Deflationary nominalism and puzzle avoidance[1]

David Mark Kovacs

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Abstract: In a series of works, Jody Azzouni has defended deflationary nominalism, the view that certain sentences quantifying over mathematical objects are literally true, although such objects do not exist. One alleged attraction of this view is that it avoids various philosophical puzzles about mathematical objects. I argue that this thought is misguided. I first develop an ontologically neutral counterpart of Field's reliability challenge and argue that deflationary nominalism offers no distinctive answer to it. I then show how this reasoning generalizes to other philosophically problematic entities. The moral is that puzzle avoidance fails to motivate deflationary nominalism.

1. Introduction

In a series of works, Jody Azzouni has defended *deflationary nominalism*:[2] the view that (1) reality contains no numbers and other mathematical objects, but (2) existentially quantified sentences (including those quantifying over mathematical objects), as well as their natural language counterparts, can be strictly and literally true without conferring ontological commitment to what they quantifier over (the "separation thesis").[3] The view is a version of *nominalism* because it dispenses with ontological commitment to mathematical objects, and it is *deflationary* because it maintains that

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[2] The moniker is due to Bueno 2014.

[3] In later work, Azzouni (2017) also refers to this general view as "quantifier neutralism".

we can existentially quantify over such objects neutrally (albeit in a non-committal way). By way of illustration, the following sentences are all literally true on Azzouni's view:

> "Some rational numbers are not natural numbers"
>
> "There is exactly one even prime number"
>
> "Some but not all natural numbers are divisible by 16"

On the face of it, these sentences contain standard objectual quantifiers ranging over numbers, which might encourage us to think that they could not be true unless numbers existed. But Azzouni rejects Quine's (1948) quantificational criterion of ontological commitment and maintains that 'There is'-sentences can be strictly and literally true without implying the existence of the objects quantified over. Thus, deflationary nominalism promises to satisfy two seemingly contradictory desiderata in the philosophy of mathematics: like standard forms of nominalism it dispenses with commitment to abstract objects, but like standard forms of platonism it "takes mathematics seriously".[4] Although deflationary nominalism is a view about mathematical objects, it can be (and has been) extended to other kinds of things in obvious ways, for example to hallucinations (Azzouni 2010: Ch. 2), fictional characters (Azzouni 2010: Ch. 3), holes (Azzouni 2012: 957), and even material objects (Azzouni 2017: Ch. 6). In what follows, my main focus will be on numbers,

[4] See Bueno and Zalta 2005 and Bueno 2009 for presentations of platonism and nominalism as different ways of prioritizing these desiderata.

though many of the points I will raise are easily generalizable to other kinds of things.[5]

One widely acknowledged reason to accept deflationary nominalism is its ability to avoid philosophical puzzles about numbers and other problematic kinds of entities. These entities raise several familiar puzzles: How can we individuate them? How can we have knowledge of or even justified belief in them, given that they are not in space and time and are not accessible to perception? And so on. On the face of it, deflationary nominalism offers the best of two worlds: it allows us to agree that certain sentences we are strongly inclined to accept are strictly and literally true, but it avoids positing strange philosophical entities, and along with that, the various puzzles surrounding them. Call this the *puzzle avoidance motivation*.

I think this consensus is mistaken: puzzle avoidance is a poor motivation for deflationary nominalism. To streamline the discussion, in the rest of the paper I will

[5] Deflationary nominalism should be distinguished from neighboring positions whose initial statement might look superficially similar. First, deflationary nominalism is not a Meinongian view (cf. routley 1980, Priest 2005, Berto 2015), since according to Azzouni non-existent things have no properties and lack any ontological status whatsoever – otherwise quantification over them could not be *neutral*. Second, deflationary nominalism differs from the view that the existential quantifier or its natural-language counterpart is polysemous between an ontologically committing and an ontologically neutral notion (Hofweber 2016). Rather, Azzouni thinks that the existential quantifier and natural language expressions such as 'there is', 'exists', etc., are univocal as they stand but do not by themselves confer ontological commitment. In fact, Azzouni thinks that *no* linguistic device in natural languages expresses ontological commitment unambiguously, and that both 'there is' and 'exists' can be read either committingly or non-committingly (see also Parent 2014 for a similar view). For stylistic reasons, in what follows I will often use 'exists' to convey ontological commitment, but I do not thereby mean to go against anything Azzouni says.

focus on one particular puzzle for one particular kind of philosophical entity: Field's reliability challenge to belief in numbers. In section 2, I will present the original challenge and explain why deflationary nominalism might be thought immune to it. In section 3, I will make my initial case that when properly understood, deflationary nominalists do not automatically escape the challenge. In section 4, I will show that while deflationary nominalism itself does not address the challenge, its adherents can nonetheless help themselves to any of the familiar responses from the literature. Azzouni's contention that mathematical objects are mind-dependent is a case in point: it is a potential answer to the challenge but is logically independent from his unorthodox views on quantification and ontological commitment. I will argue that this greatly undercuts the motivation for deflationary nominalism, which turns out to be an empty-spinning wheel in any complete solution to the reliability challenge. In section 5 I will close with the surprising conclusion that the ontological import of 'There is'-sentences is ultimately irrelevant to Field's reliability challenge. I will also briefly sketch how the argument generalizes to other puzzles about other entities.

2. Field's reliability challenge

Field originally proposed his reliability challenge as an improved version of Benacerraf's argument against mathematical realism.[6] It goes roughly as follows. If there are numbers – non-spatiotemporal, causally inert abstract objects –, then that's

[6] See Benacerraf 1973. I will use "mathematical realism" for the view that numbers and other mathematical objects exist and "platonism" for the stronger view that they are mind-independent. The distinction will be relevant in section 4.

what mathematical truths are about. And in that case, there should be an explanation of how mathematicians can reliably track truths about such objects. But precisely because numbers are non-spatiotemporal and causally inert, there can be no causal explanation of this. Moreover, it is unclear what an appropriate non-causal explanation would look like. So if there are numbers, there is no explanation of mathematician's reliability about them, contrary to our starting assumption. This is implausible, so we should conclude that there are no numbers. In premises and conclusion form:

> (F1) If there are numbers, then there is either a causal or a non-causal explanation of why mathematicians are reliable with respect to forming beliefs about them
>
> (F2) There is no causal explanation of why mathematicians are reliable with respect to forming beliefs about numbers
>
> (F3) There is no non-causal explanation of why mathematicians are reliable with respect to forming beliefs about numbers
>
> (F4) So, there are no numbers[7]

Field's reliability challenge has generated a large and impressive body of literature, and various different solutions have been proposed in response to it. Some philosophers reject F1 on the basis that mathematical statements are necessarily true, if true at all, and since they could not be false, there is no need to explain why we are reliable about them. There is no possible scenario in which the mathematical beliefs

[7] Cf. Field 1989: 25–30, 230–233. My formulation closely resembles that of Liggins 2010.

we actually have are false because the mathematical truths are different (Lewis 1986: 108–115).[8] Others reject F2 and insist that there is a causal explanation of mathematical reliability, since we are in causal contact with states of affairs that also involve numbers. For example I can *see* that I have ten fingers, and thereby acquire perceptual access to the number ten (Maddy 1990: Ch. 2). Another strategy (which I will discuss in more detail in section 5) is to argue that there is a causal explanation of mathematical reliability in which the direction of causation is the opposite of what is typically assumed: since mathematical objects are our own creations, they have the features we decided to ascribe to them in the creative act (Cole 2009). More recently, there have been serious attempts to reject F3 by constructing a substantive non-causal explanation of our mathematical reliability; the basic idea is that we have intellectual access to mathematical entities, which in many ways works analogously to our perceptual access to concrete objects (Chudnoff 2013, Bengson 2015). Finally, some philosophers attempt to partially meet the demand for explanation and partially reject it as illegitimate. For example, Rosen and Burgess (1997: 42–49) argue that there is a kind of causal explanation of our mathematical reliability, in which the development of mathematics in past centuries and the evolutionary forces that influenced of our mathematical beliefs play a significant role, but that any further pressure for a deeper explanation ought to be resisted. Since the causal explanation they invoke does not feature causally efficient mathematical states of affairs, it seems best to interpret Rosen and Burgess as rejecting F1 rather than F2.[9] A different solution in this spirit is that of Balaguer (1995), who argues that mathematical

[8] Cf. Clarke-Doane 2016

[9] See also Clarke-Doane 2016 for a related strategy.

reliability is an easy epistemic achievement because for any consistent set of mathematical beliefs, there are *some* mathematical objects of which those beliefs are true. Again, it is not fully obvious whether we should take this as a rejection of F1 or F3. The former seems more appropriate to me, since Balaguer denies that we either have or need an explanation in terms of non-causal access to mathematical objects.[10]

So far, so good. But how should a deflationary nominalist respond to Field's challenge? A seemingly obvious answer is that she should simply accept the argument as sound. F1-F4 are naturally given an ontologically committing reading, and on the committing reading the sentence 'There are no numbers' is *true*. After all, the deflationary nominalist is a *nominalist* and hence rejects the existence of abstract objects; there is no need for Azzouni to respond to Field's argument, since his own position is a version of the view stated in the argument's conclusion.

This response would no doubt fit Azzouni's own thinking about the matter, since he thinks of the reliability challenge as just one of the many puzzles we can avoid by accepting deflationary nominalism:

> "The main advantage of the separation thesis [the thesis that ontological commitment is independent from quantification], overall, is that it simplifies so many metaphysical tangles. […] The separation thesis provides so many simplifications in metaphysics simply because it eliminates the need to postulate something as existing just because certain truths prove indispensable; many metaphysical entanglements arise because this is taken for granted." (Azzouni 2004: 5)

[10] Balaguer does take himself to have given an explanation of mathematical reliability, so he would not accept my classification of his view. This is just a matter of bookkeeping, and depends on how much we demand from an account to count as an explanation.

On this point, even Azzouni's critics tend to agree. For example, in the *Stanford Encyclopedia of Philosophy* entry on mathematical nominalism, Otávio Bueno mentions a number of problems for realist interpretations of mathematics (§2) and presents Azzouni's interpretation of mathematical sentences as true but ontologically non-committing as a potential solution to them (§5). And while deflationary nominalism has been criticized on many other grounds, there seems to be a consensus that at least it avoids the problems usually thought to beset views that posit abstract objects.[11]

Yet this cannot be the end of the story. For even if Azzouni interprets F1-F4 as containing ontologically committing quantification over numbers, nothing prevents *me* from presenting a version of the argument that uses neutral quantifiers through and through. On the assumption that Field proposed F1-F4 with an ontologically committing intent, what the neutral interpretation strictly speaking gives us is a *homophonic counterpart* of Field's own argument. Throughout the rest of the paper I will focus on this reinterpreted argument, and will refer to it as the "Field-homophonic argument".

A clarification is in order here. When I distinguish between Field's own argument and the Field-homophonic argument, I grant two non-trivial assumptions. First, I grant that Field's argument *has* an ontologically non-committing reading in the first place. This is a legitimate assumption: if it does not have such a reading, the Field-homophonic argument does not even get off the ground, but neither does

[11] For example Bueno and Zalta (2005), Colyvan (2005, 2010, 2012) and Raley (2009), while objecting to other aspects of deflationary nominalism, do not question its puzzle-solving potential.

deflationary nominalism itself. Second, I grant that Field's own argument is not already the Field-homophonic argument and that the correct reading of his own version is the ontologically committing one. While this is a natural assumption, it is far from obvious. Field introduces his presentation of the argument with a preliminary discussion of which notion of truth needs to be assumed for it. He then puts to the side correspondence theories as needlessly involved and emphasizes that he needs nothing stronger than a disquotational notion of truth (1989: 228–230). This is as non-committing as it gets: while assuming the correspondence theory would plausibly force a committal reading of the argument, the disquotational theory does not tie our hands in this way. So, although I will henceforth speak of the Field-homophonic argument as distinct from Field's own, it is not at all obvious that it is not simply Field's original argument.

The Field-homophonic argument's significance lies in the fact that by the deflationary nominalist's own light, this argument cannot be sound. When read non-committingly, the sentence 'There are no numbers' is *false*. Thus, it is reasonable to ask what a distinctively deflationary nominalistic response to the *Field-homophonic argument* would look like. In the next section, I will argue that there is no such response; there is nothing in deflationary nominalism that makes it immune to Field-style puzzles.

3. The Field-homophonic argument and the problems of shmontology

In this section, I will argue that deflationary nominalism offers no distinctive solution to the Field-homophonic argument. My case for this will be pretty straightforward. If the view did offer such a solution, it would have to provide us with some guidance as to which of F1-F3 has to be rejected. But it gives us no such guidance, and therefore it fails to address the challenge. To be clear, I do not deny that Azzouni can reject one premise or other of the argument – after all, I'm not assuming that the Field-homophonic argument (let alone Field's original argument) is *sound*. All I say is that deflationary nominalism gives us no special reason to reject any of its premises. To see this more clearly, let's go over them.

F1 is plausible: assuming that there are numbers, it should not be a brute fact that mathematicians are reliable in forming beliefs about them. Moreover, it is an analytic truth that if mathematicians' number beliefs are explained, they are explained either causally or non-causally. Of course, the deflationary nominalist will think that these beliefs do not confer ontological commitment to numbers. This may be so, but it does not change the fact that the reliability of those beliefs requires explanation. The deflationary nominalist may of course hope that it will be easier to provide an explanation on an ontologically non-committal interpretation of those beliefs (I will argue against this below), but the *need* to provide one does not go away merely by switching to a non-committal interpretation. So, deflationary nominalism does not affect the plausibility of F1.

In Field's own version, F2 derives its plausibility from the idea that we are causally isolated from abstracta: if there are numbers, they are outside space and time, while we are spatiotemporally located beings; how could two such radically different kinds of things stand in causal relations? In the Field-homophonic argument, F2 receives a non-committal reading. But notice that this makes F2 not one iota less plausible. Surely if it is true that there are numbers but we are not ontologically committed to them, this does not make it *easier* to have causal access to them. Numbers that do not exist are still not in space and time and still cannot stand in causal relations. So, whatever we think of Field's reasons for maintaining that mathematicians' reliability has no causal explanation, those reasons remain at least as good on the assumption that although 'There is'-sentences about numbers are true, numbers do not exist.

What about F3? When Field set out his reliability challenge, little work has been done on non-causal explanation. This topic has since become a burgeoning area of research, most of which is taking place in the contemporary grounding literature. Either way, Field himself clearly held little hope for a non-causal explanation of our access to truths involving numbers. Are the prospects any better once we switch to the non-committal interpretation? It does not seem that they are. As we have seen, some reject F3 on the basis that we have intuitive/rational access to abstract objects. But presumably, if numbers do not exist, this does not make it easier to have such access to them (plausibly it is at least as hard to have access to non-existent things as to existing ones). I expect that at this point, Azzouni would complain that he does have a special non-causal explanatory story about mathematician's reliability: our epistemic access to numbers is "ultrathin", as he often puts it, roughly because there is nothing for us to discover about their nature.[12] I will consider this response in the next section; in the meantime, we can at least conclude that there is no *obvious* distinctively deflationary response to Field's reliability challenge.

The above line of reasoning will no doubt strike many as deeply confused. After all, deflationary nominalism was all but *designed* to avoid problems like Field's challenge, and that challenge concerned mathematical *ontology*. By contrast, when read

[12] Azzouni 2004: Ch. 6; cf. Azzouni 1997.

non-committingly, F1-F3 are not about ontology at all. It might appear, then, that one can be tempted to think of these sentences as relevant to deflationary nominalism only by tacitly reading into them things that are not there. However, I just do not see that. Azzouni frequently warns us that with the tool of neutral quantification under our belt, we should not automatically think of existential/particular quantification as having a domain with *objects* in it.[13] But I'm assuming no such thing; I'm happy to grant that the numbers the Field-homophonic argument talks about are not objects in whatever sense of 'object' Azzouni has in mind. All I assumed above was *that there were numbers*, a trivial consequence of the truth of the sentence (when read non-committingly) 'There are numbers'. Nor do I assume, as Azzouni warns us not to, that this proposition makes true the sentence that expresses it; I assert nothing more than that since the sentence is true, so is the proposition it expresses.[14] It is just that F1-F3 strike me as prima facie plausible even when I merely focus on the literal truth of these sentences, rather than their ontological content (or lack thereof). Similarly, Azzouni identifies what he calls the "aboutness illusion" as a pernicious influence on much metaphysical theorizing: we easily slip into assuming that in talking and thinking about non-existents, we stand in relations to some entities that our thought and talk are *about*.[15] Elsewhere, he even distinguishes notions of aboutness and reference (he calls them "aboutness^{e}" and "reference^{e}") that carry no such implication.[16] But, again, I'm not presupposing that

[13] Azzouni 2017: xxiv, xxviii-xxix.

[14] Azzouni 2017: Ch. 4.

[15] Azzouni 2017: xi-xii

[16] Azzouni 2010: 43–44. See also Azzouni 2004: 61 – 62, where the preferred terminology is 'refer*'.

F1-F3 are "about" numbers in any sense in which Azzouni thinks they are not. I do not take them to be anything other than what they uncontroversially are: true (if non-committing) sentences that imply that there are numbers. In short, I'm not questioning here any of the machinery Azzouni introduces to "de-ontologize" number-talk. What I question is that the machinery helps address the Field-homophonic argument. If deflationary nominalism gives us an interesting response to the Field-homophonic argument, this needs to be shown by pointing at the faulty premise and carefully spelling out why the view gives us special reasons to reject it. Generic reminders about the content of the view will not do.

In the next section I will consider a more specific (and more promising) response on the deflationary nominalist's behalf. But first, I want to give a general diagnosis of why there is no quick route from deflationary nominalism to puzzle-avoidance. Azzouni's separation thesis is a view about the relation between ontological commitment and quantificational statements. As such, it essentially appeals to certain pieces of philosophical jargon (the "ontological jargon", as I'll refer to it in what follows): 'ontological commitment', 'out there in reality', and the like. Azzouni frequently asks what gives 'There is'-statements "ontological force" [17], which of them are "ontically saturated" [18], how to determine the folk's "ontic predilections" [19], and what the "furniture of the universe" [20] includes. Now, some have

[17] Azzouni 2004: 77

[18] Azzouni 2017: Ch. 3

[19] Azzouni 2007: 210

[20] Azzouni 2007: 206

complained that the way he uses the ontological jargon makes his position obscure.[21] However, I'm *not* making this complaint. I simply observe that Azzouni's position cannot even be *stated* without appealing to the jargon. By contrast, F1-F3 of the Field-homophonic argument require no appeal to the ontological jargon. They are phrased plainly in terms of what there is (once again, consistently with the alleged ontological neutrality of 'there is'); considerations pertaining to "ontological commitment", "ontological status" or the "furniture of reality" do not enter the argument at any point. For this reason, it is hard to see how deflationary nominalism *could* help address the Field-homophonic argument. The deflationary part (Azzouni's acceptance of existentially quantified mathematical sentences as literally true, albeit non-committal) does not help; rather, it is exactly what gives rise to the problem, since it is these very sentences whose truth the argument threatens. And the nominalist part (the denial of ontological commitment to numbers) does not help either, since the Field-homophonic argument does not concern such commitment; it simply attempts to show that it is false that there are numbers, where 'there are' can be as non-committal as it gets. Note that the problem has nothing do with whether the ontological jargon is in good standing. For all I know it is; it just does not help with the argument at hand.

If you prefer, call truths like the one expressed by 'There are prime numbers' in its non-committing use *shmontological* truths. I'm happy to give Azzouni the word 'ontology' and allow that ontology concerns the entities we are ontologically committed to. Shmontology, by contrast, is simply the study of literally true 'There

[21] Hofweber (2007) raises such a concern when he worries that it is not clear what question Azzouni's deflationary nominalism is trying to answer.

is/are…'-sentences and their consequences. The deflationary nominalist can emphasize all he wants that the truth of 'There are prime numbers' has no ontological import. It still has shmontological import, since it disquotationally entails that there are prime numbers, and therefore numbers (and, recall, Field himself did not assume anything stronger than a disquotational notion of truth when presenting his argument). It does not matter that these numbers are not objects or things, that they have no properties, or that they are not among the furniture of reality; what matters is that *it is true that there are numbers*. The Field-homophonic argument threatens to undermine this claim, and with it the truth of 'There are prime numbers' (on its non-committing reading). It is therefore generally unclear how a position like deflationary nominalism could possibly help with Field-style puzzles.[22]

We can refer to the kind of problem I introduced in the preceding paragraphs as "the puzzles of shmontology". The puzzles of shmontology pose a serious challenge to would-be deflationary nominalists moved by the puzzle avoidance motivation. However, this is not the end of the matter. Although a blanket acceptance of deflationary nominalism does not make the Field-style puzzles go away, Azzouni may well have extra theoretical resources that go a long way toward addressing them. In the next section, I will argue that even if there are such resources, they are

[22] This moral is analogous to a point recently made by Korman (2015) about material object ontology: reformulating once-familiar thesis about which material objects there are as views about which material objects fall into the domain of the most joint-carving existential quantifier will fail to solve the puzzles that originally motivated these positions. In short: which objects exist in the most fundamental sense is irrelevant to puzzles concerning which objects exist, in the plain old English sense of 'exist'.

independent from deflationary nominalism itself and are also available to those who reject the view. Thus, deflationary nominalism offers no *distinctive* answer to Field-like puzzles.

4. The irrelevance of ontological commitment to responses to the Field-homophonic challenge

Above I argued that deflationary nominalism gave us no special reason to reject any of the Field-homophonic argument's premises. But of course, this does not mean that the argument *refutes* the view: deflationary nominalists are as free as anyone else to reject one of F1-F3 on independent grounds. For example, they could insist (like Lewis) that mathematical truths are necessary and that reliability about them requires no explanation. They could also adopt one of the "lightweight" explanations I mentioned above: either the Burgess-Rosen account that relies on the history of mathematics but assigns no explanatory role to mathematical objects or Balaguer's strategy of making mathematical reliability an easy epistemic achievement. (In the latter case, instead of a plenitude of mathematical objects one would need to posit a plenitude of ontologically non-committing true 'there is' sentences.) Even the more metaphysically involved strategies have non-committal analogues that could be adopted to address the Field-homophonic argument. For example, similarly to Maddy, deflationary nominalists could argue that we have perceptual access to facts involving numbers (but emphasize that when uttering this sentence, she means to confer no ontological commitment to *entities* such as numbers); and similarly to Chudnoff and Bengson, they could also propose that we have some kind of non-causal access to mathematicalia, albeit, again, without ontological commitment to them.[23]

Azzouni himself does not appeal to any of these solutions; instead, he argues for deflationary nominalism on the basis of what he calls the *independence criterion of existence*, and as we will see, this criterion does go some way to addressing the Field-homophonic argument. However, recognizing that the aforementioned solutions are also available to the deflationary nominalist will help us see more clearly that Azzouni's independence criterion is essentially in the same boat with them: *if* it constitutes a proper response to Field's reliability challenge, it does so independently of deflationary nominalism. It can be combined, of course, with deflationary nominalism, but just like all the other solutions, its puzzle-solving potential in no way depends on that thesis. This is surprising, since (as we have seen above) deflationary nominalism is usually treated as an independent solution to the reliability challenge, while the independence criterion has received comparatively little attention.

[23] For the sake of those who are skeptical of the last two strategies, I note that it is possible to express causation in a way that does not even *seem* ontologically committal: we can replace the relational expressions 'causes' and 'explains' with sentential connectives, for example 'and as an effect' (in the causal case) or 'and as a consequence' (in the non-causal case). While causation and explanation are usually construed as relations between events or facts, connectives do not confer ontological commitment to anything. Thus, 'I have ten fingers, and as an effect I know I have ten fingers' or 'I had an intuitive experience as of 2+2=4, and as a consequence I know that 2+2=4' should be acceptable even to those who disagree with Azzouni on the neutral interpretation of the quantifiers. See van Inwagen 2012 and Fine 2012 for more on causation and non-causal explanation (respectively) without causal or explanatory relata.

According to the independence criterion, something exists just in case it does not depend on mental and linguistic phenomena.[24] Azzouni uses the independence criterion to give a fairly straightforward argument for deflationary nominalism. Given the independence criterion of existence, mathematical objects exist only if they are mind-independent. But if mathematical objects are mind-independent, we are not justified in assenting to our mathematical sentences. However, we *are* justified in assenting to them; and so, we are justified in believing that ordinary mathematical statements are true, although the objects they are about do not exist. Which is to say, deflationary nominalism is true. It is worth quoting the argument in Azzouni's own words:

> "Here's an argument that we shouldn't take mathematical abstracta to exist any more than we take fictional items to exist: Mathematical abstracta are ontologically dependent on our linguistic practices in just the same way that fictional items are […] This argument, however, requires establishing that mathematical abstracta are ontologically dependent on us in the appropriate sense. How do we manage that? It is easy. Mathematical objects can't be ontologically independent of us because then we wouldn't be justified in claiming that the statements we take to be true of such mathematical objects are true. But that we're so justified is simply a datum of mathematical practice" (2004: 103)

Azzouni moves somewhat freely between the *literal truth* of mathematical sentences and our *justification* for believing that they are literally true. I will not dwell on this

[24] Azzouni 2004: Ch. 4. Azzouni often uses the expression 'ontological dependence', but he does not have in mind the relation usually at issue in the specialized literature on ontological dependence. To avoid confusion, henceforth I will use the phrases 'mind-dependence' and 'mind-independence'.

point, since none of what follows will hang on it; I will simply follow Azzouni and phrase the argument in terms of justification:

> (A1) If mathematical objects exist, they are mind-independent and our mathematical sentences are about them
>
> (A2) If our mathematical sentences are about mind-independent mathematical objects, we are unjustified in asserting them
>
> (A3) We are justified in asserting our mathematical sentences, which are also true
>
> (A4) So, mathematical objects do not exist, but our mathematical sentences are true. So, deflationary nominalism is true

The first half of A1 (that if mathematical objects exist, they are mind-independent) is a straightforward consequence of the independence criterion, and its second half (that if mathematical objects exist, our mathematical sentences are about them) is independently plausible. To establish A3, Azzouni relies on the Quine-Putnam indispensability argument. Of course, he does not believe that the argument establishes the existence of mathematical objects, but he thinks it secures the literal truth of (at least a large number of) mathematical sentences.[25]

Azzouni's remarks on A2 are somewhat condensed, but he clearly accepts it on the basis of epistemological problems in the vicinity of the Field-Benacerraf

[25] Azzouni concedes that the argument does not immediately secure the truth of *unapplied* mathematics (2004: 48).

challenge.[26] Why does he think that his own view is immune to the challenge? The answer lies in the special epistemic significance he attributes to mind-dependence:

> "If something is [mind-dependent] on an author, for example, then he or she isn't seen as needing to square the properties of that thing with something else: There is no requirement that he or she justify the claim that the item in question has the properties attributed to it. If someone isn't making something up, then he or she *is* trying to square its properties with something else and thus *is* required to provide an explanation for why the properties attributed to the item must be the properties it has." (Azzouni 2004: 99)

The upshot is that mind-dependent things are epistemically cheap because they do not have the properties that we could go wrong about; our epistemic access to them is "ultrathin".[27] Depending on the exact construal of the relation between minds and mind-dependent things as causal or non-causal, we can take Azzouni to deny either F2 or F3 of the Field-homophonic argument. Either way, there is an explanation of mathematician's mathematical reliability: the things their beliefs are about are mind-dependent (and therefore non-existent) things; in Azzouni's words, we "make them up".

[26] Azzouni usually focuses on Benacerraf's argument rather than Field's. He also adds an argument of his own, the "epistemic role puzzle". Azzouni takes this to be different from Benacerraf's problem (Azzouni 1994: 55–64), but his assessment has ben challenged; see McEvoy 2012 and Azzouni 2016 for discussion.

[27] Azzouni 2004: 127–128.

Now, it is not obvious that mind-dependence automatically shields something from Field-style concerns.[28] But never mind. Notice instead that Azzouni's radical view about the relation between quantification and ontological commitment plays no role in his proposed way out of the problem. All the heavy lifting is done by the claim that numbers are mind-dependent; the independence criterion of existence is an optional add-on of no epistemological significance.

This becomes especially clear when we direct our attention to views that also treat numbers as mind-dependent objects but make no revisionary claims about their ontological status. Julian Cole (2009), for instance, holds just such a view. In line with platonist accounts, Cole maintains that numbers and other mathematical objects are abstracta, but contrary to platonists, he also thinks that they are mind-dependent things that owe their existence to our creative powers. Since mathematical objects are abstract, Cole does not think that their dependence on us is causal. Instead, he categorizes them (along with games and social groups) as *constitutive social constructs*: constructed entities that exist "in virtue of a group of individuals having granted some item a normative role in certain of their activities".[29] Moreover, just like games but unlike social groups, mathematical entities are *pure* constitutive social constructs

[28] Suppose, for example, that we have *one-way* causal access to created abstract objects: we can cause them to exist, but they cannot cause anything. In that case, how can we know that the abstracta we created are the way we intended them to be? (Cf. Field 1989: 27, Liggins 2010: 74-75.)

[29] Cole 2009: 597. See also Thomasson 1999 for socially constructed, mind-independent artifacts in general.

in that they do not require the antecedent existence of any mind-independent entity that members of the group would only need to endow with certain features.[30]

Cole claims that his view (which he calls Practice-Dependent Realism) addresses the Field-Benacerraf challenge, and his story is essentially the same as Azzouni's: since our mathematical practices determine the features of mathematical objects, and we have causal access to these practices, there is no deep puzzle about the reliability of our mathematical beliefs.[31] The availability of Practice-Dependent Realism makes especially vivid why deflationary nominalism as such is irrelevant to the reliability challenge: what matters is the idea that mathematical objects are dependent on thought and language; whether such dependence implies that mathematical objects do not exist is neither here nor there. In this regard, the claim that mathematical objects are mind-dependent is not substantially different from any of the other things that could be said in response to the Field-homophonic argument: that they are mind-independent but causally or non-causally accessible, that they are plenitudinous, that the (pure mathematical) truths about them are necessary, or what have you. Whether these options work as solutions to the reliability challenge is controversial; the point is that if they work, they do so independently of deflationary

[30] Bueno (2009: 70–71) has independently developed a similar view, which he calls "fictionalism": mathematical statements can be interpreted as referring to human-created abstract artifacts. Unlike in the case of Cole, it is not entirely clear if Bueno thinks it is these mind-dependent things that mathematical statements *already* refer to according to our standing practices, since he is merely agnostic about (but does not deny) the existence of mind-independent platonic abstracta (see Azzouni 2015 for a criticism of this aspect of Bueno's view).

[31] Cole 2009: 607

nominalism. Deflationary nominalism itself plays no role whatsoever in addressing the reliability challenge.

It might be objected that we should still prefer deflationism to Practice-Dependent Realism on independent grounds. Indeed, Azzouni offers extensive linguistic evidence to the effect that our conventional criterion of existence is mind-independence. If he is right, then (barring other differences between the two views) Practice-Dependence Realism should give way to Azzouni's view. This may well be true, but it is irrelevant to my main point. My point has been merely that it is misguided to accept deflationary nominalism on the basis of its potential to help us get rid of philosophical puzzles. Even if linguistic evidence favored deflationary nominalism over structurally isomorphic Quinean views, that would not tell against this point. I therefore conclude that Azzouni's deflationism is not properly motivated by puzzle avoidance considerations and offers no distinctive solution to the Field-homophonic argument.[32]

5. Conclusion

Azzouni's deflationary nominalism tells us that 'There is'-sentences can be strictly and literally true without conferring ontological commitment to what they quantify over. It is prima facie reasonable to expect such a radical view to have interesting consequences for Field's reliability challenge for numbers. But when we take a closer

[32] In this regard, my conclusion agrees with the letter of Schechter's (2010: 439), who argues that the Field-Benacerraf challenge does not concern mathematical *ontology*. However, he would also make a stronger claim I disagree with: that the challenge does not even concern mathematical "shmontology" in my sense. See also Clarke-Doane 2016: 17–25 for further discussion.

look, we find this not to be the case: surprisingly, deflationary nominalism leaves the puzzle – or at least a very similar homophonic counterpart of it – entirely intact. It is another matter that the view can be combined with some other doctrine (e.g., that mathematical objects are mind-dependent), which might go a long way to solving the puzzle. But the doctrine in question is independent from deflationary nominalism; we can easily verify this by noticing that the solution it suggests would work just as well (or just as badly) even if we dropped the nominalist component, and that more generally, any of the standard solutions on the market would work just as well (or just as badly) whether we combine them with deflationary nominalism or not.

Although I did not try to demonstrate this in any detail, it is not hard to see that my argument could be generalized to other puzzles about other philosophical entities. First, it can be extended to puzzles similar to Field's about properties, propositions, possible worlds, and the like. These, too, are non-spatiotemporal, causally inert objects on their standard construal, so any set of beliefs about such entities faces a challenge similar to Field's about mathematical objects.[33] Moreover, the familiar arguments concerning them also have counterparts that use homophonically indistinguishable (albeit ontologically non-committal) premises. Deflationary nominalism offers no obvious response to these modified arguments, and for the same reason I discussed above: it is a thesis about ontological commitment in ordinary discourse, but the puzzles in question can be formulated without any appeal to ontological commitment, ontological status, or the ontological jargon in general. Moreover, deflationary nominalists could argue that these objects,

[33] See Liggins 2010: 74-75. For attempts to generalize the Field-Benacerraf challenge, see Swoyer (1996: 250-252) on properties and Divers (2002: Ch. 9, 230-234, 272-274) on possible worlds.

too, are mind-dependent (and so, by the independence criterion, do not exist). But as in the case of numbers, these responses would be entirely independent from deflationary nominalism itself and could be co-opted by realists about the requisite objects.

The argument could be extended not only to similar puzzles about different entities, but also to different kinds of puzzles about those entities. I cannot defend this claim in any detail here, but by now, the argument should be predictable. Take Benacerraf's individuation problem about numbers[34], Bradley's regress about relations[35], or any other familiar puzzle about entities that are considered philosophically problematic. The setup of none of these problems requires us to use the ontological jargon. Hence, they (or their homophonic counterparts) can be formulated as puzzles resulting from the traditional realist's view of which 'There is'-sentences are true. Moreover, the deflationary nominalist *agrees* with the realist on which of these sentences are true, provided that they are interpreted non-committingly. The most the deflationary nominalist can do is transform these ontological problems into shmontological ones, but this does not make the need to solve them any less pressing. And of course, the deflationary nominalist is free to adopt supplementary theses to solve the puzzles. Azzouni's preferred supplementary thesis would presumably be to say that the relevant entities are mind-dependent, but as we have seen, this is an optional add-on: one could be a deflationary nominalist and adopt any of the other solutions to these puzzles on the market. This means that

[34] Benacerraf 1965

[35] See Maurin 2012 for a helpful overview.

radical as Azzouni's separation thesis is, it ultimately plays no role in solving these puzzles.

Where does this leave us? There may be perfectly legitimate reasons to accept deflationary nominalism. Perhaps the view gives us a better understanding of which questions we should count as genuinely ontological. Perhaps there is also strong empirical evidence for some version of the view, because attention to ordinary speakers' linguistic behavior reveals their underlying non-committing tendencies. Any of these claims, and perhaps some more, might provide strong motivation for deflationary nominalism. But you should not endorse the view on the basis that the view offers a distinctive solution to puzzles that is not open to its realist rivals. Any appearance to this effect is an illusion arising from excessive reliance on the ontological jargon.

References

Azzouni, Jody (1994), *Metaphysical Myths, Mathematical Practice*, Cambridge: Cambridge University Press

Azzouni, Jody (1997), "Thick Epistemic Access: Distinguishing the Mathematical from the Empirical," *Journal of Philosophy*, 94: 474–484

Azzouni, Jody (2004), *Deflating Existential Consequence: A Case for Nominalism*, Oxford: Oxford University Press

Azzouni, Jody (2007), "Ontological Commitment in the Vernacular," *Noûs*, 41: 204–226

Azzouni, Jody (2010), *Talking about Nothing. Numbers, Hallucinations, and Fictions*, New York: Oxford University Press

Azzouni, Jody (2012), "Taking the Easy Road Out of Dodge," *Mind*, 121: 951–965

Azzouni, Jody (2015), "Why deflationary nominalists should not be agnostics," *Philosophical Studies*, 172:1143–1161

Azzouni, Jody (2016), McEvoy on Benacerraf's Problem and the Epistemic Role Puzzle, in Fabrice Pataut (ed.), *Truth, Objects, Infinity: New Perspectives on the Philosophy of Paul Benacerraf*, pp. 3–16, Springer

Azzouni, Jody (2017), *Ontology without Borders*, New York: Oxford University Press

Balaguer, Mark (1995), "A Platonist Epistemology," *Synthese*, 103: 303–325

Benacerraf, Paul (1965), "What numbers could not be," *Philosophical Review*, 74: 47–73

Benacerraf, Paul (1973), "Mathematical Truth," *Journal of Philosophy*, 70: 661–79

Bengson, John (2015), "Grasping the Third Realm," *Oxford Studies in Epistemology*, 5: 1–38

Berto, Francesco (2015), "There Is an 'Is' in There Is: Meinongian Quantification and Existence," In A. Torza (Ed.), *Quantifiers, Quantifiers, and Quantifiers: Themes in Logic, Metaphysics, and Language* (Synthese Library) (pp. 233-256), Dordrecht: Springer

Bueno, Otávio (2009), "Mathematical Fictionalism," Otávio Bueno & Øystein Linnebo (eds.), *New waves in philosophy of mathematics*, pp. 59–79, Hampshire: Palgrave McMillan

Bueno, Otávio, "Nominalism in the Philosophy of Mathematics," *The Stanford Encyclopedia of Philosophy* (Spring 2014 Edition), Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/spr2014/entries/nominalism-mathematics/>.

Bueno, Otávio and Edward N. Zalta (2005), "A Nominalist's Dilemma and its Solution," *Philosophia Mathematica (3)*, 13: 294–307

Burgess, John P. and Gideon Rosen (1997), *A Subject with No Object: Strategies for Nominalistic Interpretation of Mathematics*, Oxford: Clarendon Press

Chudnoff, Elijah (2013), "Awareness of Abstract Objects," *Noûs*, 47: 706–726

Clarke-Doane, Justin (2016), "What is the Benacerraf Problem?," in Pataut, pp. 17–43

Cole, Julian (2009), "Creativity, Freedom, and Authority: A New Perspective on the Metaphysics of Mathematics," *Australasian Journal of Philosophy*, 87: 589–608

Colyvan, Mark (2005), "Ontological Independence as the Mark of the Real," *Philosophia Mathematica (3)*, 13: 216–225

Colyvan, Mark (2010), "There is No Easy Road to Nominalism," *Mind*, 119, 285–306

Colyvan, Mark (2012), "Road Work Ahead: Heavy Machinery on the Easy Road," *Mind*, 121: 1031–1046

Divers, John (2002), *Possible Worlds*, London: Routledge

Field, Hartry (1989), *Realism, Mathematics and Modality*, Oxford: Blackwell

Fine, Kit (2012), "Guide to Ground," in Fabrice Correia and Benjamin Schnieder (eds.), *Metaphysical Grounding: Understanding the Structure of Reality*, Cambridge: Cambridge University Press, 37–80

Hofweber, Thomas (2007), Review of *Deflating Existential Consequence*, by Jody Azzouni, *Philosophical Review*, 116: 465–467

Hofweber, Thomas (2016), *Ontology and the Ambitions of Metaphysics*, Oxford: Oxford University Press

Korman, Daniel Z. (2015), "Fundamental Quantification and the Language of the Ontology Room," *Noûs*, 49: 298–321

Lewis, David K. (1986), *On the Plurality of Worlds*, Malden: Blackwell

Liggins, David (2010), "Epistemological Objections to Platonism," *Philosophy Compass*, 5: 67–77

Maddy, Penelope (1990), *Realism in Mathematics*, Oxford University Press

Maurin, Anna-Sofia (2012), "Bradley's Regress," *Philosophy Compass*, 7: 794–807

McEvoy, Mark (2012). Platonism and the 'Epistemic Role Puzzle', *Philosophia Mathematica (3)*, 20: 289–304

Parent, Ted (2014), "Ontic terms and metaontology, or: on what there actually is," *Philosophical Studies*, 170: 199–214

Priest, Graham (2005), *Towards Non-Being*, Oxford: Clarendon Press

Quine, W.V.O. (1948), "On What There is," *Review of Metaphysics*, 2: 21–38

Raley, Yvonne (2009), "Deflating Existence Away? A Critique of Azzouni's Nominalism," *Philosophia Mathematica (3)*, 17: 73–83

Routley, Richard (1980), *Exploring Meinong's Jungle and Beyond*, Australian National University RSSS, Canberra

Schechter, Joshua (2010), "The Reliability Challenge and the Epistemology of Logic," *Philosophical Perspectives*, 24: 437–64

Swoyer, Chris (1996), "Theories of Properties: From Plenitude to Paucity," *Philosophical Perspectives*, 10: 243–64

Thomasson, Amie L. (1999), *Fiction and Metaphysics*, Cambridge: Cambridge University Press

van Inwagen, Peter (2012), "Causation and the Mental," in Kelly James Clark and Michael Rea (eds.), *Reason, Metaphysics, and Mind*, Oxford: Oxford University Press, pp. 153–170