10

The Constituents of the Propositions of Logic

Kevin C. Klement

1 Introduction

Many founders of modern logic—Frege and Russell among them—bemoaned the tendency, still found in most textbook treatments, to define the subject matter of logic as "the laws of thought" or "the principles of inference". Such descriptions fail to capture logic's objective nature; they make it too dependent on human psychology or linguistic practices. It is one thing to identify what logic is *not* about. It is another to say what it *is* about. I tell my students that logic studies relationships between the truth-values of propositions that hold in virtue of their form. But even this characterization leaves me uneasy. I don't really know what a "form" is, and even worse perhaps, I don't really know what these "propositions" are that have these forms. If propositions are considered merely as sentences or linguistic assertions, the definition does not seem like much of an improvement over the psychological definitions. Language is a human invention, but logic is more than that, or so it seems.

It is perhaps forgiveable then that at certain times Russell would not have been prepared to give a very good answer to the question "What is Logic?", such as when he attempted, but failed, to compose a paper with that title in October 1912. Given that Russell had recently completed *Principia Mathematica*, a work alleging to establish the reducibility of mathematics to logic, one might think this overly generous. What does the claim that mathematics reduces to logic come to if we cannot independently speci-

Published in *Acquaintance, Knowledge and Logic: New Essays on Bertrand Russell's Problems of Philosophy*, edited by Donovan Wishon and Bernard Linsky. Stanford, CA: CSLI Publications, 2015, pp. 189–229. (Author's typesetting with matching pagination.)

fy what logic is? But a response might be that we know what logic is when we see it, whether or not we can put its essence into words. Still it is puzzling that less than a year prior to attempting "What Is Logic?", Russell professed to have an understanding both of the nature of logical truths and even of our knowledge of them. In *The Problems of Philosophy* (chaps. VII, X especially), written in 1911 and published in 1912, Russell argued that logical propositions are general propositions that assert relations between "certain abstract logical universals" (Russell 1912, 109), and that our knowledge of logic and mathematics consists of intuitive or direct knowledge of truths about these universals. The same view is found in other 1911 works by Russell, including "The Philosophical Importance of Mathematical Logic" and "Analytic Realism." In the former, he writes:

> Logic and mathematics force us, then, to admit a kind of realism in the scholastic sense, that is to say, to admit there is a world of universals and of truths which do not bear directly on such and such a particular existence. This world of universals must *subsist*, although it cannot *exist* in the same sense as that in which particular data exist. We have immediate knowledge of an indefinite number of propositions about universals: this is an ultimate fact, as ultimate as sensation is. Pure mathematics—which is usually called "logic" in its elementary parts—is the sum of everything that we can know, whether directly or by demonstration, about certain universals. (Russell 1992e, 39–40)

Russell comes across as brazen, taking himself to have shown more or less conclusively that not all knowledge is empirical, and not all of what is known is mind-dependent.

I am not convinced that Russell was as confident about these issues as he pretended to be at the time. He certainly *shouldn't* have been very confident. The view that logic and pure mathematics concern themselves with knowledge of certain universals fits reasonably well with the views he held early on in his logicist years, such as when composing *The Principles of Mathematics* (hereafter *PoM*), published in 1903. However, his views changed quite a lot between then and the publication of *Principia Mathematica* (hereafter *PM*), mostly as a result of his attempts to deal with the logical paradoxes. These changes in most cases brought him further away from a Pythagorean or Platonist metaphysics of special logical and mathematical entities. Even when it comes to the primitive logical constants, the changes make it much more difficult to think of them as standing for anything like universals.

A close examination, especially of Russell's manuscripts written prior to *PM*, show some awareness of some of the difficulties that arise for maintaining his original view about the particular nature of the propositions of logic and their constituents. This leaves an interpretive difficulty as to why Russell shows little hesitation in *Problems* and in other works of the period in writing as if these difficulties do not exist. Unfortunately, there is not enough in Russell's writings to provide a definitive solution to this interpretive difficulty. But there are some clues from which we can speculate. I shall argue that there is reason to think that although Russell did not have a fully worked out view of the nature of "logical universals" in this period, he had a variety of ideas about what some of them might be. These ideas include some rather strange ones such as an understanding of negation and implication as "multiple relations" between *constituents* of propositions, much like his view of the nature of judgment or belief during this period. However, I think he was never fully satisfied with these ideas, and soon came to abandon them. If these speculations are correct, they also shed light on certain other changes to Russell's metaphysics, especially his understanding of general facts and higher-order truths. In the end, however, he was still uncertain as to the nature of the constituents of the propositions of logic for quite some years to follow, until finally settling on a purely linguistic conception of logic later in his life.

2 The Earlier Development of Russell's Views

My focus in this paper is primarily Russell's views around the time of *Problems of Philosophy*. It is important, however, to contrast these views with those that came before. In the opening chapter of *PoM*, Russell characterized a proposition of logic as one containing no constants but logical constants, and a proposition of pure mathematics as a proposition of logic taking the form of a formal implication (quantified conditional). The notion of a logical constant, Russell argued, was too primitive to be defined, and so the logical constants could only be given by enumeration. In 1903, Russell's list included formal and material implication ($\supset$), the membership relation (ϵ), the "such that" class abstraction operator ($\ni$) and the notion of a relation (Rel). With at least some of these, it is fairly obvious how early Russell might have seen them as representing universals. Early Russell understood classes realistically, and hence ϵ could easily be taken as a relation holding between an individual and a class of which it is a member. In *PoM*, Russell similarly took material implication as a relation. The relation $\supset$ holds p and q when p and q are both propositions, and either both are true, both are false, or p false and q true (cf. Russell 1906, 162).

Russell's views on philosophical logic, however, changed drastically in between *PoM* and *PM*, as he struggled to devise a solution to the class-theoretic and other logical paradoxes plaguing his work on the foundations of mathematics. These changes left Russell with a much sparser metaphysics of abstracta.[1] As a result, the candidate "relata" for possible purely logical relations begin to disappear. Firstly, and perhaps most importantly, according to Russell's "no class" theory, apparent terms for classes must be analyzed away using higher-order quantification. Classes are not taken as genuine *things*, and hence cannot enter into basic relations. The membership sign ϵ is no longer taken as a primitive logical constant in *PM*, and a formula of the form

$$a \epsilon \hat{x}(\phi x)$$

is, according to the stipulations of *PM*'s *20, merely an abbreviation of one of the form

$$(\exists f)((x)(f!x \equiv \phi x) \;.\; f!a)$$

In the full rendering, the only logical constants left are truth-functional connectives and quantifiers, and nothing represents any relation or property of a class treated as a genuine thing. In Russell's vocabulary, a class is a "logical fiction" and a class-term an "incomplete symbol" having no meaning in isolation.

Russell had toyed with versions of a "no class" theory as early as May 1903 (see Frege 1980, 158), but seems to have definitively settled on it as a response to the paradoxes in late 1905 (Russell 1973d, 64). By 1906, he had come to the conclusion that something very much like a "no classes" theory (see, e.g., Grattan-Guinness 1977, 89) must be applied to deal with talk of "propositions" as well. Prior to this, Russell had understood propositions as objectively real complex entities similar to states of affairs, containing the entities they are about. However, taking propositions realistically led to various paradoxes of propositions (see, e.g., Russell 1931, §500, Russell 2014d, 131–185 and *passim*; for discussion see Landini 1998, Klement 2010b). These included contingent paradoxes such as the liar paradoxes, as well as logical antinomies stemming from violations of Cantor's powerclass theorem. By it, there must be more classes of propositions than propositions. But it seems possible to generate a distinct proposition for each class thereof, for instance, the proposition that all members of that class are true. These paradoxes, and other considerations, led Russell to become increasingly wary of his realism about propositions, but his abandonment of them proceeded in stages.

---

[1] I discuss the development of Russell's views on such matters in more detail in Klement (2004) and Klement (2014).

In the 1906 "On 'Insolubilia' and their Solution by Symbolic Logic" (Russell 1973c, 207), Russell took the intermediate position of a realism about non-quantified propositions, but posited only quantified or general "statements". An manuscript from the same period summarizes:

> The philosophical ground for this view is that judgments only have objective counterparts when they are *particular*; the *general* is purely mental; all *facts* involve no apparent [bound] variables. Much to be said for this. E.g. "I met a man"; the *fact* is "I met Jones" (Russell 2014a, 562)

Russell's willingness to consider the idea that there is no objective counterpart to quantification may have been in part a result of his having abandoned the view of *PoM* according to which quantifier phrases such as "everything" or "anything" represented special entities called "denoting concepts" in favor of the new theory of meaning of 1905's "On Denoting" (1994b; cf. Russell 1994d, 385–86).

Through the next few years, (see e.g., Russell 1907, Russell 2014e) Russell's views on propositions seem somewhat up in the air. By 1910, however, Russell had settled on his new "Multiple Relation Theory of Judgment" according to which a belief is not a dyadic relation between a believer and a proposition, but a polyadic relation between a believer and the various constituents of the would-be fact potentially making it true (Russell 1992d). On this view, all propositions, quantified or elementary, are taken as mere *façons de parler*, much like classes. They too, then, could no longer be taken as entering in as relata of basic or unanalyzable relations. This is clearly incompatible with Russell's former understanding of $\supset$ and other truth-functional connectives. Indeed, while the views of *PoM* are readily compatible with the view that logical constants represent certain kinds of universals, the same is not quite so clear for Russell's post-*PM* views.

3 The Beings of the World of Logic

So if when Russell claimed in *Problems* and other works of that period that logic was concerned with "certain abstract universals" he couldn't have meant it in quite the same way he might have had he made the same claim in 1903, what did he mean?

The most extended discussion in *Problems* of a logical or mathematical truth which he claims asserts a relation between universals concerns "two and two are four":

> It is fairly obvious, in view of what has been said, that this proposition states a relation between the universal 'two' and the universal 'four'. This suggests a proposition which we shall now endeavour to establish: name-

> ly, *All* a priori *knowledge deals exclusively with the relations of universals.* (Russell 1912, 103)

In this example, the universals that this proposition is supposed to assert a relation between are *two* and *four*. But what are these? Clues come in the next paragraph, where Russell claims that the proposition may be rephrased as "any two and any other two are four" or "any collection formed of two twos is a collection of four" (104). This suggests interpreting *two* as a property certain collections have. Russell goes on to claim that the proposition must be interpreted as about the *property* rather than the collections which exhibit this property on the grounds that we are not acquainted with all couples or groups of two, and if it were about them, we could not understand it.

It is striking how simple-minded this description is in comparison to the kind of complicated analysis that would be given to "$2 +_c 2 = 4$" in his technical work. Therein, "2" and "4" would be taken as typically ambiguous representations of certain classes of classes, which would themselves require elimination by means of the contextual definition of all class-talk in terms of higher-order quantification. The fully analyzed form of this proposition is therefore *much* more complicated than the discussion in *Problems* lets on, and this disguises difficulties with the contention that what are involved here are universals. In Russell's considered view, there are no such "things" as classes or collections, so how could there be properties thereof? Number terms are incomplete symbols and numbers are "logical fictions"; all truths about them are supposed to reduce to truths about simpler entities. They cannot enter into the acquaintance relation in any direct fashion; in some sense they simply *aren't there* to do so.

There might be at least two broad kinds of explanations for why Russell might allow himself to write in this simplistic way in *Problems*. According to the first, Russell is simply meeting his audience half way. *Problems* was meant as a popular general introduction to philosophy with a large target audience; it is not a treatise for specialists in mathematical logic. In similar fashion, Russell sometimes temporarily ignored his view of historical proper names like "Socrates" and "Plato" according to which they ought to be treated as "truncated descriptions" (Russell 1992b, 152ff., Russell 1956d, 242–43) and spoke of something like "Socrates loves Plato" as if it were an atomic proposition, at least until he had explained enough of the basics of his philosophical logic for the reader to follow along with the complications that were developed later (Whitehead and Russell 1925–1927, 45). This practice is excusable, because the claims made about "Socrates loves Plato" won't depend on anything in particular about Socrates or Plato as individuals, and hence what is said about this case will transfer over to the true atomic propositions of the form *aRb*, whatever those turn out to be. One might suspect that Russell is similarly making use of a familiar, cognitively friendly, example with "two and two are four", and ignoring, for the sake of presentation, that its full analysis would bring in further complications. Again, this will be excusable provided that what is said about "two and two are four" will remain true of the more technically correct instances of the phenomena in question. But of course, this prompts us to ask: is this the case? When it comes to *fully analyzed* propositions of logic and pure mathematics, can they too be understood as asserting relations of universals?

Before taking up that question, let us consider the other sort of explanation that might be proferred for why Russell allows himself to speak here in such simple terms. On some ways of interpreting Russell's metaphysics of "logical constructions",[2] it is perhaps not quite correct to say that Russell denies whole-scale the reality of collections, or of properties, identifiable as complex universals, that would hold of collections just in case they have a certain number of members. Such things are simply non-fundamental, or derivative in some sense. Whitehead and Russell give many instances of "primitive ideas" in *PM*, but in the modern sense, the undefined logical constants of *PM* are just disjunction ($\vee$), negation ($\sim$) and the universal and existential quantifiers for the various types. Perhaps these are the only *simple* logical notions, but one can define more complicated notions in terms of them, including, according to the logicist program, class-theoretic and mathematical notions, such as, e.g., the higher-type propositional function of type $n + 1$ satisfied by all and only those (predicative) propositional functions of type n which are satisfied by exactly two things. This is more or less how numbers were described in Russell's 1911 lectures on logic at Cambridge, where Moore wrote in his notes:[3]

> Number is a property (= prop. function of) of prop. functions:
> E.g. (x is an even prime) (prop. function): 1 is a property of this; i.e. it is satisfied by 1 value of x & no more. (Moore forthcoming)

Could such propositional functions be the "universals" Russell thinks is involved in "two and two are four" as analyzed in *Problems*?

---

[2]For interpretations that might follow these lines, see, e.g., Linsky (1999, chap. 2) and Levine (2013), Levine (forthcoming).

[3]It is perhaps worth noting that these notes also contain the more usual "Frege-Russell" definition of numbers as classes of equinumerous classes, and the view taking numbers to be properties of propositional functions is even objected to on the grounds that properties are intensionally individuated. There are many distinct, but equivalent, way to formulate a higher-type propositional function that will be satisfied by $\phi\hat{x}$ just in case $\phi\hat{x}$ is itself satisfied by exactly two arguments, but there would seem to be only one number two.

Exactly how Russell understood propositional functions, and indeed, whether or not he took them to be genuine entities at all, is a matter of some controversy.[4] In at least some pre-*PM* manuscripts, Russell argues that its being nonsense to speak of a propositional function taking itself as argument is evidence that "[a] function must be an incomplete symbol" (Russell 2014f, 498) and "not a new thing over and above its values" (Russell 2014e, 363). When he first considered dropping propositions from his ontology, he warned himself "not to let functions creep back into being" (Russell 2014c, 265), intimating that taking them realistically would be as bad as a realism about propositions. At any rate, a propositional function for him would not have been taken as a *simple* or *fundamental* entity, but instead at best as a kind of complex or constructed entity. While he does sometimes use the word "property" interchangeably with "propositional function", there is significant evidence that Russell did not equate single-argument propositional functions with the kinds of simple universals he called "predicates" or "qualities".[5]

In the 1911 piece "Knowledge by Acquaintance and Knowledge by Description", Russell gives a characterization of universals compatible with the existence of complex universals:

> Among universals I include all objects of which no particular is a constituent. Thus the disjunction "universal-particular" includes all objects. We might also call it the disjunction "abstract-concrete." (Russell 1992b, 150)

As near as I can tell, Russell makes the distinction this way so that *facts* about particular existents, such as the fact that a certain sense-datum is a certain color or that one sense-datum is to the left of another, will count as particulars. Facts, for Russell, are not "logical constructions" or "logical fictions", and can enter into relations. At this time, Russell understood perception to involve a relation between a perceiver and a fact (Russell 1992d, 122–23, Whitehead and Russell 1925–1927, 43), and hence believed that perceptions were always veridical. All objects of perception would be particulars. However, those facts involving only a relation holding between relations, such as the truth maker of the proposition "priority implies diversity" (Russell 1992a, 135) would count as a complex universal on this definition, as no particular is a constituent. To my knowledge, however, Russell never speaks of any other kind of complex universals.

[4] I develop my own views in Klement (2010a) and Klement (2013).

[5] I have argued this elsewhere; see Klement (2004). For what it's worth, Linsky (1999, chap. 2) too thinks Russell is committed to differentiating between propositional functions and universals.

There is something to be said for the suggestion that it is possible to understand "two and two are four" without having a full understanding of all the logically simple entities involved in its full *PM*-style analysis. If that were required, it seems unlikely that any lay person could grasp even such a simple mathematical truth. Russell is at times open to the possibility that we might be acquainted with something complex and not be aware of its complexity. In that case, it would be a blessing to be able to read "complex universals" into the lay person's understanding of mathematical statements, and to posit some epistemological method of gaining direct insight into the properties and relations of such complex universals that doesn't require a full understanding of their complexity. According to Russell's epistemology of mathematics (see, esp. Russell 1973e), "2 + 2 = 4" is known more directly than the more general logical axioms from which it is deduced in a system such as *PM*. Indeed, he claims that the seeming obviousness of "2 + 2 = 4" can be used as epistemological evidence in favor of a certain set of fundamental logical axioms from which it and other obvious results can be deductively derived, rather than vice versa.

During this period, Russell often speaks of a distinction between "the world of logic" and "the actual world". He talks of the "world of logic" as if it were made up of a special kind of inactual or non-existent object. Consider the following passages:

> Instead of talking about "entities", we will talk about "individuals". Then propositions, classes, relations, etc. are "Gegenstände höherer Ordnung" [objects of higher-order]. As opposed to individuals, they may be called "logical objects". They are all essentially incapable of existence. (Russell 2014d, 197)

> [Individuals are s]uch objects as constitute the real world as opposed to the world of logic. They may be defined as whatever can be subject of any proposition not containing any apparent variable. (Russell 2014b, 529)

> Here the word *individual* contrasts with class, function, proposition, etc. In other words, *an individual is a being in the actual world, as opposed to the beings in the logical world*. (Russell 1992c, 44)

Because Russell *also* describes universals as entities which "subsist" rather than "exist" in the sense that particulars do during this period (Russell 1912, 100, Russell 1992e, 39, Russell 1992a, 135), some commentators have been led to the conclusion that Russell simply equates the particular/universal distinction with the individual/higher-order object distinction,[6] a view no doubt reinforced by the fact that he uses the words "particular" and "individual" interchangeably in later works (roughly those from 1918 on, e.g., Russell

[6]See, e.g., Levine (2013) and Levine (forthcoming).

1919a, 141, Whitehead and Russell 1925–1927, 2nd ed., p. xix). But I think this interpretation is mistaken. The distinction between individuals and "higher-order objects" is a distinction between those genuine entities or logical atoms which make up the irreducible building blocks of facts and those *apparent* entities which seem to be involved in various truths due to an unfinished analysis of "incomplete symbols". This is clearer perhaps in other descriptions of the difference from the period:

> We may define an individual as something destitute of complexity. (Russell 1956b, 76)
>
> For this purpose, we will use such letters as *a*, *b*, *x*, *y*, *z*, *w*, to denote objects which are neither propositions nor functions. Such objects we shall call *individuals*. Such objects will be constituents of propositions or functions, and will be genuine constituents, in the sense that they do not disappear on analysis, as (for example) classes do, or phrases of the form "the so-and-so." (Whitehead and Russell 1925–1927, 51)
>
> We may explain an individual as something which exists on its own account; it is then obviously not a proposition, since propositions . . . are incomplete symbols, having no meaning except in use. (Whitehead and Russell 1925–1927, 162)

Russell's use of the German phrase "Gegenstände höherer Ordnung" for non-individuals in the quotation above is almost certainly a reference to Meinong (1899). Meinong's "objects of higher order" are objects that are completely dependent or, to use contemporary vocabulary, "supervenient upon" or "grounded in" simpler or more basic objects. The closest one has to this in Russell's metaphysics is the notion of a logical construction, an "apparent" entity which is not really an entity at all but just a convenient way of talking about other things. For Russell, classes, functions and propositions are such things; statements that (as Russell puts it) "verbally employ classes" (Russell 1992f, 357) upon analysis, turn out "really" to be about some or all of their members, and their members' properties and relations.

I think if we want to make sense of Russell's understanding of the nature of logical truths during this period, we cannot avoid posing it eventually in terms of the nature of the "ultimate" or "primitive" notions of logic rather than the derivative or definable ones. Russell's claim that mathematical or logical "intuition" provides us with knowledge about the relations of certain universals cannot *merely* mean that it provides us with knowledge "about" derivative or higher-order "apparent" entities that disappear on analysis, like classes and numbers. Both in *Problems* (chap. VIII) and in "Analytic Realism" he writes as if, by providing this account of *a priori* knowledge, he is striking a blow against both those empiricists who deny any kind of knowledge of "abstract ideas" as well as those idealists (e.g., Kant) who think that *a priori* knowledge is knowledge only of our own forms of understanding and not of any kind of mind-independent reality. Russell's argument that our knowledge of universals is knowledge of things that are independent of the mind requires that these universals be *simple* universals, which, in virtue of their simplicity, *must* be independent of the mind:

> Universals . . . do not depend on us in any way. In the case of particulars, we have a causal dependence, but there could not be a causal dependence in the case of universals, since they do not exist in time. A logical dependence is equally impossible, since simple things do not logically depend on anything, and complex things logically depend only on their constituents. Therefore, universals are completely independent of the mind, as is everything else which exists, in the narrow sense. The laws of logic, for example, while they are customarily called "laws of thought", are just as objective, and depend as little on the mind as the law of gravity. Abstract truths express relations which hold between universals; the mind can recognize these relations, but it cannot create them. (Russell 1992a, 136)

If Russell's argument for the objectivity of logic has any bite, then the universals involved must be ones which subsist "on their own account" and are "destitute of complexity". They *must*, in effect, be individuals and not higher-order, complex or derivative entities.

The difference between particulars and universals then is not the same as the difference between individuals and the "beings of logic". Aside from the kinds of complex universals (and particulars) Russell makes room for in "Knowledge by Acquaintance and Knowledge by Description", Russell usually explains the difference between particulars and universals as the difference between those entities that can occur only as terms of a relation and those that can occur in a relational or predicating way:

> You will observe that in every complex there are two kinds of constituents: there are terms and the relation which relates them: or there might be (perhaps) a term qualified by a predicate. Note that the terms of a complex can themselves be relations, as, for example, in the statement that priority implies diversity. But there are some terms which appear only as terms and can never appear as predicates or relations. These terms are what I call *particulars*. The other terms found in a complex, those which can appear as predicates or relations, I call *universals*. Terms like diversity, causality, father, white, etc., are *universals*. (Russell 1992a, 135; cf. Russell 1911, 170)

Notice that this is essentially the same distinction as that drawn between *things* and *concepts* in §48 of *PoM*. Notice, moreover, that Russell still maintains that universals have a "two-fold nature", which is essential to Russell's doctrine of acquaintance, as explained by Landini (this volume).

A relation may occur in a relating way in a complex, but it may also occur as one of the "terms" being related in the complex. Particulars lack this two-fold nature. We saw earlier that Russell defined "individuals" as "whatever can be subject of any proposition not containing any apparent variable [i.e., an elementary proposition]" (Russell 2014b, 529; cf. Russell 1956b, 76). Individuals include whatever *may* occur as a relatum in a complex; particulars are those individuals that can *only* occur that way. These definitions leave room for (at least some, arguably all) universals to be individuals as well.[7]

4 Logical Constants and Variables

The above clarifies what sorts of universals the propositions of logic would have to assert relations between in order for Russell's account of *a priori* logical and mathematical knowledge to work. But bearing in mind the changes to his logical views after *PoM*, we are not really any closer to an understanding of how any of the "primitive ideas" found in Russell's technical writings could reasonably count as standing for such universals.

This brings us to 1911's "The Philosophical Importance of Mathematical Logic" (1992e). There Russell sketches an account not only of what distinguishes a proposition of logic from others, but also of what makes something a logical constant. The account is not far from the view of *PoM*. A proposition of pure logic is one that "does not contain any other constants than logical constants" (35), and a mathematical proposition will "only contain variables and logical constants" (38). Similarly, he explains that pure mathematical propositions typically take the form of quantified conditionals, which can then be *applied* by finding particular instances of the variables which will affirm the antecedents. One difference is that, unlike in *PoM*, in this work he attempts to provide at least a partial definition of the notion of a logical constant:

> To obtain a proposition of pure mathematics . . . we must submit a deduction of any kind to a process [of generalization] . . . that is to say, when an argument remains valid if one of its terms is changed, this term must be replaced by a variable, i.e. by an indeterminate object. In this way we finally reach a proposition of pure logic, that is to say a proposition which

[7]Russell later changes his mind on these issues and comes to the conclusion that universals can *only* occur in complexes in a relating way and never as subject, but as he himself tells us (Russell 1956d, 204–5), this is a view he adopted under the influence of Wittgenstein. Notice that in earlier writings Russell himself claims that predicates (by which he means monadic universals, not anything linguistic) are individuals, e.g., (Russell 1931, §499). Clearly, the particular/universal distinction for him is not the same as the individual/higher-order object distinction in his early writings, as I have also argued elsewhere (Klement 2004, Klement 2005).

> does not contain any other constant than logical constants. The definition of the *logical constants* is not easy, but this much may be said: A *constant* is *logical* if the propositions in which it is found still contain it when we try to replace it by a variable. More exactly, we may perhaps characterize the logical constants in the following manner: If we take any deduction and replace its terms by variables, it will happen, after a certain number of stages, that the constants which still remain in the deduction belong to a certain group, and, if we try to push generalization still farther, there will always remain constants which belong to the same group. This group is the group of logical constants. The logical constants are those which constitute pure form; a formal proposition is a proposition which does not contain any other constants than logical constants. (Russell 1992e, 35–36)

Russell illustrates with an example. One begins with a deduction such as:

All humans are mortal.
Socrates is a human.
Therefore, Socrates is a mortal.

One then forms a hypothetical proposition with the premises of the deduction as "hypotheses" (antecedents) and the conclusion as "thesis" (consequent):

> If all humans are mortal, then if Socrates is a human, then Socrates is a mortal.

One then replaces whatever constants one can with variables provided that by doing so, the result remains "valid" (presumably this means true for every value of the variable). In this case, this yields:

If all α are β, then if x is-a α, then x is-a β.

Any constants remaining after this process count as logical constants. For this example, Russell writes: "The constants here are: *is-a*, *all*, and *if-then*. These are logical constants and evidently they are purely formal concepts" (Russell 1992e, 36). What sets them apart from the non-logical constants is that the conditional would no longer be true for all values of the variable if we attempted to replace *them* with a variable. Suppose we replaced "is-a" with "R" to obtain:

If all α are β, then if $xR\alpha$, then $xR\beta$.

In that case, if we gave "R" the value "is-not-a" instead of "is-a", "α" the value "cat", "β" the value "animal" and "x" the value "Lassie", we'd have a false instance of the conditional, which shows that we cannot replace "is-a" with a variable while preserving the validity of the argument, and hence, it is a logical constant.

I find this description unhelpful. The most natural interpretation of how we are to apply it presupposes prior knowledge of what counts as a "valid" deduction.[8] Suppose someone thought "2 + 2 = 4. Therefore, snow is white" were a valid deduction. It could then be argued that "snow" and "white" are logical constants, because if we were to replace either of them with variables in "if 2 + 2 = 4 then snow is white" we'd get something no longer true for every instance of the variable. A natural account of validity—one to which Russell himself might have been attracted—presupposes a *prior* way of distinguishing logical from non-logical constants. According to the popular Tarskian account of logical consequence, $A_1, \ldots, A_n, \therefore B$ is valid just in case there is no interpretation of the non-logical constants which make all of $A_1, \ldots, A_n$ true but B false. If we let $A^*[x_1, \ldots, x_m]$ be obtained by conjoining $A_1, \ldots, A_n$ and replacing each non-logical constant with an appropriate variable (of the appropriate type) and let $B^*[x_1, \ldots, x_m]$ be obtained in similar fashion from B, then the argument will be valid just in case $\ulcorner \forall x_1 \ldots \forall x_m (A^*[x_1, \ldots, x_m] \rightarrow B^*[x_1, \ldots, x_m]) \urcorner$ is true in those models where the domain of quantification for the variables includes all possible interpretations for constants of the same type. But as Tarski was himself aware,[9] this conception of logical consequence presupposes a prior way of differentiating logical from non-logical constants. It is not clear to me which is prior—conceptually or epistemologically—my understanding of the special nature of logical constants or my understanding of in what cases an argument is valid *in virtue* of them alone, or rather *in virtue of its form*. Perhaps Russell would be sympathetic to a hybrid approach,[10] in which one seeks to identify both what the valid deductions are and what the logical constants are by attempting to achieve a kind of "reflective equilibrium", balancing the demands of both; if so, however, there is no clear indication of this in Russell's 1911 paper.

At least we here have confirmation concerning the sort of thing Russell had in mind when thinking of "purely logical concepts"—they are the sorts of things which are taken as the "primitive ideas" or undefined symbols of the formal language of *PM*, things such as truth-functional operations (e.g., *if-then*) and quantifiers (e.g., *all*). (The inclusion of *is-a* is perhaps a bit strange, since it disappears on analysis of class-talk *à la PM* *20.02, but again, this can be chalked up to the attempt to avoid delving into complex analyses for the purposes of presentation.) Frustratingly, while he concludes the essay with the remark that logic and mathematics "force us" to recognize truths "about universals", he never explicitly claims that these universals are

[8] Proops (2007, 18) gives similar reasons for worrying about Russell's definition of a logical constant.

[9] See Tarski (1983), Tarksi (1986). There's a fair bit of secondary literature here that Russell's discussion prefigures, though an even earlier anticipation of these issues is found in Bolzano (1972).

[10] Thanks to an anonymous referee for this suggestion.

the "purely formal concepts" that logical constants represent. Even more frustratingly, he does not clarify in what ways these "formal concepts" are similar to or different from other universals. As we have seen, his usual way of formulating the universal/particular distinction make universals those constituents of complexes that can occur "as predicates or relations". Are *all* and *implication* then qualities or relations, and if so, what kinds of things have these qualities or stand in these relations? "The Philosophical Importance of Mathematical Logic" does not help us answer these questions.

I shall return to the status of truth functional operations in the next section. For the moment, let us consider how Russell might have thought that quantification could make a proposition "involve" universals. A universally quantified statement typically takes the form "$(x)(\phi x \supset \psi x)$", which is naturally read as as "if x has property ϕ, then x has property ψ, for all x". In *Problems*, he discusses "all men are mortal", and claims of it that it asserts that "if x is a man, then x is mortal" so that the universals *men* and *mortal* are invoked. To understand the proposition, one must be acquainted with these universals (Russell 1912, 106). At the level of meaning, he claims that this is just like the case of "two and two are four" analyzed as making a general claim about instances of the universals "couple" and "four membered collection". We don't need acquaintance with the *values* of the variable, only the concepts or universals which the values of the variable would have to exemplify to be relevant to the truth or falsity of the quantified statement. In "Analytic Realism", he writes:

> Pure mathematics, if I am not mistaken, is concerned exclusively with propositions which can be expressed by means of universals. Instead of having constants as terms in relations, we have *variables*, i.e. we only have the concept of an entity of a certain kind instead of a particular entity of this kind. Thus to know the universal which defines a kind is to know what is necessary for pure mathematics. It follows that pure mathematics is composed of propositions which contain no actual constituents, neither psychological as idealists believe, nor physical as empiricists believe. There are two worlds, the world of existence and the world of essence; pure mathematics belongs to the world of essence. (Russell 1992a, 137–38)

And later, in discussion, he claims that "it is the variable which makes the transition from the universal to the particular" (144). Unfortunately, however, it is hard to see how the case of "all men are mortal" is supposed to be like propositions that *only* contain variables and logical constants. The word "mortal" represents "the concept of an entity of a certain kind", but the word "mortal" could not be used in pure mathematics. In pure mathematics or logic, we'd have to use *nothing but* variables, truth-functional operators and quantifiers. Instead of "$(x)(x \text{ is human} \supset x \text{ is mortal})$" one might have instead, e.g., "$(\phi)(x)(\phi!x \supset \phi!x)$". Are there any "universals which define a kind" in this latter example? One might suggest that the *type* of the variable is what is involved, but treating these as universals seems to violate the core thought in their type theory that type-restrictions are *internal* restrictions on meaningfulness (Whitehead and Russell 1925–1927, 4). "$(\phi)(x)(\phi!x \supset \phi!x)$" cannot be taken to mean "$(x)(\phi)((x$ is an individual . $\phi!\hat{x}$ is a predicative first-level propositional function) $\supset (\phi!x \supset \phi!x))$" without violating some of the basic ideas of type theory. Perhaps Russell can be read as already holding something like the *Tractatus* conception of a "formal concept" which is properly expressed by the variable itself, rather than by any kind of constant (Wittgenstein 1922, §4.127), but this may be reaching. In a 1910 letter to Bradley (quoted in Slater 1992, 350), Russell claims that "the conception of the variable is the conception of something standing midway between particular and universal; I do not pretend to have solved all the difficulties in this conception". This does not sound like someone with a firm view in mind.

More than once in *Problems* (52, 93), and also in "Knowledge by Acquaintance and Knowledge by Description" (1992b, 161), Russell claims that every complete sentence must contain at least one word for a universal. In context, Russell seems to be thinking of ordinary language non-compound ("atomic") sentences, so it is not entirely clear he'd extend the claim also to cover all closed quantified formulæ of a formal language. Let us consider a formula made up of nothing but quantifiers and variables, e.g.:

$$(\exists\phi)(\exists x)\phi!x$$

This second-order proposition is *true*[11], but which, if any, universals needed for its proper interpretation? When Russell writes in the quotation from "Analytic Realism" above, that "pure mathematics is composed of propositions which contain no actual constituents" I take it that he means that none of the entities of which propositions of pure mathematics are composed are actual (i.e., existent, or as he says there, physical or psychological), rather than that, actually, they have no constituents at all. Unfortunately, here we

[11]Indeed it is a theorem of the formal system of *PM*. It is perhaps not altogether clear that it ought to be, as it requires there to be at least one individual. Russell eventually came to regard it as a "defect in logical purity" (Russell 1919a, 203n) that one can derive results in *PM* requiring any given number of individuals, even one. But this is not important for present purposes. Whether or not it's *logically necessary*, it's certainly *true*. Our interest here lies in whether or not the presence of quantifiers or variables alone suffices to make it the case that the propositions of logic involve universals. Whether or not this counts as a proposition of logic, its proper interpretation is still relevant to the question as to whether or not quantification is always to be understood as involving universals.

run up against an unfortunate turn of phrase Russell often uses, the "constituents of a proposition."[12] He employs this turn of phrase also when formulating his "principle of acquaintance"—one must be acquainted with all the "constituents of any proposition" which one understands. Of course, Russell no longer believes in propositions as mind-independent complexes with parts, so what does it mean for something to be a constituent of a proposition? Understood as a piece of language, the parts of a proposition would just be the words or symbols making it up, but clearly that is not what Russell has in mind. He tries to clarify in "Knowledge by Acquaintance and Knowledge by Description" by invoking his multiple relations theory of judgment, writing "the constituents of the judgment are simply the constituents of the complex which is the judgment" (Russell 1992b, 154). Presumably, then, the constituents of a proposition are the constituents of the judgment which the assertion of the proposition would indicate (cf. Russell 1992d, 117). It is natural then to frame the question regarding how universals are involved in the proper understanding of quantified formulæ of a formal logical language in terms of what sorts of things are involved in the judgment complexes that subsist when we make general judgments.

Unfortunately, prior to the *Theory of Knowledge* manuscript, the multiple relations theory of judgment was only clearly formulated for elementary judgments. If I judge that aRb where "aRb" is an atomic formula, it is clear what Russell believed the relata to the judgment relation are supposed to be: me, a, R and b. But if I judge, say that "$(\exists\phi)(\exists x)\phi!x$", what are the relata to the judgment relation, and are any of them universals? All we have to go on is a brief and tortured passage from the introduction to *PM*:

> We do not mean to deny that there may be some relation of the concept *man* to the concept *mortal* which may be *equivalent* to "all men are mortal," but in any case this relation is not the same thing as what we affirm when we say that all men are mortal. Our judgment that all men are mortal collects together a number of elementary judgments. It is not, however, composed of these since (*e.g.*) the fact that Socrates is mortal is no part of what we assert, as may be seen by considering the fact that our assertion can be understood by a person who has never heard of Socrates. In order to understand the judgment "all men are mortal," it is not necessary to know what men there are. We must admit, therefore, as a radically new kind of judgment, such general assertions as "all men are mortal." We assert that, given that x is human, x is always mortal. That is, we assert "x is mortal" of *every* x which is human. Thus we are able to judge (whether

[12] I have unfortunately replicated this sad phrase in the title of my paper. Russell himself acknowledged that he had given no very exact definition to the notion of "occurring in" a proposition (Russell 1931, 2nd ed., xi).

> truly or falsely) that *all* the objects which have some assigned property also have some other assigned property. That is, given any propositional functions $\phi\hat{x}$ and $\psi\hat{x}$, there is a judgment asserting ψx with every x for which we have ϕx. Such judgments we shall call *general judgments*. (Whitehead and Russell 1925–1927, 45)

Part of what makes this passage so obscure is that there seems to be a systematic confusion of the notion of judgment with the notion of *assertion*. Can't someone make a general judgment without "asserting" anything (save perhaps in some kind of metaphorical, inward sense)? What Russell means by "collecting together" elementary judgments without actually making them individually is not adequately clarified. He seems only to have in mind quantified propositions of the form "$(x)(\phi x \supset \psi x)$", and not more or less complex forms. Existential quantification is not addressed at all. Personally, I cannot glean from this any clear reason to think that *merely* in virtue of making use of variables or quantifiers, the propositions of logic ought to be understood as somehow providing access to special logical universals.

Russell's subsequent discussion of different notions of truth that apply to quantified as opposed to elementary propositions is somewhat clearer. Whatever the make-up of the judgment complexes for general judgments, Russell is explicit that "truth makers" (in contemporary vocabulary) for general judgments are just the truth makers of their instances:

> But now take such a proposition as "all men are mortal". Here the judgment does not correspond to *one* complex, but to many, namely "Socrates is mortal," "Plato is mortal," "Aristotle is mortal," etc. (Whitehead and Russell 1925–1927, 44–45)

> If ϕx is an elementary judgment, it is true when it *points to* a corresponding complex. But $(x).\phi x$ does not point to a single corresponding complex: the corresponding complexes are as numerous as the possible values of x. (Whitehead and Russell 1925–1927, 46)

Russell at this time uses "complex" and "fact" more or less interchangeably. It is natural to think that Russell's metaphysics is exhausted by what facts there are and their components. If only elementary propositions/judgments correspond to facts, this seems to suggest that there quite simply is no metaphysical phenomenon corresponding to the logical notion of quantification.[13] Elementary complexes, which involve no quantifiers, make elemen-

[13]In the "On Substitution" manuscript, while Russell is exploring the view that there are no quantified propositions, only quantified statements, Russell writes:

> The case of $(x).\phi x$ is queer. Suppose $f(a, b, c, d)$ is a proposition of which a, b, c, d are all the constituents. Then

$$(x, y, z, w).f(x, y, z, w)$$

tary propositions true. These in turn make first-order propositions true, and they in turn make second-order propositions true, and so on up the hierarchy of different senses of truth. But then if there is nothing in reality corresponding to *all* or the variable, it seems at best to be a feature of our psychology. This seems to put him back to the view of "On 'Insolubilia' " that "the general is purely mental". At the same time he seems to all but define a logical proposition as a *fully* general one, so that the use of variables and quantifiers is not only unavoidable in logic and pure mathematics, the use thereof is fundamental to what sets logic apart from other areas of study. As Whitehead and Russell themselves write, "[t]he ideas and propositions of logic are all *general*" (Whitehead and Russell 1925–1927, 93). If the "general is purely mental", it would appear that logic is too. This sits *very uncomfortably* with Russell's ambition to cite our knowledge of logic to bolster a case for "realism in a scholastic sense".

5 A "Multiple Relations Theory" of Truth Functions?

When it comes to quantification, I do not know how to resolve these tensions in Russell's philosophy of logic circa 1910–12. However, I think there is *more hope* that Russell may have had an understanding of truth-functional operations (disjunction, negation, implication, etc.) during the *PM* period and that immediately following which could support the claim that our knowledge of logic involves knowledge of certain abstract universals. However, I must admit from the outset that what I have to say is *highly speculative*, and derives largely from hints left behind in unpublished manuscripts, quite often not even in the context of arguing for a view but explaining his misgivings about pursuing a certain hypothesis.

> is a proposition which has no constituents . . . (Russell 2014d, 136)

It was also explicit in his 1912 lectures on logic at Cambridge that he did not believe in objective counterparts of quantification. Moore's notes contain:

> Well, this being so, there is *nothing meant* by words "all" or "some" in these [quantified] props.: there is no constituent of the prop. corresponding to them. *Also* 'all men' means nothing: if there were such a thing, it is certainly not it which is asserted to be mortal.
>
> So too in 'I met *a man*' there is no *separate* thing called '*a man*' over & above the men there are; & no *man* is a constituent of the prop. And this explains how you can say there is no such thing as a centaur.

It is possible, however, that Russell had already changed his mind at the time of these lectures from *PM* itself; see the beginning of sec. 6 below.

Again, the early view of $\supset$ was that it was a dyadic relation that held between propositions *p* and *q* when both are true, both are false, or *p* false and *q* true. Russell was not unaware that this precise understanding of implication could not survive the abandonment of the old view of propositions countenancing "objective falsehoods". Indeed, in the pre-*PM* manuscripts, in his "arguments with himself" about whether or not "propositions are entities", he cited this as a reason, and indeed this comes to the fore just as much as the issue of explaining what is involved with erroneous beliefs. In "The Paradox of the Liar", he notes that on the view that propositions are not entities, negation cannot represent the property of *falsehood*, and for awhile explores the idea that "there is no such thing as 'not' ", only disbelief. He continues:

> We may suppose this a satisfactory answer, and proceed to other difficulties. The proposition "*p* implies *q*" will be all right when *p* and *q* are true, but will need a new interpretation when *p* is false. The proposition is then true, but the constituent *p* is a non-entity. We must substitute "not-*p* or *q*". But there is still a difficulty. If we hold to the view that negative statements express disbeliefs, not beliefs in negative propositions, "not-*p* or *q*" expresses either a belief or a disbelief. But this is plainly false. We *neither* believe *q* nor disbelieve *p* when we assert "not-*p* or *q*". Thus we shall have to admit the objectivity of true negative propositions. (Russell 2014e, 321)

Russell then tables the suggestion of dismissing propositions as non-entities temporarily.

In the published 1907 paper "On the Nature of Truth", Russell discusses the nature of disjunctive facts where only one disjunct is true as an argument in favor of "objective falsehood". The sign $\vee$ clearly cannot stand for a relation between facts on a view according to which there are no "false facts" or "objective falsehoods", since disjunctions are sometimes true when one disjunct is not. In such cases there would be no "fact" or "complex" to occupy the other relation spot of the $\vee$-relation. Russell writes:

> There is, however, another argument in favor of objective falsehood, derived from the case of true propositions which contain false ones as constituent parts. Take, *e.g.,* "Either the earth goes round the sun, or it does not." This is certainly true, and therefore, on the theory we are considering, it represents a *fact*, *i.e.,* an objective complex, which is not constituted by our apprehension of it. But it is, at least apparently, compounded of two (unasserted) constituents, namely: "The earth goes round the sun" and "the earth does not go round the sun" of which one must be false. Thus our fact seems to be composed of two parts, of which one is a fact, while the other is an objective falsehood. (Russell 1907, 48)

Russell's tantalizingly brief sketch of a *response* to this argument immediately follows:

> If this argument is to be rejected, it can only be on the ground that, given a fact, it cannot always be validly analysed into subordinate related complexes, even when such analysis *seems* possible. A valid analysis we shall have to contend, must break up any apparent subordinate complexes into their constituents, except when such complexes are facts. (Russell 1907, 48)

Russell is usually read here as talking about the analysis of belief or judgment facts,[14] but given the context, it is more likely that Russell is speaking about the analysis of what we would now call "molecular complexes" or "molecular facts".

The thought would be something like this. Consider a disjunction where only one disjunct is true, e.g.:

$$\text{Desdemona loves Cassio} \vee \text{Desdemona loves Othello}$$

Desdemona does not love Cassio so there is no such complex as Desdemona-loving-Cassio. Desdemona does love Othello so there is a complex Desdemona-loving-Othello. The disjunction above is true so it is a complex or fact as well. But what is the logical structure of this fact? Here we have the case of a complex fact where it might seem like its structure is one fact relating to another fact, i.e., something like $(d - L - c) - \vee - (d - L - o)$, where $(d - L - c)$ is one complex, $(d - L - o)$ is another, and $-\vee-$ is a relation that relates these two complexes. Obviously, that cannot be right here, as there is no such complex as $(d - L - c)$. So instead, a valid analysis has to "break up" the "apparent subordinate complexes into their constituents". So in this case, $-\vee-$ is not a *dyadic* relation, but a multiple relation with even greater polyadicity. Russell's precise wording in the quotation above seems to suggest it ought to be something like $(d, L, c) - \vee - (d - L - o)$ where $-\vee-$ is then a relation with *four* relata, one of which is a complex, and the others are Desdemona, Love, and Cassio, each entering in separately. Another possible (and perhaps in some ways better[15]) view would be one that treated the disjuncts in parallel fashion, so we'd have rather $(d, L, c) - \vee - (d, L, o)$, or, if you prefer, $\vee(d, L, c, d, L, o)$, i.e., a six-place relation that forms a complex with certain relata just in case either the

[14]Indeed, the paper sketches instead a very different alternative theory of judgment on which a belief consists of ideas standing in relation to one another (cf. Russell 2014d, 185), and in some ways better prefigures Russell's views in 1919 (see Russell 1919b).

[15]This view seems better in the sense that it does not require disjunctive facts to have different logical forms depending on which disjunct is true. Often times, we have knowledge of disjunctions without knowing which disjunct is true. If the logical form of what we knew was different depending on which disjunct were true, it would seem possible to determine which disjunct were true simply by analyzing the form of the disjunction.

first three, or its last three, of those relata form a simpler complex. A similar kind of analysis could be given for conjunctions and implications. I think it would be natural to call this "the multiple relations theory of truth functions". Notice that it continues to think of $\vee$, $\supset$, etc., as relations. They're just not dyadic relations between propositions, but rather multiple relations between the constituents thereof. This is completely analogous to the change his views underwent from thinking of belief as a dyadic relation between a believer and a proposition to thinking of belief as a multiple relation between a believer and the constituents of a complex (when true) instead. Like that view, it gives rise to "direction problems" (see, e.g., Griffin 1985) but those direction problems appear to be no worse here than in the case of judgment.

Russell seems halfway to this view in the "Fundamentals" manuscript, where he writes:

> The chief difficulty in the view that there are no false complexes is ... subordinate (false) complexes in true propositions, e.g., p in $p \supset q$ when $\sim p$. It seems as though, for the sake of homogeneity, we must allow that a proposition differs from a complex, and subsists equally when true and when false, but is plural, not singular: the corresponding singular (if any) is the complex, which only subsists when the proposition is true. (Russell 2014a, 542)

What does Russell mean when, here, he suggests that a proposition is "plural"? I think he means merely that a proposition is not one entity, but many entities. Those entities sometimes form a complex, but when we speak about "the proposition", we are not speaking about that complex, but rather about those entities, plural. A "property" of a proposition is not really a monadic quality of one thing but just a misleading way of describing what would now be called a "plural property", i.e., a property of *many* things. A relation between propositions "p" and "q" is not a relation between two things, but a relation between several things, as many things as p and q together make up. This is rather like Russell's distinction in *PoM* between "a class as many" and "a class as one" and is consistent with the idea that a proposition is not "an entity", just as Russell explained to Jourdain, "a *class as many* is not an entity" (Grattan-Guinness 1977, 68). It is not that propositions are nothing, it is rather that they are not *individual* things; a proposition is not an "it", but a "they".

In the manuscript "Logic in Which Propositions are Not Entities", Russell writes:

> Roughly speaking, the view that propositions are not entities amounts to this, that the predicates that can be significantly asserted of propositions are different from those that can be asserted of entities. "The Law of Con-

> tradiction is fond of cream cheese" is to be as inadmissible as "the number 1 is fond of cream cheese." I can't help thinking this would solve some problems as to the nature of truth, also the *Epimenides* and kindred puzzles. All significant propositions about propositions, on this view, will really be propositions about entities; just as propositions about classes are. A proposition about a proposition, if it can't be reduced to the form of a proposition about entities, is to be meaningless. (Russell 2014c, 265)

The words "the number 1" do not name one individual thing, and thus claiming that "it" is fond of cream cheese involves treating a monadic property as if it had a different polyadicity, and the similar claim about "the Law of Contradiction" is nonsense for precisely the same reason.

Compare what Russell writes in *PM* (48):

> A proposition is not a single entity, but a relation of several; hence a statement in which a proposition appears as subject will be significant if it can be reduced to a statement about the terms which appear in the proposition. A proposition, like a phrase of the form "the so-and-so", where it grammatically appears as subject, must be broken up if we are to find the true subject or subjects. But in such a statement as "p is a man," where p is a proposition, this is not possible. Hence "$\{(x).\phi x\}$ is a man" is meaningless.

A similar, but different in detail, approach along these lines is explored elsewhere in the "Fundamentals" manuscript. Here the "heavy lifting" seems to be done by negation rather than by the dyadic operators such as $\vee$ and $\supset$. He writes:

> We can't say "A believes ϕx" unless ϕx is true, for there is no such proposition as ϕx unless it is true. And then "believes" is used in a derivative sense. But A can believe that x has the property ϕ even when x does not have property ϕ. We shall have to say that $\phi x \supset \psi x$ means an implication when ϕx and ψx are true, but means $\sim \phi x . \vee . \psi x$ when ϕx is false and ψx is true, and means "$\sim \psi x . \supset . \sim \phi x$" when both are false, and has no independent meaning when ϕx is true and ψx is false. (Russell 2014a, 543)

I interpret this as follows. In the first part of the quotation, Russell is just sketching the basics of the multiple relations theory of judgment. Belief cannot be a relation between the believer A and the *complex* ϕx, but it can be a relation between A and ϕ and x. Negation $\sim$, rather than being a property of a single entity, a proposition, is now a *multiple relation* which forms a complex with its relata just in case its relata do not form a complex (in the right way) on their own. The relation $\supset$ holds between complexes, but which complexes? Take $dLc \supset dLo$. If this is a fact, then there are three possibilities. The first is that there are two complexes $d - L - c$ and $d - L - o$, and the complex $dLc \supset dLo$ is a relation holding between them having the form $(d{-}L{-}c){-}{\supset}{-}(d{-}L{-}o)$. Another is that there is a complex $\sim -d - L - c$ and another one $\sim -d - L - o$ and the complex corresponding to $dLc \supset dLo$ is a complex that has those two complexes as its parts, though related in the reverse order as before. Finally, we might have the one complex $\sim -d - L - c$ and the complex $d - L - o$ and the complex corresponding to the conditional statement is the holding of a different relation between these complexes, better written as $\vee$, so we have $(\sim -d - L - c) - \vee - (d - L - o)$. If none of these three possibilities obtain, i.e., if there is neither a complex $\sim -d - L - c$ nor a complex $d - L - o$, then there is no complex, i.e., no fact that $dLc \supset dLo$; in that case, one may speak of "the proposition", but when one does, one will not be naming some "thing".

These little tidbits from the manuscripts are not much to go on. Russell is constantly experimenting with various ideas in these manuscripts, and seldom do these ideas become his considered view or make it into his published writings. I admit it is highly speculative to suggest that anything like these views should be considered the official view of *PM* or the immediate period afterwards. In *PM*, when negation and disjunction are introduced they are called "the Contradictory function" and "the Disjunctive function" (6), and nowhere are they called relations (of any sort), though this appears in the first chapter of the introduction, written by Whitehead (Russell 1948, 138), who may not have seen things the same way.

In a letter to Jourdain dated 2 January 1911, Russell writes:

> I no longer think it significant to deny
> $$x \supset q,$$
> where x is not a proposition. I think that, strictly, one ought not to use a single letter for a proposition, but always some such symbol as ϕx. But so long as this is remembered, it is not necessary always to do what strictly ought to be done. (Grattan-Guinness 1977, 136)

Landini (1998, 258) takes this to mean that *PM*'s $\supset$ and $\vee$ should not be read as any kind of relation symbols, but rather as statement connectives in the modern sense, flanked by formulæ to form formulæ. But this is by no means the only way of interpreting this letter. The reason one cannot write "$x \supset y$", where these are variables for individuals, is that $\supset$ is not a *dyadic* relation; "$x \supset y$" does not give it enough relata. In order to "do what strictly ought to be done", one must make use of a *complex* symbol for a proposition so that one has an indication of *all* the relata that enter into the relation. In practice, however, Russell seems to think it safe to ignore this "strict method" to make it, e.g., easier to state axiom schemata and rules in a uniform way, as is done in *PM* itself. This is not unlike the use of single letters as variables for classes which can be misleading with regard to the strictly correct philosophical analysis of classes, but is convenient in practice. The suggestion that propositions are *best* not represented by single letters was not a new idea. Russell claimed the same thing as early as the "Logic in Which Propositions are Not Entities" manuscript, where immediately after writing the passage quoted above, he wrote:

> Formally, propositions must not be expressed, to begin with, by simple letters, but by ϕx ... (Russell 2014c, 265)

I think this is perhaps evidence that Russell's way of thinking of the matter in 1911 had not changed much since these pre-*PM* manuscripts.

Another bit of confirming evidence can be found in the fact that Whitehead and Russell never speak of a distinction between "molecular" and "atomic" in *PM*. There are elementary propositions, and then there are quantified propositions. They describe "elementary judgments" as follows:

> We will give the name of "a *complex*" to any such object as "*a* in the relation *R* to *b*" or "*a* having the quality *q*", or "*a* and *b* and *c* standing in the relation *S*". Broadly speaking, a *complex* is anything which occurs in the universe and is not simple. We will call a judgment *elementary* when it merely asserts such things as "*a* has the relation *R* to *b*", "*a* has the quality *q*" or "*a* and *b* and *c* stand in the relation *S*". Then an elementary judgment is true when there is a corresponding complex, and false when there is no corresponding complex. (44)

The use of negation and disjunction is not enough to raise a proposition above the level of an elementary proposition, only quantifiers can do that. So why don't Whitehead and Russell mention molecular forms when introducing elementary judgments? One obvious answer is that they thought that those too could be considered as asserting relations. On the first view considered in this section, consider what an embedded disjunction would be, if true:

$$aRb \vee (cSd \vee eTf)$$

Suppose now that e bears relation T to f. Then there is a complex $e - T - f$. But *this* is not a component of the fact that $cSd \vee eTf$; that complex instead has the form $\vee(c, S, d, e, T, f)$. But *that* complex isn't a component of the fact for the whole disjunction either. It rather has the form $\vee(a, R, b, \vee, c, S, d, e, T, f)$![16] We could, if we wish to use later terminology, claim that this complex is still "atomic", but it would be less misleading

[16]The internal $\vee$ here is just like the internal J one would get if one were to analyze a judgment about judgment under the multiple relations theory of judgment. If the form of the fact that Mary judges that aRb is $J(m, a, R, b)$, then the form of my judging that Mary judges that aRb has the form $J(k, J, m, a, R, b)$.

to say that there simply is no atomic/molecular distinction: there are only elementary propositions and complexes.

At first, it might seem as if this understanding of truth functions would only be adequate to molecular propositions where quantifiers do not appear subordinate to truth functions. Especially if it is right, as I argued in the previous section, that there is no ontological correlate of quantification for Russell during this period, it might be unclear what the relata to the disjunctive relation would be in a case such as:

$$aRb \vee (x).xSd$$

Indeed, Whitehead and Russell claim that "negation and disjunction and their derivatives have a different meaning" when applied to quantified formulæ (Whitehead and Russell 1925–1927, 127). However, the way they proceed is to define negation and disjunctions of quantified formulæ in terms of negations and disjunctions of non-quantified formulæ, so that ultimately, all quantifiers are "pulled out" to the front of formulæ, resulting in prenex normal forms. Thus, according to the definition *9.04 of *PM*, the above disjunction is definitionally equivalent with:

$$(x)(aRb \vee xSd)$$

There is then nothing preventing us from understanding *this* ∨ as a multiple relation. In this way, disjunctions and negations with subordinate quantifiers are wholly eliminated. It is plausible to suppose that the definitions of *9 are designed at least in part to preserve the understanding of truth functions as multiple relations and to explain how it might be extended to cover what appear to be other kinds of cases.

If this, again, highly speculative reading of truth functions in *PM* is correct, it has the advantage of vindicating Russell's contention that there are at least *some* logical universals with which we are acquainted and are involved in the analysis of the propositions of logic. Here, ⊃, ~, ∨ and friends, although not understood quite as simply as they had been in *PoM*, still count as *relations* not in any significant way different from other relations. There nothing keeping us from understanding them as mind-independent, as far as I can tell.

6 "What is Logic?" and *The Theory of Knowledge*

Our efforts to make sense of the claim in *Problems* that logical knowledge consists of knowledge about certain abstract logical universals have garnered mixed results. When it comes to truth functions, Russell's understanding seems at least compatible with this conclusion. When it comes to other notions important for logic—quantification and variables—Russell seems to have inconsistent commitments. It may be that Russell was misled by simpler cases of quantification such as "all men are mortal" into thinking that the final analysis of all quantified statements would reveal universals, or entities "standing between" universals and particulars, to be the entities which their understanding would require. As he made clear in the letter to Bradley, he had not thought his way completely through all the issues to which the themes of quantification and variation give rise, but he was confident enough for the purposes of a popular piece to outline, in a programmatic sort of way, a basic account of *a priori* knowledge that gave pride of place to our acquaintance with universals. However, he realized the matter needed further thought, and intended to launch further investigations when he had the leisure.

In February 1912, Russell lectured on the nature of logic at Cambridge, and attending were Moore and Wittgenstein (a new figure on the scene at this time). Therein, he summarized the view of logical constants from "The Philosophical Importance of Mathematical Logic" according to which they are those for which variables cannot be substituted without spoiling the validity of a deduction. Moore's notes continue (Moore forthcoming):

> *Logical* constants are *not* the sort of constants wh[ich] can be substituted:
> e.g., *or*, *not*, *true*, 0, 1, 2 etc.
> All of these are incomplete symbols. (I think, but am not sure).
> But this = all the ideas of logic & mathem. are *meaningless*.

It seems that Russell had already begun to doubt that there are any specifically logical entities, a trend that would become more pronounced over the next year. Given the professed uncertainty, it would probably be unwise to read much into this remark, though it is intriguing.

Russell began seriously to think of writing a piece entitled "What is Logic?" in September 1912 and made an abortive attempt to do so in October. All that remains is a rather short manuscript in which Russell does little more than reveal his own confused state of mind. Apparently, whatever confidence he had in the basic view outlined in *Problems* and works of that period was gone. It shows, however, that Russell had spotted the core tension in his former position. Putting variables and variation at the centerpiece of his characterization of logic, while at the same time thinking of quantification or generality as a purely linguistic or mental phenomenon, was inconsistent with his basic realist leanings. To avoid a position on which logic would collapse back into mere "laws of thought", it would have to be possible to state the subject matter of logic in objective terms, as involving complexes, not beliefs or judgments or "propositions" understood in a linguistic fashion. Given his theory of truth at the time this meant that truth could not be a central concern for the logician:

> Difficulties of supposing there are objective falsehoods compels us to suppose that what *can* be *false* must be judgments or forms of words. Logic is not concerned with forms of words. Hence logic is not concerned with propositions.
> *True* and *False* are extra-logical. (Russell 1992g, 55)

Russell had always connected the subject matter of logic with "forms" but the exact relationship remained to be spelled out. We saw, for example, that in "The Philosophical Importance of Mathematical Logic", Russell had claimed that the "logical constants are those which constitute pure form". His prior understanding of the relationship between forms and the nature of the truths of logic seems to have been tied to the importance of generality in logic. In pure mathematics, one "generalizes" as much as possible, replacing whatever constituents of a proposition or judgment one can with a variable, so that the result depends not on the particular subject matter or content of the proposition, but only what remains when that particular content has been abstracted way from—the form. Quantified conditionals are called "formal implications" because they identify a group of propositions all having a common form and assert all of them; the propositions of pure mathematics are formal implications of the highest degree of generality.

But what exactly is a form, and is it an "entity" distinguishable from those propositions (early on) or complexes (later on) that have it? This is not something Russell had discussed much in published writings, and there is no indication that he would have had a consistent answer over the years. In 1904 manuscripts, Russell spoke of "modes of combination" in the following way:

> A complex is determined by its constituents together with their mode of combination; it is not determined by the constituents alone. E.g. "*A* is greater than *B*" and "*B* is greater than *A*" have the same constituents, but differently combined.
>
> The mode of combination of the constituents of a complex is not itself one of the constituents of the complex. For if it were, it would be combined with the other constituents to form the complex; hence we should need to specify the mode of combination of the constituents with their mode of combination . . .
>
> A mode of combination, like everything else, is an entity; but it is not one of the entities occurring in a complex composed of entities combined in the mode of combination. Thus e.g., in the case of "*A* is greater than *B*", the mode of combination may be denoted by $\hat{x}\hat{R}\hat{y}$ . . . (Russell 1994c, 98)

As might be evident from the circumflection notation he uses, Russell at this time thought of modes of combinations in terms of propositional functions. Propositional functions were understood as proposition-like complexes where one or more constituents is replaced by a variable, and modes of combination are those where *all* constituents are replaced by variables. Prior to 1905, Russell had a realist understanding of in variables, taking them as something similar to denoting concepts (Russell 1931, §93, Russell 1994e, 330, 335). On this view, he could maintain that speaking about a mode of combination itself as opposed to a complex having it is much like speaking of a denoting concept or denoting complex itself rather than what it denotes (Russell 1994a, 128–29). But Russell abandoned his former view of denoting concepts in "On Denoting" (1994b), arguing in its infamous Gray's Elegy passage that any attempt to disambiguate between a denoting complex and what it means must fail.

At any rate, by the time of "What is Logic?" Russell no longer had a non-linguistic understanding of variables to employ in understanding the nature of forms. Thus he wrote:

> A *form* is not a mere symbol: a symbol composed entirely of variables symbolizes a form, but is not a form. (Russell 1992g, 55–56)

However, echoing his 1904 understanding of modes of combination, he claims that forms are not constituents of the complexes that have them:

> In a complex, there must be something, which we may call the *form*, which is *not* a constituent, but the way the constituents are put together. If we made this a constituent, it would have to be somehow related to the other constituents, and the way in which it was related would really be the form; hence an endless regress. Thus the form is not a constituent. Take e.g. "Antony killed Brutus". We may put *a* for Antony, etc., and get "*aRb*". (Russell 1992g, 55)

A view Russell might have considered, both early and late, is one according to which a form is just a *sui generis* entity. But I think Russell realized he needed to be careful. To reify forms would be very close to reifying propositional functions, to treat them as genuine things—individuals—rather than constructions of some sort, which might reintroduce the paradoxes. One must imagine Russell thinking to himself: "Consider the form of any complex which is the fact that a form does not stand in itself to itself. Does it stand in itself to itself?" Given his treatment of classes, functions, propositions, etc., as logical constructions, it seems that Russell was interested in thinking of forms that way too. A cardinal number, for example, is just a roundabout way of talking about all those collections of "like cardinality" as if that were a single thing. Perhaps a form is just a roundabout way of talking about all those complexes of "like form". So in "What is Logic?", Russell considers defining not forms, but the relation sameness-of-form, and using that to define what would make a form a "logical" one:

> Two complexes "have the same form" if one can be obtained from the other by mere substitution of new terms in other places. Df.
>
> . . .
>
> A complex is *logical* if it remains a complex whatever substitutions may be effected in it. Df. (Russell 1992g, 55)

But he immediately saw objections to the approach. What could it mean for a complex to "remain" when substitutions are effected in it? If substitution is a relation $C\frac{x}{y}!C'$ holding when exchanging x for y in C yields C', then this relation is one that only holds between complexes, so it will not be possible for C' not to "remain" a complex, unless the substitution relation is put in terms of symbols or propositions-as-pluralities, etc., instead. But this would make logic again a study of language or thought, which Russell wanted to avoid. Moreover, if forms are "logical fictions" derived from speaking at once about all those complexes having the same form, it would be impossible to speak of forms that no complex has, such as $x \neq x$.

In the end, Russell abandoned work on the manuscript, despairing in a letter to Ottoline Morrell:

> I can't get on with "what is logic?", the subject is hopelessly difficult, and for the present I am stuck. I feel very much inclined to leave it to Wittgenstein. (Quoted in Slater 1992, 54)

Try as he might, however, Russell could not completely escape the issue. Wittgenstein had developed a keen interest in the themes involved here, and one suspects it would have been a frequent subject of discussion between them whether Russell would have wanted it to be or not. Moreover, Russell had been attempting to deal with issues in epistemology, and needed to give an account both of our acquaintance with "logical data" and of the role our understanding of logical form plays in understanding and judgment. Russell's struggles are evident throughout in the *Theory of Knowledge* manuscript (hereafter *ToK*).

Part I of *ToK* gives the impression of a Russell who is still deeply uncertain about the exact status of logical notions, i.e., whether or not there are any "entities" of logic, and if so, whether or not they stand in relations to other entities of the normal sort. Russell is perfectly happy even to admit his ignorance when it comes to such issues:

> It should be said, to begin with, that "acquaintance" has, perhaps, a somewhat different meaning, where logical objects are concerned, from that which it has when particulars are concerned. Whether this is the case or

> not, it is impossible to decide without more knowledge concerning the nature of logical objects than I possess. (Russell 1984, 97)
>
> Such words as *or*, *not*, *all*, *some*, plainly involve logical notions; and since we can use such words intelligently, we must be acquainted with the logical objects involved. But the difficulty of isolation here is very great, and I do not know what the logical objects involved really are.
>
> In the present chaotic state of our knowledge concerning the primitive ideas of logic, it is impossible to pursue this topic further. (Russell 1984, 99)

Somewhat surprisingly, however, as the book progresses, Russell seems to "find his feet" and adopts a more and more committal position especially with regard to the notion of a form in Part II.[17]

One change of mind evident during this period is that Russell has scrapped the idea that there are any "logical universals" even when it comes to truth-functional operators. Most likely, Russell came to appreciate that there was a significant tension in his earlier views according to which logic is fully general, and therefore not about any *specific things*, and the view that there are certain specific relations or other universals of special interest to the logician, such as $\sim$, $\vee$ and $\supset$. It seems likely that Wittgenstein was an influence here as well. He wrote the following to Russell as early as June 1912:

> ... one thing gets more and more obvious to me: The propositions of Logic contain ONLY *apparent* variables and whatever may turn out to be the proper explanation of apparent variables, its consequences *must* be that there are NO *logical* constants.
>
> Logic must turn out to be a *totally* different kind than any other science. (Wittgenstein 1979, 120)

Logic must be totally different in the sense that it must not be about any specific individuals, whether universal *or* particular: in short, it must not have its own subject matter in the sense that other sciences do. The upshot of this point for Russell's views at the time is expressed in the *Tractatus* this way:

> At this point it becomes manifest that there are no 'logical objects' or 'logical constants' (in Frege's and Russell's sense). (§5.4)
>
> It is self-evident that $\vee$, $\supset$, etc. are not relations in the sense in which right and left etc. are relations. (§5.42)

Whether it was due to Wittgenstein, or his own realization of the prior tension, in *ToK*, Russell finally rejects his earlier position (assuming my inter-

---

[17]Cf. Griffin (1980, 167–68) on the apparent change of attitude that seem to occur as the book progresses.

pretation in the last section was correct) that truth-functional operators stand for a kind of relation. Russell now writes:

> A proposition which mentions any definite entity, whether universal or particular, is not logical: no one definite entity, of any sort or kind, is ever a constituent of any truly logical proposition. "Logical constants", which might seem to be entities occurring in logical propositions, are really concerned with pure *form*, and are not actually constituents of the propositions in the verbal expression of which their names occur. (Russell 1984, 97–98)

Landini (this volume) takes these passages as *ToK* that there are no constituents in facts corresponding to logical particles even as early as *PM*. I think instead that they show a change in mind. Whereas, as I stressed earlier, in *PM* he spoke only of elementary complexes, now Russell begins to mark a distinction between "atomic" and "molecular" complexes (Russell 1984, 80) and along with this distinction, a distinction between atomic and molecular thought (176), though he abandoned the project before writing everything he initially intended about molecular thought.

Another change involves Russell's attitudes about general or quantified facts. As I argued earlier, the theory of truth for quantified formulæ of various orders in *PM* seems to leave in place the result of "On 'Insolubilia' " that "judgments only have objective counterparts when they are particular" and that "the general is purely mental". Russell, however, wishes to maintain the insight of "What is Logic?" that logic must concern itself not with forms of judgments or propositions but rather forms of things which are fully objective—complexes and facts. Although not much is said in *ToK* about the precise nature of general or existential facts, Russell now believes there must be such things. This is especially important for the new "form-centric" (rather than "universal-centric") conception of logic Russell wishes to develop. Russell suggests that forms might be identified with certain existential facts. I.e., the form that atomic complexes have when they involve two things related by a dyadic relation, e.g., Desdemona loving Cassio, is identified with the *fact* that $(\exists x)(\exists y)(\exists R).xRy$ (Russell 1984, 114). Although the linguistic representation of this fact is complex, Russell does not conceive of this fact as complex. All the constituents of the complex made up of Desdemona, Love and Cassio have been generalized away. Forms, despite being objective facts, are simple entities.

There is still quite a bit that is obscure and problematic here. Russell does not address the problem mentioned in "What is Logic?" with regard to *impossible forms*. There is no such fact as $(\exists x).x \neq x$. At first, one might think this is not such a problem after all. Russell is interested in forms of *complexes*, not forms of judgments or propositions. There is no such complex as "Socrates ≠ Socrates" (or anything else of this *so-called* form), and so there is no objective entity which one would be tempted to think "has" this form. Nonetheless, however, it would seem that contradictions or other impossibilities would fall under the purview of what logic studies, which is surely in part what Russell hoped to invoke forms to explain. Another puzzle he mentions is "[w]hy, if pure forms are simple, is it so obviously inappropriate to give them simple proper names such as John and Peter?" (130), but puts it aside with the excuse that his interest in that work is epistemological rather than logical.

Another worry I have is that by reintroducing objective realities corresponding to quantified formulæ, Russell is forgetting the reason he first abandoned quantified propositions in 1906. Are there as many "objective-complexes" as there are classes thereof, or other "paradoxes" of complexes? It scarcely seems to matter whether we phrase the problems in terms of propositions or in terms of facts, and it hardly seems to matter that there are no "false facts". For each class of facts *m*, for example, there will either be a fact *all members of m are subject-predicate in form* or a fact *not all members of m are subject-predicate in form*, and hence, it seems, as many facts as classes thereof. What blocks the resulting diagonal contradiction? Is there a hierarchy of facts about which the same things cannot meaningfully be said? (E.g., is it not even *meaningful* to say that a quantified fact is subject-predicate in form?) There are all sorts of responses Russell might have given to this worry, but it is hard to tell what his answer would have been from what is written in *ToK*.

7 Later Views

We know that Russell abandoned the *ToK* project in mid-1913, largely due to criticisms made by Wittgenstein.[18] Most of the secondary literature has focused on Wittgenstein's criticisms of Russell's theory of judgment. These are important and I would not wish to downplay them. However, I think Wittgenstein's attack was probably more broad based, and involved also whether or not the conception of logical forms and their connection to logical constants Russell gave in *ToK* was tenable.[19] In *ToK*, Russell attempted to

[18]The evidence for this comes mainly from Russell's letters to Ottoline Morrell, summarized nicely by Eames in her introduction to *ToK* (xvii–xx).

[19]These issues are of course not unrelated, especially if, in the background, is something like a "multiple relations" theory of logical particles as I sketched in sec. 5. I think it would be not unfair to say that Wittgenstein's attack was on Russell's account of *propositions* as incomplete symbols as it played out both in his theory of judgment and in his account of the nature of logic.

make good on his understanding of logic as fully general by denying that logical constants represent definite entities which might be thought of as constituents or components of facts (or judgments). Instead, we are now told, their contribution is purely formal. But forms themselves, Russell tells us, can be identified with certain facts, and this, Russell tells us fulfills the desideratum that a form "is not a mere incomplete symbol" (Russell 1984, 114). Facts are there all right, they are not mere *façons de parler*, and so, neither are forms. It would appear that by attempting to eliminate any kind of specifically logical objects, Russell has simply swapped out one kind of logical object for another. Forms may themselves have properties and stand in relations to one another, or to our minds (through, e.g., acquaintance or judgment). If one were to catalog all the facts there are, one would have to include in the catalog facts which are themselves *about* forms. But what then does logic study—all the facts in the catalog equally, or principally those facts which are *about* forms? Is logic a special science or not? If there are forms, and there are facts about them separate from the facts about concrete individuals, *they* would appear to be the subject matter of logic. We seem to have the following paradox: logic is the science which has no specific subject matter, and hence its subject matter is form. This appears contradictory. The subject matter of logic cannot *both* be "nothing in particular" *and* be form, unless forms are themselves nothing.

In later works, Russell seems to be aware of the difficulty, but finds it difficult to reconcile the tension. In *Our Knowledge of the External World*, Russell still endorses what might be called a form-centric account of the subject matter of logic, but at least at some points wants to describe logical knowledge as a kind of knowledge of something. At the same time, however, it is supposed to be a very *different* kind of knowledge:

> In every proposition and in every inference, there is, besides the particular subject-matter concerned, a certain *form*, a way in which the constituents of the proposition or inference are put together. ... It is forms, in this sense, that are the proper object of philosophical logic.
>
> ...
>
> It is obvious that the knowledge of logical forms is something quite different from knowledge of existing things. ... some kind of knowledge of logical forms, though with most people it is not explicit, is involved in all understanding of discourse. It is the business of philosophical logic to extract this knowledge from its concrete ingredients, and to render it explicit and pure. (Russell 1956c, 41)

Elsewhere he specifically mentions the issue of how logic differs from other sciences.

> If the theory that classes are merely symbolic is accepted, it follows that numbers are not actual entities, but that propositions in which numbers verbally occur have not really any constituents corresponding to numbers, but only a certain logical form which is not a part of propositions having this form. This is in fact the case with all the apparent objects of logic and mathematics. Such words as *or*, *not*, *if*, *there is*, *identity*, *greater*, *plus*, *nothing*, *everything*, *function*, and so on, are not names of definite objects, like "John" or "Jones," but are words which require a context in order to have meaning. All of them are *formal*, that is to say, their occurrence indicates a certain form of a proposition, not a certain constituent. "Logical constants," in short, are not entities; the words expressing them are not names, and cannot significantly be made into logical subjects except when it is the words themselves, as opposed to their meanings, that are being discussed.[*Footnote: "In the above remarks I am making use of unpublished work by my friend Ludwig Wittgenstein."] This fact has a very important bearing on all logic and philosophy, since it shows how they differ from the special sciences. But the questions raised are so large and so difficult that it is impossible to pursue them further on this occasion. (Russell 1956c, 161–62)

It seems clear that something has changed from *ToK*. He claims not only that logical constants are not names of entities, which had already been his view in *ToK*, but that, in some sense, it is not even *possible* to speak of their meanings in a direct fashion. This would seem to imply that not only are there no "logical universals" which can be spoken of, there are not even any "logical forms" which can be spoken of. This is dangerously close to the "logical mysticism" that is sometimes read into the *Tractatus*.[20] However, the thought is not developed.

In works of the 1918–19 period, he seems more content to state the problem than attempt to solve it. E.g., in his "Philosophy of Logical Atomism" lectures, he writes:

> It is not a very easy thing to see what are the constituents of a logical proposition . . . it seems as though all the propositions of logic are entirely devoid of constituents. I do not think this can be quite true. But then the only other thing you can seem to say is that the *form* is a constituent, that propositions of a certain form are always true: that *may* be the right analysis, though I much doubt whether it is.
>
> . . .
>
> I can only say, in conclusion, as regards the constituents of logical propositions, that it is a problem which is rather new. There has not been much opportunity to consider it. I do not think any literature exists at all which

[20]There are of course different interpretations of the *Tractatus*'s final position on this matter. I do not mean to be taking sides in such disputes here.

> deals with it in any way whatever, and it is an interesting problem. (Russell 1956d, 239)

Similarly, in *Introduction to Mathematical Philosophy*, he poses the subject matter of logic as a "problem which is easier to state than solve" (198). He describes, in more or less the same way he always has, how propositions of logic are derived from generalization by substituting variables for definite terms. Again, he claims that logical constants do not represent constituents of propositions, but are purely formal. He surmises that words *for* forms can always be dispensed with (200). However, he refuses either to assert that propositions of logic have no constituents or to assert that forms are constituents of propositions of logic, intimating that this is an unsolved problem. In both works, he adds a new wrinkle, which is that he no longer thinks that a proposition being fully general and true is enough to make it a logical truth, as in the case of propositions claiming that the universe contains a certain number of individuals. He claims that there must be some additional property of being a *tautology*, which he demurs from attempting to define, and simply claims it is a concept Wittgenstein had been working on and perhaps might eventually be able to explain.

Even though Russell is not prepared in this period to admit that one simply cannot speak of forms as being "constituents of" certain propositions, it seems that he would now reject the view of *ToK* that forms are themselves facts. Russell now claims that facts are not the sorts of things one can "name" (Russell 1956d, 187), a view he also claims to have been the result of the influence of Wittgenstein. If forms were facts, then he would certainly be committed to denying that they entered into such relations as "being constituent of", or were "there" to name somehow. He also claims that no facts are simple, all are complex (Russell 1956d, 202), which is again, clearly incompatible with the *ToK* view.

Nonetheless, during this period, Russell still seems to maintain that, somehow, logic studies objective reality, that it "is concerned with the real world just as truly as zoology, though with its more abstract and general features" (Russell 1919a, 169). However, he seems to have thought that this study could only be carried out successfully in a kind of *oblique* way. Rather than attempting to study form directly, we instead focus our attention on symbols. The necessity for this is perhaps evident already from the quotation from *Our Knowledge* above; since forms cannot be named, we can perhaps at best discuss the symbols that represent these forms. Yet he still seems to think it possible, though only "perhaps once in six months for half a minute", at least in thought if not in language, to break through and think about the real subject-matter of logic, adding that "[t]he rest of the time you think about the symbols, because they are tangible" (Russell 1956d, 185).

At some point, Russell gave up even the hope for such semiannual communions with the world of form, and concluded that mathematics and logic were purely linguistic in nature. It is difficult to pin down exactly when this might have been. Ray Monk (1996, 568–69) dates this change to Russell's meetings to discuss the (then published) *Tractatus* in December 1919. Monk draws our attention to a review Russell published soon thereafter in which he claims that the laws of logic "are concerned with symbols" and that they involve "different ways of saying the same thing" (Russell 1988d, 405). It seems somewhat hasty to me to conclude that Russell then thought that logic was *purely* linguistic, even if it primarily dealt with symbols. Through the 1920s, Russell believed that when two logical expressions have the same meaning that this owed to a "relation between their forms" (Russell 1988c, 129), and he continued to think that it is "easier to think about words than about what they stand for" (Russell 1988b, 169), suggesting that he perhaps continued to believe that investigation of symbols obliquely brought us knowledge about form. In the introduction to the *Tractatus* (Russell 1988a, 111), he suggested that ascending a hierarchy of languages might provide a "loophole" making it possible to say things Wittgenstein claimed could not be said, which includes statements about logical form.

At least by the 1930s, however, he seems to have concluded that there is nothing at all in objective reality corresponding to logical particles. He then held that they contribute to the syntax of expressions, not their extra-linguistic meaning, writing that they "must be treated as part of the language, not as part of what the language speaks about" (Russell 1931, 2nd ed. intro., xi). He adds elsewhere that the "non-mental world" can be "completely described" without use of the words "or", "not", "all" and "some" (Russell 1996, 362). In the early 1950s he wrote that "All the propositions of mathematics and logic are assertions as to the correct use of a certain small number of words. This conclusion, if valid, may be regarded as an epitaph on Pythagoras" (Russell 1973b, 306). Elsewhere, he admits that "I have therefore ceased to hope to meet 'if' and 'or' and 'not' in heaven" (Russell 1951, 41). The reasons he gave in these later works for these conclusions, however, are sketchy and not very convincing. It is one thing to suggest that these issues *can* be approached by discussing linguistic phenomena rather than objective phenomena. There are facts in language, and those facts have logical forms just as any other facts would. It is quite another to insist that when we do study the structure of language we must not at the same time be gaining knowledge of something extra-linguistic as well, something the structure of language has *in common* with reality as a whole. To my knowledge, Russell never fully explained why he moved away from such a position.

I think the paradox that logic seems at once to have no specific subject matter and that *this therefore is* its subject matter is one to which there is no obvious answer, and *because of this* it is one to which philosophers must continue to give serious thought.

References

Bolzano, Bernard, 1972. *Theory of Science*. Oxford: Oxford University Press. Translated by Rolf George; originally published in 1837 as *Wissenschaftslehre* (Sulzbach: J. E. von Seidel).

Frege, Gottlob, 1980. *Philosophical and Mathematical Correspondence*. Edited by Hans Kaal. Chicago: University of Chicago Press.

Grattan-Guinness, Ivor, ed., 1977. *Dear Russell–Dear Jourdain*. New York: Columbia University Press.

Griffin, Nicholas, 1980. "Russell on the Nature of Logic (1903–1913)." *Synthese* 45: 117–88.

———, 1985. "Russell's Multiple Relation Theory of Judgment." *Philosophical Studies* 47: 213–48.

Klement, Kevin C., 2004. "Putting Form Before Function: Logical Grammar in Frege, Russell and Wittgenstein." *Philosopher's Imprint* 4: 1–47.

———, 2005. "The Origins of the Propositional Functions Version of Russell's Paradox." *Russell* 24: 101–32.

———, 2010a. "The Functions of Russell's No Class Theory." *Review of Symbolic Logic* 3: 633–664.

———, 2010b. "Russell, His Paradoxes, and Cantor's Theorem: Part II." *Philosophy Compass* 5: 29–41.

———, 2013. "*PM*'s Circumflex, Syntax and Philosophy of Types." In *The Palgrave Centenary Companion to Principia Mathematica*, edited by Nicholas Griffin and Bernard Linsky, pp. 218–46. New York: Palgrave Macmillan.

———, 2014. "The Paradoxes and Russell's Theory of Incomplete Symbols." *Philosophical Studies*, forthcoming.

Landini, Gregory, 1998. *Russell's Hidden Substitutional Theory*. Oxford: Oxford University Press.

Landini, Gregory, this volume. "Types* and Russellian Facts".

Levine, James, 2013. "On the Interpretation of *Principia Mathematica* and the Multiple–Relation Theory of Judgment." In *The Palgrave Centenary*

Companion to Principia Mathematica, edited by Nicholas Griffin and Bernard Linsky, pp. 247–304. New York: Palgrave Macmillan.

———, forthcoming. "Russell on Types and Universals."

Linsky, Bernard, 1999. *Russell's Metaphysical Logic*. Stanford: CSLI Publications.

Meinong, Alexius, 1899. "Über Gegenstände höherer Ordnung und deren Verhältnis zur inneren Wahrnehmung." *Zeitschrift für Psychologie und Physiologie der Sinnesorgane* 21: 182–272.

Monk, Ray, 1996. *Bertrand Russell: The Spirit of Solitude 1872–1921*. New York: The Free Press.

Moore, G. E., forthcoming. "Russell on Philosophy of Mathematics: Notes 1911–12." Edited by James Levine.

Moore, G. H., ed., 2014. *The Collected Papers of Bertrand Russell*, vol. 5. London: Routledge.

Proops, Ian, 2007. "Russell and the Universalist Conception of Logic." *Noûs* 41: 1–32.

Russell, Bertrand, 1906. "The Theory of Implication." *American Journal of Mathematics* 28: 159–202.

———, 1907. "On the Nature of Truth." *Proceedings of the Aristotelian Society* 7: 28–49.

———, 1911. "On the Relations of Universals and Particulars." In Russell (1956a), pp. 105–124. (First published 1911.)

———, 1912. *The Problems of Philosophy*. London: Home University Library.

———, 1919a. *Introduction to Mathematical Philosophy*. London: George Allen & Unwin.

———, 1919b. "On Propositions: What They Are and How They Mean." In Russell (1956a), pp. 283–320. (First published 1919.)

———, 1931. *The Principles of Mathematics*. Cambridge: Cambridge University Press. (First edition 1903.)

———, 1948. "Whitehead and *Principia Mathematica*." *Mind* 57: 137–38.

———, 1951. *Portraits from Memory*. New York: Simon and Schuster.

———, 1956a. *Logic and Knowledge*. London: Allen and Unwin. Edited by R. C. Marsh.

———, 1956b. "Mathematical Logic as Based on the Theory of Types." In Russell (1956a), pp. 57–102. (First published 1908.)

———, 1956c. *Our Knowledge of the External World*. New York: Mentor Books, 2nd ed. (First edition 1914.)

———, 1956d. "The Philosophy of Logical Atomism." In Russell (1956a), pp. 175–281. (First published 1918.)

———, 1973a. *Essays in Analysis*. Edited by Douglas Lackey. New York:

George Braziller.
———, 1973b. "Is Mathematics Purely Linguistic?" In Russell (1973a), pp. 295–306. (Written 1951–52.)
———, 1973c. "On 'Insolubilia' and their Solution by Symbolic Logic." In Russell (1973a), pp. 190–214. (First published 1906 in French as "Les paradoxes de la logique".)
———, 1973d. "On Some Difficulties in the Theory of Transfinite Numbers and Order Types." In Russell (1973a), pp. 135–64. (First published 1906.)
———, 1973e. "The Regressive Method of Discovering the Premises of Mathematics." In Russell (1973a), pp. 272–83. (First published 1907.)
———, 1984. *Theory of Knowledge: The 1913 Manuscript*. Edited by E. R. Eames and K. Blackwell. London: Routledge.
———, 1988a. "Introduction to *Tractatus Logico-Philosophicus*, by L. Wittgenstein." In Slater (1988), pp. 101–12. (First published 1921.)
———, 1988b. "Logical Atomism." In Slater (1988), pp. 162–79. (First published 1924.)
———, 1988c. "Physics and Perception." In Slater (1988), pp. 126–33. (First published 1922.)
———, 1988d. "Wisdom of our Ancestors." In Slater (1988), pp. 403–6. (First published 1920.)
———, 1992a. "Analytic Realism." In Slater (1992), pp. 132–146. (First published 1911.)
———, 1992b. "Knowledge by Acquaintance and Knowledge By Description." In Slater (1992), pp. 147–66. (First published 1911.)
———, 1992c. "On the Axioms of the Infinite and of the Transfinite." In Slater (1992). (First published 1911.)
———, 1992d. "On the Nature of Truth and Falsehood." In Slater (1992), pp. 115–24.
———, 1992e. "The Philosophical Importance of Mathematical Logic." In Slater (1992), pp. 32–42. (First published 1911.)
———, 1992f. "Some Explanations in Reply to Mr. Bradley." In Slater (1992), pp. 349–57. (First published 1910).
———, 1992g. "What is Logic?" In Slater (1992), pp. 54–56. (Written 1912.)
———, 1994a. "Fundamental Notions." In Urquhart (1994), pp. 111–263. (Written 1904.)
———, 1994b. "On Denoting." In Urquhart (1994), pp. 414–427. (First published 1905.)
———, 1994c. "On Functions." In Urquhart (1994), pp. 96–110. (Written 1904.)
———, 1994d. "On Fundamentals." In Urquhart (1994), pp. 359–413.

(Written 1905.)

———, 1994e. “On Meaning and Denotation.” In Urquhart (1994), pp. 314–358. (Written 1903.)

———, 1996. “The Relevance of Psychology to Logic.” In *The Collected Papers of Bertrand Russell*, edited by John G. Slater, vol. 10, pp. 362–70. London: Routledge. (First published 1938.)

———, 2014a. “Fundamentals.” In Moore (2014), pp. 536–570. (Likely written 1907.)

———, 2014b. “Individuals.” In Moore (2014), pp. 529–535. (Likely written 1907–1908.)

———, 2014c. “Logic in which Propositions are not Entities.” In Moore (2014), pp. 262–267. (Likely written 1906.)

———, 2014d. “On Substitution.” In Moore (2014), pp. 126–235. (Likely written 1906.)

———, 2014e. “The Paradox of the Liar.” In Moore (2014), pp. 317–374. (Likely written 1906.)

———, 2014f. “Types.” In Moore (2014), pp. 498–514. (Likely written 1907.)

Slater, John G., ed., 1988. *The Collected Papers of Bertrand Russell*, vol. 9. London: Unwin Hyman.

———, 1992. *The Collected Papers of Bertrand Russell*, vol. 6. London: Routledge.

Tarksi, Alfred, 1986. “What are Logical Notions?” *History and Philosophy of Logic* 7: 143–154.

Tarski, Alfred, 1983. “The Concept of Logical Consequence.” In *Logic, Semantics, Metamathematics*, pp. 409–20. Indianapolis: Hackett. (First published 1936.)

Urquhart, Alasdair, ed., 1994. *The Collected Papers of Bertrand Russell*, vol. 4. London: Routledge.

Whitehead, A. N. and Bertrand Russell, 1925–1927. *Principia Mathematica,* 3 vols. Cambridge: Cambridge University Press. (First edition 1910–1913.)

Wittgenstein, Ludwig, 1922. *Tractatus Logico-Philosophicus*. London: Routledge. Trans. D. Pears & B. McGuinness, 1961.

———, 1979. *Notebooks 1914–1916*. Chicago: University of Chicago Press, 2nd ed. Ed. G. H. von Wright and G. E. M. Anscombe.