consisting of one individual and one class, let alone greater numbers of singletons.

Recall Lewis's (albeit weak) support of these two theses quoted in my section 1.5, which also reveals his priority inclinations for members and proper parts.[19] Even with his vague expectation there that somehow the individuals "come first" and that classes are a "superstructure" Lewis does not really have the option to insist that the Priority Thesis trumps the general sense of membership inclusiveness that I have argued is the standard interpretation, which permits all small things to be members. What we can do in the Priority Thesis's stead is to repurpose the sentiment behind the asserting of priority for individuals, into an assertion of the priority of members.

While, on the surface Lewis's Priority Thesis looks to be the ontological-priority-of-individuals-to-classes thesis, going deeper it can easily be understood to be more about the ontological-priority-of-members-to-their-classes thesis, as if it is an implied Membership Priority Thesis, that the members of classes come before their classes. As noted with the iterative conception of set formation, this thesis is not without precedent in the literature. The utility of such a refinement for Lewis is found in its maintaining the gist of the reasoning given for asserting the

[18] In this regard Lewis specifically cites Pollard (1986) on plural quantification and the iterative conception of set, which explicitly builds on the just mentioned Boolos (1971).

[19] Mixed fusions themselves further betray the plausibility of whole priority being Lewis's position. Consider Lewis's assessment that "Mixed fusions are forced upon us by the principle of Unrestricted Composition" (1991, 8) and not 'individuals and classes are forced upon us by the principle of Unrestricted Decomposition.'

Priority Thesis, while also being consistent with the understanding that both individuals and some classes are members of classes. Considering what Lewis promotes, the Membership Priority Thesis is one he should accept.

If we have an ontological priority for members to classes *and* an ontological priority for proper parts, however, we look to be in a terrible bind. With priority relations understood in this way how can a member such as Reality be prior to its classes, while those classes are prior proper parts of it? The reason that there is a problem here is that the priority relations need to be complementary (or at least not in conflict), which is to say that if members are prior to their classes as the orthodox iterative conception of sets looks to entail, that the whole which is Reality needs to be prior to any of its parts which it is a member of, to avoid contradiction. That is, for such an account to avoid some conflict over the direction of priority relations, it looks to at least need to be priority monist.

2.5.3 CAI and priority relations

One line of response that might be taken here is that, even if Lewis seems to prefer priority pluralism, that his other avowed commitments allow some flexibility on this matter. Consider Lewis's view of CAI: "I say that composition – the relation of part to whole, or, better, the many-one relation of many parts to their fusion – is like identity. The 'are' of composition is, so to speak, the plural form of the 'is' of identity. Call this the Thesis of Composition as Identity. It is in virtue of this thesis that mereology is ontologically innocent: it commits us only to things that are identical, so to speak, to what we were committed to before." (1991, 82)[20]

What CAI could mean is that there are no relations of priority between parts and their wholes.[21] For instance, Bricker writes that

> if composition is a kind of (total) identity, and kinds of (total) identity are all symmetric, then the debate [over whether the parts are prior to the whole or the whole is prior to the parts] makes no sense; the mereological relations between objects, by themselves, are irrelevant to questions of priority. The only way in which it could make sense to speak of some objects being prior to others is

[20] For a wide-ranging overview of this concept, see the volume of Cotnoir and Baxter (2014).

[21] For a contrary view, see Loss (2016). For some concerns with the combination of CAI, priority pluralism and irreflexive grounding, see Bailey (2011).

> derivatively from the properties and relations that they instantiate, and whether those properties and relations are fundamental. (2016, 285)[22]

This allows that so long as the properties and relations which Reality instantiates are fundamental, that *it* could be the prior entity. Yet this only seems to further put us in the position that for the account of Reality to be at least consistent in this aspect, that it requires a type of priority monism. Yet this is no good for Lewis, as it seems from what was cited earlier that he is not inclined towards accepting that Reality-the-whole has such fundamental properties. There is one relatively underexplored other option here, at least to be acknowledged, which is known as *metaphysical coherentism*, the view that dependence chains can form loops, and which can be contrasted with the more familiar *metaphysical foundationalism*, which asserts a terminal priority base (Tahko 2018).

Beside the fact that, as far as I know, the type of double non-well-founding I have described for Lewis's Reality is an unprecedented form of metaphysical coherentism, because Lewis is not himself a known proponent of metaphysical coherentism it is not feasible that his account could stand for such a view intentionally. Additionally, because no singleton for Reality is postulated by Lewis, he clearly does not expect the account in *Parts of Classes* to endorse such an option. Which is all to say, if Lewis's account ends up being an inadvertent promotion of metaphysical coherentism, it is not well defended as such. Accordingly, because it appears unlikely that this type of ontology would be acceptable on Lewis's own expressed grounds anyway, we can also choose to reject this and other loopwholes as a plausible option for him to pursue.

So we are left with an unjustified priority monism. Is it really so bad? What are some of the consequences of this option? The first follows from Bricker's statement, which is that mereological relations are no determinant of ontological priority. As far as I am aware this is actually an unusual position for a priority monist to take, in that some of what their priority position is about is specifically to do with the whole having priority, not just being a fundamental entity. To avoid this, let's consider what the landscape looks like if, instead of denying mereological priority relations, whole-to-part priority is accepted. To help discuss some of the consequences of this option, I introduce some formalism.

[22] Following a similar line as the first is Varzi, who states, "[The] truth of Composition as Identity does not depend at all on one's specific views about such matters. Whether you give priority to the wholes or to the smaller parts that compose them, you are dealing with the same "portions of Reality" and you can talk about those portions either way. That's what the thesis says." (2014, 65)

2.5.4 Over-determining priority relations

Let $x \mapsto_M y$ represent the relation that x is membership prior to y, where x is a member of y; and let $x \mapsto_P y$ represent the relation that x is parthood prior to y, whether x is a proper part of y or x is a composite for y (such that x's improper part is prior to y's improper part). Also let directional variations of the arrows represent their respective relational variants. Now to consider a simple model of the view which Lewis appears to actually hold: Presume that some mereologically atomic individual d has a singleton $\{d\}$, such that $d \mapsto_M \{d\}$. By unrestricted composition there is a mereological fusion of d and $\{d\}$, which we can call D. According to what Lewis's priority expectation appears to be we get the following, where d is prior to $\{d\}$, as its member, and these are both prior to D, as its proper parts

$$\begin{array}{ccc} & D & \\ \uparrow_P & & \uparrow_P \\ d & \mapsto_M & \{d\} \end{array}$$

Figure 2.1

Our original problem with Lewis's Reality, or R, was that the account of it being a member which still followed Lewis's preferences resulted in incompatible priority relations, whereby Reality was prior to its singleton, which was also prior to it, i.e., $R \mapsto_M \{R\}$ *and* $\{R\} \mapsto_P R$. What has been anticipated is that Lewis needs to commit to Reality being prior to its proper parts, especially because it is a member of so many of them, for the system to be at least consistent in this aspect. Therefore, we expect that Lewis needs to endorse his account showing that both $R \mapsto_M \{R\}$ and $R \mapsto_P \{R\}$, among other parts. But what about all of the other mereological composites in Reality? If whole-to-part priority is accepted, it is absolute, which means that for *any* composite whole that whole is prior to its proper parts. This beckons two problematic cases.

The first case arises if we suppose that there is some mereologically atomic individual z which has a singleton $\{z\}$, such that $z \mapsto_M \{z\}$. By unrestricted composition there is a mereological fusion of z and $\{z\}$, which we can call Z. According to our newly anticipated priority directions we get the following, where z is prior to $\{z\}$, as its member, and Z is prior to both of z and $\{z\}$, as their composite:[23]

[23] Here two part-priority arrows for the whole have been given, one for each of its proper parts. This occurs in the next case too.

$$z \mapsto_{\mathrm{M}} \{z\}$$

$$\uparrow_{\mathrm{P}} \qquad \uparrow_{\mathrm{P}}$$

$$Z$$

Figure 2.2

The next case arises if we suppose that there are the unrestricted composition of some singletons $\{p\}$ and $\{q\}$ of some entities p and q, of a composite we can call Q. In this case we get the following, where p and q are each prior to their respective singletons, and Q is prior to both of $\{p\}$ and $\{q\}$:

$$p \qquad q$$

$$\downarrow_{\mathrm{M}} \qquad \downarrow_{\mathrm{M}}$$

$$\{p\} \qquad \{q\}$$

$$\uparrow_{\mathrm{P}} \qquad \uparrow_{\mathrm{P}}$$

$$Q$$

Figure 2.3

The problem, which should be made clearer by the visualisation, is that in each case certain entities look to be over-determined by things with priority relations to them. Recalling our earlier characterisation that the ontological priority of x to y implies that y exists "in virtue of" x, this is a severe problem. In the first case the over-determination arises where $\{z\}$ exists both in virtue of z as well as in virtue of Z. In the second case the over-determination arises where $\{p\}$ exists in virtue of both p and Q, and $\{q\}$ exists in virtue of both q and Q. Note that it is not the case that the problem is because of multiple "in virtue of" relations in general. Collective multiple "in virtue of" relations are exactly the type of relations expected under a part-to-whole priority ontology, where the proper parts of some whole are necessary and sufficient for the whole to exist in virtue of them only collectively.

The problem here is that several of the "in virtue of" relations in the two cases appear to be sufficient for the parts or the singletons to exist, yet in virtue of either the whole or the members separately, which makes neither of them necessary, and therefore separately rejectable. For instance, we can easily imagine $\{z\}$ existing without Z, as it would still have z, and thereby still be founded on something, just as we can imagine either $\{p\}$ or $\{q\}$ existing without Q, as they would

still have *p* and *q*, respectively, which completely undermines the claim to their dependence on Q. This would be the case if there were no unrestricted composition, or there were some other composition relation which permitted fusions like those in the two cases. In this way the members of the classes appear to be 'enough' for the classes to exist, making some priority composite with the classes as parts gratuitous.

In the second case alone the other option is worse, in that were *p* and *q* not to exist, {*p*} and {*q*} presumably still would, because of *Q*. Notice that it would be inappropriate to strip the first case of *z* and leave all else as is, as *z* is supposed to exist in virtue of *Z*. Otherwise it would be apparent that *z* never genuinely existed in virtue of *Z* in the first place. If we consider that in the first case {*z*} seems on par with *z* in terms of priority, in that they both exist in virtue of *Z*, it is apparent that both of these types of possibilities serve to undermine any plausibility that the "in virtue of" relations literally pertain to members and their classes. Although it could solve the issue to reverse the member-to-class priority relation direction to be that of class-to-member this both has no precedent in the literature and is terribly counter-intuitive.

I believe the issue is better met by Lewis in this instance by being eliminative of the competition between the priority relations, rather than reversing member priority to be class priority instead,[24] because otherwise everything will at least still be founded on Reality for needing to make whole priority universal due to issue with classes. Yet this elimination is not an option for Lewis in the main text of *Parts of Classes* because there he gives precedence to singleton formation as a primitive, stating a belief that "The notion of a singleton, or unit set, can serve as the distinctive primitive of set theory." (1991, vii) This ends up providing us with further evidence that Lewis is committed to the proper parts of things being prior to their wholes, following his redefinition of a class. "If we take the notion of a singleton henceforth as primitive, we have this new definition of a class [...] A *class* is any fusion of singletons." (1991, 16)[25]

This redefinition runs contrary to the notion of the wholes of plural classes being prior to their singleton proper parts, as it is based on the idea of composition leading to the existence of an entity. This is especially apparent when we compare it with what Lewis says in a later chapter: "begin with singleton rather than membership as the primitive notion of set theory. We can leave it to mereology to make many-membered classes by fusing together singletons." (1991, 61)

[24] Obviously if this is the option taken, the predicament of looping vanishes. Elimination is to be examined below.

[25] This redefinition comes after Lewis's proof for the Second Thesis and replaces the earlier, informal definition of a class as a thing which has members (1991, 4).

In the current context, this belief in singleton formation as primitive appears untenable. This is because if wholes of plural classes are prior to their singleton proper parts, such that the singletons exist in virtue of the plural classes that they are parts of, it would entail that no such primitive operation as singleton formation plays a role in their existence. To avoid this issue involving singleton formation as primitive it appears that there is at least one other move available by turning towards Lewis's own structuralist re-interpretation of his program, supplied after the original ending to *Parts of Classes* in the joint "Appendix on Pairing" with Burgess and Hazen, and which was written after Lewis had finished most of the book proper.

2.5.5 Eliminating competing priorities?

On the structuralist interpretation of set theory which is given in the Appendix of *Parts of Classes*, the primitive notion of singleton can be eliminated by Ramsifying it out, so long as there are the systems of plural quantification and mereology, and the entities of infinitely (inaccessibly) many atoms in Reality (and, if any, not too much atomless gunk).[26] Here quantification over relations is simulated by constructing a suitable method of pairing entities one with another in an order (cf., 1991, 53), whereby these pairing relations can stand in for the set-theoretic ones, to the effect that "there is no primitive notion [of singleton formation] that needs understanding; [as] all the suitable relations are equally 'the' member-singleton relation" (1991, 142).

In this way, it appears that there is no longer an issue about member priority as discussed, because whichever atom stands in for the singleton of Reality under one relation is the singleton of something else under another relation. As Lewis, Burgess and Hazen explain "If all suitable relations alike are member-singleton relations, then a singleton does not have its member once and for all. For anything that can be a member, for anything that can be a singleton, there will be some member-singleton relation that pairs the one with the other. The atom that is [e.g., Lewis's cat] Possum's singleton under one suitable relation is [e.g., Lewis's other cat] Magpie's singleton under another." (1991, 143)

This looks to settle the problem of member priority, because by the invocation of structuralism classes do not ontologically depend in any sense on their members to themselves exist. Rather, the entities which exist do so already in their given manner, and merely provide a relational basis that can be co-opted by the structuralism for it to manifest an overlaid, virtual system of membership across them, as it were. This in turn simply allows certain entities to *serve* as classes by fiat, instead of requiring certain entities to naturally and solely be such creatures. In this manner, this structuralism is able to eliminate any bona-fide objects of classes altogether, leaving

[26] For all of the details see the full appendix from Lewis with Burgess and Hazen (1991, 121-149).

every entity as genuine anti-members, and only having some of these function as classes through the prism of the system. While such a move is available to later Lewis especially,[27] it is a step he is reluctant to take earlier for not wanting to go against mathematical orthodoxy (e.g., 1991, 50, 58, 61). Perhaps, if I'm not mistaken about the tension in priority relations, the issues I've described in this regard could prove to be a further motivating factor to make that switch into a form of structuralism.

Yet the significant ontological infrastructure and outlay that is required to arrive at such an alternate system, namely proper class many atoms to serve as the basis for the structuralist system, ensures that making this move does not come without its own controversy. For example, such a system looks like it would require a significant multitude of possible worlds, and not just a single one, as it is the case that large transfinite cardinalities of physical or concrete entities are difficult to still regard as (single) concrete or physical systems (Parsons 1990).[28] Similarly, "We seem to have no reason to believe that there could be physical worlds that contain highly transfinitely many entities" (Horsten 2017, 4.4), which appears to leave us compelled to entertain a multitude of concrete possible worlds to serve as the basis of the structuralism. (If we were to just have these entities as some kind of abstracta, there would be less reason to structuralise classes after all.) Fortunately for Lewis, he already has his controversial (1986) system of infinite, concrete possible worlds at hand, which could provide the needed infrastructure (as explained in Nolan's work, as mentioned in a recent footnote).

For our purposes, and for the rest of this thesis, we do need to note that, besides removing the issues of member priority that we have been considering, this strategy does also eliminate natural mixed fusions from the equation. What remains of these types of things in this instance is that they are only able to be residual, 'pseudo-entities', arising only insomuch as that on any given member-singleton relation construal some things will serve as classes and some will not, and by GEM will form a composite which can be regarded as a mixed fusion under that construal.

Gladly, this outcome does not suffice to undermine the rest of the work of this thesis, however, because maintaining sets or classes as their own, natural beings is still the norm in philosophy. This allows us to continue our analysis of mixed fusions along the lines of the main text

[27] See Nolan 2002, Chapter 7 and 2004 for the detailed requirements for Lewis to be able to achieve such a system, which also build upon Lewis's 1998 deeper commitment to structuralism than he approached in the main text of *Parts of Classes*.

[28] Cf., Tallant saying that "if there is gunk, then there is an actual infinity of proper parts that exist. We have some reasons to think that actual infinities, though mathematically describable, cannot be realized on pain of contradiction." (2013, 435)

of *Parts of Classes*, for still being an analysis of mixed fusions under that interpretation. In addition to this, some of my arguments in Chapter 5 look to challenge GEM and unrestricted composition solely on the basis of trans-world fusion anyway, which a structuralist revision of Lewis's mereological class theory (as per Nolan 2002, Chapter 7 and 2004) is still committed to. (I also remark upon this situation again directly in Chapter 7.)

So, although I accept that this type of structuralist move underpinned by a certain ontological system seems like it could be effectively used to avoid my proposed issues over priority relations in this chapter, some of my arguments in later chapters show that this move still has its own problems involving unrestricted composition, and even if mixed fusions are not necessarily the target of concern in that instance.

2.6 Conclusion

At the outset of this chapter I asked a question about how mereological hybridism involving fusions of member and non-member entities could result in those fusions being either membered or non-membered. From all that I have said, whether something is a member or not is apparently only pseudo-intrinsic to the thing itself, in that the restrictions on membership which a system such as MKU entail in the context of Lewis's theory are justified more on the grounds of the entities within the system in its entirety, rather than the specific content of any particular entity in terms of the parts it has or even the members (modulo those few specific ones which cannot be members, such as the class of ordinals, for example). This makes claims to there being an issue involving mereological hybridism here somewhat obscure, as the member types of the components of such mereological sums have little to no bearing on the member types that the sums themselves are or could be. Instead, the collective problem for large entities is that not all of them can be members without paradox ensuing. Yet when regarded on a singular scale, these appear to be mostly innocuous, allowing for some of them to be members, especially if they are understood to be important entities in their own right. Restrictions on membership look to stem mostly from there not being 'enough' singletons, rather than from the size of the entities specifically.

From here, however, I was able to entertain the prospect that the mereological Universe in Lewis's system, Reality, is a member, primarily because it is an important mixed fusion, and from the fact that its being a member does not entail any known set-theoretical paradoxes in a domain which includes proper classes. In examining the potential consequences of such an arrangement by following the standard views of mereological priority we ended up with the undesirable situation of the priority relations either being contradictory or over-determined, or largely unprecedented and derivative of an already minority view, making it rather undefended and unwelcome to accept.

However, I did acknowledge that there is a structuralist strategy available to avoid my challenges involving priority, even though I expect that some arguments in my later chapters will still result in significant problems for the view.

Although parthood does play a role in the priority issues, by way of something having its own singleton as a proper part leading to trouble, I did not arrive at any direct rejections of mereological hybridism specifically from these discussions, however. This is because it is not clearly a problem stemming from a mixture of categories of entities, but rather a problem stemming from how parthood and membership can operate without any regard for the other. As such, what I have shown in arguing beyond similar lines as Uzquiano (2006) are some of the consequences able to befall such a combination of systems of relations, rather than consequences specifically from a mereological combination of entities with incompatible properties. In this way, the situations described are not presented as defeating of the overall project of a mereological Universe such as Reality in terms of it being a transcategorial sum, per se. However, as it *would* be baseless to restrict membership for the mereological Universe in this case for the reasons I presented alone, as they do not lead to set-theoretical paradox, the motivation to restrict composition instead to avoid the problems is at least increased, even if not given outright. This is, of course, a consequence which leads to a rejection of GEM.

If this motivation for rejecting GEM is not compelling, I believe I have still in some measure shown that such an object as Reality is one that is more difficult to accept given the costs of certain paths and outcomes being made explicit. In this case, this chapter at least stands as an exposing of some of the challenges which accepting such a mereological hybridist account as Lewis's might entail, and reveals how his conception and presentation did not adequately account for these varied issues that do need to be met when determining whether to accept such a system.

In the next chapter we move on to consider a reason to outright reject mereological hybridism like that in *Parts of Classes*, a rejection involving the question of whether mixed fusions themselves *have* members or not.

Chapter 3

How mixed fusions confound Composition as Identity and unrestricted composition

3.0 Introduction

CAI is a thesis about composition which generally asserts that the parts of a whole are in some sense identical to the whole, and vice versa, and it has recently been defended by a number of philosophers (cf., Cotnoir and Baxter (eds) 2014, Smid 2015). One influential proponent of CAI is David Lewis (1991) who argues for the thesis as supportive of GEM in the context of detailing a system of parthood for the mathematical objects of classes.

In this context Lewis introduces us to mixed fusions of classes and individuals as a consequence of GEM's principle of unrestricted composition, and in this chapter I demonstrate a significant problem for CAI with regards to the prospects of there being mixed fusions. The problem is simple – if CAI is true, then mixed fusions are themselves classes, because for mixed fusions to be plausibly identical with the classes which are their parts they need to have all of the same members as their largest class parts. This is a consequence which I argue violates the standard set-theoretic axiom of Extensionality, thereby ruining the project of mereologising set theory as Lewis sets out to do.

Although I assess that Lewis (and any others who hold both a CAI thesis and some principle of composition which permits mixed fusions) is most obliged to drop CAI rather than GEM in this case, I show that a further, related, challenge to GEM remains. This next problem is that, under widely accepted and non-CAI dependent notions of property inheritance – whereby composite wholes are understood to inherit their own properties from those of their parts – some entity having parts with members is sufficient for that entity to itself have those members, such that any entity with class parts is itself a class, and, as before, the same class as its largest class part.

As this too violates Extensionality we are left with a choice between rejecting property inheritance or rejecting the composition principle which facilitates this problem (rejecting Extensionality is not an option here). I will argue in favour of retaining the far more widely accepted notions of property inheritance in this instance, and thereby call for the rejection of any composition principles which permit mixed fusions, including GEM's principle of unrestricted composition, so as to avoid mixed fusions and their attendant violations of a standard set-theoretic axiom. My conclusion, in turn, represents an instance of rejecting mereological hybridism in the case of the fusion of membered and non-membered things.

3.1 Lewis on whether mixed fusions have members

In the last chapter I asked whether mixed fusions could themselves *be* members of classes in the context of the merging of set/class theory and GEM à la Lewis's *Parts of Classes*. I concluded *yes*,

so long as not all of them are members, as this would lead to set-theoretic paradox. In this chapter I ask the complementary question of whether mixed fusions can *have* members – a question we need to consider to properly determine the nature of mixed fusions for the sake of evaluating their plausibility as beings. As it turns out, while I do demonstrate that mixed fusions *having* members actually leads to contradicting a set theoretical axiom, I also explain how mixed fusions *not having* members leads to the problem of contravening widely accepted notions of property inheritance from part to whole. My solution to avoid these issues is to reject unrestricted composition, and any other principles of composition like it which permit mixed fusions.

As in Chapter 1, we first look to the originating position which Lewis (1991) provides about the nature of mixed fusions with regards to their having members, and consider it to determine what to retain or discard. When it comes to the earliest sections of *Parts of Classes*, we see that for an answer to the question, "Can mixed fusions have members?" Lewis's presentation is more of an inclination that they do not have members, rather than any direct argument that they do not.

We see this inclination implied in several statements, such as when Lewis says, "the Division Thesis[1] permits there to be a mixed thing [i.e., composed of individual/s and class/es] which is *neither* an individual nor a class," (1991, 8), and, "Reality itself – the mereological fusion of everything – is mixed. It is *neither* individual nor class" (1991, 8) (my emphasis in both).[2] Beyond these indirect suggestions that mixed fusions do not have members, Lewis makes some more explicit statements further along in the text which are a little more direct, albeit fairly weak. This is with his redefinition of a 'class' as "any fusion of singletons" (1991, 16), and his surrounding statement that "a mixed fusion, part individual and part class ... does have singletons as parts, yet probably we would not want to say that it has members." (1991, 16)

Recalling that, by Lewis's own lights, if something has members it is thereby a class (cf., 1991, 4), his reluctance to be definitive here is strange, as it stands at odds with several of the other things he has said and done by this point in the text. For starters, just prior to this statement Lewis gives a redefinition of a class as any fusion of singletons. Now, as a mixed fusion is more than any fusion of singletons, we would not want to say that these have members if we have defined classes as fusions of singletons. Even more strongly than this, though, is that a few pages earlier Lewis provides a proof for his Second Thesis, that no class has any part that is not a class. As mixed fusions by definition *do* have parts that are non-classes (in their individual-type parts), Lewis should

[1] That "Reality divides exhaustively into individuals and classes." (1991, 7)

[2] Although these statements also advise that mixed fusions are not individuals either, this claim has been discounted for many mixed fusions in Chapter 2.

state that we definitely would not want to say that any mixed fusion has members, as a mixed fusion with members would be a class and would thereby contravene his Second Thesis.

By reading Lewis's Second Thesis with the prospects of mixed fusions having members in mind, we can see that if the thesis were true then, indeed, mixed fusions could not have members and thereby be classes, due to their proper parts which are individuals. So it is apparent that Lewis is not clear on the possibility of mixed fusions having members being a point of issue when he says, as quoted earlier, "a mixed fusion, part individual and part class [does] have singletons as parts, yet probably we would not want to say that it has members." (1991, 16) I think it should be obvious that defenders of the Second Thesis would *definitely* not want to say that a mixed fusion has members, because these would then be classes with non-class parts. My suspicion is that Lewis's too early and too easy dismissal of mixed fusions from his consideration in *Parts of Classes*, in relegating them to only "our most unrestricted quantifying" (1991, 8) ensured that he wrote them out of his concerns for the most part, which I think is what unfortunately led to him not properly considering whether mixed fusions could have members either.

I say this is unfortunate, because, clearly, as one of the consequences of the Second Thesis being true is that mixed fusions cannot be classes, it would be interesting to know Lewis's more definite and explicit view on the matter of whether mixed fusions could have members. I also say this is unfortunate because what I will mainly be arguing in the next section is that, by following Lewis's other express commitments in *Parts of Classes*, mixed fusions are more naturally seen as *having* members than not having them. The short version of my argument for this conclusion is as follows: mixed fusions are composites which have parts which have members, therefore mixed fusions have members. In the coming section I will explain how this conclusion is supported by Lewis's avowed view of CAI.

3.2 Can mixed fusions have members? What Lewis's account implies

In this section I argue that Lewis's avowed CAI thesis supports the conclusion that mixed fusions have members and are thereby classes. This follows from a consequence of the thesis that composite wholes are in some sense identical to their proper parts, such that if their proper parts have members, the wholes do too, so as to properly retain a suitable sense of identity with them. Beyond this I also argue that a widely accepted notion of property inheritance – which entails that composites inherit properties from their parts – also implies that mixed fusions have members and are thereby classes. As the notion of property inheritance is something which can itself stand apart from CAI (although the CAI style views are committed to it), this makes the idea that mixed fusions

have members one which should be widely accepted, so as not to contravene a notion of property inheritance which "everyone accepts" (Sider 2007, 75).

Although I think mixed fusions having members is a view which Lewis is implicitly committed to because of his commitment to CAI, and that anyone else who accepts mixed fusions otherwise is committed too, my argument later in section 3.3 is to show that mixed fusions having members is a consequence which should be rejected, as it leads to contradictions of the set theoretic axiom of Extensionality. My strategy in this main section, then, is to show that virtually everyone who accepts mixed fusions is obliged to accept that mixed fusions would have members based upon either CAI or property inheritance, with my strategy in the next section to then show that mixed fusions having members is actually a disastrous consequence for the very theories which provide their class parts. With this dilemma in place the solution to the problem which I advocate is to withdraw the entities which make trouble – and so I argue to reject any composition principles which permit mixed fusions in the first place. As a specific target, I focus on the principle of unrestricted composition which Lewis endorses and utilises in *Parts of Classes*, and argue that it, and thereby the classical mereological system in which it is contained, should be rejected.

3.2.1 CAI

Lewis introduces the CAI thesis as follows: "I say that composition – the relation of part to whole, or, better, the many-one relation of many parts to their fusion – is like identity. The 'are' of composition is, so to speak, the plural form of the 'is' of identity." (1991, 82) To notice here that Lewis is saying that composition is only *like* identity is to notice the relative strength of Lewis's identity claim. Indeed, Lewis is typically taken to uphold a so-called 'weak' form of CAI (e.g., Smid 2015, 9 fn 3, Yi 1999, van Inwagen 1994, Sider 2007, 78, Bennett 2015, 256, Cotnoir 2014, 9), because his position, as he describes it, is one asserting that composition is "strikingly analogous" to identity (Lewis 1991, 84), rather than saying that composition *is* identity, which is clearly a much stronger claim.[3]

These differences reflect at least two of the main variations of the CAI thesis which are present in the literature. Following Cotnoir (2014, 9) these are, as follows

Analogous CAI	The relationship between the parts taken collectively and the whole is *analogous* to identity.
Numerical CAI	The relationship between the parts taken collectively and the whole is *numerical* identity.

[3] The weakening of the commitment is also expressed in Lewis's "so to speak" in the quotation's last sentence.

To be clear Cotnoir actually labels these 'Weak' and 'Strong' CAI respectively. However, due to divergences in the literature in applying such labels it is helpful to avoid the 'strength'-relative names of the theses, and make them wear their commitments on their sleeves. Lewis clearly endorses at least Analogous CAI, both in his own words and because endorsing CAI at all seems to require at least this position at a minimum. Lando (2017) breaks Lewis's view up into the holding of the following four subsidiary views

(LA) a fusion is nothing over and above the fused entities.

(LB) a fusion and the fused entities are the same portion of reality.

(LC) a fusion is not a further ontological commitment over the fused entities.

(LD) GEM is ontologically innocent.

For our purposes in this chapter we will be focussing mainly on the first two, but will return to address ontological innocence and commitment in Chapter 7. While other defenders of CAI might split this package up in different ways by only endorsing some of the views and not others, Lando advises that Lewis avoids holding Numerical CAI by specifically not endorsing

(LE) a fusion is identical to the fused entities.

This is because Lewis says, "What's true of the many is not exactly what's true of the one. After all they are many while it is one." (1991, 87) Lando (2017, 205) takes this as an admission that the fusion is discernible from the fused entities, in such a manner that treating the fusion as strictly identical to the fused entities would not make sense of this discernibility. Lando thinks that this lack of indiscernibility is further backed up by Lewis stating that CAI has its limits and does not establish "a generalized principle of indiscernibility of identicals." (1991, 87)

Contrast Lando's understanding of Lewis's position with that of Bohn (2011), however, who argues that composition, for Lewis, is only analogous to identity taken as a *one-one* relation, as in "x=y", in distinction to when it is taken as a *many-one* relation, as in "xx=y", where it is to be regarded as literally, and strictly, identity (Bohn 2011, 154). We can see this when Lewis says that he remains unconvinced that composition involves this "ordinary one-one kind" of identity (Lewis 1991, 84 fn 12; cf., Bennett 2015, 256) which he describes as "the special, limiting case of identity in the broadened sense." (1991, 84) Bohn understands Lewis as offering a more general notion of identity to which composition belongs here, and takes Lewis's claim to composition being analogous to identity to mean that "composition is analogous to ordinary one-one identity, which is not saying that composition isn't really identity. Composition is literally and strictly speaking

identity in the general sense, but it is not literally and strictly speaking ordinary one-one identity." (Bohn 2011, 154)

I believe that for either way of understanding Lewis's own commitment to CAI that the consequence of mixed fusions having members is still obtained. Following Bohn's understanding of Lewis's view of CAI, the claim that mixed fusions have members appears to be granted immediately. We can see this if we treat the property of having members as a cumulative one, similar to weight. So, for every part of the mixed fusion which has members, these members add up to be the members of the mixed fusion, avoiding any double counting for the members given by any other membered parts already. Similarly to how the class part's members add up to be the members of mixed fusions, all of the individual parts of mixed fusion contribute zero members each to this total. Now, in terms of membership specifically, the relationship between the parts of the mixed fusion taken collectively and the whole mixed fusion is one of numerical identity, as the mixed fusion is identical to the fused entities. In this context, by Numerical CAI, mixed fusions have members.

3.2.2 Property Inheritance

To follow Lando's way of understanding Lewis's commitment to CAI, however, more discussion is in order. As we encountered in terms of LC and LD, one consequence of CAI which Lewis expects is the *ontological innocence* of GEM. Lewis expects that this is secured by CAI because then "[GEM] commits us only to things that are identical, so to speak, to what we were committed to before." (1991, 82) The concept of 'innocence' in play here portrays the notion that composite entities come at no additional cost to our ontology, and that electing to accept the existence of such entities as composite ones is no different from electing to accept the existence of their parts. (In Chapter 7 I look more into Lewis's claims to the 'ontological innocence' of GEM and dispute it.)

Lewis describes the support for CAI as consisting in five aspects (1991, 85-6) with the most important one for our purposes being what Lewis portrays as 'the ease of describing fusions', and which Bennett (2015) more succinctly calls 'property inheritance'. It will be instructive to quote both Lewis's[4] and Bennett's presentation in succession.

> The ease of describing fusions is a fourth aspect of the analogy [i.e., of CAI]. Describe Possum fully, and thereby you fully describe whatever is identical to Possum. Describe Magpie and Possum fully – the character of each, and also their interrelation – and thereby you fully describe their fusion. (Lewis 1991, 85)

[4] Recall that 'Possum' and 'Magpie' are the names of Lewis's cats.

> Both composition and identity involve *property inheritance*. Each *x*'s properties supervene (trivially) on *x*'s properties, intrinsic and relational. Similarly, each *x*'s properties supervene on the properties of its parts, intrinsic and relational. (Bennett 2015, 257)

Bennett continues on to say that of the five aspects of the analogy of CAI, there are three which Lewis "feels at least some obligation to [defend]. In contrast, two of them – aspects 4 [the notion of property inheritance we are looking at] and 5 [the blocking of multi-location notion] – are fairly uncontroversial and not really in need of argument. They are interesting phenomena that can legitimately be taken as data; it is hard to deny that composites inherit their properties and locations from those of their parts."(Bennett 2015, 257)

The claim that property inheritance can legitimately be taken as data is a strong one, and clearly interesting for our concerns. This expectation that it is hard to deny that composites inherit their properties from those of their parts finds expression in other recent works too, to the point that no-one asks *whether* property inheritance occurs, but instead are more concerned with *why* it does. We can see this in a question Cameron (2014) asks: *Why* do wholes inherit their part's properties?[5] On the notion that a whole's intrinsic nature supervenes on its part's intrinsic natures, Sider (2007) even goes so far as to say that "*everyone* agrees [that] the inheritance principles are true" (2007, 74), with the understanding that if they are true, then this intimacy of the part-whole connection needs to be explained (2007, 75). In addition, Cameron also considers that any question of why parts and their wholes are intimately located is "a special case of a more general puzzle" of why wholes inherit the properties of their parts – another instance of taking it for granted that property inheritance occurs and that it is a feature to be explained, not established. (As one illustration of why it might be a common and ordinary assumption that wholes inherit the properties of their parts, consider that if a part of you has a bruise, then it appears that you have a bruise too, and it's the same bruise.)

As for the question of 'why' composites inherit their properties from their parts, Bennett says

> we would have an answer if composition literally *is* identity.[6] But, again, we do not obviously have an answer given the weaker claim that composition is merely *like* identity. In fact – and this is the important point – we certainly do not have an answer in light of the fact that property inheritance and the ban on

[5] Cf., Bennett asking "*Why* do composites inherit their properties [from] their parts?" (2015, 257)

[6] À la Numerical CAI.

> multi-location are given as part of what it *means* to say that composition is like identity. It would be circular to 'explain' those phenomena in terms of composition as identity. (2015, 257)[7]

Bennett too notes that Lewis disavows the indiscernibility of the whole and its parts, even though in his "rough characterization of the fourth analogy [he] makes it sound as though he thinks that both identity and composition obey Leibniz's Law: as though he thinks that the properties of a thing are the very same as the properties of its parts." (Bennett 2015, 257) Her statement of the notion of property inheritance is intended to reflect that Lewis does affirm that "What's true of the many is not exactly what's true of the one." (1991, 87)

If someone thought that Lewis's retreat into Analogous CAI and not-quite-identity here might help him avoid the conclusion that mixed fusions have members, it is time to emphasise that with regards to mixed fusions it is unclear *what* Lewis thinks is true of them that is relevantly related to their parts at all, and it has to be at least *something* for the claim of analogy to even hold. Recall that Lewis explicitly doesn't think that mixed fusions can *be* members (as seen in Chapter 2), so they must be relevantly unlike individuals in some respect, and he does not explicitly go along with any notion that they can *have* members, so they must be relevantly unlike classes in that respect.

The former is not such a big deal because being a member is more of an external relation than an intrinsic property, but the latter is incredible to accept because sets and classes as we usually understand them have nothing else about them that is significant in terms of their intrinsic properties *besides* their members. So even if what's true of the many is not *exactly* what's true of the one we are left to wonder what else CAI could be doing for Lewis in the instances of any mixed fusion's composition situation being like identity, other than entailing that mixed fusions have members. If mixed fusions do not have members in this instance, the claim to any of the forms of CAI being the case are severely undermined because there seems to be no other features of sets/classes that could be employed to make even the *analogy* of identity stick.

To see a case where what's true of the many is not exactly what's true of the one, the easy discernibility any mixed fusion has to any purely class parted class, in this context, is that each has the property of having individual(s) for part(s), where each of the pure classes do not. Similarly, the easy discernibility any mixed fusion has to any purely individual parted individual is that each has

[7] To be clear, property inheritance is not necessarily a consequence solely of CAI views, as one could assert property inheritance without resorting to CAI type claims, even if they are good candidates for explaining it. They would just need an alternate justification, which is the point Bennett makes here.

the property of having class parts, where each of the pure individuals do not. This is enough to go along with Lando and pare back Lewis's position to one of Analogous CAI. It also seems to be a case where the collective properties of the proper parts are still inherited by the whole. But considering that this sort of situation is all that might reasonably be implied by Lewis's denial of the sharing of all of the same truths for a one composed by many, it is not as if any of the significant properties of the proper parts are the ones being unmatched in the whole.

In this instance, look again at Sider (2007), who says that the property inheritance principles, which "everyone accepts", means that the "intrinsic nature" of the part is "reflected" in the whole (2007, 70). The only credible way for the intrinsic natures of the class parts to be reflected in mixed fusions is for the mixed fusion to have the members too. To put mixed fusions on the memberless side of the division here is to plainly *not* reflect the intrinsic natures of their class parts. Even if their not having members is not the sole intrinsic nature of some individuals, or even something of their intrinsic nature at all, having members certainly is the intrinsic nature of classes. Classes do not really have any other intrinsic features besides which members are theirs. They will be related to many other classes if they are their members, but this is generally understood to be an external relation, and not something intrinsic to the things which are the members (and hence why any individual's membered/member status may not be considered intrinsic for it either). Recalling our tables of the four logical divisions into the things which have members and are members, to put mixed fusions in the fourth division as anti-members is akin to obstructing the natures of their parts from 'shining through'.

Besides this, it is the case that how Lewis portrays the plural classes as having their members is something that is also applicable to how mixed fusions would be understood to have members. First I will show how plural classes have members under Lewis's system, and then finish this main section off explaining how mixed fusions should be said to have them too, so as to be consistent with the understanding for how plural classes have their members as put forth by Lewis.

3.2.3 How Lewis says plural classes have members

On the idea of plural classes having their members via their singleton parts, Lewis's discussion of one of the consequences of the Main Thesis – that the parts of a class are all and only its subclasses – is revealing.

> A class has its singleton subclasses as atomic parts, one for each of its members. It usually has other parts that are not singletons, namely subclasses with more than one member. However, if *x* is part of a class *y*, then *x must have one or more singletons as parts, else x could have no members, could not be a*

> *class*, and so could not be a subclass of *y*. In fact *x* must consist entirely of singletons, else the rest of *x* would be a part of *y* with no singletons as parts, so a part of a class that is not a subclass. A class is the union, and hence the fusion, of the singletons of its members. (1991, 15-6) (my emphasis)

While the penultimate sentence is a consequence of the Main Thesis, the portion I have emphasised is the one which I think suggests that plural classes inherit their members from their singleton parts. Here, it is the expectation that some *x* could have no members *unless* it has one or more singletons as *parts* which is suggestive that having singletons (which have members) for parts is why plural classes have members. Lewis furthers this suggestiveness later on when he characterises the singletons "that are the building blocks of all classes" as that which is "distinctively set-theoretical" (1991, 31), as opposed to the gathering of 'many into one' in the case of plural classes which is distinctively mereological. Lewis also expressly treats the formation of plural classes as a result of GEM coupled with the existence of their component singletons, saying that "all classes are fusions of singletons, and nothing over and above the singletons they're made of [...] We understand how bigger classes are composed of their singleton atoms. That's the easy part: just mereology." (1991, 31) As Lewis even incorporates some of a common CAI slogan here that composites are 'nothing over and above' their parts when it comes to describing plural classes, we are brought directly to the idea that the plural classes inherit the collective members of their singleton parts.

Yet if a plural class only has members as a result of a mereological phenomenon involving what its parts are, then it is not because it is a class that it has members, but rather it is because it has members that it is a class[8] – and on Lewis's account it has members only because some of its other parts do too. This notion is also conveyed in Lewis's supplied (re)definitions for classes and membership, which come in the text after the proof given for the Second Thesis, and which are meant to be in line with his Main Thesis following the aforementioned proof. While his earlier informal definition of a class was as a thing that has members (Lewis 1991, 4), Lewis's redefinition relies on taking the notion of a singleton henceforth as primitive.

Redefinition A *class* is any fusion of singletons. (1991, 16)

Following this redefinition, Lewis then proposes a definition of membership, which has been up to this point taken as primitive, and writes in a restriction to classes as just defined.

Membership *x* is a *member* of *y* iff *y* is a class and the singleton of *x* is part of *y*. (1991, 16)

[8] Singletons are a good example of this sentiment.

So, reading this via the redefinition of classes, we get *x* is a member of *y* iff *y* is a fusion of singletons and the singleton of *x* is part of *y*. It is this last clause which I find telling, as it affirms that it is the singletons which are a part of some fusion which 'bring the members', as it were. It is not a mereological phenomenon which supports the singletons having their own members, however. While these are said to be mereologically atomic and only have themselves as parts, under Lewis's system, they are seen by Lewis to have their members "mysteriously" (Lewis 1991, Chapter 2), as it is notoriously difficult to metaphysically pin down why any singleton has the specific member that it does, even as we do somehow understand what it means to speak of singletons (1991, 59).[9] This notion of mystery fits the idea that plural classes inherit their members from the singletons which are their proper parts especially well when Lewis affirms that 'the trouble with classes'[10] is something which afflicts the plural classes in virtue of singletons. As Lewis puts it, "Singletons, and therefore all classes, are profoundly mysterious." (1991, 57)

Although Lewis has restricted his definition of membership in such a way as to preclude mixed fusions from having members, the restriction is unfounded without a good defence of the Main Thesis, which Lewis's proof for the Second Thesis was supposed to secure, as the Main Thesis is expected to be a conjunction of the First and the Second Thesis (1991, 6). Putting aside Nolan's (2002) very acceptable explanation of how Lewis's proof for the Second Thesis fails, my issue with Lewis having defined membership by writing in a restriction to classes as fusions of singletons is that this restriction is inconsistent with the attitude which Lewis clearly gives for how plural classes are said to have their members via a form of CAI, and is thereby inconsistent with Lewis's view of composites and their properties in general.

3.2.4 How mixed fusions have members

If we sought to fix this inconsistency we might propose a general principle 'G', that if a class is a part of something, that any thing(s) which it is a part of has its member(s) as member(s) too. We might also give a definition of membership which essentially says that for one thing to have a singleton as a part is for that thing to have that singleton's member as its own. While Lewis's restriction makes sense if, as he asserts, we probably would not want to say that mixed fusions have members, the restriction does not make sense with regards to how the plural classes are said to have their members. Without the restriction to classes as defined above, a definition of membership which is similar to Lewis's would easily cover mixed fusions having members. For instance: *x* is a member of *y* iff the singleton of *x* is part of *y*. What this definition amounts to is that some *x* is a member of some *y* iff *y* has some part that has *x* as a member. And this is indeed the sentiment

[9] This is why singletons have been taken as primitive in the system, after all.

[10] The title of his Chapter 2.

Lewis seeks to endorse for what I will call, for the lack of a better word, the expected 'plain' classes, that is, classes which do not have any individuals as parts.

The general principle G serves to illustrate a further point about membership inheritance. If an entity with a member composes some composite whole *with* that member, then an entity without a member also composes a composite whole *without* that member. Which is both to say that class parts bring only their own members with them in cases of fusion; and that the so-called 'plain' individuals – which we will say are things which do not have any classes as parts – already inherited the members of their plain individual parts, all zero of them.[11] This conception entails that, when the plain entities mix, either with the plain things of their type, or with the plain entities of another type, that their fusions inherit the members of each of their parts too. We already know that the other things with singletons for parts, at least, are the mixed fusions, which too are purely a result of unrestricted composition, in the given context, and the existence of their components. Such an idea as the one given here enables mixed fusions to be classes via Lewis's presentations, because they inherit *any* of the members of their parts.

This idea also lets us see how we can prevent the obstruction of the natures of certain composites parts from 'shining through'. In the case of a mixed fusion, which is the composite of at least one class and one individual, for the individual part to 'shine through' would be for that to not add any members to the mixed fusion. We don't add that it would also mean that the mixed fusion *is* a member just like its individual part, because this is an external function which follows strict conditions, as seen in Chapter 2. For the class part to 'shine through' would be for that to add its members to the mixed fusion. Which is to say, similarly to before, where classes have the property of having m members, and $m > 0$, and individuals have the property of having n members, and $n = 0$, mixed fusions of classes and individuals will inherit the property of having $m+n$ members, taking numerical values of m and n from all of its proper parts both additively and idempotently,[12] just as it does in the case of classes with only class parts under Lewis's system.

In this sense, for the property of having zero members in an individual part to conjoin with a property of having $m > 0$ members in a class part, and for both to be properly reflected in the whole of a mixed fusion, is for the mixed fusion to have the property of having $m > 0$ members, which entails that the mixed fusion is a class.

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[11] Compare Lewis describing the null set and the non-mereological notion of inclusion, "To be sure, the null set is included in any class x: all its members – all none of them – are among x's members." (1991, 10)

[12] Idempotence in the mathematical sense of when an operation is performed by an element on itself, and it returns the element itself, i.e., an unchanged value. For more on the concept of idempotence in mereological contexts, see Cotnoir (2015).

We can *almost* hear Lewis express the idea behind this himself if we transpose the noun of his famous passage (1991, 81-2) on the ontological innocence of mereology – which he expects to follow on from the thesis of CAI – from 'cats' to 'sets'.

> if we accept [classical] mereology, we are committed to the existence of all manner of mereological fusions. But given a prior commitment to [sets], say, a commitment to [set]-fusions is not a *further* commitment. The fusion is nothing over and above the [sets] that compose it. It just *is* them. They just *are* it. Take them together or take them separately, the [sets] are the same portion of Reality either way. Commit yourself to their existence all together or one at a time, it's the same commitment either way. If you draw up an inventory of Reality according to your scheme of things, it would be double counting to list the [sets] and then also list their fusion. In general, if you are already committed to some things, you incur no further commitment when you affirm the existence of their fusion. The new commitment is redundant, given the old one.

It is only *almost* hearing because it is really both individuals and sets/classes that fit the bill for mixed fusions. But the point stands. If a composite is nothing 'further', nothing 'over and above' its parts – if it 'just *is'* the parts – then how could composites which have singletons as parts not have the very things which are the members of those singletons as their members also? As this is more or less how Lewis portrays the plain plural classes having *their* members, as inheriting them from their atomic singleton proper parts, it is not credible to think that mixed fusions wouldn't be doing the same thing. This is because the conditions claimed for compositional identicality, analogous or otherwise, for the property of having members across the plural classes are the same conditions for compositional identicality as they are for the mixed fusions. Which means that, on Lewis's account and yet contrary to the Second and Main Thesis, mixed fusions have members and are thereby classes.[13]

3.3 Why mixed fusions should not have members

Having established that Lewis's CAI thesis permits there to be classes with non-class parts, what are we to say now about his Second Thesis? Is he not entitled to object to my claim that via his views on composition there are mixed fusions that have members, and object based on his advocating the Second Thesis? The trouble with this line of response, of course, is that the Second Thesis being true would work further against the plausibility of the CAI theses we have

[13] I note here that if this consequence is to hold that much of what is discussed in Chapter 2 needs to be re-evaluated to see how the issues highlighted there could be affected.

encountered, because there would be a serious mismatch in the properties of the whole and of the parts amongst mixed fusions. The Second Thesis being true makes CAI fail.

So we come to a dilemma. Are mixed fusions the way Lewis baldly asserts – non-members, and memberless – or are they the way his CAI thesis implies? If mixed fusions *do* exist in the way Lewis attests then concepts of property inheritance presumed to be of a piece with CAI are false, or at least, are not universal – a serious problem for accepting CAI in general. If mixed fusions do not exist in the manner that Lewis attests, and are in fact classes in virtue of having members, then a significant problem arises involving the contradiction of the Extensionality axiom of set theory, a problem which I will turn to now.

3.3.1 Sets offender

Even if Lewis's proof of it fails, this of course does not mean that the Second Thesis is not true. So, our next step is to offer a new argument for its acceptance. Before doing so it is helpful to note that even though Lewis's defence of the Second Thesis tends towards accepting the expanded version of the Main Thesis, in defending the Second Thesis alone this is not required, as will be shown below.

The argument I give for the truth of the Second Thesis (in terms of proposals for Lewis-style parthood structure for classes) can be aided easily by a small mereological model of some composite entities which have both class and non-class parts, as shown

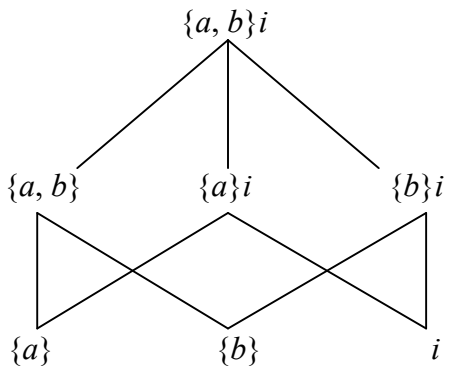

Figure 3.1 Model A.

Like with the models from an earlier chapter, read the lines going up the page as representing relations of proper parthood from the underlying elements to the overlying elements of the model.

On Model A[14] the mixed fusion $\{a, b\}i$ of the singletons $\{a\}$ and $\{b\}$ and the individual i, is a class on the supposition that the Second Thesis is false. This means that while $\{a, b\}i$ and $\{a, b\}$ are different classes mereologically, in being different entities, they are in fact identical classes set

[14] Which violates the First Thesis in that $\{a, b\}i$ is a subclass of $\{a, b\}$, *per hypothesi*, yet not a part of it.

theoretically, because both of them only have the members *a* and *b*. This is unacceptable due to a key set-theoretic axiom of Extensionality being violated. This is an axiom which Lewis endorses, rendering it as the principle which states that no two classes have the same members (1991, 100), and it is the key axiom for the identity of classes. In this instance two (mereologically) different classes have the same members, violating Extensionality.

Perhaps, if there were one, some proponent of the Second Thesis being false in this context[15] might resist my conclusion here, and they might insist that although these are different entities mereologically, for having different parts, they are still the same class, for having the same members. And this proponent might follow through on this insistence as being revelatory, in that it leads to new frontiers of thinking about classes and mereology. Yet, although this case of a mixed fusion having the same members as its largest plain class part does appear to open up the possibility of the divorce of mereological identity from class identity, I leave it up to that proponent of the Second Thesis being false to explore this notion. I instead insist in opposition that some facet of difference is enough to make this a clear case of a violation against the axiom of the adopted class theory.

My objection here, then, is that following GEM's principle of mereological uniqueness, the mixed fusions and their largest plain class parts are different entities because of their different parts, *tout court*, which means that they are different classes but with the same members, a clear violation of the axiom of Extensionality. Lewis informally describes the principle of the Uniqueness of Composition as that it never happens that the same things have two different fusions (1991, 74), which, in some of the more formalised presentations of GEM (which Lewis 1991, 73 does cite),[16] is known as *Strong Supplementation*. This is the condition that if *y* is not part of *x*, then *y* has some part *z* that does not overlap *x* – which results in mereological fusions being extensional and unique. I offer that we take the spirit of class Extensionality at face value, and recognise that even if it is not explicit in its mereological considerations, that it is meaning to suggest that the entity is unique, in being the only entity with such members. With all of this I think it is fair to say that mixed fusions having the members that their largest plain class parts have is a conclusion that nobody should want to reach (and as far as I can tell, nobody currently does endorse such a conclusion), unless they are willing to completely redefine set/class theory, and thereby much of modern mathematics.

[15] The qualification of 'in this context' is important here because there are many proposals of non-GEM mereologisations of set theory where classes are said to have non-class parts. See, e.g., Katz (1998), Fine (2010), Caplan et al. (2010) and McElhoes (2017).

[16] See, e.g., Simons "SA5" (1987, 1.4).

Therefore the best defence that I can think to give of the Second Thesis is that it prohibits Extensionality violations in the case of Lewis-style parthood structure for classes. If its falsehood were to allow the sorts of cases as represented by Model A, then because these violate the extensionality of sets and classes, the Second Thesis is best offered as a means to avoid such violations. To be sure, the Second Thesis is not something which can be shown to govern other class conditions on purpose, however. So although it may be best to hold it following a requirement to avoid violations of Extensionality, the Second Thesis does not directly prevent these violations if there are other conditions already in place which achieve the same effect. This is seen in the fact that the Second Thesis is satisfiable in less structured cases of parthood among classes, such as if all classes are only mereological atoms (e.g., The only part any class has is itself), or if classes are not mereological at all (e.g., No class has any part that is not a class, because no class has any parts). So, if classes were to have only improper parts, or even no parts at all, the Second Thesis would still be able to be true. This shows that any prohibitive effect of the Second Thesis on Extensionality violations is only valuable in cases where this is even plausible, such as in Lewis's proposed mereological structure for classes.

I stated earlier that a host of new difficulties arise if mixed fusions have members, and this is that the standard set and class theories would be declared literally false, and not the whole story about how their sort of membership operates. Clearly Lewis should have no truck with this, as throughout *Parts of Classes* he states an intention to take mathematics as it is found, rather than mutilating it to suit philosophy (e.g., 1991, 58-59). This is in addition to stating that the mathematical aspect of the work, and its mereological enterprise, is not supposed to run afoul of ordinary mathematical practice in any way (1991, vii). Rather, the point is to show that there "is more mereology in set theory than we usually think." (1991, vii) In his promoting realism about classes and their parts, Lewis's aim is to show that the philosopher's mereology is globally compatible and consistent with the mathematician's set/class theory, and that neither system needs to be revised in light of the other.

But if it is not permissible for mixed fusions to have members without violations of set/class theory, what are we to make of the evidence from CAI which suggests that they do?

3.4 Resisting mixed fusions as classes

By now it should be clear that the problem I am having here is with how something like a mixed fusion could be so radically different to its parts – especially in the context of CAI. To go along with Lewis's express attitude towards mixed fusions, however, would actually undermine his endorsement of composition as many-one identity. This is because he describes mixed fusions as at

least not really like and at most nothing like their parts, especially by saying things about them which imply that they are anti-members, and also by asserting that they are the lone type of thing in the fourth division of membership, the sole anti-members (following our membership division Table 2.4 displaying Lewis's positions in Chapter 2). I concur with Lando (2017) that "if the analogy between composition and identity has any content—it seems unavoidable that the features of the whole and the features of the parts are strictly connected, either by Indiscernibility or in a weaker ... way." (2017, 206)

I think the dilemma faced by proponents of CAI in conjunction with any mereology which accommodates mixed fusions – so not just GEM if some other systems do accommodate them – is whether to restrict the mereology so that mixed fusions are not formed by composition, such as by denying that GEM is absolutely general/universal/topic-neutral; or otherwise to reject CAI. I suspect that there is no plausible option to restrict CAI from being absolutely general/universal/topic-neutral, and this is because the identity relation itself is the pinnacle of claims to absolute generality, universality and topic-neutrality (cf., Uzquiano 2006). Also, aside from it being an *ad hoc* response, to permit CAI to be non-absolute under these circumstances seems like it would amount to restricting identity to be non-absolute too, making this an implausible move for proponents of CAI. This to me suggests that CAI ought to be rejected, if the mereology is not otherwise restricted.

For Lewis, to reject CAI would also be to lose one of his defensive supports for the acceptance of GEM.[17] For example, he argues that

> whenever there are some things, they have a fusion. *Whenever*! It doesn't matter how many or disparate or scattered or unrelated they are. [...] There is still a fusion. So I am committed to all manner of unheard-of things: trout-turkeys, fusions of individuals and classes, all the world's styrofoam, and many, many more. [...] It is no problem to describe an unheard-of fusion. It is nothing over and above its parts, so to describe it you need only describe the parts. Describe the character of the parts, describe their interrelation, and you have *ipso facto* described the fusion. [...] Its character is exhausted by the character and relations of its parts. (1991, 79-80)

Lando (2017) offers an interpretation of "a fusion is nothing over and above the fused entities" as meaning "that the fusion has no feature that its parts lack." (2017, 204) Further to this is the more

[17] Not that CAI is necessary for GEM (Lewis 1991 just uses it as a key support for it). See Lando (2017) for discussion and criticism of the notion that CAI is necessary for GEM.

positive aspect of the interpretation, that the fusion also has features of its parts, which is echoed above in Lewis saying "to describe [a fusion] you need only describe the parts" – which to me is another clear indication that mixed fusions should be seen to have the members of their class parts under CAI. To not forget the other facets of the CAI claim, here, however – if the mixed fusion is a composite of a plain class and a plain individual which is a cat, say, then the identity of the cat will contribute to the identity of the mixed fusion too. In this case the mixed fusion will be a cattish thing which has members, yet not solely a cat: a cattish class, perhaps. No doubt this fits the bill as being an unheard-of-thing, much like a trout-turkey.

Considering that the domains in which mereology is usually or more commonly applied to are those involving material objects, or otherwise spatial domains, it is understandable that one may seek to eliminate any problems involving mixed fusions by restricting GEM's absolute universality, thereby losing mixed fusions altogether, rather than rejecting CAI. Especially when this specific problem for CAI is not being found within more ordinary mereological domains, and only in more exotic mixed ones.

Lando (2017), for instance, remarks that if absolutely universal applications of GEM lead to undesirable consequences it is worth restricting mereological universality, which is what he opts to do for reasons not involving mixed fusions. Lewis, however, offers no such restriction, and GEM has no explicit restrictions of this kind intrinsic to it. Here it appears that Lewis does not have a credible out under these circumstances that does not involve dropping one of his major commitments. As the attitude we're following here is that of Lewis explicitly saying "of course, nothing has two singletons" (1991, 15), I advance the rejection of CAI as illegitimate to hold universally, and thereby at all, because of class Extensionality violations. For one, if the mereology denies the entry of mixed fusions, then there is nothing left to talk about. For another, I think the problem we have encountered, of mixed fusions having the same members as their class parts, thereby violating class Extensionality, actually arises due to other notions supporting the CAI thesis, and not due to GEM specifically. Therefore it is CAI which is to be rejected in this contrastive instance, for being the culprit.

We can hear echoes of Lewis again in this proposal "there is no good independent reason to restrict composition. [Classical] Mereology *per se* is unproblematic, and not to blame for the set-theoretical paradoxes; so it would be unduly drastic to stop the paradoxes by mutilating mereology, if there is any other remedy." (Lewis 1991, 19) The better remedy, to start with, is to restrict not composition, but rather the CAI thesis, and in full. By this I mean that CAI should not be seen as something which is universal, and as universality is tied to it via claims to explaining identity, CAI cannot be said to hold in any case. But I don't think this is the whole story here. The reason I said

CAI should be rejected to start with is because of the choice between CAI and GEM, and, in this instance, CAI looks to lose out as being the major reason class extensionality is violated. Yet, I have also suggested that property inheritance *in general* suggests that mixed fusions have the members of their class parts, whether or not this property inheritance is conditioned by a CAI thesis. To be clear, all CAI theses are understood to entail property inheritance, but not all property inheritance conceptions are understood to entail CAI.

This makes the problem of mixed fusions having the members of their class parts rear again anyway, but in slightly different circumstances. In this case we are now faced with a choice between GEM and property inheritance. In this instance, even though I would still say that property inheritance is the culprit, in that it is the reason why mixed fusions have the same members as their class parts, I advance that we restrict GEM instead, by restricting its composition principle, because it is the reason we even have to contend with mixed fusions. I think that this is an acceptable strategy in this case specifically because property inheritance is supposed to be a "fairly uncontroversial" notion which is "not really in need of argument" (recalling Bennett 2015), and one which "everyone accepts" (recalling Sider 2007). Contrast this with GEM and its principle of unrestricted composition which are fairly controversial[18] and in need of significant justification – hence Lewis himself trying to support GEM by bringing in CAI (1991, 81-7).

To be clear, Lewis's version of CAI consists in unrestricted composition as "the second aspect of the analogy" (1991, 85) with composition and identity, but as I have already mentioned it can be the case that other forms of composition could constitute a CAI-thesis together with other principles. The more straightforward version of my argument here, then, is that, as a package of principles, CAI is sufficient for mixed fusions with members – a situation which should be rejected by rejecting CAI. To elaborate, CAI is sufficient for mixed fusions with members by consisting in both unrestricted composition (or some other suitably liberal composition principle) and also property inheritance, among other principles. The suitable composition principle is necessary for the formation of mixed fusions but not sufficient for them to have members, whereas property inheritance is insufficient for the formation of mixed fusions on its own as it makes no claims to composition. However, property inheritance is sufficient for mixed fusions to have members in the case that mixed fusions are formed. Hence CAI is to be rejected primarily in being sufficient for the problem of mixed fusions having members, in being the package of theses that it is.

Consequently, if we consider a context which does not have the greater package of CAI in play, but still has both the right sort of composition principle and property inheritance remaining,

[18] Witness the literature on the Special Composition Question, and the rejections of Universalism therein.

the question then arises of whether to reject the principle which is mereologically necessary but property-wise insufficient for mixed fusions having members (the composition principle) or to reject the principle which is only property-wise sufficient for mixed fusions having members, just in case the right sort of composition abounds. In the case of unrestricted composition I propose that we stick with the widely accepted property inheritance in favour of the controversial composition principle. Moreover, to rule out the problem of mixed fusions having members altogether, I propose that we deny any composition principle which will permit them – as I imagine any candidate principle which is able to permit mixed fusions will still be less acceptable than property inheritance. In this way, if we start in a context of property inheritance being accepted then we should brook no composition principle which will result in mixed fusions having members.

The further difference between the two dilemmas given here is that CAI can itself be controversial in other contexts, so the dilemma present between it and GEM is more on par. But because GEM can be endorsed without CAI (Lando 2017), one can be rejected in place of the other too. And although GEM could be endorsed without property inheritance too, it does not appear to be the case that anybody does so, and I see no good reason to challenge such a conventional notion as property inheritance by upholding GEM instead. This is even though, without property inheritance, set and class theory are not violated in the manner described, and things like mixed fusions are able to be radically different to their parts. To insist to keep GEM after rejecting CAI, then, is tantamount to rejecting property inheritance, and this looks to be far less justifiable than restricting GEM in this instance. Therefore, the problem of mixed fusions having the same members as their class parts sounds like a good, independent reason to restrict composition principles which permit mixed fusions, such as GEM's unrestricted composition. In this way, mixed fusions do not have members, and thereby do not violate class extensionality, because mixed fusions do not exist.

3.5 Conclusion

In this chapter I argued against a kind of mereological hybridism found in mixed fusions of classes and individuals, their key difference of one type of entity being membered, and the other type not being membered. I presented an argument to the effect that if one thing has zero members, and another thing has one member, their fusion should have one member – a conclusion which I claimed follows on from our ordinary conception of property inheritance. The problem with this conclusion, however, is that it results in violations of standard membership theory axioms of Extensionality, where classes are uniquely defined by their members. To maintain the restriction against more than one thing having the same collection of members I advanced that composition principles and systems which allow the fusion of memberless and membered things should be

rejected. Throughout the chapter I also discussed Lewis's assessment that mixed fusions do not have members, given his Second Thesis which engenders that result, and showed how these do not succeed against his claims to the thesis of CAI. From this I argued that Lewis is forced to deny this composition thesis, and thereby lose one of his main supports for accepting GEM in the first place, a system which I further ended up rejecting anyway for its permitting of mixed fusions.

Although I believe these issues over memberless and membered composites are a compelling reason for the rejection of mixed fusions and their supporting conditions on their own – as this feature of theirs is one which is guaranteed – this situation only presents one form of mereological hybridism worth resisting. Other aspects of mereological hybridism that merit consideration and rejection, and which are plausibly found in the case of mixed fusions, are presented in the chapters to follow, which together reinforce the rejection of mixed fusions here as one of the many components of a robust cumulative argument against a variety of mereological hybridisms in general.

In the next chapter I argue against a recent claim that mixed fusions are grounded by their proper parts, and claim that they are better off regarded as grounding entities instead, if at all.

Chapter 4
Grounding the mixed mereological Universe

4.0 Introduction

In Chapter 1 I explained that when it comes to mixed fusions that these are rarely, if ever, given more than a cursory treatment in the literature, and that they are frequently treated in a dismissive manner – even by the one who introduced them, David Lewis. In this chapter we are going to look at one of the rare cases which go against this norm, and one that is very recent. This comes in Trogdon and Cowling's (2019) defence of *abstract atomism*, which is the view that abstract entities, insofar as they have proper parts, are grounded in their proper parts. That paper goes against the normal attention given to mixed fusions because it explicitly relies on a positive conception of mixed fusions to argue for the abstract atomist position.

In this chapter, what I set out to do is provide a critical assessment of Trogdon and Cowling's claims, and to point out where I think their discussion goes right, where it goes wrong, and what I think they miss. Roughly, I will show that, although they mostly focus on mixed fusions with only abstracta for proper parts, and which they claim are grounded in their proper parts, Trogdon and Cowling's arguments also carry over to mixed fusions in general, including cases where these would have concreta proper parts too. From here I argue that if mereological gunk – i.e., any entity which has proper parts all of which have proper parts (so no mereological atoms) – is included in the account then we arrive at an ontology with conflicting grounding directions. This is because, in this case, and due to unrestricted composition, there would be things grounded in their proper parts (the abstract mixed fusions) fused with things that are not grounded in their proper parts (the gunk), resulting in the contradiction of mixed fusions which are both grounded and not grounded by their proper parts. With Trogdon and Cowling admitting that they "take the possibility of concrete gunky objects seriously" (2019, 2037 fn 22), my claim is that by following the consequences of their grounding proposal, along with fairly expanding the examination of mixed fusions to their general case across the entirety of Reality, and also by allowing for concrete gunk in the system, that their position is turned on its head.

Specifically I will argue that as this kind of expansion would either lead to mixed fusions not being properly grounded in their proper parts (contradicting Trogdon and Cowling's proposal), or worse – mereological gunk which grounds entities (which goes against the usual conception of gunk as having no basis to ground from) – that the best remaining grounding result is one wherein the mereological Universe as a mixed fusion is grounding of everything else. Such that it is a grounder of, rather than grounded by, its proper parts. In addition to this I will demonstrate how, regardless of gunk, the generalised expansion of Trogdon and Cowling's proposal actually leaves their view open to other criticisms which undermine their grounding scheme entirely – along with

any prospects of there being such a mixed fusion as the mereological Universe that is grounded or grounding as described.

4.1 The projects of foundationalism

To start, Trogdon and Cowling set out what the expected task of *ontological foundationalism* is, that of identifying which existents ground other existents but are not themselves grounded by anything. Trogdon and Cowling note that usually the project of ontological foundationalism focuses specifically on the thesis of *concrete foundationalism*, which has the task of identifying not just what existents ground what other existents, but specifically which concreta ground other concreta but are not themselves grounded by any concreta. They claim that there is potentially more to the story of ontological foundationalism when it comes to the consideration of abstract entities, and they offer another thesis of *abstract foundationalism*, which has the task of identifying which abstracta ground other abstracta but are not themselves grounded by any abstracta.

Amongst both foundationalisms runs the common thread of inquiry over whether the grounding elements are either mereological atoms or mereological wholes, with the usual positions involving what the grounding elements are being called *atomic* or *monist* positions respectively. So while an *atomic foundationalist* will claim that the existents which ground other existents but are not themselves grounded by anything are mereological atoms, which are the proper parts of those other existents, a *monist foundationalist* will claim that the existent which grounds other existents but is not itself grounded by anything is a mereological whole, which is the mereological sum of all of the other existents.

What Trogdon and Cowling argue for explicitly is the superiority of an abstract atomic foundationalist position over an abstract monist foundationalist one. To get there, they first introduce category-relative conceptions of fundamentality and grounding for both abstracta and concreta. With regards to concreta, they say that an entity is *c-grounded* just in case it is grounded by concreta, and an entity is *c-fundamental* just in case it is not c-grounded (2019, 2030). Similarly, with regards to abstracta, they say that an entity is *a-grounded* just in case it is grounded by abstracta, and an entity is *a-fundamental* just in case it is not a-grounded (2019, 2031). Note that this clearly entails both that any concreta which is not a-grounded is a-fundamental and that any abstracta which is not c-grounded is c-fundamental. This will be important later.

With these conceptual distinctions in place, they then say that an entity is *complex* just in case it has proper parts, and an entity is *simple* just in case it isn't complex (2019, 2033). Having assumed what I will (in Chapter 5) call the homeomereity theses with regards to concreta, wherein

these only have concreta for parts, and also for abstracta, wherein these only have abstracta for parts (2019, 2033 fn 11), Trogdon and Cowling's next step is to say that, with regards to a totality concretum, *x* is the "cosmos" just in case *x* is concrete and, for any *y*, *y* is concrete and numerically distinct from *x* just in case *y* is a proper part of *x*. In terms of a totality abstractum they say, *x* is the "amalgam" just in case *x* is abstract and, for any *y*, *y* is abstract and numerically distinct from *x* just in case *y* is a proper part of *x*. From here they formulate the theses of atomism and monism in both their abstracta and concreta variants.

Concrete atomism there are multiple simple concreta, any simple concrete entity is c-fundamental, and any complex concrete entity decomposes into simple concreta such that the latter ground the former. (2019, 2033)

Concrete monism there is a cosmos that is c-fundamental, and any concrete entity numerically distinct from the cosmos is grounded by it. (2019, 2033)

Abstract atomism there are multiple simple abstracta, any simple abstract entity is a-fundamental, and any complex abstract entity decomposes into simple abstracta such that the latter ground the former. (2019, 2033)

Abstract monism there is an amalgam that is a-fundamental, and any abstract entity numerically distinct from the amalgam is grounded by it. (2019, 2034)

To be precise, Trogdon and Cowling actually name the concrete theses 'priority' atomism and monism, respectively, following the usual naming in the literature which comes from Schaffer (2009; 2010). However, these are names which I find a bit misleading in the context of there being a proposal for both a concrete atomism/monism and an abstract atomism/monism, because these theses already do actually specify what types of entity they are making claims about. I reserve the less specific 'priority atomism' and 'priority monism' for the following theses, which make no explicit commitment to things being either concrete or abstract specifically.

Priority atomism there are multiple simple entities, any simple entity is fundamental, and any complex entity decomposes into simple entities such that the latter ground the former.

Priority monism there is a unique mereological Universe that is fundamental, and any entity numerically distinct from the mereological Universe is grounded by it.

All of these theses can be accommodated by the two main views of composition found in the literature, that of mereological nihilism, which denies the existence of composite entities, and that

of mereological universalism, which asserts that for any two things there is a composite of them. Nihilism and universalism, for short, clearly diverge, but they are each able to accommodate atomistic and monistic theses. Nihilism does so by distinguishing between plurality nihilism and singular nihilism. Plurality nihilism takes the atomistic clause that there are multiple simple entities yet has an empty value for the term 'complex entity', such that even if it is true that "any complex entity decomposes into simple entities", as there are no complex entities on this view the consequence is irrelevant. Singular nihilism accommodates the monistic theses by taking the monistic clauses that there is either a cosmos, an amalgam, or a unique mereological Universe, and it is just that this is the only existent, making it mereologically atomic and without proper parts.

Although usually a mereological Universe is described as a complex entity which is the singular totality whole which has every other existent as a proper part, all it needs to be is the thing which has all existents as parts, either proper or improper. In having only itself as a part, and an improper one, a single mereological atom can be a mereological Universe under this conception, even if it is only one thing which has no proper parts, as in Horgan and Potrč's (2000; 2008) 'blobject'. Such an atom is still a mereological Universe insofar as it is the singular mereological totality of all things. As there are no entities which are numerically distinct from the mereological Universe on such a view, the consequence of these being grounded by the mereological Universe is irrelevant.

Universalism can accommodate any of the atomistic and monistic theses by the fact that it permits both simple and complex entities. As a thesis about composition, universalism is most commonly associated with the standard theory of parthood relations, GEM, with its presentation of the principle of unrestricted composition therein. What sets GEM and universalism apart from mereological nihilism, besides the acceptance of composites, is their ability to accommodate either atoms *or* gunk, or a mixed system of both atoms and gunk. Any entity which is *entirely gunky* is mereologically infinitely divisible, as all of its proper parts have proper parts, to the point that there are no mereological atoms or simples among its parts. Yet it is supposed that some things may be only *partially gunky*, in that they have at least some atoms as parts, yet are otherwise composed of other entirely gunky elements. As stated, GEM is able to be formalised to accommodate any of these options (for details see Simons 1987; Varzi 2016).

4.2 Abstracta and sets

To return towards explaining how Trogdon and Cowling explicitly argue for the superiority of an abstract atomic foundationalist position over an abstract monist foundationalist one, we note that their next step is to embrace as a working hypothesis a set-theoretic conception of abstracta (2019,

2035). "On such a view, any abstract entity is either a set or something constructed from sets. As for the categories of abstracta, we assume that there are properties (monadic and *n*-adic), propositions, and mathematical entities." (2019, 2035)

What this view allows is that while some abstracta are sets, not all abstracta need to be, but only if these are at least composed of sets. Which means that from the outset they deny that there are any mereologically simple abstracta which are not sets. I do not believe this is a necessary condition for their argument for abstract atomic foundationalism, however, as that argument only needs for there to be mixed fusions of non-empty sets and non-membered things, it is actually irrelevant to their argument whether the non-membered part of a mixed fusion is the empty set or not. We will come across this idea again in more detail later. For now, from embracing a set-theoretic conception of abstracta Trogdon and Cowling go on to refer to Lewis's (1991) proposal for the mereology of sets.

> We begin with [some] terminology. The *empty set* is the unique set without members. A *non-empty* set is any set distinct from the empty set. So a non-empty set is either a *singleton*—a set with only one member—or a set with more than one member. Let one set be a *subset** of another just in case the former is a subset of the latter and both are non-empty. So, while the empty set is a subset of any set, the empty set isn't a subset* of any set; and, while there is one and only one set that's a subset of the empty set (itself), no set is a subset* of the empty set. Lewis plausibly suggests that subsethood* is a special case of parthood—the parts of a non-empty set are all and only the non-empty subsets of that set. [...] The picture that emerges is this: any set is either complex or simple; a set is simple just in case it's a singleton or the empty set; a set is complex just in case it's a multi-membered set; and, since multi-membered sets decompose into singletons of their members, any complex set decomposes into simple sets. Given Lewis' metaphysics of sets and our assumption that abstract reality is set-theoretic in nature, all complex abstracta decompose into simple abstracta. *Gunky* abstracta—abstract entities such that all their parts have proper parts—are impossible. (2019, 2036-7)

Note that within and around this nice summary of Lewis's mereological system for sets, as explained in more detail in my earlier Chapter 1, Trogdon and Cowling diverge from Lewis's account in several ways. For one, they focus on sets and do not incorporate classes (2019, 2036 fn 20), and for another they treat the empty set as an abstract mereological atom while Lewis treated it as the mereological sum of *all* individuals, as in all non-membered members. They explicitly opt for

this view of the mereological character of the empty set for at least two reasons. The first being that, while singletons have no non-empty proper subsets, the empty set "has no proper subsets period, [so] it's plausibly taken to be simple as well." (2019, 2036) The second being that it comports with their proposal of abstract atomic foundationalism, whereby the empty set, along with all other abstract mereological atoms in the singletons form the grounds of the amalgam as the sum of all sets.

Indeed, Trogdon and Cowling explain how both Lewis's and their acceptance of unrestricted composition allows for there to be a mereological sum of all abstracta, viz., the 'amalgam' as defined earlier. One mischaracterisation of Lewis they make, however, is to say that he suggests that the empty set is the 'cosmos' (2019, 2031 fn 26), as in the mereological sum of all and only concreta (as met earlier). Yet Lewis makes no such claim. Instead, Lewis opts for a characterisation of the empty set as the maximal sum of non-membered members – whether these be abstract or concrete or whatever – and does not constrain this to only be a singular cosmos at all.[1] Lewis settles on this characterisation of the empty set because he thinks that at least *something* would exist to be able to stand for it/fulfil the role. Whereas if we were to select some arbitrary individual as the empty set, like one of his cats, say, then the contingency of that feline's existence would be too precarious to base the entire foundation of set-theory upon (see 1991, 1.4 for the full discussion). This is especially in terms of the iterative conception of sets wherein the hierarchy of sets begins at the null set and goes through iterative stages of that being a member of a singleton, which is then a member of a singleton, and so on. But Lewis does not overtly commit to the individuals that exist being *only* concreta, and in fact makes no final determination either way (even if he has apparent preferences). He at best accepts the following

> Remember what an individual is: not necessarily a commonplace individual like Magpie or Possum, or a quark, or a space-time point, but anything whatever that has no members but is a member. If you believe in some remarkable non-classes – universals, tropes, abstract simple states of affairs, God, or what you will–it makes no difference. They're still individuals, however remarkable, so long as they're members of classes and not themselves classes. (Lewis 1991, 8-9)

[1] This is expected to follow from Lewis's (1986) account of the plurality of concrete worlds, in which there are infinitely many concrete cosmoi. Moreover, Lewis does not prohibit these concrete worlds from having abstract immanent universals as parts, making this allowance for the sake of not inadvertently precluding different theories of properties from the outset. All of which is to be discussed further in Chapter 5.

That being said, Lewis does distinguish between the maximal sum of individuals and the maximal sum of singletons/sets/classes, going so far as to say that "Reality itself – the mereological fusion of everything – [divides] into one part which is the most inclusive individual and another which is the most inclusive class." (1991, 8) This means that one conceptual distinction between Trogdon and Cowling's account and Lewis's amounts to what they think the mereological Universe is maximally composed of. In Lewis's case he thinks that it is composed of the most inclusive individual and the most inclusive class,[2] whereas in Trogdon and Cowling's case the mereological Universe appears to be composed of the maximal abstractum – the amalgam – and the maximal concretum – the cosmos.

While it is not clear from Lewis that the most inclusive class is the maximal abstractum nor that the most inclusive individual is the maximal concretum,[3] it is clear for Trogdon and Cowling that the cosmos is the most inclusive composite of non-membered members (sans the empty set, and modulo my arguments for any mixed fusions as members). For them this means the cosmos is the sum of all individuals because they don't classify the empty set as an individual (2019, 2036 fn 20), whereas their amalgam is *not* the most inclusive class, nor even the most inclusive set, as this has nothing to do with their avoiding specifically class-theoretic terminology (2019, 2036 fn 20).[4] The amalgam is neither of these things for Trogdon and Cowling, and this is because the amalgam is a mixed fusion on their view – even without the empty set part being classified as an individual. Here is where an alternate view of mixed fusions as, at least, the sums of classes and singulates, as per my terminology in Chapter 2, looks to arise. Even with their terminological differentiation, however, because Trogdon and Cowling still treat the empty set as a held singulate, their view still incorporates mixed fusions in the manner of Lewis's original account, viz., as the sums of individuals/held singulates and classes.

Unlike Lewis who accepts there being a most inclusive class, Trogdon and Cowling properly avoid the notion of there being a most inclusive set, permitting that the mereological sum of all non-empty sets may be something which isn't itself a non-empty set (2019, 2037). They would still view this as an abstract entity, however, hence their specification earlier that any abstract entity is either a set or something constructed from sets (2019, 2035). Given a restricted version of my arguments in Chapter 3 though, I would insist that anything which resolves into only atomic singletons parts itself has members, pressuring Trogdon and Cowling's account to accept that the

[2] Which I dispute in Chapter 2 by arguing that some mixed fusions of individuals and classes are themselves properly characterised as individuals, in that they are non-membered, yet members of sets/classes.

[3] Nor does he appear to even be trying to endorse any position on this issue.

[4] Recall my discussion of the technical differences between sets and classes in Chapter 2.

mereological sum of all non-empty sets, even if it may be something which isn't itself a non-empty *set*, is at least going to be a free class, instead of another kind of anti-member (which to me look similar to Nolan 2002's mentioned fusions of classes which are not themselves classes). I expect that whether such a sum is a proper class or not depends on how many members it has, as well as other factors such as the limitation of size of sets, as discussed in Chapter 2.

The most important point for our purposes here though is that Trogdon and Cowling would not posit this sum of all non-empty sets as the most inclusive abstractum, inclusive in terms of parthood, and this is because it does not have the empty set as one of its parts. This entails that the sum of all non-empty sets is in fact a proper part of the amalgam, which *is* the most inclusive abstractum on their account. Trogdon and Cowling agree with this, and put forth that the amalgam is a mixed fusion which has abstract, atomic parts in the empty set and all singletons, and no other atomic parts (2019, 2037-8).

4.3 Existence grounding, and grounding as a relation

We are almost now in the territory of Trogdon and Cowling's arguments for abstract atomism. Before we cross over we should look at an aspect of their discussion which *would* be an argument against abstract monism, if it weren't for one of their express ideological commitments as well as what I take to be a flawed view they hold on the membership eligibility of mixed fusions. Early on in their paper Trogdon and Cowling advise that they make the assumption that grounding is not a relation, in the sense of it not being an *entity* which exists alongside other entities like tables and chairs (or numbers and classes), for instance. Instead they say they help themselves "to grounding as an element of our best metaphysical theories and an indispensable tool for stating facts about metaphysical structure". (2019, 2030)

Why *not* treat grounding as a relation *qua* entity? For Trogdon and Cowling it is better to see it as a piece of ideology, in the vein of Kment, who says "It shouldn't be assumed that all ingredients of reality must be individuals, properties, or relations—or entities of any kind, for that matter. For example, it's possible that in order to describe reality completely, we need to use some primitive piece of ideology that relates to some aspect of reality that doesn't belong to one of these three ontological categories, and which may not be an entity at all" (2014, 150). This allows them to avoid the expected questions of whether grounding as an entity faces a vicious regress, in terms of what grounds grounding, i.e., the "question of what abstracta if any ground the grounding relation itself" (2019, 2038 fn 26). It also means that for any particular grounding claims, these are not to be understood as making a claim that the grounder(s) stand in any particular relation to the groundee(s). As Trogdon and Cowling explain, "remember that the abstract monist claims that the

amalgam *grounds* its proper parts. So in making this claim, we shouldn't interpret the abstract monist as claiming that the amalgam stands in a particular relation to its proper parts." (2019, 2038)

Beyond the understandable desire to avoid difficult questions of regress management, Trogdon and Cowling also favour not treating grounding as a relation because were they "to treat grounding as a relation then abstract monism would be false, as the amalgam doesn't stand in any relations." (2019, 2038) They believe that the amalgam doesn't stand in any relations because they believe that no mixed fusion stands in any relations, and they believe this notion because they think that no mixed fusion is eligible to be a member of any set. This is important following their set-theoretic conception of properties which follows Lewis (1986), where to have a property or to stand in any relations is to be a member of some set (2019, 2038). Yet by the rules of set-formation which they heed in following Lewis's (1991) formulation, it is implausible that no mixed fusions are members of sets. In Chapter 2 I argued that actually many mixed fusions can be members under Lewis's system, even as he himself implausibly denies it, and as Trogdon and Cowling have followed suit here.

Although I only say "many" mixed fusions, the ones to be excluded would presumably not be the amalgam, because there is nothing clearly about it being a member which would lead to a set-theoretic paradox. Which, in the context of accepting set/membership-theory, is the most sensible reason to restrict membership for any given thing anyway. Besides this, it is just implausible to say that mixed fusions do not have properties or stand in relations. Here's one example: each mixed fusion has the property of having a part which has a member. Here's another: each mixed fusion has the property of having two or more parts. Or even: each mixed fusion stands in either the relation of proper part or of whole to some other thing. You get the idea.

This all being said, Trogdon and Cowling's actual argument *for* abstract atomism does not appear to be affected by these issues of mixed fusions being members or not (and regardless of whether another view is prematurely excluded or not). But discussion of these issues helps frame the notion that Trogdon and Cowling seek to convey in their atomistic grounding claims. On their view the grounders of things are going to be entities that don't stand in grounding relations to the things they ground, but rather are to be said to ground the things that they do by making them exist. Here Trogdon and Cowling explain the distinction they make between existence grounding and content grounding, in the context of abstracta.

> Suppose that there are propositions and that they're abstract in nature. Given what we've said so far about grounding, the following question is coherent:
>
> (1) What a-grounds the proposition that Socrates is a philosopher?

> In posing (1), however, there are at least two questions you might have in mind:
>
> (2) What abstracta make it the case that the proposition that Socrates is a philosopher exists?
>
> (3) What abstracta make it the case that Socrates is a philosopher?
>
> When someone asks (2) they want to know what abstracta, if any, make it the case that the proposition exists regardless of whether the proposition is true. So let's say that (2) concerns *existence* a-grounding. In contrast, when someone asks (3) they want to know what abstracta, if any, make it the case that Socrates is a philosopher. Hence, (3) doesn't target the existence of propositions in the same way that (2) does. So let's say that grounding claims like (3) target *content* a-grounding rather than existence a-grounding, where questions of this sort concern what abstracta make a proposition true. (2019, 2032)

With their focus being on existence grounding rather than content grounding, Trogdon and Cowling's claim that abstract atomic entities are the grounds for all abstract composite entities is the claim that the abstract composite entities exist because of the abstract atomic ones. If it was the other way around, as an abstract monist foundationalism would attest, the claim that an abstract composite totality entity is the grounds for all abstract atomic entities would be the claim that the abstract atomic entities exist because of the abstract composite totality. Trogdon and Cowling's argument for abstract atomism hinges on this difference in attitude towards the existence 'direction', and the plausibility that they find in favour of the abstract atoms making the abstract composites exist in this case.

4.4 Narrowing the focus

For their argument, although Trogdon and Cowling say "let's narrow our focus to the abstract atomist's claim that the singletons ground the amalgam, and the monist's claim that the amalgam instead grounds the singletons" (2019, 2039), they should actually be including in each claim that it is the singletons plus the empty set – i.e., that the amalgam is a mixed fusion, as discussed earlier in section 4.2. This is because those component parts are what they take to be *all* of the atomic abstracta, and so Trogdon and Cowling are not entitled to make the claim that it is only the singleton atoms which ground the amalgam (qua sum of all abstracta), as a key component would be missing. If the empty set is not atomic, nor an abstracta, however, as they think Lewis (1991)

holds, the empty set should still feature in a grounding claim about the sum of all singletons and the empty set (which is also what Trogdon and Cowling identify the amalgam with). However, the empty set being a concretum, simple or complex, would at least complicate, if not ruin, their proposal here, of the singletons (plus the empty set) grounding the amalgam.

To help differentiate the two ways to construe the entity, which may be rendered as two distinct entities depending on the categories its parts belong to, let's keep the amalgam as whatever is the sum of all abstracta, and let's call the sum of all singletons and the empty set the 'set-sum'. If Trogdon and Cowling are right to treat the empty set along with each singleton as all of the atomic abstracta, which are the parts of all of the abstracta, then the set-sum will be the amalgam. However, if the empty set is instead concrete, then the set-sum will not be the amalgam. Nor will the amalgam be a mixed fusion in this latter case, as it will be the sum of all abstract singletons only. This is actually a problem for Trogdon and Cowling's upcoming argument for the grounding of mixed fusions to be seen in the next section, as they rely on the amalgam being a mixed fusion to make the case for abstract atomism.

Which to me suggests that Trogdon and Cowling's choice to make the empty set an abstractum is not so much an optional choice, but is rather a requirement for their proposal. It also means that their call to narrow our focus also spoils the argument they end up making for abstract atomism because it is the fact that the amalgam is a mixed fusion that their main argument for abstract atomism rests upon, and we cannot both exclude the empty set from view as well as keep it there implicitly when it comes to the question of whether the fusion of all sets, empty or not, is either grounded in its proper parts or grounds them.

So, rather than allowing that Trogdon and Cowling keep their 'narrow focus', as it looks to beg the question, we can just extend the grounding discussion to be the general one which does not focus on the categories of the things involved. This also makes sense for the bigger picture for how this fits into the context of this thesis, because Lewis's system of sets and mixed fusions still mostly mirrors what Trogdon and Cowling are doing here. Not the grounding directional claims, to be sure, just the mereological structures and co-ordination of sets among them. Which means it can therefore be understood along similar lines, at least with respect to the grounding claims, even if not specifying what is and what is not abstract or concrete.

4.5 The argument for the grounding of mixed fusions

For the sake of discussing Trogdon and Cowling's actual argument for abstract atomism we will keep the empty set in view explicitly and put aside the other concerns I have raised surrounding

their inclusion of it. (It is worth keeping the emtpy set in view because it is the only part of the composite amalgam that Trogdon and Cowling can guarantee is non-membered from the outset – which helps ensure that the amalgam is a mixed fusion as required for their argument, and a mixed fusion in virtue of it having other (non-empty) sets as parts too.) Here is the main quote of their argument.

> Consider the following substantive general grounding claim: mixed fusions (fusions of non-empty sets and either the empty set or individuals) are grounded by the entities that compose them. As the amalgam is a mixed fusion, the [abstract] atomist's claim is an instance of this general grounding claim. And we think that this general grounding claim is highly plausible.
>
> Consider, for a moment, exotic entities such as *trout-turkeys*—individuals composed of trout and turkeys. Notice that the only reason we have to think that there are such things is because we already think that there are trout and turkeys and we accept universalism, the thesis that composition is unrestricted. There are no independent reasons to believe in trout-turkeys. [...] As such, trout-turkeys simply aren't candidate grounds for either trout or turkeys. More generally speaking, they play a substantive role in neither ordinary discourse, our best scientific theories, nor our philosophical theorizing.
>
> This moral extends to mixed fusions. The only reason we have to think that there are such things is because we already think that certain other things exist and we accept universalism. We lack independent reasons to posit them. Consider, for example, *cat-singleton-whiskers*, fusions of cats and singletons of their whiskers. We think that there are such things only because we already think that there are cats and singletons of their whiskers and we accept universalism. Turning to the amalgam, we think that there is such a thing only because we're already on board with the singletons and the empty set and again we accept universalism. As such, mixed fusions are no more candidate grounds for their proper parts than trout-turkeys are. [...] Provided that mixed fusions don't ground the things that compose them, we get the desired conclusion: mixed fusions are grounded by the entities that compose them. (2019, 2039-40)

What makes Trogdon and Cowling's argument troubled from what I said before is that we can see the places in their argument where they bring the empty set back in for the sake of making their

points about mixed fusions, although this was supposed to be cut off for the discussion. Although troublesome, I already said that this is a technicality we would put aside. In terms of plausibility, I should say that I think the gist of this argument is satisfactory – it is difficult to imagine that composite mixed fusions ground their proper parts in general, in the sense of ground as make them exist, when these entities have never been entertained outside of a few philosophy texts, and only seem to be an 'artefact' of unrestricted composition and the independent commitments to non-membered entities and membered entities. Indeed, a rejection of sets would entail that no mixed fusions exist, and as the existence of sets as abstracta is contentious in the face of anti-Platonist or nominalist views concerning existents, it seems fair to think it would be unwise to permit mixed fusions as the objects which all others depend for their existence on.

The recognition of the existence of mixed fusions makes much more sense in the direction of starting with common entities, and seeing how a powerful principle has certain logical consequences. That is, it looks to be the case that we learn about the existence of mixed fusions via some principles of mereology and our previous commitments, rather than it looking to be the case that mixed fusions are what account for the existence of the things we are committed to. Yet I said 'all others' above because even though Trogdon and Cowling try to focus on abstracta only, everything that they say still accords with the picture of the mereological Universe as a whole, and in the context of there being both sets and non-membered entities other than the empty set, the mereological Universe itself is a mixed fusion. This leads to a big problem for their argument. Although it is supposedly constrained for its conclusion, it is clearly presented with general applicability, such that it actually has some bearing on the questions of both priority atomism and priority monism in general. However, Trogdon and Cowling do nothing to engage with the problems that stem from the general case.

Consider that under their system and proposal the mereological Universe is a mixed fusion. This is the same as in Lewis's (1991) system, where Lewis calls this mereological Universe 'Reality'. The mereological Universe is a mixed fusion here because it is the sum of all things – all the membered things and all the non-membered things. Going along with Trogdon and Cowling's substantive general grounding claim, that mixed fusions are grounded by the entities that compose them, it follows that the mereological Universe is grounded by the entities that compose it.

Yet there is a key difference to be found in our attitudes towards the mereological Universe and any old mixed fusions. While it may be the case that the only reason we have to think that there are such things as *mixed fusions* is because we already think that there are sets and non-membered things and we accept universalism, there are independent reasons to believe in the existence of a *mereological Universe*, even if not many use this name or terminology. One such instance is the

ordinary belief that there is something that we and everything else are all a part of. While there are several mereological criteria which need to be met to make this ordinary idea more rigorous – such as which conditions enforce the uniqueness of that mereological Universe, and just which composition principles can support one, especially as not everyone who accepts that there are composites accepts that composition is unrestricted – it is still such an ordinary idea that it isn't something that might be said to be only cooked up by a certain combination of metaphysical principles, unlike how mixed fusions themselves can be seen to be.

Further to this, unlike the situation with trout-turkeys we cannot say that the mereological Universe simply isn't a candidate grounds for either trout or turkeys, or anything else, because this is the very question at issue in the general case of priority atomism vs. priority monism! That is, whether the proper parts of the whole ground the whole or whether the whole grounds its proper parts is exactly the question at issue. And more generally speaking, the mereological Universe *does* play a substantive role in each of our ordinary discourse, our best scientific theories, *and* our philosophical theorizing. Across these situations it is an ordinary posit that finds its way into how people place themselves amongst existents, modulo the details; in any nominalist science it is just the cosmos or the physical universe; and in philosophy it is a key item in the very debate at hand. So one of my contentions is that Trogdon and Cowling cannot make the argument they make for abstract atomism without begging the question against a priority monism involving a mixed fusion as the mereological Universe, and this is clearly unacceptable.

4.6 What about gunk?

My other contention is that Trogdon and Cowling's argument also begs the question against the possibility of mereological gunk, that entity which is infinitely divisible into more and more proper parts. We can see this if we consider that gunky things are paradigm non-well-founded entities in the context of their proper parts founding them, and this is because there is no decompositional entity/entities at which their mereological structure terminates. While this is not to say that gunk is not *grounded*,[5] however, it is the case that on the ordinary conception of metaphysical or ontological foundationalism – one particular view of which Trogdon and Cowling espouse in terms of grounding and abstract foundationalism – that this thesis at least requires well-founding (Bliss 2014).

This is an issue for their view, then, which asserts that mixed fusions are grounded in their proper parts. This is because for some of the proper parts of the mereological Universe to be gunky

[5] For discussion of well-founding in grounding, see Dixon (2016), and for distinctions between well-founding and grounding see Trogdon (2013, 108).

parts is for these, in being non-well-founded, to not provide appropriate *foundations* for the mereological Universe which they are supposed to ground. As Trogdon and Cowling advocate a kind of *priority pluralism* (cf., Schaffer 2010) in which the mereological Universe is said to be more than the usual "fully *grounded* by any proper parts which together compose it" (Dixon 2016, 444) (my emphasis), because it is also *founded* by them – a priority pluralism I will dub *proper part foundationalism* – the prospects of there being gunk in the cosmos in this scenario are cut off. This, of course, will not do, as Trogdon and Cowling say that they "take the possibility of concrete gunky objects seriously" (2019, 2037, fn 22).[6]

This makes sense – one of the reasons to favour GEM over mereological nihilism is that it leaves the question of gunk open, whereas nihilism precludes it from the outset. Additionally, Trogdon and Cowling maintain much of Lewis's system, wherein he says, in describing the potential mereological makeup of Reality, "Maybe there isn't any gunk; or maybe there aren't any atomic individuals; or maybe there are both." (1991, 21) All of which leaves their proposal in a precarious position, because of what it implies for the general case beyond just abstracta.

As Dixon highlights "Schaffer [2010] discusses several strategies the pluralist might employ in an attempt to accommodate the possibility of gunky objects. Each of these strategies, Schaffer points out, puts the pluralist in an awkward position. Of relevance to the issue at hand is the strategy in which the pluralist admits the possibility of non-terminating grounding chains. But this, Schaffer notes, contradicts the claim that grounding is well-founded. As Schaffer does not think any of the other strategies are successful either, he finds reason to prefer monism to pluralism." (2016, 445)

Although it is available for Trogdon and Cowling to forgo the well-foundedness of grounding, such that their claims to a general priority pluralism could still hold in the case of a partially gunky mereological Universe which is a mixed fusion, this is not in keeping with their foundationalism as presented, and would require a much greater defence of the grounding conditions than the one they provide. Worse than this is that, even leaving aside these issues surrounding well-founding, there are still other problems for their proposal. We see this if we ask after the a- and c-grounding status of a mereological Universe such as Lewis's Reality, or such as one that follows their strictures, which I will call the 'cosmalgam'.

[6] Recall, however, that they already preclude gunk in the abstract domain via their claims that the only abstracta are formed from sets, and that all of the singleton sets and also the empty set are atomic.

4.7 What is the grounding status of the cosmalgam?

Recall Trogdon and Cowling's definitions of a- and c-grounding: "let's say that an entity is c-grounded just in case it's grounded by concreta, and an entity is c-fundamental just in case it isn't c-grounded ... let's say that an entity is a-grounded just in case it's grounded by abstracta, and an entity is a-fundamental just in case it isn't a-grounded" (2019, 2030 and 2031).

Now consider their express commitment that "reality divides, *exclusively* and *exhaustively*, into the abstract and the concrete" (2019, 2036) (my emphasis). From this package we have it that every entity is *either* abstract or concrete, and there are no other possibilities, and that every entity is at least one of (but maybe more) a-grounded, c-grounded, a-fundamental and c-fundamental. I say 'but maybe more' here because it appears that via the definitions, and also according to Trogdon and Cowling's discussion, there can be a variety of combinations. For instance, from the definitions we can tell that if something isn't c-grounded that it is c-fundamental, which seems to allow that an abstractum which is a-grounded may be c-fundamental if it is not c-grounded.

Trogdon and Cowling endorse this general sentiment when they say "It's an open question whether any ground for a concrete[/abstract] entity is itself concrete[/abstract]. So, from the fact that a concrete[/abstract] entity is grounded, it doesn't follow that it's grounded by concreta[/abstracta] in particular, at least in the absence of further argument." (2019, 2030 and 2031) Now, it would be the case that any a-grounded thing is c-fundamental if being a-grounded precluded it from being c-grounded. This is following the condition that "an entity is c-fundamental just in case it isn't c-grounded". Yet this preclusion doesn't hold up under Trogdon and Cowling's openness. Indeed, they even offer that "Singletons of concreta such as {Socrates} are potential examples of abstracta that have concrete grounds but lack abstract grounds." (2019, 2031) However, there is further argument that such *grounding-kind heterogeneity*, as I will call it, is implausible, and is not in fact what Trogdon and Cowling articulate in terms of the amalgam.

We will start with the latter issue of this not being what Trogdon and Cowling articulate, and cover the implausibility claim after. Take the given example of singletons of concreta such as {Socrates}. According to Trogdon and Cowling, by their use of 'grounds' "we always mean 'fully grounds'. Intuitively, a full ground provides a fully satisfactory explanation of what it grounds, while a mere partial ground does not." (2019, 2030) This is one of their five starting assumptions regarding grounding (2019, 2030). Another one is that grounding is transitive, such that if A grounds B and B grounds C it follows that A grounds C (2019, 2030). In the example of singletons of concreta such as {Socrates} being grounded on their concrete members, the claim that the singletons and the empty set fully ground the amalgam does not hold. This is because, even if all singletons of concreta are a-fundamental, in that they have no a-grounds – which Trogdon and

Cowling hold of all singletons, that they are a-fundamental – these are still c-grounded. If to be grounded is just to be fully grounded, then all singletons of concreta are fully grounded in the concreta which are their members. But if all singletons of concreta are fully grounded in concreta, and the amalgam is fully grounded in every abstracta membered set and concreta membered set and the empty set, and grounding is transitive, then the amalgam is fully grounded in both abstracta and concreta.

Yet, as detailed earlier, Trogdon and Cowling argue for the thesis of abstract atomism: there are multiple simple abstracta, any simple abstract entity is a-fundamental, and *any complex abstract entity decomposes into simple abstracta such that the latter ground the former*. (2019, 2033) (my emphasis) Remembering that "the latter ground the former" is to be read as "the latter fully ground the former", as in "the latter provides a fully satisfactory explanation of the former" we run into the issue that it does not look like it can be properly said that the simple abstracta, of which the singletons of concreta are supposed to be some, genuinely fully ground the amalgam, let alone less maximal complex abstracta. How can they, together with the abstracta membered singletons be said to fully ground the amalgam, if they themselves are fully grounded in concreta?

This suggests that the notion of something fully grounding something else is not that it provides a fully satisfactory explanation of what it grounds. Which is to say that if we are told that the sets fully ground the amalgam, as Trogdon and Cowling do tell us, and we are told that this is a fully satisfactory explanation of the amalgam's grounding situation, it sounds like we are being misled. Key information is missing – most notably the fact that the concreta which are members of singletons fully ground their singletons, thereby fully grounding the amalgam in concert with the abstracta only membered sets and the empty set. Even if we think that some of what this means is that the notion of full grounding being transitive is troubled, the bigger problem for Trogdon and Cowling is that their abstract atomism thesis looks to be obviated as is.

Turning to the former issue, we now examine my claim that grounding-kind heterogeneity is implausible in this context in general. Notice that because of the exclusivity and exhaustivity conditions with regards to things being only either abstracta or concreta, that the mixed fusion which is the mereological Universe – the cosmalgam – must be either abstract or concrete. Presuming that it is not abstract, because Trogdon and Cowling want the amalgam to be just the sum of all sets, empty or not, then the cosmalgam must be concrete on their view. Following the assumption that grounding is transitive, let's say that according to priority pluralism in this case that as the concreta proper parts of the cosmalgam all ground the cosmos proper part, and the abstracta proper parts of the cosmalgam all ground the amalgam proper part, that thereby the cosmos along with the amalgam grounds the cosmalgam. As before, if we are to understand by 'grounds' that

what is always meant is 'fully grounds', and remembering that every entity is at least one of a-grounded, c-grounded, a-fundamental and c-fundamental, and that the cosmalgam is being claimed to be grounded, we would like to find out whether the cosmalgam is fully a-grounded or fully c-grounded or both or neither.

Prima facie it sounds like to ask of something that is said to be grounded in its array of abstract and concrete proper parts whether it is either fully a-grounded or fully c-grounded is absurd, because the 'full' in each grounding type appears to deny the grounding contribution of the proper parts which are not of such types. That is, when the amalgam and the cosmos jointly ground the cosmalgam, neither can "fully" a- or c-ground it, for an entity is a-grounded just in case it is grounded by abstracta, and an entity is c-grounded just in case it is grounded by concreta. The cosmalgam fits neither of these bills, as it is not grounded by either solely concreta or abstracta.

If we were to generalise the grounding statements here, they would only amount to 'an entity is grounded just in case it is grounded by _____", which just means "an entity is grounded just in case it is grounded". This tautology does not sound informative at all. Now we have a big problem. If the cosmalgam is not a-grounded, then it is a-fundamental, and if the cosmalgam is not c-grounded, then it is c-fundamental, by our earlier definitions. If those are the exhaustive types of fundamentality as supposed, in that being either a- or c-fundamental or both is the only type of 'fundamental' a thing can be then the cosmalgam is simply *fundamental*. But anything that is fundamental is not supposed to be grounded at all. Clearly this goes against the claim of the priority pluralist, whether atomic or gunky, that the proper parts of the mereological Universe are the grounds for it. So we are left with the terrible situation that a generalisation of an argument for the grounding of the amalgam on its proper parts to the grounding of the cosmalgam on its proper parts, ends up contradicting the thesis that it should be defending.

4.8 Conclusion

The mereological Universe as a mixed fusion that is a cosmalgam, then, does not plausibly look to be grounded in its proper parts. This, and the other issues presented, also suggests that Trogdon and Cowling's proposal is inadequate for defending the claim that the amalgam is grounded in its proper parts. Whether this conclusion leads to it being the case that the mereological Universe as a mixed fusion is better seen as the grounds for its proper parts, or whether grounding claims (in any direction) involving such a Universe should instead be rejected, is work for another time.

For the purpose of this thesis, though, this result leaves us in a similar place as that at the end of Chapter 2. That is, while it is not apparently paradoxical or contradictory for the

mereological Universe to ground everything else, it definitely appears to be less plausible that the mereological Universe qua mixed fusion is the reason everything else exists given the conceptual contingency of mixed fusions. Moreover, it also presents another situation which is largely unprecedented in the literature. As such, this outcome still has an aporetic feel to it, much in the same way as how the double non-well-founding in Chapter 2 does not look to be paradoxical but yet is still very undesirable for presenting only bad options going forward if we don't drop unrestricted composition. I do think that if one were to insist on sticking with GEM and unrestricted composition here, it just seems like it again would leave us at a crossroads of unpalatable decisions to make, either doubling down on an implausible grounding by the mereological Universe, or discarding the concept of grounding entirely (because minimising it to small, mereological pockets also seems farfetched). For proponents of grounding I expect this to be an unpleasant choice, and so I offer that the better way out really is to reject the mereology and composition principles which seem to force it.

In fact, I think that if Trogdon and Cowling do wish to maintain their grounding scheme, that they had better reject unrestricted mereological composition/universalism after all. Moreover, their scheme looks to be one that *requires* such a restriction (and could maybe be put forth as an argument *for* restricting composition). This is because their view actually strictly prohibits composition between abstracta and concreta by their holding that concreta only have concrete parts, and abstracta only have abstract parts, and these being exhaustive kinds. In that case sums of abstracta and concreta can be *neither* of these kinds, and therefore cannot exist (a type of situation I detail further in Chapter 5). I suppressed this issue earlier for the sake of my argument, to show the problem of the cosmalgam upending their grounding claims, yet my argument only really works if we ignore the fact that the cosmalgam shouldn't be concrete or abstract anyway, according to their proposals.

With regards to the subtle mereological hybridism which reared in this chapter, via the common kinds of parts being either abstract or concrete, I think we have learned too that when it comes to grounding in the context of a mereological Universe which is a transcategorial sum of abstracta and concreta, that things are more complicated than they may at first appear. Furthermore, there's also the problem that given the usual view of the ontology of classes as abstracta, and the usual view of ourselves and the natural universe as concreta, for instance, that in the case that the mereological Universe is the ground of all things, there could be a mixed kind of grounding taking place, from something which is not clearly either abstract or concrete itself.

Although the question of a form of mixed grounding won't be pursued in this thesis, in the next chapter I do go on to specifically examine the issues surrounding mereological hybridism in

the case of abstract-concreta fusions, such as mixed fusions having both abstract and concrete parts, with these being issues that do not involve notions of grounding at all.

Chapter 5
Mereology, modality, and the abstract–concrete distinction

5.0 Introduction

To make a case against unrestricted mereological composition principles, in this chapter I adapt and analyse a conception of the abstract–concrete distinction as one which distinguishes between the kinds of *parts* that entities belonging to these different categories may have. The conception being, roughly, that abstract entities have only abstracta for parts, and concrete entities have only concreta for parts.

While this conception is able to satisfy the traditional notion stemming from Plato that abstracta and concreta belong to two fundamentally distinct realms, the trouble is that its mereological character engenders a dilemma when coupled with a theory of part-whole relations which permits arbitrary fusions of things from any category of entity whatsoever, like the standard classical mereology. As these sorts of parthood theories allow mereological sums consisting of parts from both the alleged abstract and concrete realms, questions about such composites naturally arise. Are wholes with both abstracta and concreta for parts themselves abstract? Or are they concrete? Or do they instead undermine the dualism and not belong to either of these categories?

The dilemma arises because the standard abstract–concrete distinction at least involves conceptions of both mutual exclusivity and exhaustiveness across the two realms of beings (Katz 1998, Kaufman 2002, cf., Cowling 2017, 70-1). This appears to preclude such transcategorial composites, let alone there being any further category for them to fall under. Clearly we have a choice to make – do we restrict our mereology to avoid such violations or do we reject the standard abstract–concrete distinction?

As I see it, significant instances of incidentally taking the latter strategy are to be found within the ontology deployed in both David Lewis's (1986) *On the Plurality of Worlds* and his (1991) *Parts of Classes*. These works make substantial use of distinguishing between the abstract and the concrete; and also contain influential defences of GEM, and importantly in respect to its principle of unrestricted composition. Although such objects are not at all his main focus in these texts, therein Lewis acknowledges that the unrestricted composition principle entails certain supra-world fusions. The two varieties he describes either fuse parts from within multiple concrete worlds or fuse parts from both within concrete worlds and outside of them.

It is these latter entities in particular that can make good representatives of abstracta and concreta fusions, as Lewis's express views on concrete worlds imply that entities entirely outside of these worlds are abstract. Specific instances of such in/out concrete/abstract composites can be found in some of Lewis's mixed fusions. This is because mixed fusions are allegedly composed

from set-theoretical classes – which are abstracta and outside of worlds on his view – and set-theoretical individuals – such as ordinary concrete beings like cats and crows. (I only say 'some' mixed fusions here because there could be abstracta that are individuals, after all.) In contrast, the former type of entity – what Lewis calls 'trans-world individuals' – may serve as a different species of transcategorial entity, insofar as their being outside of concrete worlds results in them being abstracta, even with their concreta parts.

Considering that Lewis is a leading proponent of GEM, the fact that each of these kinds of entities appear to have cases whereby they comprise both abstract and concrete parts suggests that the clear line for him to take is to reject my mereologised version of the standard account of the abstract–concrete distinction. However, as he neither addresses the abstract/concrete status of either kind of supra-world fusion, nor treats the general issue of transcategorial fusion directly, we will need to determine what his rejection of this distinction might entail, and whether it is justified.

On my assessment, how Lewis applies an abstract/concrete division in his work, including how he characterises the consequences of entities being concrete and also the relations between in-world and out-world entities, leads to major problems for his modal system. By way of trying to reconcile Lewis's professed views in these works, some of the point of this chapter is to demonstrate and examine various constraints which arise for mereological views in the context of their employing such categorial delineation. Moreover, I seek to show that Lewis's mixed fusions and trans-world individuals are worth rejecting on the grounds that the troubles stemming from their alleged supra-world fusion instead justify a restriction on composition, specifically as a relation which does not cross world boundaries. Accordingly, this looks to overcome the well-known Lewis/Sider objections that any restriction on composition is unwarranted for leading to unintelligible vague existence/vagueness in how many things exist (Lewis 1986, 212-13 for the left side of the slash; Sider 1997 and 2001, 4.9 for the right).

To start making my case for these claims, in section 5.1 I introduce the standard abstract–concrete distinction, explaining some of the ways abstracta and concreta have been contrasted and conceived. I also give a summary of a variety of entities that fuse along the abstract/concrete dimension to indicate that some of the ramifications of my analysis may extend beyond Lewis's systems alone. In section 5.2 I suggest various abstracta and concreta parthood theses that are plausibly connected to the standard distinction, and I begin to examine how these can constrain mereological relations between the two kinds of entities. Additionally, I discuss how Lewis's concrete modal realism does not quite fit the standard distinction and start to consider his options for making the distinction in other ways.

Following this, in section 5.3 I present an argument that for Lewis's views to be consistent, he had better reject an element of the claims to the exclusivity of the two realms. I claim that he should instead accept concreta as able to have abstracta for parts (but not vice versa). Yet I also argue that due to the plausible notions of property inheritance that Lewis explicitly advances which support this outcome – namely a principle of concrete inheritance such that anything with any concrete parts at all is itself concrete – that he is forced to accept concrete impossible objects. This is a result which runs contrary to his well-known aversion to such entities, and which I further argue serves to undermine his case for concrete modal realism. (Note that such entities are not simply a contradiction in terms for technical reasons, as is made clearer by section 5.6.4.) To be clear, this principle of concrete inheritance is not one which Lewis can simply dispense with, both because he defends it directly and indirectly in a variety of ways, and also because he admits that without this inheritance his concrete worlds are not concrete. Which means that to dispense with it would be a death blow for his view of an infinite plurality of *concrete* possible worlds.

After demonstrating in section 5.4 that certain mereological outlooks are incompatible with the standard abstract–concrete distinction – and that some attempts to avoid this incompatibility are flawed – in section 5.5 I conclude my assessment of the prospects for abstracta and concreta fusions. In particular, I argue for restricting mereology to prohibit various claims of supra-world summation, such as that which permits mixed fusions and trans-world individuals. To be clear, I do not present an argument against abstracta and concreta fusions altogether. This is because the principle of concrete inheritance from Lewis's account that was used to make the previous case only suffices to prohibit abstracta from having concrete parts, and not the other way around. Instead, I claim that in the case of an exhaustive distinction between abstracta and concreta understood along the usual lines of their being fundamentally different that, at the least, every abstractum is concreta-part free. Even though I am inclined to believe the converse, and to thereby maintain the standard abstract–concrete distinction mereologically overall, it is not something I try to defend herein.

5.1 Beginnings of an abstract–concrete distinction

Making a distinction between abstract entities and concrete ones is a longstanding project in philosophy. On one understanding, the Platonic dualism between ideal Forms and sensible objects is seen as the origin of the distinction. Here Plato classified existents into two overarching domains, realms or worlds: the allegedly imperfect realm of mutable and particular sensible things, and the supposedly perfect realm of the immutable and universal, non-sensible Forms, which transcends it (Katz 1998, 125-6; Kaufman 2002).

For the sake of illustrating their expected ontological separation as concreta and abstracta, we might stylise this two realm view as follows (with no spatial or mereological implications intended).

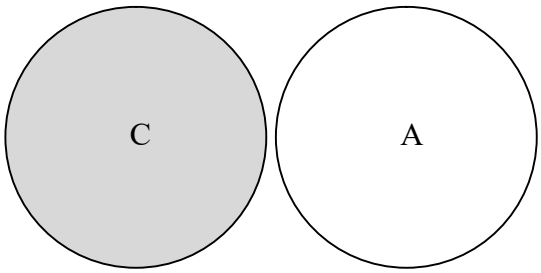

Figure 5.1 Stylised two realm view of concreta and abstracta.

Beyond its rejection of the Forms, the modern view of the abstract–concrete distinction mainly differs from Plato's by disagreeing on the causal status of abstracta. Where Plato viewed the Forms as exerting causal influence on the way the things are in the sensible realm, the prevailing contemporary notion of abstract entities regards them as having zero causal influence on the way things are anywhere (Rosen 2017). This negative conception of abstract entities is dominant when it comes to presentations of what their other general features might be, in that abstract entities are customarily depicted in terms of the features they lack.

For instance, abstracta are routinely presented as being, at the least: non-spatial, non-temporal,[1] non-mental, non-sensible, non-physical, non-material, non-corporeal, non-changing, and as we have already seen in the modern sense, non-causal.[2] Besides the Forms, some of the usual candidates for abstract entities are: numbers, sets, classes, properties, universals, relations and propositions, among other things.[3]

Concrete entities, on the other hand, are commonly regarded as having at least some of the features which abstracta are said to lack, if not all of the positive counterparts from their negative list above (modulo the mental for the obvious reasons). The usual candidates for concrete entities are physical/material entities, like what animals, plants and geological formations are normally

[1] Non-spatiotemporal also.

[2] However, one positive description commonly associated with abstracta is that they are *necessary* (cf., Tennant 1997).

[3] For the much longer list from which this one is derived, see Oppy (2014, 170). Whether alleged (supernatural or otherwise) gods are also abstracta candidates is highly contentious, even if they are considered to share some similarities to abstracta in being, for example, necessary, or non-corporeal, or 'beyond' space-time (all potential characteristics of proposed gods which require explicit specifications depending on the theology, of course) (cf., Leftow 1990). This can also be because gods are often understood to stand in causal relations to the natural world, which sets them apart from the way the usual abstracta candidates are normally conceived (cf., Cowling 2017, 83). As expected, within an exhaustive and exclusive two category view any gods would need to belong to one or the other category.

taken to be, which are kinds of entities ordinarily understood to have both spatial and temporal aspects. Other candidates include mental entities (such as minds), for importantly having at least temporal attributes. Across these cases, a key expectation for attributing concreteness to something is often that the entity exists within space and time, either as a part of the physical universe, or as one of the entities amongst which the physical universe could be understood to operate.

Concreta	**Abstracta**
Physical	Non-physical
Spatial	Non-spatial
Temporal	Non-temporal
Causal	Non-causal
Changeable	Non-changeable

Table 5.1 Some common attributions of concreta (sufficient) and abstracta (necessary).

As is usual, just as there is varying consensus on what all of the concrete entities are, there is no unanimous agreement on what to accept as any of the abstract entities.[4] In fact, some philosophers find it unreasonable to even consider that there are things that exist outside space and time, and therefore reject completely the conception of abstract entities from a purportedly transcendent realm.[5] This means that for any candidate entities that they wish to keep in their ontology which are usually taken to be abstracta, they have to explain how these are not really abstract entities, contrary to the standard expectation. Philosophers who elect to deny the existence of abstracta are generally called 'nominalists', whereas those who accept abstract entities along with concreta are usually called 'platonists'.[6]

5.2 Distinguishing realms

Following Plato's historical lead, a tendency among philosophers (and platonists in particular) when subscribing to an abstract–concrete distinction is to invoke two disparate realms of entities. For instance, according to Edward Zalta, "in our metaphysical foundations, there are two fundamentally

[4] For a very informative book-length treatment on conceptions of and debates regarding abstracta, see Cowling (2017).

[5] Witness Goodman and Quine "We do not believe in abstract objects [...] Any system that countenances abstract entities we deem unsatisfactory as a final philosophy" (1947, 105); Kotarbiński's view that there are only material bodies (Woleński 2016); and his friend Leśniewski, the progenitor of modern mereology, who "in elaborating his logic he emphatically rejects anything which is not concrete, individual, located in space and time" (Simons 2015, 4.3).

[6] With a lowercase 'p' to signify that it is not Plato's view of transcendent objects that they specifically share, but rather one with something of the same flavour, viz., an accepting of entities which are in addition to the familiar concrete ones we engage with on a daily basis, additional entities which exist in a radically different manner from those ordinary ones.

different kinds of objects, abstract objects and ordinary objects, constituting mutually exclusive domains." (2006, 678)[7] Along this line, Cowling (2017) says that questions about the abstract–concrete distinction may be of interest "because they demarcate explanatory projects and theoretical roles undertaken by the abstract realm and its inhabitants." (2017, 100) And besides the fact that Sider seeks to reject the abstract–concrete distinction as "a relic of a certain theory" which thwarts the simplification of ideology, he too conveys it as promoting that "reality divides into two realms—abstract and concrete—in a way that is significant on various fronts. Epistemic: we know about the abstract a priori. Modal: facts about the abstract are necessary. Causal: the abstract is causally inert. Spatial: abstract entities are not in space and time." (2013, 287)

Usually this presumed separation of the abstract and the concrete arises precisely from following some contrastive account of the expected differences in features that either kind of entity is supposed to have in comparison to the other.[8] As Kaufman points out, after describing the traditional distinction as something that "for [platonists] has been a standard dualistic ontology ... in which reality consists exclusively of two types of objects, *abstract* and *concrete*" (2002, 216), the defining of abstracta as "anticoncrete" in the manner of negative features given above ends up establishing the two as substances in fundamental opposition to each other, thereby enforcing their mutual separation (2002, 230-31). Indeed, this dualistic conception is what further serves to separate the abstract from the concrete by putting them not just under different categories, but also into different realms, or 'worlds'. As Jerrold Katz explains it,

> Early in the development of the theory of Forms, Plato seems to have held a "two world" view. In addition to the world we inhabit together with other spatiotemporally located and causally interrelated things, there is an independent world of nonspatial, atemporal, and causally inert Forms.[9] This view explains the objectivity and autonomy of the concepts examined in dialectic by taking the abstract to be transcendent (dwelling outside our world) and the concrete to be immanent (dwelling in our world). Plato's account of how we know the Forms would seem to commit him to the "two world" view. (Katz 1998, 125-6)

From its usual depiction, David Lewis's (1986) modal realism appears to be intuitively recognisable as a modern take on this two realm conception of an abstracta/concreta divide – even if it might not

[7] What Zalta means by 'ordinary' is our 'concrete' (cf., Wildman (2016, 184)).

[8] From a nominalist point of view this separation is, of course, guaranteed.

[9] This last piece of the description is flawed on Plato's account, however, because his Forms are supposed to contribute to the being of things in the sensible world as "causes par excellence" (Rosen 2017).

at first be understood to assert a *singular* concrete realm. This qualification is because, according to Lewis, there exists an infinite plurality of mereologically disjoint[10] concrete possible worlds, each causally independent and isolated from the other, each with their own self-contained spatiotemporal relations and world inhabitants. What exists in their entirety outside of all of these worlds, for Lewis, are the infinite number of mathematical objects posited by set theory, each of which counts as an abstractum on his view, for not being a part of any concrete world.

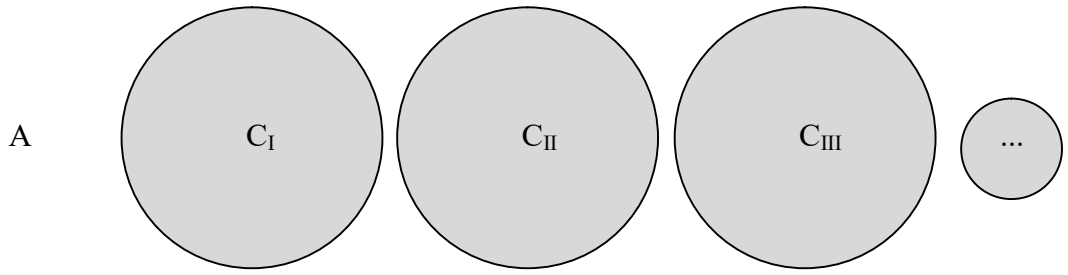

Figure 5.2 Stylised view of a small number of Lewis's plurality of concrete worlds stretching onwards, with transcendent abstracta.

With its multitude of concrete worlds, Lewis's system helps us put into focus a need to clear up potential ambiguity surrounding the notion of 'realm' in the context of the abstract–concrete distinction – and a clarification which I believe is worth making when discussing realms altogether. The ambiguity I refer to arises because at first one might assume that an exhaustive division of reality into two such realms entails that reality has two major parts, one abstract realm part and one concrete realm part, where a decomposition of each realm into constituent parts gives the denizens of said realm. Indeed, that is what the two world view modelled earlier in Figure 5.1 can suggest. Yet for this to be the case (especially under GEM) requires that each realm be a mereological unit, either as a simple or a sum, consisting only of the things which belong to the category which the realm represents.

However, it is not required that for a realm to be counted as one that it needs to be a *mereological* unity.[11] There are other options, two of which I find pertinent to consider for Lewis's system specifically. The first we might contemplate is that the realms are classes, whereby, e.g., the class of all concreta is identified as the concrete realm because it has all of the concrete entities (i.e., the worlds and their concrete parts) as members. Indeed, as Lewis accepts a class of all individuals

[10] I.e., non-overlapping by not sharing any parts.

[11] This is an important point to consider for the sake of not foreclosing realms under restricted composition mereologies, such as mereological nihilism which denies *all* composition. As other mereologies might still use the concept, such as to assert that the concrete realm contains e.g., all of the concrete simples (or otherwise all of the restricted concrete composites), they should be able to make this claim without the realm needing to be a whole.

qua concreta/possibilia,[12] this seems promising. Where it falls apart, however, is in identifying the abstract realm with a class too – presuming, as is plausible, that both of the purported realms are the same kinds of unities. The obvious problem arises because if the realm of abstracta is supposed to be the class of all abstracta it would therefore need to contain itself, in violation of the standard set-theoretic rules against self-containment for classes.[13]

I think the better option at least, for Lewis, is to see both of the single realms as genuine pluralities.[14] In this case, the concrete realm is to be identified with the plurality of concrete things, and the abstract realm is to be identified with the plurality of abstract things. Provided they have some different occupants,[15] each realm can be unique. If they together comprise all entities, then the realms are jointly exhaustive, and if they further share no occupants, then they are mutually exclusive. Although a myriad of pluralities in general can satisfy each of those three conditions, Lewis has the convenience of anchoring his identification of the occupants of the plurality of concreta to all of those things which have parts in possible worlds, leaving the rest which do not have such parts to be the abstracta.[16]

To be clear, the realms of the abstract and the concrete on this suggestion are not singular objects, like a class or a mereological composite would be. Rather they are literally unique pluralities of things, the occupants of which satisfy the requisite conditions for being abstract or concrete, and which would comprise either of those other two kinds of unities – either as the members of a class, or the parts of a whole, respectively – provided this was ontologically consistent. So, considering that Lewis is a major proponent of unrestricted composition, thereby allowing distinct fusions of every concrete object and every abstract one, why can't he just identify the realms with mereological sums?

The problem here lies precisely with the composition principle which affords those fusions, as it also yields so much more besides. To see it, recall that with unrestricted composition entailing that any plurality of entities forms a whole, it will follow that on Lewis's account of possible worlds there is a composite (let's call it 'C') with all of the concrete possible worlds (and transitively their

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[12] "[W]e have no notion what could stop any class of individuals – in particular, the class of all worlds – from comprising a set. [...] I continue to accept a set of all worlds, indeed a set of all individuals." (1986, 104)

[13] Lewis is well aware of this, e.g., "we dare not say that whenever there are some things, there is a class of them" (1991, 8).

[14] See Simons (1982, especially section 7) for how pluralities (what he calls 'manifolds') may be *counted* as one, without literally *being* one.

[15] While the term 'member' may be fitting in this context, I use 'occupant' instead to contrast with classes.

[16] Furthermore, this should be acceptable as Lewis is favourable towards plurals (cf., 1991, 3.2).

parts[17]) as its parts. This is in addition to a composite (let's call it 'A') which has none of the parts of any possible world as its parts. At first it seems we need look no further than these two mereological sums to see two realms on Lewis's account – one composed of all of the parts of concrete worlds, the composite C; and one composed of all of the parts disjoint from concrete worlds (whereby all of those parts disjoint from concrete worlds are themselves abstract) the composite A.

However, while this may give us an intuitive depiction of two distinct realms, one which has a large fusion of abstracta and the other which has a large fusion of concreta as parts (but not necessarily 'large' in the set-theoretic sense discussed in Chapter 2), such a portrayal is insufficient to capture the complete picture of all entities which exist under this system. Specifically, this conception neglects the abstracta-concreta fusions which unrestricted composition entails. In addition to not accounting for such 'cabs', as I will sometimes call the abstracta-concreta fusions, there is also the issue of trans-world sums which are not part of any concrete worlds also being a part of the proposed concrete realm, composite C. The trouble here is that, on the common reading of Lewis's modal realist system, concreta are only parts of worlds. So, treating trans-world sums as concreta is erroneous as they are not parts of worlds, which makes their inclusion as parts of C undermine the prospect of that composite being appropriately identified as the concrete realm.

What's more, these two kinds of supra-world entities, the ones which are cabs (whether mixed fusions or otherwise) and the trans-world sums, further complicate the legitimacy of even my 'realms as plurals' conception. This is because it is unclear whether cabs should themselves be identified as abstracta or concreta, as well as it being dubious that the common reading of Lewis's 'out-world' entities as abstracta is consistent considering that a great number of these will have some concrete parts and therefore, presumably, some concrete properties.

5.3 Mereological hybrids meet mereological models

Besides the examples from Lewis's work, there are a variety of other potential cases of abstracta/concreta based mereological hybridism to be found across the metaphysical literature. Immanent universals, pantheistic gods, the universe as a whole, impure sets, and structured Russellian/singular propositions all appear as candidates for entities which might contain both abstract and concrete parts.[18] Not all of these candidates are presented with the intention of them

[17] Remembering that the concrete worlds are parts of themselves too – i.e., improper parts.

[18] There have also been variously considered what Parsons (2007a) calls 'mixed objects', which have both spatiotemporal entities and non-spatiotemporal entities as parts. Here the former could be ordinary material things like a

being recognised specifically as instances of such mixed creatures, however, rather they may only be proposed within the greater context of whatever theory it is that they are used to support (or are supported by, or are consequences of, etc.), and just so happen to also be cases of transcategorial composites – much like those examples sourced from Lewis's work above.

To be sure, the way that these entities can be regarded as cross-category composites depends on their specific mereological construal. For instance, a god regarded as mereologically simple will not be a transcategorial composite,[19] whereas a pantheistic one as the sum of all things, or perhaps similarly as the sum of all possible worlds, very well may be.[20] Similarly, if all propositions are paradigmatically understood to be abstracta, then whether propositions about particular concrete individuals are literally structured by having these individuals among their constituent parts (along with e.g., properties and relations (à la Soames 1987), and these taken as abstracta) will warrant regarding them as transcategorial sums. If propositions are not so structured, it might not be. In a different manner, impure sets[21] need not be accepted as having either their members or the null set as parts, which hinders impressions that impure sets are indeed abstracta[22] with concrete parts, and thereby transcategorial composites.[23]

Although Lewis himself proffers scepticism about what the abstract–concrete distinction exactly amounts to, he does accept that these two categorisations mark "fundamentally different kinds" (1986, 1.7), while also himself deploying a specific conception of concreteness for possible worlds wherein their proper parts all have ('analogically' or familiar) spatiotemporal relations with distinct parts within the same world (1986, 1.6). Therefore, as some distinction between the two kinds is still essential for Lewis's defence of concrete possible worlds, it is worth examining how best to reconcile the aforementioned difficulties in abstractum/concretum identification to see how Lewis's options affect the prospects of his systems plausibility. This is a task which should also prove useful for considering such issues more generally, as with the abundance of other potential cabs. It is also worth considering some of these concepts in more detail, and in general, to further understand how the varying conceptions constrain our options overall, whether or not a framework like Lewis's is adopted.

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human, a cat or a river, and the latter could be numbers, presumably, or places qua immaterial things (2007a, 205 and 226-7; cf., Lewis 1986, 212 fn 9; Armstrong 1997, 13; Oppy 1997, 327; and Effingham 2011, 458 fn 5 and 466).

[19] Or a composite at all, cf., Lombard (2007–2010), vol. 1, p. 108.

[20] See Oppy (1997) and Nagasawa (2014; 2016; 2017) on such topics of divine mereological complexity.

[21] I.e., those sets which have ordinary things as (at least some of their) members.

[22] As sets in general are paradigmatically understood to be.

[23] For a variety of non-GEM strategies for treating sets as having their members as parts see Katz (1998), Johnston (2006), Fine (2010), Caplan et al. (2010) and McElhoes (2017).

To illustrate specifically the problems Lewis's system faces with regards to the two realm view, though, and to assist with examining the associated consequences, consider the following 4-atom GEM style model consisting of two abstracta atoms and 2 concreta atoms.[24]

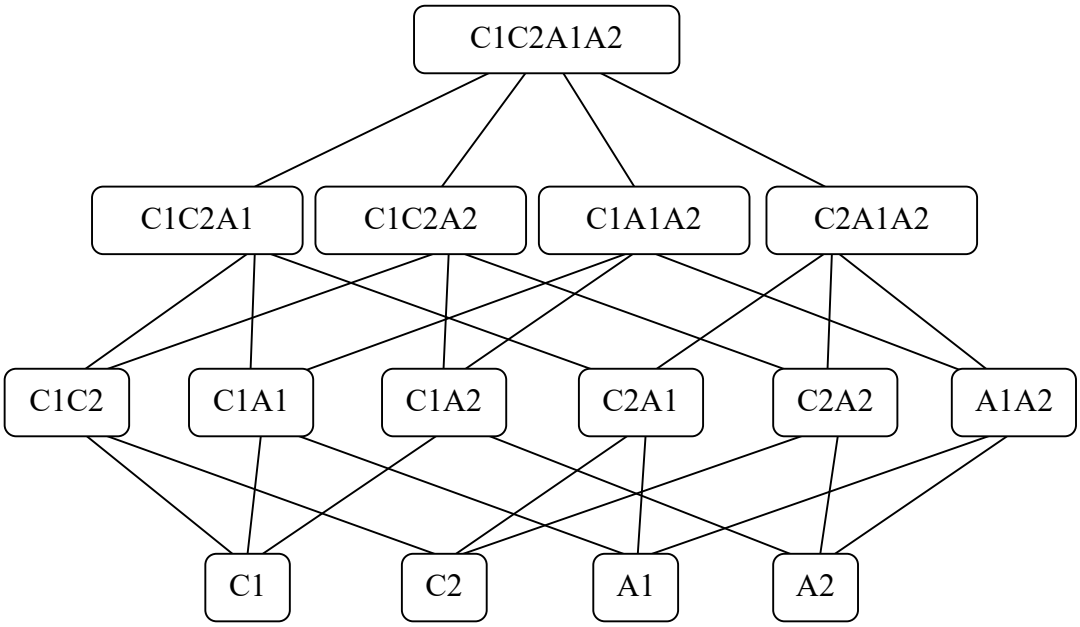

Figure 5.3 A four atom model of concreta/abstracta fusion.

If we treat C1 and C2 each as the sole distinct possible worlds for the sake of argument, then their sum C1C2 is the sum of all possible worlds on the model – paralleling the composite C above. And the sum A1A2 is itself a parallel of the composite A above, as it has none of the parts of any possible world as its parts.[25] Following this understanding of the model, the most we can say with confidence is that C1 and C2 are each concrete, and that A1, A2 and A1A2 are all abstracta. But what about the rest? The model reveals how we can't treat the two composites C and A as exhaustive in terms of everything being either abstract or concrete, because there would be many entities unaccounted for that are not parts of either of those sums. (Indeed, on the model most of the entities are cabs.) What's more, the idea that C1C2 is itself the concrete realm actually flounders against the expectation that any non-world entities are abstracta. This is also an expectation which, if followed through completely, would make the totality C1C2A1A2 the best candidate for the abstract realm as a whole, for its having all abstracta as its parts, yet also the worst candidate for being the abstract realm, for having all concreta as its parts too.

This sort of model clearly reveals my concern with how unrestricted mereological fusion does not seem compatible with the standard abstract–concrete distinction, especially as it operates

[24] Where a mereological atom is an entity which has no proper parts, and only has one (improper) part – itself.

[25] Notice too that because A1A2 is the sum of the things which overlap no parts of worlds, it only has the kind of not-world parts which themselves do not have any parts from possible worlds as its parts.

alongside realms and/or worlds. In the next section I go into greater detail about what exactly the problem stems from and makes troubles with, while starting to precisify some of the complex arrangements of theses proposed for this situation.

An important collection of questions to resolve are whether cabs are in fact abstracta or concreta? Or even whether the abstract–concrete distinction needs to be reformed, if not outright rejected, in the presence of certain mereological principles? Or whether, instead, it is the mereological principles which prove unworthy of being maintained? To consider this series of questions we keep our model in Figure 5.3 above in mind, but also focus on the epitome of abstracta and concreta sums given in Lewis's mixed fusions of classes and concrete individuals.

To keep things mereologically uncomplicated, let's consider a mixed fusion which is a composite sum of just two mereologically simple entities – something like C1A1 from Figure 5.3. For the abstract part of this mixed fusion we will treat 'unit classes' or 'singletons' (i.e., classes with only one member), as Lewis (1991) defends them, viz., mereological atoms which have no other parts besides themselves. For the sake of this example, for the concrete part of the mixed fusion we will imagine electrons as having no other parts besides themselves. Then, with the singleton 'S', of the null set, and an electron 'E', from the moon, say, together forming a composite 'MF' by unrestricted composition, some of our earlier questions arise: is the mixed fusion MF an abstract entity, a concrete entity, or neither? Let's examine the prospects for these answers in turn.

5.4 Are fusions of both abstracta and concreta abstract or concrete?

For the two questions of whether MF is an abstract entity or whether it is a concrete one, we need to consider if it is feasible that any abstract entity can have any concrete parts (and vice versa). We have already encountered the expectation that this is implausible, both because of the negative contrast usually given to abstract things when compared to concrete things and because of the two realm view which puts abstracta in a domain separate from concreta. Consequently, the suppositions that abstracta are uniformly abstract and that concreta are uniformly concrete are ones which it seems fair to say would not allow that either abstracta or concreta have the other kind of thing specifically as parts.

As Katz avers, following the convention that concreta are spatiotemporal objects, "It is implicit in the traditional[26] abstract/concrete distinction that the abstract has no taint of the concrete and the concrete no taint of the abstract. An abstract object can have nothing about it that is spatially or temporally locatable, and a concrete object can have nothing about it that is neither

[26] Modulo our previous correction regarding the causal aspect of Plato's Forms.

spatially nor temporally locatable." (1998, 121)[27] In this vein, Katz seeks to explicate the standard abstract–concrete distinction in terms of his:

> (D) An object is abstract just in case it lacks both spatial and temporal location and is homogeneous in this respect. An object is concrete just in case it has spatial or temporal location and is homogeneous in this respect. (1998, 124)[28]

Katz characterizes homogeneity in the following way "I shall say that an object is *homogeneous* when, roughly speaking, its constituents each have the same ontological nature as the object itself. Abstract objects are, as it were, abstract all the way down, and concrete objects are concrete all the way down." (1998, 121) We can precisify Katz's characterisation of homogeneity here using Seibt's stricter definition of things when they are uniformly like-parted as these things being 'homeomerous'.[29]

> (H1) A whole of kind K is homeomerous just in case all its parts are *of the same kind as the whole, i.e., of kind K*. (2017, 256)

From here we can distinguish homeomereity from homogeneity which varies by recognising that a whole may be identified as of a different kind to its proper parts, while these parts are all of the same kind as each other.

> (H2) A whole of kind K is homogeneous[30] just in case all the parts of a whole of kind K *are all of kind K*, which may differ from kind K*.[31]

For example, a whole which is of the kind 'cardboard box' can be identified as homogeneous with respect to[32] all of its proper parts which are of the kind 'cardboard'. This whole is not

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[27] Cf., Hodes (1984: 127) "Numbers are so pure, so unstained by the cement of the universe, that reference to them and their ilk seems quite *sui generis*."

[28] Note that this is clarified to include mental entities which, if not spatial, are at least temporal. Compare Frege taking abstract objects as belonging to a third realm separate from both a distinct "external world" and realm of consciousness. (Beaney 1997, 336-7)

[29] This follows a transliteration and simplification of the ancient Greek adjective 'όμοιομερής' (homoeomerēs) and the associated noun 'όμοιομέρεια' (homoeoméreia), terms which may have first been used systematically by Aristotle (Seibt 2017, 255). In the distinction between homeomerous and homogeneous, roughly, 'homoe' (like) and 'meros' (part) are being contrasted with 'homo' (same) and 'genos' (kind).

[30] Seibt mistakenly writes 'homeomerous' again for this definition of 'homogeneous'.

[31] Across both definitions the term 'part' is meant to be read in the restricted sense of 'proper part' only.

[32] Note here that both definitions may be employed in more restricted senses which relate to what is called the 'granularity' of the parts concerned. In this case, proper part recognition is restricted to something like 'cardboard piece' grain-size, while things like the molecules that are proper parts of the cardboard box (on ordinary views) which

homeomerous, however, with respect to the kind 'cardboard box', as not all of its proper parts are cardboard boxes, if any.

Looking back at Figure 5.3 we can therefore identify the composite C1C2, if an abstractum, as homogeneous (and not homeomerous) with respect to the kind 'concrete', as all of its proper parts are concreta. This situation presents a further issue we need to consider alongside the plausibility of cabs when it comes to unrestricted composition – that of whether a whole which is an abstractum can be homogeneously concrete. I think the instinctive and natural response is no, because it's absurd, a response which may be shared by many. Consider Kaufman (2002) making clear that – despite some differences between the more traditional conception of the abstract–concrete distinction and Katz's version given in (D) – "the two have one common feature: the abstract and the concrete are taken to represent two *fundamentally different* orders of things, metaphysically homogeneous[33] things with incompatible properties." (Kaufman 2002, 218)

What I plan to do, using the definitions given, is precisify this sort of objection, and to discuss the strategies some might take in response. Before we continue, however, further clarification of the usual understanding of the abstract–concrete distinction as involving at least the joint exhaustiveness and mutual exclusivity of the two categories is in order. Here, on my understanding, to assert that the abstract–concrete distinction is exhaustive is to accept

Exhaustivity Every entity is either abstract or concrete, or both.

While my Exhaustivity thesis here asserts that no entity can be *neither* abstract nor concrete – as that would be a further option beyond what has been exhausted – it additionally does not imply that each entity is *only* one of either abstract or concrete, as that is what the claim of exclusivity is for.[34] Compare, e.g., Cowling who instead offers, "(i) *Exhaustiveness*. The abstract–concrete distinction is exhaustive. It applies to absolutely all entities and guarantees that each entity is either abstract or concrete." (2017, 70)

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are not themselves cardboard pieces have been ignored. For questions involving abstracta-concreta fusions the size of parts is irrelevant, because of how broad these categorisations of kinds are supposed to be.

[33] As Kaufman's paper primarily discusses Katz's views here, it is expected that what Kaufman means by 'homogeneous' follows Katz's definition too.

[34] It should be noted that Exhaustivity operates solely along the dimension of the abstract and concrete here. This means that this thesis does not preclude there being other category delineations which are either supersidiary or subsidiary to it, such as, e.g., 'object' as a category covering both of them (cf., Rettler, Bailey 2017, 1.1; Katz 1998, 145), or, e.g., class and proposition being subsidiaries of the category abstract, or, e.g., physical and mental being subsidiaries of the category concrete (cf., Katz 1998, 145). Also note that Exhaustivity is still compatible with either purely concreta (à la reist or nominalist) or purely abstracta (à la Pythagoreanist) ontologies.

This slight difference in our definitions results in Cowling's succeeding statement of exclusivity being redundant because its main point, that no entity belongs to both of the two categories, is already implied. "(ii) *Exclusivity*. The abstract–concrete distinction is exclusive. No entity belongs to the category *abstract* entity and to the category *concrete* entity. When paired with Exhaustiveness, Exclusivity partitions all entities into the two categories of *abstract* entity and *concrete* entity." (2017, 70)

Besides this, Cowling-Exclusivity fails to ensure that an important criterion for the platonist is met, namely that not only should no entities belong to both of the categories, but also that each entity is completely either abstract or concrete and does not harbour any traces of the other category – in other words, that both of these kinds of entities are uniformly their own kinds.[35] (After all, this is both why inhabitants of the two realms are kept apart and not integrated on this view, as well as why the properties attributed to either kind of entity are set in distinction to the other.)

With this in mind, on my understanding, to assert that the abstract–concrete distinction is exclusive is to accept

Exclusivity Abstracta and concreta are each uniformly their own kinds.

How exactly the term 'uniformly' is to be understood in this context is something elaborated upon below. For now we have, at least, that Exhaustivity guarantees there are no other categories (along the same dimension) beyond the abstract and the concrete, whereas Exclusivity ensures that where something is of one of those categories that it is of it uniformly, and thereby also unable to belong to both categories.

Following all of the foregoing lines of thinking, the standard abstract–concrete distinction therefore appears to imply that the two categories are both mutually exclusive and jointly exhaustive, with exclusivity understood in the sense that the things of each category do not have things of the other category as their parts, and with exhaustivity understood in the sense that there are no other categories besides these two. From this perspective, then, it seems like the abstract–concrete distinction can be understood to entail that the things that exist are exactly either entirely abstract or entirely concrete (in the homeomerous sense), and nothing else.[36] I call this the Ex-Ex thesis:

Ex-Ex Anything is only either entirely abstract or entirely concrete.

[35] Cf., Katz (1998, 121 et alibi).

[36] 'Nothing else' along this specific dimension, to be sure.

For our question of whether mereological composites like MF (which recall has an abstractum singleton S and a concretum electron E as its only proper parts) are themselves abstract, the consequence of the Ex-Ex thesis is that they are not. Nor are they concrete. These consequences come from the Ex-Ex thesis's two implicit abstracta and concreta homeomereity theses, that anything which is an abstract entity only has abstract entities as parts, and that anything which is a concrete entity only has concrete entities as parts.

Abstracta Homeomereity Any abstract entity only has abstract entities as parts.[37]

Concreta Homeomereity Any concrete entity only has concrete entities as parts.[38]

With such homeomereity conditions prevailing, it is not possible for things composed of both abstract and concrete entities to be either abstract or concrete themselves. Along with the remaining exhaustiveness commitment of the Ex-Ex thesis, this means that, as understood from the point of view of the standard dualistic ontology inherited from Plato through to its slight revision in Katz's (D), mereological composition between entities from these two different categories is denied, because such composites can belong to neither of the two available categories, and there are no other categories available for these to belong to.

The consequence of the Ex-Ex thesis being true is thereby that composition cannot occur between things from the two different categories of the abstract and the concrete. Which means, in this case, the existence of entities like MF is denied. Such a conclusion constitutes a clear obstruction of GEM by restricting its principle of unrestricted composition from operating beyond single-category domains in contexts of there being both abstracta and concreta. Although this sort of limitation may already be adhered to by GEM supporting nominalists, such as Nelson Goodman (1951) who already expect only a concrete domain for mereology to operate in anyway, or also by those mereologists that follow the common practice of restricting attention to the concrete domain for many typical applications of GEM, such as when seeking to utilise the theory to explicate things about spacetime and our surrounding concepts therein (cf., in Kleinschmidt 2014, xxii and Lando 2017), there are still views which do treat it as an absolutely general theory of composition,

[37] Cf., Sajama and Vihjanen's claim that "abstract entities cannot have parts that are concrete entities." (2013, 88); and, somewhat similarly, Berto and Jago reporting Zalta's view that "Abstract objects do not have concrete things as parts." (2019, 62)

[38] Cf., Trogdon and Cowling assuming both homeomereity theses for the sake of their defence of abstract atomism (discussed in Chapter 4) "We assume that any complex concrete entity only has further concreta as proper parts, and any complex abstract entity only has further abstracta as proper parts." (2019, 2033 fn 11)

intended to be applied completely transcategorially and cross-domain without restriction (cf., the discussion in Varzi 2006, discussed in Chapter 6).

Obviously such a restriction on mereology constitutes a rejection of Lewis's (1986 and 1991) views and systems, especially the mereological relations he advances as holding between concrete individuals and abstract classes with his proposed mixed fusions. Yet it also goes further than this if we maintain the conception of concrete objects as only being the parts of possible worlds, and for there not to be any concreta outside worlds, as Lewis (1986) looks to do. To see the problem, think back to Figure 5.3 where C1 and C2 are supposed to be distinct possible worlds. The above conception of concreta means that the mereological fusion, C1C2, is *not* a part of any possible worlds, which results in the conclusion that C1C2 is not itself a concretum.

This seems absurd if we consider that *all* of C1C2's proper parts are concrete – i.e., it seems absurd to accept that C1C2 is homogeneously concrete without being homeomerously concrete, especially when we have understood homeomereity in the case of abstracta and concreta to be absolute, and not relative to something like the 'grain' size of the parts involved. Although we already saw that homogeneity does not entail homeomereity in general, we also recognise that it is Lewis's claimed modal organisation coupled with a conception that entities outside worlds are abstract which results in this consequence here, as it is the case that sums of homeomerous concreta confined within single concrete worlds are both homogeneously and homeomerously concrete on his view.

All of this means that, even though Lewis's system might at first appear to provide a fair division into abstract and concrete realms, the candidate concrete realm within the system cannot genuinely be identified as itself a concretum without undermining a distinction of the concrete and the abstract as between that which is inside of worlds and that which is outside of worlds. In this case we are left with a supposedly concrete realm which is itself an abstractum, and also populated with an overwhelming majority of trans-world composite objects which are not themselves concrete – which clearly invalidates the claim that such an entity is indeed a concrete realm.

With Lewis being a major modern defender of GEM, especially understood as operating with absolutely generality, he is unlikely to accept any restriction on composition from these apparent problems due to mixed fusions and trans-world sums. Instead, I think that Lewis would be forced to reject both my Ex-Ex thesis and the conception of entities outside of worlds as only able to be abstracta. While I think Lewis may be able to accept a weaker version of the Ex-Ex thesis which uses Cowling-Exclusivity here, I do not think the latter option is a strategy which is available to him. In the next section I will explain why.

5.5 Out-of-world Abstracta and Concrete Inheritance

Let us first consider the latter issue of the abstract/concrete status of entities outside of worlds. Prima facie it is understandable to expect that Lewis generally maintains a concreta and abstracta division along an inside-of-world and outside-of-world dimension. We can surmise this first by his portrayal of possible worlds and their parts as that which is concrete, with the parts of possible worlds, including their improper parts, as the only possibilia (note that there are some caveats here to be addressed later though). This then contrasts with Lewis's assessment that every entity outside of possible worlds – whether or not they have parts from within them – are impossibilia. In this case, both the mereological sums composed of parts from different worlds (Lewis's trans-world fusions), and things without parts in any worlds (for Lewis, at least sets/classes) are thereby impossibilia.

Additionally mixed fusions can represent a further type of impossibilia composite here – as these can be fusions of individual parts from within worlds and class parts not from any worlds, such that no mixed fusion is a part of any possible world either, whether the individual parts are from worlds or not. All together, this possible/impossible split conveys an expectation that what is possible is in worlds and what is in worlds is concrete, whereas what is not in worlds is impossible and what is not in worlds is not concrete – i.e., those are abstract.

In addition to this, Lewis contrasts his plurality of concrete worlds view with what he calls the 'ersatz' view of possible worlds (with 'ersatz' basically meaning 'abstract' for Lewis in this context).

> The ersatzers say that instead of an incredible plurality of concrete worlds, we can have one world only, and countless abstract entities representing ways that this world might have been. [...] According to ersatzism [there] is one concrete world, and one only. *It includes all the concrete beings there are*. There are no other worlds, and no other-worldly possible individuals – not, at any rate, if those are understood as concrete (or partly concrete) entities. (1986, 136) (my emphasis)[39]

The emphasised notion that concrete worlds include all the concrete beings that there are is not one that Lewis disputes – the key difference in this context being that Lewis just explicitly proposes more worlds than one for concreta to be in, and not for there to be more concreta that are outside of worlds altogether.

[39] This 'partly concrete' qualification is to be addressed with the aforementioned caveats later.

To introduce some further distinguishing terms, within this framework we can continue to describe objects as either 'in' or 'out' of worlds. By leaving 'in' to mean 'pure in', in the sense that all of such thing's parts, proper and improper, are in a world, we get 3 types of composite (via GEM) which have their improper part outside of every world: (1) 'out but with all proper parts in' – such as trans-world sums comprising 2 mereological atoms; (2) 'out with some but not all proper parts in' – such as mixed fusions with in-world individual parts, as well as trans-world sums with 3 or more proper parts; and (3) 'out with no proper parts in' – such as how Lewis treats sets and classes. We might call these 'whole outs', 'half outs', and 'pure outs' respectively.

While Lewis is explicit that he takes classes as abstract, he does not explicitly state whether he also takes trans-world sums and mixed fusions to be abstracta, even though these are both out-world types of objects. (This is regrettable, and I will argue later that it is exactly at these points of fusion from within each world to outside it that Lewis's system faces serious problems.) I think it is reasonable to presume from the context of Lewis's discussion of the abstract–concrete distinction (1986, 1.7) that he does not expect there to be a third kind of entity beyond abstracta or concreta, so from the fact that trans-world sums and mixed fusions have their improper part outside of worlds, it is reasonable to assess that by Lewis's account they are indeed abstract objects. We can characterise this as the thesis that all 'out-of-world' entities are abstracta.

Out-of-world Abstracta Entities which are not parts of possible worlds are abstracta.

Yet, although Lewis's concrete/abstract divide appears to follow an in/out or possible/impossible arrangement, there are several aspects of his presentation, along with other views which Lewis promotes, that I believe compel him to generally accept the non-pure outs as concreta, rather than abstracta, leading to violations of the above thesis.

First, Lewis appears to accept a fusion principle that any fusion of concreta is itself concrete.

Concreta Fusion Any fusion of concreta is itself a concretum.[40]

We can see Lewis accepting this when querying the abstract/concrete status of worlds themselves. Here he says "Whole worlds ... cannot stand in spatiotemporal and causal relations to anything outside themselves, and it seems that nothing can stand in such relations to its own parts" (1986,

[40] Cf., Lewis's (1991) explicit fusion thesis with regards to individuals and implicit fusion thesis with regards to classes, as detailed earlier in Chapter 1.

84). According to Lewis's criteria for things being worldmates and proper parts of worlds,[41] these are conditions which imply that the improper part of possible worlds, the worlds themselves, are not themselves concrete. From this, Lewis asks "Should we conclude that worlds – including the one we are part of – are abstract wholes made of concrete parts? Perhaps, indeed, divisible exhaustively into concrete parts?" (1986, 84) He objects to this, and offers instead that "wholes can inherit concreteness from their parts." (1986, 84)

The Concreta Fusion Thesis clearly entails that whole out trans-world sums such as C1C2 are indeed concrete entities. What's more, Lewis's response also signals an acceptance of an even more general concreteness inheritance (as without it, his purportedly concrete worlds are not concrete after all).

Concrete Inheritance Anything with any concrete parts at all is itself concrete.[42]

Concrete Inheritance clearly entails that half out sums – like trans-world sums with 3 or more proper parts and mixed fusions with concrete individual parts – are indeed concrete entities,[43] which means that Lewis's view entails that there are concrete impossibilia. In addition to the principle of Concrete Inheritance, Lewis's commitment to the thesis of CAI (as discussed in Chapter 3) also looks to force him into accepting concrete impossibilia, following the terminological distinction over entities that are parts of worlds or not, on pain of inconsistency. We can see this especially recalling that CAI asserts that composites are collectively identical to their parts, such that any

[41] That, roughly, things are proper parts of worlds if they are 'worldmates' (1986, 69), where proper parts are worldmates iff they are ('analogically' or familiarly) spatiotemporally related (1986, 71 definition combined with the qualification on 75–76).

[42] Compare Berto and Jago wondering "how can a wholly nonexistent world have a part which exists? [...] a whole with an existent part is surely something that *in part* exists" (2019, 54-55). On this kind of query, Lewis insists that existence is a binary matter – that things either do or do not exist. For instance "existence cannot be a matter of degree. If you say there is something that exists to a diminished degree, once you've said 'there is' your game is up." (1991, 80-81) The expectation here is that concreteness follows a similar principle.

[43] Although Katz's (1998) system does not permit supra-world fusion, it is worth noting that he attempts to reject Exhaustivity by introducing a third kind of entity, yet which are ultimately composed only of both abstracta and concreta parts, and so are not purely alternate to the abstract–concrete distinction in their own manner. I think his attempt fails, however, as his system includes a form of Concrete Inheritance. This serves to defeat his rejection of Exhaustivity by entailing that his purported third kind of entity are, in fact, concrete. This expected entailment is not peculiar to me either. Kaufman (2002, 233) slightly mistakenly complains that Katz asserts a sole world and universe as a physical universe, thereby undermining the proposal of a third kind of category by doing so, presumably from also expecting such inheritance to follow. ('Mistakenly' only insofar as Katz does not actually treat his one universe qua whole or improper part as physical in any *explicit* sense at all. If he did it would indeed make his alleged third kind of entity even more undeniably concrete.)

spatiotemporal extension properties that any proper part has will also be had by the wholes which they are parts of.

Returning to Figure 5.3, to be collectively identical to both C1 and C2 along the dimension of their concreteness is clearly for their sum C1C2 to itself be concrete. Even if we relaxed the CAI thesis, I find Concrete Inheritance agreeable, as I think it is natural to expect that a whole has some of its properties which amount to being cumulative properties of its proper parts, such as mass or length.[44] For example, considering that concreta are generally understood to have *some* extension, whether spatial or temporal, or both,[45] C1C2 should weigh the combined mass of C1 and C2, as well as have their combined spatial volumes and (perhaps overlapping) temporal duration.

For a more familiar example, think of a red handled screwdriver. Even though the steel shaft is grey in colour, making the totality of the colour of the screwdriver a red-grey combo, we can still say that the whole thing has *some* primary colour to it (and you would not be wrong to ask someone to pass you the red screwdriver even though it is not entirely red). Similarly, if the handle of the screwdriver has some mass, then we can say that the screwdriver itself has some mass, without even considering the mass of the steel. The plausibility of Concrete Inheritance thereby rests on this kind of notion, that where a property like mass can be a marker of concreteness, that having a part which has mass is sufficient for something being concrete, because it too has mass – the mass of its parts. From this, then, just like in the screwdriver case where we say it has some primary colour to it, when a trans-world sum has some part which has some mass, this means that the whole has some mass too, and at least the mass of that part. And because to have some mass is thereby to simply have mass, the trans-world sum is concrete due to the mass that it has.

In the case of mixed fusions with concrete parts we get the same result – they share the mass/length/whatever concreta sufficient property of their concrete parts which makes their concrete part concrete, and are themselves thereby concrete too. The consequence for Lewis here is that by accepting Concrete Inheritance *within* concrete worlds it is therefore inconsistent and unjustified for him to deny it in the case of either trans-world or supra-world sums.

What can be derived from Lewis's various assertions in this context, then, is that he holds an implicit package of at least 5 theses which – within his framework of mereology, classes and possible worlds – are together incompatible. The positive side of the package includes the four of

[44] For a dissenting opinion on Concrete Inheritance like entailments see Gaskin (2008, 58 ff) who holds both that propositions with concrete constituents are still abstracta, and that the world too is "essentially abstract", even with concrete parts.

[45] Cf., Lewis (1986, 73) remarking on some spatiotemporal variety across worlds.

Exhaustivity, Out-of-world Abstracta, Concreta Homeomereity, and Concrete Inheritance (which itself entails Concreta Fusion anyway). The negative side includes the implied rejection of Abstracta Homeomereity which follows from the combination of Out-of-world Abstracta with Lewis's acceptance of mixed fusions and trans-world sums.

My contention is that it is this implied rejection of Abstracta Homeomereity via Out-of-world Abstracta which is at odds with Concrete Inheritance in this context. This is because this rejection of Abstracta Homeomereity permits abstracta with concrete parts, whereas Concrete Inheritance entails that any candidate abstracta with concrete parts is actually concrete. This results in the overall inconsistency of Lewis's implicit package of theses, because both of the theses of Concrete Inheritance and Out-of-world Abstracta are upheld, making his system incapable of determining either trans-world sums or mixed fusions with concrete parts as either abstract or concrete, and thereby violating Exhaustivity.

To see what his options are for avoiding this inconsistency, in the next section I examine whether Lewis has any viable options left for salvaging a consistent package of these theses under the conditions of plural concrete worlds, GEM, and pure out classes, and determine his best overall strategy for denying my Ex-Ex thesis.

5.6 Lewis's options for rejecting my Ex-Ex thesis

As I have argued, if my Ex-Ex thesis as the basis of the standard abstract–concrete distinction is to be accepted, GEM is incompatible with a varied ontology consisting of only abstract and concrete entities – even if there is only a single concrete world and some abstracta which transcend it. This means that a commitment to GEM in such a context requires a rejection of my Ex-Ex thesis, something which I think David Lewis, as both a proponent of GEM and an ontology populated by abstracta and concreta, is compelled to do.

A rejection of the Ex-Ex thesis can be complete or partial, however. This is because the Ex-Ex thesis consists in three subsidiary theses – the two homeomereity theses already met, and the Exhaustivity thesis. As such, Lewis looks to have seven options to consider in this context so as to be able to maintain a system of concrete worlds and transcendent abstracta which is compatible with GEM: reject the complete package of theses, i.e., reject all three of them, or reject some other combination of one or more of the theses.

In the previous section I showed that Lewis's assertions established his position as one which already implicitly rejects Abstracta Homeomereity, yet I also showed that this rejection is inconsistent with another positive thesis which he endorses, viz., Concrete Inheritance. Therefore,

in this section I seek to examine the issue of Lewis's obligation to reject my Ex-Ex thesis within the context of determining which of his theses Lewis is plausibly able to maintain.

It should be noted from the outset here that, as the foregoing section also reveals, deciding upon the best choice for Lewis in this case is complicated by the fact that it is difficult to determine precisely what the premises of his views entail in comparison to what he characterises as the consequences of these premises. To try and follow the various potential threads, I will switch between examining those of Lewis's options which stem from endorsing Out-of-world Abstracta, and those options which permit its violation. To be upfront, I do think Lewis is most obliged to reject Out-of-world Abstracta over the explicitly given Concrete Inheritance. Not just because he admits he needs the latter for his worlds to be concrete, but also by following the principles of property inheritance that he endorses, whether via CAI or even in the independent sense as discussed in Chapter 3, and by the end of my analysis this should be clear. It is this heavily obliged Concrete Inheritance that also results in Lewis's system endorsing concrete impossibilia, however, a result I will be demonstrating later in section 5.6.4. For now, though, we start to examine Lewis's options for rejecting my Ex-Ex thesis, beginning with rejecting Exhaustivity.

5.6.1 Rejecting Exhaustivity

If we start by first sticking with the original view of Lewis's concrete/abstract division as one more or less following the in–out world distinction, then merely rejecting Exhaustivity won't permit the existence of both trans-world fusions and mixed fusions. This is because these cannot be a third (or other) kind of entity that is not abstract nor concrete, a third kind being what a rejection of Exhaustivity would allow, and they cannot because of Out-of-world Abstracta. On this thesis, mixed fusions and trans-world sums cannot be concreta for being out of worlds but must be abstracta, yet following Abstracta Homeomereity they also cannot be abstracta due to their having concrete parts.

This all means that for a rejection of Exhaustivity to work in this case, Out-of-world Abstracta would also at least either need to be rejected outright or otherwise weakened so as to only exclude out-of-world things from being concreta, rather than be an assertion that out-of-world things are *only* abstracta. If Out-of-world Abstracta is only rejected by this weakening, however, Concrete Inheritance would need to be rejected too, as the mixed fusions and trans-world sums could not be concreta under the weakening (which still prohibits out-world concreta), nor following Concreta Homeomereity. If Out-of-world Abstracta is instead rejected, rather than weakened, though, Concrete Inheritance would still need to be rejected here too as keeping it would obviate the aim of rejecting Exhaustivity (to allow for kinds beyond concreta and abstracta). This is because both mixed fusions with concrete parts and trans-world sums would still be concreta after all.

Additionally, if Concrete Inheritance is not rejected here we would also end up with a violation of Concreta Homeomereity (which is being kept in this example) due to mixed fusions being concrete via inheritance, yet still with abstracta class parts.

In the case of rejecting only Exhaustivity, then, Lewis will be obliged to justify and defend an at least three category approach, presumably maintaining pure outs as abstracta, and worlds and their parts as concreta, yet while also rejecting his inheritance principles and modifying his implied out-of-world views to allow for the third or more kinds. However, I don't think that denying Exhaustivity is a cogent strategy for Lewis here. This is for several reasons.

One, because even though Lewis does express reservations regarding the overall aptness of the abstract–concrete distinction (1986, 81), he still manages to maintain one in connection with possible worlds, i.e., the distinction of what appears entirely in possible worlds being concrete and what appears entirely outside of possible worlds being abstract. That troubles with this presumption arise when supra-world fusions are scrutinised says more about there being a problem with supra-world fusions than with the initial presumption that the abstract–concrete distinction tracks the mereological in-world/out-world one.

Two, rejecting the property inheritance of concreteness undermines Lewis's claim that possible worlds (the improper parts) are concrete after all, and this is a problem worth avoiding in the advancement of concrete modal realism. In addition to this, that Lewis advances CAI (which brings property inheritance) as a reason to accept GEM (as discussed in Chapter 3), and not the other way around, suggests that he should be rejecting the mereology with priority instead. This is especially the case when, as I have argued in this context, CAI actually results in problems for the mereology rather than properly supports it.

Lastly, as Lewis actually gives evidence of a preference for both Out-of-world Abstracta and Concrete Inheritance, whereas he gives no evidence of a preference for rejecting Exhaustivity, pursuing this option in isolation from considering the choice between these other two principles seems unwarranted. This is even more so when rejecting Exhaustivity to properly allow other kinds of entities besides abstracta and concreta requires rejecting both Out-of-world Abstracta and Concrete Inheritance lest this purported rejection of Exhaustivity alone become a de facto rejecting of a homeomereity thesis as well.

Taking the rejection of Exhaustivity off the menu for Lewis then leaves him with 3 options to deny the Ex-Ex thesis: to either reject Abstracta Homeomereity, Concreta Homeomereity, or both.

5.6.2 Rejecting Abstracta Homeomereity

In the context of Lewis's initial concrete/abstract division as an apparent in–out world distinction via Out-of-world Abstracta the rejection of Abstracta Homeomereity alone is insufficient for maintaining compatibility between GEM and Lewis's system. While this allows mixed fusions to be abstracta with concrete parts, and the same for trans-world sums, unless Lewis also rejects Concrete Inheritance (and Concrete Fusion along with it) the principles are in tension over the status of supra-world fusions.[46]

This means a rejection of Abstracta Homeomereity here amounts to deciding between rejecting Out-of-world Abstracta or rejecting Concrete Inheritance. The trouble with the former option is that Abstracta Homeomereity is no longer rejected in practice, as any alleged abstractum with concrete parts is actually instead concrete (and any actual abstractum could only have abstract parts), and this becomes rather a rejection of Concreta Homeomereity instead because mixed fusions would be concrete with abstract parts.

Therefore, for a rejection of Abstracta Homeomereity to be consistent in this context, Lewis needs to also reject Concrete Inheritance while maintaining Out-of-world Abstracta. As I stated before, this is not really amenable to Lewis's defence of GEM, nor his claim that possible worlds are actually concrete wholes. Beyond this, however, is the further wrinkle that Lewis makes allowances for there being either immanent universals or tropes qua in-world abstracta, even though he neither specifically endorses nor rejects them herein.

> [If] ordinary particulars contain universals or tropes as (non-spatiotemporal) parts, then worlds composed of ordinary particulars will in turn have universals or tropes as parts; in which case not all the parts of worlds are paradigmatically concrete. Indeed we might contemplate a theory of numbers – one which says, for instance, that the number three is the structural universal of tripartiteness – according to which some parts of worlds would turn out to be paradigmatically abstract. (1986, 82)

Recalling that Out-of-world Abstracta does not entail that all abstracta are out of worlds, but only that all out-world entities are abstract, the fact that Lewis does not strictly exclude abstracta from being in-world too thereby results in the potential for violation of Concrete Homeomereity as well here, as any concrete thing with abstract immanent universals or tropes for parts would entail.

[46] Note well that there is no room here for out-world entities to be both abstract and concrete as Exhaustivity alone allows. After all, Out-of-world Abstracta says out-world entities are abstracta, not both, and Concrete Inheritance says that things with any concrete proper parts are concrete, not both.

In this case, because a rejection of Abstracta Homeomereity is off the table for Lewis here unless he makes even further independent and undefended rejections again, including the additional rejection of either Concreta Homeomereity or both immanent universals and tropes, I conclude that this is also not a plausible option for Lewis to pursue.

5.6.3 Rejecting Concreta Homeomereity

In the context of Lewis's initial concrete/abstract division as an apparent in–out world distinction via Out-of-world Abstracta the rejection of Concreta Homeomereity alone is also insufficient for maintaining compatibility between GEM and Lewis's system. This is because, in this instance, neither trans-world sums nor mixed fusions can be concreta (for being out of worlds) or abstracta (for having concreta parts under Abstracta Homeomereity), and yet there is no other kind for them to be following Exhaustivity. This leaves such hybrid entities incompatible with the remaining package of theses.

Therefore, for a rejection of Concreta Homeomereity to get towards permitting supra-world fusions, Lewis needs to go beyond the initial division and reject Out-of-world Abstracta outright. (Merely weakening it to be Out-of-world Not Concreta is inadequate because without a denial of Exhaustivity here any out-world entities can still only be abstracta, and Concrete Inheritance is still contradicted.) Under Exhaustivity this allows that out-world entities may be either concreta, abstracta, or both. Following the implications of Concrete Inheritance, we hereby end up with a reason to reject Concreta Homeomereity in the case of mixed fusions with concrete parts, as these would be concrete entities with abstract parts (the class parts) – which invites an assessment of abstracta as precisely those things with no concreta for parts.

This is good, because here we have a principle which looks to support a homeomereity thesis rather than for such a thesis to be merely a stipulation. That is, Concrete Inheritance looks to entail Abstracta Homeomereity precisely because any alleged abstractum could not have concrete parts and yet still be abstract, leaving things with only abstract parts as abstracta, a homeomerous result. In this case too, the inclusion of either abstract immanent universals or tropes make no trouble as concreta are permitted to have abstracta parts once Concreta Homeomereity is rejected, it being only the converse that is prohibited with Abstracta Homeomereity being maintained. And we are also not rejecting any property inheritance principles which Lewis shows support for in multiple places, making this the most attractive option for Lewis to pursue thus far.

There are two further complications worth discussing with regards to Lewis's allowances for certain in-world abstracta here. The first is that he initially describes these as non-spatiotemporal (as above in 1986, 82), which is difficult to reconcile with his assertion that the parts of concrete worlds

are spatiotemporal worldmates. Yet it's likely that Lewis initially describes them as non-spatiotemporal because they are either paradigmatically abstract, or commonly taken to be such. In this case, Lewis remains clearly open to there being concreta with abstract proper parts, as he says "if universals or tropes are non-spatiotemporal parts of ordinary particulars that in turn are parts of worlds, then here we have abstractions that are parts of worlds. So, by and large, ... it seems that indeed I should say that worlds as I take them to be are concrete; and so are many of their parts, but perhaps not all." (1986, 86) Because Lewis accepts that particulars can still be concrete even with abstract parts, I think we are safe to insist that this means that any mereological combination of abstracta and concreta should be understood to form a concrete composite from his views too.

The second complication to be discussed actually upends the first one, however, and also serves to reduce Lewis's account from being conclusively non-concrete homeomerous. This is because Lewis goes on to argue that spatiotemporal location is heritable, either from part-to-whole or from whole-to-part, so long as one of them is the primary bearer of a location. In the latter case, after all, "if a universal is wholly present in each of many located particulars, as by definition it is, that means that it is where its instances are. It is multiply located, not unlocated." (1986, 83) We met the former case of heritability previously when Lewis insisted that possible worlds inherit concreteness from their concrete proper parts – with this being an instance of these worlds at least inheriting the collective location from their proper parts. In the context of these claims, Lewis attests that "worlds and their parts – including the universals, if such there be! – are concrete." (1986, 84)[47]

Although tropes and other candidate abstracta weren't specified here, the generality of Lewis's attitude, that parts are located where their wholes are, and (at least collectively) vice versa, achieves the same result. This brings with it some problematic consequences, however. Consider that by Lewis's account there is a mereological Universe, 'Reality' which has everything that exists as its parts. By Concrete Inheritance Reality is concrete, and by the locational aspect of that inheritance it is located at the collective locations of all of its located parts. In the case that there are in-world and out-world entities, and even if the pure outs are meant to be abstract, these are all

[47] Note that the difference in the complications here is not from Lewis being inconsistent – rather he is surveying the consequences of a variety of 'Ways' the abstract–concrete distinction has been made, such as by distinguishing paradigm cases or by making reductions to them, or by denying certain features for abstracta, or by treating abstracta as special forms of generalisations from specific instances, i.e., as abstractions. What this means is that rather than revealing Lewis's views here as expressly problematic, then, we learn instead that he has no specific objection to certain conditions being upheld, like non-Concreta Homeomereity.

going to be located where Reality is. Indeed, everything is going to be located where Reality is, whether via a sub-location of Reality's total location, or sharing it.

Yet if this is the correct analysis of this situation, whereby everything would have spatiotemporal location after all, insofar as anything at all has some and mereology is classical, then spatiotemporality can no longer serve as a means to distinguish between abstracta and concreta in this context. For a small consequence: Lewis then loses the option to claim that immanent universals etc are concreta after all. What we might say instead is that some apparently non-spatiotemporal things are only derivatively located (but not thereby spatiotemporal!) in a weak subsidiary sense by being parts of things which are the primary bearers of a location.[48] Additionally, we might say that composite wholes either properly derive their locations from primary bearers of them which are their proper parts, in a strong supersidiary sense, or are only derivatively located in a weak subsidiary sense at the location of the things that they are a part of too (provided that there is even some location to derive).

Under this interpretation, possible worlds, trans-world sums and mixed fusions will all strongly inherit collective locations from their located proper parts, and thereby be concreta, whereas candidate non-spatiotemporal proper parts of located concreta will only weakly be associated with the locations of any concreta that they are parts of, and thereby be abstracta. A rejection of Concreta Homeomereity can then be maintained on two fronts here – via pure out abstracta class parts of concrete mixed fusions, and by in-world abstracta homeomerous parts of concrete worlds.

To begin to wrap up this sub-section, there are two more outstanding issues to be mentioned, one resolved fairly simply, and one which will require further examination of Lewis's claims. The first issue is to resolve the denial of Lewis's more explicit view of Out-of-world Abstracta. Here I think we are justified in insisting that Lewis needs to choose between that or Concrete Inheritance, and that because he gives greater defence of the latter in multiple contexts, and it has greater import for his view of concrete possible worlds, that he is better off maintaining the inheritance principle rather than the out-of-world one.

Besides this point, Out-of-world Abstracta appears to be more of a generalised assumption derived from following the common notion that mathematical objects like classes and sets are abstracta and outside worlds to infer that all things outside of worlds are abstracta too. This is especially so when the mereological misfits of trans-world sums and mixed fusions are not the usual

[48] Here I ignore any complications from the debate over substantivalist vs relationalist views of location that there may be, hopefully without issue.

suspects on first consideration and only come to mind as out-world beings under the conditions of absolutely unrestricted composition. So I think it is legitimate, in this sense at least, to deny Lewis as having any reasonable warrant to insist that out-world entities can only be abstracta, when he both has principles in operation against it and lack of justification for it.

The second problem to be mentioned here is that, with the in/out-world split being a possible/impossible object split, Lewis's best view ends up committing him to concrete impossibilia – something which he apparently rejects. Before we think that maybe this is not the best option for Lewis after all we need to examine what this rejection of concrete impossibilia properly amounts to. This is because, as I will show, it is not strictly a view against just concreta being outside of worlds, but rather, against there being concreta which instantiate logical contradictions.

One further caveat – although I do say that this is the best option for Lewis, this claim is based upon the amount of defence he gives to certain views and principles in comparison to the others, and the sort of overall project he has in mind being maintained rather than discarded, even with all of these acknowledged deficits. So even though I will show how Lewis can end up accepting some concrete impossibilia after all, I still think that the overall collection of options available to Lewis along with this one are all too problematic to maintain in general, and that in the end rejecting GEM by restricting composition to avoid supra-world fusions is still the most justified option amongst them.

5.6.4 Isn't Lewis against concrete impossibilia?

The issue to be addressed here is that, as Lewis specifically argues against concrete impossible *worlds* (1986, 7 fn 3; 1983, 21), whether he is also against concrete impossibilia in general, as some have taken him to be (cf., sentiments like this in Vacek 2013a, 298 and 2016, 84; Kiourti 2010, 34 and 167-8; and Yagisawa 1988, 182-183 via Vacek 2020, 27). If so, this would put some obstacles in the way of my claim that his views lead him to accepting some concrete impossibilia. However the evidence for his, at least obliged rather than clearly endorsed, acceptance of these is in my favour.

To start, we can see that even though Lewis does not explicitly set it up in terms of contrastive cases, he clearly distinguishes between two kinds of impossibilia – impossibilia which are contradictions (as discussed in 1986, 7 fn 3), and impossibilia which are (merely) out-world parts (1986, 211 ff). While Lewis denies the existence of impossibilia which are contradictions in any form, whether these are any of the four varieties found in being out-world/in-world and impossible/concrete, he does not deny the existence of impossibilia qua out-world entities and non-contradictions. In fact he accepts them both as of a piece with his commitment to unrestricted

composition in any supra-world sums (e.g., 1991, 7), and also the kind of objects that mathematical classes are (e.g., 1986, 94 fn 60).

Because Lewis does not deny that impossibilia exist, and instead only denies the existence of contradictory impossibilia in or out of worlds, his arguments should be understood to be strictly only against contradictory existents (whether in worlds or not) rather than against any "free-floating" (cf., 1986, 230) or otherwise less-than-maximal out-world impossibilia, concrete or abstract. However, as much of the succeeding discussion on this topic and concerning the strengths and weaknesses of Lewis's stance on concrete impossibilia has been with regards to (concrete) impossible worlds alone, it is easy to see why this distinction is not frequently associated with Lewis's views on impossibilia in general.[49]

Following this, as Lewis only provides arguments against certain kinds of concrete impossible worlds, and does not make any case against concrete impossibilia in general, I claim that even within Lewis's own system we do not need something like Vacek's indispensability argument for concrete impossibilia – which concludes that "We ought to have ontological commitments to concrete impossibilia" (2013b, 148) – to assess that Lewis, or anyone else who accepts supra-world fusion and Concrete Inheritance, is obliged to commit to these. Even if only of a specific kind, Lewis already directly serves up concrete impossibilia all by himself anyway.

My assessment that Lewis is at a minimum implicitly committed to concrete impossibilia, at least in the out-world sense, goes beyond Concrete Inheritance entailments to be furthered by how he discusses a strategy to conceive of trans-world individuals as 'possible' in virtue of the status of their proper parts.

> If we thought they [i.e., trans-world individuals] should not be ignored, perhaps because we thought that we ourselves were trans-world individuals, it would be appropriate and easy to give 'possible individual' a more inclusive sense. We could say that an individual exists at a world iff, quantifying only over parts of that world, *some part* of that individual exists – that way, the trans-world individuals would count as possible. [Continuing in a footnote:]

[49] We can see the influence of this focus in inexact statements like Vacek's that "According to David Lewis impossible individuals do not exist as it is contradictory to postulate them" (2016, 84), especially when they are contrasted with Lewis's own characterisation that "Parts of worlds are *possible* individuals; trans-world individuals are therefore *impossible* individuals." (1986, 211) Cf., Lewis in an earlier work saying that "the possible individuals are not all the individuals. I wish to impose no restrictions on mereological summation of individuals, hence I must grant that there are individuals consisting of parts from several worlds. But such a cross-world sum is not a *possible* individual." (1983, 39)

> Indeed, I think that in this sense, there would be no impossible individuals. No individual is wholly distinct from all the worlds; so every individual is divisible into parts which are parts of worlds. (1986, 211)

Here we see Lewis adverting to a different sense of 'being in' a world that goes beyond something being a part of a world to include something merely having parts in worlds as sufficient for being in a world (as discussed among other options for interpreting 'being in' in (1986, 96) and (1983, 39-40)). In the case of trans-world individuals, all of their parts resolve to some part(s) in worlds, whereas for mixed fusions, some of their parts only resolve to worlds in the case that the individual-type part of the mixed fusion is a part of a world.

Besides the generic possibilia as concreta assimilation, what makes this interpretation friendly to the proposal of concrete, mereologically out-world impossibilia, on my view, is that Lewis does not immediately discredit the notion that "we ourselves [are] trans-world individuals".[50] This is even while throughout the text he clearly inclines towards the understanding, and is also very amenable to the notion, that we, like donkeys, are concrete beings, and usually deemed to also be parts of concrete worlds.[51] The implication here, of course, is that if concrete out-world impossibilia were absolutely objectionable to Lewis, he would be unlikely to entertain them, and especially not us ourselves as some of them. Hence I think that to deny that we would also be concreta if indeed we thought that we ourselves were trans-world individuals is not really an option for Lewis. I think this all suffices to subvert the ordinary understanding of Lewis's view as limiting out-world parts to only be abstracta, even if he does not intend such a consequence.

Overall, on the question of whether Lewis is against concrete impossibilia, I would say that while he plainly disfavours them by some of his statements, it is by others that he is obliged to

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[50] An idea which he also expresses earlier in the piece as well: "individuals may be partly actual – big individuals, composed of parts from more worlds than one, and so partly in each of several worlds. If there are any such trans-world individuals that are partly in this world, hence partly actual, should we call them 'actual' *simpliciter*? – That depends. We needn't, if we think of them just as oddities that we can mostly ignore. I think they are exactly that. (See section 4.3.) But if we were reluctant to ignore them in our quantifying, perhaps because we thought that we ourselves were among them, then we might appropriately call them 'actual'." (1986, 96)

[51] For instance, Lewis (1986) calls donkeys paradigmatically concrete (83; 168 ff) and says that we are their worldmates (110). He also commonly uses phrases like "the world I am part of" (94), as in "I, and the world I am part of" (208); or more inclusively as in "worlds – including the one we are part of" (84) and "my indexical theory of actuality, on which the worlds are all of a kind and to call one world actual is only to say that it is the one we are part of." (123) Cf., "the concrete totality we are part of" (140) and "we do know for certain that the world we are part of is the actual world – just as certainly as we know that the world we are part of is the very world we are part of." (93) (Interestingly, this last quote appears to claim knowledge at odds with us being trans-world individuals after all).

accept them, in case his account be less supported from the theses he promotes. However, with the implications from Lewis's principles and other assertions being that his system includes concrete impossibilia regardless of how he seems to characterise what follows from his stated views, what remains to be seen is whether this is a problem or not for his system in itself, beyond its characterisation. In the next section I argue that a key problematic situation is out-world entities having in-world properties, as well as their potential for multiple in-world causal involvement, and even the latter regardless of their abstract/concrete status.

5.6.5 Problems from out-world entities having in-world concrete parts

Consider that a consequence of Concrete Inheritance appears to be that out-world parts can have paradigmatically in-world properties, such as mass and length. And not in some simulated or merely implicitly representational sense either – like what Lewis objects to in the case of a 'pictorial ersatzist' claim that, roughly, abstract donkeys are just like concrete donkeys, only abstract (1986, 3.3 for many details). No, for it is by Concrete Inheritance that the concrete properties had by out-world entities are had by them in the exact manner that the in-world composites do. (This also seems to be the case if universals are immanent parts of concrete worlds, because the out-world composites will have those universals as their parts too, even regardless of whether the out-world entities are abstracta or not.)

This consequence thereby suggests a simple reason to deny such supra-world fusions – that they undermine the natural expectation that out-world entities do not have in-world properties. In doing so, they also seem to force an expansion of ordinary conceptions surrounding the means for demarcating worlds and properties which serves no other purpose than permitting these unfamiliar out-world beings to be included in the ontology. Although we are no longer calling these entities abstracta due to Concrete Inheritance, Lewis makes a parallel point with regards to demarcation for the abstract–concrete distinction, when discussing the prospects of an ersatz donkey being just like a concrete donkey, yet abstract. Here he says "The sharing of properties across the divide between concrete and abstract must have its limits." (1986, 168) While the divide Lewis mentions here is of course not the in/out-world under the current conditions, the point made is very similar even still. There is supposed to be something fundamentally different between in-world and out-world beings, and abstracta and concreta (cf., Lewis 1986, 81), otherwise why entertain the distinctions?

This objection works against both trans-world sums and mixed fusions when they are recognised as concrete via Concrete Inheritance, and it works against the latter too even in the case of only a single concrete world making mereological connections with things out-world to it. The trouble remaining for unrestricted composition here is that even if one thought a retreat back to Out-of-world Abstracta was now more justified, we are still left with the unappealing consequence that

putatively abstract composites like mixed fusions and trans-world sums, with proper parts that have mass or some form of extension, do not themselves have those properties, because of some purported metaphysical barrier.

While Lewis at least provides workable criteria for the concreteness of our own world (i.e., regardless of other concrete possible worlds), we have already seen that his account still has the limitation that world wholes themselves are not viably concrete unless there is Concrete Inheritance. This serves to make a restriction on Concrete Inheritance at the boundary between in-world and out-world terribly ad-hoc. Clearly, if we find in-world Concrete Inheritance plausible, denying it at this boundary needs greater motivation than for the sake of avoiding uncomfortable consequences. Besides – if the problems of out-world entities having in-world properties (or abstracta having concreta properties) is a reason to invoke Out-of-world Abstracta, why not just restrict supra-world fusion instead?

Beyond this, the retreat to Out-of-world Abstracta faces a disagreeable causal situation anyway. Consider that in the context of mixed fusions there are proper class many singletons which each independently fuse with every single concrete object. Following the natural intuition that to causally interact with a part is sufficient to causally interact with a whole which it is a part of,[52] we reach the undesirable consequence that proper class many abstracta (at least!) are causally interacted with, severely violating the understanding that abstract entities do not enter into causal relations (or at least the same causal interactions as concreta, more on this sort of issue below).

For some in-worldly examples of part relevant cause and effect think of a cat knocking a glass off a table with its tail. Here we can recognise a distinction between the cat as a whole with a tail for a part, and accept its tail as where the force for the knocking is primarily exerted from, yet still also accept that it was the cat knocking the glass. Consider too that eating an apple which is part of a trans-world individual is also consuming some part of a trans-world sum. Or imagine if I get COVID-19 and one of my lungs collapses. That would be an event which happens to both my lung and to me. And if the disease manages to kill me and I'm also part of many mixed fusions, then what happens to these mixed fusions is that one of their parts passes away.

I believe that this notion of causal interaction is importantly different, however, to those which can face impure sets which have in-(single)-world concreta for members. Due to space limitations I cannot explain the situation thoroughly, but the difference mostly follows from Lewis

[52] Thanks to Claudio Mazzola for suggesting this line of inquiry.

upholding that the members of classes are not parts of classes.[53] In this situation, although Lewis argues that impure sets can have both *some* sort of causal involvement and spatiotemporality,[54] a case can be made that these instances are more derivative and indirect for the impure sets than they are for any composite wholes. This allows that the impure set cases can be resisted in a way that the supra-world sum cases cannot.

5.6.6 Out of options

It appears that out of all of Lewis's options for rejecting my Ex-Ex thesis to save unrestricted composition in the context of abstracta/concreta fusion both across and transcending concrete possible worlds, his best option, ostensibly, is to reject Concreta Homeomereity. This is especially considering the inter-working of the other commitments he appears to uphold, most notably Concrete Inheritance. Yet after analysis, I think that this option leads to problems worth restricting composition for too. In addition to this, because many supra-world fusions will be located at multiple worlds – with the mereological Universe being in every world – all of these are also causally interacted with regardless of it being some of their parts only that are located in worlds. This is also regardless of whether they are, as single entities, abstract or concrete (or even otherwise, like some third kind).

So the problem I see for Lewis's account, and any with a similar understanding, is that regardless of which of the many out-world entities considered here, all of them lead to problems once involved causally in-world, to the point that it is worth prohibiting such an involvement by rejecting the formation of out-world entities with mereological links to in-world entities. Without much incentive for radicalising the notion of causal involvement besides 'saving the phenomena' of supra-world fusions I think we are left to better deny such fusions and the principles which support them to avoid such a requirement.

In addition to this, from what I have argued, I believe the proposal for rejecting Concreta Homeomereity is spoiled in the case of out-world concreta. However, my arguments do not seem to

[53] As before, see Johnston (2006), Fine (2010), Caplan et al. (2010) and McElhoes (2017) for a variety of non-GEM views which *do* treat members as parts of sets. It would be worth determining whether any of these proposals face the troubles I am describing for supra-world fusions if they were to include such entities in their ontologies, however I have not evaluated them here due to limitations of space. Note that Katz (1998) treats members as parts of sets following a non-GEM view too, yet explicitly rejects supra-world fusion, as mentioned earlier.

[54] See Lewis (1986, 83-4 and 94-95); cf., Maddy (1990, 59). However, Lewis (1991, 33) does walk the location claim back, to say that we haven't a clue whether classes are in space and time or whether they are not, even though he is himself explicit in keeping them outside of space and time in the sense of not being parts of any possible worlds (Lewis 1986, 94 and 1991, 142; cf., Nolan 2002, 155).

have any bearing on there being in-world abstracta such as what immanent universals could be. So although this leaves a rejection of Concreta Homeomereity still available in the case of in-world concreta having abstracta parts, this is no solution for Lewis and other supra-world fusers. That is, I think Concreta Homeomereity can be rejected only insofar as supra-world fusion is rejected too. Because we were looking at rejections of my Ex-Ex thesis for the sake of maintaining such unrestricted composition, this is a rejection which does not actually achieve such an end.[55]

The final strategy for Lewis, for the sake of completeness, is to consider the rejecting of both homeomereity theses. The implausibility of such a strategy should by now be clear, however, in that out-world causal involvement will still occur in this instance regardless of whether out-world entities are abstract or concrete. Similarly, the implausibility of Lewis rejecting Concrete Inheritance has also already been addressed, which suggests that this dual rejection is not a strategy with any great purchase. Finally, even if it doesn't matter for the system whether out-world entities are specifically abstract or concrete, what does matter is how they are connected to worlds and what this kind of connection usually is thought to entail. As we have seen, it is the mereological connection which invites the problems, so prohibiting that looks to be the best option to avoid them. With the rejection of both homeomereity theses being the last of his options I think that Lewis is doomed to having an obligation to restrict composition after all.

5.7 Conclusion

In this chapter I examined the prospects of there being cabs, i.e., mereological sums of abstract and concrete entities. One of the questions was whether any such sums could be acceptable, or whether composition ought to be restricted to avoid them. In discussing the conception of possible worlds and various cabs from Lewis (1986 and 1991), I concluded that supra-world fusions like mixed-fusions with concrete parts and trans-(concrete)world sums are objectionable for a variety of reasons, and ultimately because of these composite wholes being out-world entities with unjustifiable in-world involvement. I claimed that there is no good reason to maintain supra-world

[55] As an aside, I believe that if one wanted to maintain a version of an Ex-Ex thesis in the case of in-world abstracta, one could swap my Exclusivity out for Cowling-Exclusivity. In that case, concreta can have abstracta parts (and vice versa) because the thesis only requires that an entity on its own belong to only one of those two categories (if at all). That is, Cowling-Exclusivity allows singular entities to be either abstracta or concreta regardless of the kinds of their proper parts. While this establishes a variant Ex-Ex thesis which is significantly different from my own in not being constrained by homeomereity, I don't think it makes out-world entities any less problematic in terms of the ways I have described. I thereby think it is best left as an in-world variant option for those who want to make an abstract–concrete distinction without there being supra-world fusions, and which seeks to at least permit in-(concrete)-world abstracta.

fusions involving in-world concreta, whether fused with concreta or abstracta in other worlds or with any of either merely outside altogether.

This is therefore an argument against unrestricted composition, and against the classical mereology which holds it, drawing a line for composition at the boundary of worlds. I expect much of what I said can generalise to be against other forms of composition in different mereological systems, to be prohibitive of supra-world fusion generally. However, because I did not determine a serious problem with abstracta being parts of concreta after all, in this chapter I have made no claim against restricting abstracta and concreta fusion *absolutely*. Yet this does not mean that I endorse abstracta being parts of concrete objects and worlds. It's only that I think that if there are to be any cabs at all under a concept of concreteness involving worlds, they will be in-world cabs alone. The further consequence here is that for mixed fusions to exist, sets would have to be (presumably abstract) parts of concrete worlds (which Lewis does not accept). Combined with my other arguments against mixed fusions, however, I think this is a form of mereological hybridism worth resisting, and we would be better off leaving abstract sets outside, and denying that they form composites with concrete, in-world individuals at all.

In the next chapter we look towards a more generalised discussion of mereological hybridism in the case of the Universe being a mereological one and thereby combining all things into one, irregardless of the categories which any entity may belong to, and apply some of the lessons learned from this chapter.

Chapter 6

Invitation to an Ontological Beheading: Transcategorial sums and the mereological Universe

That which does not have a name does not exist. **Unfortunately everything had a name.**

Nabokov (1959)

6.0 Introduction

The name that we might be inclined to give to everything that exists, as some sort of totality, is the Universe. Yet exactly what this name is supposed to refer to is unclear, because the sort of totality which it names on one understanding is a different sort of totality than what it names on another. For instance, according to the view of physicalism "the universe and everything in it conforms to [the] condition of being physical" (Stoljar 2015). In contrast, and as discussed in the last chapter, according to the view of modern platonism there are also abstract entities – with numbers, propositions and universals being common candidates for such beings – which are said to be entirely non-physical because they are not taken to exist in space or time (Balaguer 2016).

Besides the potential for disagreement over what we might call the 'contents' of the Universe, there is also room for disagreement over what sort of entity the Universe itself actually is, in terms of the kind of totality it exemplifies. Is it a single entity, or is it a genuine plurality of entities? If it is a single entity, is it some sort of maximal whole with everything else as one of its parts, or is it a type of unique collection with all other things as its members? And if it is a multiplicity of entities, are these singular elements each such that they satisfy a certain substantive condition? Or do they not necessarily have anything in common, and are merely many single entities which the plurality consists in?

The debate over whether all that exists is properly characterised as either strictly singular or strictly plural can be understood to be a debate over whether the Universe as this totality of all things is one that is monistic or pluralistic – over whether the Universe is either properly one or properly many. One prominent view in favour of the referent of the term 'Universe' being a strictly singular entity of course comes via GEM, wherein both the uniqueness and unrestrictedness of composition within it result in a single, maximal and unique totality whole.

Yet because GEM makes no claims as to what the specific contents of its Universe are, knowing that the system guarantees one is very different from knowing what that Universe itself is like. This is because what it is that is said to be mereologically unified in such a way depends on what it is that actually exists. If it is the case that there are only physical things, then the Universe as a singular whole must itself be a physical thing. If it is the case that there are only both physical and abstract things, for example, then the Universe as a singular whole must itself be one of these kinds

of entities – similarly to how Exclusivity operated in the previous chapter by limiting what categories every separate thing belongs to.

Following GEM's non-commitment to the nature of the contents of any composite, we know that the principle of unrestricted composition is able to live up to its name by entailing that there are composites with parts from any kind or category of entity that there may be. And at this point in the thesis we have seen transcategorial sums explicitly exemplified in several ways, including fusions of the membered with the non-membered; the abstract with the concrete; the possible with the impossible, and even combinations of the three pairings via some mixed fusions as in the last chapter.

Yet this small selection of occasions of mereological hybridism is not all that is available for consideration. For instance we may like to consider the prospects of cross-categorial sums of the material and the immaterial; the physical and the non-physical; spatial/non-spatial; temporal/non-temporal; necessary/contingent; extended/unextended; located/unlocated, and so on. Admittedly, some of those mentioned – depending on one's view of things – may be extensionally equivalent categorical divides, whereas others may only be acceptable provided that these divisions are deemed to be suitably *categorial* splits. (It may also be the case that mixed fusions can fit into some of these divisions and exemplify a large variety of prospective transcategorial sums, but this too will depend on the other characteristics of each of the mixed fusions proper parts, beyond just these parts having members, and being members, or not). The point is that, as it stands, mereological hybridism is as widespread as what both the scheme of categories and the style of composition allows.

By now in this thesis it is clear that I think that there are certain types of transcategorial sums which, if actually formed, would either contradict the very conditions which are supposed to accommodate them or otherwise lead to serious problems which make their host theories less plausible. Yet, while my main line of argument throughout this thesis has been to restrict composition to avoid mixed fusions and other troublesome mereological hybrids that have similar category content, it is not strictly the case that I have been arguing for a Universe as a plurality. After all I haven't strictly been endorsing nor denying the existence of abstracta, say. Rather, I have been reflecting on what I see as the consequences of certain entities existing, given the principles which have been said to apply to them. Due to the fairly conditional nature of my discussion (i.e., "if x exists and y applies, what do we end up with? Is it acceptable?") it may well be that I could support a mereological Universe so long as there are only concreta and sums of them.

With this in mind, the aim in this chapter is of a more general nature. So, while in previous chapters I covered some of the ways in which I think the formation of mixed fusions lead to all sorts of different issues, it is in this chapter that I examine more of the conceptual prospects surrounding mereological hybridism generally. In doing so, I still take aim at an entity which may or may not be understood to be a mixed fusion (and the biggest and probably most significant mixed fusion of them all, i.e., Lewis's 'Reality'), but only depending on auxiliary commitments. And even if the mereological Universe at the heart of this discussion is not understood to be a mixed fusion, I show that there is still room for concern over mereological hybridism in other ways. Such as if, e.g., numbers are said to exist outside of space and time and are not sets, or there are said to be impossibilia which are not sets either, or there are claims to non-extended minds alongside extended matter, and so on, then we still need to properly determine just how a mereological Universe comprising such disparate kinds of entities is able to itself have unity.

In doing so, I also contribute to and enhance the debate between Simons (2003) and Varzi (2006) over how transcategorial sums could be seen to affect the plausibility of the notion of a mereological Universe, and argue that, given the great potential for problems as I have demonstrated in my thesis, that a mereological Universe qua transcategorial sum is an entity worth, at the least, being very cautious about, and in many instances, worth rejecting. As I will argue, I think this is ultimately because we are not entitled to simply assume the compatibility of mereological hybrids. Instead, we actually have to work to confirm that the compatibility is genuine.

To start we look at the concerns Simons (2003) presents against the mereological Universe being a transcategorial sum.

6.1 Simons against unrestricted composition

Simons (2003) thinks that the main reason for doubting "the unbridled existence of mereological sums is that individuals [i.e., single things] come from different categories: things, properties and relations (if one takes these to be individuals), events, abstract objects, and perhaps more. If even two of these categories are occupied, and the principal of mereological summing applies everywhere ... then we have to contend with *transcategorial sums*." (2003, 237)

Simons abjures polycategorial ontologies which also incorporate mereological Universes. This is by following the expectation that with there being many categories, that any absolute totality which has parts from each of these will in some way make problems for the purported system of categories itself. Although Simons only specifically presents what he takes to be problems in the

case that a mereological Universe is a transcategorial sum, the concerns he raises can be generalised to transcategorial sums in general, as in the following[1]:

(i) If transcategorial sums are in one of the categories occupied by their proper parts, why does this category trump the rest?

(ii) If transcategorial sums are in a different category unshared by any of their proper parts, why do they get a new category?

(iii) If transcategorial sums are in more than one of the categories occupied by their proper parts, and possibly in all of them, then wouldn't this be an *ad hoc* exception to the idea that categories are mutually exclusive?

(iv) If transcategorial sums are in no category at all, why are they exempt?

Unfortunately Simons does not go further to show why the concerns he raises are altogether conclusive reasons for rejecting the prospects of there being a transcategorial mereological Universe, or even any transcategorial sums in general. Although Varzi (2006) agrees with Simons that (iii) and (iv) are unacceptable for the lone reason that they are utterly *ad hoc* (2006, 109), he picks up on the lack of overall conclusiveness and offers several objections to Simons's claim that the universe is not a single mereological being.

6.2 Varzi's resistance

To start, Varzi resists Simons's conclusion by offering that the mereological Universe being transcategorial in many cases would not be unique to it alone.

> Now, I agree that if the universe were the only entity with parts in different categories, each of these options would be embarrassing and ultimately unacceptable. However, the universe cannot be the only such entity. If the universe has parts in different categories, then so does the universe minus Chisholm's left foot [e.g., a physical object], or the universe minus Sebastian's stroll in Bologna [e.g., an event], or the universe minus the number 42 [e.g., an abstractum]. If the universe has parts in different categories, then so do lots of its proper parts: there is no obvious reason to rule out those mereological sums if we are considering countenancing the universal sum. [...insofar as parthood is closed under least upper bounds.] (2006, 109 and fn 5)

[1] Also modifying Varzi (2006, 109).

Varzi finds no problems with option (i) – insofar as it applies to the Universe – provided that the mereological system which guarantees the mereological Universe also entails that there are composites which are exactly like it in terms of the multitude of categories which their parts are from.

> Whatever category hosts that thing that is the universe minus Chisholm's left foot can host the universe as a whole, and that category gets the privilege [i.e., of hosting the universe] precisely because its members have parts in all categories. There would be a problem if the universe minus Chisholm's left foot, the universe minus Sebastian's stroll in Bologna, the universe minus the number 42, and so on, were themselves in different categories. (In that case we would be forced to put the universe in more than one category, as per [(iii)].)[2] But there is no reason why that should be the case. Those parts of the universe are equally representative of all there is and will therefore be in the same category. And the whole universe can be in that category, too. (2006, 109-110)

Yet Varzi's plausible claim that the mereological Universe would not likely be unique in being a transcategorial sum with parts from every category does nothing to appease the generalised version of Simons's problem with transcategorial sums in (i), it merely delays it.

To see this all we have to do is consider some fusion F which is a proper part of the mereological Universe, and which has parts from every category, yet is such that it itself has no proper parts which also have parts from every category. In this case F may face the concern raised by (ii) in having its own category, and the natural concern is why does it occupy a category which none of its proper parts do? It makes no sense to reply that because a lot of other entities could be in the same position with regards to their own categorisation that F itself is off the hook due to the fact that the problem is not unique to F. This reply merely avoids answering the concern, which should count against F and any other entity like it, and not give them some plausibility just because they are not alone in doing so.

What I think Varzi has done in his response to Simons here is focus on the least important component of Simons's objection. To see how this is the case I will need to back up a little bit and provide both Simons's and Varzi's mereological Universe specific renditions of problem (i). Where Simons asks "why does this category trump the rest and get to "bag" the universe?" (2003, 237), Varzi rephrases it as "why should a single category 'trump' all the others and get this exclusive

[2] This is a non-sequitur. From all that Varzi has said, the Universe in this case could just as easily be in its own category too.

privilege?" (2006, 109)[3] What we can see is that Varzi's angle on resisting Simons here focuses on the 'exclusive privilege' clause which can be omitted for the general case. This I think shows what the critical problem Simons poses really is, which is that of why one of the categories of some thing's proper parts of which it similarly occupies is the category of the transcategorial sum itself.

The issue I am raising here is that it is not at all answered by Varzi why a specific category which a composite occupies along with only at least some of its proper parts is able to trump any of the other categories that some one of its other parts occupy. Varzi only offers a way to see that the category which a transcategorial sum occupies might not be a category which it exclusively occupies in comparison to its proper parts. Furthermore, even if the category which hosts a transcategorial sum is not an exclusive category which it alone occupies in comparison to its proper parts – which we would *not* expect a trumping one to be anyway, as a trumping one trumps all other categories its proper parts occupy *with* one of the categories of at least one of its proper parts – this still does not explain why this one in particular is allowed to trump the other categories. That is, in offering how it might at least not be exclusive, Varzi has done nothing to show why it is plausible that this category is the trumping one.

6.3 Resisting Varzi

Considering that Varzi advocates for transcategorial sums as able to belong in different categories to their proper parts, how does Varzi meet the concern expressed by (ii), over why a transcategorial sum should get a new category to some of its proper parts? Regrettably, he doesn't, and in fact goes as far as to say that he expects transcategorial sums to belong to new categories in some cases: "it seems clear that in a polycategorial ontology at least *some* [transcategorial] sums call for unheard-of categories." (2006, 111) Instead of defending this expectation or trying to explain why it should stack up, Varzi offers scepticism towards the idea of being critical of novel category proposals or consequences, saying "Who are we to decide what there is and what there isn't on the basis of ... our modest apparatus of categorial distinctions?" (2006, 112)

But this is more a diversion from the issues than an answer to our questions of whether transcategorial sums are acceptable or not. Certainly we are able to note that any system of categories that we propose and anticipate may be fallible, yet we are still also free to analyse the plausibility of certain situations with those frameworks in mind. That is, we can still put forth a series of ontological conditions and check whether a claimed posit is viable under those conditions, even if we do not have absolute confidence that these conditions hold. In the end what we get from

[3] I.e., of being the category which 'hosts' the mereological Universe.

Varzi here is that it seems most likely that a mereological Universe which is an every-category-transcategorial-sum will not be unique in its transcategorial fusing, and that some of its proper parts will also be every-category-transcategorial-sums too.

But this does not put us any closer towards answering how a composite wholes category can trump any of the categories of its proper parts. Therefore let's consider what Varzi offers as a motivation for this nonchalance towards transcategorial fusion. With regards to whether transcategorial sums have their own, perhaps 'new', categories he asks

> What do we do in such cases? Shall we declare non-existent all those sums for which our palette of ontological categories is too poor? [...] I for one am inclined to be quite liberal in this regard. For one thing, not only do I think that a thing is identical with the mereological sum of its constituent parts. I also share the view that a thing is 'nothing over and above' its parts, hence I don't think the question of the existence of a mereological sum makes much sense in case we already agree on the existence of the pieces. The sum is just the pieces 'counted loosely', as some like to say. (2006, 111-112)

The view that Varzi is referring to here is CAI.[4] In later work, Varzi explicitly states that he broadly and loosely understands CAI as the thesis that a composite whole is nothing over and above its parts, and the parts nothing over and above the whole. (2014, 47) Presuming his conception has remained quite the same over that period, it is very unclear to me how this is supposed to help Varzi's case that concern over transcategorial sums is overblown, as it looks to be question-begging.

Consider that Varzi offers that what CAI amounts to is the assumption that there are not irreducible composite wholes which "exemplify properties and relations that are not reducible to or grounded on the properties and relations exemplified by the things that compose them" (2014, 63) If the key element of the CAI thesis according to Varzi is that there are no irreducible composite wholes we should then be able to look at certain posited wholes and evaluate whether they are plausibly irreducible or not. And while we may not already *specifically* agree on the existence of the pieces, the point of the discussion at hand is able to be conditional too – *if* certain pieces exist, *and* they are of certain categories, is composition between them plausible?

The reason Varzi's claim to CAI looks to be question-begging then is that he assumes that it can suitably override issues of what sort of category a composite could be said to belong to, because

[4] A thesis which I resist in Chapter 3.

the composite is just identical to its parts counted loosely. But clearly this runs in to immediate problems, some of which even Varzi himself takes issue with. On one reading this might sound like the transcategorial composite occupies every category which its proper parts belong to – after all it is identical to all of them altogether and is nothing over and above them. Yet Varzi explicitly rejected this option, which is Simons's (iii), as being "utterly *ad hoc*" (2006, 109). As I cannot see what other way this identity claim could work, I believe that we are owed an explanation of how it makes sense.

Without option (iii) we are left to look at a trumping option for the category of the composite, with it either being in a completely new category that none of its proper parts are in, or the composite itself being in one of the categories that at least one of its proper parts is in, but one that not all of them are in. There is no clear way that the composite could then be said to be identical to its proper parts in this case either – after all it is either only identical to some of them, on the metric of 'sameness of category', or identical to none of them, for having its own category.

So how does the CAI thesis help Varzi here? I don't think that it does, and he doesn't show *how* it could. In fact the claim to CAI seems to work against the idea that the sum could even have its own category – how can it if it is 'no more than' (say) its proper parts? A composite entity with its own 'new category' (2006, 111) which is distinct from its proper parts looks like a simple case of something being over and above its proper parts. What is even more confusing about all of this is an example which Varzi gives which is supposed to show that

> in some cases we may even feel at home with a transcategorial sum. Take a tornado – say hurricane Gloria. An object? Not really: the water molecules, the leaves, and the other stuff that is involved in the hurricane might all be objects, but *the hurricane is something over and above all that*. An event, then? No – because hurricanes move, or so we find it natural to say, while events are temporally extended and thus it makes no sense to say that they move. There are lots of little events involved in Gloria, but *the hurricane itself is something over and above such events*. So maybe we can say that Gloria includes both objects and events as parts – it is a transcategorial sum. (2006, 112) (my emphasis)

Looking at this quote, I don't see how Varzi is properly upholding CAI here. In the context under discussion – because we are discussing the mereological Universe of which everything is supposed to be a part – it is the case that everything is mereological, in having at least one part, itself. Which means that the proposed hurricane is mereological. It also happens to be a composite whole, with

the kind of parts offered for this thing by Varzi being one of only either object or event. What confuses me is that Varzi tries to argue that such a transcategorial sum is *neither* an object *nor* an event because the hurricane is both *something over and above* objects and *something over and above* events.

But that does not make any sense in terms of the CAI thesis as given. With the agreed upon parts of the hurricane in the example being some objects and some events, surely the hurricane, in being composed of objects and events, cannot be something over and above its parts. I expect that what Varzi must have meant instead is that the hurricane is *not just* the objects which are its parts, and *not just* the events which are its parts. But of course! The CAI theorist would say it is just both – both *all* of the objects and *all* of the events which are its parts. That Varzi then permits the hurricane itself to be neither object nor event is what leaves us with the question of how this third unknown category trumps the other two, but it also leaves open again the problem that this third category is not clearly reducible to either of the other two categories, something which immediately confounds any claims to the plausibility of CAI.

Putting this another way, if to maintain Varzi's CAI endorsement we grant that what he really means is that the hurricane does not have only objects as parts nor only events as parts – which is a required for us to grant anyway considering that is what is being offered as the case in the example – we still have the problem that he offers that the hurricane *itself* belongs in neither of the categories of which its parts do. On Simons's question (ii) of how could a composite belong to a category which none of its parts do, we are no further towards understanding why it is able to.

6.4 Smid's commitments

Nor are we given much help in a similar case following Jeroen Smid's discussion surrounding the potential implications for ontological commitment under conditions involving GEM. Here Smid (2015) spells out an argument that one might use to object to GEM (but which he does not actually endorse). The issue is that from the categories one is already committed to GEM appears to entail that there are extra ontological categories, thereby obliging an increase in ontological commitments. He calls this the Argument from Qualitative Extravagance:[5]

L1. UMC [i.e., unrestricted composition] is an axiom of [classical] mereology;

L2. UMC entails the existence of sums, objects that were previously not in one's ontology;

[5] Smid (2015, 3), with the square bracket text added by me throughout L1-L5.

L3. These objects are (sometimes) of a different ontological kind or category [than the one's originally in one's ontology];[6]

L4. Such a different ontological kind or category is an additional [ontological] commitment;

L5. Hence, mereology commits one to *additional kinds* of objects.

Smid accepts that this argument does seem to have persuasive force if it is the case that accepting GEM "entails accepting cross-categorial entities that were (by assumption) previously not there." (2015, 11) Yet he resists the notion that this is a problem, as "the defender of mereology can hold that she simply accepts one, big cross-categorial universe, and that the ontological commitment to this (very special?) kind of entity entails an ontological commitment to the other ('pure') categories." (2015, 11)

Here Smid relies on the situation at hand being relevantly similar to another one which he addresses in the case of purported additional ontological commitments when it comes to the parts of a whole and the whole itself. Smid's argument for the *quantitative* parsimony of GEM follows on from his claim that proper parts are not additional ontological commitments to the wholes which they are proper parts of (2015, 7-11).

Smid's argument for this claim is roughly as follows: the truth-conditions for a sentence such as '*O* exists' (where *O* is a composite object) are no different from the truth-conditions for a sentence '*x*, *y*, and *z* exist' (where *x*, *y*, and *z* are the only proper parts of *O*). And these truth-conditions are no different because e.g., '*x* exists' is true iff there is an object in the world that is *x*. (2015, 6) Because ontological commitments can be seen as certain demands that we put on the world – such that we plausibly only make such commitments if we think they are supported by truth-conditions relevant to them, Smid asserts that "If the truth-conditions are the same, then—given the demand-talk analysis—the ontological commitment of '*O* exists' is the same as that of '*x*, *y*, and *z* exist'." (2015, 6) In this way a commitment to any proper parts just is a commitment to any wholes which they are proper parts of, and vice versa. This is given that the truth-conditions for either ontological commitment are satisfied in the same way in that if a whole exists its proper parts exist, and that if some things exist which are proper parts, these are proper parts of at least some whole.

The way this is meant to mirror the scenario for categories and ontological commitment is that, in the case of a mereological Universe which is an all encompassing transcategorial sum, that

[6] For our purposes the terms 'kind' and 'category' can be used interchangeably without issue.

the commitment to the mereological Universe entails a commitment to whatever its category may be, and also whatever categories its proper parts belong to. To be clear, Smid does not deny that mereology might commit us to entities belonging to different categories. Instead he seeks to show how we might not treat these different categories as further commitments. As he says "Cross-categorial entities are probably different from pure kinds. However, the point is that these pure categories are not to be counted as additional commitments after accepting the cross-categorial entity that is their sum." (2015, 12)

Here Smid explicitly seeks to deny L4 of the Argument from Qualitative Extravagance, that such a different ontological kind or category is an additional ontological commitment. Yet Smid also offers that one could instead deny L3, that these objects might be of a different ontological kind or category than the one's originally in one's ontology, on the grounds that "one could hold that fusions that have parts in different categories do not belong in any category; they belong partly to each category to which their parts belong." (2015, 12)

On the face of it this looks to be both a contradiction in itself and a blend of the problems raised by both of Simons's (iii) and (iv). Namely that something being in more than one of the categories occupied by its proper parts, and possibly in all of them, looks to be an *ad hoc* exception to the idea that categories are mutually exclusive. As well as the problem that the exemption of something being in no category at all is unfounded, as each thing is supposed to be able to be categorised in some manner. The apparent contradiction I see is that composites are being said to *not* belong in any category as well as to *partly* belong in some categories. This seems like a contradiction because belonging in a category appears to be a binary affair – something either belongs to a category or it does not, and if it belongs to a category, then it does not *not* belong to that category.

Here I'm reminded of Lewis talking about existence (which I previously referred to in Chapter 5), of which he says "existence cannot be a matter of degree. If you say there is something that exists to a diminished degree, once you've said 'there is' your game is up. Existence is not some special distinction that befalls some of the things there are. Existence just *means* being one of the things there are, nothing else." (1991, 80-1)

My point here is that just as we cannot say that some transcategorial composite T somewhat 'faintly' belongs to each of the categories which it's proper parts belong to without the implication that T just simply belongs to each of the categories, we also cannot interpret Smid as meaning to say something more along the lines of 'a proper part of the composite T belongs to category G'. This is because this says more about the category which the proper part of T belongs to than any

category which the composite T itself does. Besides this, because Smid allows that things which are composite are of the category 'complex' – in distinction to things which are mereologically atomic being of the category 'simple' – it is not plausible that he could treat transcategorial sums as not belonging to any category (2015, 12). So I'm more inclined to understand Smid as meaning to say that 'fusions that have parts in different categories do not belong in any *one* category'. (Which also lessens the plausibility of the reading that such fusions are only 'faintly' belonging in any category.)

But now we appear squarely back at Simons's objection (iii) to transcategorial sums, that occupying more than one category runs afoul of the notion that categories are mutually exclusive. To precisify this objection slightly, it is not the case that *all* categories are mutually exclusive, and this is both because not every way of devising them requires only that each category be an 'ultimate' one, and also because categories which are mutually exclusive are usually only properly exclusive of another category which is their opposite pair, rather than any other variety of categories.

For instance, on E. J. Lowe's (2006) system of four fundamental categories, one of them – objects – further divides into the non-fundamental categories of substances and non-substances; Ingvar Johansson (1989) has a main category of quality which subdivides into substance and property; Roderick Chisholm (1996) offers the main categories of contingent and necessary, which respectively divide into states (events) and individuals (boundaries; substances), and states and nonstates (attributes; substance); and Joshua Hoffman and Gary Rosenkrantz (1994) divide entities into the abstract and concrete, both of which have several subdivisions – the abstract: property, relation, proposition; and the concrete: event, time, place, substance, to name but a few (see Thomasson 2018 for a detailed overview).

So although the way in which Simons explicitly phrases his objection shows that he expects categories to be ultimate, saying "If the universe [as a transcategorial sum] is partly in one category and partly in another, what does this mean, and does it not undermine the very notion of a category as a system of mutually exclusive ultimate types of object?" (2003, 237-8) it may be clearer to state the objection for the generalised case in the following manner: transcategorial sums being in more than one of the different super- or subsidiary categories occupied by their proper parts, and possibly in all of them, is an *ad hoc* (and therefore unjustified) exception to the idea that different super- or subsidiary categories are mutually exclusive.

Gladly, this objection does not appear to be question-begging, because the assumption that different super- or subsidiary categories are mutually exclusive is actually not what is at issue. What is at issue is how something could plausibly straddle multiple categories simultaneously, in a way

that does not undermine our original expectation of the strict mutual exclusivity of belonging to certain pairs of categories. The very fact that something cannot do so without a contradiction of that original assumption is what makes that rhetorical objection, i.e., (iii), coherent.

While all of this means that Smid is wrong about denying L3 on the grounds that transcategorial composites need not be of a different ontological category to any of their parts, and can belong to the categories of each of them, how does his claim that premise L4 can be resisted fare? Unfortunately for our purposes here it is irrelevant. Recalling that I started off this section by saying that Smid's discussion isn't much help, I meant this specifically with regards to the question of *how the category of a transcategorial composite is supposed to trump the category of at least one of its proper parts* – and either by being in a completely different category to *all* of its proper parts, or by being in a completely different category to *at least one* of its proper parts. While Smid's arguments may provide grounds for not treating certain 'new' categories[7] as further ontological commitments, they do nothing to justify why such composites are able to belong to such different categories in the first place. Nor do Smid's arguments justify why a transcategorial composite might belong to any one category which some one of its proper parts does, to the exclusion of a different category some one of its other proper parts belongs to.

To be fair, Smid's arguments weren't expressly supposed to provide these justifications. Yet his tendered grounds for denying L3 coupled with the general implication that such new or trumping categorial belongings are in some way acceptable (by at least not being extra ontological costs), suggest that he does not seem to have any quarrels with Simons's objection (i) or (ii), although he could have encountered these objections in their original form.[8] From what we have been given by both Varzi and Smid, then, I conjecture that it is inconclusive and still not in favour of overcoming the questions raised by Simons, as both Varzi and Smid seem to elide the rhetorical objections present in (i) and (ii) without actually showing the manner in which this avoidance is achieved, and with considerable additional problems incurred along the way.

I think that although Simons's questions are also inconclusive as *rejections* of transcategorial sums, they still stand as plausible criteria to meet to make such sums more credible, and to help us understand some important obstacles to overcome if we are to accept that they somehow fit within our ontological frameworks.

[7] 'New' as in newly recognised, not as in some sort of new creation.

[8] Smid (2015) does cite Simons (2003) after all.

With this in mind, what I offer in the next section is an explanation of how I think some cases could be made which do meet Simons's objections, even with mixed fusions representing some of the proposed transcategorial composites.

6.5 Ontological division

We are now back to the earlier concerns about transcategorial sums: how might categorial trumping from composite to parts be acceptable? And how would it be plausible? For these concerns it will be helpful to review the two remaining challenges which Simons's generalised (i) and (ii) pose. Starting with (i) we can look towards mixed fusions as an example. In this instance we can consider a mixed fusion which has only two atomic parts – one which is a singleton, and thereby a class, and one which is an individual, and thereby a singulate. On the question of whether the mixed fusion is a class or a singulate we do not look to the natures of the parts for guidance but rather the formal systems and principles which have been said to entail and frame them. Imagine that I'm wrong about property inheritance with regards to what I argued for in Chapter 3, and that instead we simply insist that no mixed fusion can have members on pain of contradicting the set theoretic axiom of Extensionality. By that argument, mixed fusions are singulates, and either individuals or anti-members. (As I argued in Chapter 2 some mixed fusions are able to be members, but not all.) We therefore have a reason for why any mixed fusion belongs to the category singulate – if they exist, mixed fusions are necessarily singulates to avoid contradicting the set theory which jointly entails them. So we have an example of justified trumping in the sense of (i).

In the case of Simons's (ii) we will look at a situation which can involve mixed fusions, but doesn't necessarily need to. Here we look at how Lewis's (1986) ontology of possible worlds gives grounds for plausible trumping claims. As we saw in Chapter 5, for Lewis possible worlds are concrete maximal sums, such that the mereological whole which any possible world is only has parts which are possible too (e.g., 1986, 211 inter alia). Lewis advances that there are an infinite multitude of possible worlds, with each possible world necessarily mereologically disjoint from every other one, none of them sharing any parts, and thereby none of them overlapping.

However, again as we saw in Chapter 5, because Lewis accepts infinite possible worlds and also accepts GEM, he admittedly also accepts that there are trans-world fusions, which have proper parts from multiple possible worlds (1986, 211). Lewis acknowledges that this commits him to the existence of impossible objects, as the fusion of things that exist in different possible worlds itself does not exist in any possible world. Even though 'impossible' has a special technical meaning here, as these fusions aren't impossible in the sense that they cannot be formed, rather they are

impossible objects for not belonging to any possible world, they still allow for a category divide between the impossible and possible.

Because of trans-world fusion and also the existence of impossible objects, for Lewis there are actually three types of impossible objects in terms of their mereological structure. The first properly decompose only into proper parts which are not from possible worlds; the second properly decompose into both parts which are from possible worlds and parts which are not from possible worlds; and the last properly decompose only into parts which are from possible worlds.

I think it is from some of these cases that we can have examples of category trumping which are justified on the grounds that in this context being possible means existing as part of a possible world. On the first type of impossible object, though, as expected, fusions of impossibilia are also themselves impossible objects because they belong to no world. Why do they belong to no world? Because possible worlds only have possibilia as parts. So no category trumping here.

With the second type, however, because the impossible parts aren't part of a possible world, their fusion with parts from possible worlds is an object that is not in a possible world either. Again this is because possible worlds only have possibilia as parts, which means that the impossible part cannot overlap a possible world entity without thereby being a part of a possible world. Therefore such fusions are impossible objects. (As we know, on Lewis's account many mixed fusions are examples of this type of impossible object, with all singletons being impossibilia, and all the inhabitants of possible worlds being individuals.) In this second case we have the category of the composite trumping the category of one kind of its parts over another, and for logical reasons within the system of rules and conditions given.

The last type of impossible objects appears at first surprising, but it is understandable when we have trans-world fusions. Because trans-world fusions are fusions of things which are themselves possible, it appears counter-intuitive at first that they should compose objects which are impossible. But this is occasioned only because the possible worlds that each of the possible things are from are disjoint, such that there can be no fusion which overlaps both of them which is itself possible – as that would negate the possible worlds being disjoint.

We can see this is because each possible world is its own maximal sum. What this means is that any two possibilia which are each in their own distinct possible worlds belong to and are a part of their own maxima. By this reasoning, w1 (world 1) overlaps possible proper part 1 (p1) and world 2 (w2) overlaps possible proper part 2 (p2). As defined w1 does not overlap w2, and p1 does not overlap p2. If p1 overlapped p2 they would share a part, and as p1 is fully overlapped by w1, w1 would overlap p2. But w2 overlaps p2 and w1 and w2 do not overlap, so this cannot be the case.

The proposed impossible composite i3 which has p1 and p2 as parts overlaps them both, of course, just as they overlap it. For the same reasons that p1 and p2 do not overlap, if i3 was in a possible world, i3 could not overlap either p1 or p2. This would mean they could not be i3's parts, which contradicts the assumption that they are. As it's possible that p1 and p2 could be mereological atoms, here we have the category of the composite trumping the category of all of its parts, along the possible/impossible divide. So we have an example of justified trumping in the sense of (ii).

To answer Simons's questions, then, asking how a transcategorial composite's category either trumps the category of all of its parts or at least some its parts, the answer I give is: it depends on how the systems which harbour such composites are compatible with the trumping. Which is not to say that in every case of proposed transcategorial composition that the harbouring systems will actually be compatible. Far from it. It is just to say that we cannot outright dismiss the potential compatibility by assuming that an incompatibility is certain, solely on the grounds that the composition is transcategorial.

As I have shown in this section, it's not like the composites which are compatible pose any problems for the categorial systems in which they have been placed – obviously, as this is some of what being compatible means. But I think that's the point, that compatibility is something which is to be evaluated, rather than assumed. So this analysis actually cuts against Varzi's claims as well, the ones which say that we shouldn't have any reservations about transcategorial sums at all. How are we to know in advance that some proposed transcategorial composites aren't incompatible?

The point to contrast here is that in Chapter 5 I explicitly rejected transcategorial fusions of abstracta and concreta in the case their fusion is supra-world, by variously arguing that such sums can be neither abstract nor concrete without contradicting theses supporting the categorical divisions. As the result back there and the recent result here endow us with both negative and positive instances of transcategorial sum category trumping, I make my case that category compatibility for mereological hybrids is something to be verified and not presumed, with a plea to play it safe and not assume that liberal forms of composition across categories are assured to have no troubling consequences.

6.6 Conclusion

Although I've shown that Simons's challenge questions can be met in some cases, I still think they offer an important guide to the sorts of questions we should be willing to ask ourselves when faced with novel ontological considerations. Even though I find Simons's line of questioning inconclusive towards whether there is a mereological Universe or not, I also find Varzi's defence of a

mereological Universe wanting, on the same grounds that we just don't know what is and what isn't compatible until we take a good look.

I think the takeaway here is that categories come with compatibility issues built into the very premise of categories – naturally throwing up questions like 'does what we already believe in fit them?' and 'can the things said to be from different categories harmoniously co-exist?'. Much of this chapter (and also this thesis) has been about a variant on these questions, one which asks 'are mereological fusions of these legitimate?' I hope to have shown that before we can make such determinations that we need to look at the proposed entities and their respective categories and make a proper case for whether they are compatible or not. As I have argued in this chapter, I think that in some cases they can be compatible, but as I argued in Chapter 5 (via the abstract/concrete distinction and across the boundaries of worlds) in some cases they are not. Following these variable results I further offer that we are not entitled to assume compatibility and that we therefore have to work to verify it. To me this means that we cannot assume that transcategorial sums exist in general, from which I think it follows that we are obliged *not* to commit to them, pending review.

As such, I think the answer to the question of whether or not there is a mereological Universe is – it depends. And what it looks to depend on is the wide array of other ontological infrastructure that it is supposed to sit upon. Because we cannot plausibly be certain in advance, the onus is on us to provide good reasons why our proposed entities truly fit the systems that we also expect to be in place. Further to this, I think our lack of knowing whether the compatibility will work until we check things out should make us hesitant to commit to the things which clearly need some analysis to verify their compatibility before we indeed perform such analyses. Which means that if I had to give a more decisive answer to the question of whether there is a mereological Universe, I would say no, because the work required to fulfil such analyses may never cease. After all, the mereological Universe is supposed to have everything else which exists as its proper parts, and if we're not even clear on what all of these other things could be, or whether we should accept them on independent grounds, how can we plausibly infer that the mereological Universe exists, let alone accept it?

...and I had hoped so much that everything would be orderly, all simple and neat.

(Nabokov 1959)

Chapter 7
Mixed fusions, classical mereology, and ontological innocence

7.0 Introduction

In this chapter I offer an argument for why GEM is not 'ontologically innocent' (OI). While this is a concept which has been glanced at in previous chapters, it has not been directly addressed, and the issue presented here with regards to it is an important piece of my cumulative argument against unrestricted mereological hybridism, mainly as it is supported by unrestricted composition and GEM.

The notion of OI that is at stake is one which involves ontological commitment, wherein claims to the existence of certain entities or claims to the acceptability of certain theories are said to be OI insofar as the commitments one is obliged to hold in maintaining the theories or the existence claims are not to be counted as *extra* commitments (Smid 2015). The characterisation of a claim as OI is thereby one with regards to the cost of accepting that claim as true – a theory which is purported to be OI is one that is supposed to come at no additional costs to the ones which are already accepted up front, even if this may not at first glance appear to be the case.

In mereological debates, claims of OI often involve claims of innocence for composite entities. In his defence of GEM, David Lewis (1991) famously argues for its OI in the sense that when it comes to our ontological commitment to the number of entities that exist that we do not incur numerically more commitments by accepting mereological sums or fusions of the things that we are already committed to. As we have seen Lewis put it "given a prior commitment to cats, say, a commitment to cat-fusions is not a *further* commitment." (1991, 81) The notion of OI on offer here, then, is mainly focused along quantitative lines – to the effect that if we are to count our commitments it would be redundant to count a fusion in addition to its parts, as this would be a form of double-counting (Varzi 2000). For Lewis, if we count the fusion we already 'have' the parts, and vice versa, so GEM should not be seen as permitting a form of ontological non-parsimony for permitting unrestricted summation or composition of any two or more things.

I think Lewis is wrong about the OI of GEM. This is because it can be seen to permit composite wholes which are irreducible to their parts – specifically the mereological hybrids of mixed fusions – with these irreducible entities needing to be counted as additional ontological commitments to their parts insofar as they cannot be properly represented by anything else. The essence of my argument that GEM is not OI is that because it permits something which is not OI, viz. mixed fusions, it is not OI itself.

The argument I make here, however, depends on suppressing my claim in Chapter 3, namely the claim that property inheritance entails that mixed fusions have members. As Lewis rejects

mixed fusions having members anyway he should in some ways be supportive of this situation (modulo the property inheritance concerns), but what I will be arguing in this chapter is that mixed fusions not having members is precisely what leads to them being irreducible to their proper parts, and thereby being additional ontological commitments.

Because having less ontological commitments is seen as a theoretical virtue, and claims to the OI of GEM have played an important role in its defence, after this issue of irreducibility I am able to present the Lewisian with a difficult trilemma, to either accept: mixed fusions having members and violating Extensionality; mixed fusions being irreducible and thereby coming at an unjustified cost; or, rejecting unrestricted composition to avoid mixed fusions. As is usual, I opt for the rejection of unrestricted composition and GEM to avoid mixed fusions, and, in turn, the rejection of unrestricted mereological hybridism.

7.1 Lewis's depictions of mixed fusions as irreducible to their proper parts

As has been the case throughout this thesis, my position in this chapter is framed against the background of the presentation of GEM and set theory given in Lewis (1991). Recall the variety of different characterisations I have made about mixed fusions over the course of this thesis when compared to those given by Lewis (as in Table 1.3 in Chapter 1). Although there is clearly a great disparity between what Lewis thinks of mixed fusions and what I think of them, we do have two key points of explicit agreement about what mixed fusions are like, one being that they are irreducible to their parts (7a) and the other being that they lack members (3). However, as these points of agreement are undermined by other claims which Lewis makes, it is not clear that we actually agree upon anything to do with mixed fusions at all. For our purposes in this chapter, the key point of *dis*agreement between Lewis and I here is that Lewis thinks mixed fusions are OI, whereas I do not (7b).

As I think that Lewis upholding mixed fusions as irreducible is inconsistent with some of his other views, I need to spell out the details which show this. My claim is that although Lewis makes several explicit statements about the nature and character of mixed fusions which imply their irreducibility to their parts, it is in fact the case that many of these statements are undermined by Lewis's other express commitment to CAI. Which is to say that, although the notion of CAI implies reducibility (as explained in 7.2), and Lewis explicitly endorses CAI, it is the case that he also makes explicit statements about mixed fusions which entail their *ir*reducibility. I believe that this inconsistency is a great problem for Lewis's view, and my focus in this section is to show that if mixed fusions are to be accepted at all, that they are indeed best seen as irreducible to their parts.

To make the case that Lewis presents mixed fusions explicitly as entities which are irreducible to their parts we need to see what Lewis explicitly says about mixed fusions in this regard. To be clear, Lewis does not make any explicit claims to the irreducibility of mixed fusions. The situation I describe, however, is one which I believe is implied by his explicit statements on the characters of mixed fusions.

Recall that on the topic of whether mixed fusions can *be* members of classes, Lewis is dismissive (see Chapter 2 for my discussion and rejection of Lewis's argument for this conclusion). The way in which Lewis's view here entails that mixed fusions are irreducible, is that if mixed fusions cannot be members, yet in many cases their parts can be (such as all of their individual parts, and any of their class parts which are not proper classes for Lewis), then this not-a-member feature of mixed fusions is clearly not derived from the is-a-member feature of their parts. When compared with my view of mixed fusions being eligible for membership from Chapter 2, this irreducibility also looks to apply to at least some mixed fusions, namely the ones which cannot be members due to their size. However, perhaps this is not a very strong case of irreducibility, because something's being a member could be seen as being an external relation, such that it may not be completely correct to suggest that this is a genuinely irreducible property, if these are not the sorts of properties which can ever be properly reduced to.

On the topic of whether mixed fusions are themselves abstract or concrete if at least some of their parts are abstract and some of their parts are concrete (as classes may be abstract and some individuals may be concrete), Lewis's preferred view is clear, although not entirely explicit in the sense of a direct answer to such a question. As discussed earlier in Chapter 5, we can surmise that no mixed fusion for Lewis is explicitly a concrete entity, which only leaves them to be abstracta under the presumed dualism.

The way in which Lewis's views entail that mixed fusions are irreducible here is that if mixed fusions have both abstract and concrete components, yet the composites are only one of these, and in this case abstracta, then the composites have a property that is not found in each of their proper parts, namely that of being abstract. So any mixed fusion with both abstract and concrete parts which is itself an abstractum cannot be reduced to its concrete parts, because the abstract property of the whole is not derived from such parts. Now, if there are some abstract entities which are not classes or mixed fusions, then there may be mixed fusions which have all and only parts which are abstracta. In this case a mixed fusion that is abstract would be fully reducible, in terms of its abstractness, to its proper parts. However, as there are plausibly many things which are non-abstracta, if there are mixed fusions in the way that Lewis proposes, at least some of them would be fusions of abstracta and concreta.

If instead mixed fusions were to be claimed as neither abstract nor concrete, even while having both of those types of parts, then these as wholes would have a new type of property which is not found in their parts. This would make mixed fusions irreducible too, as the new type of property that the composite is would not reduce to the properties of its proper parts. As such, there appears to be no way of taking mixed fusions with both abstracta and concreta parts as one of either abstract or concrete or even a novel type and to maintain that they are reducible to a variety of types of parts – and this is because the abstractness property of an abstract entity with concrete components would not be reducible to those concrete components, just as the concreteness property of a concrete entity with abstract components would not be reducible to those abstract components, and the novel property of a novel type with both abstract and concrete components would not be reducible to those abstract and concrete components.

As I argued in Chapter 5 that such fusions of abstracta and concreta are best understood to be concreta, my claim to mixed fusions' irreducibility here mostly rests on classes being abstracta. As I mentioned earlier in Chapter 2, Daniel Nolan (2002, Chapter 7; 2004) has shown how to resist this being a necessary situation, however, especially in the context of Lewis's plenitudinous modal realism. He does this by building upon a form of structuralism about classes which only requires a sufficiently infinitely large amount (proper class many) of concrete individuals as parts of Lewisian possible worlds to serve as the structural basis for set theory – a strategy which serves to eliminate both pure-out impossibilia and classes as abstracta within Lewis's system. (Arguably Lewis could accept this modification of his system, as he does propose going structuralist in both the Appendix of his 1991, as well as relatedly in 1998 – the works which Nolan himself builds upon to make the modification.)

While I think Nolan's strategy is admirable for its elimination of pure-out impossibilia (insofar as classes are the only pure-out entities) I expect that it still faces some of the problems I describe in Chapter 5 for supra-world fusion, such as those in the case of trans-world sums. Which means I would still call for a restriction of composition in that context, even with the elimination of out-world abstracta. Although, if my objections in Chapter 5 are resisted, I believe there is still at least one further case which suggests the irreducibility of mixed fusions – their not having members (a conclusion I reach in Chapter 3).

On this topic of whether mixed fusions have members, Lewis's presentation is more of an expectation that they do not have members, rather than any direct argument that they do not, as I discussed in section 3.1. Even still, there I acknowledged that Lewis's Second Thesis, that no class has any part that is not a class, indirectly suffices to set Lewis as against mixed fusions having members, as any of these would be a class and would thereby contravene his Second Thesis. So,

even though Lewis (weakly) explicitly asserts that mixed fusions do not have members, his Second Thesis is a stronger indication that he accepts they should not. And I think that if it is the case that mixed fusions do not have members, then it is this that makes them irreducible to their class parts.

This is because any mixed fusion's property of not having members is a property which is not reducible to any singleton's (which are parts of mixed fusions) property of having members. Even where mixed fusions have parts which do not have members – which they must to be mixed fusions – a mixed fusion not having the property of having members is still not reducible to its singleton parts. This is because for the reduction to be plausible, it needs to be total, and not selective.

To recap, although none are entirely direct statements of mixed fusions being irreducible to their proper parts, there appear to be at least three ways that Lewis indirectly shows that many mixed fusions are irreducible to their proper parts. This is by Lewis denying that they are members, by denying that they have members, and by implying that they are abstracta even in the case they have some parts which are concrete (see Chapter 5 for further discussion of this last situation). As I have argued that mixed fusions cannot have their class parts' members without violating the very class or set theory that they are caught up with in Chapter 3 – which is a consequence of CAI that I see arising from the notion of property inheritance which is embedded in it – I agree with Lewis that mixed fusions do not have members, and are therefore irreducible to their class parts. Yet as I mentioned earlier, I think that Lewis's presentation of mixed fusions as irreducible to their proper parts is undermined by his commitment to CAI. In the next section we see how.

7.2 How CAI precludes the irreducibility of composites

The point of promoting a commitment to a theory or entity as one that is OI is to give a reason to not reject the theory or entity for the additional theoretical costs it brings. On the topic of claims to the OI of GEM, Hawley says "Other things being equal, a theory which involves fewer ontological commitments is preferable to one which involves more. If mereology is ontologically innocent, then accepting mereology does not add to our ontological commitments; this removes one obstacle to accepting mereology." (2014, 72)

This idea of removing obstacles to the acceptance of GEM is indeed why Lewis makes the claim that GEM is OI. As he says in his classic statement of the idea, which also involves an argument for CAI,

> [Classical] Mereology is ontologically innocent. To be sure, if we accept mereology, we are committed to the existence of all manner of mereological

> fusions. But given a prior commitment to cats, say, a commitment to cat-fusions is not a *further* commitment. The fusion is nothing over and above the cats that compose it. It just *is* them. They just *are* it. Take them together or take them separately, the cats are the same portion of Reality either way. Commit yourself to their existence all together or one at a time, it's the same commitment either way. If you draw up an inventory of Reality according to your scheme of things, it would be double counting to list the cats and then also list their fusion. In general, if you are already committed to some things, you incur no further commitment when you affirm the existence of their fusion. The new commitment is redundant, given the old one. [...] It is in virtue of [CAI] that mereology is ontologically innocent: it commits us only to things that are identical, so to speak, to what we were committed to before. (1991, 81-2)

There are a couple of things to unpack from this paragraph for the purpose of showing how CAI precludes irreducibility. The main ones for our purposes are the claim that prior commitments can preclude seemingly new commitments from being truly new, and the claim that fusions can be 'nothing over and above' their composing parts. Both can be understood as claims to the reducibility of fusions to their composing parts. In the first case, the commitment to the fusion can be reduced to a commitment to the parts – in this way the commitment to the fusion is not an additional commitment, because it is only really a commitment to the parts after all.

In the second case, it is the interpretation of the phrase 'nothing over and above' which can be understood as endorsing the reducibility of fusions to their proper parts. Smid (2017) helpfully shows that the phrase 'nothing over and above' has at least 5 interpretations that are commonly involved in making arguments for and of understanding GEM, one of which directly refers to reducibility, or reduction. These interpretations include

No Additional Commitment	X is nothing-over-and-above$_{NAC}$ the Ys: X is not an additional ontological commitment relative to the ontological commitment to the Ys.
No Condition	X is nothing-over-and-above$_{NC}$ the Ys: The (mere) existence of the Ys is sufficient for the existence of X.
No Other	X is nothing-over-and-above$_{NO}$ the Ys: X is the only object that is R-related to the Ys.

Reduction	X is nothing-over-and-above$_R$ the Ys: X has no properties that are irreducible to the properties of the Ys.
Identity	X is nothing-over-and-above$_I$ the Ys: X is identical with the Ys.

Smid explains that although GEM does justify No Condition, in that if some things exist they have a fusion, and No Other, in the context of parthood extensionality or uniqueness (a notion discussed in Chapter 3), that it is only compatible with No Additional Commitment given the counting policy for ontological commitment which is in place (2017, 11). He further explains that in the literature there are at least two counting policies, the standard one, which holds that we should count ontological commitments using identity and non-identity, and Varzi's 'Minimalist View' (2000) which instead advises to count via overlap and disjointness (2017, 11).

As Smid explains it, it is only via the Minimalist View which No Additional Commitment holds, because the standard view ensures that we increase our count of ontological commitments for each non-identical thing, whereas the Minimalist View permits that non-identical things are not counted as additional beings so long as one overlaps the other. For example, on the standard view I would count my shoe, my shoes sole and its lace as three separate commitments. Yet on the Minimalist view I could count just one commitment to my shoe, and as the sole and the lace are proper parts of it, and hence are fully overlapped by the shoe, I could see my commitment to these parts as non-additional.

It is in this way that we can understand Lewis's claim to the OI of GEM for bringing no additional commitments via his use of the phrase 'nothing over and above'. However, we can also understand Lewis's phrasing as supporting Reduction, that a fusion has no properties which are irreducible to its proper parts. We can see this especially in Lewis's earlier use of the phrase 'nothing over and above' in his statement of support for GEM's principle of unrestricted composition, where he says that because an unheard-of fusion "is nothing over and above its parts ... to describe it you need only describe the parts." (1991, 80; quoted fully in section 3.4)

Clearly, Lewis saying that to describe a fusion one needs only to describe its parts is a case of Reduction, because what would be described in any case is the properties each has. However, as Smid explains, Reduction is not a principle of GEM. "Mereology is silent on properties, reduction, and supervenience. Consequently, it does not force one to accept or deny [Reduction] in the context of parts and wholes. However, the extensionality of mereology is often associated with [Reduction]." (2017, 10)

Of the five senses of 'nothing over and above' which Smid sets out, Lewis is explicitly committed to the first four. His commitment to CAI is not quite a commitment to Identity, and this is because of Lewis's weakening of the identity claim to be not quite strict identity, but rather something which is analogous to it, as discussed in Chapter 3. (Recall that this is roughly the weakening that composition is not strictly 1-1 identity, but rather a form of many-1 identity.) In the context of this discussion, we might interpret Lewis as at least holding the following variation

Analogous Identity X is nothing-over-and-above$_{AI}$ the Ys:
X is analogously identical with the Ys.

Taking Analogous Identity as supported by the other four interpretations of the phrase 'nothing over and above', and all of these senses of the phrase as indicative of Lewis's view of CAI and his claim to its OI, we have a pretty clear picture that Lewis explicitly holds that all composites are reducible to their proper parts – because the many parts are many-1 identical with the one composite – and that this reducibility is of a piece with his claim to GEM's innocence via CAI.

This picture receives some support from Varzi (2014) in arguing for the OI of GEM via the CAI thesis, as if consonant with his Lewisian instincts (2014, 67).

> I am not saying that the argument above *establishes* the innocence of mereology. Obviously, one may still think that some composite wholes exemplify properties and relations that are not reducible to or grounded on the properties and relations exemplified by the things that compose them, in which case commitment to the former would be a significant ontological addition to commitment to the latter. Rather, the point of the argument is to clarify the idea that mereology *can* be innocent: that it is innocent on the assumption that there *aren't* irreducible wholes like that, which is what Composition as Identity amounts to (even on the weak understanding of "as" that I have endorsed)." (2014, 63)

Smid too says "Failure to reduce an entity can be a reason to count it as a further ontological commitment." (2017, 6) It is this, along with Varzi's last point, from which CAI amounts to the claim that there aren't irreducible wholes, which supports the idea that CAI precludes composite irreducibility and instead promotes Reduction.

7.3 GEM's loss of innocence

Appropriating some of Lewis's words above, we can see that with regards to claims to CAI, that it *is* a problem to describe a mixed fusion under those conditions. This is because a mixed fusion *is* something over and above its parts. It's not that to describe it you need only describe the parts – because if you describe the character of the parts you have the description 'has members' for its class parts. Yet, as mixed fusions cannot have members, to describe their class parts we cannot have *ipso facto* described the fusion. Therefore a mixed fusion's character is *not* exhausted by the character and relations of its parts. It is something more than them.

The trouble with Lewis's presumption of OI for all composites, including mixed fusions, is that irreducibility is a hallmark of additional ontological commitment, and thereby any irreducible entity comes at a cost to one's theory. Unfortunately for Lewis, I don't think there is a viable strategy out of this predicament without forsaking theses and principles which are important to his views. For instance, in Chapter 3 I already argued that Lewis has to forego both CAI and property inheritance from parts to wholes to maintain the existence of mixed fusions, because otherwise class theory is violated. If this is done, however, my point in this chapter stands, that in the case of mixed fusions their subsequent irreducibility leads to their being additional ontological commitments, because without properly accounting for or describing their properties, mixed fusions would be missing from one's 'inventory of reality'. So, in this sense, GEM loses any claims to innocence in the case that it permits mixed fusions. As Lewis sought to justify GEM via both CAI and OI, with the loss of both of these supporting notions, such a defence of GEM is severely weakened.

As I argued in Chapter 3, with property inheritance being something we would rather hold on to, as otherwise many properties of wholes are something like emergent, my strategy here is to restrict composition and to thereby treat GEM as falsified by this case. I think the takeaway here is that any claims to GEM being OI is constrained by context – specifically the context of the kinds of entities that are fed into the system, as it were, and what they are supposed to be like according to our best theories. As is known, GEM itself makes no initial claims as to what exists to even initiate composite formation. Where it becomes powerful, and for some, unforgivable, is what it can permit when given a specific collection of entities. Usually this is in terms of the number of things it outputs, but from what I have argued here, it can also be in terms of the types of things that it outputs. While the former problem is one involving quantitative concerns – are there too many things? – the latter problem is one involving qualitative concerns – are there really things like that?

We have seen this sort of idea already in earlier chapters, where certain kinds of entities forming composites, such as mixed objects consisting of abstracta and concreta, look to confound the principles which support claims to the initial inputs. The blame, as it were, in such cases can

only be put on the blind operation of fusion which GEM asserts, much how just as in the significant quantitative cases GEM can be seen as the culprit for an excessive number of things in existence. What I want to suggest, then, is the sense in which GEM is not OI by dint of what it permits, and that is by its capacity to facilitate the formation of properly problematic entities, and not just ones which do not sit right with our intuitions (such as odd trout-turkey monsters), but ones which literally break our pre-established conventions. I think it is by being both blind to its inputs and mute regarding its outputs, as it were, that GEM is vulnerable to generating such confounding outputs from independently plausible and/or separate inputs, exactly because the initial inputs themselves host incompatible features. In this regard, I believe this makes GEM unable to be classified as OI precisely because it itself offers no protection from troubled entailments, where such entailments come at great cost.

I think what also may have been misleading about GEM applied in restricted domains, is that in these domains troubles similar to the ones I have ventured do not arise. I think that in these restricted cases although it may appear that GEM is innocent that rather this is just another example of how the system itself has no control over how the outputs compare to the inputs, and it caught a lucky break. To me, it appears that it is the automatic generation of composites itself, as in Smid's No Condition, which makes the system of GEM non-innocent, and this is because the system *can* lead to problems, not that it necessarily always will, as there is no internal restriction to stop this. I think this means that GEM can only then be seen as contextually innocent, and not absolutely. Which thereby means that it is not simply innocent, but conditionally so, and this appears to be by fortune rather than anything fundamental to GEM itself.

7.4 Conclusion

With the previous general picture in mind, I wanted to show how vulnerable GEM really is in the case of mixed fusions, which requires that the mereological domain be one which includes both classes and non-classes. To really emphasise this vulnerability, I have shown how mixed fusions cannot be seen as reducible without themselves contravening the system of standard set or class theory, because for them to properly be reducible they need to have their class parts' members after all. This demonstrates the vulnerability of GEM by showing that because it has no say in what its inputs and outputs are (in terms of their kinds of beings, beyond sums), it is rather just a conduit for making composites out of what it is given – even if these are troublemakers for the theoretical environment that they are posited in.

Because of GEM's inherent vulnerability in this regard, and because mixed fusions provide an exemplary case of irreducibility which thwarts more traditional claims to the OI of GEM, I

assess that GEM is not itself OI. To avoid the costs that mixed fusions would bring in the sense of irreducibility I think it is best to restrict composition here, to prohibit mereological hybrids like them. In addition to this, it appears that we ought to remain wary that holding unrestricted composition principles are risky in any instance where we have not properly evaluated all candidate composites which may form in their presence. To further prevent GEM's vulnerability from being inadvertently exploited by positing *separately* more-or-less harmless existents, then, I think it is best to restrict composition in general and avoid unrestricted mereological hybridism altogether.

Chapter 8

Epilogue

8.0 The end of this whole story

In this thesis I have argued against unrestricted mereological hybridism. This has been primarily within the context of there being singulates and classes, and with some of the kinds of entities that these might be recognised as (e.g., concreta and abstracta; possibilia and impossibilia) put forth as contrastively exemplifying categorical differences in a variety of ways. Specifically, I focused on mereological hybridism under the condition of unrestricted composition as it is found within classical mereology operating absolutely, and I have shown how several unfavourable consequences arise when classes and singulates are expected to form composite wholes – depending, of course, on just what the entities said to be fused together separately are.

The biggest general take away from the arguments that I have made, I think, is that any proposal of arbitrary mereological fusion needs to be reconsidered, because there could be more to it in terms of the resultant fusions than what may at first appear. In this sense, I have been advocating a strategy of mereological modesty, where instead of permitting widespread, unconstrained summation of any and every entity whatsoever from the outset, that we look towards only instances of composition across categories which we have properly defended, to first try and ensure that such transcategorial composites operate consistently within the framework in which we expect them to be deployed.

One of the subtle issues which I think has been lurking in the background of this study surrounds the fact that unrestricted composition alone (but within a mereological framework) is insufficient for making mereological hybrids. As a principle of generation which does not supply its own inputs, unrestricted composition needs independently provided ingredients to bind and combine to even begin to arrive at the sorts of odd and objectionable wholes that I have surveyed over the course of this work. And because our reasons for positing such ingredients are incredibly varied and contested throughout the entire philosophical corpus it becomes quite difficult to definitively pin down just what combinations should be most resisted, or whether some combinations work or not only in the presence of other kinds or under other conditions or conceptions.

So, although I have called for a restriction on composition, I have not made a positive assessment of how strictly minimal or maximal I think this restriction should be altogether. This is because the existence of problematic composites in the presence of unrestricted composition is highly conditional on the kinds of entities which are their proper parts, and the variety of options philosophers have claimed for kinds makes the explicit decisions for composition restriction need to operate on a case by case basis. While I do accept that unrestricted composition is independently

insufficient to beget mereological hybrids, I still think unrestricted composition (or, more properly, a composition principle strong enough to facilitate the sorts of mereological fusions under contention in this thesis, which unrestricted composition of course is) is necessary for their formation, to the effect that we can fairly point towards this composition principle as the primary 'culprit' with regards to such objectionable entities.

So instead of proposing any explicitly positive principle of composition, I have pointed towards several problematic consequences involving the mereological combination of certain dominantly considered kinds that I think may arise given the prevalent ontological commitment to them (or at least instances of them). This is in the hope that it is recognised that much more needs to be said in the case that we posit certain kinds of entities that form novel wholes in the presence of certain composition principles. To me it seems that if we are lucky, the composites we might encounter as residual artefacts of the systems we propose and the primary entities that we commit to can themselves be harmless additions to our ontology, perhaps things that we only have to consider if we are quantifying broadly or speaking in an appropriately absolute manner as to cover all that there is, or may be. I don't believe Lewis is so lucky in this regard, however.

As I have demonstrated, mixed fusions, given that they require a class-like membership system in play to even have the major parts which make them up, and a mereology to do their mixing, immediately throw up challenges to the systems in which they arise. This is by compelling questions like: how are they supposed to properly fit in? What role might they play in an ontology? What are their consequences? Why accept them after all? In response to these sorts of questions, a lot of what I have shown is that mixed fusions generally don't seem to properly fit in without us making serious compromises in other areas of metaphysics which seem to me far more important (or at least, that are treated with more urgency and concern) than having an absolutely operating composition principle – and more important especially given that these compromises are to be made in light of the attitude that mixed fusion can just be ignored, even if not denied.

With my arguments over the course of this thesis regarded as a cumulative batch of problems facing the prospects of mixed fusions understood in a variety of ways with respect to the status of some of their proper parts, I would like to go further. I think that instead of ignoring mixed fusions – which I think has led to ignoring their potential consequences – we ought to deny them instead. If, in the manner of our rejecting mixed fusions here we only lose entities that we aren't supposed to really care about anyway, it is clearly no loss on an entity front. The remaining problem, however, is that the hopeful general ontologist loses a formal system with pretensions to absolute coverage, following concerns over the consequences of its composition principle operating universally.

From what I can tell, there are multiple strategies here to take in the face of the problems I have raised, each of which looks to remove mixed fusions from the ontological menu. The most direct, of course, is to reject unrestricted composition or any principle appropriately like it which is sufficient to permit mixed fusions in the manner given throughout this thesis. While I do not have a replacement restricted principle of composition in mind, it should be noticed that certain problems with mixed fusions that I have articulated do only arise insofar as composition is able to generate a mereological Universe (as in Chapter 2). That is, mixed fusions that aren't the sum of all things can look more harmless when the focus is on troubles that the Universe brings. But if I'm right about mixed fusions inheriting the members of their class parts (as in Chapter 3), then there is a significant problem which affects *all* of them, and not just the totality whole, within the confines of the class theory that they partially stem from.

With this as a reason for an absolute restriction on the formation of mixed fusions the prospect of an abstract atomism (as in Chapter 4) which focuses solely on composition in an abstract domain of sets is ruined. This is because the situation would be such that with the null set being both an abstractum and a mereological simple it is thereby the sole singulate in that context, due to all the mixed fusions having members and by that violating the extensionality of class membership.

Even if member inheritance as a consequence is resisted, the prospects of a GEM-style mereological situation for abstracta qua sets is further challenged once the range of the composition relation employed for them expands to include concreta as well. Firstly, the prospective proper part foundationalism within the abstract atomism is thereafter less justified due to proper part foundationalism being less justifiable for concreta. This is following the possibility of concrete mereological gunk. Additionally, with oppositional foundational directions being impossible for a mereological Universe which has the proposed abstracta amalgam as a part, the most apparent options of either an implausible mixed fusion foundation or a rejection of foundations remain. Although I came to no determinate conclusion in this regard, I accept that future work could be done to figure out which foundation option is better here if this project were to be further pursued. I imagine it would also need to incorporate a response to the issue of loopwholes that I raised in Chapter 2.

Secondly, the additional problems that I argue arise with regards to the prospects of abstracta/concreta fusion in general (as in Chapter 5) also serve to undermine such a mereologising of classes qua abstracta. With the more general problems here opening up the scope of my criticism of mereological hybridism beyond issues involving membership, I still did not reach an absolute restriction in the case of cabs. Instead, once again, I argued for a restriction that was conditional,

and this time on the relational matrix surrounding abstracta, concreta and possible worlds. I definitely think it would be worth doing more work to further consider the prospects of notions like all abstracta and concreta being in a single, unified world, or if Concreta Homeomereity could be better justified to resist such a world.

The other strategies to avoid the formation of mixed fusions and some other mereological hybrids, besides directly restricting monistic universal composition, are: to restrict the field of the parthood relation (as in the domain of its applicability), to advance mereological pluralism, to reject parthood relations altogether, or to reject the existence of classes. The first of these can be done by restricting the field to a single minimal domain – such as only within a one-world view of concrete worlds, and it being a world which is class-free, perhaps leaving classes to be non-mereological entities. This is a strategy akin to that of Uzquiano (2006), who, although is not there addressing problems with mixed fusions or mereological hybrids, is still looking to avoid problems with set-theoretical violations in a ZFCU domain under GEM. Because Uzquiano's arguments show that there are significant incompatibilities with these two systems due to how membership is supposed to work in ZFCU, he suggests that "the burden of argument should be not on those who [do not] think that objects of a certain sort enter into the part–whole relation but rather on those who think that they do." (2006, 162)

I have made a similar suggestion to this in the context of the proposals for a mereological Universe in general (in my Chapter 6), by offering that category system incompatibilities need to be shown to *not* arise, rather than assume that they do not, for us to be justified in asserting the existence of such a totality fusion. However, unlike Uzquiano, I don't believe that the topic-neutrality of parthood needs to be the condition to go. This is because, with such a rejection leading to a de facto restriction on composition anyway, there is still room here to instead just restrict the problematic composition principle directly, and not do it by limiting the entire field of composition. This allows for consideration of a range of different composition principles each restricted to particular domains, and principles which do not cross categorial boundaries that result in the kinds of problems noted.

In this way a mereology which has all of the same principles as GEM except for the universality of composition could still be in place, if desired, and one such mereology for multiple, mereologically disconnected domains. Consider here the case of single-world bound GEM systems, but with an equivalent mereological system for each of many worlds. Or separate systems of GEM-concreta alongside GEM-abstracta (that aren't classes), for example. I am aware that at least McDaniel (2017) is a proponent of a mereological pluralism which might be able to operate in this

manner, and so think it would also be worth pursuing questions of mereological hybridism further in the context of non-classical mereological systems like this in future work.

Beyond having a plurality of segregated composition relations, another option to explore may be one which has some domains which are compositional and some which are compositionally nihilistic, such as for a two-realm view which has composition relations in the one concrete world, and nihilism for abstract classes. A committed parthood generalist may at least prefer this, because all things are still mereological, although it would preclude all composition among classes in that domain. Although this option is unexplored in any detail in the literature as far as I know, it is one which Lewis acknowledges when he reflects upon the fact that in his earlier work he claimed that the part-whole relation applies to individuals and *not* classes, "I might at least have granted that the part-whole relation applies to classes in a *trivial way*: even if a class has no proper parts, as I then thought, at least it should have itself as an improper part." (1991, 3)

Trivial or not, this option is still able to support the topic-neutrality of parthood, even in the cases that the generality of composition is either rejected or made domain specific. The remaining two options to avoid the formation of mixed fusions and their ilk are the strongest, and therefore likely to be more controversial than some compromising alterations or modifications which may still maintain less than maximal GEM systems, or other non-classical mereological frameworks. The first of these options is found in Cowling's (2014) case for mereological anti-realism, wherein he presents an Eleatic argument against the existence of mereological relations – essentially offering that because mereological relations are not causally active, we ought to reject the existence of any of them. The second of these options also results in no mixed fusions, but does not necessarily preclude other mereological hybrids from forming, because this option is the denial of the existence of classes altogether, which concrete nominalists, for example, are willing to do.

While both of these options remove the possibility of at least one problematic entity from the landscape, in mixed fusions, they remove a lot else besides, and so any weighing up of these stronger, rejecting positions will need to be considered against the viability of more moderate options which look only to meet the problems posed in a minimal manner. And because these two positions are normally offered for their own merits anyway, their inclusion here is mainly to note that the avoidance of bad sums can be met in many ways. This further seems to leave the prospects for a definitive answer on how best to positively, rather than negatively, avoid mixed fusions and other transcategorial sums open-ended.

Ultimately, I think the worst problem that I have posed for mixed fusions is in Chapter 3, as it comes from something essential to their class parts, and so from a mereological hybridism arising

from entities being classes or singulates, a hybridism which leads to mixed fusions having members and contravening the very class theory their parts come from. This is the worst because it also follows mixed fusions wherever they go, in or out of worlds, and for whatever else they are in addition to their necessarily being sums of individuals and classes. Furthermore, it also affects *all* mixed fusions equally – rather than only something which affects however many mixed fusions have abstract and concrete parts under certain conditions, or if a mixed fusion is a mereological Universe, or whatever. What to me serves as a further decisive indictment against mixed fusions in this regard, is that even if the property inheritance of membership that I propose in Chapter 3 is resisted, mixed fusions are then clear instances of additional ontological commitments because they are therefore irreducible to their proper parts (as in Chapter 7). Which leads to them needing to be counted distinctly if included in one's ontology, and thereby coming at an unjustified ontological cost.

With this lack of ontological innocence for mixed fusions receiving its justification due to the opposing option of mixed fusions as extensionality violating classes being resisted, the acceptance of mixed fusions looks to me to definitely come at too great a cost to bear. With options on the mereological horizon other than those which breed mixed fusions, I think that the many failures of mereological hybridism that I have presented in this thesis in the case of mixed fusions serve as a powerful motivation to look beyond the limits of the rejected classical mereology and unrestricted composition, and also beyond other views which could allow the same kinds of problems with entities, toward a more secure mereological point of view.

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