Causal interpretation of Gödel's ontological proof

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1   Introduction. Gödel's causal philosophy

Gödel states in his philosophical notes (from 1954, according to Hao Wang) that cause is "the fundamental philosophical concept" ([27, p. 432–433], [54, p. 119, 294, 315]). In discussions with Wang in the 1970s, Gödel confirms the fundamental role of causality with respect to time: "the real idea behind time is causation" [54, p. 320, 168], as well as with respect to the general and the particular: "causation is fundamental: it should also explain the general and the particular" [54, p. 312], which themselves in turn make a "fundamental fact of reality" [54, p. 295].[1]

For Gödel, philosophy includes metaphysical and theological worldview. The theological worldview has it as its principle that "the world and everything in it has meaning and sense". This principle is analogous to the "the principle that everything has a cause", which is "the basis of the whole of science" (letter to his mother dated October 6, 1961, see [53] and [54, p. 108]). Hence, if causality is the fundamental philosophical concept, then theological "meaning and sense" should be, in some way, founded on and explained in causal terms. Consequently, Gödel also holds that philosophy, which is for him "rationalistic" and "theological" [54, p. 290], should be transformed into an exact theory, a science. As he emphasizes (in a letter to his mother, 1961), one day we will be able to scientifically justify the theological worldview. Gödel mentions several times, in his discussions with Wang, that

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[1]"It is a fundamental fact of reality that there are two kinds of reality: universals and particulars (or individuals)" [54, p. 295].

what should be done in philosophy is something similar to what Newton had done for physics:

> The beginning of physics was Newton's work of 1687, which needs only very simple primitives: force, mass, law. I look for a similar theory for philosophy or metaphysics.[2]

Gödel expresses his programme of scientific philosophy quite succintly: when he says (1975, according to Wang) that

> the meaning of the world is the separation of force and fact,

he is actually explaining a specific philosophical-theological concept ('meaning of the world') in what he takes to be scientific causal terms ('force', 'fact').

Gödel expresses the principles that the world and everything in it has a meaning, and that meaning has always a causal sense (everything has a cause), by saying, more neutrally, that the world is rational (statement no. 1 in his "My philosophical viewpoint", around 1960, according to Wang). In this sense, as confirmed by Wang, Gödel can also conceive the principle of sufficient reason ('ratio') as "a given fundamental truth" [54, p. 63].

It is clear that although Gödel's concept of science is deeply grounded in the results of science of his time, it is not a common one: for example, it should include phenomenological introspection[3] (which includes a change of personality, too [54, p. 167]). These phenomenological procedures should be transformed into an exact methodology by means of which we could clearly and

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[2]Cf. [54, p. 167, cf. p. 233, 288, 319, 309, 332]. Here are some other confirmations, in various contexts, of Gödel's conception of philosophy as a science: "Husserl reached the end, arrived at the science of metaphysics", "philosophy is persecuted science" [54, p. 166]; on philosophy as "exact theory" and on a new science of concepts, see [54, p. 229–230]; cf. "the transformation of certain aspects of traditional philosophy into an exact science" (quotation according to [7]).

[3]Cf. [28, 5, 52]. Gödel speaks of "a deeper and deeper self knowledge of reason [. . . a more and more complete rational knowledge of the essence of reason (of which essence the faculty of self knowledge is itself a constituent part)]", Gödel's letter to Tillich, quoted according to van Atten [6].

precisely perceive primitive concepts of philosophy and "set up" axioms about them (see, e.g., [54, p. 288, 289]). Here, mathematical logic has a crucial role:

> The significance of mathematical logic for philosophy lies in its power to make thought explicit by illustrating and providing a frame for the axiomatic method. Mathematical logic makes explicit the central place of predication in the philosophical foundation of rational thought. ...[4]

From Gödel's ascribing to causality a fundamental role in explaining the general and the particular, and from the fact that predication relates the general and the particular, it follows that even the logical concept of predication could and should be further explained by means of causal terms. However, Gödel's axiomatic onto-theological system for his proof of God's existence (see [26]) does not contain explicit concept of causality. In [34] we have proposed a possible way of how necessity operator of the system could be interpreted as causality.

It should be emphasized that for Gödel causality does not come into philosophy only from science, as from the outside. He is quite aware that causality is also an intrinsically philosophical concept, a basic one for philosophers like Aristotle or Kant. For example, it is one of Kant's categories (see Gödel's note from 1954), and, as Gödel stresses, also a concept by means of which Kant describes the influence of things in themselves on our receptivity [25]. (For Aristotle, see for instance Gödel's description of the causal work of Aristotelian intellect (*nous*) [54, p. 235]).

As to the role of the concept of causality in Gödel's philosophy, Wang reports that Gödel said "on several occasions" in the seventies that he "was not able to decide what the primitive concepts of philosophy are" [54, p. 120, cf. p. 288]. However, the fundamental role of causality in philosophy is confirmed by Gödel's statements from different periods of his life, and seems to have remained, for him, largely undisturbed. On the other side, on the ground of the published Gödel's philosophical texts and remarks, it is far

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[4][54, p. 293].

from certain how Gödel might have thought about the details in carrying out the programme of a causally founded philosophy. Here, we will concentrate on a possible causal sense and formalization of Gödel's ontological system and his ontological proof of the necessary existence of God.

In the following, we, first, introduce Gödel's preferred, constructivity criteria of knowledge, and connect them with the concept of causality. Secondly, we give an historical example of a constructive approach to the causal knowledge and analyze the fulfilment of constructivity criteria in Gödel's ontological proof. Thirdly, we re-axiomatize Gödel's ontological system by introducing quantifiable causal terms (system QCGO) and give the system an appropriate semantics. Finally, we prove some additional interesting theorems of QCGP.

2 Gödel's constructivism and causality

Gödel's programme of scientific philosophy by the use of axiomatic method is in accordance with at least some aspects of his constructivistic epistemological views. On the example of Gödel's axiomatizing of logic of proofs (a sort of formalized epistemology), we show what kind of axiomatization Gödel might have in mind, and, at the same time, what obstacles in defining a fully constructivist system he was aware of. A similar axiomatic approach we will apply to the theory of causality.

Gödel emphasizes in his Zilsel Lecture from 1938 [24] the epistemological advantage of the reduction of our knowledge to "constructive systems", in the sense that "constructive systems are better than those that work with the existential 'there is'" [24, p. 83]. Constructive knowledge is more evident and reliable in that it gives more secure foundations (of mathematics, on which Gödel is focused in Zilsel Lecture, 1938, [24, p. 91]). Gödel enumerates the following criteria of constructivity (where as the second item the elimination of existential quantifiers is included):

1. computability and decidability of the "primitive operations and relations" (by means of propositional calculus and recursive definitions);

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2. $\exists x p(x)$ is used only as an abbreviation for $p(c)$; propositional operations "should not be applied to" $\forall x p$ – only statements with free variables should be used instead;

3. the use of axiomatic systems (recursive definitions, propositional calculus, substitution) with "ordinary complete induction" (Gödel leaves open the question about further axioms and rules);

4. "objects should be surveyable (that is, denumerable)" (according to an earlier draft, *Konzept*, objects are individuals, functions, and relations).

Let us first see how Gödel meant to apply these criteria to the concept of proving. Since this concept is, according to some Gödel's reflections, a special case of causation, we shall thereafter generalize our approach and propose a possible appropriate formalization of the concept of causality.

As to the causal conception of proofs and proving, Gödel mentions that axioms cause theorems, and theorems cause their consequences.[5] Morover, Gödel considered the possibility of an axiomatic deduction of logical and set-theoretical axioms themselves in terms of causality:

> Perhaps the other Kantian categories (that is logical ⟦categories⟧, including necessity) can be defined in terms of causality, and the logical (set-theoretical) axioms can be derived from the axioms of causality" [27, p. 432–435].

As to the constructivity of proofs, Gödel himself points out that his translation of intuitionistic propositional logic, being interpreted as a logic of provability, into S4 modal system [21](1933)

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[5]"Causation in mathematics, in the sense of, say, a fundamental theorem causing its consequences, is not in time, but we take it as a scheme in time" [54, p. 320]. "He once said to me that there is a sense of cause according to which axioms cause theorems. It seems likely that Gödel has in mind something like Aristotle's conception of cause or *aitia* which includes both causes and reasons" [54, p. 120]. It is important to distinguish here causality in a logical sense (say of logical consequence, or of a structure of logical forms), from the causality in our cognition of concepts and axioms, i.e. from the influence of concepts and logical truths on our mind.

is not constructive. The reason is that operator B (in place of $\Box$) has the meaning: "it is provable that" ("there is a proof that"), i.e. it implicitly contains existential quantification over proofs. Thus Gödel tried in his Zilsel Lecture to come to some version of logic of proofs (a sort of what is today called "justification logic") that will better satisfy the constructivity criteria. Instead of a provability operator (B), he uses formats zBq and zBp, q, with the meaning 'z is a proof of q' and 'z is a proof of q from p', respectively, operations on proofs $f(z, u)$ and z', and mentions axioms $zBp, q \wedge uBq, r \rightarrow f(z, u)Bp, r$, $zB\phi(x, y) \rightarrow \phi(x, y)$, as well as $uBv \rightarrow u'B(uBv)$.

A striking feature of both Gödel's S4 logic of provability (1933) and his logic of proofs from the Zilsel Lecture is that they can prove the unprovability of inconsistency. In the S4 logic of provability, we can prove $B\neg B0 = 1$ by means of modal axioms (on the ground of the contraposition of a **T** axiom $B0 = 1 \rightarrow 0 = 1$) [21]. On the other side, Gödel mentions that in his justification logic (1938) it follows from $\vdash aBq$ that $\vdash aB\forall u \neg uB0 = 1$. That is, in causal terms, it can be proved that nothing causes contradiction if some q is provable, and that this fact has its own cause. Gödel notes that B is not applied here to $\forall u$ (which would violate the second condition of constructivity listed above) since $\forall u \neg uB0 = 1$ "occurs here in *suppositio materialis* as an object, in quotation marks" [24, p. 101]. We reconstruct this proof in a modern format of justification logic. We extend the classical first-order basis with the justification logic counterparts of modal axioms **K** and **T** and the rule of axiom necessitation:

> **JK** $t\colon (\phi \rightarrow \psi) \rightarrow (u\colon \phi \rightarrow t \cdot u\colon \psi)$,
> **JT** $t\colon \phi \rightarrow \psi$,
> **ANec** $\vdash \phi \Longrightarrow \vdash c\colon \phi$,
> ϕ is an axiom, c its associated proof term,

where t and u are proof terms (see [4]), and add the following justification counterpart of a modal rule:

> **NecU** $\vdash \phi_1 \rightarrow t\colon (\phi_2 \rightarrow \ldots \rightarrow u\colon (\phi_n \rightarrow z\colon \psi)\ldots)$

$$\Longrightarrow \vdash \phi_1 \to t\colon (\phi_2 \to \ldots \to u\colon (\phi_n \to \mathsf{gen}_x(z)\colon \forall x\psi)\ldots),$$
$$x \notin \mathsf{free}(\phi_{m\leq n}).$$

(For the latter rule in a general modal setting, see [29, p. 293] and [51, p. 63] (**R5**).) We give a proof of $\mathsf{gen}_u((e \cdot ((c \cdot c') \cdot d)) \cdot a)\colon \forall z\neg z\colon \bot$ (instead of simply $a\colon \forall z\neg z\colon \bot$), where a is a proof of some theorem.[6]

1	$a\colon q$	any theorem q, a its proof
2	$c\colon ((z\colon \bot \to (\bot \to \neg q)) \to ((z\colon \bot \to \bot) \to$	CPC axiom,
	$\quad (z\colon \bot \to \neg q)))$	ANec, z not occurring in $a\colon q$
3	$c'\colon (z\colon \bot \to (\bot \to \neg q))$	CPC, ANec
4	$c \cdot c'\colon ((z\colon \bot \to \bot) \to (z\colon \bot \to \neg q))$	$2,3$ JK
5	$z\colon \bot \to \bot$	JT
6	$d\colon (z\colon \bot \to \bot)$	ANec
7	$(c \cdot c') \cdot d\colon (z\colon \bot \to \neg q)$	$4,6$ JK, MP
8	$e\colon ((z\colon \bot \to \neg q) \to (q \to \neg z\colon \bot))$	CPC, ANec
9	$e \cdot ((c \cdot c') \cdot d)\colon (q \to \neg z\colon \bot)$	$7,8$ JK, MP
10	$(e \cdot ((c \cdot c') \cdot d)) \cdot a\colon \neg z\colon \bot$	$1,9$ JK
11	$\top \to (e \cdot ((c \cdot c') \cdot d)) \cdot a\colon \neg z\colon \bot$	10 CPC
12	$\top \to \mathsf{gen}_z((e \cdot ((c \cdot c') \cdot d)) \cdot a)\colon \forall z\neg z\colon \bot$	11 NecU
13	$\mathsf{gen}_z((e \cdot ((c \cdot c') \cdot d)) \cdot a)\colon \forall z\neg z\colon \bot$	12 MP

CPC is classical propositional calculus, a and c' are assumed to be composed proof terms obtained by means of a successive application of **ANec**, and z is a proof variable. Gödel simply says that a, instead of $\mathsf{gen}_v((e \cdot ((c \cdot c') \cdot d)) \cdot a)$, is the proof of $\forall z\neg z\colon \bot$, thus leaving the proof generalization implicit, and reducing the finally obtained proof term to the starting proof part.

In the light of Gödel's second incompleteness theorem, by means of an arithmetically encoded statement that is precisely analogous to the above sentences expressing the unprovability of inconsistency, it is clear that the two Gödel's logics mentioned are not formalizations of a logic of proofs for any defined system containing arithmetics. Both logics formalize a proofs concept in some sense which is prior to the concept of proof in any particular system (provability in the "absolute sense").

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[6]Cf. a proof in [13], which I encountered afterwards.

Evaluating the constructivity of his logic of proofs, Gödel mentions that the third and the fourth of the conditions for constructivity are not satisfied (additional axioms about B are not reducible to definitions, there is no finite procedure for generating proofs, so proofs are "unsurveyable", see [47, p. 67]). As Gödel supposes, one way to repair these defects, as far as it is possible, would be to restrict logic L of proofs to the logic of proofs of L itself. Artemov fulfilled the idea of a logic of arithmetical proofs, without provable $aB\forall u\neg uB\bot$ and with a denumerable domain of (arithmetical) proofs (see [2]). Building upon semantics by Mkrtytchev [41] and including features of Kripkean models, Fitting described a general semantics in the sense of a logic of evidence (or justifications) in a broader sense than just arithmetical proofs [15]. However, if Gödel's 1938 logic of proofs is not interpretatively restricted to arithmetical proofs or to evidence in general, but is interpreted in a causal way,[7] the (causal) counterpart of $aB\forall u\neg uB\bot$ need not necessarily be a problematic one. Following this idea, we will apply features of Gödel's justification logic to his ontological proof, with modalities interpreted causally (in some stronger sense than just proof causality). Before doing that, we will generalize our discussion extending it to the principle of sufficient reason, and adding a historical note about the (non-)constructivity of the principle.

3   The principle of sufficient reason

Gödel's attempts to formalize a logic of provability and proofs are as such essentially interconnected with his later philosophical considerations about the principle of sufficient reason, and with his study of Leibniz's and Kant's philosophy. "Reason" can be in general conceived either in a logical sense (proof) or in a real sense (cause), independently of whether we conceive proofs themselves causally (as Gödel), or separate proofs and causality from one another (as e.g. Kant). Gödel often refers to Leibniz in connection with the principle of sufficient reason, but it was in fact Rugjer J.

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[7]Cf. a proposal for a justification logic of counterfactuals, mentioned in [3].

Bošković,[8] who gave a sort of constructive turn to the principle.

The principle of sufficient reason, which is a "fundamental truth" for Gödel (as reported by Wang), in one of Leibniz's formulations reads:

> ...no fact can ever be true or existent, no statement correct, unless there is a sufficient reason why things are as they are and not otherwise [38, p. §32].

If we use $\square$ in the existential (non-constructive) sense of 'there is a reason (proof, cause) for ...', the principle of sufficient reason amounts to the following:

$$\phi \rightarrow \square\phi$$

Of course, other ways of formalizing the principle of sufficient reason are available (see, e.g. [43, 44, 10][9]). Leibniz was aware of our limitations in the use of this principle: "in most cases we can't know what the reason is" (ibid).[10] However, the problem with what we may today call the non-constructivity of the principle of sufficient reason was pointed out more precisely by Bošković (then by Leibniz) in his *De continuitatis lege* (1754) [11]. Bošković criticizes the negative use of this principle in the sense of the inference from the "lack of reason" (*ex defectu rationis*), where the contrapositive of the principle of sufficient reason is a premise:

$$\{\neg\square\phi, \neg\square\phi \rightarrow \neg\phi\} \vdash \neg\phi.$$

Note that in $\neg\square\phi \rightarrow \neg\phi$, universal quantification (over reasons), $\neg\square$, occurs under a connective ($\rightarrow$, $\square$ being read existentially as "there is a reason", and occurring in *suppositio personalis*, not in *suppositio materialis*), thus violating the second above mentioned Gödel's criterion of constructivity. As Bošković argues, since "all" reasons are not in general available to human reasoning agents

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[8]Dubrovnik 1711 – Milan 1787.

[9]Let us mention that $\phi \rightarrow \square\phi$ is an axiom of the intuitionistically based propositional provability logic KM (see [42]).

[10]Cf. a commentary in [p. 201–202, 217–222, 282–284][55], where Leibniz's principle of sufficient reason is interpreted as a "completeness claim" about axiom systems in logic, mathematics, and metaphysics.

and there may be some reason for ϕ not being known to us, this sort of reasoning is not reliable (the exception is, for example, the case of the absurdity of ϕ). Instead, Bošković proposes a general inductive-deductive procedure of accumulating and verifying singular available reasons in order to establish a probable knowledge (e.g. a natural law), which should take into account also non-sensible basic elements of matter. At the same time, this constructive shift from the existential to the instantial conception of the principle of sufficient reason provides a justification of the empirical application of the concept of causality in science (in the form of probable causal laws).[11] In a sense, Bošković's criticism of the non-constructive use of the principle of sufficient reason can be considered as a precursor of Gödel's attempt at a constructive theory of reasons in 1938.

4 A glimpse on GO ontological proof

On inspection of Gödel's ontological proof in a form present in Gödel's sketch from 1970 and in Scott's slight variation of the proof, as well as on inspection of additional Sobel's modal collapse proof,[12] we can notice that Gödel's criteria of constructivity are, at least to some extent, respected. Following Hájek (e.g. [30]), we will use 'GO' to name Gödel's formal ontological system as presented by Scott. (1) In GO, there is a decidable modal propositional basis. (2) Regarding first-order quantifiers, we remark the following: (a) $\exists x \phi(x)$ can be read as if it is skolemized ($\phi(c)$, $\phi(f(x))$), (b) $\forall x$ can be omitted in the proof leaving x free, except where it occurs under $\square$ in *de dicto* position (*suppositio materialis*) – we notice that nowhere in the ontological proof $\forall x$ appears in *de re* position (bounding into a modal context). However, no exception is present for $\forall X$, e.g. in $\forall x(Gx \rightarrow \forall X(Xx \rightarrow \mathcal{P}(X))$, that is, $\forall X$ normally occurs under connectives and need not be in *suppositio materialis*. (3) A properly defined axiomatic system

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[11]See Hunter [31], who favours Bošković's theory in opposition to Humean skepticism. Cf. [36].

[12]See [26, 48]. Cf. [50] for a detailed axiomatic and model-theoretic reconstruction, and [1] for an introductory commentary.

is used. Although the concept of "positiveness" is not defined (in distinction to G, $\mathcal{E}\!\int\!\!\int$ and N), it is described by means of axioms, obviously having the concept of ultrafilter as a paradigm.[13] (4) Objects, including causes, can be surveyable – at least one object suffices for the whole ontological proof. This in distinction to the "unsurveyable" number of proofs if "proof" is taken in an "absolute sense" (see [24]).

Replacing $\Box$ of GO with causal terms in analogy with Gödel's proof terms of [24] and in connection with the discussion about non-constructivity in the use of the principle of sufficient reason, makes a further move towards a more constructive (in the Gödelian sense of [24]) ontological theory. Since Gödel conceived mathematical theoremhood, too, as a causal matter, formula ϕ in $t\colon \phi$, in causally (ontologically) reformulated justification logic, can be a logical/mathematical theorem, and t its proof in the sense of a logical cause of ϕ. – Let us add two examples of how ontological causality can exceed a logical/mathematical one. First, under the supposition of the causation $a\colon q$ and of the presence of a (Ea), it is plausible to have $a'\colon \forall u(\neg u\colon (0 = 1))$ as a consequence in a causal sense: if a actually causes q then some a' (not necessarily a proof) prevents that anything causes $\neg q$, since otherwise we could have a contradictory causal event (with some b actually causing $\neg q$). Hence, a implies the causation that exceeds mathematical formalism. Another example is Axiom **C5** (see next section), $\neg t\colon \phi \to ?t\colon \neg t\colon \phi$. Here, $?t$, cannot have a sense of one proof of infinitely many propositions of the form $\neg t\colon \phi$ (cf. [3]), though it could have a sense of some unique ontological cause with infinitely many effects of the form mentioned.

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[13]This may be connected with Gödel's quest for a non-mechanical procedure in finding new axioms, not only for set theory, but for philosophy, too, in the sense of finding and defining (by means of axioms) primitive philosophical concepts. Gödel meant that such a procedure could be obtained through some future improvement and refinement of phenomenological introspective method (up to an exact, scientific theory) in investigating our use of concepts.

5 Formalization of a causal Gödel ontology (QCGO)

5.1 Language $\mathcal{L}$QCGO

In [34] we proposed a combination of Gödel's onto-theological formal system GO with justification logic. We conceived justification terms causally, with the intention to describe the concept of causation constructively, in the sense of working with ("more evident") causal instances instead of merely with the presence of causation ('there is a cause that'). We now relax this constraint and generally allow the (first-order) quantification over causes, too, in order to enable universally quantified causation in *de dicto* position, as at first allowed by Gödel in his Zilsel Lecture. Similarly, Gödel nowhere in his onto-theological system (GO) explicitly restricted first-order quantification (although we see that first-order quantification *de re* never occurs in his ontological proof).

We now describe a quantificational causal system QCGO and its language $\mathcal{L}$QCGO.

The vocabulary of $\mathcal{L}$QCGO is a modification and extension of the vocabulary of first-order justification logic FOLP (see [4, 17]). The vocabulary of $\mathcal{L}$QCGO consists of first- and second-order variables (x, y, z, x_1, ...; $X^n, Y^n, Z^n, X^n_1, \ldots$), first-order constants ($c, c_1, \ldots$), second-order constants (relation symbols $P^n, P^n_1, \ldots$; = and function symbols $+, \cdot, !, ?$, gen_x, gen_y, ..., gen_X, gen_Y, ..., abs_x, $\mathsf{abs}_y, \ldots$, exs), third-order constant $\mathcal{P}$ (positivity), and parentheses. Operators are $\neg$, $\rightarrow$, $\forall$ (other propositional and quantification operators classically defined), $\imath$, λ, and : .

By the following two definitions we jointly define terms and formulas of $\mathcal{L}$QCGO.

Definition 1 (Terms). *First-order variables, first-order constants, and terms of the form $\imath x \phi(x)$ are first-order terms (for ϕ, see Definition 2 below). Compound first-order terms are built in an analogous way as proof terms of FOLP, with the addition of the two last cases in the following list (t and u are first-order terms):*

- *$(t + u)$ – causal sum,*

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- $(t \cdot u)$ – *the application of cause t (causal nexus) to cause u (distal cause), proximate cause,*

- $!u$ – *affirmation of t,*

- $?u$ – *limitation of t,*

- $\mathsf{gen}_x(t)$, $\mathsf{gen}_X(t)$ – *general cause (with respect to cause t as its special case),*

- $\mathsf{abs}_x(t)$ – *abstract cause, cause of having a property (with respect to t as a cause of the corresponding state of affairs; see syntax and semantics below),*

- $\mathsf{exs}(t)$ – *cause of existence (actualization of cause t).*

Familiar second-order grammar is used, with first-order identity sentences, λ abstraction, and causal formulas of the shape $t \colon \phi$.

Definition 2 (Formula).

$$\phi ::= R^n t_1 \ldots t_n \mid t = t' \mid \mathcal{PT} \mid t \colon \phi \mid \neg\phi \mid (\phi_1 \to \phi_2) \mid \forall x\phi \mid \forall X\phi$$
$$\mid (\lambda x.\phi)(t)$$

where R^n is a first-order relation symbol. We will use $(\lambda x_1 \ldots x_n.\phi)$, or simply ϕ, if there is no ambiguity, as short for $(\lambda x_1.(\lambda x_2.(\ldots (\lambda_n. \phi)\ldots)))$. λ abstract, $(\lambda x.\phi)$, is a second-order term.

All and only first-order terms are causal terms (in FOLP, a special set of proof variables and constants is disjoint from the set of first-order terms; however, in quantified logic of proofs QLP by M. Fitting [16], all and only proof terms are first-order terms).

In addition, we will use r, s as meta-variables for first-order variables or first-order constants, t, u, v for first-order terms, and T, U, V for second-order terms (R, S for second-order variables or constants).

The following definitions will be used (the last two of them are slight modifications of abbreviations in GO):

$$Et =_{def} \exists x x = t,$$
$$\overline{T^n} =_{def} (\lambda x_1 \ldots x_n.\neg T x_1 \ldots x_n),$$
$$X^n = Y^n =_{def} \forall x_1 \ldots \forall x_n (X x_1 \ldots x_n \leftrightarrow Y x_1 \ldots x_n),$$

$Gt =_{def} \forall X(\mathcal{P}X \to Xt)$ ("most positive", "God-like"),

$\mathcal{E}\!\int\!\int_t(X, u) =_{def} Xu \land \forall Y(Yu \to t\colon \forall z(Xz \to Yz))$ (essence),

$N_t u =_{def} \forall X(\mathcal{E}\!\int\!\int_t(X, u) \to \text{exs}(t)\colon \exists x Xx)$ (necessary existence).

According to the above definitions, we note that an essence of x is supplied with a causality by means of which the essence brings about all the properties of x. Necessary existence of x is a sort of actualization of x's essence.

5.2 System QCGO

Axioms are classical propositional tautologies and the following three groups of axioms: general logical axioms, general causal axioms, and causal positivity axioms (onto-theological).

General *logical* axioms are the following ones:

∀1a $\forall x\phi \to (Et \to \phi(t/x))$, t is rigid and substitutable[14] for x in ϕ (for "rigid" see Axiom $=\mathbf{R}$ and Definition 3 below),

∀1b $\forall x(\phi \to \psi) \to (\phi \to \forall x\psi)$, $x \notin \text{free}(\phi)$,

$=\mathbf{1}$ $t = t$,

∀2a $\forall X\phi \to \phi(T/X)$, T is substitutable for X in ϕ,

∀2b $\forall X(\phi \to \psi) \to (\phi \to \forall X\psi)$, $X \notin \text{free}(\phi)$,

$\lambda\mathbf{Cnv}$ $\phi(t/x) \leftrightarrow (\lambda x.\phi)(t)$, t is substitutable for x in ϕ,

E $\forall x Ex$ (see Def. of E below),

Subs $t_1 = t_2 \to (\phi(t_2/x) \to \phi(t_1/x))$, where t_1 and t_2 are substitutable for x in ϕ,

D $\forall y(y = \imath x\phi(x) \leftrightarrow (\forall x(\phi \to x = y) \land \phi(y/x)))$, x and y are different variables.

We adopt the following general *causal* axioms:

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[14] We say that t is substitutable for x in ϕ if t, or a free variable that occurs in t, does not become bound by $\forall, \lambda,$ or $\imath$ operator if t is substituted for x in ϕ.

CAE Et if $t\colon \phi \in \mathcal{CS}$ or $t = \mathsf{exs}(u)$ with provably possible u (i.e. $\mathsf{QCGO} \vdash \neg x\colon \neg \exists y\, y = u$[15] (for $\mathcal{CS}$ see below),

CE $(Et \wedge Eu) \to E(t * u)$, $Et \to E * t$, $Et \to E * (t)$,
$(Eexs(t) \wedge Eexs(u)) \to Eexs(t * u)$, $Eexs(t) \to Eexs(*t)$, $Eexs(t) \to Eexs(*(t))$
($*$ is a causal function symbol),

CMon $t\colon \phi \to (t + u)\colon \phi$, $\qquad u\colon \phi \to (t + u)\colon \phi$,[16]

CK $t\colon (\phi \to \psi) \to (u\colon \phi \to (t \cdot u)\colon \psi)$,

CT $t\colon \phi \to (Et \to \phi)$,

C4 $t\colon \phi \to\, !t\colon t\colon \phi$,

C5 $\neg t\colon \phi \to\, ?t\colon \neg t\colon \phi$,

C∀ $t\colon \phi \to \mathsf{gen}_x(t)\colon \forall x\phi$, $x \notin \mathsf{free}(\phi)$,
$t\colon \phi \to \mathsf{gen}_X(t)\colon \forall X\phi$, $X \notin \mathsf{free}(\phi)$,

Cλ $t\colon \phi(u/x) \to \mathsf{abs}_u(t)\colon (\lambda x.\phi)(u))$,

=R $r = s \to \exists z\colon r = s$.

CMon–**C4**, and **C∀** are second-order generalizations of the corresponding schemes of FOLP [4], with modified **CT** requiring the presence of the cause. According to **CT**, cause t is conceived as a "sufficient reason" (see a comment in footnote 18 below).

System QCGO also includes causal Gödelian axioms for the *positivity* of properties (onto-theological axioms, cf. **GA1 − 5** of GO, with modifications in **CGA2** and **CGA4**, see [34]):

QCGA1(=GA1) $\forall X(\mathcal{P}\overline{X} \leftrightarrow \neg\mathcal{P}X)$,

QCGA2 $\forall X\forall Y((\mathcal{P}X \wedge t\colon \forall y(Xy \to Yy)) \to \mathcal{P}Y)$,

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[15]Applications of **CAE** for exs in such a proof should be ultimately based on cases without **CAE** for exs.

[16]Note that a cause is assumed to be an (at least) sufficient cause (it may include redundant components, and may be a sufficient and necessary cause as well). Hence, if t is a sufficient cause of ϕ, the fact that, possibly, u is a sufficient cause of $\neg\phi$ does not in any way change anything in the first fact (that t is a sufficient cause of ϕ). Hence, their sum continues to be a cause of ϕ (as well as of $\neg\phi$). What will actually be the case depends on which cause exists (they cannot both actually co-exist, since then $\phi \wedge \neg\phi$ would be the case). For instance, if t exists (Et), then ϕ will actually obtain (see Axiom **CT**). We remark that Axiom **CE** does not imply $E(t + u)$ from Et (or Eu).

QCGA3(=GA3) $\mathcal{P}G$,

QCGA4 $\forall X(\mathcal{P}X \rightarrow \imath xGx \colon \mathcal{P}X)$,

QCGA5 $\mathcal{P}N$.

Definition 3 (Rigid term)**.** *A term t is rigid iff $\vdash x = t \rightarrow$ $\exists y\, y \colon x = t$.*

As in justification logic, a *constant specification* set, $\mathcal{CS}$, is needed – it is a set of formulas $k \colon \phi$ (k being a first-order constant) for each axiom ϕ (for each axiom there is a corresponding cause). – In the examples below, we will often use certain letters as abbreviations to indicate a cause term that is composed by causal term operations on basic causal terms assigned by $\mathcal{CS}$ to the axioms.

Rules of inference are as follows:

MP $\vdash \phi \rightarrow \psi, \phi \implies \vdash \psi,$

U1 $\vdash \phi \implies \vdash \forall x \phi$,

U2 $\vdash \phi \implies \vdash \forall X \phi,$

ACau (axiom causation): if $\vdash \phi$, then $\vdash k \colon \phi$, where ϕ is an axiom, and k a cause constant with $k \colon \phi \in \mathcal{CS}$.

Quantificationally bound and free variables are conceived as usual. In addition, we say that t in $t \colon \phi$ binds free x (X) of ϕ if and only if x (X) is not free in t.

Remark 1. *Let us notice that even proper logical axioms of* QCGO *(taken from general first-order or second-order logic) have their causes, which will make possible, by means of other axioms (like* **CK***) and rules, to derive propositions about logical causality. This is in accordance with the so-called Gödel's Platonism and his statements like the ones mentioned above about axioms causing theorems, and theorems causing their consequences (see the beginning of the next section for a closer explanation). Our intended interpretation of this logical causation of* QCGO *is that logical axioms and theorems are not justified merely by their epistemological reasons (evidence, proofs), but have also their ontological foundation in some fundamental features of (not epistemically conditioned) objectivity.*

We assume a familiar definition of inconsistency of a set Γ of formulas by means of derivability of a contradiction (ϕ and $\neg\phi$) from Γ, and a definition of Γ being consistent as Γ not being inconsistent.

6 Some propositions and theorems

6.1 General features of ontological causation

From the form of a causal formula $t\colon \phi$ it can be seen that an effect, which is expressed by a formula (ϕ), cannot as such be a cause, expressed by a first-order term (t), of any further effect. Strictly, it is not so that an axiom causes a theorem, but the cause of an axiom ($t\colon \phi$), combined with a causal nexus between the axiom and a theorem ($s\colon (\phi \to \psi)$), causes the theorem ($t{\cdot}s\colon \psi$).[17] Thus, for example, the transitivity of causality does not obtain in a literal sense. What is transitive is causal nexus, expressed as a causal conditional $t\colon (\phi \to \psi)$.

Proposition 1 (Transitivity of causal nexus). QCGO $\vdash t\colon (\phi \to \psi) \to (u\colon (\psi \to \chi) \to ((a \cdot t) \cdot u)\colon (\phi \to \chi))$, *where a is taken to be the cause of $(\phi \to \psi) \to ((\psi \to \chi) \to (\phi \to \chi))$.*

Dokaz.

1	$(\phi \to \psi) \to ((\psi \to \chi) \to (\phi \to \chi))$	PC
2	$a\colon ((\phi \to \psi) \to ((\psi \to \chi) \to (\phi \to \chi)))$	1 ACau
3	$t\colon (\phi \to \psi) \to (a \cdot t)\colon ((\psi \to \chi) \to (\phi \to \chi))$	2 CK
4	$t\colon (\phi \to \psi) \to (u\colon (\psi \to \chi) \to ((a \cdot t) \cdot u)\colon (\phi \to \chi))$	3 CK

Type text h

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[17]Thus, on the present account, cause and effect are inhomogeneous. In the light of the discussion between eventualist and factualist accounts of causality (see, e.g. [9], in connection with Gödel's "slingshot" argument), we propose a mixed account, where cause would be, in a way, similar to an event, and effect to a fact. Actually, Gödel's distinction between *force* and *fact* seems to be even more appropriate: cause could be conceived as a (causal) force, whereas effect is simply a fact resulting from the work of causal force. See the introductory part of this chapter and [54, p. 309–313] for Gödel's informal distinction of force (thesis) and fact (antithesis of force).

PC stands for the propositional fragment of QCGO.

Further, to say that ϕ causes χ if ϕ causes ψ and ψ causes χ, means, more precisely, that the cause of ϕ, only if taken together with the needed causal nexuses (of $\phi \to \psi$ and $\psi \to \chi$), causes χ:

Proposition 2. *Suppose that* QCGO $\vdash a' \colon (\phi \to ((\phi \to \psi) \to ((\psi \to \chi) \to \chi)))$. *Then* QCGO $\vdash t \colon (\phi \to \psi) \to (u \colon (\psi \to \chi) \to (v \colon \phi \to (((a' \cdot v) \cdot t) \cdot u) \colon \chi))$.

Dokaz. In a similar way like the proof of Proposition 1. $\qquad\square$

Causal nexus is not asymmetric. If the left-to-right as well as the right-to-left directions have their respective causes, the cause of the biconditional can easily be calculated:

Proposition 3 (Symmetry). *Let* QCGO $\vdash b \colon ((\phi \to \psi) \to ((\psi \to \phi) \to (\phi \leftrightarrow \psi)))$. *Then,*

$$\text{QCGO} \vdash t \colon (\phi \to \psi) \to (u \colon (\psi \to \phi) \to ((b \cdot t) \cdot u) \colon (\phi \leftrightarrow \psi))$$

Dokaz. By a successive application of axiom **CK** like for Proposition 1. $\qquad\square$

In addition, causal nexus is reflexive since $\phi \to \phi$ is provable in QCGO.[18]

From the above examples it is clear that a causal nexus can be decomposed into intermediate causes that are indicated in the causal prefix, and that the number of intermediate causes is finite (since a causal prefix is of a finite length). The decomposition of a causal nexus ends with the first, immediate causes, which are by CS ascribed to the axioms. Let us notice that the origins of such a causal theory, as build upon a paradigm of logical reasoning and logic of proofs, can be traced back to Aristotle. In [35, 37], we

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[18]As commented by B. Žarnić, it follows that beside some cause t of ϕ there is always a compound cause $u \cdot t$ of the same fact ϕ (if $u \colon (\phi \to \phi)$ holds). In this sense, we make a further precision in the concept of sufficient cause. Cause t is sufficient even though not all those causal components are explicitly included in t which, according to the system, provably follow to be included in the causation of ϕ. Similarly, if existent t causes ϕ, we say that $t + u$ (u being existent but redundant), too, causes ϕ in the same causation event.

showed how a significantly similar style of formalization can be applied to Aristotle's causal conceptions of logic and science.[19]

In the following propositions the subscripts of causal prefixes indicate the axioms or theorems associated with the prefixes. Sometimes, just a sublogic to which axioms or theorems belong is indicated (PC, SOL – propositional and second-order fragment of QCGO, respectively, sub – substitution, λ – λ conversion).

Proposition 4.

(1) $\mathsf{QCGO} \vdash t \colon (\phi \wedge \psi) \to ((a'_{\wedge E} \cdot t) \colon \phi \wedge (a''_{\wedge E} \cdot t) \colon \psi)$

(2) $\mathsf{QCGO} \vdash\ !t \colon t \colon {}_{\mathcal{T}}\phi \to (Et \to \phi)$

(3) $\mathsf{QCGO} \vdash (t \colon \phi \vee t \colon \psi) \to (b_{\vee I} \cdot\, !t + b'_{\vee I} \cdot\, !t) \colon (t \colon \phi \vee t \colon \psi)$ [2].

Dokaz. We prove (1):

1	$a'_{\wedge E} \colon ((\phi \wedge \psi) \to \phi)$	PC, ACau
2	$a''_{\wedge E} \colon ((\phi \wedge \psi) \to \psi)$	PC, ACau
3	$t \colon (\phi \wedge \psi) \to (a'_{\wedge E} \cdot t) \colon \phi$	CK, MP
4	$t \colon (\phi \wedge \psi) \to (a''_{\wedge E} \cdot t) \colon \psi$	PC, MP
5	$t \colon (\phi \wedge \psi) \to ((a'_{\wedge E} \cdot t) \colon \phi \wedge (a''_{\wedge E} \cdot t) \colon \psi)$	PC

□

6.2 Specific onto-theological causal features

Many of the proofs of the following propositions are similar to the proofs in system CGO of [34] (without the quantification on causes and without definite descriptions) or in some non-causal variants of the axiomatization of Gödel's ontology as [50]. We elaborate some specific proofs of QCGO.

Proposition 5. $\mathsf{QCGO} \vdash \neg u \colon \neg \exists x G x$

Dokaz. Cf. [34] for CGO. □

Theorem 1. $\mathsf{QCGO} \vdash \forall x (G x \to \mathcal{E}\!\!\int\!\!\int_{c_{SOL} \cdot \imath x G x}(G, x))$

Dokaz. Like in [34], where it is sketched for CGO. □

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[19] For comparison, see [56, p. 209–234] on general properties of causality in physics.

Proposition 6.

$$\mathsf{QCGO} \vdash Gx \to \forall X(Xx \to \mathcal{P}X)$$
$$\mathsf{QCGO} \vdash Gx \to (Gy \to x = y)$$

Dokaz. See [50] for GO. $\qquad\qquad\square$

Proposition 7 (The uniqueness of Gx.)**.** $\mathsf{QCGO} \vdash (Gx \wedge Ex) \to x = \imath x Gx$

Dokaz.

1	$Gx \wedge Ex$	ass.
2	$\forall y(Gx \to (Gy \to x = y))$	Proposition 6, U1
3	$Gx \to \forall y(Gy \to x = y)$	1, 2 $\forall$1a
4	$\forall y(Gy \to x = y)$	1, 3 MP
5	$Gx \wedge \forall y(Gy \to x = y)$	1, 4 PC
6	$x = \imath x Gx$	5 D, MP
7	$(Gx \wedge Ex) \to x = \imath x Gx$	1–6, Ded. Th.

(Cf. also Chapter "Gödel's 'slingshot' argument and his onto-theological system" in this book, for logic $\mathsf{fGO}_{\imath}$.) $\qquad\square$

Proposition 8. $\mathsf{QCGO} \vdash \exists x Gx \to exs(c_{SOL} \cdot \imath x Gx)\colon \exists x Gx$

Dokaz. Like Scott's proof for GO, with $exs(c_{SOL} \cdot \imath x Gx)$ for $\square$. $\quad\square$

Theorem 2. $\mathsf{QCGO} \vdash \exists x Gx$

Dokaz.

1	$\neg \exists x Gx$	ass.
2	$\neg exs(c_{SOL} \cdot \imath x Gx)\colon \neg \exists x Gx$	Prop. 5
3	$\neg exs(c_{SOL} \cdot \imath x Gx)\colon \exists x Gx$	1 CT, CAE CE, Prop. 5
4	$?exs(c_{SOL} \cdot \imath x Gx)\colon \neg exs(c_{SOL} \cdot \imath x Gx)\colon \exists x Gx$	3 C5
5	$a\colon (\exists x Gx \to exs(c_{SOL} \cdot \imath x Gx)\colon \exists x\colon Gx)$	4 ACau, Prop. 8
6	$b\colon ((\exists x Gx \to exs(c_{SOL} \cdot \imath x Gx)\colon \exists x\colon Gx) \to$ $(\neg exs(c_{SOL} \cdot \imath x Gx)\colon \exists x Gx \to \neg \exists x Gx))$	contrapos., ACau
7	$(b \cdot a)\colon (\neg exs(c_{SOL} \cdot \imath x Gx)\colon \exists x Gx \to \neg \exists x Gx)$	5, 6 CK, MP

8 $?exs(c_{SOL} \cdot \imath xGx)\colon \neg exs(c_{SOL} \cdot \imath xGx)\colon \exists xGx$ 7 CK,
   $\to (b \cdot a) \cdot ?exs(c_{SOL} \cdot \imath xGx)\colon \neg\exists xGx$ MP
9 $\neg(b \cdot a) \cdot ?exs(c_{SOL} \cdot \imath xGx)\colon \neg\exists xGx \to$ 8
   $\neg?exs(c_{SOL}\cdot\imath xGx)\colon \neg exs(c_{SOL}\cdot\imath xGx)\colon \exists xGx$ contrapos.
10 $\neg(b \cdot a) \cdot ?exs(c_{SOL} \cdot \imath xGx)\colon \neg\exists xGx$ Prop. 5
11 $\neg?exs(c_{SOL} \cdot \imath xGx)\colon \neg exs(c_{SOL} \cdot \imath xGx)\colon \exists xGx$ 9, 10 MP
12 $\exists xGx$ 1-11 Ded.Th.
                             4,11 contrd.

$\square$

Corollary 1. QCGO $\vdash exs(c_{SOL} \cdot \imath xGx)\colon \exists xGx$

Dokaz. From Proposition 8 and Theorem 2. $\square$

Informally, the theorems on God's existence say that a God-like being exists because it (its essence = being God) is actualized, i.e., in an essential sense, it exists of itself (*a se*). This seems to correspond to Gödel's idea of God as something which is "necessary in itself" (*an sich notwendig*) [27, p.431].

Proposition 9. QCGO $\vdash y = \imath xGx \to \exists x\, x\colon y = \imath xGx$

Dokaz. Like the proof of $y = \imath xGx \to \Box y = \imath xGx$ (Rigidity of God) in the "slingshot" chapter of this book, with $\exists x\, x\colon$ instead of $\Box$. $\square$

Existentially quantified cause in Proposition 9 will be instantiated below in Theorem 3.

Proposition 10. QCGO $\vdash \exists x\, x = \imath xGx$ *(i.e. $E\imath xGx$)*

Dokaz. Follows from Proposition 7 and Theorem 2. $\square$

Theorem 3 (Universal Cause). QCGO $\vdash \phi \leftrightarrow ((c_{SOL} \cdot \imath xGx) \cdot exs(c_{SOL} \cdot \imath xGx))\colon \phi$.

Dokaz. For from left to right direction, see [34] for CGO. Specifically for QCGO, the proof depends on $Eexs(c_{SOL} \cdot \imath xGx)$ (like Theorem 2). Cf. also axiomatic non-justificational proof; see [14, p. 163–164]. From right to left: we need $E((c_{SOL} \cdot \imath xGx) \cdot exs(c_{SOL} \cdot \imath xGx))$, which follows from axioms **CAE** and **CE**, Proposition 7

and Theorem 2. (Causal term $c_{SOL} \cdot \imath x G x$ is in this last case, for simplicity, taken as short for $c_{FOL} \cdot (c_{SOL} \cdot \imath x G x)$, with *FOL* for first-order fragment of QCGO.) $\hspace{1cm}\square$

According to Theorem 3, universal ontological cause contains (second-order) logic and its application to positive ("real") properties (comprised by $\imath x G x$). Let us call the application of logic on "real" properties "real logic". Thus, we could say that the activation ("affirmation of being") of real logic is a universal, onto-theological cause of each fact (and is "superposed" on possible natural causes of facts; see Proposition 11, Case 3 below). In addition, by **C4, CAE**, and Proposition 10, it follows

$$QCGO \vdash \phi \leftrightarrow !((c_{SOL} \cdot \imath x G x) \cdot exs(c_{SOL} \cdot \imath x G x)):$$
$$((c_{SOL} \cdot \imath x G x) \cdot exs(c_{SOL} \cdot \imath x G x)): \phi.$$

(Since we proposed a reading of ! as "affirmation", this could have some connection with Gödel's reflexion that "the affirmation of being is the cause of the world" [27, p. 433].) – Eventually, it can be noticed that, due to Theorem 3, each designating first-order term is rigid.

Proposition 11.

1. $QCGO \vdash \mathcal{P}(\lambda x. x = x)$.

2. $QCGO \vdash \phi \rightarrow \mathcal{P}(\lambda x. \phi))$ *Positivity of facts.*

3. $QCGO \vdash t: \phi \rightarrow ((c_{SOL} \cdot \imath x G x) \cdot exs(c_{SOL} \cdot \imath x G x)): (Et \rightarrow \phi)$ *Reduction of causes to the universal cause.*

4. $QCGO \vdash ((c_{SOL} \cdot \imath x G x) \cdot exs(c_{SOL} \cdot \imath x G x)): \phi \leftrightarrow \mathcal{P}(\lambda x. \phi)$ *Positivity of universal causation.*

5. $QCGO \vdash t: \phi \rightarrow (Et \rightarrow \mathcal{P}(\lambda x. \phi))$ *Positivity of causation.*

See [34] for Cases 2, 5.

Dokaz.

1. Like for CGO in [34].

2. Follows from the first clause of Proposition 6, with $(\lambda x.\phi)$ for X, and Theorem 2. (Cf. proof in [34] as a modification of the modal collapse proof in CGO).

3. From **CT** and Theorem 3.

4. From **CT** with axioms **CAE**, **CE**, Theorem 3, Proposition 7, and Case 2 above (Proposition 11).

5. From **CT** and Case 2 above (Proposition 11).

$\square$

Proposition 12. QCGO $\vdash t\colon \forall x\phi \to (Et \to \forall x((c_{SOL} \cdot \imath xGx) \cdot exs(c_{SOL} \cdot \imath xGx))\colon \phi$ (Universal Converse Barcan).

Dokaz. From $t\colon \forall x\phi$ and from Et, $\forall x\phi$ follows, and hence $Ey \to \phi(y/x)$ for $y \notin \mathsf{free}(t\colon \forall x\phi)$. By Theorem 3 we obtain $Ey \to ((c_{SOL} \cdot \imath xGx) \cdot exs(c_{SOL} \cdot \imath xGx))\colon \phi(y/x)$, and thus (by first-order logic and **E**) $\forall x((c_{SOL} \cdot \imath xGx) \cdot exs(c_{SOL} \cdot \imath xGx))\colon \phi$. $\square$

Proposition 13. QCGO $\vdash \forall xt\colon \phi \to (Et \to ((c_{SOL}\cdot\imath xGx)\cdot exs(c_{SOL}\cdot \imath xGx))\colon \forall x\phi)$ (Universal Barcan).

Dokaz. From $\forall xt\colon \phi$, it follows $Ey \to t\colon \phi(y/x)$ ($y \notin \mathsf{free}(t\colon \forall x\phi)$), and from there $Et \to (Ey \to \phi(y/x))$. Thus, by first-order logic, we obtain $Et \to \forall x\phi$. By Theorem 3, $Et \to ((c_{SOL} \cdot \imath xGx) \cdot exs(c_{SOL} \cdot \imath xGx))\colon \forall x\phi$ follows. $\square$

Proposition 14 (Ass1). QCGO $\vdash \phi \leftrightarrow \imath xGx = \imath x(x = \imath xGx \wedge \phi(x/\imath xGx))$

Dokaz. Cf. chapter "Gödel's 'slingshot' argument and his onto-theological system" for $\mathsf{fGO_{\imath}}$ in this book. $\square$

From the above propositions we can see that God, being the cause of positivity of properties, is also the cause of the positivity of each fact. It follows that even worst facts should contain an essential positivity aspect – being, self-identity, necessary part of the whole (however difficult, otherwise, this may be to understand). Further, God is involved in all causation (since in the "universal cause"), together with specific causes of facts (Et, see

Case 3 of Proposition 11),[20] and thus sustains the positive side in each causal event. Hence, finally, in no fact or causal event the negative can prevail. In this way, if we conceive the positive as "affirmation of being", Leibniz's question "why is there something rather than nothing?" [39, p. 602], too, receives its answer – in the fundamental primacy of the positive. In addition, according to this Gödelian system, nothing in the "world" disappears, nothing new appears (see Propositions 12 and 13) – all objects are, in a sense, omnipresent, although they are not all equally fundamental.[21]

We will now show how Gödel's slingshot argument from his [22] [22] can be extended to QCGO, where a precise onto-theological cause for each pair of asserted equivalent propositions can be constructed.

Proposition 15. $QCGO \vdash X\imath xGx \rightarrow (d_\imath \cdot \imath xGx) \colon X\imath xGx$, with $d_\imath$ assumed to be a compound cause of $\mathcal{P}X \rightarrow X\imath xGx$.

Dokaz. Like the proof of $X\imath xGx \rightarrow \Box X\imath xGx$ in the "slingshot" chapter of this book. After we derive $X\imath xGx \rightarrow \mathcal{P}X$, formula $\mathcal{P}X \rightarrow \imath xGx \colon \mathcal{P}X$ follows by **CGA4**, and $X\imath xGx \rightarrow \imath xGx \colon \mathcal{P}X$ by propositional calculus. From Definition of G and from $G\imath xGx$ we have $d_\imath \colon (\mathcal{P}X \rightarrow X\imath xGx)$ (with $d_\imath$ as an appropriate cause). From the last two derived sentences the proposition follows by **CK**. $\square$

Theorem 4. $QCGO \vdash \phi \rightarrow (\psi \rightarrow ((d'_{PC} \cdot ((d_{PC} \cdot ((e'_{subs} \cdot ((e_{subs} \cdot (d'_\lambda \cdot (d'_\imath \cdot \imath xGx))) \cdot (d''_\lambda \cdot (d''_\imath \cdot \imath xGx)))) \cdot c)) \cdot a)) \cdot b) \colon (\phi \leftrightarrow \psi))$

Dokaz. Terms a and b in the following proof are assumed to be causal terms for the instances of Proposition 14. Terms $d'_\imath$ and $d''_\imath$ are meant as originating from $d_\imath$ of Proposition 15 for particular

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[20]Gödel mentions in his notes the involvement of creatures in a sort of "secondary" creation: "God created [schuf] things so that they can at their turn still 'create' ['erschaffen'] something" (K. Gödel, *Max-Phil* X, p. 12, quotation according to [40]). This moment was also pointed out to me in discussion by Paul Weingartner (2013, 2015), see [56].

[21]For broader philosophical explanations and motivations of a Gödelian philosophical view with temporal and modal "collapse", see [57, 58, 33, 34].

[22]See [49] and [46], as well as "Gödel's 'slingshot' argument and his onto-theological system" in this book.

instantiations of the proposition. c is causal term for $\imath xGx = \imath x(x = \imath xGx \land \phi(x/\imath xGx))$. Subscripts of other cause terms indicate axioms or other formulas to which they belong as their respective causal terms.

1   $a\colon (\phi \leftrightarrow \imath xGx = \imath x(x = \imath xGx \land \phi(x/\imath xGx)))$ Prop.14, ACau

2   $b\colon (\psi \leftrightarrow \imath xGx = \imath x(x = \imath xGx \land \psi(x/\imath xGx)))$ as for 1

3   $\phi \to \imath xGx = \imath x(x = \imath xGx \land \phi(x/\imath xGx))$ Prop.14

4   $\phi \to (\lambda x.x = \imath x(x = \imath xGx \land \phi(x/\imath xGx)))(\imath xGx)$ 3 λCnv

5   $\phi \to (d'_\lambda \cdot (d'_\imath \cdot \imath xGx))\colon$ $\imath xGx = \imath x(x = \imath xGx \land \phi(x/\imath xGx))$ 4, Prop. 15, λCnv

6   $\psi \to \imath xGx = \imath x(x = \imath xGx \land \psi(x/\imath xGx))$ Prop. 14

7   $\psi \to (\lambda x.x = \imath x(x = \imath xGx \land \psi(x/\imath xGx)))(\imath xGx)$ 6 λCnv

8   $\psi \to (d''_\lambda \cdot (d''_\imath \cdot \imath xGx))\colon \imath xGx = \imath x(x = \imath xGx \land \psi(x/\imath xGx))$ 7, Prop. 15, λCnv

9   $e_{subs}\colon (\imath xGx = \imath x(x = \imath xGx \land \phi(x/\imath xGx)) \to (\imath xGx = \imath x(x = \imath xGx \land \psi(x/\imath xGx)) \to \imath x(x = \imath xGx \land \phi(x/\imath xGx)) = \imath x(x = \imath xGx \land \psi(x/\imath xGx))))$ Subs, ACau

10   $\phi \to (e_{subs} \cdot (d'_\lambda \cdot (d'_\imath \cdot \imath xGx)))\colon$ $(\imath xGx = \imath x(x = \imath xGx \land \psi(x/\imath xGx)) \to \imath x(x = \imath xGx \land \phi(x/\imath xGx)) = \imath x(x = \imath xGx \land \psi(x/\imath xGx)))$ 5, 9 CK, PC

11   $\phi \to (\psi \to ((e_{subs} \cdot (d'_\lambda \cdot (d'_\imath \cdot \imath xGx))) \cdot (d''_\lambda \cdot (d''_\imath \cdot \imath xGx))))\colon$ $\imath x(x = \imath xGx \land \phi(x/\imath xGx)) = \imath x(x = \imath xGx \land \psi(x/\imath xGx)))$ 8, 10 CK, MP

12   $e'_{subs}\colon (\imath x(x = \imath xGx \land \phi(x/\imath xGx)) = \imath x(x = \imath xGx \land \psi(x/\imath xGx)) \to (\imath xGx = \imath x(x = \imath xGx \land \phi(x/\imath xGx)) \leftrightarrow \imath xGx = \imath x(x = \imath xGx \land \phi(x/\imath xGx)) \to (\imath xGx = \imath x(x = \imath xGx \land \phi(x/\imath xGx)) \leftrightarrow \imath xGx = \imath x(x = \imath xGx \land \psi(x/\imath xGx)))))$ Subs, ACau

13   $\phi \to (\psi \to ((e'_{subs} \cdot ((e_{subs} \cdot (d'_\lambda \cdot (d'_\imath \cdot \imath xGx))) \cdot (d''_\lambda \cdot (d''_\imath \cdot \imath xGx)))) \cdot c)\colon$ $(\imath xGx = \imath x(x = \imath xGx \land \phi(x/\imath xGx)) \leftrightarrow \imath xGx = \imath x(x = \imath xGx \land \psi(x/\imath xGx))))$ 11, 12 CK, PC

14 $d_{PC}\colon ((\imath xGx = \imath x(x = \imath xGx \wedge \phi(x/\imath xGx))$ PC,
   $\leftrightarrow \imath xGx = \imath x(x = \imath xGx \wedge \psi(x/\imath xGx)))$ ACau
   $\rightarrow ((\phi \leftrightarrow \imath xGx = \imath x(x = \imath xGx \wedge \phi(x/\imath xGx)))$
   $\rightarrow (\phi \leftrightarrow \imath xGx = \imath x(x = \imath xGx \wedge \psi(x/\imath xGx)))))$

15 $\phi \rightarrow (\psi \rightarrow ((d_{PC} \cdot ((e'_{subs} \cdot ((e_{subs}$ 1, 13, 14
   $\cdot (d'_{\lambda} \cdot (d'_{\imath} \cdot \imath xGx))) \cdot (d''_{\lambda} \cdot (d''_{\imath} \cdot \imath xGx)))) \cdot c)) \cdot a)\colon$ PC, CK,
   $(\phi \leftrightarrow \imath xGx = \imath x(x = \imath xGx \wedge \psi(x/\imath xGx))))$ MP

16 $d'_{PC}\colon ((\phi \leftrightarrow \imath xGx = \imath x(x = \imath xGx \wedge \psi(x/\imath xGx)))$ PC, ACau
   $\rightarrow ((\psi \leftrightarrow \imath xGx = \imath x(x = \imath xGx \wedge \psi(x/\imath xGx)))$
   $\rightarrow (\phi \leftrightarrow \psi)))$

17 $\phi \rightarrow (\psi \rightarrow ((d'_{PC} \cdot ((d_{PC} \cdot ((e'_{subs} \cdot ((e_{subs}$ 2, 15, 16
   $\cdot (d'_{\lambda} \cdot (d'_{\imath} \cdot \imath xGx))) \cdot (d''_{\lambda} \cdot (d''_{\imath} \cdot \imath xGx)))) \cdot c))$ PC, CK,
   $\cdot a)) \cdot b)\colon (\phi \leftrightarrow \psi))$ MP

$\square$

In short, the cause of the equivalence of each pair of formulas is nicely composed of logic and $\imath xGx$.

7 Semantics

At first, second-order domain and the meanings of λ abstracts and $\mathcal{P}$ will be defined by means of frames in a general and possibly arbitrary way. Thereafter, we will be able to determine their meanings in models in a specific interconnected way.[23]

Definition 4 (QCGO frame and variable assignment). *Frame, $\mathfrak{F}$, and variable assignment, a, are ordered set $\langle W, R, D, D(n), I, q, a \rangle$, such that*

1. $W \neq \varnothing$,

2. $R \subseteq W \times W$, *reflexive, euclidean,*

3. $D \neq \varnothing$,

4. $\varnothing \neq D(n) \subseteq \wp(D^n)^W$,

5. (a) $I(c) \in D$,

   (b) $I(C^n) \in D(n)$,

---

[23]For such a procedure in general, see e.g. [14, 12].

(c) $I(\mathcal{P}, w) \in \wp D(1)$,

(d) $I(!), I(?), I(\mathsf{gen}_x), I(\mathsf{gen}_X), I(\mathsf{abs}_u), I(\mathsf{exs}) \in D^D$,

(e) $I(\cdot), I(+) \in D^{D^2}$,

6. $q(w) \in \wp D$, *closed under causal functions, includes the set of all* $I(\mathsf{exs})(d) \in q(w)$ *closed under causal functions with respect to* d, *based on the set of all* $I(c)$ *with* c *associated by* $\mathcal{CS}$ *to an axiom,* $\bigcup_w q(w) \neq D$,

7. $a(x) \in D$, $a(X^n) \in D(n)$.

Where there is no ambiguity, $!, ?, \mathsf{gen}_x, \mathsf{gen}_X, \mathsf{abs}_u, \mathsf{exs}, \cdot$, and $+$, are used for $I(!), I(?), I(\mathsf{gen}_x), I(\mathsf{gen}_X), I(\mathsf{abs}_u), I(\mathsf{exs}), I(\cdot)$ and $I(+)$, respectively.

In the following definition, the influence function (In) is introduced, which is needed for the definition of the satisfaction of causal formulas and is a counterpart of the evidence function (stemming from [41]) in Fitting models for justification logic.[24] In case of constants and variables, the notation $[\![t]\!]_a^{\mathfrak{F},w}$ will be used for $a(t)$ if t is a variable, and for $I(t)$ if t is a constant (analogously, $[\![T^n]\!]_a^{\mathfrak{F}}$ will be used for second-order constants and variables). Further, $[\![t]\!]_a^{\mathfrak{F},w}$ will be used for $*[\![t']\!]_a^{\mathfrak{F},w}$, or $[\![t']\!]_a^{\mathfrak{F},w} * [\![t'']\!]_a^{\mathfrak{F},w}$ if $*$ is a one-place or two-place causal function, and finally, it will be used in the way defined in Case 8 of Definition 5 if t is a definite description. For a second-order term, T, $[\![T^n]\!]_a^{\mathfrak{F},w}$ is the value of the function $[\![T^n]\!]_a^{\mathfrak{F},}$ at w.

Definition 5 (Satisfaction, influence, and the denotation of description in $\mathfrak{F}, a, w$)**.**

1. $\mathfrak{F}, w \models_a T t_1 \ldots t_n$ *iff* $\langle [\![t_1]\!]_a^{\mathfrak{F},w}, \ldots, [\![t_n]\!]_a^{\mathfrak{F},w} \rangle \in [\![T^n]\!]_a^{\mathfrak{F},w}$, *we include here also the formulas of the shape* $(\lambda x.\phi)(t)$ *(as if there are no parantheses around* t*)*,

2. $\mathfrak{F}, w \models_a \mathcal{P}T$ *iff* $[\![T]\!]_a^{\mathfrak{F}} \in I(\mathcal{P}, w)$,

3. $\mathfrak{F}, w \models_a t = u$ *iff* $[\![t]\!]_a^{\mathfrak{F},w} = [\![u]\!]_a^{\mathfrak{F},w}$,

---

[24]Gödel sometimes mentions "influence" (germ. 'Einwirkung') in a narrow connection with his concept of cause. For example, when explaining why cause involves space: "being near = possibility of influence" [27, p. 434-435].

4. $\mathfrak{F}, w \models_a \neg\phi$ iff $\mathfrak{F}, w \not\models_a \phi$,

5. $\mathfrak{F}, w \models_a \phi \wedge \psi$ iff $\mathfrak{F}, w \models_a \phi$ and $\mathfrak{F}, w \models_a \psi$,

6. $\mathfrak{F}, w \models_a \forall x\phi$ iff for each $d \in Q(w)$, $\mathfrak{F}, w \models_{a[d/x]} \phi$,

7. $\mathfrak{F}, w \models_a \forall X^n\phi$ iff for each $d^n \in D(n)$, $\mathfrak{F}, w \models_{a[d^n/X^n]} \phi$,

8. $\llbracket \imath x\phi \rrbracket_a^{\mathfrak{F},w} = \begin{cases} d \in q(w)\} & \text{if for any } d' \in q(w), \\ & \mathfrak{M}, w \models_{[d'/x]} \phi \Longleftrightarrow d' = d, \\ a \text{ member of } D\backslash\{q(w)\} & \text{otherwise} \end{cases}$

9. $\llbracket (\lambda x.\phi) \rrbracket_a^{\mathfrak{F}} \in D(1)$,

10. $In(\phi, \mathfrak{F}, w, a) \in \wp D$ *(influence), with the following conditions:*

    (a) *for some $d \in q(w)$, $d \in In(\phi, \mathfrak{F}, w, a)$ if ϕ is an axiom,*

    (b) $\llbracket t \rrbracket_a^{\mathfrak{F},w} \in In(\phi \rightarrow \psi, \mathfrak{F}, w, a)$ & $\llbracket u \rrbracket_a^{\mathfrak{F},w} \in In(\phi, \mathfrak{F}, w, a) \Longrightarrow$ $\llbracket t \rrbracket_a^{\mathfrak{F},w} \cdot \llbracket u \rrbracket_a^{\mathfrak{F},w} \in In(\psi, \mathfrak{F}, w, a)$,

    (c) $\llbracket t \rrbracket_a^{\mathfrak{F},w} \in In(\phi, \mathfrak{F}, w, a) \Longrightarrow \llbracket t \rrbracket_a^{\mathfrak{F},w} + \llbracket u \rrbracket_a^{\mathfrak{F},w} \in In(\phi, \mathfrak{F}, w, a)$, $\llbracket u \rrbracket_a^{\mathfrak{F},w} \in In(\phi, \mathfrak{F}, w, a) \Longrightarrow \llbracket t \rrbracket_a^{\mathfrak{F},w} + \llbracket u \rrbracket_a^{\mathfrak{F},w} \in In(\phi, \mathfrak{F}, w, a)$,

    (d) $\llbracket t \rrbracket_a^{\mathfrak{F},w} \in In(\phi, \mathfrak{F}, w, a) \Longrightarrow {!}\llbracket t \rrbracket_a^{\mathfrak{F},w} \in In(t\colon \phi, \mathfrak{F}, w, a)$,

    (e) *if for some w' with wRw', $\llbracket t \rrbracket_a^{\mathfrak{F},w'} \notin In(\phi, \mathfrak{F}, w', a)$ or $\mathfrak{F}, w' \models_{a'} Et, \neg\phi$, then $?\llbracket t \rrbracket_a^{\mathfrak{F},w} \in In(\neg t\colon \phi, \mathfrak{F}, w, a)$, where a' differs from a at most with respect to the free variables not occurring in t,*

    (f) $\llbracket t \rrbracket_a^{\mathfrak{F},w} \in In(\phi, \mathfrak{F}, w, a) \Longrightarrow \mathsf{gen}_x(\llbracket t \rrbracket_a^{\mathfrak{F},w}) \in In(\forall x\phi, \mathfrak{F}, w, a)$, *where x does not occur free in t,* $\llbracket t \rrbracket_a^{\mathfrak{F},w} \in In(\phi, \mathfrak{F}, w, a) \Longrightarrow \mathsf{gen}_X(\llbracket t \rrbracket_a^{\mathfrak{F},w}) \in In(\forall X\phi, \mathfrak{F}, w, a)$, *where X does not occur in t,*

    (g) $\llbracket t \rrbracket_a^{\mathfrak{F},w} \in In(\phi(u/x), \mathfrak{F}, w, a) \Longleftrightarrow \mathsf{abs}_u(\llbracket t \rrbracket_a^{\mathfrak{F},w}) \in In((\lambda x.\phi)(u), \mathfrak{F}, w, a)$,

    (h) $\llbracket \imath xGx \rrbracket_a^{\mathfrak{F},w} \in In(\mathcal{PT}, \mathfrak{F}, w, a)$.

    (i) $\llbracket t \rrbracket_a^{\mathfrak{F},w} \in In(\phi, \mathfrak{F}, w, a)$ & $wRw' \Longrightarrow \llbracket t \rrbracket_a^{\mathfrak{F},w'} \in In(\phi, \mathfrak{F}, w', a)$

    (j) $\llbracket t \rrbracket_a^{\mathfrak{F},w} \notin In(\phi, \mathfrak{F}, w, a)$ & $wRw' \Longrightarrow \llbracket t \rrbracket_a^{\mathfrak{F},w'} \notin In(\phi, \mathfrak{F}, w', a)$

11. $\mathfrak{F}, w \models_a t\colon \phi$ iff for all w' with wRw', and for all a' that differ from a at most with respect to the free variables not occurring in t,

(a) $\mathfrak{F}, w' \models_{a'} Et \to \phi$,

(b) $[\![t]\!]_a^{\mathfrak{F},w'} \in In(\phi, \mathfrak{F}, w', a)$.

See evidence function conditions for first-order logic of proofs in [17]. For propositional conditions corresponding to (10e) and (10j) in epistemic setting, see [45]. Compare Case 8 with [19, p. 400–401].

Definition 6 (QCGO model and variable assignment)**.** *A* QCGO *model and variable assignment, i.e. a pair $\langle \mathfrak{M}, a \rangle$, are a frame and a variable assignment $\langle \mathfrak{F}, a \rangle$ such that for each ϕ,*

$$[\![(\lambda x.\phi)]\!]_a^{\mathfrak{F},w} = \{\, d \mid \mathfrak{F}, w \models_{a[d/x]} \phi \,\} \ \ and \ \ [\![(\lambda x.\phi)]\!]_a^{\mathfrak{F}} \in D(1),$$

and such that the following conditions for $I(\mathcal{P}, w)$ hold:

1. $[\![T]\!]_a^{\mathfrak{F}} \in I(\mathcal{P}, w) \iff [\![\overline{T}]\!]_a^{\mathfrak{F},w} \notin I(\mathcal{P}, w)$ *(complementarity),*

2. $[\![T]\!]_a^{\mathfrak{F}} \in I(\mathcal{P}, w) \ \& \ \forall w' \ with \ wRw'([\![T]\!]_a^{\mathfrak{F},w} \subseteq [\![U]\!]_a^{\mathfrak{F},w} \ \& \ In(\forall x (Tx \to Ux), \mathfrak{F}, w', a) \neq \varnothing) \implies [\![U]\!]_a^{\mathfrak{F}} \in I(\mathcal{P}, w)$ *(causal closure),*

3. $[\![T]\!]_a^{\mathfrak{F},w} = \bigcap \{\, [\![U]\!]_a^{\mathfrak{F},w} \mid [\![U]\!]_a^{\mathfrak{F}} \in I(\mathcal{P}, w) \,\} \implies [\![T]\!]_a^{\mathfrak{F}} \in I(\mathcal{P}, w)$ *(closure under intersection),*

4. $[\![T]\!]_a^{\mathfrak{F}} \in I(\mathcal{P}, w) \& wRw' \implies [\![T]\!]_a^{\mathfrak{F}} \in I(\mathcal{P}, w') \ \& \ [\![\imath x Gx]\!]_a^{\mathfrak{F},w'} \in In(\mathcal{P}T, \mathfrak{F}, w', a)$ *(positivness causality),*

5. $[\![N]\!]_a^{\mathfrak{F}} \in I(\mathcal{P}, w)$ *(essence-existence causation),*

with $[\![T]\!]_a^{\mathfrak{F}}, [\![\overline{T}]\!]_a^{\mathfrak{F}}, [\![U]\!]_a^{\mathfrak{F}}, [\![N]\!]_a^{\mathfrak{F}} \in D(1)$.

Definition 7 (Designation and satisfaction in $\mathfrak{M}, a$)**.** *Like definition 5, but restricted to models ($\mathfrak{M}$) and associated variable assignments.*

Definition 8 (Validity)**.** QCGO $\models \phi$ *iff for each $\mathfrak{M}$, w, and a, $\mathfrak{M}, w \models_a \phi$.*

8 Outline of adequacy

We outline the soundness and completeness proofs.

Theorem 5 (Soundness). *In the class of* QCGO *models, axioms of* QCGO *are valid and rules truth-preserving.*

Dokaz. For the proof, we take several axioms as examples.

CT Follows from the reflexivity of R (Definition 4), and Definition 5, Case 11.

C5 Let $\mathfrak{M}, w \models_a \neg t \colon \phi$. Hence, there are w' with wRw', and a' differing from a at most with respect to the valuation of the variables not occurring free in t, such that $\mathfrak{M}, w' \models_{a'} Et, \neg\phi$, or, alternatively, such that $[\![t]\!]_a^{\mathfrak{M},w'} \notin In(\phi, \mathfrak{M}, w', a)$. Thus, for each w'' with wRw'', and a', $\mathfrak{M}, w'' \models_{a'} \neg t \colon \phi$ (because of euclidean R, and Case 11 of Definition 5). Also $[\![?t]\!]_a^{\mathfrak{M},w''} \in In(\neg t \colon \phi, \mathfrak{M}, w'', a)$ (Definition 5, Case 10e). Therefore, $\mathfrak{M}, w \models_a ?t \colon \neg t \colon \phi$ (see Definition 5, Case 11).

C∀ (first-order case) Let $\mathfrak{M}, w \models_a t \colon \phi$, with $x \notin \text{free}(t)$. Thus, for each w' with wRw', and each a' differing from a at most with respect to the valuation of the variables not occurring free in t, $\mathfrak{M}, w' \models_{a'} Et \to \phi$ and $[\![t]\!]_a^{\mathfrak{M},w'} \in In(\phi, \mathfrak{M}, w', a)$. Hence, since $x \notin \text{free}(t)$, $\mathfrak{M}, w' \models_{a'} Et \to \forall x\phi$ (Definition 5, Case 6). In addition, $\text{gen}_x([\![t]\!]_a^{\mathfrak{M},w'}) \in In(\forall x\phi, \mathfrak{M}, w', a)$ (Definition 5, Case 10f). Therefore, $\mathfrak{M}, w \models_a \text{gen}_x(t) \colon \forall x\phi$ (Definition 5, cases 11 and 10f).

Cλ Suppose that $\mathfrak{M}, w \models_a t \colon \phi(u/y)$. Hence, for each w' with wRw', $[\![t]\!]_a^{\mathfrak{M},w'} \in In(\phi(u/y), \mathfrak{M}, w', a)$, and thus $\text{abs}_u([\![t]\!]_a^{\mathfrak{M},w'}) \in In((\lambda y.\phi)(u), \mathfrak{M}, w, a)$ (Case 10g of Definition 5). Finally, on the ground of Cases 1 and 9 (λ formula satisfaction) of Definition 5, and Case 11 of Definition 5, $\mathfrak{M}, w \models_a \text{abs}_u(t) \colon (\lambda y.\phi)(u)$.

CGA1-5 Follow straightforwardly from Definition 6 (denotation of $\mathcal{P}$).

D We just note that in case of $[\![\imath x\phi]\!]_a^{\mathfrak{F},w} \notin q(w)$, as well as in case of more than one $d \in q(w)$ satisfying ϕ, the biconditional is vacuously satisfied.

$\square$

The following outline of a completeness proof is given for closed formulas (sentences) and is partly built on a Gallin style completeness proof for intensional logic [18] (but see also Fitting's completeness proof for FOLP [17]) and is accommodated for causal interpretation. For simplicity, we will assume universal accessibility in models.

Since in the course of the proof saturated (maximal consistent and ω-complete) supersets of sentences should be build, we introduce an infinite supply of new first-order and second-order constants ("witnesses") outside $\mathcal{L}\mathsf{QCGO}$ to be able to consistently instantiate each existential sentence in the set. We call this extended language $\mathcal{L}\mathsf{QCGO}'$. Also, constant specification $\mathcal{CS}$ should be extended to $\mathcal{CS}'$ in order to take into account all new constants and terms containing the new constants: we extend $\mathcal{CS}$ with $k : \phi'$ for each $k : \phi$ so that ϕ' is like ϕ except for possibly containing terms in which new constants occur.

First, we need to show that for each consistent set of sentences of $\mathcal{L}\mathsf{QCGO}$, a sequence W of saturated sets, w, of sentences of language $\mathcal{L}\mathsf{QCGO}'$ can be build, on the ground of which the function INF can be defined:

Definition 9. $t \in INF(\phi, w)$ *iff for some w' in W, $t : \phi \in w'$,*

and which has the following property of "$\neg t$: -completeness"

for each w in W, $\neg t : \phi \in w$ iff, for some w' in W,
$Et \wedge \neg\phi \in w'$ or $t \notin INF(\phi, w')$.

We assume that a modified and extended Gallin-style construction of W for a consistent set Γ of sentences of $\mathcal{L}\mathsf{QCGO}$ is possible in the following way. The construction starts from a sequence W^0 of sets w_i of sentences of $\mathcal{L}\mathsf{QCGO}'$ in such a way that $\Gamma = w_0$ and each other $w_{i \in \omega} = \varnothing$. We build each W^{n+1} by comparing W^n with

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the pair $\langle \phi^n, n \rangle$ so that we extend w^n by ϕ^n if and only if this extension can be accomplished in a consistent way, i.e. if and only if the sequence W^{n+1} that is obtained in this way is relatively consistent – we say that W^n is relatively consistent if and only if for each w_i^n in W^n and each finite $w' \subseteq w_i^n$, the set $\{\forall x \neg x \colon \neg \wedge (w' \subseteq w_i^n)\}_i$[25] is consistent (cf. [18, p. 25]). Finally, we define $w_i = \bigcup_{n \in \omega} w_i^n$, and $W = \{w_i\}_{i \in \omega}$.

Let us remark that in the first-order case, for ω-completeness, we add to w_i^n, for each $\exists x \phi$, not only an instantiation of ϕ, by substitution k/x with a new k, but also Ek. Let us also note that, for instance, $\exists x \, t \colon Pxy$, with $x \notin \mathsf{free}(t)$, does not have any further instantiation, since x is not bound by $\exists x$, but already by t (and can be generalized by $\mathsf{gen}_x(t)$ replacing t). Also, in order to achieve the $\neg t \colon$ -completeness of W, we add $\neg t \colon \phi$ to w_i^n if and only if we add $Et \wedge \neg \phi$, too, as a member to the alphabetically first empty w_j^n or put $t \notin INF(\phi, w_j^n)$.

Proposition 16.

$$t \in INF(\phi, w) \Longleftrightarrow t \colon \phi \in w.$$

Dokaz. From left to right. Assume $t \in INF(\phi, w)$. Then for some w', $t \colon \phi \in w'$ (Definition 9). Hence, for no w'', $Et \wedge \neg \phi \in w''$, since otherwise, $\neg t \colon \phi \in w'$ (according to the rules of the construction of W: relative consistency and the conditions for the membership of $\neg t \colon \phi \in w_i$). For the same reason $\neg t \colon \phi \notin w$, and hence $t \colon \phi \in w$ (maximality). – From right to left. From Definition 9 (the right to left direction) it follows that $t \in INF(\phi, w)$ for each w, if for some w, $t \colon \phi \in w$. $\qquad\square$

Now, the following proposition is provable:

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[25]We denote by $\wedge(w')$ the conjunction of all members of w'.

Proposition 17.

1. *If ϕ is an axiom, then there is a constant k such that $k \in INF(\phi, w)$.*

2. $t \in INF(\phi \to \psi, w)$ & $u \in INF(\phi, w, u) \implies (t \cdot u) \in INF(\psi, w)$,

3. $t \in INF(\phi, w) \implies (t + u) \in INF(\phi, w)$, $\quad u \in INF(\phi, w) \implies (t + u) \in INF(\phi, w)$,

4. $t \in INF(\phi, w) \implies {!t} \in INF(t : \phi, w)$,

5. *for some w', $t \notin INF(\phi, w')$ or $Et \wedge \neg\phi \in w' \implies {?t} \in INF(\neg t : \phi, w)$,*

6. $t \in INF(\phi, w) \implies \mathsf{gen}_x(t, w) \in INF(\forall x\phi, w)$, *where x does not occur free in t,*
   $t \in INF(\phi, w) \implies \mathsf{gen}_X(t \in INF(\forall X\phi, w)$, *where X does not occur in t,*

7. $t \in INF(\phi(u/x), w) \iff \mathsf{abs}_u(t) \in INF((\lambda x.\phi)(u), w)$,

8. $[\![\imath x G x]\!]_a^{\mathfrak{F}, w} \in INF(\mathcal{P}T, \mathfrak{F}, w, a)$,

9. $\imath x G x \in INF(\mathcal{P}T, w)$ *for each w and each positive second-order term T,*

10. $t \in INF(\phi, w) \implies$, *then for each w', $t \in INF(\phi, w)$.*

Dokaz. We prove some cases as examples. For Case 1, the proposition follows from Definition 9 on the ground of $\mathcal{CS}'$ and maximality (for each axiom ϕ and each w, it holds that $\phi \in w$ as well as $k : \phi \in w$, where $k : \phi \in \mathcal{CS}'$). For Case 2, assume that $t \in INF(\phi \to \psi, w)$ and $u \in INF(\phi, w)$. Thus, $t : \phi \to \psi \in w$ and $u : \phi \in w$ (Proposition 16), and hence, $(t \cdot u) : \psi \in w$ (**CK**, maximality). Therefore, $(t \cdot u) \in INF(\psi, w)$ (Proposition 9). We take Case 5 as a further example. According to the construction of w in W, if for some w', $t \notin INF(\phi, w')$ or $Et \wedge \neg\phi \in w'$, then $t : \phi \notin w$ and hence $\neg t : \phi \in w$. It follows that $?t : \neg t : \phi \in w$ (**C5**, maximality). Thus, $?t \in INF(\neg t : \phi, w)$ (Proposition 16). Case 9 follows from Axiom **QCGA4** and Definition 9. For Case 10, if we assume the antecedent, $t : \phi \in w$ follows (Proposition 16). Therefore, for each w', $t \in INF(\phi, w'$ (Definition 9, from the right to left direction). $\qquad\square$

We skip the proof that, by means of the construction des-
cribed, a sequence W satisfies all the proposed properties (each
w saturated, $\neg t$: -completeness of W). Provided these properties
obtain, the following holds:

$$t\colon \phi \in w \Longleftrightarrow \text{ for each } w', Et \to \phi \in w'$$
$$\text{and } t \in INF(\phi, w')$$

After establishing the sequence W with the required properties,
the second step is to build a canonical frame and, thereafter,
a canonical model associated with W. In the canonical frame
and model, the ground-domain is the set of equivalence classes
of individual constants of $\mathcal{L}\mathsf{QCGO}'$. An equivalence class $[k] =$
$\{k' \mid k = k' \in w, \text{ for any } w\}$.

Definition 10 (Canonical frame, variable assignment). *Canoni-
cal frame, $\mathfrak{F}^W$, for a set W of saturated sets of sentences, and
a variable assignment a, taken together, are ordered set $\langle W, D,$
$D(n), I, q, a\rangle$ such that:*

1. *W: the set of all worlds w of the sequence W (w is a saturated
   set of sentences of $\mathcal{L}\mathsf{QCGO}'$),*

2. *$D = \{[k] \mid k \text{ is an individual constant of } \mathcal{L}\mathsf{QCGO}'\}$,*

3. *$\varnothing \neq D(n) \subseteq \wp(D^n)^W$ ',*

4. *(a) $I(k) = [k]$,*

   (b) $I(K^n) \in D(n)$,

   (c) $I(\mathcal{P}) \in \wp D(1)$,

   *(d) $I(*^1, w) = \{\langle [k_1], [k_2]\rangle \mid *k_1 = k_2 \in w \text{ or } *(k_1) = k_2 \in w\}$,
   where $* \in \{!, ?, \mathsf{gen}_x, \mathsf{gen}_X, \mathsf{abs}_u, \mathsf{exs}\}$,*

   *(e) $I(*^2, w) = \{\langle [k_1], [k_2], [k_3]\rangle \mid (k_1 * k_2) = k_3 \in w\}$, where $* \in$
   $\{\cdot, +\}$,*

5. *$q(w) \in \wp D$, with the corresponding conditions from Defini-
   tion 4,*

6. *$a(x_i) \in D$, $a(X_i^n) \in D(n)$.*

Definition 11 (Canonical model, variable assignment)**.** *Canonical model, $\mathfrak{M}^W$, and variable assignment are a special canonical frame and variable assignment (Definition 10), defined in analogy to Definition 6 by means of the appropriate designation of λ terms and $\mathcal{P}$, with $I(K^n, w) = \{\langle [k_1], \ldots [k_n]\rangle \mid Kk_1 \ldots k_n \in w\}$, and $I(K^1) \in I(\mathcal{P}, w)$ if and only if $\mathcal{P}K \in w$, and having the corresponding second-order domains.*

We assume the proof that a canonical model is a QCGO model.

Theorem 6 (Canonical satisfaction, denotation, and influence)**.** *For a world $w \in W$ in canonical model $\mathfrak{M}^W$,*

- *(1) $\mathfrak{M}^W, w \models \phi$ iff $\phi \in w$,*
- *(2) $[\![\imath x\phi]\!]^{\mathfrak{M},w} = [k] \in q(w)\}$ if $Ek \in w$ and $\forall x(\phi(x) \leftrightarrow x = k) \in w$, otherwise $[\![\imath x\phi]\!]^{\mathfrak{M}^W,w} \notin q(w)$,*
- *(3) $[k] \in [\![(\lambda x.\phi)]\!]^{\mathfrak{M}^W,w}$ iff $\phi(k/x) \in w$,*
- *(4) $[\![T]\!]^{\mathfrak{M}^W} \in [\![\mathcal{P}]\!]^{\mathfrak{M}^W,w}$ iff $\mathcal{P}T \in w$,*
- *(5) $[\![t]\!]^{\mathfrak{M}^W,w} \in In(\phi, \mathfrak{M}^W, w)$ iff $t \in INF(\phi, w)$.*

Dokaz. Let us give some examples.

- $\mathfrak{M}^W, w \models t\!: \phi$
  $\Longleftrightarrow$ for each w', $\mathfrak{M}^W, w' \models Et \to \phi$, that is for each w', $\mathfrak{M}^W$, $w' \not\models Et$ or $\mathfrak{M}^W, w' \models \phi$, and $[\![t]\!]_a^{\mathfrak{M}^W,w'} \in In(\phi, \mathfrak{M}^W, w')$,
  $\Longleftrightarrow$ for each w', $Et \to \phi \in w'$ (since $Et \notin w'$ or $\phi \in w'$), and $t \in INF(\phi, w')$ (inductive hypothesis),
  $\Longleftrightarrow t\!: \phi \in w$ (Definitions 10 and 11).

- $[\![\imath x\phi]\!]^{\mathfrak{M}^W,w}$. Let $Ek \in w$ and $\forall x(\phi(x) \leftrightarrow x = k) \in w$; then $\imath x\phi = k \in w$ (**D**, $\forall 1a$, maximality); hence $[\![\imath x\phi]\!]^{\mathfrak{M}^W,w} = [k] \in q(w)\}$ (inductive hypothesis). – Let $\forall x(\phi(x) \leftrightarrow x = k) \notin w$; then $\exists x \neg(\phi(x) \leftrightarrow x = k) \in w$ and for some k', $\neg(\phi(k'/x) \leftrightarrow k' = k) \in w$; hence, either $\phi(k/x) \notin w$ or $k' = k \notin w$; accordingly (inductive hypothesis), either $\mathfrak{M}^W, w \not\models \phi(k'/x)$ or $\mathfrak{M}^W, w \not\models k' = k$; therefore $[\![\imath x\phi]\!]^{\mathfrak{M}^W,w} \notin q(w)$.

- $[k] \in [\![(\lambda x.\phi)]\!]^{\mathfrak{M}^W, w} \iff \mathfrak{M}^W, w \models_{a[[k]/x]} \phi \iff \mathfrak{M}^W, w \models \phi(k/x) \iff \phi(k/x) \in w$ (inductive hypothesis) $\iff (\lambda x.\phi)(k) \in w$ (Definitions 10 and 11).

$\square$

The completeness can be then proved from the satisfiability of any consistent sentence of QCGO. This finishes our outline on adequacy.

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