Idle Questions

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Abstract

In light of the problem of logical omniscience, some scholars have argued that belief is *question-sensitive*: agents don't simply believe propositions but rather believe answers to questions. Hoek (2022) has recently developed a version of this approach on which a belief state is a "web" of questions and answers. Here, we present several challenges to Hoek's question-sensitive account of belief. First, Hoek's account is prone to very similar logical omniscience problems as those he claims to address. Second, the link between belief and action he proposes is too rigid. We close by sketching a generalization of the account that can meet these challenges.

1 Introduction

The classical picture of belief due to Hintikka (1962) represents an agent's belief state as a set of possible worlds—intuitively, those worlds compatible with what they believe. An agent believes a proposition just in case it is true at every world in this set. This picture has the advantage that it can capture the systematicity and holistic nature of belief as well as relate the contents of an agent's beliefs to rational action and communication.

Yet, the classical picture also faces the well-known problem of logical omniscience. In particular, it predicts that belief is closed under logical consequence: if ϕ entails ψ, then an agent who believes ϕ must also believe ψ. Of course, this is highly unrealistic. Most agents don't, it seems, believe every logical consequence of what they believe.

There have been a large number of approaches to this problem.[1] Recently, the *question-sensitive* approach has gained some traction (Koralus and Mascarenhas, 2013; Pérez Carballo, 2016; Yalcin, 2018; Hoek, 2022).[2] Its main in-

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[1]See, e.g., Hintikka 1975; Stalnaker 1976a,b, 1984; Duc 1997; Alechina et al. 2004; Berto 2010; Ripley 2012; Bjerring 2013; Jago 2007, 2014a, 2015; Bjerring and Schwarz 2017; Bjerring and Skipper 2019; Hawke et al. 2019; Skipper and Bjerring 2020; Elga and Rayo 2021; Soysal 2022.

[2]Schaffer (2007) employs a question-sensitive approach to knowledge to analyze knowledge-wh attributions ("I know whether it is raining"). Berto and Hawke (2021) employ a topic-sensitive approach to knowability (topics being closely related to questions). Beddor and Goldstein (forthcoming) develop a question-sensitive approach to intentionality.

novation is to model belief not as a mere set of worlds, but rather as a partition, or a set of partitions, on worlds, which are commonly interpreted as questions (Hamblin, 1958, 1973; Lewis, 1988). It is postulated that (1) agents only believe propositions *relative to a question*, and (2) they believe a proposition relative to a question only if the proposition is at least a partial answer to that question. In this sense, beliefs are "sensitive" to questions. This means, among other things, that agents only believe a logical consequence of their beliefs if that consequence answers a question that the agent is sensitive to.

Recently, Hoek (2022) has defended a version of this approach on which a belief state is a "web" of questions and answers meeting certain constraints. Hoek argues that this model has two advantages over the classical picture of belief. First, it provides a better account of the connection between belief and action. Second, it explains not only how agents can fail to be logically omniscient, but also why deduction is useful given that it doesn't involve acquiring any new information.

While Hoek's account has many merits, in this paper, we present several challenges that suggest it doesn't achieve its intended goals. In particular, we argue that, first, Hoek's model of belief is prone to very similar logical omniscience problems as those he claims to address, and second, the link between belief and action he proposes is too rigid.

Here's a brief outline. In §2, we give an overview of Hoek's question-sensitive account of belief. In §3, we argue that Hoek's model of belief entails several problematic closure principles. In §4, we raise a general worry for Hoek's inquisitive belief-action principle and suggest a way towards a more promising account of belief.

2   An overview of Hoek's account

Hoek's account can be split into two parts. The first part is a formal model of belief in terms of a set of partitions obeying certain constraints. The second part is a general principle concerning how belief relates to action. In this section, we give a brief overview of both parts.

2.1   Modeling belief

To explain Hoek's question-sensitive model of belief, we first need to review some ideas from the semantics of questions. Given a background set of worlds W, a *proposition* over W is a set of worlds from W, i.e., a subset of W. A *partition* over W is a set of propositions $\pi \subseteq \wp W$ such that (i) $\varnothing \notin \pi$, (ii) $\bigcup \pi = W$, and (iii) $A \cap B = \varnothing$ for all $A, B \in \pi$. The members of π are called *cells* of π, or π-*cells*.

As a first pass, we can model a *question* as a partition on the set of possible worlds (Hamblin, 1958, 1973; Groenendijk and Stokhof, 1984). Intuitively, a partition represents the exhaustive set of complete answers to the question. An *answer* to a question Q is any proposition $A \subseteq W$ that can be obtained by

taking the union of cells of the partition, i.e., $A = \bigcup X$ for some $X \subseteq Q$.[3] (A *complete answer* to Q is a cell of Q; a *partial answer* to Q is the union of two or more cells of Q.) For example, the question *Who came to the party?* is modeled as a partition where any two worlds in the same cell agree on who came to the party. The answer *Sam came to the party* is modeled as the set of worlds where Sam came to the party, which can be obtained by taking the union of the cells with worlds where Sam came to the party.

On Hoek's account, the content of a belief is modeled as a pair $\langle Q, A \rangle$, written as A^Q, of a question Q and an answer A to Q, also called a *quizposition*. An agent's belief state is modeled as a set of quizpositions satisfying certain constraints. To explain these constraints, we need to take a detour into what Hoek calls "quizpositional mereology", i.e., an exploration of the relations of containment, or "parthood", amongst quizpositions.

A question R is a *part of* a question Q, which we'll write as $R \leq Q$, if every R-cell is a union of Q-cells, i.e., every complete answer to Q entails a complete answer to R. For example, the question *What's the month?* is part of the question *What's the date?* since a complete answer to the latter determines a complete answer to the former. A quizposition B^R is a *part of* a quizposition A^Q, which we'll also write as $B^R \leq A^Q$, if R is a part of Q and A entails B, i.e., $A \subseteq B$. Thus, the quizposition $\langle$*What's the month?*, *It's July*$\rangle$ is a part of $\langle$*What's the date?*, *It's July 5, 2022*$\rangle$.

The first constraint Hoek places on belief states is that they must be closed under parthood: if an agent believes A^Q, and $B^R \leq A^Q$, then the agent also believes B^R. For example, an agent who believes *It's July 5, 2022* relative to the question *What's the date?* also believes *It's July* relative to the question *What's the month?*.

The second constraint is that belief states are partially closed under conjunction: if an agent believes A^Q and B^R, where $R \leq Q$, then the agent also believes AB^Q (where AB^Q is short for $(A \cap B)^Q$). For example, an agent who believes *It's 2022* relative to *What's the date?* and believes *It's July* relative to *What's the month?* also believes *It's July, 2022* relative to *What's the date?*.

Putting this together, Hoek defines an *inquisitive information state* as a set of quizpositions I satisfying the following constraints:

1. *Closure under Parthood:* if $A^Q \in I$ and $B^R \leq A^Q$, then $B^R \in I$;

2. *Partial Closure under Conjunction:* if $A^Q, B^R \in I$ and $R \leq Q$, then $AB^Q \in I$.

An agent *believes A relative to Q* if A^Q is in that agent's inquisitive information state. For brevity, we say an agent *believes A* (simpliciter) if they believe A relative to the question $A? = \{A, \overline{A}\}$.

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[3]Technically, Hoek defines an answer to a question to be a set of cells, rather than the union of cells. For our purposes, this difference doesn't matter. We simply define answers as propositions for ease of formal exposition (so that we don't have to write "$\bigcup A$" every time we want to discuss the proposition that A represents).

On this model, an agent's beliefs may fail to be closed under consequence. For example, Chip may believe that the CVV of his debit card is 107 (relative to *Is the CVV 107?*) while failing to believe that the CVV is prime (relative to *Is the CVV prime?*) even though 107 is prime.[4]

2.2   Belief and Action

According to the classical account, an agent who believes a proposition is disposed to, in some sense, act on that information. But, as Hoek points out, an agent may act on a piece of information only in some circumstances. Here's an example Hoek uses:

> *Romeo Recall:* Juliet comes home to find a note saying "Somebody called for you—didn't catch a name but he sounded upset." There is a phone number below it, but the beginning is smudged and Juliet can only read the last digits "6300". She instantly recognises Romeo's number, and decides to go see him. When no one answers the door, she rushes to a phone booth. She dials 2-1-2-5-2-9-... only to realize she doesn't remember the final four digits. (p. 114)

From a classical perspective, it is hard to determine whether Juliet believes Romeo's number ends in -6300. On the one hand, she acts on that information when she reads the note. So, from a classical perspective, it seems as though she believes Romeo's number ends in -6300. On the other hand, she fails to act on that information when she is in the phone booth. So it seems as though she doesn't believe Romeo's number ends in -6300.

Hoek uses question-sensitivity to explain such cases. As Hoek notes, there is a close connection between decision problems and questions. Generally, decision theorists represent decisions in terms of payoff matrices, where the columns are states of the world, the rows are options the agent can choose from, and the cells contain the utility the agent would receive in that state were they to choose that option. Hoek proposes thinking of these states (i.e., the columns) as *answers* to the questions that the decision problem *raises*. An agent is only disposed to act on the propositions they believe relative to the questions that are raised by the decisions they face.

Let's illustrate with the *Romeo Recall* example. When Juliet reads the note, she must decide whether to go see Romeo. In making this decision, she faces the question: *Was it Romeo who called?*.[5] If it was, she should find a phone booth and call him, since he seemed upset. If it wasn't, she should stay home

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[4]It's easy to verify that $\{\langle R, B \rangle \mid R \leq \text{\textit{Is the CVV 107?}} \text{ and } \text{\textit{The CVV is 107}} \subseteq B\}$ satisfies Closure under Parthood and Partial Closure under Conjunction. Since the question *Is the CVV prime?* isn't part of *Is the CVV 107?* (for example, the answer "no" to the former doesn't entail a yes-answer or a no-answer to the latter), the quizposition $\langle \text{\textit{Is the CVV prime?}}, \text{\textit{The CVV is prime}} \rangle$ isn't in this set.

[5]Hoek says the question Juliet faces at home is *Whose number ends in -6300?*. But this seems to conflict with his account of what it is for a decision problem to raise a question (p. 123). Let w_1 be a world where Romeo's number ends in -6300 but someone else whose number also happens to end in -6300 called Juliet. Let w_2 be a world that agrees with w_1 on

given the inconvenience of rushing to the phone booth over nothing. We might represent this decision in a payoff matrix like the one below.

	Romeo called	Romeo didn't call
Go to phone booth	5	-1
Stay home	-20	0

Now, the question *Was it Romeo who called?* is distinct from the question *Does Romeo's number end in -6300?*—after all, the call could have come from another number that coincidentally ends in -6300. But, for the sake of illustration, let's suppose that no one else has a phone number ending in the same four digits as Romeo and that Juliet knows this. Then given her other beliefs (e.g., that someone whose number ends in -6300 called), we can replace the question *Was it Romeo who called?* with the question *Does Romeo's number end in -6300?* for the purposes of this decision. Relative to this question, Juliet believes the answer is yes. Later, in the phone booth, Juliet is faced with a very different decision: she must decide which number to call. To determine which option is best, she must address the question: *What is Romeo's number?*. Relative to this question, she believes his number starts with 212-529, but she doesn't believe Romeo's number ends in -6300. This, for Hoek, explains why Juliet acts on her belief when she sees the note but not when she's in the phone booth: Juliet faces different questions in each case. In the first, she faces the question *Does Romeo's number end in -6300?*, whereas in the second, she faces the question *What is Romeo's number?*.

Hoek spells this out more formally as follows. An *option* is a function $\mathbf{a}\colon W \to \mathbb{R}$ from worlds to utility values. A *decision problem* Δ is a finite set of options. We say Δ *raises* a question Q if every option assigns a constant utility to worlds in the same Q-cell: for all $\mathbf{a} \in \Delta$, all $q \in Q$, and all $w, v \in q$, $\mathbf{a}(w) = \mathbf{a}(v)$. Given a proposition A, we say an option $\mathbf{a}$ *(strictly) A-dominates* an option $\mathbf{b}$ if $\mathbf{a}(w) > \mathbf{b}(w)$ for all $w \in A$. Given A is an answer to Q, we say $\mathbf{a}$ *strictly A^Q-dominates* $\mathbf{b}$ if $\mathbf{a}$ strictly A-dominates $\mathbf{b}$ in any decision problem raising Q.

With this terminology, Hoek states his account of how belief relates to action as follows (p. 125):

> **Inquisitive Belief-Action Principle (IBAP).** A belief that A^Q manifests in action as a disposition to forego A^Q-dominated options in any decision problem that raises Q.

Now, just because an agent is disposed to forego A^Q-dominated options, it doesn't immediately follow that the agent believes A^Q. To bridge this gap, Hoek postulates an additional principle (p. 130):

> **(Quacks like a) Duck Principle (DP).** If an agent has the behavioral dispositions that are associated with a belief with a certain

whose number ends in -6300 but where it was Romeo who called. If this decision problem is to raise the question *Whose number ends in -6300?*, then Juliet's option to stay home (say) must assign the same utilities to w_1 and w_2, which seems incorrect here.

content, and moreover that agent has those dispositions in virtue of their beliefs, then that agent actually has a belief with that content.

Together, IBAP and DP give us a way to infer what a person believes from their dispositions for action:

> **Hoek's Principle (HP).** An agent believes A^Q iff they are disposed (in virtue of their beliefs[6]) to forego A^Q-dominated options in any decision problem that raises Q.[7]

So because Juliet is disposed to act on the information that Romeo's number ends in -6300 in decision problems raising the question *Does Romeo's number end in -6300?* (e.g., she rushes off when she sees the note), HP predicts that Juliet believes Romeo's number ends in -6300 relative to that question. But because she isn't disposed to act on this information in decision problems raising the question *What is Romeo's number?* (e.g., she doesn't finishing dialing '6300' in the phone booth), HP predicts Juliet doesn't believe Romeo's number ends in -6300 relative to that question.

3   Against the model of belief

Having outlined Hoek's question-sensitive account, we now present several problems for it. The first two problems concern Hoek's closure conditions, viz., Closure under Parthood (§3.1) and Partial Closure under Conjunction (§3.2), while the third concerns any model of belief that is closed under equivalence (§3.3). We consider some responses on Hoek's behalf and argue that none are successful (§3.4).

3.1   Closure under Parthood

Recall that the problem of logical omniscience concerns failures to believe the necessary consequences of one's beliefs. For example, Chip can believe that the CVV of his debit card is 107 without believing it is prime. For Hoek, this is because an agent is only required to believe a consequence of one of their beliefs when the questions associated with those beliefs are related by parthood. So if Chip believes that the CVV is 107 relative to *Is the CVV 107?*, it doesn't follow that Chip believes that the CVV is prime relative to *Is the CVV prime?* since neither question is part of the other (see footnote 4).

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[6]In what follows, we suppress mention of this clause since in the cases that are under consideration, the agent has (or lacks) the relevant dispositions in virtue of their beliefs.

[7]Hoek appeals to both directions of this biconditional in his arguments on page 136. For example, in the proof that Closure under Parthood follows from these principles, he says, "Suppose an agent X believes A^Q, *and hence* avoids A^Q-dominated actions in any choice that raises Q" (emphasis added), which is the left-to-right direction. Having then established that X is disposed to forego B^R-dominated options, he says, "X has the disposition associated with believing B^R. By the Duck Principle, X does believe B^{R}", which is the right-to-left direction.

Unfortunately, the problem doesn't entirely go away. For Hoek's account still predicts the following: if Chip believes the CVV is 107 relative to *What is the CVV?*, then he also believes the CVV is prime relative to *Is the CVV prime?*. The culprit is Closure under Parthood. Recall, R is a part of Q if every complete answer to Q necessarily entails a complete answer to R. So Closure under Parthood says if an agent believes A^Q where (i) A necessarily entails B, (ii) B is an answer to a question R, and (iii) every complete answer to Q necessarily entails a complete answer to R, then the agent must believe that B^R. Now, (i) *The CVV is 107* (A) necessarily entails *The CVV is prime* (B). And, (ii) *The CVV is prime* (B) is an answer to *Is the CVV prime?* (R). Moreover, (iii) every complete answer to *What is the CVV?* (Q) necessarily entails a complete answer to *Is the CVV prime?* (R). Hence, by Closure under Parthood, if Chip believes the CVV is 107 relative to *What is the CVV?* (A^Q), then Chip must also believe the CVV is prime relative to *Is the CVV prime?* (B^R). By the same type of reasoning, to believe that the CVV is 107 relative to *What is the CVV?*, Chip must believe the CVV is a twin prime, a safe prime, the 28th prime, etc., relative to the corresponding questions—in other words, he must be omniscient about the mathematical properties of 107.[8]

This, we submit, is an unwelcome result. Imagine Chip is looking at the back of his debit card and sees very clearly the number '107' written for the CVV. He has the CVV memorized by heart. When a bank teller asks, "What's your CVV?", he doesn't hesitate in answering, "107". It seems for all the world that Chip has the relevant dispositions to act on the information that the CVV is 107. Yet, for Hoek, this isn't enough to conclude that Chip believes the CVV is 107 relative to *What is the CVV?*: he must, in addition, act on the information that it is prime, that it is a safe prime, etc. This just seems much too demanding: one doesn't need to know all the mathematical properties of one's CVV to know what one's CVV is. Hoek could bite the bullet and insist that in our example, Chip doesn't believe that the CVV is 107 relative to *What is the CVV?* unless he has all these other beliefs about his CVV. But, since any proposition has non-obvious logical consequences, this would make it extremely difficult to believe anything, especially relative to questions that have a large number of possible answers. This includes questions Hoek suggests we can have beliefs relative to, such as *Whose number ends in -6300?*, *What's the date?*, *What are the two biggest cities in Brazil?*, *How many murders were there in Michigan last year?*, and so on. We believe that the question-sensitive account would lose most of its appeal if one were to embrace this consequence.

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[8]Teague (forthcoming) has recently criticized Closure under Parthood using two examples from Stalnaker (1984). The first involves lack of concepts: According to Teague, William III can believe *England can avoid all war* relative to *Which kinds of war can England avoid?* without believing *England can avoid nuclear war* relative to the same question. The second involves lack of attention: According to Teague, the absent-minded detective can believe *The butler is the culprit* relative to *Who is the culprit?* without believing *The chauffeur isn't the culprit* if they haven't considered the possibility. Our counterexample to Closure under Parthood, by contrast, doesn't rely on lacking concepts or attention: Chip may possess the concept PRIME and attend to the possibility that the CVV is prime without believing it.

3.2 Partial Closure under Conjunction

One instance of the problem of logical omniscience concerns failures of closure under conjunction. For example, Juliet can believe that Romeo's number starts with 212-529 and that it ends with -6300 without believing that Romeo's number is 212-529-6300.[9] For Hoek, this is because an agent's beliefs are only closed under conjunction when the questions of the conjuncts are related by parthood. So, for example, even if Juliet believes that Romeo's number starts with 212-529 relative to *Does Romeo's number start with 212-529?*, and that Romeo's number ends with -6300 relative to *Does Romeo's number end in -6300?*, nothing follows about what else Juliet believes, since neither question is a part of the other. In particular, it doesn't follow that Juliet believes that Romeo's number is 212-529-6300 relative to *What is Romeo's number?*.

Unfortunately, the problem doesn't entirely go away. This time, the culprit is Partial Closure under Conjunction. Recall that this principle says if an agent believes A^Q and believes B^R and every complete answer to Q necessarily entails a complete answer to R, then the agent believes AB^Q. Now, every complete answer to *What is Romeo's number?* (Q) necessarily entails a complete answer to *Does Romeo's number end in -6300?* (R). Moreover, *Romeo's number starts with 212-529* (A) is an answer to *What is Romeo's number?* (Q), albeit a partial one. Thus, Hoek's account still predicts the following: if Juliet believes that Romeo's number starts with 212-529 (A) relative to *What is Romeo's number?* (Q) and that Romeo's number ends with -6300 (B) relative to *Does Romeo's number end in -6300?* (R), then Juliet believes Romeo's number is 212-529-6300 (AB) relative to *What is Romeo's number?* (Q).[10]

This, we submit, is an unwelcome result. Recall in the initial setup of the *Romeo Recall* case that when Juliet is in the phone booth, she's confronted with the question *What is Romeo's number?*.[11] She dials 2-1-2-5-2-9, but then stops. This suggests that Juliet believes that Romeo's number starts with 212-529 relative to *What is Romeo's number?*. At any rate, it is easy to imagine that Juliet does have such a belief state while still failing to recall the last four digits in the phone booth. But by Partial Closure under Conjunction, Juliet should know Romeo's number in the phone booth. So it looks like Partial Closure under Conjunction undermines Hoek's own analysis of the *Romeo Recall* example—an example that was introduced to motivate the account in the first place.

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[9]In what follows, we use 'Romeo's number' as short for 'Romeo's 9-digit phone number'.

[10]A similar problem affects Hoek's *Trivial Trouble* case (p. 117).

[11]Hoek says the question Juliet faces in the phone booth is *What are the last four digits of Romeo's number?*. But for reasons similar to the ones given in footnote 5, this seems incorrect. There are worlds that agree on what Romeo's last four digits are but where the option of typing '6300' yield different utilities (e.g., the world where Romeo's number is 212-529-6300 yields a higher utility than a world where his number is 323-630-6300). At any rate, this would not help Hoek here since *Does Romeo's number end in -6300?* is part of *What are the last four digits of Romeo's number?*. So if Juliet has *some* belief relative to the latter question (even a trivial one), Partial Closure under Conjunction still entails she believes the last four digits of Romeo's number are -6300 relative to the latter question.

3.3 Closure under Equivalence

Another instance of the problem of logical omniscience concerns failures of closure under necessary equivalence. For instance, Chip might believe that the CVV is among 101, 103, 107,... (listing all the three-digit primes) without believing the (three-digit) CVV is prime, or vice versa.

Hoek's account doesn't explain this *even if* we dropped both closure principles from his model of belief. The reason is simply that Hoek analyzes both questions and answers in terms of sets of *possible* worlds (p. 121). This has two problematic consequences.

First, beliefs are closed under equivalent *answers* to a question. On the possible worlds analysis, necessarily equivalent propositions are identical: if A and B are true at the same possible worlds, then $A = B$. So since the proposition *The CVV is a three-digit prime* is necessarily equivalent to *The CVV is among 101, 103, 107,...*, it follows that if Chip believes *The CVV is a three-digit prime*, then he also believes *The CVV is among 101, 103, 107,...*.[12]

Second, for Hoek, beliefs are also closed under equivalent *questions*. Let's say Q and R are necessarily equivalent if every complete answer to one is necessarily equivalent to a complete answer to the other. In that case, necessarily equivalent questions are identical: if Q and R have the same complete answer at every possible world, then $Q = R$. So since *Is the CVV a three-digit prime?* is necessarily equivalent to *Is the CVV among 101, 103, 107,...?*, it follows that if Chip believes *The CVV is a three-digit prime* relative to the former, he must believe it relative to the latter.

Putting these closure principles together, then, we obtain the following principle:

3. *Closure under Equivalence:* if $A^Q \in I$ and A is necessarily equivalent to B and Q is necessarily equivalent to R, then $B^R \in I$.

Again, this falls out of the possible worlds analysis of questions and answers independently of the other closure principles Hoek assumes. Thus, Chip believes *The CVV is a three-digit prime* iff he believes *The CVV is among 101, 103, 107,...*. This seems just as problematic as the other logical omniscience problems.

To draw out the counterintuitive nature of this principle, consider noncontingent propositions. On the possible worlds analysis, there is only one necessarily true proposition (W) and one necessarily false proposition ($\varnothing$). So the proposition *107 is prime* and *107 either is or isn't prime* are identical, as are the propositions *107 isn't prime* and *107 is and isn't prime*. By Closure under Equivalence, if Chip believes 107 either is or isn't prime (relative to *Is 107 prime?*), then Chip believes 107 is prime; and if Chip believes that 107 isn't prime, then Chip also believes it both is and isn't prime. This is exactly the kind of prediction that we sought to avoid in the first place.

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[12]It doesn't help to construe answers to questions as sets of cells as opposed to sets of worlds (see footnote 3). For relative to the same question, necessarily equivalent answers are still identical: if $X, Y \subseteq Q$ and $\bigcup X = \bigcup Y$, then $X = Y$ since Q is a partition.

3.4   Potential replies

Here, we consider three replies on Hoek's behalf to the counterexamples above and argue that none of them is satisfactory.

Reply 1: The metalinguistic strategy.   One response to these cases, the "metalinguistic" strategy, is to explain the agent's dispositions to act in terms of their beliefs about language.[13] So regarding our case from §3.1, this strategy says that while Chip believes the CVV is prime, he doesn't believe the sentence 'The CVV is prime' is true, and, moreover, it is the latter fact that explains his dispositions (e.g., his inability to answer correctly when asked "Is your CVV prime?"). This is consistent with Closure under Parthood since *Is 'The CVV is prime' true?* isn't part of *What is the CVV?*: two possible worlds can agree on the answer to the latter while disagreeing on the answer to the former (e.g., by assigning different truth conditions to this sentence). Likewise, in the case from §3.2, this strategy says that while Juliet believes that Romeo's number is 212-529-6300, she doesn't believe that 'Romeo's number is 212-529-6300' is true. This is consistent with Partial Closure under Conjunction since *What is Romeo's number?* and *Is 'Romeo's number is 212-529-6300' true?* aren't related by parthood. Similarly for the examples in §3.3.

Let us note that a defender of the classical picture could equally employ the metalinguistic strategy to diagnose counterexamples Hoek presents to it. Thus, adopting this strategy would potentially undermine the initial motivations for Hoek's alternative picture. This problem needn't be fatal. Hoek might try to argue that his account nevertheless provides a more principled explanation of at least some phenomena than the classical picture, even if his account, too, has to rely on the metalinguistic strategy at some point. The following considerations strongly suggest, however, that using the metalinguistic strategy within Hoek's account doesn't yield a viable view.

Firstly, this strategy ascribes highly counterintuitive combinations of beliefs to the agents involved. Assuming, for example, that Juliet understands the relevant expressions and knows elementary disquotational principles, it is implausible to hold that Juliet believes Romeo's number is 212-529-6300 but not that 'Romeo's number is 212-529-6300' is true. Likewise, it is implausible to say that Chip believes the CVV is prime, but not 'The CVV is prime' is true.

Secondly, this strategy only works if the agents in these cases *always* face metalinguistic questions about the truth-values of certain sentences. Not only is this move ad hoc, it isn't supported by Hoek's definition of "facing" a question provided that we fill in the details of the case accordingly. For example, if Chip is deciding whether to change his CVV to a prime number for security reasons, then on Hoek's definition, Chip doesn't face the metalinguistic question *Is 'The CVV is prime' true?*, since his options may assign different utilities to worlds that agree that the sentence 'The CVV is prime' is true but not on whether the CVV is *actually* prime. In this case, Chip primarily cares about whether his

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[13]See, e.g., Lewis 1986, 1996; Stalnaker 1976b, 1984, Braddon-Mitchell and Jackson 2007, 200f.

CVV is actually prime, not whether the English sentence 'The CVV is prime' is true.

Lastly, this strategy doesn't adequately explain the cases in §3.3. Observe that the sentence '107 is prime' is derivable from the axioms and rules of Peano Arithmetic (PA).[14] Moreover, *Is '107 is prime' true?* is part of *What are the truths of arithmetic?*. So by Closure under Parthood, if Chip doesn't believe '107 is prime' is true, he must also not believe relative to *What are the truths of arithmetic?* that all of the sentences expressing the axioms of PA are true and that its rules are truth-preserving. This is hard to accept. We could imagine, for instance, that Chip is a mathematician who has a list of the axioms and rules of PA in front of him and explicitly endorses all of them. It doesn't seem that his inability to determine whether '107 is prime' is true is due to a lack of belief in the truth of one of the axioms or in the truth-preservingness of one of the rules.[15]

Reply 2: Introducing impossible worlds. One might suggest that these problems can be avoided by introducing impossible worlds.[16] On common impossible worlds accounts, for any sentence ϕ, there is an impossible world in which ϕ is true. Accordingly, since there are impossible worlds in which Chip's CVV is 107 but not prime and in which Romeo's number is 212-259-6300 but doesn't end in -6300, and worlds in which Chip's CVV is prime without being 101 or 103 or 107 . . . , etc., this would allow him to avoid our counterexamples.

Let us note that adding impossible worlds to the classical picture is already enough to solve the problem of logical omniscience. Thus, just as with the metalinguistic strategy, adopting this strategy would potentially undermine the initial motivations for Hoek's question-sensitive picture. Hoek might argue that an impossible-worlds model that is integrated into a question-sensitive account of belief can have more explanatory power than an impossible-worlds account by itself. However, the impossible-worlds strategy cannot be sensibly applied to Hoek's account.[17] This is because introducing impossible worlds breaks Hoek's notion of parthood, even for cases that are supposed to motivate Hoek's account. For instance, there are impossible worlds in which it is July 5 without it being July. On this strategy, then, *What's the month?* isn't part of *What's the date?*: two impossible worlds can agree on the date without agreeing on the month. Introducing impossible worlds, in other words, ensures that no two questions (at least, no two questions we can express in language) will be parts of one another, rendering the closure principles vacuous.

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[14]This is how Stalnaker (1984, 76) raises the problem of deduction, for which he then introduces the fragmentation strategy.

[15]Field (1986), Jago (2014b), and Soysal (2022) raise a similar problem for the fragmentation strategy.

[16]For a recent survey of theories and uses of impossible worlds, see, e.g., Berto and Jago 2019.

[17]There are also general reasons to doubt that one can give an adequate account of belief within an impossible-worlds framework. See, e.g., Bjerring and Schwarz 2017 for criticism.

Reply 3: Dropping the problematic principles. Finally, could Hoek respond by simply dropping these problematic principles from the account?

It's difficult to see how Hoek would give up Closure under Equivalence, which follows directly from the definition of questions and answers on his account. But perhaps he could give up Closure under Parthood and Partial Closure under Conjunction.[18] On the resulting picture, belief is still question-sensitive, but there is no systematic connection between the question-answer pairs that form a belief state: a belief state can be any set of question-answer pairs.

However, this approach requires giving up the main motivating principle relating question-sensitive belief to action—what we called Hoek's Principle (HP). Hoek himself presents a formal proof that HP entails both principles (p. 136). For example, suppose an agent believes A^Q. By the IBAP, they are disposed to forego A^Q-dominated options. However, agents who are disposed to forego A^Q-dominated options are also thereby disposed to forego B^R-dominated options whenever $B^R \leq A^Q$. By the DP, this implies such an agent believes B^R. Thus, HP entails Closure under Parthood. Hoek also presents a proof that HP entails Partial Closure under Conjunction.[19]

As a logical matter, then, Hoek cannot give up these principles without giving up the very principle motivating the account, HP, leaving us once again without an account of how belief relates to action on the question-sensitive picture. Indeed, this is precisely the worry Stalnaker (1991) raises to question-sensitive accounts of belief: he says, "[It is not] clear how to generalize [this model] to an account of knowledge and belief in terms of capacities and dispositions to use information (or misinformation) to guide not just one's question-answering behavior but one's rational actions generally" (p. 438). Hoek explicitly frames his project as a response to Stalnaker's worry (p. 116). But his response to Stalnaker is precisely HP. So without HP, we are right back where we started.

More generally, modeling belief states as arbitrary sets of question-answer pairs undermines one of the main motivations for introducing question-sensitivity in the first place, viz., to avoid the problem of logical omniscience while preserving the theoretical benefits of the classic account. This includes explaining the holistic nature of belief update (i.e., how changing one belief systematically impacts other beliefs) and explaining how we can attribute to agents beliefs in propositions they haven't considered (Yalcin, 2018). Allowing any arbitrary set of question-answer pairs to count as a belief state would nullify these benefits. For example, updating one's belief with a question-answer pair will not automatically have ramifications for one's other beliefs, thereby undermining the utility of Hoek's definition of update (p. 138). We are thus left without a clear motivation for introducing question-sensitivity in an account of belief.

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[18]We thank an anonymous referee for suggesting this possibility.

[19]We flag, however, that this latter proof (footnote 31) implicitly assumes a questionable principle, viz., if an agent picks an option $\mathbf{b} \in \Delta$, then they will continue to do this in any composite decision problem consisting of Δ, e.g., they will choose $\langle \mathbf{b}, \mathbf{t} \rangle$ or $\langle \mathbf{b}, \mathbf{o} \rangle$ in $\Delta \times \{\mathbf{t}, \mathbf{o}\}$. So it might be that Hoek's question-sensitive account could consistently drop Partial Closure under Conjunction. Still, Hoek appeals to this principle in multiple places to address the practical problem of deduction (e.g., pp. 135 and 140).

4 Towards an adequate account of belief

In the previous section, we presented problems for Hoek's formal model of belief. In closing, we present a worry for Hoek's inquisitive belief-action principle and sketch the beginnings of a more promising account.

Hoek's account is supposed to capture the stability of behavior. This is reflected in HP, which suggests that whenever there is a dominant option in a decision problem, an agent will choose it. Accordingly, Hoek criticizes accounts that make an agent's behavioral dispositions subject to masking, or dependent on the presence of elicitation conditions, such as Elga and Rayo's (2021) account (pp. 119f.). But in our view, the relation between belief and action postulated by Hoek's account is too rigid. Whether an agent acts on the information they possess can depend on many factors. It can depend on how a decision problem is presented to them—for instance, Juliet might know the answer to *What is Romeo's phone number?* if someone asks her, but not if she has to dial it in the phone booth.[20] And it can depend on an agent's cognitive state or on external circumstances—for instance, Juliet might normally recall the last four digits of Romeo's number, but not when she's in a state of panic or distracted by loud music.[21] We thus think that in trying to capture the stability of behavioral dispositions, Hoek's account leaves too little room for their instability.

This suggests the following weakening of Hoek's account. Dispositions are generally subject to normal or *ceteris paribus* conditions for their manifestation. For instance, a normal condition for a match to manifest its disposition to light if struck (where striking is the trigger) is the presence of oxygen in the match's environment. It is thus natural to assume that the same goes for the dispositions associated with beliefs. Juliet can act on the information that Romeo's number ends in -6300 in normal conditions, but not in all conditions.

Perhaps, then, belief isn't just *question*-sensitive but, more generally, *circumstance*-dependent, i.e., dependent on the set of normal conditions being considered. These circumstances can concern not only the question an agent faces, but anything about their cognitive or physiological functioning, or their external environment. On the view we suggest, beliefs are thus associated with both a possible-worlds content and a set of circumstances.[22] Hoek might object that this makes the relation between belief and action too fragile to be of pre-

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[20]Similarly, research in social science suggests that an agent's choices can depend on how a question is phrased (this is the *framing effect*—see, e.g., Tversky and Kahneman 1981), on the order in which options are presented (this is the *order effect*—see, e.g., Schuman and Presser 1981), etc. There is some controversy over how significant these effects are. But for our purposes, it suffices if agents' choices are sometimes influenced by such factors, which is very difficult to deny.

[21]Soysal (2022) argues that an account of belief should take into account the algorithms that an agent employs. We find it plausible that appeal to algorithms can have predictive and explanatory power, since the circumstances associated with a belief partly depend on which algorithms an agent uses.

[22]This suggestion bears similarities to the account endorsed by Rayo (2013, 113–115) and Elga and Rayo (2021). But Elga and Rayo's account is supposed to capture the phenomenon of fragmentation, whereas our suggestion aims to capture many types of cases that don't involve fragmentation. See Kipper et al. 2022 for further discussion.

dictive or explanatory value. But note that we appeal to the fragility of vases, the flammability of matches, the solubility of sugar, etc. to explain and predict their behavior even though it is generally understood that these dispositions depend on normal conditions. Similarly, beliefs can be highly useful in predicting and explaining an agent's behavior even if they are dependent on circumstances, which at any rate is independently plausible. Our suggestion thus allows for the existence of systematic but defeasible relations between belief and action.

We believe that our suggestion is also suitable for solving the problem of logical omniscience. For instance, an agent can fail to have a belief whose content is necessarily equivalent to the content of something they believe, because the associated circumstances differ. Our suggested view thus allows making sufficiently fine-grained distinctions between belief states to avoid modeling agents as logically omniscient. Since questions are only one aspect of the circumstances associated with a belief, it also allows making finer distinctions than Hoek's account, and hence, it has the potential to escape the problems we raised for this account.

To be clear, these are just tentative suggestions. More work will need to be done to develop a circumstance-dependent account of belief. Our point is simply that an adequate account of belief must accommodate the ways in which belief is sensitive to circumstances beyond the questions the agent faces.[23]

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