Bilateralism, Collapsing Modalities, and the Logic of Assertion and Denial

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Abstract

Rumfitt has given two arguments that in unilateralist verificationist theories of meaning, truth collapses into correct assertibility. The present paper I give similar arguments that show that in unilateral falsificationist theories of meaning, falsehood collapses into correct deniability. According to bilateralism, meanings are determined by assertion and denial conditions, so the question arises whether it succumbs to similar arguments. I show that this is not the case. The final section considers the question whether a principle central to Rumfitt's first argument, 'It is assertible that *A* if and only if it is assertible that it is assertible that *A*', is one that bilateralists can reject and concludes that they cannot. It follows that the logic of assertibility and deniability, according to a result by Williamson, is the little known modal logic K4 studied by Sobociński. The paper ends with a *plaidoyer* for bilateralists to adopt this logic.

Keywords. Theories of Meaning, Assertion, Denial, Bilateralism, Verificationism, Falsificationism, Moore's Paradox

1 Introduction

In his reply to Dummett's comment on '"Yes" and "No"',[1] Rumfitt gives two arguments to show that the kind of verificationism in the theory of meaning endorsed by Dummett, the view that the sense of a declarative sentence is determined by the conditions for its correct assertibility, faces troubles distinguishing truth from assertibility. The first argument depends on a schema that equates the assertibility of the assertibility of a declarative sentence with its assertibility. The second is a version of Moore's paradox. The equation of truth with assertibility leads to an anti-realism many find unpalatable. In the absence of a retort, Rumfitt's argument must force philosophers with a weaker stomach to reject Dummett's verificationism. In this paper I shall not

[0]The research in this paper was funded by the Alexander von Humboldt Foundation.
[1]See (Rumfitt, 2000), (Dummett, 2002) and (Rumfitt, 2002).

consider whether such a retort may be given.[2] Rather, I shall show that parallel problems afflict a kind of falsificationism in the theory of meaning, according to which the sense of a declarative sentence is determined by the conditions for its correct deniability: such a position faces troubles distinguishing falsehood from deniability. Absent a retort, it, too, should therefore be rejected by those averse to an extreme anti-realism that equates falsity with correct deniability. I shall then address a question that immediately arises. Bilateralism proposes that the sense of a declarative sentence is determined by the conditions of both, its correct assertibility and its correct deniability. It is thus an amalgamation of these kinds of verificationism and falsificationism. The question that immediately arises is whether it won't succumb to similar arguments.

The matter is of some importance. Although in his comment Dummett cheerfully admits that truth is to be equated with correct assertibility (Dummett, 2002, 294),[3] and in his falsificationist moments (cf. (Dummett, 1993c, 81ff)) may have equally cheerfully admitted that falsity is to be equated with correct deniability, the prospect that a bilateralist theory of meaning can draw a distinction between truth and correct assertibility, while largely remaining within Dummettian confines, presents a core motivation for bilateralists to espouse their position. This is clear from Rumfitt's reply. It is also crucial to Price, who was the first to defend 'an account which takes the fundamental notion for a recursive theory of sense to be not assertion conditions alone, but these in conjunction with rejection, or *denial*, conditions. [. . .] One feature of the proposed account of sense will be of central importance: its ability to distinguish the notions of truth and assertibility (or the descriptions ". . . is true" and ". . . is assertible").' (Price, 1983, 162) This, Price explains, is because 'it may be deniable that "S" is assertible, without it also being deniable that S.' (Price, 1983, 167) Thus '"S" is true', which Price takes to be equivalent to 'S', has different deniability conditions from '"S" is assertible', and hence they have different senses. Equating '"S" is false' with '"¬ S" is true' and taking into account the equivalence of '"¬ S" is assertible' with '"S" is deniable' accepted by bilateralists, a distinction between falsity and correct deniability may also be drawn with Price's example: it may be that it is deniable that 'S' is deniable, without it also being assertible that S. Thus '"S" is false' has different deniability conditions from '"S" is deniable', and hence they have different senses.

The kinds of verificationism and falsificationism under consideration are unilateral theories of meaning: only one speech act is taken as fundamental and meaning explained in terms of it. Bilateralism is more complicated than them. It builds on two fundamental speech acts and needs to take account of their interactions. Methodological considerations urge us to favour simplicity. The complications of bilateralism are justified, bilateralists claim, because they yield the *desideratum* of permitting a distinction between truth and assertibility,

[2] One that immediately comes to mind is to forswear the use of classical logic. But even on an intuitionist account of the meanings of the logical constants, similar problems persist, as Dummett is first to point out. See (Dummett, 1978).

[3] It should be mentioned that in earlier work Dummett did distinguish truth from correct assertibility. See in particular (Dummett, 1993a, ch. 7, sec. 2) and (Dummett, 1993b). There Dummett argues that the distinction between the truth and the correct assertibility of a sentence, or, as he then says, the existence of grounds for it or its justifiability, is necessary in the light of complex sentences in which they occur as constituents, in particular as antecedents of conditionals. The question remains how to reconcile this claim with other of Dummett's meaning-theoretical convictions, and his later retraction suggests that he came to see the tension as insurmountable.

while staying within the confines of a Dummettian theory of meaning that explains meaning directly in terms of the competence that enables speakers to use language.[4]

Price's line of thought has considerable intuitive appeal. But in the light of Rumfitt's arguments against verificationism and their parallels against falsificationism, intuition had better be backed up by argument. What guarantees that truth and assertibility, falsity and deniability do not collapse, as there cannot be such undeniable sentences the assertibility of which is deniable, such unassertible sentences the deniability of which is deniable? After all, some verificationists may well have thought that truth is distinguishable from assertibility, and maybe cognately for some falsificationists. What protects bilateralists from Rumfittian arguments being advanced against them?

In the present paper I shall show that no arguments cognate to the Rumfittian arguments against verificationism and its parallels against falsificationism can be levelled against bilateralism. I'll first rehearse Rumfitt's two arguments, and accompany them by brief analyses. Using this material, I'll reconstruct parallel arguments against falsificationism. I then show that no similar arguments can be given against bilateralism. The final section considers the question of the logic of assertibility and deniability on the bilateralist plan. It shows that there are strong, maybe even inescapable reasons for bilateralists to accept the equation of the assertibility of the assertibility of a sentence with its assertibility, and therefore, consequent upon a result of Williamson's, that they should accept the little known modal logic K4 studied by Sobociński as the logic of assertibility and deniability. This is interesting, for here we have an argument for a modal logic that is not one of the usual suspects. The section considers briefly two lines of response to scepticism that this can be so.

2 Verificationism, Bilateralism, Falsificationism

Rumfitt summarises the unilateralist verificationist and bilateralist positions in two principles on how meanings are determined (Rumfitt, 2002, 305):

(α) 'The conditions under which a sentence may correctly be asserted determine its sense.'

(β) 'The conditions under which a sentence may be correctly asserted, together with the conditions under which it may be correctly denied, jointly determine its sense.'

The following summarises the kind of falsificationism under consideration here:

(γ) The conditions under which a sentence may be correctly denied determine its sense.

Rumfitt equates the sense of a declarative sentence with its truth conditions (Rumfitt, 2002, 305). This equation is commonly accepted and I shall not question

[4]Bilateralism has been attacked from various directions. It has been argued that bilateralism is circular, if the aim is to establish bivalence (Dickie, 2010), or that it cannot make good its claim that denial is a speech act recognisably different from assertion of negation (Textor, 2011) or that its logic cannot make sense of assumptions (Kürbis, 2023). I shall not question the foundations of bilateralism here.

it. In presenting arguments against falsificationism, I shall similarly equate the sense of a declarative sentence with its falsity conditions. Rumfitt avails himself of a notion of falsity, the assumption being that verificationists have a notion of falsity even if truth is prior. I shall similarly assume that falsificationists have a notion of truth, even if falsity is prior.

3 Rumfitt's Arguments against Verificationism

Rumfitt's first argument for the co-extensiveness of truth and assertibilty appeals to a 'plausible assumption about correct assertibility' (Rumfitt, 2002, 315):

(AA) 'It is correctly assertible that it is correctly assertible that A if and only if it is correctly assertible that A.'

The argument then runs as follows:

> Consider an arbitrary sentence A together with the sentence $\Box A$ which is got from A by applying to it an operator, '$\Box$', which translates 'it is correctly assertible that' into the language of A. By the (AA) schema, A and $\Box A$ may be correctly asserted under the same conditions. So by thesis (α) they share their truth conditions. It follows that A is true if and only if $\Box A$ is true. The sentence $\Box A$, however, was so constructed as to be true if and only if the sentence A may be correctly asserted. Accordingly, thesis (α) and the (AA) schema jointly entail that an arbitrary sentence is true if and only if it is correctly assertible. (Rumfitt, 2002, 315)

Rumfitt's argument turns on the following: sameness of assertibility conditions means sameness of sense, which in turn means sameness of truth conditions; the assertibility of a sentence A is expressed by $\Box A$, its truth by plain A; thus if the (AA) schema is accepted, $\Box A$ and A have the same truth conditions: if one is true, so is the other, and truth collapses into assertibility:

1. $\Box A$ is true iff A is correctly assertible.	Construction of $\Box$
2. $\Box A$ is correctly assertible iff A is correctly assertible.	(AA)
3. $\Box A$ is true iff A is true.	From 2 by thesis (α)
4. A is true iff A is correctly assertible.	From 1 and 3

We can also reconstruct Rumfitt's argument as appealing to a principle isolated by Williamson as encapsulating the verificationist claim: sameness of assertibility conditions entails sameness of truth conditions, which corresponds to the rule of proof 'from $\vdash \Box A \leftrightarrow \Box B$, infer $\vdash A \leftrightarrow B$' (Williamson, 1988, 303f). Taking the (AA) schema as axiomatic, that is $\vdash \Box\Box A \leftrightarrow \Box A$, immediately yields $\vdash \Box A \leftrightarrow A$.

Although this assumption is not required in Rumfitt's first argument, he operates with a 'factive conception of assertibility and a cognate conception of deniability': 'a sentence is true if assertible and false if deniable.' (Rumfitt, 2002, 317) More on the latter later. The former is expressed by:

(Fact$\Box$) $\Box A \to A$

Rumfitt's second argument also appeals to the distributitivity of assertibility over conjunction:

(Dist$\Box$) $\Box(A \wedge B) \rightarrow (\Box A \wedge \Box B)$

The argument runs as follows:

> let A be a given sentence and consider the pair of sentences $A \wedge \neg\Box A$ ('A but it is not assertible that A'; cf. Moore's paradox) and $A \wedge \neg A$ ('A and $\neg A$'). By factivity, $\Box(A \wedge \neg A) \rightarrow A \wedge \neg A$, so that it is a consequence of the theory of assertibility that $\neg\Box(A \wedge \neg A)$. The second of our pair of sentences, then, is not assertible under any condition whatever. By distributivity, however, $\Box(A \wedge \neg\Box A) \rightarrow (\Box A \wedge \Box\neg\Box A)$, and by factivity $\Box(A \wedge \neg\Box A) \rightarrow (\Box A \wedge \neg\Box A)$, so that it is also a consequence of the theory of assertibility that $\neg\Box(A \wedge \neg\Box A)$. Thus the first of our pair of sentences is also not assertible under any condition whatever. The sentences $A \wedge \neg\Box A$ and $A \wedge \neg A$, then, may be correctly asserted under exactly the same conditions (namely: never), so by thesis (α) they must share their truth conditions. Since the latter sentence is invariably false, the same must go for the former, so we have $\neg(A \wedge \neg\Box A)$, i.e. $A \rightarrow \Box A$. This combines with factivity to yield $A \leftrightarrow \Box A$. On the assumption that assertibility is factive and that it distributes over conjunction, then, thesis (α) again entails that a sentence's assertibility coincides with its truth. (Rumfitt, 2002, 316f)

The strategy of this argument is to present a (meaningful) sentence X that must by force of logic lack conditions in which the meaning determining feature applies to it, i.e. assertibility, from which it follows by thesis (α) that it must be logically false. If $\vdash \neg\Box X$, then X has the same truth conditions as $\bot$.

It may be objected that assertibility, as it is generally understood, is not the notion Rumfitt makes it out to be. Assertibility is relative to a speaker and a time. Rumfitt's argument does not go through, as he treats assertibility as a property of sentences, when it is really a relation between a speaker, a time and a sentence. Although s cannot be in a position where it is assertible for s at t that both, p and it is not assertible for s at t that p, it may be the case that for another speaker s' it is assertible at t that both, p and it is not assertible for s at t that p. No contradiction ensues.[5]

It may furthermore be objected that usually assertibility is considered to be defeasible, and hence not factive.[6] It concerns the availability of warrants or justifications for the assertions of sentences, and speakers may be warranted or justified in asserting sentences at a time even though they are or turn out to be false. That is, the usual notion of assertibility is not Rumfitt's, which, as will become plain in section 6, is tied to knowledge, from where its factivity stems.

My concern here is not, however, to question whether Rumfitt and the bilateralists are entitled to their notion of assertibility. It may not be the common notion, but I take it to be coherent and an appeal to it defensible. It is a robust, objective notion of assertibility, and, just like truth, not relative to speakers. It is

[5]I owe this objection to a referee. Compare with the ancient puzzle on how something can be small as well as tall (*Theaetetus* 154b-155c).

[6]As done by a referee.

this notion that bilateralism in the theory of meaning builds on, and that is my concern here.[7]

Notice also that, although Rumfitt's second argument does not appeal to the (AA) schema, if it did, (Fact$\Box$) is not required. $\vdash \neg\Box(A \wedge \neg\Box A)$ can be proved if instead a weaker consistency principle is adopted: if it is assertible that A, then it is not assertible that $\neg A$. This is the D axiom: $\Box A \to \neg\Box\neg A$. Suppose $\Box(A \wedge \neg\Box A)$, then by (Dist$\Box$), $\Box A \wedge \Box\neg\Box A$. By D, $\Box\neg\Box A \to \neg\Box\Box A$. Contraposing the 4 axiom, $\neg\Box\Box A \to \neg\Box A$, so $\Box\neg\Box A \to \neg\Box A$. Contradiction. This argument, then, goes through for a non-factive, defeasible notion of assertibility.

4 Rumfittian Arguments against Falsificationism

For verificationists, truth is tied to the meaning determining feature: assertibility conditions determine truth conditions. For falsificationists this notion is falsity: deniability conditions determine falsity conditions. For verificationists, falsity is determined by truth: it is unverifiability. For falsificationists, truth is determined by falsity: it is unfalsifiability.[8]

Falsificationists are likely to accept double negation elimination, just as verificationists accept double negation introduction. For verificationists, falsity consists in the impossibility of verification. For falsificationists, truth consists in the impossibility of falsification. If $\neg\neg A$, then it is impossible that A is falsified, hence A is true:[9]

(DNE) $\neg\neg A \to A$

I take the following to be uncontroversial:

(FNT) A is false iff $\neg A$ is true

To falsify A just is to establish the truth of $\neg A$.

Let $\boxtimes A$ symbolise 'It is correctly deniable that A'. Principles cognate to the factivity of assertibility and distribution are:

(Fact$\boxtimes$) $\boxtimes A \to \neg A$

(Dist$\boxtimes$) $\boxtimes(A \vee B) \to (\boxtimes A \wedge \boxtimes B)$

Maybe the deniability condition of a disjunction just is the deniability of each disjunct, but only the direction displayed above is required below.

The following is a falsificationist parallel of the (AA) schema:

(DD) It is correctly deniable that it is not correctly deniable that A if and only if it is correctly deniable that A.

Symbolically: $\boxtimes\neg\boxtimes A \leftrightarrow \boxtimes A$. The left to right direction is plausible, because by (Fact$\boxtimes$), if it is deniable that it is not deniable that A, then it is not not deniable that A, so by (DNE) it is deniable that A. The right to left direction is plausible because if it is deniable that A, surely A's failure to be deniable must be deniable.

[7] I say a few more things on a related topic in (Kürbis, 2022).

[8] Kapsner develops such a falsificationism that follows up on Dummett's ideas (Kapsner, 2014, Ch 6). The present considerations add to Kapsner's qualms about the acceptability of this position.

[9] See again (Kapsner, 2014, Ch 6) for further argument.

For further confirmation that (DD) is exactly what one should expect, observe that bilateralists following Price and Rumfitt accept that it is deniable that A iff it is assertible that $\neg A$:

(AND) $\Box\neg A \leftrightarrow \boxtimes A$

Then (DD) repackages (AA) when A is replaced by $\neg A$, and the other principles are also immediate. It is, however, important that it is possible to give them an independent motivation: falsificationism in the theory of meaning is a position that is independent of verificationism. The coincidence of deniability with assertibility of negation may be useful heuristically, but it need not form part of the formulation or even merely the motivation of falsificationism.

We can now run an argument almost parallel to Rumfitt's:

1. $\boxtimes A$ is true iff A is correctly deniable.	Construction of $\boxtimes$
2. $\neg \boxtimes A$ is correctly deniable iff A is correctly deniable.	(DD)
3. $\neg \boxtimes A$ is false iff A is false.	From 2 by thesis (γ)
4. $\neg \boxtimes A$ is false iff $\neg\neg \boxtimes A$ is true.	(FNT)
5. If A is false, then A is correctly deniable.	From 3, 4, (DNE) and 1

The converse follows by (Fact$\boxtimes$), and so falsity and correct deniability coincide. It is worth noting that, due to the appeal to negation, this argument requires slightly more resources than Rumfitt's.

This is a result worth noting: it is sometimes claimed that falsificationism lends itself to the justification of classical logic, and classical logic is often characterised as the logic for an account of truth and falsity in which these notions outstrip verifiability and falsifiability. The argument just given shows that this is not quite so straightforward. The falsificationist who accepts (γ) should probably opt for a logic with falsity preserving rules of inference that is dual to intuitionist logic.[10]

The Williamsonian rule corresponding to this argument is 'from $\vdash \boxtimes A \leftrightarrow \boxtimes B$ infer $\vdash \neg A \leftrightarrow \neg B$', encapsulating the thought that sameness of deniability conditions entails sameness of falsity conditions. Applied to (DD) taken axiomatically immediately yields $\vdash \neg\neg \boxtimes A \leftrightarrow \neg A$, and hence $\vdash \boxtimes A \leftrightarrow \neg A$ by classical logic, as Williamson and Rumfitt might argue. In fact it suffices to observe that then $\vdash \neg A \rightarrow \neg\neg \boxtimes A$, hence $\vdash \neg A \rightarrow \boxtimes A$ by (DNE), hence $\vdash \boxtimes A \leftrightarrow \neg A$ by (Fact$\boxtimes$).

To construct a falsificationist version of Rumfitt's second argument against verificationism, consider the sentences $A \vee \neg A$ and $A \vee \boxtimes A$. Denying the first entails a contradiction, by (Fact$\boxtimes$): $\boxtimes(A \vee \neg A) \rightarrow \neg(A \vee \neg A)$, so it is never deniable and it is a theorem of the theory of deniability that $\neg \boxtimes (A \vee \neg A)$. By (Dist$\boxtimes$), $\boxtimes(A \vee \boxtimes A) \rightarrow (\boxtimes A \wedge \boxtimes \boxtimes A)$, where the second conjunct of the consequent entails $\neg \boxtimes A$, by (Fact$\boxtimes$). So $\boxtimes(A \vee \boxtimes A)$ too entails a contradiction, so another theorem of the theory of deniability is $\neg \boxtimes (A \vee \boxtimes A)$. Thus $A \vee \neg A$ and $A \vee \boxtimes A$ share their deniability, conditions, namely, never, and thus must share their truth conditions. As the latter is invariably true, so is the former, and hence $\neg A \rightarrow \boxtimes A$. Equivalence follows once more by (Fact$\boxtimes$).

[10] Once more see Kapsner for discussion (Kapsner, 2014, Ch. 7).

5 Bilateralism is Safe against Rumfittian Arguments

5.1 Rumfitt's First Argument

For bilateralism to succumb to a parallel of the first Rumfittian argument, bilateralists would have to accept, not just (AA) and (DD), but also complementary schemata that state that the denial conditions of A and $\Box A$ and the assertion conditions of $\neg \boxtimes A$ and A coincide:

(BiAA) $\boxtimes\Box A \leftrightarrow \boxtimes A$

(BiDD) $\Box\neg \boxtimes A \leftrightarrow \Box A$

Only then do A and $\Box A$, and $\neg \boxtimes A$ and A share their truth conditions according to thesis (β). (BiDD) is equivalent to $\boxtimes \boxtimes A \leftrightarrow \boxtimes\neg A$ which captures the claim that the deniability conditions of $\boxtimes A$ and $\neg A$ coincide. Rewriting both according to (AND) gives:

(BiAA′) $\Box\neg\Box A \leftrightarrow \Box\neg A$

(BiDD′) $\Box\neg\Box\neg A \leftrightarrow \Box A$

The two schemata are equivalent.

Given (AA), the logic of assertibility is at least **S4**, so one half of them already holds, as, using the familiar definition of $\Diamond$, $\Box A \to \Box\Diamond A$ is valid in **S4**.

Given the T axiom, the other half, $\Box\Diamond A \to \Box A$, entails $\Box\Diamond A \to A$, the converse of the B axiom. Adding it to **T** collapses the modality. For in **T**, $\Diamond(A \to \Box A)$, hence by necessitation $\Box\Diamond(A \to \Box A)$, so by the converse of B, $A \to \Box A$.

There is, however, no danger here for the bilateralist. Translated back into bilateralist idiom, $\Box\Diamond A \to \Box A$ says that if it is deniable that A is deniable, then A is assertible. Bilateralists are under no pressure to accept this.

The dialectics of Rumfitt's first argument is to produce a *prima facie* plausible principle, the (AA) schema, and show that its acceptance should lead verificationists also to accept what amounts to Williamson's rule 'from $\vdash \Box A \leftrightarrow \Box B$ infer $\vdash A \leftrightarrow B$', which lets the modality collapse. This is mirrored in the first argument against falsificationism: (DD) is as plausible as (AA), and its acceptance should lead falsificationists to accept what amounts to the rule 'from $\vdash \boxtimes A \leftrightarrow \boxtimes B$ infer $\vdash A \leftrightarrow B$', which also lets the modality collapse. The bilateralist case is different in that $\Box\Diamond A \to \Box A$ is not even *prima facie* plausible, and the argument cannot get off the ground.

5.2 Rumfitt's Second Argument

To reconstruct an argument against bilateralism along the lines of Rumfitt's second argument, it would not be sufficient to exhibit a sentence X that is never assertible, or a sentence Y that is never deniable. The first does not suffice for truth, the second not for falsehood. Rather, we should have to exhibit a sentence X that is never assertible and always deniable, or a sentence Y that is never deniable and always assertible. Only then could we conclude that X is logically false, or that Y is logically true.

Suppose, then, that $\vdash \neg\Box X$ and $\vdash \boxtimes X$. Then by (Fact$\boxtimes$), $\vdash \neg X$. Conversely, if $\vdash \neg X$, then $\vdash \boxtimes X$ by necessitation and (AND), and also $\vdash \neg\Box X$, by (Fact$\Box$) and contraposition. Hence $\vdash \neg\Box X$ and $\vdash \boxtimes X$ iff $\vdash \neg X$, and so the sentences X we should be looking for coincide with the provable contradictions of the theory of assertibility and deniability.

Suppose, next, that $\vdash \neg \boxtimes Y$ and $\vdash \Box Y$. Then by (Fact$\Box$), $\vdash Y$. Conversely, if $\vdash Y$, then $\vdash \Box Y$ by necessitation and $\vdash \neg\Box\neg Y$ by (Fact$\Box$) and reductio, i.e. $\vdash \neg \boxtimes Y$ by (AND). Hence $\vdash \neg \boxtimes Y$ and $\vdash \Box Y$ iff $\vdash Y$, and so the sentences Y we should be looking for coincide with the theorems of the theory of assertibility and deniability.

Let's recapitulate the result in the context and language of normal modal logic. In **T** and its extensions there are sentences A such that $\vdash \neg\Box A$ but $\nvdash A$, and sentences B such that $\vdash \neg\Box\neg B$, i.e. $\vdash \neg \boxtimes B$, but $\nvdash \neg B$. Rumfitt points out that for some such A, verificationists nonetheless must accept A as false, and the parallel argument suggests that falsificationists must nonetheless accept some such B as true. A and B are of a kind that entail that truth implies assertibility, falsity deniability. The result of the previous two paragraphs show that there are no sentences X such that $\vdash \Diamond\neg X$ and $\vdash \Box\neg X$, but $\nvdash \neg X$, and no sentences Y such that $\vdash \Diamond Y$ and $\vdash \Box Y$ but $\nvdash Y$, as for any sentence X fulfilling the first condition, $\vdash \neg X$, and for any sentence Y fulfilling the second condition, $\vdash Y$. Hence neither is it possible to exhibit sentences X and Y that have a similarly awkward effect for bilateralism as the sentences A and B have for verificationism and falsificationism exhibited by Rumfitt and myself. Thus no collapse of assertibility and deniability into truth and falsity can ensue.

5.3 The Strength of Bilateralism

The two previous sections show that bilateralism does not succumb to versions of the Rumfittian arguments against unilateral verificationism and falsificationism.

It may be objected that bilateralism has been defended only against very weak arguments.[11] The first argument is weak, because $\Box\Diamond A \rightarrow \Box A$ is not plausible, the second because it demands rather a lot of sentences X and Y. But this merely shows the strength of bilateralism. For sameness of sense, bilateralism demands sameness of assertibility as well as sameness of deniability conditions. It thus demands more than verificationism and falsificationism. This stronger demand means that, in the case of the first Rumfittian argument, (BiAA')/(BiDD') would be required for a similar argument to get off the ground. A potential argument against bilateralism is weak because the left to right direction of the principle it would have to build on is implausible, but that this is so is due to the strength of bilateralism. Similarly for the second argument: the stronger demands bilateralism makes on sameness of sense mean that rather more must be demanded of sentences if they were to be used in an attempt to run a version of Moore's paradox against bilateralism, as Rumfitt does against verificationism and as I do against falsificationism. Indeed, so much more is demanded that it is impossible for there to be such sentences, as was shown. Once more it is the strength of bilateralism that ensures that no Moorean assault against it is possible.

[11] As done by a referee.

6 Bilateralism and the Logic of Assertion and Denial

Rumfitt concludes from his first argument: 'Contraposing, a philosopher who wishes to make room for a difference in extension between the predicates "true" and "correctly assertible" must either reject the (AA) schema, or join me in rejecting thesis (α)' (Rumfitt, 2002, 315). This conclusion is slightly misleadingly put. Rumfitt does not explicitly reject the (AA) schema and sometimes sounds as if he accepts it. The disjunction is, however, inclusive. Furthermore, there is, *prima facie* at least, no direct link between accepting or rejecting the one and accepting or rejecting the other. The (AA) schema is independent of thesis (α): one may accept both, accept one and reject the other, or, evidently, reject both.[12]

Although compelling and accepted by many, Rumfitt makes clear that on a knowledge-based account of assertibility, such as the one Dummett and he endorse, the (AA) schema 'from right to left is dubious' (Rumfitt, 2002, 315). This is because on such an account, 'a sentence is (objectively) correctly assertible if and only if knowledge is available which would warrant a speaker in asserting it' (Rumfitt, 2002, 316), but it is doubtful whether 'the availability of knowledge that *p* suffices for the availability of knowledge that knowledge that *p* is available' (Rumfitt, 2002, 316). Rumfitt points out a likeness of the (AA) schema to the KK principle, rejected by many: knowledge that *p* is no guarantee for knowledge of knowledge that *p*.

Other places show that Rumfitt rejects the (AA) schema. Rumfitt refers to a result of Williamson's (Williamson, 1988, 311) that a rule of modal logic that corresponds to thesis (β) yields 'a quite bizarre consequence' (Rumfitt, 2002, 317) for the relation between truth, assertibility and deniability when the modal logic of these notions is at least **S4**, i.e. when the (AA) schema holds:

(BiTC) From $\vdash \Box A \leftrightarrow \Box B$ and $\vdash \Diamond A \leftrightarrow \Diamond B$ infer $\vdash A \leftrightarrow B$

where by (AND) the second premise is equivalent to $\vdash \boxtimes A \leftrightarrow \boxtimes B$. (BiTC) thus captures the bilateralist thesis (β) that sameness of assertibility and deniability conditions entails sameness of truth conditions. Williamson shows that **S4**+(BiTC) is equivalent to a system Sobociński (Sobociński, 1964) called K4[13]. This system has the rather curious feature that A is equivalent to a truth function of modalisations of itself:

(TAD) $\vdash A \leftrightarrow \Box A \vee (\Diamond A \wedge \neg\Diamond\Box A)$

In the present context, this is equivalent to $\vdash A \leftrightarrow \Box A \vee (\neg \boxtimes A \wedge \boxtimes\Box A)$: A is true iff it is either assertible or it is not deniable but its assertibility is deniable. It is this consequence that strikes Rumfitt as bizarre.[14]

Referring to another result of Williamson's that the smallest normal modal logic closed under (BiTC) is **T** (Williamson, 1990), Rumfitt remarks 'that thesis (β) is already implicit in that theory of assertibility' (Rumfitt, 2002, 317) that takes this to be its modal logic, that is when the (AA) schema fails.

It is plausible independently of a knowledge based account of assertion that bilateralists should reject the (AA) schema. Even if bilateralists accepted

[12]As pointed out by a referee.

[13]Not to be confused with the system that results from adding the 4 axiom to modal logic **K**.

[14]Williamson shows that K4 can also be axiomatised by adding (BiTC) to **S4**. For proofs see the appendix to (Williamson, 1988).

that A and 'It is assertible that A' share their assertibility conditions, the expectation from Price's indication of how to draw a distinction between truth and assertibility is that they may still differ in deniability conditions. Thus on a bilateralist account, the (AA) schema should fail together with the failure of the deniability conditions of A and 'It is assertible that A' to coincide.

Or so one would hope. The question is whether bilateralists can indeed evade accepting the (AA) schema. There are very good reasons to conclude that they can't. To show this requires some preparation.

In the bilateral logic developed by Rumfitt (Rumfitt, 2000, 800ff), following Smiley (Smiley, 1996), every formula is prefixed by either +, indicating its correct assertibility, or −, indicating its correct deniability.[15] + and − indicate speech acts, and therefore cannot be iterated: $+ - A$, e.g., is illformed. + and − are coordinated by two structural rules, where $\alpha*$ is the result of reversing the sign of α:

Reductio: If $\Gamma, \alpha \vdash \bot$, then $\Gamma \vdash \alpha*$
Non-Contradiction: From α and $\alpha*$ infer $\bot$

Negation and implication are governed by the rules:

$$+\!\rightarrow\! I : \frac{\begin{array}{c}[+A]^i\\ \Pi\\ +B\end{array}}{+A \rightarrow B}\,i \qquad +\!\rightarrow\! E : \frac{+A \rightarrow B \qquad +A}{+B}$$

$$-\!\rightarrow\! I : \frac{+A \qquad -B}{-A \rightarrow B} \qquad -\!\rightarrow\! E : \frac{-A \rightarrow B}{+A} \qquad -\!\rightarrow\! E : \frac{-A \rightarrow B}{-B}$$

$$-\neg I : \frac{+A}{-\neg A} \qquad -\neg E : \frac{-\neg A}{+A} \qquad +\neg I\!: \frac{-A}{+\neg A} \qquad +\neg E\ \frac{+\neg A}{-A}$$

Suppose we add Rumfitt's operator □ to bilateral logic. Weiss proposes two principles governing □ (Weiss, 2018):

(□+) From + □A infer + A

(+□) From + A infer + □A

□ translates 'it is correctly assertible that' into the language of A. Hence it translates what is represented by +, which immediately suggests (□+) and (+□). (□+) is uncontroversial. Despite the immediacy of the suggestion, below are two brief arguments for (+□). + and − of bilateral logic are not sentential operators and cannot be iterated, so in order to distinguish them from □ and ⊠, which are sentential operators and can be iterated, I shall continue to render the latter as 'It is correctly assertible that' and 'It is correctly deniable that', and the former as 'is correctly assertible' and 'is correctly deniable'. The latter require noun phrases as their subjects to form sentences. For simplicity I'll use formulas as names of

[15]Humberstone developed a similar logic around the same time as Rumfitt in (Humberstone, 2000), a paper also referenced by Rumfitt.

themselves, as does Rumfitt. Context makes clear how a formula is used. Recall that $\Box A$ is constructed so as to be true if and only if A is correctly assertible.

First Argument for $(+\Box)$. Suppose (1) A is correctly assertible, but (2) $\Box A$ is correctly deniable. From (1) it follows that $\Box A$ is true, by construction of $\Box A$. From (2) it follows that $\Box A$ is false, by the factivity of correct deniability. Contradiction. Thus $+A, -\Box A \vdash \bot$, and so $+A \vdash +\Box A$, by bilateral reductio.

Second Argument for $(+\Box)$. Rumfitt compares 'A but it is not assertible that A' to Moore's Paradox (Rumfitt, 2002, 316). But to consider A to be assertible and its assertibility to be deniable is also a version of Moore's Paradox. That is, 'A is assertible' and '$\Box A$ is deniable' are paradoxical, hence from 'A is assertible' it follows that '$\Box A$ is assertible' by bilateral reductio.

Complementary principles for $-$ and $\boxtimes$ akin to the (DD) schema and arguments for them may be formulated along parallel lines. I shall leave this as an exercise to the interested reader.

Weiss argues that $(+\Box)$ has dire consequences. It is vital for the bilateralist account of meaning that the obtaining of the conditions for the correct deniability of a sentence should not be consequent upon the failure of the conditions for its correct assertibility to obtain. However, it follows trivially from $(+\Box)$ that $+\neg\Box A \vdash -A$. Thus 'bilateralism collapses into an implausible unilateralism' (Weiss, 2018, 99), because denial conditions are, after all, determined by assertion conditions—that is, the denial conditions of a sentence obtain when its assertion conditions fail to obtain, contrary to the core bilateralist claim that assertion and deniability conditions are independent and jointly required for a specification of sense. Accordingly, Weiss observes, while $(\Box+)$ is uncontroversial, bilateralists will want to reject $(+\Box)$. The problem is that the bilateral formalism leaves them no room for doing so. All formulas are signed by either $+$ or $-$, indicating asserted or denied formulas. The distinction that Price and Rumfitt want to draw between truth and assertibility just cannot be drawn in bilateral logic.

I set Weiss's worry that bilateralism collapses into unilateralism aside. It is fair to say, as Weiss observes, too, that strictly speaking, his argument shows that denial conditions follow from assertion of failure of assertion conditions, not merely failure of assertion conditions. Whether this opens up a way out for the bilateralist or not, it underlines once more a point Weiss stresses, namely that we are often not interested in what is assertible or deniable, but rather in what is and is not the case. For this reason, Weiss rejects the formalism of bilateral logic: logic concerns inferential relations between propositions, not between propositions asserted or denied. Again, I will not follow him in this here.[16] As in previous sections, I shall take the framework of bilateralism for granted. But I take away two points from Weiss's discussion: (a) bilateralism has no room to resist $(\Box+)$ and $(+\Box)$; (b) the logic of truth and falsity cannot be represented in bilateral logic. My aim is to explore the options bilateralism has in the light of these two points.

Concerning point (b), a case can be made that bilateralists are entitled to a different formalisation of logic when truth and falsity are concerned. The point is not that there is no logic of truth and falsity over and above the logic of assertion

[16]Elsewhere I raise doubts about the coherence of the bilateral formalism (Kürbis, 2023).

and denial. To the contrary. Truth is to be distinguished from assertibility, falsity from deniability. Let's grant that this distinction can be drawn. Bilateralist may then simply agree that bilateral logic is not suited to the formalisation of the logic of truth and falsity, and that this logic requires the standard formalism also employed by Rumfitt in his reply to Dummett.

It remains the case that truth and falsity are secondary to and to be explained in terms of assertibility and deniability. Thus bilateral logic is more basic, and the logic of truth and falsity subservient to it. Anything established in the former should be available in the latter, and the latter must cohere with the former.

($\Box$+) and (+$\Box$) together entail $\vdash +A \leftrightarrow \Box A$ and the modality collapses in bilateral logic. Adding a truth operator to unilateral propositional logic is vacuous, where writing plain A represents the truth of A, so it would not be surprising that an assertibility operator is vacuous in bilateral logic, and cognately for a deniability operator. The question is whether this collapse also carries over to a system that does not operate on asserted or denied formulas, but on their truth and falsity.

(+$\Box$) need not contradict Price's claim that it may be the case that $\Box A$ is deniable, while not being the case that A is deniable, so that truth does not entail assertibilty, i.e. $A \to \Box A$ fails. (+$\Box$) does not concern this principle, but a weaker one, namely whether the assertibility of A, not its truth, entails the assertibility of $\Box A$. And as we have see, it is hard to see how bilateralists can refrain from accepting that this is so.

($\Box$+) and (+$\Box$) correspond in a clear sense to the (AA) schema. And not just that, if (+$\Box$) is accepted, then the (AA) schema follows. (1) Suppose $\Box A$ is true. Then by construction A is assertible, so by factivity of assertibility, A is true, hence $\Box A \to A$. (2) Suppose $\Box A$ is true. Then by construction A is assertible. But then by (+$\Box$), so is $\Box A$, and hence $\Box\Box A$ is true, again by construction of $\Box\Box A$. Hence $\Box A \to \Box\Box A$. The converse follows by (1). Thus we have shown that the (AA) schema holds.

Notice that we cannot argue that $A \to \Box A$: if A is true, nothing need to follow about its assertibility. Hence the modality does not collapse in the logic of truth and falsity.

To come back to the question whether bilateralists can reject the (AA) schema, the last few pages make a very good case that they can't. It has been argued that (+$\Box$) holds in bilateral logic. Bilateral logic being more basic than the logic of truth and falsity we should expect to be allowed to appeal to it also in the logic of truth and falsity. Doing so establishes the (AA) schema. There is thus a clear tension between bilateralism and its rejection.

Rumfitt remarks that the (AA) schema is dubious, not that it is false, that its consequences for the logic of assertion and denial are bizarre, not that they are absurd. The tension would ease if the (AA) schema is accepted and the consequences are endorsed. As Rumfitt makes clear, this would put bilateralists in good company. Doubts about the KK principle may motivate doubts about the (AA) schema, but they are not conclusive: the (AA) schema concerns a specific kind of knowledge, and, as Rumfitt points out, some philosophers have no qualms about it. He refers to Crispin Wright as one who seems to regard it as evident (Wright, 1992, 13). Others will follow suit. Even if the (AA) schema is accepted, the knowledge based account of assertibility remains defensible. I shan't attempt to do so here, but rather consider the question whether accepting this result is really as much of a problem for bilateralism as

Rumfitt and Williamson present it to be.

It should be underlined once more that there are strong reasons to accept Sobociński's K4 as the logic of assertion and denial within bilateralism, and hence the move is well motivated. Rumfitt argues that their logic is at least **T**. (BiTC) captures a fundamental thought of bilateralism, thesis (β), and is not negotiable. The (AA) schema has been argued for appealing to fundamental aspects of bilateralism. Reasoning from first principles of bilateralism, the logic of assertion and denial presents itself as K4.

Bizarre or not, it may reasonably be questioned whether anyone would come up with (TAD) upon reflection on the concepts of truth, assertibility and deniability independently of the bilateralist considerations voiced here. Many will join Williamson in his complaint that he knows of no concept of assertibility that validates it (Williamson, 1988, 312). But this is no objection to adopting K4. Two responses may be given. Both rely on the fact that it is a result of sustained philosophical reflection that K4 is the logic of assertion and denial.

(1) Theorising, and theorising in philosophy in particular, may result in a revision of our concepts. If our ordinary concepts of assertion and denial do not support K4, then bilateralists may propose a revision of these concepts. Compare Dummett on negation: philosophical reflection on meaning and use led Dummett to propose that our ordinary concept of negation, which is unrestrictedly subject to the principle of bivalence, should be revised so as to admit its validity only for decidable sentences or predicates. Similarly bilateralists can respond that philosophical reflection has forced upon them a revision of the ordinary concepts of assertion and denial.

(2) K4 is indeed the logic of our ordinary concepts of assertion and denial. It is just that not all aspects of our concepts are immediately or intuitively evident, and some aspects of our concepts may be accessible only through thorough reflection on them. Starting from first principles, philosophical investigations have established that the logic of truth, assertion and denial on the bilateralist account of meaning is a normal modal logic in which $\Box$ and $\boxtimes$ satisfy factivity (Fact$\Box$), its cognate (Fact$\boxtimes$), the (AA) and (DD) schemata and rule (BiTC). It is a philosophical and logical discovery that the truth of A is equivalent to $\Box A \vee (\neg \boxtimes A \wedge \boxtimes \Box A)$.[17]

The bizarreness of K4 can be turned into an advantage: In K4 the truth of a proposition turns out to be equivalent to a truth function of propositions concerning its assertibility and deniability. Thus K4 really does permit not only that a distinction between truth and assertibility, falsity and deniability may be drawn, but also that truth may actually be *defined* in terms of assertibility and deniability.

Finally, (TAD) may look bizarre, but consider an equivalent form that results from replacing A by $\neg A$, contraposing and repackaging:

(TAD′) $\vdash A \leftrightarrow \neg \boxtimes A \wedge (\boxtimes \boxtimes A \rightarrow \Box A)$

[17] If no sense can be made of the possible worlds semantics for K4, as Williamson complains, so much the worse for its possible worlds semantics. $\Box$ and $\boxtimes$ have been endowed with sense. No further possible worlds semantics is required for their intelligibility.

A is true iff it is not deniable and if its deniability is deniable, then it is assertible. I'm not going to speculate whether this may strike anyone as less bizarre or whether it is more palatable to intuition than (TAD). But it is certainly quite interesting. If A is true, it already follows by (Fact⊠) that it is not deniable. By (TAD') in addition a kind of double deniability elimination holds: its double deniability implies its assertibility. If the knowledge-based account of assertibility and deniability is retained, this means that the true sentences are those where the deniability of their deniability implies that knowledge is available that would warrant their assertion. It may well be possible to cash this out as being of some epistemological importance.

More could be said about the implications of the line of thought pursued in this section, but this is not the place to do so. I shall end this *plaidoyer* for K4 as the logic of assertion and denial to be adopted by bilateralists with a final observation. It follows that for truth not to collapse into assertibility there must be sentences such that we cannot put ourselves into a position to recognise that they have the property that equates to their truth: for if we were, we could deduce their truth and hence assert them, collapsing truth and assertibility once more. Thus there must be sentences of which we are not in a position to recognise that they are not deniable and that their double deniability implies their assertibility. For if we were, we could apply (TAD') and infer their truth, which would make these sentences assertible. There must also be sentences the assertibility of which is deniable, while they themselves are not deniable, as in Price's example, but we cannot put ourselves into a position to recognise this, for otherwise we could deduce their truth by (TAD), and hence assert them. Bilateralists may simply accept this consequence. A *desideratum* of bilateralism is to permit verification transcendent truth. Hence even though truth is defined in terms of assertibility and deniability, the property that equates to truth must sometimes be forever beyond our apprehension, and so be it. On the other hand, rather than bizarre, maybe this is absurd.

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