4 A Cantorian Argument Against Frege's and Early Russell's Theories of Descriptions

Kevin C. Klement

It would be an understatement to say that Russell was interested in Cantorian diagonal paradoxes. His discovery of the various versions of Russell's paradox—the classes version, the predicates version, the propositional functions version—had a lasting effect on his views on philosophical logic. Similar Cantorian paradoxes regarding propositions—such as that discussed in §500 of *The Principles of Mathematics*—were surely among the reasons Russell eventually abandoned his ontology of propositions.[1] However, Russell's reasons for abandoning what he called 'denoting concepts', and his rejection of similar 'semantic dualisms' such as Frege's theory of sense and reference—at least in 'On Denoting'—made no explicit mention of any Cantorian paradox. My aim in this paper is to argue that such paradoxes do pose a problem for certain theories such as Frege's, and early Russell's, about how definite descriptions are meaningful. My first aim is simply to lay out the problem I have in mind. Next, I turn to arguing that the theories of descriptions endorsed by Frege and by Russell prior to 'On Denoting' are susceptible to the problem. Finally, I explore what responses a contemporary 'semantic dualist' with commitments similar to Frege or early Russell could give that might have some plausibility.

AN OVERVIEW OF THE PROBLEM

Here's the core difficulty: by Cantor's powerset theorem, every set has more subsets than members. Given a sufficiently abundant metaphysics of properties, concepts, 'propositional functions' or suchlike, a result similar to Cantor's theorem applies to them: for a given logical category of entity, the number of properties applicable (or not) to entities in that category must exceed the number of entities in that category (see Russell 1903: §§102, 348). If, like Frege and early Russell, we believe that a descriptive phrase of the form ⌜the φ⌝ has a *sense* or *meaning* which is a distinct entity from its denotation, and believe that such a sense exists for every property φ, we come to the brink of violating Cantor's theorem. If we now consider those properties applicable or not to senses or meanings, and are committed to a distinct sense, as a self-standing entity, for each such property, we risk positing as many senses as properties applicable to them, in violation of Cantorian principles.[2]

In order to state the problem more precisely, it is worth listing those principles that together generate the problem. In stating the principles, let us use the label *descriptive sense* for those senses—assuming there are any—that either *are* the senses of definite description phrases or are appropriate for playing this role, that is, they have as part of their nature some property φ such that they present as denotation the unique entity of which φ holds, if there is such a unique entity, and otherwise lack denotation (or present some special chosen object as denotation).[3] Let us furthermore use the notation '⟦the φ⟧' to speak about such a sense itself (as opposed to its denotation). Hence, according to the usual story, (A) is true but (B) is false:

(A) the author of *Frankenstein* = the second daughter of Mary Wollstonecraft
(B) ⟦the author of *Frankenstein*⟧ = ⟦the second daughter of Mary Wollstonecraft⟧

I use the word 'expresses' for the relation between a phrase and its sense, the word 'presents' for the relation between a sense and the denotation it picks out, and the word 'designates' for the relation between a phrase and its denotation, so we have:

(C) 'the author of *Frankenstein*' expresses ⟦the author of *Frankenstein*⟧
(D) ⟦the author of *Frankenstein*⟧ presents the author of *Frankenstein*
(E) 'the author of *Frankenstein*' designates the author of *Frankenstein*

Consider then, the following set of assumptions:

Comprehension Principle (CP): For every open sentence '... *x* ...' not containing 'φ' free, the corresponding instance of the following schema is true:

> There exists a property φ such that any entity *x* has φ if and only if ... *x* ...

Descriptive Sense Principle (DSP): For every property φ, there is at least one descriptive sense, that is, at least one sense taking the form ⟦the φ⟧.

Sense Uniqueness Principle (SUP): A descriptive sense involves only one property, that is: for any descriptive senses ⟦the φ⟧ and ⟦the ψ⟧, ⟦the φ⟧ is identical to ⟦the ψ⟧ only if φ is the same property as ψ.

> *Sense Property Principle (SPP)*: (i) Descriptive senses themselves have or lack properties and can be presented by other descriptive senses in virtue of these properties, and (ii) descriptive senses are not divided into logical subtypes: those properties applicable (or not) to some descriptive senses are applicable (or not) to all of them.

Applying Cantorian diagonal argumentation, from these principles we obtain a Russell-style paradox. Certain descriptive senses do not have their corresponding properties. For example, ⟦the author of *Frankenstein*⟧ did not author Frankenstein, and ⟦the sense that presents the Eiffel Tower⟧ does not present the Eiffel Tower.[4] On the other hand, ⟦the sense⟧ is a sense, and ⟦the self-identical thing⟧ is a self-identical thing. Let us now introduce terms for certain properties, *autopredicability* and *heteropredicability*, as follows:

> Definition: x is heteropredicable if and only if there is a property φ such that x is identical to a descriptive sense of the form ⟦the φ⟧ and x does not have φ.

> Definition: x is autopredicable if and only if x is not heteropredicable, or equivalently, for all properties φ, if x is identical to a descriptive sense of the form ⟦the φ⟧, then x has φ.

Notice that in order to be autopredicable, a sense does not have to present itself: a descriptive sense ⟦the φ⟧ can be autopredicable even if other entities besides it have the property φ. For example, assuming a plurality of senses, ⟦the sense⟧ is autopredicable, because it is a sense, and this does not require it to be the *only* sense. However, if there *are* any senses that present themselves, they are autopredicable. If the sense Sally loves most is ⟦the sense Sally loves most⟧, then ⟦the sense Sally loves most⟧ is autopredicable. All descriptive senses derived from uninstantiated properties, for example, ⟦the King of France in 1905⟧ are heteropredicable.

You may have guessed where this is going, but let's walk through it carefully:

(1) By (CP), there is a property H such that any entity x has H if and only if there is a property φ such that x is identical to a descriptive sense of the form ⟦the φ⟧ but x does not have φ.
(2) By (1) and (DSP), there is a descriptive sense, ⟦the H⟧.[5]
(3) By (SPP), the question as to whether or not ⟦the H⟧ has property H arises.

However, both assumptions are impossible:

(4) First let's show that ⟦the H⟧ cannot have property H.
    (4a) Assume for *reductio ad absurdum* that ⟦the H⟧ has property H.

(4b) By (4a) and (1), there is a property φ such that ⟦the *H*⟧ is identical to a descriptive sense of the form ⟦the φ⟧ but ⟦the *H*⟧ does not have φ.

(4c) Let us call the property posited in (4b) '*G*'. Hence, ⟦the *H*⟧ is identical to a descriptive sense ⟦the *G*⟧, and ⟦the *H*⟧ does not have *G*.

(4d) By (SUP) and the first conjunct of (4c), *G* is the same property as *H*.

(4e) By (4d) and the second conjunct of (4c), ⟦the *H*⟧ does not have *H*, which contradicts our assumption at (4a).

(5) Likewise ⟦the *H*⟧ cannot *not* have *H*.

(5a) Assume that ⟦the *H*⟧ does not have *H*.

(5b) By (1) and (5a), it is not the case that there is a property φ such that ⟦the *H*⟧ is identical to a descriptive sense of the form ⟦the φ⟧ but ⟦the *H*⟧ does not have φ.

(5c) By logical manipulations on (5b), we get: for all properties φ, if ⟦the *H*⟧ is identical to a descriptive sense of the form ⟦the φ⟧, then ⟦the *H*⟧ has φ.

(5d) By (5c), if ⟦the *H*⟧ is identical to a descriptive sense of the form ⟦the *H*⟧, then ⟦the *H*⟧ has *H*.

(5e) Certainly, ⟦the *H*⟧ is identical to a descriptive sense of the form ⟦the *H*⟧, and so by (5d), ⟦the *H*⟧ has *H*, which contradicts (5a).

(6) *Conclusion*: At least one of the assumptions (CP), (DSP), (SUP) or (SPP) must be false.

The process of reasoning given above is difficult to counter. Alleging that a *logical* mistake is made somewhere along the way would require, I think, toeing a fairly revisionist line with regard to certain logical principles, denying, for example, the indiscernibility of identicals, the principle of bivalence or the validity of *reductio* argumentation. There are more subtle concerns one might have, particularly with regard to the correct logic of the notation '⟦the φ⟧' and whether it is legitimate to 'quantify in' to such a construction.[6] However, when scrutinized, I believe such worries in the end would amount to a rejection of (CP), (DSP), (SUP) or (SPP), or at least advocacy of a precisification of one of these principles. The argument at least establishes that it would be naïve to accept all of these principles without further scrutiny.

FREGE AND EARLY RUSSELL

Next, I want to argue that Frege and early Russell either were committed to the principles (CP), (DSP), (SUP) and (SPP), or were at least committed to those instances necessary to generate the antinomy just described. (CP) is roughly the standard principle of comprehension for second order logic; both Frege and Russell accepted a version of this principle within their logical systems. In Frege's logical system, higher-order quantification is quantification over functions. Certain functions, called 'concepts', were understood as functions onto truth-values. Frege believed that if a proper name is removed from a sentence, what remains is a name of a concept (see Frege 1891, passim). Hence, if we take 'property' in 'CP' to mean Fregean concepts, Frege would have endorsed (CP). A caveat should be entered that Frege would only have accepted (CP) as a general principle for those logical languages that avoided vagueness and ensured that every well-formed name had a reference. Frege believed that a complex expression does not refer whenever it contains a subexpression that doesn't refer. So in natural language, where nonreferring expressions are used, Frege might have demurred from something such as (CP). For example, he might have claimed that the open sentence 'x = Hercules' would not correspond to any function. However, (CP) is only involved in the antinomy sketched above at step (1), where it is concluded that there is such a property as *H*, or hetero-predicability. Even if Frege would have insisted upon exceptions to (CP) for ordinary language, these exceptions are not relevant to the issue under consideration, because there is nothing about our definition of *H* that involves expressions that do not refer, vague concept words or anything similar.

In the case of early Russell, there is a complicating factor regarding whether (CP) should be taken as positing what he called 'class-concepts' or 'predicates', or what he called 'propositional functions'.[7] Unfortunately, Russell sometimes used the word 'property' synonymously with 'predicate', and sometimes synonymously with 'propositional function' (see Linsky 1988). On my own interpretation, by 'predicate' Russell meant the ontological correlate of an adjectival phrase, which he understood roughly as a Platonic universal devoid of complexity; whereas, by 'propositional function' he understood a complex proposition-like unity containing variables in place of definite terms (see Klement 2005). In *The Principles of Mathematics*, when responding to the very similar antinomy involving predicates that are not predicable of themselves, Russell came to the conclusion that there is no such predicate as 'not predicable of oneself' (Russell 1903: §101). This might make it seem as if he would have denied (CP) and reject any such predicate as *H*. However, early Russell distinguished between predicates and propositional functions. While he denied a common *predicate*, Russell admitted that there was a propositional function satisfied by all and only predicates not predicable of themselves, and also admitted a denoting concept derived from this propositional function denoting the class of all such predicates (Russell 1903: §84 chiefly, but compare §§77, 96, 488). Although the primary chapter on denoting concepts in *Principles of Mathematics* deals principally with denoting concepts derived from predicates, Russell's final position admitted denoting 'complexes' derived from propositional functions. This is clearer in Russell's 1903 through 1904 manuscripts, where a denoting complex was explicitly described as derived from an indefinable 'denoting function', written '*ı*', along with a propositional function (Russell 1994: 87–89, 355–56). Indeed, a denoting complex is portrayed in these manuscripts as a complex formed from the function *ı* and a propositional function. Hence, the relevant version of (CP) would involve propositional functions rather than simple predicates. Here, just as Russell accepted a propositional function satisfied by all and only predicates not predicable of themselves, there seems no barrier to supposing that he would have allowed a version of (CP), interpreted to deal with propositional functions, according to which *H* would be admitted as a propositional function.

Given the description we have just given of a Russellian denoting complex as a complex formed by the operator '*ı*' and a propositional function, it is clear that he would have posited such a denoting complex for each propositional function; and hence, interpreting 'descriptive sense' to mean the same as 'denoting complex', early Russell would have accepted (DSP).

With regard to (DSP), in the case of Frege the situation is a bit complicated in that the properties, or at least the properties posited by his version of (CP), are functions located at the level of reference. Frege applied the sense/reference distinction to concept and other function expressions as well. According to the little known theory of definite descriptions presented in Frege's *Grundgesetze,* §11, an ordinary language description of the form the φ would be analysed with a term of the form ⌜$\setminus\dot{\varepsilon}\,\varphi(\varepsilon)$⌝, where ⌜$\dot{\varepsilon}\,(\ldots \varepsilon \ldots)$⌝ represents Frege's abstraction operator for forming a name of the extension of a concept, and '\' represents a function on extensions that yields the sole object included in the extension if there is such a sole object, and returns its argument as value otherwise. Frege regarded the sense of a complex expression as a whole containing the senses of its subexpressions as parts; so the sense of the phrase '$\setminus\dot{\varepsilon}\,F(\varepsilon)$' would contain the sense of the concept phrase '*F*( )' as part. Presumably, Frege would have regarded it as generally the case that a descriptive sense would contain as a part a sense presenting a concept, but would not contain the function or concept *itself*. Arguably, then, Frege might have room for claiming that only those concepts that are presented by some sense have descriptive senses corresponding to them. Certainly, Frege might have claimed that certain concepts, for example, those that map objects to the True or the False in an irregular fashion not corresponding to any condition specifiable in a finitely complex way, are not presented by any sense, and, hence, have no corresponding descriptive sense. Again, however, even if Frege is not committed to (DSP) in its strongest form, this will not provide a solution to the antinomy described above unless *H* is one of the concepts having no corresponding sense. Yet *H* does map objects to the True in a finitely specifiable way; indeed, the expression we used above to define *H* presumably expresses a sense, and this sense would present *H*. It is unlikely that Frege could have plausibly claimed that there is no such descriptive sense as ⟦the *H*⟧ without further argumentation.

With regard to (SUP), it seems clear that Russell would be committed to a one-to-one correlation between denoting complexes and propositional functions. Since a denoting complex simply is a complex containing the description operator and a propositional function, the denoting complex so-formed would be different for different propositional functions. In Frege's case, again the situation is slightly more complicated, though not much. A Fregean descriptive sense would be a composite containing as a part a sense presenting some concept. Different descriptive senses would result for different concept senses. Since presumably a given concept sense would present at most one concept, (SUP) would hold for Frege as well.

Moving to (SPP), part (i) is part and parcel of adopting a realism about meanings, denoting complexes or senses. If there are such entities, they must have properties and be presented by other senses. If not, then it would seem impossible for us to speak about them. The more interesting question would be whether or not Frege or Russell would have room for denying part (ii) of (SPP) by drawing upon some theory of logical types. Frege's only division of logical type was that involved in the distinction between functions and objects and the hierarchy of functions of different levels. Descriptive senses are presumably all objects on Frege's approach: an object was characterized by Frege as an entity the expression for which does not contain an empty spot (Frege 1891: 140–41). According to Frege's theory of indirect speech, descriptive senses are sometimes the referents of descriptive phrases, which would count as object expressions in Frege's theory. There does not seem to be much room for dividing them into logical types.

Russell's views on logical types changed through his career, but *early* Russell (at least) was explicit that all *singular entities*—all 'individuals', as he would say—form a single logical type and can all be logical subjects in propositions. Indeed, he found it 'self-contradictory' to deny of any entity that it is not a logical subject (Russell 1903: §49). The short-lived theory of types presented in Appendix B to the *Principles* made a distinction between pluralities, or classes-as-many, and individuals, but classes-as-many were not regarded by Russell as singular entities at all.[8] Denoting complexes, if taken to be unified entities, and not—as in Russell's later thought—just roundabout ways of talking about multiple entities entering into propositions in more complicated ways, would presumably all count as the same logical type for Russell.

POSSIBLE ALTERNATIVE RESPONSES

I have just argued that the historical Russell and Frege were committed to the principles giving rise to the antinomy sketched earlier. It is nevertheless worth considering what routes would be available to those who might wish to attempt to preserve the core of a semantic dualist position similar to Frege's or early Russell's yet avoid the antinomy. Because the antinomy is generated by accepting the conjunction of (CP), (DSP), (SUP) and (SPP), any adequate response would require rejecting one or more of these principles. Let us consider them in turn and, for each, consider the immediate benefits and costs of abandoning it. Again, (CP) is roughly the standard principle of comprehension of second-order logic. Of course, the merits and demerits of deviating from first-order logic are still matters of philosophical controversy.[9] There are many rival views on the nature of properties, their existence conditions and the appropriate corresponding logic worth exploring. I cannot survey them here. However, it is not clear that the general problem wouldn't also arise for those who think of properties as, say, functions from possible worlds to classes or some such. Whatever one's view of properties, avoiding the contradiction above by rejecting (CP) would require saying something more about the relationship between properties and open sentences and discovering some principled reason not only why not every open sentence corresponds to a property, but *in particular*, why there is no such property as *H*. Of course, there are other reasons to prefer a 'sparse' rather than 'abundant' theory of properties besides blocking this paradox, but in the context of the present discussion, one must be willing to accept even that there is no *derivative* or less fundamental notion of 'property' relevant to the meaningfulness of descriptions that would generate the problem. After all, it would not be plausible to suppose that there is no such sense as ⟦the author of *Frankenstein*⟧ simply because the trait of *authoring Frankenstein* is not metaphysically simple or fundamental. Still, this warrants further exploration.[10]

We have already seen that there might be room within a Fregean perspective for denying (DSP) in its full generality. Doubts about (DSP) multiply further if we deviate from the Fregean conception of a sense as an abstract object, and instead portray a sense as something psychological, linguistic or psycho-linguistic (for example, an item of the language of thought). Then surely, if our metaphysics of properties is abundant, we would not be committed to a sense for every property. This definitely makes the situation more Cantor-friendly. But as with the case of Frege himself, the advantages of scrapping (DSP) in the present context, are not what they might seem. Again, for this to serve as a solution to the paradox sketched above, we would have to conclude that *H* is one of the properties to which there corresponds no descriptive sense. It is difficult to exclude ⟦the *H*⟧ except by means of some ad hoc restriction. For example, without further elaboration, it is unclear how appeal to a psychological or linguistic conservatism about the existence of senses could help avoid ⟦the *H*⟧, since we at least *seem* to be able to have thoughts making use of this sense, and form linguistic expressions that express it. Someone wishing to use the falsity of (DSP) to avoid the problem needs a more subtle response.[11]

In examining (SUP), it is important not to confuse it with something like its converse. If properties are located at the level of denotation, or otherwise are given identity conditions governed by their extensions either in this world or even across all possible worlds, then it would be implausible to suppose that a property could have only one descriptive sense. One might hold, for example, that the property *being the 8th planet from the sun* is the same property as *being the (5 + 3)rd planet from the sun*, and yet deny that the sense ⟦the 8th planet from the sun⟧ is identical to the sense ⟦the (5 + 3)rd planet from the sun⟧. I have tried to formulate the argument above without assuming that for a given φ there is only one descriptive sense of the form ⟦the φ⟧. However, notice that if the problem is that, by Cantor's theorem, there must be more properties of descriptive senses than descriptive senses, allowing there to be *more than one* ⟦the φ⟧ for a given φ, if anything, makes the problem worse, not better.

(SUP) itself would be very difficult to deny while maintaining both that descriptive senses present their denotation in virtue of the denotation's unique possession of a certain property, and that the relationship between sense and denotation is determinate. Suppose there were a descriptive sense *S**, such that *S** took both the form ⟦the φ⟧ and the form ⟦the ψ⟧, but $\varphi \neq \psi$. It would then seem possible for there to be one entity uniquely φ and a distinct entity uniquely ψ, and it would be indeterminate which entity would be presented by *S**. The best hope for making good on a denial of (SUP) might come from a more sophisticated understanding of the nature of senses/intensions generally, whereupon a sense, understood as a linguistic meaning, does not by itself fix a denotation, and only does so in conjunction with features of the context in which a linguistic act is performed.[12] One might then maintain, for example, that the description 'the person I most admire' has the same sense regardless of who utters it. However, if uttered by Sally, it has its denotation in virtue of that denotation having one property (*being admired most by Sally*), but if uttered by Raoul it has its denotation in virtue of that denotation having a different property (*being most admired by Raoul*). However, it is doubtful that this would avoid the problem altogether. The paradox above might still be reformulable by choosing one context of evaluation to consider throughout, or by focusing more narrowly on a certain subclass of descriptive senses ('eternal senses'?) that are not so context-dependent. Again, in order for this strategy to solve the contradiction, we'd have to say that the descriptive sense ⟦the *H*⟧ from the argument above is one of the senses corresponding to more than one property. Yet there does not seem to be anything context-dependent about it.

Of the four principles, (SPP) is the most difficult to fully evaluate. A realist about senses would be hard pressed to deny part (i). Indeed, even for a view on which senses are derivative or constructed entities—reducible to something more fundamental—it would be odd to claim that they don't have properties, or cannot be presented by other senses. It would then become hard to see how the statements we make about senses when philosophizing about them could be possible.[13] Part (ii) of (SPP) is more open to criticism. Notice, however, that it does not suffice simply to place descriptive senses in a different logical category from concrete objects. All talk of properties in the argument above can be restricted to 'properties of descriptive senses' and the argument goes through just as well. One would need, instead, to divide descriptive senses and/or the properties applicable to them, into various ramified subtypes. Notice that the definition given of *heteropredicability* is what is sometimes called an 'impredicative' definition:[14] it involves quantification over a range that includes that which is being defined. Someone might, therefore, claim either that the definition is illegitimate (which amounts to a rejection or modification of (CP)), or that the property so defined falls into a separate logical category from those it quantifies over. To solve the above contradiction, one would need argue, *on the basis of this*, either that the question as to whether or not ⟦the *H*⟧ is *H* does not arise (thus blocking step (3)), or that something goes wrong at steps like (5d), where a quantifier over properties using φ and ψ is instantiated to *H*. Typically, ramified type-theories derive their philosophical justification by appeal either to a Tarskian hierarchy of languages, or to some sort of vicious-circle principle.[15] Without delving more into the nature of properties and their relationship to descriptive senses, the philosophical viability of ramification cannot be fully assessed. I, for one, am skeptical. While the paradox we have been discussing is a semantic paradox, involving *meanings*, the paradox deals most directly with properties and senses, and only indirectly with language. This makes it quite different from the Grelling or Liar paradoxes. If properties or senses are abstracta, it is not clear how a Tarskian hierarchy of languages would be relevant, nor exactly what would make impredicativity viciously circular.[16]

Although I am still somewhat undecided about the issue, I think this sort of paradox poses a definite challenge for the would-be defender of intensional entities such as senses or denoting concepts. It represents yet another reason in favour of theories such as that of the mature Russell's theory of descriptions, in which differences between descriptive phrases intuitively having different meanings are respected without positing a special class of intensional entity.

NOTES

1. For further discussion, see Landini 1998: Ch. 8.
2. This paradox is a new instance of a general category of Cantorian problems regarding senses which I discuss in Klement 2003.
3. In this paper, I do not assume anything about whether or not proper names or other individually referring expressions besides descriptions should be understood as expressing descriptive senses. For those who accept a Fregean descriptivist theory of names, the points made in this paper will have even more importance, but the argument itself does not presuppose any such thing. The last clause, with regard to 'presenting a chosen object', is meant to accommodate views such as Frege's use of the sign '\' as a 'substitute for the

definite article' in his logical language (see Frege 1893: §11), Carnap's use of the sign '$\imath$' (see Carnap 1956: 32ff.), Church's use of the sign 'ι' (see Church 1951: 14), and similar devices.

4. This example may take some thought to figure out. Notice, we are not asking whether or not the sense that presents the Eiffel tower presents the Eiffel tower, but something more like whether or not the sense of the phrase 'the sense that presents the Eiffel tower' presents the Eiffel Tower. Presumably, if this sense were to present anything, it would present a sense, and not a monument in Paris. However, I think a Fregean *should* say that it lacks denotation altogether, since the uniqueness requirement is not fulfilled.
5. I should note that we need not assume that this sense is unique; if there are multiple descriptive senses for H, the contradiction results for any of them. More on this below.
6. Notice that I did not introduce '$[\![\text{the } \varphi]\!]$' as shorthand for 'the sense of the phrase $\ulcorner$the $\varphi\urcorner$. If I had, then surely quantifying in would be illegitimate. I mean something closer to a function mapping properties to their descriptive senses. However, given the possibility of the *falsity* of the *converse* of (SUP), this way of understanding $[\![\text{the } \varphi]\!]$ might also be overly simplistic. This is merely an instance of the general sort of difficulty Russell himself drew attention to in 'On Denoting' regarding speaking about meanings as opposed to their denotations. (For a general discussion, see Klement 2002b.) The argument can be captured without using this notation and instead making use of a relation sign representing the *presentation* relation, such as Church's 'Δ' (see Church 1951: 16). However, the argument is much easier to follow for informal discussion with the notation '$[\![\text{the } \varphi]\!]$'.
7. Unfortunately, it is still widely believed that Russell equated predicates/concepts and propositional functions. I have argued against this common misreading elsewhere. See Klement 2004, 2005.
8. See, e.g., his remarks to Jourdain, quoted in Grattan-Guinness 1977: 78.
9. In particular, Quine's criticism of the higher-order logic of *Principia Mathematica* as having been born in the sin of confusing metalinguistic schematic variables with object-language variables for 'propositional functions' understood realistically as complex attributes comes to mind. Quine suggests instead that the innocuous part of second-order logic can be reconstructed in a first-order set theory (see, e.g., Quine 1961, 1969). Of course, it is highly unlikely that someone wishing to maintain a Fregean intensionalist view of the meaningfulness of descriptions would look to a hyper-extensionalist like Quine for salvation; the theory of descriptions was one of the few Russellian doctrines Quine liked.
10. The sort of conservatism with regard to reifying universals and other intensional entities found in Cocchiarella's work (e.g., Cocchiarella 1987, 2000) might represent a compromise worth exploring. I cannot do the issue justice here.
11. For further discussion of related issues, see Klement 2003.
12. Versions of such sophisticated understanding of senses, or intensions, abound in the philosophy of language literature from the past 30 years: see, e.g., Perry 1977, Stalnaker 1978, Burge 1979, Chalmers 2002, etc. The details of their views, and the terminology they use, vary widely.
13. Perhaps an adherent to a modified version of the early Wittgensteinian saying/showing distinction (see Wittgenstein 1922) could accept that we can't actually 'say anything' about senses. The issue is too difficult to be broached here, and if there's something to it, we can't really discuss it anyway.

14. This notion generally comes from Whitehead and Russell 1925 (see Introduction, Ch. 2). For discussion, see Chihara 1973; Hazen 1983; Goldfarb 1989; Urquhart 2003 and others.
15. See Tarski 1933; Whitehead and Russell 1925; Church 1974, 1976; Anderson 1987 and others.
16. For influential criticisms of certain forms of ramification along these lines, see Gödel 1944 and Quine 1966; for a defence of the historical Russell, see Landini 1998: Ch. 10. For a discussion of the plausibility of ramification for Fregeans, see Klement 2002a: Ch. 7.

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