

Conceivability, Essence, and Haecceities

David Elohim*

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Abstract

This essay aims to redress the contention that epistemic possibility cannot be a guide to the principles of modal metaphysics. I introduce a novel epistemic two-dimensional truthmaker semantics. I argue that the interaction between the two-dimensional framework and the mereological parthood relation, which is super-rigid, enables epistemic possibilities and truthmakers with regard to parthood to be a guide to its metaphysical profile. I specify, further, a two-dimensional formula encoding the relation between the epistemic possibility and verification of essential properties obtaining and their metaphysical possibility or verification. I then generalize the approach to haecceitistic properties, and examine the Julius Caesar problem as a test case.

1 Introduction

In this essay, I endeavor to provide an account of how the epistemic interpretation of two-dimensional semantics can be sensitive to essential and haecceitistic properties. Let a model, M , be comprised of a set of epistemically possible worlds C ; a set of metaphysically possible worlds W ; a domain, D , of terms and formulas; binary relations defined on each of C and W ; and a valuation function mapping terms and formulas to subsets of C and W , respectively. So, $M = \langle C, W, D, R_C, R_W, V \rangle$. A term or formula is epistemically necessary or apriori iff it is inconceivable for it to be false ($\Box \iff \neg\Diamond\neg$). A term or formula is negatively conceivable iff nothing rules it out apriori ($\Diamond \iff \neg\Box\neg$). A term or formula is positively conceivable only if the term or formula can be perceptually imagined. According to the epistemic interpretation of two-dimensional semantics, the semantic value of a term or formula can then be defined relative to two parameters.¹ The first parameter ranges over the set of epistemically possible worlds, and the second parameter ranges over the set of metaphysically possible worlds. The value of the term or formula relative to the first parameter determines the value of the term or formula relative to the second parameter. Thus,

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¹See Yalcin (2007: 1001-1002) and Chalmers and Rabern (2014: 211-212).

the epistemically possible value of the term or formula constrains the metaphysically possible value of the term or formula; and so conceivability might, given the foregoing, serve as a guide to metaphysical possibility.

Roca-Royes (2011) and Chalmers (2010a; 2011; 2014) note that, on the above semantics, epistemic possibility cannot track the difference between the metaphysical modal profile of a non-essential proposition – e.g., that there is a shooting star – and the metaphysical modal profile of an essential definition, such as a theoretical identity statement – e.g., that water = H₂O. Another principle of modal metaphysics to which epistemic possibilities are purported to be insensitive is haecceity comprehension; namely, that $\Box\forall x,y\Box\exists\Phi(\Phi x \iff x = y)$.

The aim of this note is to redress the contention that epistemic possibility cannot be a guide to the principles of modal metaphysics. I will argue that the interaction between the two-dimensional framework and the mereological parthood relation, which is super-rigid, enables the epistemic possibility of parthood to be a guide to its metaphysical profile. Further, if essential properties are haecceitistic properties, then the super-rigidity of haecceitistic properties entails that the epistemic possibility of their obtaining entails the metaphysical possibility of their obtaining.

In Section **2**, I outline a novel hyperintensional, epistemic two-dimensional truthmaker semantics. In Section **3**, I examine a necessary condition on admissible cases of conceivability entailing metaphysical possibility in the two-dimensional framework, focusing on the property of super-rigidity. I argue that – despite the scarcity of properties which satisfy the super-rigidity condition – metaphysical properties such as the parthood relation do so. In Section **4**, I address objections to one dogma of the semantic rationalism underpinning the epistemic interpretation of two-dimensional semantics. The dogma states that there are criteria on the basis of which formal from informal domains, unique to the extensions of various concepts, can be distinguished, such that the modal profiles of those concepts would thus be determinate. I examine the Julius Caesar problem as a test case. I specify, then, a two-dimensional formula encoding the relation between the epistemic possibility of essential properties obtaining and its metaphysical possibility, and I generalize the approach to haecceitistic properties. In Section **5**, I address objections from the indeterminacy of ontological principles relative to the space of epistemic possibilities, and from the consistency of epistemic modal space. Section **6** provides concluding remarks.

2 Topic-Sensitive Two-Dimensional Truthmaker Semantics

Chalmers defines epistemic possibility as not being apriori ruled out (2011: 63, 66),² i.e. as the dual of epistemic necessity or apriority (65),³ $\diamond\phi$ iff $\neg\blacksquare\neg\phi$, and as being true at an epistemic scenario i.e. epistemically possible world (62, 64)⁴. I concur that epistemic possibility is the dual of epistemic necessity i.e. apriority, but argue in this paper for a novel epistemic two-dimensional truthmaker semantics which avails of hyperintensional epistemic states, i.e. epistemic truthmakers or verifiers for a proposition, which comprise a state space (Fine 2017a,b,c; Hawke and Özgün, forthcoming). Epistemic states are parts of epistemically possible worlds, rather than whole worlds themselves. Apriority is thus redefined in the hyperintensional semantics.

According to truthmaker semantics for epistemic logic, a modalized state space model is a tuple $\langle S, P, \leq, v \rangle$, where S is a non-empty set of states, i.e. parts of the elements in A in the foregoing epistemic modal algebra U , P is the subspace of possible states where states s and t comprise a fusion when $s \sqcup t \in P$, \leq is a partial order, and $v: \text{Prop} \rightarrow (2^S \times 2^S)$ assigns a bilateral proposition $\langle p^+, p^- \rangle$ to each atom $p \in \text{Prop}$ with p^+ and p^- incompatible (Hawke and Özgün, forthcoming: 10-11). Exact verification (\vdash) and exact falsification (\dashv) are recursively defined as follows (Fine, 2017a: 19; Hawke and Özgün, forthcoming: 11):

- $s \vdash p$ if $s \in \llbracket p \rrbracket^+$
(s verifies p , if s is a truthmaker for p i.e. if s is in p 's extension);
- $s \dashv p$ if $s \in \llbracket p \rrbracket^-$
(s falsifies p , if s is a falsifier for p i.e. if s is in p 's anti-extension);
- $s \vdash \neg p$ if $s \dashv p$
(s verifies not p , if s falsifies p);
- $s \dashv \neg p$ if $s \vdash p$
(s falsifies not p , if s verifies p);
- $s \vdash p \wedge q$ if $\exists v, u, v \vdash p, u \vdash q$, and $s = v \sqcup u$
(s verifies p and q , if s is the fusion of states, v and u , v verifies p , and u verifies q);
- $s \dashv p \wedge q$ if $s \dashv p$ or $s \dashv q$
(s falsifies p and q , if s falsifies p or s falsifies q);
- $s \vdash p \vee q$ if $s \vdash p$ or $s \vdash q$
(s verifies p or q , if s verifies p or s verifies q);

²One might also adopt a conception on which every proposition that is not logically contradictory is deeply epistemically possible, or on which every proposition that is not ruled out a priori is deeply epistemically possible. In this paper, I will mainly work with the latter understanding' (63). 'For example, a sentence s is deeply epistemically possible when the thought that s expresses cannot be ruled out a priori' (66).

³We can say that s is deeply epistemically necessary when s is a priori: that is when s expresses actual or potential a priori knowledge' (65).

⁴For all sentences s , s is epistemically possible iff there exists a scenario [i.e. epistemically possible world - HK] such that w verifies s ' (64), where '[w]hen w verifies s , we can say that s is true at w ' (63)

$s \dashv p \vee q$ if $\exists v, u, v \dashv p, u \dashv q$, and $s = v \sqcup u$
(s falsifies p or q, if s is the fusion of the states v and u, v falsifies p, and u falsifies q);
 $s \vdash \forall x\phi(x)$ if $\exists s_1, \dots, s_n$, with $s_1 \vdash \phi(a_1), \dots, s_n \vdash \phi(a_n)$, and $s = s_1 \sqcup \dots \sqcup s_n$
[s verifies $\forall x\phi(x)$ "if it is the fusion of verifiers of its instances $\phi(a_1), \dots, \phi(a_n)$ " (Fine, 2017c)];
 $s \dashv \forall x\phi(x)$ if $s \dashv \phi(a)$ for some individual a in a domain of individuals (op. cit.)
[s falsifies $\forall x\phi(x)$ "if it falsifies one of its instances" (op. cit.)];
 $s \vdash \exists x\phi(x)$ if $s \vdash \phi(a)$ for some individual a in a domain of individuals (op. cit.)
[s verifies $\exists x\phi(x)$ "if it verifies one of its instances $\phi(a_1), \dots, \phi(a_n)$ " (op. cit.)];
 $s \dashv \exists x\phi(x)$ if $\exists s_1, \dots, s_n$, with $s_1 \dashv \phi(a_1), \dots, s_n \dashv \phi(a_n)$, and $s = s_1 \sqcup \dots \sqcup s_n$ (op. cit.)
[s falsifies $\exists x\phi(x)$ "if it is the fusion of falsifiers of its instances" (op. cit.)];
s exactly verifies p if and only if $s \vdash p$ if $s \in \llbracket p \rrbracket$;
s inexactly verifies p if and only if $s \triangleright p$ if $\exists s' \leq s, s' \vdash p$; and
s loosely verifies p if and only if, $\forall v, s.t. s \sqcup v \vdash p$ (35-36);
 $s \vdash A\phi$ if and only if for all $u \in P$ there is a $u' \in P$ such that $u' \sqcup u \in P$ and $u' \vdash \phi$, where $A\phi$ denotes the apriority of ϕ ;
 $s \dashv A\phi$ if and only if there is a $v \in P$ such that for all $u \in P$ either $v \sqcup u \notin P$ or $u \dashv \phi$;
 $s \vdash A(A\phi)$ if and only if for all $u \in P$ there is a $u' \in P$ such that $u' \sqcup u \in P$ and $u' \vdash \phi$ and there is a $u'' \in P$ such that $u' \sqcup u'' \in P$ and $u'' \vdash \phi$;
 $s \vdash A(\forall x\phi(x))$ if and only if for all $u \in P$ there is a $u' \in P$ such that $u \vdash [u' \vdash \exists s_1, \dots, s_n, \text{ with } s_1 \vdash \phi(a_1), \dots, s_n \vdash \phi(a_n), \text{ and } u' = s_1 \sqcup \dots \sqcup s_n]$;
 $s \vdash A(\exists x\phi(x))$ if and only if or all $u \in P$ there is a $u' \in P$ such that $u \vdash [u' \vdash \phi(a)]$ for some individual a in a domain of individuals (op. cit.).

In order to account for two-dimensional indexing, we augment the model, M, with a second state space, S^* , on which we define both a new parthood relation, \leq^* , and partial function, V^* , which serves to map propositions in a domain, D, to pairs of subsets of S^* , $\{1,0\}$, i.e. the verifier and falsifier of p, such that $\llbracket p \rrbracket^+ = 1$ and $\llbracket p \rrbracket^- = 0$. Thus, $M = \langle S, S^*, D, \leq, \leq^*, V, V^* \rangle$. The two-dimensional hyperintensional profile of propositions may then be recorded by defining the value of p relative to two parameters, c,i: c ranges over subsets of S, and i ranges over subsets of S^* .

- (*) $M, s \in S, s^* \in S^* \vdash p$ iff:
(i) $\exists c_s \llbracket p \rrbracket^{c,c} = 1$ if $s \in \llbracket p \rrbracket^+$; and
(ii) $\exists i_{s^*} \llbracket p \rrbracket^{c,i} = 1$ if $s^* \in \llbracket p \rrbracket^+$

(Distinct states, s, s^* , from distinct state spaces, S, S^* , provide a multi-dimensional verification for a proposition, p, if the value of p is provided a

truthmaker by s . The value of p as verified by s determines the value of p as verified by s^*).

Epistemic (primary), subjunctive (secondary), and 2D hyperintensions can be defined as follows, where hyperintensions are functions from states to extensions, and intensions are functions from worlds to extensions. Epistemic two-dimensional truthmaker semantics receives substantial motivation by its capacity (i) to model conceivability arguments involving hyperintensional metaphysics, and (ii) to avoid the problem of mathematical omniscience entrained by intensionalism about propositions⁵:

- Epistemic Hyperintension:
 $\text{pri}(x) = \lambda s. \llbracket x \rrbracket^{s,s}$, with s a state in the epistemic state space S
- Subjunctive Hyperintension:
 $\text{sec}_{v_{\otimes}}(x) = \lambda w. \llbracket x \rrbracket^{v_{\otimes},w}$, with w a state in metaphysical state space W

In epistemic two-dimensional semantics, the value of a formula or term relative to a first parameter ranging over epistemic scenarios determines the value of the formula or term relative to a second parameter ranging over metaphysically possible worlds. The dependence is recorded by 2D-intensions. Chalmers (2006: 102) provides a conditional analysis of 2D-intensions to characterize the dependence: ‘Here, in effect, a term’s subjunctive intension depends on which epistemic possibility turns out to be actual. / This can be seen as a mapping from scenarios to subjunctive intensions, or equivalently as a mapping from (scenario, world) pairs to extensions. We can say: the two-dimensional intension of a statement S is true at (V, W) if V verifies the claim that W satisfies S . If $[A]_1$ and $[A]_2$ are canonical descriptions of V and W , we say that the two-dimensional intension is true at (V, W) if $[A]_1$ epistemically necessitates that $[A]_2$ subjunctively necessitates S . A good heuristic here is to ask “If $[A]_1$ is the case, then if $[A]_2$ had been the case, would S have been the case?”. Formally, we can say that the two-dimensional intension is true at (V, W) iff “ $\Box_1([A]_1 \rightarrow \Box_2([A]_2 \rightarrow S))$ ” is true, where “ \Box_1 ” and “ \Box_2 ” express epistemic and subjunctive necessity respectively’.

- 2D-Hyperintension:
 $2D(x) = \lambda s \lambda w \llbracket x \rrbracket^{s,w} = 1$.

Following the presentation of topic models in Berto (2018; 2019), Canavotto et al (2020), and Berto and Hawke (2021), atomic topics comprising a set of topics, T , record the hyperintensional intentional content of atomic formulas, i.e. what the atomic formulas are about at a hyperintensional level. Topic fusion is a binary operation, such that for all $x, y, z \in T$, the following properties are satisfied: idempotence ($x \oplus x = x$), commutativity ($x \oplus y = y \oplus x$), and associativity [$(x \oplus y) \oplus z = x \oplus (y \oplus z)$] (Berto, 2018: 5). Topic parthood is a partial order, \leq , defined as $\forall x, y \in T (x \leq y \iff x \oplus y = y)$ (op. cit.: 5-6).

⁵See Author (ms₁) through (ms_n) for further discussion.

Atomic topics are defined as follows: $\text{Atom}(x) \iff \neg\exists y < x$, with $<$ a strict order. Topic parthood is thus a partial ordering such that, for all $x, y, z \in T$, the following properties are satisfied: reflexivity ($x \leq x$), antisymmetry ($x \leq y \wedge y \leq x \rightarrow x = y$), and transitivity ($x \leq y \wedge y \leq z \rightarrow x \leq z$) (6). A topic frame can then be defined as $\{W, R, T, \oplus, t\}$, with t a function assigning atomic topics to atomic formulas. For formulas ϕ , atomic formulas, p, q, r (p_1, p_2, \dots), and a set of atomic topics, $\text{Ut}\phi = \{p_1, \dots, p_n\}$, the topic of ϕ , $t(\phi) = \oplus \text{Ut}\phi = t(p_1) \oplus \dots \oplus t(p_n)$ (op. cit.). Topics are hyperintensional, though not as fine-grained as syntax. Thus $t(\phi) = t(\neg\neg\phi)$, $t\phi = t(\neg\phi)$, $t(\phi \wedge \psi) = t(\phi) \oplus t(\psi) = t(\phi \vee \psi)$ (op. cit.).

If a formula is two-dimensional and the two parameters for the formula range over distinct spaces, then there won't be only one subject matter for the formula, because total subject matters are construed as sets of verifiers and falsifiers and there will be distinct verifiers and falsifiers relative to each space over which each parameter ranges. This is especially clear if one space is interpreted epistemically and another is interpreted metaphysically. Availing of topics, however, and assigning the same topics to each of the states from the distinct spaces relative to which the formula gets its value is one way of ensuring that the two-dimensional formula has a single subject matter.

The diamond and box operators can then be defined relative to topics:

$$\begin{aligned} \langle M, w \rangle \Vdash \diamond^t \phi &\text{ iff } \langle R_{w,t} \rangle(\phi) \\ \langle M, w \rangle \Vdash \square^t \phi &\text{ iff } [R_{w,t}](\phi), \text{ with} \\ \langle R_{w,t} \rangle(\phi) &:= \{w' \in W \mid t' \in T \mid R_{w,t}[w', t'] \cap \phi \neq \emptyset \text{ and } t'(\phi) \leq t(\phi)\} \\ [R_{w,t}](\phi) &:= \{w' \in W \mid t' \in T \mid R_{w,t}[w', t'] \subseteq \phi \text{ and } t'(\phi) \leq t(\phi)\}. \end{aligned}$$

We can then combine topics with truthmakers rather than worlds, thus countenancing doubly hyperintensional semantics, i.e. topic-sensitive epistemic two-dimensional truthmaker semantics:

- Topic-Sensitive Epistemic Hyperintension:

$$\text{pri}_t(x) = \lambda s \lambda t. \llbracket x \rrbracket^{s \cap t, s \cap t}$$
, with s a truthmaker from an epistemic state space.
- Topic-Sensitive Subjunctive Hyperintension:

$$\text{sec}_{v_{\oplus} \cap t}(x) = \lambda w \lambda t. \llbracket x \rrbracket^{v_{\oplus} \cap t, w \cap t}$$
, with w a truthmaker from a metaphysical state space.
- Topic-Sensitive 2D-Hyperintension:

$$2D(x) = \lambda s \lambda w \lambda t \llbracket x \rrbracket^{s \cap t, w \cap t} = 1.$$

3 Super-rigidity

Mereological parthood satisfies a crucial condition in the epistemic interpretation of two-dimensional semantics. The condition is called super-rigidity, and its significance is that, unless the semantic value for a term is super-rigid, i.e. maps

to the same extension throughout the classes of epistemic and metaphysical possibilities, the extension of the term in epistemic modal space risks diverging from the extension of the term in metaphysical modal space. Chalmers provides two other conditions for the convergence between the epistemic and metaphysical profiles of expressions. In his (2002), epistemically possible worlds are analyzed as being centered metaphysically possible worlds, such that conceivability entails metaphysical (1-)possibility. In his (2010), the epistemic and metaphysical intensions of terms for physics and consciousness are argued to coincide, such that the conceivability of physics without consciousness (i.e. zombies) entails the metaphysical possibility of physics without consciousness. Thus, the 1- and 2-intensions of an expression can converge without super-rigidity. In this paper, however, I focus just on the role of the super-rigidity condition in securing epistemic possibility as a guide to modal metaphysics. Super-rigidity ought to be replaced by the hyper-rigidity condition specified below, in hyperintensional contexts.

Chalmers defines super-rigidity thus: ‘When an expression is epistemically rigid and also metaphysically rigid (metaphysically rigid *de jure* rather than *de facto*, in the terminology of Kripke 1980), it is *super-rigid*’ (Chalmers, 2012: 239). He writes: ‘I accept Apriority/Necessity and Super-Rigid Scrutability. (Relatives of these theses play crucial roles in "The Two-Dimensional Argument against Materialism" (241). The Apriority/Necessity Thesis is defined as the ‘thesis that if a sentence S contains only super-rigid expressions, s is a priori iff S is necessary’ (468), and Super-Rigid Scrutability is defined as the ‘thesis that all truths are scrutable from super-rigid truths and indexical truths’ (474).

There appear to be only a few expressions which satisfy the super-rigidity condition. Such terms include those referring to the properties of phenomenal consciousness, to the parthood relation, and perhaps to the property of friendship (367, 374). Other candidates for super-rigidity are taken to include metaphysical terms such as ‘cause’ and ‘fundamental’; numerical terms such as ‘one’; and logical constants such as ‘ \wedge ’ (Chalmers, op. cit.).

Crucially for the purposes of this paper, there appear to be no clear counterexamples to the claim that mereological parthood is super-rigid. If this is correct, then mereological parthood in the space of epistemic modality can serve as a guide to the status of mereological parthood in metaphysical modal space. The philosophical significance of the foregoing is that it belies the contention proffered by Roca-Royes (op. cit.) and Chalmers (op. cit.) concerning the limits of conceivability-based modal epistemology. The super-rigidity of the parthood relation permits the conceivability of mereological parthood to bear on its metaphysical profile. I argue further that – supposing essential properties are haecceitistic properties, and essential and haecceitistic properties are super-rigid – the conceivability of haecceitistic properties obtaining can be a guide to the metaphysical possibility of haecceitistic properties obtaining.

In the hyperintensional setting, the super-rigidity property is replaced by a hyper-rigidity property, which is defined as follows:

(*) $M, s \in S, s^* \in S^* \vdash p$ iff:

- (i) $\forall c'_s \llbracket p \rrbracket^{c,c'} = 1$ if $s \in \llbracket p \rrbracket^+$; and
- (ii) $\forall i_{s^*} \llbracket p \rrbracket^{c,i} = 1$ if $s^* \in \llbracket p \rrbracket^+$

4 Two Dogmas of Semantic Rationalism

The tenability of the foregoing depends upon whether objections to what might be understood as the two dogmas of semantic rationalism can be circumvented.⁶

4.1 The First Dogma

The first dogma of semantic rationalism mirrors Quine's (1951) contention that one dogma of the empiricist approach is the distinction that it records between analytic and synthetic claims. The analogous dogma in the semantic rationalist setting is that a distinction can be drawn between contextual intensions – witnessed by differences in the cognitive significance of two sentences or terms which have the same extension, e.g., with $x = 2$, 'x²' and '2x' – by contrast to epistemic intensions. Chalmers (2006) delineates orthographic, linguistic, semantic, and cognitive (including conceptual) contextual intensions, and argues that they are all distinct from epistemic intensions in light of apriority figuring in the definition of the epistemic possibilities which are input to the latter functions. The distinction coincides with two interpretations of two-dimensional semantics. As noted, the epistemic interpretation of two-dimensional semantics takes the value of a formula relative to a first parameter ranging over epistemically possible worlds to determine the extension of the formula relative to a second parameter ranging over metaphysically possible worlds (cf. Chalmers, *op. cit.*). According to the metasemantic interpretation, a sentence, such as that 'water = H₂O', is metaphysically necessary, whereas assertions made about metaphysically necessary sentences record the non-ideal epistemic states of agents and are thus contingent (cf. Stalnaker, 1978, 2004). The first dogma is thus to the effect that there are distinct sets of worlds – sets of either epistemic possibilities or of contextual presuppositions, respectively – over which the first parameter ranges in the epistemic and metasemantic interpretations.

If no conditions on the distinctness between contextual and epistemic intensions can be provided, then variance in linguistic intension might adduce against the uniqueness of the epistemic intension. Because of the possible proliferation of epistemic intensions, conditions on the super-rigidity of the formulas and terms at issue might thereby not be satisfiable. The significance of the first dogma of semantic rationalism is that it guards against the collapse of epistemic and linguistic intensions, and thus the collapse of language and thought.

A defense of the first dogma of semantic rationalism might, in response, be proffered, in light of the status of higher-order distributive plural quantification in natural language semantics. Plural quantifiers are distributive, if the individuals comprising the plurality over which the quantifier ranges are conceived of singly, rather than interpreting the quantifier such that it ranges over irreducible

⁶Thanks to xx for the objections.

collections. Natural language semantics permits plural quantification into both first and second-level predicate position. However, there are no examples of plural quantification into third-level predicate position in empirical linguistics, despite that examples thereof can be readily countenanced in intended models of formal languages. As follows, higher-order plural quantification might adduce in favor of the first dogma of semantic rationalism, to the effect that linguistic and conceptual epistemic intensions can be sufficiently distinguished.

4.2 The Second Dogma

The second dogma of semantic rationalism mirrors Quine's (op. cit.) contention that another dogma of empiricism is the reduction of the meaning of a sentence to the empirical data which verifies its component expressions. The analogous dogma in the semantic rationalist setting states that individuation-conditions on concepts can be provided in order to distinguish between concepts unique to formal and informal domains. The significance of the second dogma of semantic rationalism is that whether the objects falling under a concept belong to a formal domain of inquiry will subsequently constrain its modal profile.

In the space of epistemic possibility, it is unclear, e.g., what reasons there might be to preclude implicit definitions such as that the real number of the x 's is identical to Julius Caesar (cf. Frege, 1884/1980: 56; Clark, 2007) by contrast to being identical to a unique set of rational numbers as induced via Dedekind cuts. It is similarly unclear how to distinguish, in the space of epistemic possibility, between formal and informal concepts, in order to provide a principled account of when a concept, such as the concept of 'set', can be defined via the axioms of the language in which it figures, by contrast to concepts such as 'water', where definitions for the latter might target the observational, i.e. descriptive and functional, properties thereof.

The notion of scrutability concerns 'suppositional' inferences from a base class of truths, *PQTI* – i.e. physical, phenomenal, and indexical truths and a 'that's-all' truth – which determine canonical specifications, A_{1-n} , of epistemically possible worlds, to other truths (Chalmers, 2010b: 3). Scrutability from a canonical description of an epistemically possible world i.e. scenario, characterized by the set of truths, *PQTI*, to an arbitrary sentence, fixes an epistemic intension. Chalmers (2012: 245) is explicit about this: "The intension of a sentence S (in a context) is true at a scenario w iff S is a priori scrutable from $[A]$ (in that context), where $[A]$ is a canonical specification of w (that is, one of the epistemically complete sentences in the equivalence class of w) . . . A Priori Scrutability entails that this sentence S is a priori scrutable (for me) from a canonical specification $[A]$ of my actual scenario, where $[A]$ is something along the lines of *PQTI*". However, physical, phenomenal, and indexical truths are orthogonal to truths about necessarily non-concrete objects such as abstracta.⁷ How then are the epistemic intensions for abstracta fixed? The most obvious

⁷For challenges to the indexing account of mathematical explanation, see Baker and Colyvan (2011). For more on mathematical explanation and its relation to scientific truths, see Mancosu (2008); Pincock (2012); Lange (2017); and Baron et al (2020).

maneuver would be to add mathematical truths to the scrutability base from which sentences about mathematical objects can be inferred.⁸ It is not obvious, however, which mathematical, or perhaps logical, truths would be necessary to add in order to capture all truths about formal domains. In this section, I thus provide an explanation of how formal and informal domains can be distinguished which departs from this suggestion, and where the distinction can thereby serve to determine the modal profiles of the relevant domain classes.

The concept of mereological parthood provides a borderline case. While the parthood relation can be axiomatized so as to reflect whether it is irreflexive, non-symmetric, and transitive, its status as a formal property is more elusive. The fact, e.g., that an ordinal is part of the sequence of ordinal numbers impresses as being necessary, while yet the fact that a number of musicians comprise the parts of a chamber ensemble might impress as being contingent.

The Julius Caesar problem, and the subsequent issue of whether there might be criteria for delineating formal from informal concepts in the space of epistemic modality, may receive a unified response. The ambiguity with regard to whether the parthood relation is formal – given that its relata can include both formal and informal objects – is similar to the ambiguity pertaining to the nature of real numbers. As Frege (1893/2013: 161) notes: ‘Instead of asking which properties an object must have in order to be a magnitude, one needs to ask: how must a concept be constituted in order for its extension to be a domain of magnitudes [...] a thing is a magnitude not in itself but only insofar as it belongs, with other objects, to a class that is a domain of magnitudes’. Frege defines a magnitude as the extension of a relation on arbitrary domains (op. cit.). The concept of a magnitude is then referred to as a ‘Relation’, and domains of magnitudes are defined as classes of Relations (162). Frege defines, then, the real numbers as relations on – namely, ratios of – magnitudes; and thus refers to the real numbers as ‘Relations on Relations’, because the extension of the higher-order concept of real number is taken to encompass the extension of the lower-order concept of classes of Relations, i.e., domains of magnitudes (op. cit.). The interest of Frege’s definition of the concept of real number is that explicit mention must be made therein to a domain of concrete entities to which the number is supposed, as a type of measurement, to be applied.

In response: The following implicit definitions – i.e., abstraction principles – can be provided for the concept of real number, where the real numbers are defined as sets, or Dedekind cuts, of rational numbers. Following Shapiro (2000), let F, G , and R denote rational numbers, such that concepts of the reals can be specified as follows: $\forall F, G [C(F) = C(G) \iff \forall R (F \leq R \iff G \leq R)]$. Concepts of rational numbers can themselves be obtained via an abstraction principle in which they are identified with quotients of integers – $[Q(m, n) = Q(p, q) \iff n = 0 \wedge q = 0 \vee n \neq 0 \wedge q \neq 0 \wedge m \times q = n \times p]$; concepts of the integers are obtained via an abstraction principle in which they are identified with differences of natural numbers – $[Diff(\langle x, y \rangle) = Diff(\langle z, w \rangle) \iff x + w$

⁸Chalmers (2012: 388) suggests this maneuver with regard to the problem of the scrutability of mathematical truths in general.

$= y + z]$; concepts of the naturals are obtained via an abstraction principle in which they are identified with pairs of finite cardinals – ‘ $\forall x,y,z,w[\langle x,y \rangle (=P) = \langle z,w \rangle (=P) \iff x = z \wedge y = w]$ ’; and concepts of the cardinals are obtained via Hume’s Principle, to the effect that cardinals are identical if and only if they are equinumerous – $\forall \mathbf{A} \forall \mathbf{B} [[N_{\mathbf{x}}: \mathbf{A} = N_{\mathbf{x}}: \mathbf{B} \equiv \exists R[\forall x[\mathbf{A}x \rightarrow \exists y(\mathbf{B}y \wedge Rxy \wedge \forall z(\mathbf{B}z \wedge Rxz \rightarrow y = z))] \wedge \forall y[\mathbf{B}y \rightarrow \exists x(\mathbf{A}x \wedge Rxy \wedge \forall z(\mathbf{A}z \wedge Rzy \rightarrow x = z)]]]]]$.

Frege notes that ‘we can never [...] decide by means of [implicit] definitions whether any concept has the number Julius Caesar belonging to it, or whether that same familiar conqueror of Gaul is a number or not’ (1884/1980: 56). A programmatic line of response endeavors to redress the Julius Caesar problem by appealing to sortal concepts, where it is an essential property of objects that they fall in the extension of the concept (cf. Hale and Wright, 2001: 389, 395). In order further to develop the account, I propose to avail of recent work in which identity conditions are interpreted so as to reflect relations of essence and explanatory ground. The role of the essentiality operator will be to record a formal constraint on when an object falls under a concept ‘in virtue of the nature of the object’ (Fine, 1995: 241-242). The role of the grounding operator will be to record a condition on when two objects are the same, entraining a hyperintensional type of implicit definition for concepts which is thus finer-grained and less susceptible to error through misidentification.

In his (2015a), Fine treats identity criteria as generic statements of ground. By contrast to **material** identity conditions which specify when two objects are identical, **riterial** identity conditions explain in virtue of what the two objects are the same. Arbitrary, or generic, objects are then argued to be constitutive of criterial identity conditions. Let a model, M , for a first-order language, L , be a tuple, where $M = \langle I, A, R, V \rangle$, with I a domain of concrete and abstract individuals, A a domain of arbitrary objects, R a dependence relation on arbitrary objects, and V a non-empty set of partial functions from A to I (cf. Fine, 1985). The arbitrary objects in A are reified variables. The dependence relation between any a and b in A can be interpreted as a relation of ontological dependence (op. cit.: 59-60). Informally, from $a \in A$ s.t. $F(a)$, one can infer $\forall x.F(x)$ and $\exists x.F(x)$, respectively (57). Then, given two arbitrary objects, x and y , with an individual i in their range, ‘ $[(x = i \wedge y = i) \rightarrow x = y]$ ’, such that x and y mapping to a common individual explains in virtue of what they are the same (Fine, 2015b).

Abstraction principles for, e.g., the notion of set, as augmented so as to record distinctions pertaining to essence and ground, can then be specified as follows:

- Given x,y , with $\text{Set}(x) \wedge \text{Set}(y)$: $[\forall z(z \in x \equiv z \in y) \leftarrow_{x,y} (x = y)]$

(Intuitively, where the ‘given’ expression is a quantifier ranging over the domain of variables-as-arbitrary objects: Given x, y , whose values are sets, it is essential to x and y being the same that they share the same members); and

- Given x,y , with $\text{Set}(x) \wedge \text{Set}(y)$: $[\forall z(z \in x \equiv z \in y) \rightarrow_{x,y} (x = y)]$

(Intuitively: Given arbitrary objects, x, y , whose values are sets, the fact that x and y share the same members grounds the fact that they are the same).

Combining both of the above directions yields the following hyperintensional, possibly asymmetric, biconditional:

- Given x, y , with $\text{Set}(x) \wedge \text{Set}(y)$: $[\forall z(z \in x \equiv z \in y) \leftrightarrow_{x,y} (x = y)]$.

A reply to the Julius Caesar problem for real numbers might then avail of the foregoing metaphysical implicit definitions, such that the definition would record the essentiality to the reals of the property of being necessarily non-concrete as well as provide a grounding-condition:

- Given F, G [$\mathbf{C}(F) = \mathbf{C}(G) \leftrightarrow_{F,G} \forall R(F \leq R \iff G \leq R)$], and
- $\Box \forall X_{X/F} \Box \exists Y [-\mathbf{C}(Y) \wedge \Box (X = Y)]$

(Intuitively: Given arbitrary objects, F, G , whose values are the real numbers: It is essential to the F 's and the G 's that the concept of the F s is identical to the concept of the G 's iff (i) F and G are identical subsets of a limit rational number, R , and (ii) with $\mathbf{C}(x)$ a concreteness predicate, necessarily for all real numbers, X , necessarily there is a non-concrete object Y , to which necessarily X is identical; i.e., the reals are necessarily non-concrete. The foregoing is conversely the ground of the identification.)⁹

Heck (2011: 129) notes that the Caesar problem incorporates an epistemological objection: 'Thus, one might think, there must be more to our apprehension of numbers than a mere recognition that they are the references of expressions governed by HP [Hume's Principle – HK]. Any complete account of our apprehension of numbers as objects must include an account of what distinguishes people from numbers. But HP alone yields no such explanation. That is why Frege writes: "Naturally, no one is going to confuse [Caesar] with the [number zero]; but that is no thanks to our definition of [number]" (Gl, §62)'

The condition of being necessarily non-concrete in the metaphysical definition for real numbers, as well as the conditions of essence and ground therein, provide a reply to the foregoing epistemological objection, i.e. the required explanation, by contrast to the insufficient definition, of what distinguishes people from numbers.

⁹Rosen and Yablo (2020) also avail of real, or essential, definitions in their attempt to solve the Caesar problem, although their real definitions do not target grounding-conditions. The need for a grounding-condition is mentioned in Wright (2020: 314, 318). The approach here developed, of solving the Caesar problem by availing of metaphysical definitions, was arrived at independently of Rosen and Yablo (op.cit.) and Wright (op. cit.). The examination of the relation between abstraction principles and grounding, though not essence, has been pursued by Rosen (2010); Schwartzkopff (2011); Donaldson (2017); and De Florio and Zanetti (2020). Mount (2017: ch. 5) examines the relations between essence and number and grounding and number separately. The approach here developed is novel in examining metaphysical definitions which incorporate conditions on both essentiality and grounding.

4.3 Mereological Parthood

The above proposal can then be generalized, in order to countenance the abstract profile of the mereological parthood relation. By augmenting the axioms for parthood in, e.g., classical mereological parthood with a clause to the effect that it is essential to the parthood relation that it is necessarily non-concrete, parthood can thus be understood to be abstract; and truths in which the relation figures would thereby be necessary.

- Given x : $\Phi(x) \wedge \Box \forall x \Box \exists y [\neg C(y) \wedge \Box(x = y)] \leftrightarrow_x \Gamma(x)$ where
- $\Gamma(x) := x$ is the parthood relation, $<$, which is irreflexive, asymmetric, and transitive, and where the relation satisfies the axioms of classical extensional mereology codified by the predicate, $\Phi(x)$ (cf. Cotnoir, 2014):
Weak Supplementation: $x < y \rightarrow \exists z[(z < y \vee z = y) \wedge \neg \exists w[(w < z \vee w = z) \wedge (w < x \vee w = x)]]$, and
Unrestricted Fusion: $\forall xx \exists y[F(y, xx)]$,
with the axiom of Fusion defined as follows:
Fusion: $F(t, xx) := (xx < t \vee xx = t) \wedge \forall y[(y < t \vee y = t) \rightarrow (y < xx \vee y = xx)]$

As with sets, members of which can be concrete yet for which membership is necessary, fusions are themselves abstracta, formed by a fusion-abstraction principle. The abstraction principle states that two singular terms – in which an abstraction operator, σ , from pluralities to fusions figures as a term – are identical, if and only if the fusions overlap the same locations (cf. Cotnoir, ms). Let a topological model be a tuple, comprised of a set of points in topological space, μ ; a domain of individuals, D ; an accessibility relation, R ; and a valuation function, V , assigning distributive pluralities of individuals in D to subsets of μ :

$$M = \langle \mu, D, R, V \rangle;$$

$R = R_{xx,yy}{}_{xx,yy \in \mu}$ iff $R_{xx} \subseteq \mu_{xx} \times \mu_{xx}$, s.t. if $R_{xx,yy}$, then $\exists o \subseteq \mu$, with $xx \in o$ s.t. $\forall yy \in o R_{xx,yy}$, where the set of points accessible from a privileged node in the space is said to be open; and $V = f(ii \in D, m \in \mu)$.¹⁰ Necessity is interpreted as an interiority operator on the space:

$$M, xx \Vdash \Box \phi \text{ iff } \exists o \subseteq \mu, \text{ with } xx \in o, \text{ such that } \forall yy \in o M, yy \Vdash \phi.$$

The following fusion abstraction principle can then be specified:

$$\text{Given } xx, yy, F[\sigma(xx, F) = \sigma(yy, F) \leftrightarrow_{xx, yy} [f(xx, m_1) \cap f(yy, m_1) (\neq \emptyset)]].$$

(Intuitively, given arbitrary objects whose values are the pluralities, xx, yy : It is essential to xx and yy that fusion-abstracts – formed by mapping the pluralities to the abstracta – are identical, because the fusions overlap the same nonstationary – i.e., $\neq \emptyset$ – locations. The converse is the determinative ground of the identification.)

The foregoing constraints on the formality of the parthood relation – both being necessarily non-concrete and figuring in pluralities which serve to individuate fusions as abstract objects – are sufficient then for redressing the objections

¹⁰ μ is further Alexandrov; i.e., closed under arbitrary unions and intersections.

to the second dogma of semantic rationalism; i.e., that individuation-conditions are wanting for concepts unique to formal and informal domains, which would subsequently render the modal profile of such concepts indeterminate. That relations of mereological parthood are abstract adduces in favor of the claim that the values taken by the relation are necessary. The significance of both the necessity of the parthood relation, as well as its being abstract rather than concrete, and thus being in some sense *apriori*, is that there are thus compelling grounds for taking the relation to be super-rigid, i.e., to be both epistemically and metaphysically necessary.

Finally, a third issue, related to the dogma is that, following Dummett (1963/1978: 195-196), the concept of mereological parthood might be taken to exhibit a type of ‘inherent vagueness’, in virtue of being indefinitely extensible. Dummett (1996: 441) defines an indefinitely extensible concept as being such that: ‘if we can form a definite conception of a totality all of whose members fall under the concept, we can, by reference to that totality, characterize a larger totality all of whose members fall under it’. It will thus be always possible to increase the size of the domain of elements over which one quantifies, in virtue of the nature of the concept at issue; e.g., the concept of ordinal number is such that ordinals can continue to be generated, despite the endeavor to quantify over a complete domain, in virtue of iterated applications of the successor relation, and the concept of real number is such that the reals can continue to be generated via elementary embeddings. Bernays’ (1942) theorem states that class-valued functions from classes to sub-classes are not onto, where classes are non-sets (cf. Uzquiano, 2015a: 186-187). A generalization of Bernays’ theorem can be recorded in plural set theory,¹¹ where the cardinality of the sub-pluralities of an incipient plurality will always be greater than the size of that incipient plurality. If one takes the cardinal height of the cumulative hierarchy to be fixed, then one way of tracking the variance in the cardinal size falling in the extension of the concept of mereological parthood might be by redefining the intension thereof (Uzquiano, 2015b). Because it would always be possible to reinterpret the concept’s intension in order to track the increase in the size of the plural universe, the intension of the concept would subsequently be non-rigid; and the concept would thus no longer be super-rigid.

One way in which the objection might be countered is by construing the variance in the intension of the concept of parthood as tracking interpretational modalities, rather than alethic modal properties. Fine (2005: 547) has a similar approach to the concept of set, and writes: ‘On the usual conception of the cumulative hierarchy of ZF, we think of the membership predicate as given and of the ontology of sets or classes as something to be made out. Thus given an understanding of membership, we successively carve out the ontology of sets by using the membership-predicate to specify which further sets should be added to those that are already taken to exist. Under the present approach, by contrast, we think of the ontology of classes as given and of the membership

¹¹See Burgess (2004/2008), for an axiomatization of ‘Boolos-Bernays’ plural set theory, so named after the contributions of Bernays (*op. cit.*) and Boolos (1984, 1985). See Linnebo (2007), for critical discussion.

predicate as something to be made out. Thus given an understanding of the ontology of classes, we successively carve out extensions of the membership-predicate by using conditions on the domain of classes to specify which further membership-relationships should obtain. Then, the parthood relation can be necessary while satisfying full S5 – i.e., modal axioms K $[(\Box\phi \rightarrow \psi) \rightarrow (\Box\phi \rightarrow \Box\psi)]$, T $(\Box\phi \rightarrow \phi)$, and E $(\neg\Box\phi \rightarrow \Box\neg\Box\phi)$ – despite that there can be variations in the size of the quantifier domains over which the relation and its concept are defined. Let \uparrow be an intensional parameter which indexes and stores the relevant formulas at issue to a particular world (cf. Vlach, 1973). The \downarrow -symbol is an operator which serves to retrieve, as it were, that indexed information. These arrow-operators are referred to as Vlach-operators. Adding Vlach-operators is then akin to multiple-indexing: The value of a formula, as indexed to a particular world, will then constrain the value of that formula, as indexed – via the addition of the new arrows – to different worlds. Interpreting the operators interpretationally permits there to be multiple-indexing in the array of intensional parameters relative to which a formula gets its value, while the underlying logic for metaphysical modal operators can be S5, partitioning the space of worlds into equivalence classes. Formally:

$$\uparrow_1 \forall x \exists \phi \uparrow_2 \exists y [\phi(x) \downarrow_1 \wedge \phi(y) \downarrow_2].$$

The clause states that, relative to a first interpretational parameter in which all of the x 's satisfying the parthood predicate are quantified over, there is – relative to a distinct interpretational parameter – another element which satisfies that predicate. Crucially, differences in the interpretational indices, as availed of in order to record variance in the size of the cumulative hierarchy of elements falling in the range of the parthood relation, is yet consistent with the cardinality of the elements in the domain falling in the range of the relation being fixed, such that the valuation of the relation can yet be metaphysically necessary.

4.4 Summary

In this section, I addressed objections to a dogma of the semantic rationalism underpinning the epistemic interpretation of two-dimensional semantics. In response to the objections to the dogma – according to which criteria on distinguishing formal from informal domains unique to the extensions of various concepts are lacking, which subsequently engenders indeterminacy with regard to the modal profiles of those concepts – I availed of generic criterial identity conditions, in which it is essential to identical arbitrary representatives of objects that they satisfy equivalence relations which are conversely ground-theoretically determinative of the identification, and further essential thereto that they satisfy the predicate of being necessarily non-concrete. The extensions of indefinitely extensible concepts can further be redefined relative to distinct temporal intensional parameters, despite that the background modal logic for the intensions of the concepts partitions the domain of worlds into equivalence classes, and thus satisfies S5. Thus, parthood can be deemed a necessary, because abstract, relation, despite (i) temporal variance in the particular objects on which the parthood relation is defined; and (ii) variance in the cardinality of the domain

in which those objects figure, relative to which the concept's intensions are defined.

My strategy in what follows will be to provide two-dimensional formulas for essential properties. The first dimension is interpreted epistemically and the second dimension is interpreted metaphysically. Then, supposing essential properties are haecceitistic properties (see Korbmacher, 2016), I will generalize the formula to account for the interaction between epistemic and metaphysical profiles of haecceities.

Suppose that essential properties either are super-rigid or ground super-rigidity.¹² Following Fine (2000), suppose there is an operator, \Box_F , where $\Box_F A$ is read 'it is true in virtue of the nature of the nature of (some or all) of the F's that A' where 'each of the objects mentioned in A is involved in the nature of one of the F's' (op. cit.: 543). \Box_F satisfies the axioms KTE and necessitation:

$$\begin{aligned} & \Box_F A \rightarrow A, \\ & \Box_F (A \rightarrow B) \rightarrow (\Box_F A \rightarrow \Box_F B), \\ & \neg \Box_F A \rightarrow \Box_{F,|A|} \neg \Box_F A, \text{ F rigid, where} \end{aligned}$$

F is rigid "if it is a rigid predicate symbol or is of the form $\lambda x \bigvee_{1 \leq i \leq n} A_i$, $n \geq 0$, where each formula A_i , $i = 1, \dots, n$, is either of the form Px or of the form $x = y$ for some variable y distinct from x " (545), and

$|E|$ stands for $\lambda x(x\eta E)$ x the first variable not free in E , where $x\eta E$ stands for $\bigvee_{1 \leq i \leq m} x = x_i \vee \bigvee_{1 \leq i \leq m} P_i x$,

$$\begin{aligned} & A \vdash \Box_{|A|} A, \text{ and} \\ & F \subset G \rightarrow (\Box_F A \rightarrow \Box_G A) \end{aligned} \quad (546).$$

A model M is a quadruple $\langle W, I, \preceq, \phi \rangle$, where

W is a non-empty set of worlds, I is a function taking each $w \in W$ into a non-empty set of individuals I_w , \preceq is a reflexive transitive dependence relation on $\bigcup_{w \in W} I_w$ with respect to which each world is closed ($a \in I_w$ and $a \preceq b$ implies $b \in I_w$), and ϕ is a valuation function taking each constant a into an individual $\phi(a)$ of some I_w ($w \in W$), each rigid predicate symbol H into a subset $\phi(H)$ of some I_w , and each world w and pure n -place predicate symbol F into a set $\phi(F, w)$ of n -tuples of I_w , where a pure predicate involves no reference to any object (544, 547-548).

For a subset J of $\bigcup I_w$, the closure $c(J)$ of J in M is $\{b: a \preceq b \text{ for some } a \in J\}$ (548).

M is a model with E a sentence or closed predicate whose constants are a_1, \dots, a_m and whose rigid predicate symbols are P_1, \dots, P_n (op. cit.). The objectual content $[E]^M$ of E in M is then $\{\phi(a_1), \dots, \phi(a_m)\} \cup \{\phi(P_1), \dots, \phi(P_n)\}$ and E is defined in M at $w \in W$ if $[E]^M \subseteq I_w$ (op. cit.).

Then the semantics for \Box_F can be defined as follows:

$w \Vdash \Box_F A$ iff (i) $[A]^M \subseteq c(F_w)$, and (ii) $v \Vdash A$ whenever $I_v \supseteq F_w$, where F_w is $\phi(w, F)$ (op. cit.).

\Box_F can be defined relative two parameters, the first ranging over epistemically possible worlds or truthmakers considered as actual, and the second

¹²See Fine (1994), for the locus classicus of accounts according to which essence grounds metaphysical necessity.

ranging over metaphysically possible worlds or truthmakers, such that the conceivability of it being true in virtue of the nature of the nature of (some or all) of the F's that A entails the metaphysical possibility or verification of it being true in virtue of the nature of the nature of (some or all) of the F's that A:

$$\forall c \in C, w \in W [\Box_F A]^{c,w} = 1 \text{ iff } \exists c' \in C, w' \in W [\Box_F A]^{c',w'} = 1.$$

Korbmacher (2016) argues that essential properties are haecceitistic properties. If so, then the following two-dimensional formula can be specified. If it is epistemically possible that Φx , then it is metaphysically possible that Φx . Formally:

$$\forall c \in C, w \in W [\Phi x]^{c,w} = 1 \text{ iff } \exists c' \in C, w' \in W [\Phi x]^{c',w'} = 1.$$

Thus, the epistemic possibility of haecceity comprehension constrains the value of the metaphysical possibility of haecceity comprehension, and – in response to Roca-Royes and Chalmers – there is a case according to which conceivability is a guide to a principle of modal metaphysics.

Conceivability is not a fail-safe method of alighting upon haecceities or essential properties. However, evidence about the haecceitistic or essential properties of objects can play a role in ascertaining which of a number of epistemic possibilities or truthmakers is actual. The epistemic two-dimensional method countenanced in the foregoing is such that – because haecceities and essential properties either are super-rigid or entail super-rigidity – epistemic truthmakers or possibilities about essential properties considered as actual will determine the values of their metaphysical truthmakers or possibilities. An accidental property might mistakenly be thought to be essential, in which case conceivability would not be an adequate guide to metaphysical verification or possibility. However, once essential properties are discovered in the actual world, the actuality of the epistemic verification or possibility thereof can serve as a guide to their metaphysical verification or possibility. Another way that evidence might bear on the actuality of epistemic truthmakers is via the role of apriori scrutability in defining primary intensions. Chalmers writes that '[t]he primary intension of [a sentence] S is true at a scenario [i.e. epistemically possible world] w iff [A] epistemically necessitates S, where [A] is a canonical specification of w', where '[A] epistemically necessitates S iff a conditional of the form "[A] \rightarrow S" is apriori' and the apriori entailment is the relation of scrutability (Chalmers, 2006; see also 2012: 245, quoted above). Because physical, phenomenal, and indexical truths are built into the scrutability base, and scrutability plays a central role in the definition of primary intensions, there is thus at least one viable route to the epistemology of essence via conceivability as constrained by actual evidence.

5 Concluding Remarks

One of the primary objections to accounting for the relationship between conceivability and metaphysical possibility via the epistemic interpretation of two-dimensional semantics is that epistemic possibilities are purportedly insensitive to modal metaphysical propositions, concerning, e.g., parthood and the haecceitistic properties of individuals. In this paper, I have endeavored to redress

the foregoing objection. In virtue of the super-rigidity of the parthood relation and essential properties, conceivability can thus serve as a guide to to principles of modal metaphysics.

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