Does Chance Undermine Would?

Alexander W. Kocurek* Forthcoming in *Mind*

Abstract. Counterfactual skepticism holds that most ordinary counterfactuals are false. The main argument for this view appeals to a 'chance undermines would' principle: if ψ would have some chance of not obtaining had ϕ obtained, then $\phi \mathrel{\Box\!\!\rightarrow} \psi$ is false. This principle seems to follow from two fairly weak principles, viz., that 'chance ensures could' and that $\phi \mathrel{\Box\!\!\rightarrow} \psi$ and $\phi \mathrel{\Diamond\!\!\rightarrow} \neg\psi$ clash. Despite their initial plausibility, I show that these principles are independently problematic: given some modest closure principles, they entail absurdities. Moreover, on the most promising strategy for saving these principles, they do not, in the relevant sense, entail the chance-undermines-would principle. Instead, they entail a principle that only supports *counterfactual indeterminism*, the view that most ordinary counterfactuals are chancy, i.e., not settled true. I demonstrate this by developing an indeterminist semantics that vindicates the clash and chance-ensures-could principles but not the chance-undermines-would principle. This view, I argue, offers a better account of our credal and linguistic judgments than counterfactual skepticism.

1 Introduction

Consider the following counterfactual:

(1) If I were to jump, I would come down.

Intuitively, (1) seems true. However, here is an argument that it is not. According to quantum mechanics, there is a small chance at any given moment that the particles constituting my body all quantum tunnel to a faraway place. It's a very small chance, to be sure—so small that it's almost certain that no such event has or ever will occur anywhere in the universe. Still, according to contemporary physics, there is a chance it could happen. So (1) is not true: if I were to jump, I could quantum tunnel to the moon.

Recently, Hájek (ms) has argued along these lines for **counterfactual skepticism**, the view that most of the counterfactuals we utter in ordinary conversation are false (see also Hájek 2014, 2020a,b). Whenever ψ would have a chance of not obtaining had ϕ obtained, the counterfactual $\phi \mathrel{\Box\!\!\rightarrow} \psi$ is false. But if we live in a chancy world, then unless ϕ nomologically entails ψ, there will almost always be some chance that ψ wouldn't obtain had ϕ obtained. Thus, most ordinary counterfactuals are false. In a slogan: *chance undermines would*.

*Thanks to Mathias Boehm, David Boylan, Melissa Fusco, Simon Goldstein, Alan Hájek, Harold Hodes, Ethan Jerzak, Justin Khoo, Theo Korzukhin, Matt Mandelkern, Sven Neth, Carlotta Pavese, Rachel Rudolph, Paolo Santorio, Ginger Schultheis, Will Starr, and two anonymous referees for helpful comments on earlier versions of this paper. This paper was presented at the Virtual Language Workshop (2019), the Cornell Semantics Reading Group (2019), a colloquium at Syracuse University (2020), the Cornell-Rochester Linguistics Workshop (2020), and PhLiPonLine (2021). I am thankful to the audiences at these venues for their feedback.

The chance-undermines-would principle can be justified by two further principles of counterfactual reasoning that are widely endorsed. The first states there is a 'clash' between would-counterfactuals and their might-not (or could-not) counterparts: it's impossible for both $\phi \boxright \psi$ and $\phi \diamondright \neg \psi$ to be true. The second can be summarized in the slogan *chance ensures could*: whenever ψ would have a chance of obtaining had ϕ obtained, the could-counterfactual $\phi \diamondright \psi$ is true. These two principles are extremely plausible. So it is surprising that they seem to lead to counterfactual skepticism.

There have been a number of attempts to resist these conclusions (e.g., Lewis 1986; DeRose 1999; Bennett 2003; Hawthorne 2005; Edgington 2008; Ichikawa 2011; Leitgeb 2012a,b, 2013; Moss 2013; Lewis 2016; Stefánsson 2018; Sandgren and Steele 2020). By and large, they accept the chance-undermines-would principle but deny that it entails counterfactual skepticism (by positing, e.g., ambiguity, context-shifts, primitive counterfacts, etc.). They typically do not reject the clash or chance-ensures-could principles.

In this paper, I take a different approach to resisting counterfactual skepticism. There are two notions of consequence that are relevant to assessing the argument from chance: preservation of truth and preservation of settled truth (i.e., chance 1). On the first, the argument is valid, but the clash principle fails. On the second, the argument is invalid: the clash and chance-ensures-could principles do not entail the chance-undermines-would principle. They only entail a principle that supports **counterfactual indeterminism**, the view that most ordinary counterfactuals are *chancy*, i.e., *not settled true*. On this view, counterfactuals are similar to claims about the future. Most claims about the future are unsettled, but that doesn't mean they're false. So too, I argue, for counterfactuals.

The paper has three parts. The first part raises a puzzle that challenges the principles used in the argument from chance. The other two parts constitute an abductive argument in favor of counterfactual indeterminism over counterfactual skepticism: the best response the skeptic has to this puzzle is deeply problematic, and there is a strictly better non-skeptical (indeterminist) account that avoids these problems.

In the first part (§§2–3), I show that the clash and chance-ensures-could principles used in the argument from chance are problematic independently of counterfactual skepticism. Specifically, I construct a dilemma: either ϕ is compatible with a non-zero chance that $\neg \phi$, or it's not. Either way, these principles lead to absurd results given some very modest closure principles. This presents a problem not just for counterfactual skeptics, but also for any of their rivals who endorse these principles.

In the second part (§§4–6), I present what I take to be the most promising way out of the dilemma for the counterfactual skeptic. Utilizing an analogy between chance-talk and epistemic modals, I develop a skeptical semantics that resolves the dilemma. This is done by distinguishing two notions of consequence (preservation of truth and preservation of settled truth), each rejecting a different closure principle. Both notions validate the chance-undermines-would principle by equating bare counterfactuals $\phi \boxright \psi$ with counterfactuals of the form $\phi \boxright \mathrm{ch}(\psi) = 1$.

In the third part (§§7–9), I argue for an alternative theory. Equating $\phi \boxright \psi$ with $\phi \boxright \mathrm{ch}(\psi) = 1$ turns out to have problematic consequences for the interaction between counterfactuals, chance, and credence. Instead, I propose a non-skeptical account that resolves the dilemma in the same way, viz., by distinguishing two notions of consequence.

This account does not equate $\phi \mathrel{\Box\!\!\rightarrow} \psi$ with $\phi \mathrel{\Box\!\!\rightarrow} \text{ch}(\psi) = 1$. It therefore offers a better explanation of our credal and linguistic judgments concerning counterfactuals. Moreover, the chance-undermines-would principle fails on both notions of consequence. It fails for the same reason that the corresponding principle for claims about the future fails, viz., lack of settled truth does not imply falsehood. So at best, the argument from chance shows that most counterfactuals are unsettled, not that they're false.

2 The Argument from Chance

First, let's review the argument from chance for counterfactual skepticism in more detail.[1] The argument assumes that the universe is chancy: the laws of nature plus the initial conditions of the universe do not determine a unique future state of the universe. Perhaps there are reasons to doubt this assumption, but I won't question it here.

Suppose it's objectively chancy whether a particular coin comes up heads when flipped. As it happens, the coin is never flipped; but it could have been flipped. Consider (2):

(2) If the coin were flipped, it would land heads.
Flip $\mathrel{\Box\!\!\rightarrow}$ *Heads*

Many have the intuition that there is something wrong with (2). Hájek thinks, first, that what's wrong with (2) is that it is false, and second, that the reason (2) is false applies to most ordinary counterfactuals.

To defend the first claim, Hájek appeals to the following widely accepted principle:

Clash. $\phi \mathrel{\Box\!\!\rightarrow} \psi, \phi \mathrel{\Diamond\!\!\rightarrow} \neg\psi \vDash \bot$

This principle classically follows from a more general principle that many have endorsed (though Hájek himself is explicit that he does not need to assume it in his argument):

Duality. $\phi \mathrel{\Diamond\!\!\rightarrow} \psi \mathrel{=\!\!|\!\!\vDash} \neg(\phi \mathrel{\Box\!\!\rightarrow} \neg\psi)$

It is further supported by the fact that sentences like (3) sound bad:

(3) ??If I were to jump, I would come down, and if I were to jump, I could fail to come down.

Using Clash, Hájek argues (2) is false because (4) is incontrovertibly true:

(4) If the coin were flipped, it could land tails.
Flip $\mathrel{\Diamond\!\!\rightarrow}$ *Tails*

As for the second claim, Hájek notes that (4) seems true simply because the outcome of the coin flip is objectively chancy. It does not matter whether the coin is fair: even if the coin were extremely biased towards heads, as long as there'd be *some chance* that it would

[1] Hájek presents two such arguments: one based on chance and another based on underspecificity. In what follows, I will set aside the argument from underspecificity: everything I say here is compatible with the success (or the failure) of that argument. Our main focus is the argument from chance.

land tails if it were flipped, then it *could* have done so. Though Hájek does not articulate the point in these terms, we can make this precise with the following natural principle:

Chance Ensures Could. $\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\psi) \neq 0 \vDash \phi \mathrel{\Diamond\!\!\rightarrow} \psi$

Both Clash are Chance Ensures Could are extremely plausible. Yet they classically entail the following skepticism-generating principle:

Chance Undermines Would. $\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\neg\,\psi) \neq 0 \vDash \neg(\phi \mathrel{\Box\!\!\rightarrow} \psi)$

Thus, most ordinary counterfactuals are false: there's almost always *some* chance that their consequents would not have obtained had their antecedents obtained.[2]

One objection is that the truth of $\phi \mathrel{\Box\!\!\rightarrow} \psi$ does not require that the chance of ψ (had ϕ obtained) be 1: it suffices that the chance would be very high. This amounts to rejecting either Chance Ensures Could or Clash and instead endorsing:

High Chance Ensures Would. $\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\psi) > t \vDash \phi \mathrel{\Box\!\!\rightarrow} \psi$ where $0.5 < t < 1$ is high.

But as Hájek observes (as does Hawthorne (2005)), High Chance Ensures Would leads to absurd results when combined with another principle that is nearly universally accepted:

Agglomeration. $\phi \mathrel{\Box\!\!\rightarrow} \psi, \phi \mathrel{\Box\!\!\rightarrow} \chi \vDash \phi \mathrel{\Box\!\!\rightarrow} (\psi \wedge \chi)$

Informally, the problem is that High Chance Ensures Would (with Agglomeration) predicts that if ϕ were the case, *nothing improbable would happen* since it rules out every sufficiently

[2] Hájek also gives a direct argument for Chance Undermines Would: to deny it is to misunderstand chance.

> There is no particular way that this chancy process would turn out, were it to be initiated. In the words of Jeffrey (1977), 'there is no telling whether the coin would have landed heads up on a toss that never takes place. That's what probability is all about'. To think that there is a fact of the matter of how the coin would land is to misunderstand chance. (Hájek, ms, p. 5)

However, it's unclear what exactly is being misunderstood about chance. One could give a parallel argument for claims about the future: 'To think there is a fact of the matter of how the coin will land is to misunderstand chance'. Yet Hájek seems to think otherwise for claims about the future (see the discussion of Horn 1 in §3). If it is not a mistake to think there is a fact of the matter how the coin *will* land, why should it be a mistake to think there is a fact of the matter how the coin *would* land? One answer: as Hájek puts it, '*nothing* counterfactually determines how that chance process would be resolved' (Hájek, ms, p. 7). But arguably, nothing determines how a chance process will be resolved either, so it's not clear why that's a barrier to truth. Another answer: as Hájek (2020a, p. 11) notes, future contingents 'all face their moment of reckoning' when their truth value is settled, whereas for chancy counterfactuals, there is no such moment to point to. (Thanks to an anonymous referee for pointing out this response.) While this seems to be a real disanalogy, the question is whether it explains why it's a mistake to deny Chance Undermines Would, and it is not clear that it does.

Thus, the anti-skeptic can reasonably deny that they 'misunderstand chance' if they reject Chance Undermines Would. (See also Stefánsson 2018, p. 883 for further criticism.) By contrast, it seems much harder for the anti-skeptic to deny Clash or Chance Ensures Could. For this reason, the 'indirect' argument that appeals to these principles strikes me as more forceful. So this will be version of the argument from chance discussed in what follows.

With that said, there is a sense in which the counterfactual indeterminist can agree with Hájek's line of thought here (even though it doesn't validate Chance Undermines Would): there's no *settled* fact as to how chancy processes would turn out were they initiated (see footnote 39).

improbable outcome. But it also predicts if ϕ were the case, *something improbable would happen* since the chance that nothing improbable happens is quite low. These predictions can't both be correct.

To illustrate, let's say $t = 0.999$ for the sake of argument. Suppose we have a coin that has a 0.9999 bias towards heads. Now consider the following series:

(5) a. If the coin were flipped 1 billion times, it would land heads on the first flip.
$\mathit{1\ Billion\ Flips} \mathrel{\Box\!\!\rightarrow} \mathit{Heads}_1$
b. If the coin were flipped 1 billion times, it would land heads on the second flip.
$\mathit{1\ Billion\ Flips} \mathrel{\Box\!\!\rightarrow} \mathit{Heads}_2$
c. If the coin were flipped 1 billion times, it would land heads on the third flip.
$\mathit{1\ Billion\ Flips} \mathrel{\Box\!\!\rightarrow} \mathit{Heads}_3$
$\vdots$

Since the coin's bias surpasses the threshold, High Chance Ensures Would predicts that all of the counterfactuals in (5) are true. By Agglomeration, these collectively entail:

(6) If the coin were flipped 1 billion times, it would land heads all 1 billion times.
$\mathit{1\ Billion\ Flips} \mathrel{\Box\!\!\rightarrow} (\mathit{Heads}_1 \wedge \mathit{Heads}_2 \wedge \cdots \wedge \mathit{Heads}_{10^9})$

But given the bias of the coin, the chance that it would land heads 1 billion flips in a row is $(0.9999)^{10^9} \approx 2.4 \cdot 10^{-43432}$—way below the threshold t. So High Chance Ensures Would also predicts (6) is false, contradiction. Thus, one cannot respond to the argument from chance by replacing Chance Ensures Could with High Chance Ensures Would.

At any rate, it is hard to deny Chance Ensures Could and Clash: both seem like incredibly plausible principles. And most of the responses to the argument from chance do not question these principles but instead attempt to block the move from Chance Undermines Would to counterfactual skepticism—e.g., by positing an ambiguity in could-counterfactuals (Lewis, 1986), giving counterfactuals a probabilistic semantics (Edgington, 2008; Leitgeb, 2012a,b, 2013), appealing to context-shifts when woulds and their contrary could-nots are assessed (Ichikawa, 2011; Lewis, 2016; Sandgren and Steele, 2020), or stipulating the existence of primitive counterfacts explaining the truth of would-counterfactuals (Hawthorne, 2005; Stefánsson, 2018). Almost none of the responses to the argument from chance deny Chance Ensures Could or Clash.[3]

3 Dilemma

Despite their widespread acceptance, the combination of Chance Ensures Could and Clash is problematic independently of counterfactual skepticism. In this section, I present a dilemma that demonstrates this. The horns of the dilemma turn on whether we accept the following principle, which states that 'chancy contradictions' of the form $\phi \wedge \mathrm{ch}(\neg \phi) \neq 0$ are genuine contradictions:

[3] The main exceptions are Lewis (1986), who denies Chance Ensures Could on the relevant reading, and DeRose (1999) and Moss (2013), who deny Clash and instead hold that $\phi \mathrel{\Box\!\!\rightarrow} \psi$ and $\phi \mathrel{\Diamond\!\!\rightarrow} \neg \psi$ are merely unassertible. See Mandelkern 2019; Hájek ms for criticism of the latter.

Chancy Contradiction. $\phi \wedge \text{ch}(\neg\,\phi) \neq 0 \vDash \bot$

I'll have more to say about this principle in §4. For now, what I want to show is this: whether or not we accept Chancy Contradiction, the combination of Chance Ensures Could and Clash entails absurd results given some modest closure principles.

Horn 1: Reject Chancy Contradiction. For this horn, I appeal to the following principles:

Identity. If $\phi \not\vDash \bot$, then $\vDash \phi \mathrel{\Box\!\!\rightarrow} \phi$

Separation. If $\phi \not\vDash \bot$, then $\phi \mathrel{\Box\!\!\rightarrow} (\psi \wedge \chi) \vDash (\phi \mathrel{\Box\!\!\rightarrow} \psi) \wedge (\phi \mathrel{\Box\!\!\rightarrow} \chi)$

These principles entail the following natural principle:

Distribution. If $\phi \not\vDash \bot$, then $\vDash (\phi \wedge \psi) \mathrel{\Box\!\!\rightarrow} \phi$ and $\vDash (\phi \wedge \psi) \mathrel{\Box\!\!\rightarrow} \psi$

These principles are very plausible: they have about as much claim to universal acceptance as Agglomeration. Yet Chance Ensures Could, Clash, and Distribution together entail Chancy Contradiction.[4]

Horn 1. Assume $\phi \wedge \text{ch}(\neg\,\phi) \neq 0 \not\vDash \bot$ for reductio.

1.	$\vDash (\phi \wedge \text{ch}(\neg\,\phi) \neq 0) \mathrel{\Box\!\!\rightarrow} \phi$	Distribution
2.	$\vDash (\phi \wedge \text{ch}(\neg\,\phi) \neq 0) \mathrel{\Box\!\!\rightarrow} \text{ch}(\neg\,\phi) \neq 0$	Distribution
3.	$\vDash (\phi \wedge \text{ch}(\neg\,\phi) \neq 0) \mathrel{\Diamond\!\!\rightarrow} \neg\,\phi$	Chance Ensures Could, 2
4.	$\vDash \bot$	Clash, 1, 3

[4] An anonymous referee observes a different route to Horn 1 via another widely endorsed principle:

True-True. $\phi \wedge \psi \vDash \phi \mathrel{\Box\!\!\rightarrow} \psi$.

By True-True, $\phi \wedge \text{ch}(\neg\,\phi) \neq 0 \vDash \phi \mathrel{\Box\!\!\rightarrow} \text{ch}(\neg\,\phi) \neq 0$. By Chance Ensures Could, $\phi \wedge \text{ch}(\neg\,\phi) \neq 0 \vDash \phi \mathrel{\Diamond\!\!\rightarrow} \neg\,\phi$. Already this is a bad result; but furthermore, by Clash, $\phi \wedge \text{ch}(\neg\,\phi) \neq 0 \vDash \neg(\phi \mathrel{\Box\!\!\rightarrow} \phi)$. By Identity, $\phi \wedge \text{ch}(\neg\,\phi) \neq 0 \vDash \bot$.

Since True-True and Chance Ensures Could already lead to bad results, this suggests that one of them is the culprit. Ultimately, I reject True-True (though only for formulas with chance operators, and only for what I later call 'classical' consequence; similarly for modus ponens). But even if we respond by rejecting Chance Ensures Could, we can still derive Chancy Contradiction using a weakened version of Chance Ensures Could, which does not conflict with True-True:

Chance Weakly Ensures Could. $\neg\,\phi, \phi \mathrel{\Box\!\!\rightarrow} \text{ch}(\psi) \neq 0 \vDash \phi \mathrel{\Diamond\!\!\rightarrow} \psi$.

Using Chance Weakly Ensures Could in place of Chance Ensures Could in Horn 1, we can show the following: if $\phi \wedge \text{ch}(\neg\,\phi) \neq 0$ is consistent, then it is valid. For assume $\phi \wedge \text{ch}(\neg\,\phi) \neq 0 \not\vDash \bot$. Then:

1.	$\vDash (\phi \wedge \text{ch}(\neg\,\phi) \neq 0) \mathrel{\Box\!\!\rightarrow} \phi$	Distribution
2.	$\vDash (\phi \wedge \text{ch}(\neg\,\phi) \neq 0) \mathrel{\Box\!\!\rightarrow} \text{ch}(\neg\,\phi) \neq 0$	Distribution
3.′	$\neg(\phi \wedge \text{ch}(\neg\,\phi) \neq 0) \vDash (\phi \wedge \text{ch}(\neg\,\phi) \neq 0) \mathrel{\Diamond\!\!\rightarrow} \neg\,\phi$	Chance Weakly Ensures Could, 2
4.′	$\neg(\phi \wedge \text{ch}(\neg\,\phi) \neq 0) \vDash \bot$	Clash, 1, 3′
5.′	$\vDash \phi \wedge \text{ch}(\neg\,\phi) \neq 0$	reductio, 4′
6.′	$\vDash \phi$ and $\vDash \text{ch}(\neg\,\phi) \neq 0$	conjunction elimination, 5′

Now, observe that by the laws of probability, $\text{ch}(\cdot) \neq 0$ is closed under logical equivalence. Since $\vDash \phi$, it follows that $\neg\,\phi$ is equivalent to $\bot$. And since $\vDash \text{ch}(\neg\,\phi) \neq 0$, we have $\vDash \text{ch}(\bot) \neq 0$ by closure under equivalence. But by the laws of probability, $\vDash \text{ch}(\bot) = 0$. Combining these two, we obtain $\vDash \bot$.

This horn is particularly pressing for Hájek, as he seems to reject Chancy Contradiction. Indeed, Hájek thinks that ϕ is even compatible with $\mathrm{ch}(\neg\phi) = 1$:

> There is no contradiction between a counterfactual with a probabilistic consequent and the opposite 'might' counterfactuals, even when the probability is high, just as there is no contradiction between '$P(X) = x$' and 'not X', even when the probability is high—even as high as 1... The only thing that can contradict a probability statement is a contradiction, or another probability statement (that attributes a different probability), or something that entails such a statement. (Hájek, ms, p. 63)

Rejecting Chancy Contradiction is difficult for the counterfactual skeptic, however. For one thing, if we reject Chancy Contradiction, then Horn 1 requires us to give up Clash, Chance Ensures Could, or Distribution. Arguably, Distribution is the most plausible of the three.[5] Furthermore, rejecting Chancy Contradiction would undercut the intuitions in the argument from chance. The skeptic reasons that if I were to jump, there's a chance I wouldn't come down, so it can't be true that if I were to jump, I *would* come down. But if chancy contradictions are consistent, that doesn't immediately follow: my coming down is consistent with there being a chance I don't come down. In that case, it's unclear why it's inconsistent to say that if I were to jump, I would come down and also there'd be a chance I wouldn't come down. The intuition that there's a conflict between $\phi \mathrel{\Box\!\!\rightarrow} \psi$ and $\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\neg\psi) \neq 0$ seems to be based on a conflict between ψ and $\mathrm{ch}(\neg\psi) \neq 0$ more generally. This suggests that the counterfactual skeptic ought to be at least sympathetic towards Chancy Contradiction.

Horn 2: Accept Chancy Contradiction. For this horn, I appeal to the following:

Could-Closure. If $\psi \vDash \chi$, then $\phi \mathrel{\Diamond\!\!\rightarrow} \psi \vDash \phi \mathrel{\Diamond\!\!\rightarrow} \chi$

Though this principle is not often discussed, it follows from two more basic principles that many accept (though we won't assume them below), viz., Duality and Would-Closure:

Would-Closure. If $\psi \vDash \chi$, then $\phi \mathrel{\Box\!\!\rightarrow} \psi \vDash \phi \mathrel{\Box\!\!\rightarrow} \chi$

Both of these closure principles hold on many of the standard semantic theories for counterfactuals in the literature. And in any case, Could-Closure is plausible on its own terms.[6] For instance, the reasoning from (7a) to (7b) seems impeccable:

[5] In fact, we can sharpen Horn 1 by using Chance Undermines Would directly (rather than appeal to Chance Ensures Could and Clash). Assume $\phi \wedge \mathrm{ch}(\neg\phi) \neq 0 \nvDash \bot$ for reductio. Then:

1.	$\vDash (\phi \wedge \mathrm{ch}(\neg\phi) \neq 0) \mathrel{\Box\!\!\rightarrow} \phi$	Distribution
2.	$\vDash (\phi \wedge \mathrm{ch}(\neg\phi) \neq 0) \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\neg\phi) \neq 0$	Distribution
3.	$\vDash \neg((\phi \wedge \mathrm{ch}(\neg\phi) \neq 0) \mathrel{\Box\!\!\rightarrow} \phi)$	Chance Undermines Would, 2
4.	$\vDash \bot$	1, 3

Thus, even if the counterfactual skeptic has an independent argument for Chance Undermines Would, they must still either reject Distribution or accept Chancy Contradiction.

[6] Will Starr (p.c.) points out that Could-Closure may seem less plausible when considering free choice inferences. To illustrate, observe that the following inference seems plausible:

(7) a. If I were to jump, I could fall and break my leg.
b. If I were to jump, I could break my leg.

Now, observe that Chancy Contradiction entails $\phi \vDash \text{ch}(\neg\phi) = 0$. (I'll return to the plausibility of this consequence later.) Thus, by Could-Closure, $\phi \Diamond\!\!\rightarrow \psi \vDash \phi \Diamond\!\!\rightarrow \text{ch}(\neg\psi) = 0$. But this can then be used to generate absurdities given Chance Ensures Could and Clash. Specifically, we can derive the following trivializing principle:

No Counterfactual Chance. $\phi \Box\!\!\rightarrow \text{ch}(\psi) \neq 0, \phi \Box\!\!\rightarrow \text{ch}(\neg\psi) \neq 0 \vDash \bot$

Horn 2.

1.	$\phi \Box\!\!\rightarrow \text{ch}(\psi) \neq 0 \vDash \phi \Diamond\!\!\rightarrow \psi$	Chance Ensures Could
2.	$\phi \Diamond\!\!\rightarrow \psi \vDash \phi \Diamond\!\!\rightarrow \text{ch}(\neg\psi) = 0$	Chancy Contradiction and Could-Closure
3.	$\phi \Box\!\!\rightarrow \text{ch}(\psi) \neq 0 \vDash \phi \Diamond\!\!\rightarrow \text{ch}(\neg\psi) = 0$	1, 2
4.	$\phi \Box\!\!\rightarrow \text{ch}(\neg\psi) \neq 0, \phi \Diamond\!\!\rightarrow \text{ch}(\neg\psi) = 0 \vDash \bot$	Clash
5.	$\phi \Box\!\!\rightarrow \text{ch}(\neg\psi) \neq 0, \phi \Box\!\!\rightarrow \text{ch}(\psi) \neq 0 \vDash \bot$	3, 4

Already, this is a bad result: No Counterfactual Chance entails, for instance, that the following counterfactuals are inconsistent, where they patently are not.

(8) a. If the coin were flipped, it would have some chance of landing heads.
b. If the coin were flipped, it would have some chance of landing tails.

But we can make this result worse. For if $0 < n < 1$, then $\text{ch}(\psi) = n \vDash \text{ch}(\psi) \neq 0$ and $\text{ch}(\psi) = n \vDash \text{ch}(\neg\psi) \neq 0$. So by Would-Closure and line 5, $\phi \Box\!\!\rightarrow \text{ch}(\psi) = n \vDash \bot$.[7] Not only is this implausible, it is essential to Hájek's error theory that counterfactuals of the form $\phi \Box\!\!\rightarrow \text{ch}(\psi) = n$ can be true (Hájek, ms, p. 76). His explanation for why it is useful to assert counterfactuals despite the fact that they're mostly false is that their chancy-counterparts can be true. But Horn 2 seems to show that the chancy-counterparts are never true.

Triviality Result. By combining these two horns, we can sharpen this dilemma into a triviality result.[8] The two horns can be summarized as follows:

Horn 1. Clash, Chance Ensures Could, and Distribution entail Chancy Contradiction.

(i) a. If the coin were flipped, it could land on heads or on tails.
b. $\Rightarrow$ If the coin were flipped, it could land on heads (tails).

If that's right, though, then Could-Closure must fail for reasons familiar from the literature on free choice. For by Could-Closure, $\phi \Diamond\!\!\rightarrow \psi \vDash \phi \Diamond\!\!\rightarrow (\psi \vee \chi)$. So if also $\phi \Diamond\!\!\rightarrow (\psi \vee \chi) \vDash (\phi \Diamond\!\!\rightarrow \psi) \wedge (\phi \Diamond\!\!\rightarrow \chi)$ (the $\Diamond\!\!\rightarrow$-counterpart of free choice), then $\phi \Diamond\!\!\rightarrow \psi \vDash \phi \Diamond\!\!\rightarrow \chi$ for any ϕ, ψ, and χ, which is absurd. However, the instances of Could-Closure appealed to in Horn 2 do not rely on free choice reasoning. So I will set aside these concerns with Could-Closure in what follows.

7 Even worse, substituting ϕ with $\text{ch}(\psi) = n$, we derive $\vDash \neg(\text{ch}(\psi) = n \Box\!\!\rightarrow \text{ch}(\psi) = n)$, contra Identity.

8 Thanks to an associate editor for suggesting this formulation of the problem.

Horn 2. Clash, Chance Ensures Could, Chancy Contradiction, and Could-Closure entail No Counterfactual Chance.

Moreover, No Counterfactual Chance and Distribution entail:[9]

No Chance. $\mathrm{ch}(\phi) \neq 0, \mathrm{ch}(\neg \phi) \neq 0 \vDash \bot$

Combining these observations together, we get the following triviality result:

Triviality Result. Clash, Chance Ensures Could, Distribution, and Could-Closure entail No Chance.

Yet to accept No Chance is to give up on the very idea that the world is objectively chancy.

Thus, given some modest closure principles, Clash and Chance Ensures Could lead to absurd results. This is a problem not just for counterfactual skeptics, but for everyone. These principles are all incredibly plausible and are endorsed both by skeptics and by many of their rivals. This leaves us with a puzzle: where does the dilemma go wrong? If the culprit is Clash or Chance Ensures Could, the argument from chance falls apart. But if the culprit is one of the closure principles, the argument from chance can survive.

Ultimately, I think we shouldn't give up on Clash and Chance Ensures Could just yet. Nor do I think we should embrace counterfactual skepticism, however. In the remainder of the paper, I will explain why. To do this, I first develop what I take to be the best response to the dilemma on behalf of the counterfactual skeptic—one that maintains Clash and Chance Ensures Could but rejects some of the closure principles for chance statements (§§4–6). I then argue that (i) even this response faces serious problems (§7) and (ii) counterfactual indeterminism resolves the dilemma in the same way, and still vindicates Clash and Chance Ensures Could, but does not suffer from these problems (§§8–9).

4 Analogy with Epistemic Modals

My diagnosis of the dilemma draws on an analogy between chance-talk and epistemic modals. In brief, my claim is that chancy contradictions seem to exhibit many of the same inferential patterns as epistemic contradictions. Appreciating this point will help pave a way out of the dilemma.

Let's look again at Chancy Contradiction. This principle is motivated by the observation that it sounds incoherent to say, in the same breath, that something *will* happen and also there's a chance it *won't*, as illustrated in (9).

(9) ??The coin will land heads and there's a chance it won't.

[9] Proof: Assume $\mathrm{ch}(\phi) \neq 0 \wedge \mathrm{ch}(\neg \phi) \neq 0 \nvDash \bot$ for reductio. Then:

1.	$\vDash (\mathrm{ch}(\phi) \neq 0 \wedge \mathrm{ch}(\neg \phi) \neq 0) \Box\!\!\rightarrow \mathrm{ch}(\phi) \neq 0$	Distribution
2.	$\vDash (\mathrm{ch}(\phi) \neq 0 \wedge \mathrm{ch}(\neg \phi) \neq 0) \Box\!\!\rightarrow \mathrm{ch}(\neg \phi) \neq 0$	Distribution
3.	$\vDash \bot$	No Counterfactual Chance, 1, 2

It's important to note that (9) refers to the *current* chances.[10] Chances change over time: things that were once chancy get resolved one way or the other, thereby losing their chanciness. Thus, there is nothing wrong with (10).

(10) The coin landed heads and before it did, there was a chance it wouldn't land heads.

It is also fine to say (11) if, for example, the coin was made maximally biased towards heads between 11:00 and the time of utterance. What sounds bad is (12): if there's still a chance the coin won't land heads, then it has yet to be determined whether it will.

(11) The coin will land heads at 12:00, and it had a chance at 11:00 of not landing heads at 12:00.

(12) ??The coin will land heads at 12:00, and right now it has a chance of not landing heads at 12:00.

One explanation for this is that it's just an instance of Moore's paradox. For instance, it is well known that (13) is consistent but unassertible:

(13) ??The coin will land heads and I don't know it.

But this can't be the whole story. Epistemic contradictions like (14) also sound marked.

(14) ??The coin will land heads and it might not.

It is a familiar point, however, that Moorean sentences typically sound fine in embedded contexts, such as third-person attitude reports, whereas epistemic contradictions do not (Yalcin, 2007). And chancy contradictions seem to pattern with the latter, at least for some attitudes, like certainty, suggesting (9) isn't simply Moorean.[11]

(15) Alice is certain that the coin will land heads and that I don't know it.

(16) ??Alice is certain that the coin will land heads and that it might not.

(17) ??Alice is certain that the coin will land heads and that there's a chance it won't.

However, there are also strong reasons to reject Chancy Contradiction. For one thing, chancy contradictions sound better in ordinary belief reports, such as (18).[12]

(18) I think that the coin will land heads and that there's a chance it won't.

Moreover, by reductio, Chancy Contradiction is equivalent to the following principles:

Chance Undermines Truth. $\mathrm{ch}(\neg\phi) \neq 0 \vDash \neg\phi$

Truth Undermines Chance. $\phi \vDash \mathrm{ch}(\neg\phi) = 0$

[10] Thanks to an anonymous referee for helping me clarify this point.

[11] However, chancy contradictions may sound better than epistemic contradictions in some environments; e.g., Cariani (2021, ch. 6) reports finding chancy contradictions infelicitous, but felicitous under 'suppose'.

[12] They also may be felicitous in knowledge reports, though this depends on one's view of knowledge. See Hawthorne 2004; Hawthorne and Lasonen-Aarnio 2009; Bacon 2014; Dorr et al. 2014; Moss 2018 for discussion of whether one can know something that's chancy.

Both of these principles seem to contradict our initial assumption that the world is chancy. The first says that if there's some chance that ϕ is false, then ϕ *is* false. The second leads to a kind of fatalistic argument: since $\phi \vee \neg \phi$ is a logical truth for any ϕ, it follows using proof by cases that $\text{ch}(\phi) = 1 \vee \text{ch}(\phi) = 0$ is a logical truth for any ϕ.

This tension might sound familiar. Indeed, it is well-known that similar patterns are exhibited by epistemic modals ('might', 'probably', etc.). For instance, Yalcin (2007) observes that while epistemic contradictions seem inconsistent ($\phi \wedge \Diamond_e \neg \phi \vDash \bot$), epistemic possibility does not undermine truth ($\Diamond_e \neg \phi \nvDash \neg \phi$). Similarly, even if we accept excluded middle, we should not accept 'epistemic' excluded middle ($\vDash \Box_e \phi \vee \Box_e \neg \phi$). The same goes for probability operators: while it seems plausible that $\phi \wedge \text{prob}(\neg \phi) \vDash \bot$, we should not accept $\text{prob}(\neg \phi) \vDash \neg \phi$ or $\vDash \text{prob}(\phi) \vee \text{prob}(\neg \phi)$ (Yalcin, 2012). This suggests chance operators share inferential properties with epistemic modals. So whatever explanation we give of these inferential patterns for epistemic modals, a similar explanation can be given for chancy statements.

A counterfactual skeptic might insist that the notion of objective chance is not a folk notion, but rather a technical one used by philosophers. So we cannot rely on ordinary linguistic intuitions about the word 'chance' to tell us anything about chancy contradictions in the sense that's relevant to the counterfactual skeptic. If that's right, however, then we also cannot rely on intuitions about counterfactuals embedding chance-talk to justify principles such as Chance Ensures Could. But it's hard to see what other evidence we could have for these principles besides our linguistic intuitions. Even if we're just using a purely theoretical notion of chance, we still need a bridge that links this notion to the semantics of counterfactuals. And this bridge will likely be justified by intuitions about counterfactuals embedding talk of chance in this (technical) sense. Thus, it seems unwise for the skeptic to dismiss linguistic intuitions about chance-talk: those very intuitions are what justify the principles used in the argument from chance in the first place.

Still, I should confess that intuitions about chancy contradictions are mixed. While many of the speakers I have asked agree that chancy contradictions such as (9) sound bad, some (including Hájek (p.c.)) report them sounding fine. For example, some report chancy contradictions are felicitous when the chances are small.[13] Thus, some think (19) is okay if the coin is, say, 0.9999 biased towards heads (cf. Ninan ms).

(19) The coin will land heads, but there's a small chance it won't.

However, not everyone agrees that (19) is completely felicitous even in this case.[14]

Any theory will have a hard time doing justice to everyone's intuitions here, given that they're divided. The reader who thinks (9) is completely fine is free to develop an alternative response to the dilemma. But as I mentioned in §3, the counterfactual skeptic will have a hard time developing a plausible response if they simply reject Chancy

[13] Thanks to an anonymous referee for pointing this out.

[14] The semantic account in §5 could accommodate the felicity of (19) in a variety of ways. One option is to say that people who find (19) acceptable are engaging in a kind of 'probabilistic loose speech' (cf. Moss 2018, 2019). Thus, an assertion of (19) strictly communicates that it's certain the coin will land heads, though (19) can also be asserted loosely to communicate that it's merely *close enough* to certain, which is compatible with it also being (close enough to) certain that there's a chance it won't. Another proposal is to appeal to a knowledge-norm of assertion and add that knowledge doesn't require certainty (cf. Stefánsson 2018).

Contradiction.[15] So to give the skeptic the benefit of the doubt, I will assume that chancy contradictions like (9) are, in some way, defective.

This does not mean, however, that chancy contradictions can't be *true*, at least in some sense. Rather, what I want to suggest is defective about chancy contradictions is that they can't be *settled true*—that is, they cannot receive chance 1. The key to resolving the dilemma, I argue, is to recognize that Chancy Contradiction is invalid as a principle for preserving truth, but valid as a principle for preserving settled truth.

5 The Simple Semantics for Chance Statements

Above, I drew an analogy between chancy contradictions and epistemic contradictions. In this section, I use that analogy to develop a semantics for chance statements that will help us resolve the dilemma. Both the proposal I give on behalf of the counterfactual skeptic in §6 and the non-skeptical proposal I present in §8 will build off this semantics.

The basic idea is to adapt an information-sensitive account of epistemic modals to chance operators. On information-sensitive accounts, truth conditions are assigned relative to a world and an 'information state', which is usually modeled as a set of worlds or a credence function. These sorts of accounts are often employed to explain the inferential patterns of epistemic contradictions discussed in §4. This is done by distinguishing two notions of consequence: classical and informational.

Classical consequence captures preservation of truth: an argument is classically valid iff at any world and information state where the premises are true, the conclusion is true. In this sense, epistemic contradictions are consistent: ϕ can be true at a world and a state even if $\neg\phi$ is left open by that state. Informational consequence captures preservation of certainty: an argument is informationally valid iff at any information state where the premises are certain, the conclusion is certain. In this sense, epistemic contradictions are inconsistent: no information state can be certain in ϕ while leaving open $\neg\phi$. It does not follow, however, that if an information state leaves open $\neg\phi$, it must be certain in $\neg\phi$. Thus, informational consequence is non-classical. Specifically, reductio fails: while $\phi \wedge \Diamond_e \neg\phi$ is informationally inconsistent, $\Diamond_e \neg\phi$ doesn't informationally entail $\neg\phi$.

My proposal is to adopt a structurally similar account for chance statements but to interpret the 'information state' parameter not as the credences of some agent, but rather as the objective chances at a particular moment of time. So $\text{ch}(\phi) = n$ is true at a world and probability function (representing the objective chances at a particular time) iff the probability of ϕ according to that function is n (cf. Cariani 2021, ch. 6).

[15] My impression is that (apart from Hájek) those who think (19) sounds fine tend to report that the original examples motivating the argument from chance, such as (i), also sound fine (cf. Stefánsson 2018, p. 891).

(i) If I were to jump, I would come down, but there's a small chance I would quantum tunnel.

The same people also tend to think (3) is fine, suggesting they reject Clash. The view I ultimately defend is sympathetic with this (see §§7–8): counterfactual indeterminism predicts (i) is as acceptable as ordinary chancy contradictions assuming the chances are equally small. Its predictions about the felicity of (i) depend on whether felicitous assertion requires certainty or just near-certainty (see footnote 14).

Let's spell this out formally. Given some proposition variables $\mathsf{Prop} = \{p_1, p_2, p_3, \dots\}$, our formal language is presented in Backus-Naur form as follows:[16]

$$\phi ::= p \mid \neg\,\phi \mid (\phi \wedge \psi) \mid \mathrm{ch}(\phi) = n.$$

We'll say ϕ is **chance-free** if it does not contain any chance operators. We define the following abbreviations (along with the usual ones for boolean connectives):

$$\begin{aligned} \Box\,\phi \quad &:= \quad \mathrm{ch}(\phi) = 1 \\ \Diamond\,\phi \quad &:= \quad \mathrm{ch}(\phi) \neq 0. \end{aligned}$$

Given a set of worlds W, let $\mathsf{Prob}(W)$ be the set of probability mass functions over W, which we'll call **chance functions**. A model for this language is a pair $\langle W, V \rangle$ where W is a nonempty set of worlds and $V : \mathsf{Prop} \to \wp\, W$ is a valuation function. Throughout, I leave $\mathcal{M}$ implicit. The truth conditions are given below (where $[\![\phi]\!]^{ch} = \{v \in W \mid [\![\phi]\!]^{ch,v} = \mathsf{T}\}$):

$$\begin{aligned} [\![p]\!]^{ch,w} = \mathsf{T} \quad &\Leftrightarrow \quad w \in V(p) \\ [\![\neg\,\phi]\!]^{ch,w} = \mathsf{T} \quad &\Leftrightarrow \quad [\![\phi]\!]^{ch,w} = \mathsf{F} \\ [\![\phi \wedge \psi]\!]^{ch,w} = \mathsf{T} \quad &\Leftrightarrow \quad [\![\phi]\!]^{ch,w} = \mathsf{T} \text{ and } [\![\psi]\!]^{ch,w} = \mathsf{T} \\ [\![\mathrm{ch}(\phi) = n]\!]^{ch,w} = \mathsf{T} \quad &\Leftrightarrow \quad ch([\![\phi]\!]^{ch}) = n. \end{aligned}$$

Call this the **simple semantics** for chance statements.

For technical simplicity, I'm treating the chance parameter and world parameter as independent. We could instead make the chance parameter dependent on a world and time parameter, but doing so unnecessarily complicates the formalism. Instead, we can simply interpret the chance parameter as standing in for a time parameter: given a world w, a chance function ch represents a time at which the objective chance function at world w is ch. In this sense, the truth conditions of chance statements are time-sensitive.[17]

Note, the truth conditions for $\mathrm{ch}(\phi) = n$ are independent of the world parameter. As a result, $[\![\mathrm{ch}(\phi) = n]\!]^{ch}$ is always either W or $\varnothing$. This is because one of the parameters of evaluation is the chance function itself: once you know what the chances are, the world is irrelevant to the truth of $\mathrm{ch}(\phi) = n$. Again, we could make the chance parameter depend on the world parameter, but nothing in what follows would be affected by doing this. Relatedly, on this semantics, chance functions are 'self-aware' in that chance statements never have intermediate chance: if $\mathrm{ch}(\phi) = n$ is true, then $\mathrm{ch}(\mathrm{ch}(\phi) = n) = 1$ is true.[18] This mirrors what we find in the so-called 'test semantics' for epistemic modals (Veltman, 1996; Gillies, 2001; Yalcin, 2007).

[16] This notation encodes the inductive definition of a formula in a single line. It effectively says that (i) propositional variables are formulas, (ii) the negation of any formula is a formula, (iii) the conjunction of any two formulas is a formula, and (iv) for any n, the chance operator $\mathrm{ch}(\cdot) = n$ applied to a formula is a formula.

[17] By contrast, the truth conditions of atomic formulas are insensitive to the chance parameter. This means atomic formulas represent time-insensitive (i.e., eternalist) propositions. We could allow them to represent time-sensitive (i.e., temporalist) propositions by generalizing the definition of $V(p)$ to be a set of chance-world pairs. None of the formal results would be affected by this change.

[18] The term 'self-aware' is borrowed from Schwarz (2018, p. 59).

To explain the observations from §4, we need to distinguish two notions of logical consequence. First, there's **classical** consequence, which is glossed as preservation of truth. Where $ch \in \mathsf{Prob}(W)$, let s_{ch} be the smallest $P \subseteq W$ such that $ch(P) = 1$.[19] Then Γ classically entails ϕ, written $\Gamma \vDash_c \phi$, iff for all $ch \in \mathsf{Prob}(W)$ and $w \in s_{ch}$, if $[\![\gamma]\!]^{ch,w} = \mathsf{T}$ for all $\gamma \in \Gamma$, then $[\![\phi]\!]^{ch,w} = \mathsf{T}$. Second, there's what I'll call **historical** consequence, which is glossed as preservation of settled truth, i.e., chance 1. Let's say ch **settles true** ϕ if $ch([\![\phi]\!]^{ch}) = 1$. Then Γ historically entails ϕ, written $\Gamma \vDash_h \phi$, iff for all ch, if ch settles true every $\gamma \in \Gamma$, then ch settles true ϕ. Historical consequence is weaker than classical consequence: if an argument is classically valid, it is historically valid, but not necessarily vice versa.

Chancy Contradiction is not classically valid ($\phi \wedge \Diamond\neg\phi \nvDash_c \bot$): ϕ and $\Diamond\neg\phi$ can both be true so long as there is a world with non-zero chance at which ϕ is false. Informally put: ϕ can later turn out to be true even though there's currently some chance that $\neg\phi$. This explains why it's felicitous to embed chancy contradictions under 'thinks', as in (18): there is nothing wrong with believing that the coin will land heads (if, say, the coin is 0.99 biased towards heads) while believing that there's still a chance it won't land heads.

Chancy Contradiction is historically valid ($\phi \wedge \Diamond\neg\phi \vDash_h \bot$): no chance function could *settle true* both ϕ and $\Diamond\neg\phi$. To settle true ϕ is to assign it chance 1, whereas to settle true $\Diamond\neg\phi$ is to assign $\neg\phi$ a non-zero chance. Informally put: it cannot be settled true that ϕ and, at the same time, left open by the objective chances that $\neg\phi$. This explains why it's infelicitous to embed chancy contradictions under 'certain', as in (17): one cannot be certain that ϕ if one is also certain that there's a non-zero chance that $\neg\phi$.[20]

Chance Undermines Truth is not even historically valid ($\Diamond\neg\phi \nvDash_h \neg\phi$). All it takes for a chance function to settle true $\Diamond\neg\phi$ is that it assign $\neg\phi$ a non-zero chance. But to settle true $\neg\phi$, it must assign $\neg\phi$ chance 1. Thus, a chance function can settle true $\Diamond\neg\phi$ without settling true $\neg\phi$. However, Truth Undermines Chance is historically valid ($\phi \vDash_h \neg\Diamond\neg\phi$): settling true ϕ means assigning ϕ chance 1, and so leaving no chance that $\neg\phi$. Like Chancy Contradiction, though, it is not classically valid ($\phi \nvDash_c \neg\Diamond\neg\phi$).

Since Chance Undermines Truth follows from Chancy Contradiction by reductio, and from Truth Undermines Chance by contraposition, this means reductio and contraposition are historically invalid. Furthermore, the fatalistic argument from Truth Undermines Chance fails ($\nvDash_h \Box\phi \vee \Box\neg\phi$). Since this argument is just an instance of proof by cases, this means proof by cases is also historically invalid. For chance-free formulas, however, historical validity collapses to classical validity.[21] So reductio, contraposition, and proof

[19] Note: this definition is not well-defined if regularity fails, i.e., if ϕ can be possible but have probability zero, such as in infinite dart cases. In that case, our model of chance will need to be suitably generalized; see Easwaran 2014 for discussion. Throughout, I will set aside issues with regularity for ease of exposition.

[20] As an anonymous referee points out, this reasoning implicitly relies on some principle linking credence and chance, such as the Principal Principle. Throughout, I assume some such principle holds, though I won't try to give an exact formulation here. Further, I will set aside complications arising from the possibility of oracles. It may be that (17) can sound fine in contexts where oracles are present, but this wouldn't undermine the explanation for the general infelicity of (17) given here.

[21] Proof: Suppose $\phi_1, \ldots, \phi_n \vDash_h \psi$, where $\phi_1, \ldots, \phi_n, \psi$ are chance-free. Let ch and w be such that $[\![\phi_1]\!]^{ch,w} = \cdots = [\![\phi_n]\!]^{ch,w} = \mathsf{T}$. Let ch_w be the probability function such that $ch_w(w) = 1$. Then by induction, $[\![\chi]\!]^{ch,w} = [\![\chi]\!]^{ch_w,w}$ for any chance-free χ. (Note: if $V(p)$ is taken instead to be a set of chance-world pairs,

by cases are historically valid for chance-free formulas.

In sum, here are the main features of the simple semantics for chance statements:

- Truth conditions are relativized to a world and a chance function.
- Classical consequence is preservation of truth. Historical consequence is preservation of settled truth (i.e., chance 1). The latter does not universally license reductio, contraposition, or proof by cases for formulas with chance operators.
- Chancy Contradiction and Truth Undermines Chance are historically valid but classically invalid. Chance Undermines Truth is not valid in any sense.

6 The Restrictor Semantics for Counterfactual Skepticism

Again, my goal is to present an abductive argument against counterfactual skepticism. My strategy is to first develop what I take to be the best response to the dilemma on behalf of the skeptic, and to then argue that there is a strictly better non-skeptical response available. In this section, I implement the first step of my argument: I will show how to develop a skeptical semantics for counterfactuals that validates Clash and Chance Ensures Could while avoiding the dilemma in a plausible way.[22] While I will raise problems for this semantics in §7, it will pave the way for a more promising account in §8.

To begin, here are the target truth conditions we want to get for counterfactuals whose consequents are chance statements: $\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\psi) = n$ is true at a world and probability function (representing the objective chances at some time) iff (roughly) the probability of ψ would be n were ϕ the case. Of course, this account is incomplete until we answer two questions. First, what do we mean by the probability of *ψ were ϕ the case*? And second, what about counterfactuals whose consequents aren't chance statements?

To answer the first question, I suggest we appeal to **imaging**. Roughly, imaging is to counterfactuals as conditionalization is to indicatives: while conditionalization is often glossed as 'indicative supposition', imaging corresponds to 'counterfactual supposition'. Unlike conditionalization, imaging does not preserve certainties: propositions with probability 1 are not guaranteed to have probability 1 after imaging. This feature is crucial for counterfactuals: e.g., the chance that an unflipped coin actually didn't land heads is 1, yet if it were flipped, the chance it wouldn't land heads is not 1.

There are many different ways to define imaging formally (Lewis, 1976; Nute, 1980; Gärdenfors, 1982; Katsuno and Mendelzon, 1992; Joyce, 1999). As an example, here is how Lewis (1976) defines it: imaging on a proposition P amounts to shifting the probability mass of every world to its closest P-world. More precisely, suppose we have a Stalnaker selection function f which takes a proposition P and a world w and returns the unique

$[\![\chi]\!]^{ch_w,w}$ needs to be assessed relative to $\mathcal{M}_{ch} = \langle W, V_{ch}\rangle$ where $V_{ch}(p) = \{\langle ch_w, w\rangle \mid \langle ch, w\rangle \in V(p)\}$.) Hence, $[\![\phi_1]\!]^{ch_w,w} = \cdots = [\![\phi_n]\!]^{ch_w,w} = \mathsf{T}$. Since $ch_w(\{w\}) = 1$, $ch_w([\![\phi_1]\!]^{ch_w}) = \cdots = ch_w([\![\phi_n]\!]^{ch_w}) = 1$. But then $ch_w([\![\psi]\!]^{ch_w}) = 1$, and so $[\![\psi]\!]^{ch_w,w} = [\![\psi]\!]^{ch,w} = \mathsf{T}$.

[22] This account resembles probabilistic approaches to counterfactuals (Edgington, 2008; Leitgeb, 2012a,b; Moss, 2013; Lassiter, 2017) as well as information-sensitive approaches to conditionals (Kolodny and MacFarlane, 2010; Santorio, 2017; Schulz, 2014, 2017; Mandelkern, 2019; Khoo, forthcoming).

closest P-world to w.[23] We assume minimally that $f(P, w) \in P$ (the closest P-world is a P-world) and that if $w \in P$, then $f(P, w) = w$ (the closest world to w is itself). Then where $\Pr(\cdot)$ is a probability mass function and $\Pr(\cdot \mid\mid P)$ is the result of imaging Pr on P:

$$\Pr(Q \mid\mid P) = \sum_{w \in W} \Pr(w) \cdot \begin{cases} 1 & \text{if } f(P, w) \in Q \\ 0 & \text{if } f(P, w) \notin Q \end{cases}$$

Apart from some minimal constraints, the account I'm sketching won't seriously depend on the details of how we define imaging. This is just one model to guide intuitions.

To answer the second question, I suggest the skeptic adopt a restrictor analysis of counterfactuals (cf. Kratzer 1981, 1986, 2012). On this view, the 'if'-clauses of counterfactuals at the level of logical form are restrictors on operators, usually a chance operator. This can be supported by the apparent scopelessness of chance operators with respect to counterfactuals. For example, the following seem equivalent:

(20) a. If the coin were flipped, there'd be a 50% chance it would land heads.
b. There's a 50% chance that if the coin were flipped, it would land heads.

If there is no explicit operator present, then according to the restrictor theory, there must be an unpronounced covert operator. In the case of counterfactuals, a natural suggestion is that this covert operator is a 'with chance 1' operator. This seems to capture the counterfactual skeptic's intuition: to say it *would* happen is to say it happens for sure, with chance 1, in the relevant counterfactual scenario. Counterfactuals make very strong claims according to the restrictor theory: they will typically be false unless their consequent is about the past of their antecedent or is nomologically implied by their antecedent.

Let's spell this out formally. To capture the idea that counterfactuals are restrictors on chance operators, I'll assume that counterfactuals always take the form $\text{ch}(\phi \mathrel{\Box\!\!\rightarrow} \psi) = n$. The formula $\phi \mathrel{\Box\!\!\rightarrow} \psi$ will be an abbreviation for $\text{ch}(\phi \mathrel{\Box\!\!\rightarrow} \psi) = 1$. So here is our language:

$$\phi ::= p \mid \neg\phi \mid (\phi \wedge \psi) \mid \text{ch}(\phi) = n \mid \text{ch}(\phi \mathrel{\Box\!\!\rightarrow} \psi) = n.$$

We define the following abbreviations (along with our earlier ones for $\Box$ and $\Diamond$):

$$\begin{aligned} \phi \mathrel{\Box\!\!\rightarrow} \psi \quad &:= \quad \text{ch}(\phi \mathrel{\Box\!\!\rightarrow} \psi) = 1 \\ \phi \mathrel{\Diamond\!\!\rightarrow} \psi \quad &:= \quad \phi \mathrel{\Box\!\!\rightarrow} \Diamond\psi. \end{aligned}$$

A model for this language is now a triple $\mathcal{M} = \langle W, ||, V \rangle$, where W and V are as before, and where $||\colon \mathsf{Prob}(W) \times \wp W \to \mathsf{Prob}(W)$ is an imaging function such that for all $ch \in \mathsf{Prob}(W)$ and all $P \subseteq W$:

(i) $ch(\cdot \mid\mid P)$ is defined iff $P \neq \varnothing$; if defined:
(ii) $ch(P \mid\mid P) = 1$
(iii) if $ch(P) = 1$, then $ch(\cdot \mid\mid P) = ch(\cdot)$

[23] Note: this presupposes there always is a unique closest P-world to w. While Lewis (1976) assumes this when presenting this account of imaging, it is well-known he rejected the uniqueness assumption for counterfactuals in general. See Gärdenfors 1982 for a generalization of imaging that does away with this assumption.

The truth conditions for counterfactuals are given below (where $ch_\phi(\cdot) \coloneqq ch(\cdot \mid\mid [\![\phi]\!]^{ch})$):

$$[\![\mathrm{ch}(\phi \mathrel{\Box\!\!\rightarrow} \psi) = n]\!]^{ch,w} = \top \quad \Leftrightarrow \quad ch_\phi([\![\psi]\!]^{ch_\phi}) = n$$

Classical and historical consequence are defined as in §5. Call this the **restrictor semantics** for counterfactual skepticism.[24]

In line with the observation that (20a) and (20b) seem equivalent, $\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\psi) = n$ and $\mathrm{ch}(\phi \mathrel{\Box\!\!\rightarrow} \psi) = n$ receive the same truth conditions (when defined):[25]

$$[\![\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\psi) = n]\!]^{ch,w} = \top \quad \Leftrightarrow \quad ch_\phi([\![\psi]\!]^{ch_\phi}) = n.$$

As a corollary, $\phi \mathrel{\Box\!\!\rightarrow} \psi$ is equivalent to $\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\psi) = 1$:

$$[\![\phi \mathrel{\Box\!\!\rightarrow} \psi]\!]^{ch,w} = \top \quad \Leftrightarrow \quad [\![\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\psi) = 1]\!]^{ch,w} = \top.$$

This means Chance Undermines Would holds: $\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\neg\psi) \neq 0$ contradicts $\phi \mathrel{\Box\!\!\rightarrow} \psi$.

In addition, it follows from the definition of $\mathrel{\Diamond\!\!\rightarrow}$ that:

$$[\![\phi \mathrel{\Diamond\!\!\rightarrow} \psi]\!]^{ch,w} = \top \quad \Leftrightarrow \quad ch_\phi([\![\psi]\!]^{ch_\phi}) \neq 0.$$

So $\phi \mathrel{\Diamond\!\!\rightarrow} \psi$, $\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\psi) \neq 0$, and $\mathrm{ch}(\phi \mathrel{\Box\!\!\rightarrow} \psi) \neq 0$ are all equivalent. This seems well-supported by the data. For example, the following all sound equivalent.[26]

(21) a. If the coin were flipped, it could land heads.
b. If the coin were flipped, there'd be some chance it would land heads.
c. There's some chance that if the coin were flipped, it would land heads.

Furthermore, Chance Ensures Could and Clash hold since $\phi \mathrel{\Diamond\!\!\rightarrow} \psi \coloneqq \phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\psi) \neq 0$. In fact, we have Duality:

$$[\![\phi \mathrel{\Diamond\!\!\rightarrow} \psi]\!]^{ch,w} = \top \quad \Leftrightarrow \quad [\![\neg(\phi \mathrel{\Box\!\!\rightarrow} \neg\psi)]\!]^{ch,w} = \top.$$

How does the restrictor semantics resolve the dilemma from §3? Since it validates Chance Ensures Could and Clash, it must violate one of the closure principles. Which one? The answer depends on the notion of consequence. Because classical and historical consequence disagree on Chancy Contradiction, they face different horns of the dilemma: classical consequence faces Horn 1 whereas historical consequence faces Horn 2.

[24] Note, because $[\![\mathrm{ch}(\phi) = n]\!]^{ch}$ is either W or $\varnothing$, imaging on a false chance statement (or a false counterfactual for that matter) is undefined. So a counterfactual whose antecedent is a false chance statement (or a false counterfactual) is undefined in truth value. There's a similar problem for the test semantics for epistemic modals. See Willer 2013; Starr 2014; Moss 2015, 2018; Charlow 2019; Goldstein 2019, forthcoming for some solutions. We can also get around this issue by generalizing imaging to map each chance function and set of chance-world pairs to another chance function. The constraints can likewise be suitably generalized.

[25] Proof: Suppose ch_ϕ is defined. Observe $[\![\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\psi) = n]\!]^{ch,w} = \top$ iff $ch_\phi([\![\mathrm{ch}(\psi) = n]\!]^{ch_\phi}) = 1$. Now, either $[\![\mathrm{ch}(\psi) = n]\!]^{ch_\phi} = W$ or $[\![\mathrm{ch}(\psi) = n]\!]^{ch_\phi} = \varnothing$. So if $ch_\phi([\![\mathrm{ch}(\psi) = n]\!]^{ch_\phi}) = 1$, then $[\![\mathrm{ch}(\psi) = n]\!]^{ch_\phi} = W$, which means $ch_\phi([\![\psi]\!]^{ch_\phi}) = n$. If instead $ch_\phi([\![\mathrm{ch}(\psi) = n]\!]^{ch_\phi}) \neq 1$, then $[\![\mathrm{ch}(\psi) = n]\!]^{ch_\phi} \neq W$, meaning $[\![\mathrm{ch}(\psi) = n]\!]^{ch_\phi} = \varnothing$. In that case, $ch_\phi([\![\psi]\!]^{ch_\phi}) \neq n$. Hence, $[\![\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\psi) = n]\!]^{ch,w} = \top$ iff $ch_\phi([\![\psi]\!]^{ch_\phi}) = n$.

[26] Note: these equivalences break down if regularity fails. Again, I ignore this complication in what follows.

For classical consequence, the culprit is Identity: even though $\phi \wedge \Diamond\neg\phi$ is classically consistent, $(\phi \wedge \Diamond\neg\phi) \mathrel{\Box\!\!\rightarrow} (\phi \wedge \Diamond\neg\phi)$ is not classically valid. Imaging on $\phi \wedge \Diamond\neg\phi$ results in an *unsuccessful* update, i.e., an update where the resulting probability function does not assign probability 1 to the information it was updated on.[27] This may seem counterintuitive; but counterfactuals like (22) do seem odd.

(22) ??If the coin were flipped and there were a chance it hadn't been flipped, then the coin would be flipped and there'd be a chance it hadn't been flipped.

Any counterfactual of the form $\alpha \mathrel{\Box\!\!\rightarrow} (\phi \wedge \Diamond\neg\phi)$ will sound off, even if $\alpha = (\phi \wedge \Diamond\neg\phi)$. So while it sounds radical, rejecting Identity here is not unmotivated.[28,29]

For historical consequence, the culprit is Could-Closure: $\phi \mathrel{\Diamond\!\!\rightarrow} \psi \not\models_{\mathrm{h}} \phi \mathrel{\Diamond\!\!\rightarrow} \Box\psi$, even though $\psi \models_{\mathrm{h}} \Box\psi$. Looking back, this is the weakest step. This can seen with an example:

(23) a. If the coin were flipped, it could land tails.
Flip $\mathrel{\Diamond\!\!\rightarrow}$ *Tails*
b. $\not\Rightarrow$ If the coin were flipped, it could have had zero chance of landing heads.
Flip $\mathrel{\Diamond\!\!\rightarrow} \mathrm{ch}(\mathit{Heads}) = 0$

Contrary to (23b), if the coin were flipped, it would have a non-zero chance of landing heads. Yet it is exactly this reasoning that is used to infer line 2 of Horn 2.

This doesn't mean that there are no interesting closure principles that hold on this view. For one thing, classical consequence still validates Could-Closure:

Classical Could-Closure. If $\psi \models_{\mathrm{c}} \chi$, then $\phi \mathrel{\Diamond\!\!\rightarrow} \psi \models_{\mathrm{c}} \phi \mathrel{\Diamond\!\!\rightarrow} \chi$.

Since $\psi \not\models_{\mathrm{c}} \Box\psi$, we cannot apply Classical Could-Closure in Horn 2. But we can still explain the instances of Could-Closure that seemed plausible: they were all cases where the entailment held classically. And for chance-free formulas, historical consequence reduces to classical consequence. So Could-Closure is historically valid when reasoning with chance-free formulas; but more caution must be taken when reasoning with chance statements.[30]

[27] Unsuccessful updates have also been investigated in other contexts, such as the dynamic epistemic logic literature. See, e.g., van Benthem 2004; Baltag et al. 2008; Holliday and Icard 2010. The failure of success for indicative conditionals is also discussed in Goldstein forthcoming.

[28] What happens when you update on $\phi \wedge \Diamond\neg\phi$? If $[\![\phi \wedge \Diamond\neg\phi]\!]^{ch} = \varnothing$, then $ch_{\phi\wedge\Diamond\neg\phi}$ is undefined, so $(\phi \wedge \Diamond\neg\phi) \mathrel{\Box\!\!\rightarrow} \psi$ is undefined. If $[\![\phi \wedge \Diamond\neg\phi]\!]^{ch} \neq \varnothing$, then $[\![\phi \wedge \Diamond\neg\phi]\!]^{ch} = [\![\phi]\!]^{ch}$ (since $[\![\Diamond\neg\phi]\!]^{ch} = W$). In that case, imaging on $\phi \wedge \Diamond\neg\phi$ will produce the same results as imaging on ϕ. And for chance-free ϕ, $\mathrm{ch}(\phi \mathrel{\Box\!\!\rightarrow} \phi) = 1$ holds whenever $[\![\phi]\!]^{ch} \neq \varnothing$. Thus, in most cases, $(\phi \wedge \Diamond\neg\phi) \mathrel{\Box\!\!\rightarrow} \phi$ holds and $(\phi \wedge \Diamond\neg\phi) \mathrel{\Box\!\!\rightarrow} \Diamond\neg\phi$ fails. This addresses the argument due to Williams (2010, p. 658) that the dual and would-be-possible readings of $\mathrel{\Diamond\!\!\rightarrow}$ are not equivalent. His proof (which is similar to the argument given in Horn 1) relies on the truth of $(p \wedge \Diamond\neg p) \mathrel{\Box\!\!\rightarrow} \Diamond\neg p$, which does not hold on this semantics.

[29] For readers who think (22) just sounds like a tautology, we can explain this by, first, stipulating that counterfactuals presuppose successful update (i.e., that $ch_\phi([\![\phi]\!]^{ch_\phi}) = 1$), and second, replacing classical validity with Strawson-validity (Fintel, 2001; Gillies, 2007). In that case, Identity will be Strawson-valid, but Horn 1 is blocked since Strawson-validity is not transitive (specifically, the inference to line 4 fails). Then one can explain (22) the same way one explains 'If the king of France is bald, the king of France is bald'. Thanks to Paolo Santorio for suggesting a strategy like this one.

[30] We also get a restricted form of Could-Closure for historical consequence:

We also still get a strong closure principle for $\mathrel{\Box\!\!\rightarrow}$:

Liberal Would-Closure. If $\psi \vDash_{c/h} \chi$, then $\phi \mathrel{\Box\!\!\rightarrow} \psi \vDash_{c/h} \phi \mathrel{\Box\!\!\rightarrow} \chi$.

In other words, the consequents of counterfactuals are historically and classically closed under both historical and classical consequence. This is stronger than just saying Would-Closure holds for classical and historical consequence. For instance, $\psi \vDash_h \Box\psi$ while $\psi \nvDash_c \Box\psi$. So Would-Closure for neither classical nor historical consequence guarantees that $\phi \mathrel{\Box\!\!\rightarrow} \psi \vDash_c \phi \mathrel{\Box\!\!\rightarrow} \Box\psi$, whereas Liberal Would-Closure does. Even with such a strong closure principle for would-counterfactuals, we are not forced to accept Could-Closure.[31]

In sum, here are the main features of the restrictor semantics:

- Counterfactuals always have the logical form $\mathrm{ch}(\phi \mathrel{\Box\!\!\rightarrow} \psi) = n$. $\phi \mathrel{\Box\!\!\rightarrow} \psi$ is an abbreviation for $\mathrm{ch}(\phi \mathrel{\Box\!\!\rightarrow} \psi) = 1$.
- $\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\psi) = n$ is equivalent to $\mathrm{ch}(\phi \mathrel{\Box\!\!\rightarrow} \psi) = n$. Thus, $\phi \mathrel{\Box\!\!\rightarrow} \psi$ is equivalent to $\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\psi) = 1$, and $\phi \mathrel{\Diamond\!\!\rightarrow} \psi$ is equivalent to $\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\psi) \neq 0$.
- Clash, Chance Ensures Could, and Chance Undermines Would are both classically and historically valid. Duality is also valid in both senses.
- For classical consequence, Horn 1 of the dilemma is blocked since Identity fails. For historical consequence, Horn 2 is blocked since Could-Closure fails.

7 Problems with the Restrictor Theory

Let's take stock. In §3, I presented a dilemma for anyone who accepts Clash and Chance Ensures Could. The horns of the dilemma turn on whether chancy contradictions are consistent. In §4, I suggested a way out of the dilemma for the counterfactual skeptic, viz., treat chancy contradictions similarly to epistemic contradictions. We saw how this could be done with a simple semantics for chance statements in §5. This led to the development of the restrictor semantics in §6. On this approach, $\phi \mathrel{\Box\!\!\rightarrow} \psi$ is an abbreviation for $\mathrm{ch}(\phi \mathrel{\Box\!\!\rightarrow} \psi) = 1$, which is equivalent to $\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\psi) = 1$. This semantics validates Clash, Chance Ensures Could, and Chance Undermines Would, but rejects closure principles for counterfactuals involving chance statements, thereby resolving the dilemma.

So far, things are looking up for the counterfactual skeptic. But not for long. The main problem with the restrictor semantics is that it predicts that $\phi \mathrel{\Box\!\!\rightarrow} \psi$ and $\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\psi) = 1$

Contraposed Historical Could-Closure. If $\neg\chi \vDash_h \neg\psi$, then $\phi \mathrel{\Diamond\!\!\rightarrow} \psi \vDash_{c/h} \phi \mathrel{\Diamond\!\!\rightarrow} \chi$.

Since $\neg\Box\psi \nvDash_h \neg\psi$ (even though $\psi \vDash_h \Box\psi$), we cannot apply this principle in Horn 2.

[31] Earlier, I said that Could-Closure follows from Would-Closure and Duality, both of which are historically valid. But this proof crucially relies on contraposition, which is historically invalid. The proof goes like this:

1.	$\psi \vDash \chi$	premise
2.	$\neg\chi \vDash \neg\psi$	contraposition, 1
3.	$\phi \mathrel{\Box\!\!\rightarrow} \neg\chi \vDash \phi \mathrel{\Box\!\!\rightarrow} \neg\psi$	Would-Closure, 2
4.	$\neg(\phi \mathrel{\Box\!\!\rightarrow} \neg\psi) \vDash \neg(\phi \mathrel{\Box\!\!\rightarrow} \neg\chi)$	contraposition, 3
5.	$\phi \mathrel{\Diamond\!\!\rightarrow} \psi \vDash \phi \mathrel{\Diamond\!\!\rightarrow} \chi$	Duality

The move from 1 to 2 is not historically valid. For instance, $\psi \vDash_h \neg\Diamond\neg\psi$, but $\Diamond\neg\psi \nvDash_h \neg\psi$. So the derivation of Could-Closure for historical consequence is blocked.

are semantically equivalent. This makes it difficult for the restrictor semantics to account for cases where we seem to need to distinguish the two.

One example of such a case, as Moss (2013, pp. 257–258) points out, involves rational agents' credences in counterfactuals. Suppose we have a fair coin that is never flipped. Then it seems we should have a very high credence in (24a) while having zero (or almost zero) credence in (24b).[32] Moreover, it seems we should be certain (or almost certain) in (24c) (cf. Mandelkern 2019, p. 303).

(24) a. If the coin were flipped 1 billion times, it would land heads at least once.
$\mathit{1\ Billion\ Flips} \mathrel{\Box\!\!\rightarrow} \mathit{Some\ Heads}$ rational credence: ≈ 1
b. If the coin were flipped 1 billion times, it would have a 100% chance of landing heads at least once.
$\mathit{1\ Billion\ Flips} \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\mathit{Some\ Heads}) = 1$ rational credence: 0
c. If the coin were flipped 1 billion times, it could fail to land heads even once.
$\mathit{1\ Billion\ Flips} \mathrel{\Diamond\!\!\rightarrow} \neg\, \mathit{Some\ Heads}$ rational credence: 1

As it stands, the restrictor semantics cannot account for these credal judgments. According to the restrictor theory, since (24a) does not restrict an explicit chance operator, it restricts a covert 'with chance 1' operator, i.e., it has the form $\mathrm{ch}(\mathit{1\ Billion\ Flips} \mathrel{\Box\!\!\rightarrow} \mathit{Some\ Heads}) = 1$, which is equivalent to (24b). Thus, rational agents must assign the same credence to (24a) and (24b). Likewise, (24a) and (24c) are inconsistent by Clash, so rational agents cannot assign high credence to both. Call this **the credence problem**.

Another case where we want to distinguish $\phi \mathrel{\Box\!\!\rightarrow} \psi$ and $\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\psi) = 1$ involves compounds of counterfactuals. As Khoo and Santorio (2018, pp. 46–47) observe, the restrictor analysis of indicative conditionals faces difficulties accounting for probabilities of compound conditionals. Unsurprisingly, similar problems arise for the restrictor analysis of counterfactuals. For example, suppose we have two fair coins that are never flipped. Then intuitively, the chance of (25a) is 1/4, whereas the chance of (25b) is 0.

(25) a. If the first coin were flipped, it would land heads and if the second coin were flipped, it would land heads.
$(\mathit{Flip}_1 \mathrel{\Box\!\!\rightarrow} \mathit{Heads}_1) \wedge (\mathit{Flip}_2 \mathrel{\Box\!\!\rightarrow} \mathit{Heads}_2)$ chance: $1/4$
b. If the first coin were flipped, it would have a 100% chance of landing heads and if the second coin were flipped, it would have a 100% of landing heads.
$(\mathit{Flip}_1 \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\mathit{Heads}_1) = 1) \wedge (\mathit{Flip}_2 \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\mathit{Heads}_2) = 1)$ chance: 0

Yet the restrictor semantics predicts (25a) and (25b) are equivalent, and so have the same chance. According to the restrictor theory, the counterfactuals in (25a) must be interpreted

[32] Moss notes that there is a way of reading (24a) on which we should give it fairly low credence. The reading is brought out most clearly by placing focus on 'would'. This suggests placing focus on 'would' forces a reading on which the consequent is interpreted under the scope of a 'certainly' or 'with chance 1' operator. Even so, there does seem to be a perfectly legitimate reading of (24a) on which it's rational to assign it high credence. Perhaps the counterfactual skeptic could deny this, instead holding that there is only one 'reading' of the counterfactual that is simply drawn out by the use of focus. They might then say that the appearance of another reading is due to a widespread error on the part of English speakers. But this claim would require further evidence to support—especially in light of the semantics in §8, which can accommodate both 'readings' without attributing widespread error to English speakers.

as restricting covert 'with chance 1' operators. And (25b) is just the result of making those covert operators overt. Call this **the compounding problem**.[33]

A related problem stems from the fact that the restrictor semantics rejects Conditional Excluded Middle (both classically and historically):

CEM. $\vDash (\phi \,\Box\!\!\rightarrow \psi) \vee (\phi \,\Box\!\!\rightarrow \neg\, \psi)$

One trivial reason CEM fails is that $\phi \,\Box\!\!\rightarrow \psi$ may be undefined in truth value (if $[\![\phi]\!]^{ch} = \varnothing$, then ch_{ϕ} is undefined). But we could avoid this by adding a premise such as $\phi \,\Box\!\!\rightarrow (\psi \vee \neg\, \psi)$, which ensures the truth value of $\phi \,\Box\!\!\rightarrow \psi$ is defined:[34]

Limited CEM. $\phi \,\Box\!\!\rightarrow (\psi \vee \neg\, \psi) \vDash (\phi \,\Box\!\!\rightarrow \psi) \vee (\phi \,\Box\!\!\rightarrow \neg\, \psi)$

Yet even Limited CEM fails: $(\phi \,\Box\!\!\rightarrow \psi) \vee (\phi \,\Box\!\!\rightarrow \neg\, \psi)$ is abbreviation for the formula $\mathrm{ch}(\phi \,\Box\!\!\rightarrow \psi) = 1 \vee \mathrm{ch}(\phi \,\Box\!\!\rightarrow \neg\, \psi) = 1$, which need not hold.

The failure of Limited CEM is a problem for two reasons. First, even though it has been historically controversial, CEM has received strong support in the literature recently (Higginbotham, 1986, 2003; Williams, 2010; Klinedinst, 2011; Swanson, 2012; Santorio, 2017; Mandelkern, 2019). Arguably, whatever evidence there is for CEM carries over to Limited CEM. Second, the way in which Limited CEM fails invalidates instances of the law of additivity stated in the object language.[35] For it still is true that $\mathrm{ch}(\phi \,\Box\!\!\rightarrow \psi) = n \;\Dashv\vDash\; \mathrm{ch}(\phi \,\Box\!\!\rightarrow \neg\, \psi) = 1 - n$. Thus, even if $\mathrm{ch}(\phi \,\Box\!\!\rightarrow \psi) = n$ is defined, the following will not generally hold (where '$\vDash \mathrm{ch}(\phi) = \mathrm{ch}(\psi_1) + \mathrm{ch}(\psi_2)$' means that $\mathrm{ch}(\psi_1) = n_1, \mathrm{ch}(\psi_2) = n_2 \vDash \mathrm{ch}(\phi) = n_1 + n_2$ for all n_1 and n_2):

$$\vDash \mathrm{ch}(\phi \,\Box\!\!\rightarrow \psi \vee \phi \,\Box\!\!\rightarrow \neg\, \psi) = \mathrm{ch}(\phi \,\Box\!\!\rightarrow \psi) + \mathrm{ch}(\phi \,\Box\!\!\rightarrow \neg\, \psi)$$
$$\vDash \mathrm{ch}(\phi \,\Box\!\!\rightarrow \psi) = \mathrm{ch}(\phi \,\Box\!\!\rightarrow \psi \wedge \phi \,\Diamond\!\!\rightarrow \psi) + \mathrm{ch}(\phi \,\Box\!\!\rightarrow \psi \wedge \neg(\phi \,\Diamond\!\!\rightarrow \psi)).$$

Note, chance *functions*, i.e., parameters in our points of evaluation, do not violate the laws of probability. It is just that certain instances of these laws stated in the object language fail. Call this **the additivity problem**.

Observe that these problems arise from the assumption that $\phi \,\Box\!\!\rightarrow \psi$ and $\phi \,\Box\!\!\rightarrow \mathrm{ch}(\psi) = 1$ are *classically* equivalent. According to the restrictor semantics, rational agents cannot have different credences in $\phi \,\Box\!\!\rightarrow \psi$ and $\phi \,\Box\!\!\rightarrow \mathrm{ch}(\psi) = 1$ because they have exact same semantic content. Similarly, since $(\phi_1 \,\Box\!\!\rightarrow \psi_1) \wedge (\phi_2 \,\Box\!\!\rightarrow \psi_2)$ is true at all the same chance-world pairs as $(\phi_1 \,\Box\!\!\rightarrow \mathrm{ch}(\psi_1) = 1) \wedge (\phi_2 \,\Box\!\!\rightarrow \mathrm{ch}(\psi_2) = 1)$, their chances relative to a single chance function must be the same.

[33] As an anonymous referee points out, this problem is closely related to the credence problem. We could, for example, contrast the intuitive chances of (ia–b):

(i)	a.	If the coin were flipped, it would land heads.	chance: 1/2
	b.	If the coin were flipped, it would have a 100% chance of landing heads.	chance: 0

So the use of compound conditionals may not be strictly necessary to illustrate the problem. It simply helps to draw out these intuitions further.

[34] A version of this principle is defended in Cariani and Goldstein 2018.

[35] For related results concerning probability modals, see Goldstein 2017.

But there is no problem with $\phi \mathrel{\Box\!\!\rightarrow} \psi$ and $\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\psi) = 1$ being *historically* equivalent. In fact, their historical equivalence explains our intuitions about extreme credences and chances. Thus, even though rational agents can have different credences in $\phi \mathrel{\Box\!\!\rightarrow} \psi$ and $\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\psi) = 1$, it seems plausible that they can't be *certain* in $\phi \mathrel{\Box\!\!\rightarrow} \psi$ without being certain in $\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\psi) = 1$. Similarly, if the chance of $(\phi_1 \mathrel{\Box\!\!\rightarrow} \psi_1) \wedge (\phi_2 \mathrel{\Box\!\!\rightarrow} \psi_2)$ is 1, then the chance of $(\phi_1 \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\psi_1) = 1) \wedge (\phi_2 \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\psi_2) = 1)$ must also be 1.

Relatedly, we saw that the classically validity of Clash predicts that rational agents can't have high credence in both $\phi \mathrel{\Box\!\!\rightarrow} \psi$ and $\phi \mathrel{\Diamond\!\!\rightarrow} \neg\,\psi$, which seems incorrect in light of (24a) and (24c). Even so, it seems plausible that a rational agent can't be *certain* in both $\phi \mathrel{\Box\!\!\rightarrow} \psi$ and $\phi \mathrel{\Diamond\!\!\rightarrow} \neg\,\psi$. This suggests that Clash holds historically, but not classically.

So ideally, we want an account of counterfactuals that classically distinguishes $\phi \mathrel{\Box\!\!\rightarrow} \psi$ and $\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\psi) = 1$, but renders them historically equivalent. Furthermore, we want $\phi \mathrel{\Box\!\!\rightarrow} \psi$ and $\phi \mathrel{\Diamond\!\!\rightarrow} \neg\,\psi$ to be classically compatible but historically incompatible. My goal in the next section is to show how we can achieve this in a non-skeptical framework while still broadly accepting the restrictor semantics's resolution of the dilemma.

8 The Indeterminist Semantics

In this section, I develop a non-skeptical alternative to the restrictor semantics from §6.[36] I argue this semantics is strictly better than the restrictor semantics in that it resolves the dilemma in the same way, but avoids the problems from §7.

This semantics avoids these problems by allowing counterfactuals to occur bare, i.e., not under the scope of a chance operator. So our language will now be defined as follows:

$$\phi ::= p \mid \neg\,\phi \mid (\phi \wedge \psi) \mid \mathrm{ch}(\phi) = n \mid \phi \mathrel{\Box\!\!\rightarrow} \psi.$$

The abbreviations for $\Box$, $\Diamond$, and $\mathrel{\Diamond\!\!\rightarrow}$ remain the same—in particular, $\phi \mathrel{\Diamond\!\!\rightarrow} \psi := \phi \mathrel{\Box\!\!\rightarrow} \Diamond\,\psi$. Models will be of the same form as before, except we impose new restrictions on imaging. Specifically, we require that imaging be defined as in Lewis 1976: imaging on P effectively shifts the probability of each world to its closest P-world. More precisely, in addition to the previous constraints on imaging, we require the following (where $P \neq \varnothing$):

(iv) If $ch(\{w\}) = 1$, then there is a $v \in P$ such that $ch(\{v\} \,||\, P) = 1$
(v) Where ch_w is the probability function such that $ch(\{w\}) = 1$:

$$ch(Q \,||\, P) = \sum_{w \in W} ch(w) \cdot ch_w(Q \,||\, P)$$

Say ch is **opinionated** if $ch = ch_w$ for some w. Informally, opinionated functions are the probabilistic representatives of worlds. Since $ch_w(\cdot \,||\, P)$ is also opinionated by (iv), we can think of $ch_w(\cdot \,||\, P)$ as representing the unique closest P-world to w. Thus, (v) says that imaging on P is the result of shifting the probability of every world to its closest

[36] This semantics is similar to the epsilon semantics developed by Schulz (2014, 2017). For reasons of space, I won't comment on the differences here.

P-world. So we may define $f_{ch}(\phi, w)$ (the closest $[\![\phi]\!]^{ch}$-world to w) to be that v such that $ch_w(\{v\} \mid\mid [\![\phi]\!]^{ch}) = 1$. The truth conditions for counterfactuals are stated below:

$$[\![\phi \mathbin{\Box\!\!\to} \psi]\!]^{ch,w} = \mathsf{T} \quad \Leftrightarrow \quad [\![\psi]\!]^{ch_\phi, f_{ch}(\phi,w)} = 1.$$

Call this the **indeterminist semantics** for counterfactuals. (I will explain the label in §9.) As before, $\text{ch}(\phi \mathbin{\Box\!\!\to} \psi) = n$ and $\phi \mathbin{\Box\!\!\to} \text{ch}(\psi) = n$ have the same truth conditions:[37]

$$[\![\text{ch}(\phi \mathbin{\Box\!\!\to} \psi) = n]\!]^{ch,w} = \mathsf{T} \quad \Leftrightarrow \quad ch_\phi([\![\psi]\!]^{ch_\phi}) = n \quad \Leftrightarrow \quad [\![\phi \mathbin{\Box\!\!\to} \text{ch}(\psi) = n]\!]^{ch,w} = \mathsf{T}.$$

Defining our two notions of consequence as in §5, we can block the horns of the dilemma in exactly the same way as in §6. Identity is still classically invalid, so we cannot use it to derive lines 1 and 2 of Horn 1. And Could-Closure is not historically valid, so line 2 of Horn 2 does not follow from line 1. Hence, we avoid the dilemma. However, there are important differences between the restrictor semantics and the indeterminist semantics.

First, Chance Undermines Would is neither classically nor historically valid. This is for exactly the same reason Chance Undermines Truth isn't valid. For ch to settle true $\phi \mathbin{\Box\!\!\to} \Diamond \neg \psi$, all that's required is that ch_ϕ does not settle true ψ. But to settle true $\neg(\phi \mathbin{\Box\!\!\to} \psi)$, ch_ϕ must settle true $\neg \psi$, which is a strictly stronger requirement. Moreover, even if ch_ϕ fails to settle true ψ, that's compatible with ψ being true at ch_ϕ and $f_{ch}(\phi, w)$ for some w. In other words, the truth of $\phi \mathbin{\Box\!\!\to} \Diamond \neg \psi$ does not entail the truth of $\neg(\phi \mathbin{\Box\!\!\to} \psi)$. So no counterfactual skepticism.

Duality fails for the same reason. The indeterminist semantics predicts that $\phi \mathbin{\Diamond\!\!\to} \psi$ does not entail $\neg(\phi \mathbin{\Box\!\!\to} \neg \psi)$. This prediction seems empirically well-supported. For instance, as Moss (2013, p. 253) notes, the inference from (26a) to (26b) seems bad, whereas the inference to (26c) seems good (though see footnote 32 for a caveat).

(26) a. If the coin were flipped, it could land tails.
b. $\not\Rightarrow$ It's not the case that if the coin were flipped, it would land heads.
c. $\Rightarrow$ If the coin were flipped, there'd be some chance it wouldn't land heads.

Second, Clash is historically valid, but classically invalid. The pair of $\phi \mathbin{\Box\!\!\to} \psi$ and $\phi \mathbin{\Diamond\!\!\to} \neg \psi$ (i.e., $\phi \mathbin{\Box\!\!\to} \Diamond \neg \psi$) behaves exactly like a chancy contradiction of the form $\psi \wedge \Diamond \neg \psi$.

[37] Proof: Assume ch_ϕ is defined. By the truth conditions, $[\![\phi \mathbin{\Box\!\!\to} \text{ch}(\psi) = n]\!]^{ch,w} = \mathsf{T}$ iff $ch_\phi([\![\psi]\!]^{ch_\phi}) = n$, and $[\![\text{ch}(\phi \mathbin{\Box\!\!\to} \psi) = n]\!]^{ch,w} = \mathsf{T}$ iff $ch([\![\phi \mathbin{\Box\!\!\to} \psi]\!]^{ch}) = n$. So we need to show $ch([\![\phi \mathbin{\Box\!\!\to} \psi]\!]^{ch}) = ch_\phi([\![\psi]\!]^{ch_\phi})$.

$$\begin{aligned}
ch([\![\phi \mathbin{\Box\!\!\to} \psi]\!]^{ch}) &= ch(\{v \mid [\![\phi \mathbin{\Box\!\!\to} \psi]\!]^{ch,v} = 1\}) \\
&= ch(\{v \mid [\![\psi]\!]^{ch_\phi, f_{ch}(\phi,v)} = 1\}) \\
&= \sum_{v \in W} ch(v) \cdot [\![\psi]\!]^{ch_\phi, f_{ch}(\phi,v)} \\
&= \sum_{v \in W} ch(v) \cdot ch_{f_{ch}(\phi,v)}([\![\psi]\!]^{ch_\phi}) \\
&= \sum_{v \in W} ch(v) \cdot ch_v([\![\psi]\!]^{ch_\phi} \mid\mid [\![\phi]\!]^{ch}) \\
&= ch_\phi([\![\psi]\!]^{ch_\phi}).
\end{aligned}$$

The last step follows from condition (v) on imaging.

Just as it is possible for $\psi \wedge \Diamond \neg \psi$ to be true at a chance-world pair, so too it is possible for $\phi \mathrel{\Box\!\!\rightarrow} \psi \wedge \phi \mathrel{\Diamond\!\!\rightarrow} \neg \psi$ to be true. And just as it is not possible for $\psi \wedge \Diamond \neg \psi$ to be settled true by a chance function, so too it is not possible $\phi \mathrel{\Box\!\!\rightarrow} \psi \wedge \phi \mathrel{\Diamond\!\!\rightarrow} \neg \psi$ to be settled true.

Third, Liberal Would-Closure does not hold in full. Specifically, one of the 'mixed' closure principle fails: the consequent of would-counterfactuals is not classically closed under historical consequence. For instance, $\psi \vDash_{\mathrm{h}} \Box \psi$ whereas $\phi \mathrel{\Box\!\!\rightarrow} \psi \nvDash_{\mathrm{c}} \phi \mathrel{\Box\!\!\rightarrow} \Box \psi$. However, we do still get the 'unmixed' closure principles for would-counterfactuals:

Classical Would-Closure. If $\psi \vDash_{\mathrm{c}} \chi$, then $\phi \mathrel{\Box\!\!\rightarrow} \psi \vDash_{\mathrm{c}} \phi \mathrel{\Box\!\!\rightarrow} \chi$

Historical Would-Closure. If $\psi \vDash_{\mathrm{h}} \chi$, then $\phi \mathrel{\Box\!\!\rightarrow} \psi \vDash_{\mathrm{h}} \phi \mathrel{\Box\!\!\rightarrow} \chi$

So from the fact that $\psi \vDash_{\mathrm{h}} \Box \psi$, we *can* infer that $\phi \mathrel{\Box\!\!\rightarrow} \psi \vDash_{\mathrm{h}} \phi \mathrel{\Box\!\!\rightarrow} \Box \psi$.

Finally, Limited CEM is valid. Each chance function ch gives rise to a Stalnaker selection function f_{ch}. Assuming $f_{ch}(\phi, w)$ is defined (i.e., ch_{ϕ} is defined), $\phi \mathrel{\Box\!\!\rightarrow} \psi$ is true at w relative to ch iff ψ is true, relative to ch_{ϕ}, at the f_{ch}-closest ϕ-world to w. So either ψ is true at the f_{ch}-closest ϕ-world (i.e., $\phi \mathrel{\Box\!\!\rightarrow} \psi$ is true) or else $\neg \psi$ is (i.e., $\phi \mathrel{\Box\!\!\rightarrow} \neg \psi$ is true).

In sum, here is how the indeterminist semantics differs from the restrictor semantics:

- Chance Undermines Would is not valid in any sense. Neither is Duality.
- Clash is only historically valid. (Chance Ensures Could is still valid in both senses).
- Liberal Would-Closure does not hold. However, Would-Closure holds for both classical and historical consequence.
- Limited CEM is valid in both senses.

These features of the indeterminist semantics allow it to avoid the three problems discussed in §7. Take the credence problem first. Since (24a) does not entail (24b) (because Liberal Would-Closure fails), rational agents can have a high credence in the former while having a low credence in latter. And since (24a) and (24c) are consistent (because Clash is classically invalid), rational agents can have high credence in both.[38]

(24) a. If the coin were flipped 1 billion times, it would land heads at least once.
1 Billion Flips $\mathrel{\Box\!\!\rightarrow}$ *Some Heads* rational credence: ≈ 1
b. If the coin were flipped 1 billion times, it would have a 100% chance of landing heads at least once.
1 Billion Flips $\mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\textit{Some Heads}) = 1$ rational credence: 0
c. If the coin were flipped 1 billion times, it could fail to land heads at least once.
1 Billion Flips $\mathrel{\Diamond\!\!\rightarrow} \neg$ *Some Heads* rational credence: 1

[38] Santorio (forthcoming) presents a triviality result to the effect that given some modest assumptions (all of which are satisfied in this system), $\Pr(\phi \mathrel{\Box\!\!\rightarrow} \psi) = \Pr(\phi \mathrel{\Diamond\!\!\rightarrow} \psi)$ (which does not hold in this system). The proof first establishes $\Pr(\phi \mathrel{\Box\!\!\rightarrow} \neg \psi \wedge \phi \mathrel{\Diamond\!\!\rightarrow} \psi) = 0$, and then uses this to establish the result. The indeterminist semantics blocks the first step, specifically, in the following inference, where $\Pr_{\phi \Box\!\rightarrow \neg \psi}(\cdot) = \Pr(\cdot \mid \phi \mathrel{\Box\!\!\rightarrow} \neg \psi)$:

$$\Pr(\phi \mathrel{\Box\!\!\rightarrow} \psi \mid \phi \mathrel{\Diamond\!\!\rightarrow} \psi) > 0 \quad \Rightarrow \quad \Pr_{\phi \Box\!\rightarrow \neg \psi}(\phi \mathrel{\Box\!\!\rightarrow} \psi \mid \phi \mathrel{\Diamond\!\!\rightarrow} \psi) > 0$$

While $\phi \mathrel{\Box\!\!\rightarrow} \neg \psi \wedge \phi \mathrel{\Diamond\!\!\rightarrow} \psi$ is classically consistent, and so can receive positive probability, conditionalizing on $\phi \mathrel{\Box\!\!\rightarrow} \neg \psi$ results in a probability function that assigns probability zero to $\phi \mathrel{\Diamond\!\!\rightarrow} \psi$. So $\Pr_{\phi \Box\!\rightarrow \neg \psi}(\cdot \mid \phi \mathrel{\Diamond\!\!\rightarrow} \psi)$ is undefined. This solution turns on Clash being classically invalid but historically valid.

However, rational agents cannot have credence 1 in (24a) without also having credence 1 in (24b), since Historical Would-Closure holds. Similarly, they cannot have credence 1 both in (24a) and in (24c), since Clash is historically valid.

The compounding problem is solved in a similar way. Since (25a) is not classically equivalent to (25b), the two can have different chances. But if the former has chance 1, the latter must as well.

(25) a. If the first coin were flipped, it would land heads and if the second coin were flipped, it would land heads.
$(Flip_1 \mathrel{\Box\!\!\rightarrow} Heads_1) \wedge (Flip_2 \mathrel{\Box\!\!\rightarrow} Heads_2)$ chance: 1/4

b. If the first coin were flipped, it would have a 100% chance of landing heads and if the second coin were flipped, it would have a 100% of landing heads.
$(Flip_1 \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(Heads_1) = 1) \wedge (Flip_2 \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(Heads_2) = 1)$ chance: 0

Finally, because Limited CEM is valid, the additivity problem is solved. Thus, the following hold universally:

$$\mathrm{ch}(\phi \mathrel{\Box\!\!\rightarrow} \psi \vee \phi \mathrel{\Box\!\!\rightarrow} \neg \psi) = \mathrm{ch}(\phi \mathrel{\Box\!\!\rightarrow} \psi) + \mathrm{ch}(\phi \mathrel{\Box\!\!\rightarrow} \neg \psi)$$
$$\mathrm{ch}(\phi \mathrel{\Box\!\!\rightarrow} \psi) = \mathrm{ch}(\phi \mathrel{\Box\!\!\rightarrow} \psi \wedge \phi \mathrel{\Diamond\!\!\rightarrow} \psi) + \mathrm{ch}(\phi \mathrel{\Box\!\!\rightarrow} \psi \wedge \neg(\phi \mathrel{\Diamond\!\!\rightarrow} \psi)).$$

Taken together, we have a solution to all the problems raised in §7. Moreover, we still resolve the dilemma in exactly the same way that was presented in §6. The indeterminist semantics is therefore a strictly better account than the restrictor semantics. And this is all done while avoiding counterfactual skepticism.

9 Where the Argument from Chance Goes Wrong

As we saw in §8, Chance Undermines Would is neither classically nor historically valid in the indeterminist semantics. Thus, the argument from chance must go wrong somewhere. Classically, the argument is unsound since Clash fails. Historically, however, Clash and Chance Ensures Could both hold. So where does the argument go wrong on historical consequence? The answer is that it is historically invalid: it trades on a conflation between a counterfactual's falsehood and its lack of settled truth.

To see why, it is instructive to look more closely at the derivation of Chance Undermines Would from Clash and Chance Ensures Could. As a reminder, here are the relevant principles exactly as they were stated earlier:

Chance Ensures Could. $\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\psi) \neq 0 \vDash \phi \mathrel{\Diamond\!\!\rightarrow} \psi$

Clash. $\phi \mathrel{\Box\!\!\rightarrow} \psi, \phi \mathrel{\Diamond\!\!\rightarrow} \neg \psi \vDash \bot$

Chance Undermines Would. $\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\neg \psi) \neq 0 \vDash \neg(\phi \mathrel{\Box\!\!\rightarrow} \psi)$

Derivation of Chance Undermines Would.

1.	$\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\neg\psi) \neq 0 \vDash \phi \mathrel{\Diamond\!\!\rightarrow} \neg\psi$	Chance Ensures Could
2.	$\phi \mathrel{\Box\!\!\rightarrow} \psi, \phi \mathrel{\Diamond\!\!\rightarrow} \neg\psi \vDash \bot$	Clash
3.	$\phi \mathrel{\Box\!\!\rightarrow} \psi, \phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\neg\psi) \neq 0 \vDash \bot$	cut, 1, 2
4.	$\phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\neg\psi) \neq 0 \vDash \neg(\phi \mathrel{\Box\!\!\rightarrow} \psi)$	reductio, 3

Notice, however, that the move from 3 to 4 makes use of an instance of reductio that was found to be specious for ordinary chance claims. It is the same move that we want to block in the inference from Chancy Contradiction to Chance Undermines Truth.

Chancy Contradiction. $\phi \wedge \mathrm{ch}(\neg\phi) \neq 0 \vDash \bot$

Chance Undermines Truth. $\mathrm{ch}(\neg\phi) \neq 0 \vDash \neg\phi$

If we are not licensed to use reductio to derive Chance Undermines Truth from Chancy Contradiction, it seems we are similarly not licensed to infer Chance Undermines Would from line 3. Instead, we can only infer a weakened principle, which is effectively the counterfactual version of Chancy Contradiction (cf. Santorio 2017 on indicatives):

Counterfactual Chancy Contradiction. $\phi \mathrel{\Box\!\!\rightarrow} \psi, \phi \mathrel{\Box\!\!\rightarrow} \mathrm{ch}(\neg\psi) \neq 0 \vDash \bot$

Thus, the argument from chance relies on a specious move, viz., the move from a counterfactual's *lack of settled truth* (i.e., not having chance 1) to its *falsehood*. It therefore does not establish Chance Undermines Would. All the argument establishes is the historical validity of Counterfactual Chancy Contradiction. This principle does not support counterfactual skepticism, however. At best, it supports **counterfactual indeterminism**, the view that most counterfactuals are chancy, i.e., not settled true.

Counterfactual indeterminism is hardly a surprising conclusion. It simply says about counterfactuals what ordinary indeterminism says about future contingents. Most ordinary claims about the future are not settled true. It does not follow, however, that most claims about the future are false. Counterfactual indeterminism says the same for counterfactuals: most are chancy, but it does not follow that most are false.

Counterfactual indeterminism is not simply a 'no truth value' view of counterfactuals. The indeterminist semantics assigns a truth value to counterfactuals relative to a world and chance function (which, again, we can think of as standing in for a time). At most, it says that counterfactuals can lack a 'settled truth value', where all that means is that they can be chancy. It is therefore compatible with many of the counterfactuals we use in ordinary conversation being, in some sense, true.[39]

[39] Counterfactual indeterminism actually avoids Hájek's objections to 'no truth value' views (pp. 51–53) even if we interpret 'truth' as settled truth. He presents several arguments. First: if would-counterfactuals lack a truth value, then so do could-counterfactuals, which is implausible. But this argument assumes Duality, which the indeterminist semantics rejects. Indeed, if a $\phi \mathrel{\Box\!\!\rightarrow} \psi$ lacks a settled truth value, then $\phi \mathrel{\Diamond\!\!\rightarrow} \neg\psi$ is simply true. Second: counterfactuals with true antecedents and false consequents are false. If by 'true' and 'false', we mean 'settled true' and 'settled false', the indeterminist semantics agrees: $\phi \wedge \psi \vDash_{\mathrm{h}} \phi \mathrel{\Box\!\!\rightarrow} \psi$. If we mean receiving semantic value T or F, the indeterminist semantics agrees for chance-free ϕ and ψ, though this

10 Conclusion

We started with a simple yet powerful argument for counterfactual skepticism. The argument uses two plausible principles: Chance Ensures Could and Clash. From there, we seemed to derive Chance Undermines Would, which quickly leads to skepticism. But we saw that the two principles used in this argument entail absurd results given some modest closure principles. I then proposed what I took to be the most promising resolution of the dilemma on behalf of the counterfactual skeptic. This approach allows us to maintain Clash, Chance Ensures Could, and Chance Undermines Would while denying problematic instances of closure in a principled manner. However, the proposal faced serious problems because it equated bare counterfactuals with 'chance 1' counterfactuals.

In response, I showed how we can develop a strictly better, non-skeptical account of counterfactuals that resolves the dilemma in the same manner, but avoids the problems facing the skeptic. Looking back, we observed that the argument from chance founders on an equivocation between a counterfactual's falsehood with its lack of settled truth. Once this mistake is recognized, we see that the argument for chance only licenses the less surprising conclusion that most counterfactuals are not settled true, i.e., counterfactual indeterminism. Counterfactuals, then, are chancy in exactly the way claims about the future are.

References

Bacon, Andrew. 2014. "Giving your knowledge half a chance." *Philosophical Studies* 171:373–397.

Baltag, Alexandru, van Ditmarsch, Hans P, and Moss, Lawrence S. 2008. "Epistemic logic and information update." *Handbook on the Philosophy of Information* 369–463.

Bennett, Jonathan Francis. 2003. *A Philosophical Guide to Conditionals*. Oxford University Press.

Cariani, Fabrizio. 2021. *The Modal Future*. Cambridge: Cambridge University Press.

Cariani, Fabrizio and Goldstein, Simon. 2018. "Conditional heresies." *Philosophy and Phenomenological Research* 1–32.

Charlow, Nate. 2019. "Grading modal judgment." *Mind* 1–39.

DeRose, Keith. 1999. "Can it be that it would have been even though it might not have been?" *Philosophical Perspectives* 13:385–413.

Dorr, Cian, Goodman, Jeremy, and Hawthorne, John. 2014. "Knowing against the odds." *Philosophical Studies* 170:277–287.

can fail for chance statements, which seems to be the right result; e.g., $\phi \wedge \mathrm{ch}(\neg\phi) \neq 0 \not\models_c \phi \mathrel{\square\!\!\rightarrow} \mathrm{ch}(\neg\phi) \neq 0$ (see footnote 4). Third: probabilistic accounts of counterfactuals suffer from a Frege-Geach problem. The indeterminist semantics, by contrast, is fully compositional. Finally: 'one wonders what the probability of a counterfactual *is*, if not its probability of truth'. Fair enough, but probability is not the probability of settled truth anyway. Counterfactuals have probabilities in exactly the same way claims about the future do.

Easwaran, Kenny. 2014. "Regularity and hyperreal credences." *The Philosophical Review* 123:1–41.

Edgington, Dorothy. 2008. "Counterfactuals." *Proceedings of the Aristotelian Society* 108:1–21.

Fintel, von Kai. 2001. "Counterfactuals in a Dynamic Context." In Michael Kenstowicz (ed.), *Ken Hale, A Life in Language*, 123–152. Cambridge, MA: MIT Press.

Gärdenfors, Peter. 1982. "Imaging and Conditionalization." *Journal of Philosophy* 79:747–760.

Gillies, Anthony. 2001. "A New Solution to Moore's Paradox." *Philosophical Studies* 105:237–250.

—. 2007. "Counterfactual Scorekeeping." *Linguistics and Philosophy* 30:329–360.

Goldstein, Simon. 2017. "Triviality Results for Probabilistic Modals." *Philosophy and Phenomenological Research* 99:188–222.

—. 2019. "Generalized Update Semantics." *Mind* 128:795–835.

—. forthcoming. "Epistemic modal credence." *Philosophers' Imprint* 1–34.

Hájek, Alan. 2014. "Probabilities of counterfactuals and counterfactual probabilities." *Journal of Applied Logic* 12:235–251.

—. 2020a. "Contra counterfactism." *Synthese* 1–30. doi:10.1007/s11229-020-02643-x.

—. 2020b. "Counterfactual scepticism and antecedent-contextualism." *Synthese* 1–23. doi: 10.1007/s11229-020-02686-0.

—. ms. "Most Counterfactuals are False." Unpublished manuscript.

Hawthorne, John. 2004. *Knowledge and Lotteries*. Oxford: Oxford University Press.

—. 2005. "Chance and Counterfactuals." *Philosophy and Phenomenological Research* 70:396–405.

Hawthorne, John and Lasonen-Aarnio. 2009. "Knowledge and Objective Chance." In Patrick Greenough and Duncan Pritchard (eds.), *Williamson on Knowledge*, 92–108. Oxford University Press.

Higginbotham, James. 1986. "Linguistic Theory and Davidson's Program in Semantics." In Ernie Lepore (ed.), *Truth and Interpretation: Perspectives on the Philosophy of Donald Davidson*, 29–48. Basil Blackwell Publishers.

—. 2003. "Conditionals And Compositionality." In John Hawthorne and Dean Zimmerman (eds.), *Language and Philosophical Linguistics*, volume 17, 181–194. Oxford: Blackwell.

Holliday, Wesley H. and Icard, Thomas. 2010. "Moorean Phenomena in Epistemic Logic." In L. Beklemishev, V. Goranko, and V. Shehtman (eds.), *Advances in Modal Logic*, volume 8, 178–199.

Ichikawa, Jonathan. 2011. "Quantifiers, Knowledge, and Counterfactuals." *Philosophy and Phenomenological Research* 82:287–313.

Joyce, James. 1999. *The Foundations of Causal Decision Theory*. Cambridge: Cambridge University Press.

Katsuno, H. and Mendelzon, A. O. 1992. "On the difference between updating a knowledge base and revising it." In Peter Gärdenfors (ed.), *Belief Revision*, 183–203. Cambridge University Press.

Khoo, Justin. forthcoming. *The Meaning of If*. Oxford: Oxford University Press.

Khoo, Justin and Santorio, Paolo. 2018. "Lecture notes: Probability of conditionals in modal semantics."

Klinedinst, Nathan. 2011. "Quantified Conditionals and Conditional Excluded Middle." *Journal of Semantics* 28:149–170.

Kolodny, Niko and MacFarlane, John. 2010. "Ifs and Oughts." *Journal of Philosophy* 107:115–143.

Kratzer, Angelika. 1981. "The Notional Category of Modality." In H. J. Eikmeyer and H. Rieser (eds.), *Words, Worlds, and Contexts: New Approaches to Word Semantics*, 38–74. Berlin: Walter de Gruyter.

—. 1986. "Conditionals." *Chicago Linguistics Society* 22:1–15.

—. 2012. *Modals and Conditionals: New and Revised Perspectives*. New York: Oxford University Press.

Lassiter, Daniel. 2017. "Probabilistic language in indicative and counterfactual conditionals." *Semantics and Linguistic Theory* 27:525–546.

Leitgeb, Hannes. 2012a. "A Probabilistic Semantics for Counterfactuals. Part A." *The Review of Symbolic Logic* 5:26–84.

—. 2012b. "A Probabilistic Semantics for Counterfactuals. Part B." *The Review of Symbolic Logic* 5:85–121.

—. 2013. "A Lottery Paradox for Counterfactuals Without Agglomeration." *Philosophy and Phenomenological Research* 89:605–636.

Lewis, David K. 1976. "Probabilities of Conditionals and Conditional Probabilities." *The Philosophical Review* 85:297–315.

—. 1986. "Postscripts to "Counterfactual Dependence and Time's Arrow"." In *Philosophical Papers Volume II*, 52–66. Oxford: Oxford University Press.

Lewis, Karen. 2016. "Elusive Counterfactuals." *Noûs* 50:286–313.

Mandelkern, Matthew. 2019. "Talking about Worlds." *Philosophical Perspectives* 32:298–325.

Moss, Sarah. 2013. "Subjunctive credences and semantic humility." *Philosophy and Phenomenological Research* 87:251–278.

—. 2015. "On the Semantics and Pragmatics of Epistemic Modals." *Semantics & Pragmatics* 8:1–81.

—. 2018. *Probabilistic Knowledge*. Oxford: Oxford University Press.

—. 2019. "Full Belief and Loose Speech." *The Philosophical Review* 128:255–291.

Ninan, Dilip. ms. "Easy Foreknowledge." Unpublished manuscript.

Nute, Donald. 1980. *Topics in Conditional Logic*, volume 20. Springer Science & Business Media.

Sandgren, Alexander and Steele, Katie. 2020. "Levelling counterfactual scepticism." *Synthese* 1–21. doi:10.1007/s11229-020-02742-9.

Santorio, Paolo. 2017. "Conditional excluded middle in informational semantics." In Alexandre Cremers, Thomas van Gessel, and Floris Roelofsen (eds.), *Proceedings for the 21st Amsterdam Colloquium*, 385–394.

—. forthcoming. "General Triviality for Counterfactuals." *Analysis* 1–15.

Schulz, Moritz. 2014. "Counterfactuals and Arbitrariness." *Mind* 123:1021–1055.

—. 2017. *Counterfactuals and probability*. Oxford: Oxford University Press.

Schwarz, Wolfgang. 2018. "Subjunctive Conditional Probability." *Journal of Philosophical Logic* 47:47–66.

Starr, William B. 2014. "A Uniform Theory of Conditionals." *Journal of Philosophical Logic* 43:1019–1064.

Stefánsson, H. Orri. 2018. "Counterfactual Skepticism and Multidimensional Semantics." *Erkenntnis* 83:875–898.

Swanson, Eric. 2012. "Conditional excluded middle without the limit assumption." *Philosophy and Phenomenological Research* 85:301–323.

van Benthem, Johan. 2004. "What one may come to know." *Analysis* 64:95–105.

Veltman, Frank. 1996. "Defaults in update semantics." *Journal of Philosophical Logic* 25:221–261.

Willer, Malte. 2013. "Dynamics of Epistemic Modality." *The Philosophical Review* 122:45–92.

Williams, J Robert G. 2010. "Defending Conditional Excluded Middle ." *Noûs* 44:650–668.

Yalcin, Seth. 2007. "Epistemic Modals." *Mind* 116:983–1026.

—. 2012. "A Counterexample to Modus Tollens." *Journal of Philosophical Logic* 41:1001–1024.