The grammar of philosophical discourse*

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Abstract

In this paper, a formal theory is presented that describes syntactic and semantic mechanisms of philosophical discourses. They are treated as peculiar language systems possessing deep derivational structures called architectonic forms of philosophical systems, encoded in philosophical mind. Architectonic forms are constituents of more complex structures called architectonic spaces of philosophy. They are understood as formal and algorithmic representations of various philosophical traditions. The formal derivational machinery of a given space determines its class of all possible architectonic forms. Some of them stand under factual historical philosophical systems and they organize processes of doing philosophy within these systems. Many architectonic forms have never been realized in the history of philosophy. The presented theory may be interpreted as falling under Hegel's paradigm of comprehending cultural texts. This paradigm is enriched and inspired with Propp's formal, morphological view on texts. The peculiarity of this modification of the Hegel-Propp paradigm consists of the use of algebraic and algorithmic tools of modeling processes of cultural development. To speak metaphorically, the theory is an attempt at the mathematical and logical history of philosophy inspired by the Internet metaphor. And that is why it belongs to the tradition of doing metaphilosophy in The Lvov-Warsaw School, which is continued today mainly by Woleński, Pelc, Perzanowski, and Jadacki.

Keywords: *Architectonic forms of philosophy; architectonic spaces; transformations of forms; logic of discourse; philosophical grammar*

1. Introduction: The intuitive basis for the formal theory of mental philosophical representations

The presented theory is based on the following assumptions: (1) The philosophical discourse is understood as the process of linguistic activity in which a

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subject doing philosophy creates and transforms peculiar mental representations. These representations are encoded in the philosopher's mind; (2) The creation and transformation of mental representations of philosophy is determined by various grammatical and semantic rules; (3) Between mental representations of philosophy there hold different syntactic and semantic relations; (4) There are at least four levels of mental representations of philosophy. In this paper, no application of the theory is presented; this task is realized in Krysztofiak (2006).

Since every scientific theory should satisfy explanatory purposes, so the presented metaphilosophical theory should also fulfill such an explanatory function. The range of explanation of the proposed theory may be sketched in the following way. Philosophers create their systems that are usually understood in various ways, determined by mental representations encoded in philosophical minds. Linguistic acts of doing philosophy that are always executed within some discourse stemming from some philosophical tradition are delimited in respect to their contents by processes of activation of corresponding mental representations. Thus, the grammatical and semantic machinery, encoded in philosophical mind, indirectly determines ways of understanding acts of doing philosophy. Therefore, the range of explanation of the presented formal theory may be comprehended as the area of acts of philosophical practice, executed in some cultural tradition.

One may look at the phenomenon of philosophy as stratified into two strata. The first stratum is composed of linguistic entities such as philosophical languages, acts of doing philosophy executed by real minds, philosophical discourses understood as linguistic processes comprising philosophical texts, dialogues of various types, and finally different philosophical traditions implemented in books, journals, and especially in the social memory. The second stratum of the phenomenon of philosophy is composed of mental representations encoded in philosophical minds. There are four levels of these mental representations. The first one is consisted of spaces of architectonics of philosophical systems. Every such a space corresponds to some philosophical tradition in such a way that any space is an internalized philosophical tradition in a philosophizing mind. The second level comprises architectonics of philosophical systems. They are interpreted as mental representations of philosophical discourses. They determine possible courses of a given philosophical discourse. Architectonics are set-theoretic constructions composed of derivations that constitute the third level of representations. Some constellations of derivations are representations of acts of doing philosophy. Derivations are set-theoretic structures defined on the set of lexical elements which belong to the fourth level. Lexical elements are mental representations of linguistic categories of a given philosophical language corresponding to a given philosophical tradition. Table 1 sketches the relationships between both strata of the phenomenon of philosophy.

Table 1. *The left side describes the surface strata of the phenomenon of philosophy whereas the right side describes its deep structure.*

The empirical stratum of the phenomenon of philosophy	The stratum of mental representation of the phenomenon of philosophy
(1) The philosophical tradition	The space of philosophical architectonics
(2) The region of philosophical discourses	The level of architectonics
(3) Acts of doing philosophy	The level of derivations
(4) Philosophical languages	The level of lexical bases of philosophical grammar

Now, it is easy to present explanatory purposes of the proposed formal theory in details. (1) All changes (even revolutionary ones) of philosophical traditions are explained by appropriate grammatical and semantic processes occurring on the level of spaces of architectonics encoded in philosophizing minds. Such cultural processes like amalgamations of philosophical traditions or the emergence of new traditions can be explained by some special grammatical laws determined on various formal structures involved in spaces of architectonics. (2) Conceptual and formal similarities between philosophical systems are treated as manifestations of various transformations of architectonics into others. Different ways of interpreting philosophical systems within one and the same discourse can be considered as inter-architectonic transitions, ruled by some algorithms defined on constituents of architectonics. (3) Finally, the complexity of particular acts of doing philosophy executed within some discourse may be explained by formal properties of derivations. (4) Changes in philosophical languages (creation of new notions and categories, or on the contrary, making them anachronistic ones) are understood as evoked by processes of introducing lexical representations into philosophical bases or processes of cancelling them from these representational structures.

2. The formal theory of philosophical representations

Our theory divides itself into four parts: (1) the theory of lexical representations, (2) the theory of derivations, (3) the theory of architectonics, and (4) the theory of architectonic spaces.

2.1. *The level of lexical representations and its formal model*

The level of cognitive, lexical representations is composed of the following syntactic categories: (a) the category of totality, (b) the category of ontological dimensions, (c) the category of worlds, (d) the category of regions, (e) the category of explanators, (f) the category of philosophical notions. These categories exclude each other. Categories from (a) to (e) are formal whereas the last category is material. This means that philosophical notions function as tools of interpretation of elements belonging to formal categories.

2.1.1. *The category of totality.* The category of totality is introduced into the theory on the ground of the intuition of totality that constitutes a cognitive ability of mind for executing an operation of unrestricted general quantification. In natural languages, this type of quantification occurs in syntactic structures of the type "*Everything is A,*" where "*A*" is usually an adverb but sometimes "*A*" is a verb or even a noun. It is easy to indicate many cases of such structures in philosophical languages. Many philosophers say that they try to understand or explicate the totality of what exists or even what may exist. It is sometimes said that philosophy is an activity where one tries to comprehend the whole of human experience. There are used various words for expressing this intuition of totality. The use of this formal category by a philosopher is just a starting-point of philosophical enterprise. The subsequent step in this activity consists in interpreting the category of totality.

2.1.2. *The category of ontological dimensions.* When a philosopher builds his or her philosophical system, s/he often distinguishes different ontological domains or spheres that constitute the totality of all entities. Theses domains are conceptualized variously in different systems. By virtue of these conceptualizations ontological dimensions are constituted. The peculiarity of dimensions may be reduced to the fact that two different entities belonging to two different dimensions cannot be conceptualized by common philosophical notions. This means that to one and the same dimension there belong only these entities, conceptualizations of which comprise some common notions. The practice of distinguishing ontological dimensions stems from the subject's capacity to evaluate meaningfulness or meaninglessness of our ordinary speech acts. In many situations, language users can decide whether a properly composed sentence is meaningful or meaningless. By virtue of semantic competence it is easy to formulate the appraisal that the utterance "*Love is a number that satisfies any diofantic equation and that is why it cannot be predicated of flowers and dogs*" is senseless. Therefore, flowers or people together with numbers cannot belong to a common ontological dimension.

2.1.3. *The category of worlds.* Language users possess the ability to distinguish various worlds. First of all, they discriminate their actual worlds from all fictional worlds. If one reads some criminal story, then one is aware of that the world being described in the story is not reality. It is very easy to give many different examples of distinguishing various worlds. In philosophical discourse, the capability of discriminating worlds does not disappear. A philosophical subject is able to differentiate the so-called real world from possible or fictional worlds. Between these worlds there hold relations of similarity. For instance, the world of natural numbers and the world of real numbers bear higher resemblance than, for instance, the resemblance between the world of natural numbers and the world presented by Conan Doyle's story about Sherlock Holmes. Our decisions concerned with the degree of resemblance between worlds are intuitive. It means that a philosophical subject does not use any algorithm in evaluating, estimating, and ordering degrees of similarity between various worlds. Ontological dimensions are those cognitive representations that enable a philosophical subject to compare worlds with respect to their degrees of similarity. If two worlds are located within one ontological dimension, they are comparable in respect of their resemblance. If two worlds belong two different ontological dimensions, they are incomparable in this respect.

2.1.4. *The category of ontological regions.* Language users possess the disposition to regionalize or stratify worlds. This competence manifests itself in distinguishing various regions, strata or sections within worlds. It occurs frequently in scientific discourse as well as in the ordinary life that language users try to find deep or hidden structures or sections of the world that determine phenomena or events appearing in the surface reality. A language user ascribes two different objects to one and the same world because, on the basis of his or her knowledge, these objects are entangled in some relational system. In the case of distinguishing regions within one world, two objects that are comprehended as being involved in two distinct regions must be connected one to another by some relation. The semantic mechanism of stratifying a world consists in the ascription of some properties to some class of objects. In this way, for instance, trees and animals possess a common property of being alive whereas stones do not manifest such a property. Therefore, one may assert that trees and animals belong to one and the same biological region of the world. Regions of worlds may be constituted by virtue of different properties and relations. The principles of distinguishing regions may be temporal, spatial, communicational, ecological, etc.

2.1.5. *The category of explanators.* It is the case that, for many languages, their users are able to produce complex syntactic structures called conditionals. These syntactic structures are often answers to the questions of the shape: *Why* *is it a case that α?* Many questions of this syntactic shape are understood as demanding an explanation. Content expressed in conditionals underlying the *explanans | explanandum* schema may be comprehended in the following way: That α is a case is caused or determined by that β is a case. "That α is a case" is an *explanandum.* "that β is a case" is an *explanans*. An *explanans* usually indicates that some object or objects possess some properties or they are entangled in some relations. Objects being described in an *explanans* of a given conditional are called explanators. That is why in conditionals there is expressed such a content that properties or relations defined by explanators cause or determine some facts in the reality described by a given discourse.

The disposition to distinguish explanators in worlds and in their special regions belongs to the cognitive competence of language users. In philosophical discourse, one uses the special type of explanators. They possess peculiar semantic features. Philosophical explanators are usually categorized as individuals. In science, in turn, explanators are usually categorized as classes of objects. Therefore, philosophers in opposition to scientists often use proper names for designating their explanators.

2.1.6. *The set-theoretic model of lexical representations.* Let us introduce the following terminological conventions: Let T be the set of lexical elements belonging to the category of totality, D be the set of elements falling under the category of ontological dimensions, W be the set of elements comprised by the category of worlds, R be the category of ontological regions, and E be the set of philosophical explanators. Let us assume that: (1) $t_1 \in T, \dots, t_k \in T$, (2) $d_1 \in D, \dots, d_i \in D$, (3) $w_1 \in W, \dots, w_n \in W$, (4) $r_1 \in R, \dots, r_m \in R$, (5) $e_1 \in E, \dots, e_m \in E$. The sum of sets T, D, W, R, E constitutes the category of formal lexical elements of the philosophical grammar. Let the second general category of lexical elements of the philosophical grammar be the set of philosophical concepts P. Let us adopt the convention: (6) $p_1 \in P, \dots, p_z \in P$. Let 0 be the empty lexical element. This element satisfies functions similar to those satisfied by the element stop in the theory of algorithms. The following axioms establish the set-theoretic model of lexical representations.

(AL 1) $(\forall X_i,X_j)[X_i \in \{T, D, W, R, E, \{0\}, P\} \wedge X_j \in \{T, D, W, R, E, \{0\}, P\} \wedge X_i \neq X_j \rightarrow X_i \cap X_j = \varnothing]$

(AL 2) $(\forall X)[X \in \{T, D, W, R, E, \{0\}, P\} \rightarrow X \neq \varnothing]$

Let us introduce the definition.

(Df 1) $L = T \cup D \cup W \cup R \cup E \cup \{0\}$

The axioms establish the exclusiveness and nonemptiness of distinguished lexical categories.

2.2. *The level of derivational representations and its model*

Cognitive derivational representations of philosophical discourse are structures founded upon lexical representations. The level of these representations determines two types of cognitive mechanisms of creating philosophical systems, namely, mechanisms of substructuration and mechanisms of interpretation. Mechanisms of the first type participate in processes of generation of architectonic forms whereas mechanisms of the second type are responsible for processes of fulfilling architectonic forms with philosophical concepts. Hence, one must distinguish between two kinds of derivations corresponding to the above mentioned mechanisms: substructural derivations and interpretive derivations. The first class of derivations is understood as the set of operations assigning elements of formal lexical categories to finite sets of such elements of an appropriate type. Interpretive derivations are comprehended as operations transforming formal lexical elements into sets of philosophical concepts.

2.2.1. *The general notion of derivation.*

Each derivation as a syntactic object is composed of three constituents: an argument, a value, and a way of assignment of a value to an argument. It is assumed that the only two ways of assignment take part in processes of generating philosophical systems. Some of derivations appear to be invalid or needless structures in processes of generating philosophical systems. Let us adopt the following general definition of derivation.

(Df 2) $d \in \mathrm{Der} \equiv (\exists l, l_1, \ldots, l_i)[l, l_1, \ldots, l_i \in \mathrm{L} \cup \mathrm{P} \wedge (d = l \Rightarrow \{l_1, \ldots, l_i\} \vee d = l \rightarrow \{l_1, \ldots, l_i\})]$

One may distinguish some special classes of derivations.

(Df 3) $d \in \mathrm{Der}_{\mathrm{for}} \equiv (\exists l, l_1, \ldots, l_i)[l, l_1, \ldots, l_i \in \mathrm{L} \wedge (d = l \Rightarrow \{l_1, \ldots, l_i\})]$

(Df 4) $d \in \mathrm{Der}_{\mathrm{sem}} \equiv (\exists l, l_1, \ldots, l_i)[l, l_1, \ldots, l_i \in \mathrm{P} \wedge (d = l \Rightarrow \{l_1, \ldots, l_i\} \vee (d = l \rightarrow \{l_1, \ldots, l_i\})]$

$\mathrm{Der}_{\mathrm{for}}$ is the class of formal derivations founded exclusively upon formal lexical elements. $\mathrm{Der}_{\mathrm{sem}}$ is the class of semantic derivations founded upon only philosophical concepts.

Derivations are structures that may be added, intersected or substituted. Let us define some auxiliary notions. Ω is a function assigning any derivation to its argument. Ω* is a function assigning any derivation to its value and *Ω is a function attributing any derivation to its way of assignment.

(Df 5) (i) $[d = l \Rightarrow \{l_1, \ldots, l_i\}] \rightarrow \Omega(d) = l$,
(ii) $[d = l \rightarrow \{l_1, \ldots, l_i\}] \rightarrow \Omega(d) = l$

(Df 6) (i) $[d = l \Rightarrow \{l_1, \ldots, l_i\}] \rightarrow \Omega^*(d) = \{l_1, \ldots, l_i\}$,
(ii) $[d = l \rightarrow \{l_1, \ldots, l_i\}] \rightarrow \Omega^*(d) = \{l_1, \ldots, l_i\}$

(Df 7) (i) $[d = l \Rightarrow \{l_1, \ldots, l_i\}] \rightarrow {}^*\Omega(d) = \{\Rightarrow\}$,
(ii) $[d = l \rightarrow \{l_1, \ldots, l_i\}] \rightarrow {}^*\Omega(d) = \{\rightarrow\}$

The notion of the categorial identity holding between lexical elements is introduced as follows. Two lexical elements of the philosophical grammar are identical categorially if and only if they belong to the same lexical category or at least one of them is the empty lexical element. This relation is reflexive and symmetric but it is not transitive.

(Df 8) $l_1 =_{cat} l_2 \equiv \{[(l_1 \in \mathrm{T} \equiv l_2 \in \mathrm{T}) \wedge (l_1 \in \mathrm{D} \equiv l_2 \in \mathrm{D}) \wedge$
$(l_1 \in \mathrm{W} \equiv l_2 \in \mathrm{W}) \wedge (l_1 \in \mathrm{R} \equiv l_2 \in \mathrm{R}) \wedge (l_1 \in \mathrm{E} \equiv l_2 \in \mathrm{E}) \wedge$
$(l_1 \in \mathrm{P} \equiv l_2 \in \mathrm{P})] \vee l_1 \in \{0\} \vee l_2 \in \{0\}\}$

Let us define two operations: the sum of derivations and the intersection of derivations.

(Df 9) $(\forall d_1, d_2, d_3)\{d_1 \in \mathrm{Der} \wedge d_2 \in \mathrm{Der} \rightarrow$
(1) $[\Omega(d_1) \neq \Omega(d_2) \vee {}^*\Omega(d_1) \neq {}^*\Omega(d_2) \vee \sim(\forall l_1, l_2)(l_1 \in \Omega^*(d_1) \wedge l_2 \in \Omega^*(d_2)$
$\rightarrow l_1 =_{cat} l_2) \rightarrow d_1 +_{\mathrm{Der}} d_2 = \varnothing] \wedge$
(2) $[\Omega(d_1) = \Omega(d_2) \wedge {}^*\Omega(d_1) = {}^*\Omega(d_2) \wedge (\forall l_1, l_2)(l_1 \in \Omega^*(d_1) \wedge l_2 \in \Omega^*(d_2)$
$\rightarrow l_1 =_{kat} l_2) \rightarrow (d_1 +_{\mathrm{Der}} d_2 = d_3 \equiv d_3 \in \mathrm{Der} \wedge {}^*\Omega(d_3) = {}^*\Omega(d_1) \wedge \Omega^*(d_3)$
$= \Omega^*(d_1) \cup \Omega^*(d_2) \wedge \Omega(d_3) = \Omega(d_1))]\}$

(Df 10) $(\forall d_1, d_2, d_3)\{d_1 \in \mathrm{Der} \wedge d_2 \in \mathrm{Der} \rightarrow$
(1) $[\Omega(d_1) \neq \Omega(d_2) \vee {}^*\Omega(d_1) \neq {}^*\Omega(d_2) \vee \sim(\forall l_1, l_2)(l_1 \in \Omega^*(d_1) \wedge l_2 \in \Omega^*(d_2)$
$\rightarrow l_1 =_{cat} l_2) \rightarrow d_1 \bullet_{\mathrm{Der}} d_2 = \varnothing] \wedge$
(2) $[\Omega(d_1) = \Omega(d_2) \wedge {}^*\Omega(d_1) = {}^*\Omega(d_2) \wedge (\forall l_1, l_2)(l_1 \in \Omega^*(d_1) \wedge l_2 \in \Omega^*(d_2)$
$\rightarrow l_1 =_{cat} l_2) \rightarrow [d_1 \bullet_{\mathrm{Der}} d_2 = d_3 \equiv d_3 \in \mathrm{Der} \wedge \Omega(d_3) = \Omega(d_1) \wedge {}^*\Omega(d_3) =$
${}^*\Omega(d_1) \wedge (\Omega^*(d_1) \cap \Omega^*(d_2) = \varnothing \rightarrow \Omega^*(d_3) = \{0\}) \wedge (\Omega^*(d_1) \cap \Omega^*(d_2)$
$\neq \varnothing \rightarrow \Omega^*(d_3) = \Omega^*(d_1) \cap \Omega^*(d_2))]]\}$

There are cases that the sum of derivations is the empty set. If it is not a fact it will be said that the sum of derivations is effective. The intersection of two derivations is effective if it is not the empty set. It is easy to prove some theorems:

(TD 1) $(\forall d)[d \in \mathrm{Der} \wedge (\forall l_1, l_2)(l_1 \in \Omega^*(d) \wedge l_2 \in \Omega^*(d) \rightarrow l_1 =_{cat} l_2) \rightarrow$
$d +_{\mathrm{Der}} d = d]$
(TD 2) $(\forall d)[d \in \mathrm{Der} \wedge (\forall l_1, l_2)(l_1 \in \Omega^*(d) \wedge l_2 \in \Omega^*(d) \rightarrow l_1 =_{cat} l_2) \rightarrow$
$d \bullet_{\mathrm{Der}} d = d]$
(TD 3) $(\forall d_1, d_2)[d_1 \in \mathrm{Der} \wedge d_2 \in \mathrm{Der} \rightarrow d_1 \bullet_{\mathrm{Der}} d_2 = d_2 \bullet_{\mathrm{Der}} d_1]$
(TD 4) $(\forall d)[d \in \mathrm{Der} \wedge (\forall l_1, l_2)(l_1 \in \Omega^*(d) \wedge l_2 \in \Omega^*(d) \rightarrow l_1 =_{cat} l_2) \rightarrow$
$d \bullet_{\mathrm{Der}} d = d +_{\mathrm{Der}} d]$

(TD 5) $(\forall d)[d \in \mathrm{Der} \wedge \sim(\forall l_1, l_2)(l_1 \in \Omega^*(d) \wedge l_2 \in \Omega^*(d) \rightarrow l_1 =_{\mathrm{cat}} l_2) \rightarrow d +_{\mathrm{Der}} d = \varnothing]$
(TD 6) $(\forall d)[d \in \mathrm{Der} \wedge \sim(\forall l_1, l_2)(l_1 \in \Omega^*(d) \wedge l_2 \in \Omega^*(d) \rightarrow l_1 =_{\mathrm{cat}} l_2) \rightarrow d \bullet_{\mathrm{Der}} d = \varnothing]$
(TD 7) $(\forall d)[d \in \mathrm{Der} \wedge \sim(\forall l_1, l_2)(l_1 \in \Omega^*(d) \wedge l_2 \in \Omega^*(d) \rightarrow l_1 =_{\mathrm{kat}} l_2) \rightarrow d +_{\mathrm{Der}} d = d \bullet_{\mathrm{Der}} d]$
(TD 8) $(\forall d)[d \in \mathrm{Der} \rightarrow d +_{\mathrm{Der}} d = d \bullet_{\mathrm{Der}} d]$

The important operation is a substitution of a given lexical element by another lexical element in a derivation.

(Df 11) $(\forall d_1, d_2)(\forall l_i, l_k)\{d_1 \in \mathrm{Der} \wedge l_i \in \mathrm{L} \cup \mathrm{P} \wedge l_k \in \mathrm{L} \cup \mathrm{P} \rightarrow \mathrm{Subst}(d_1, l_i, l_k) = d_2 \equiv$
(1) $(l_i \neq \Omega(d_1) \wedge l_k \notin \Omega^*(d_1) \rightarrow d_2 = \varnothing) \wedge$
(2) $(l_i = \Omega(d_1) \wedge l_i \notin \Omega^*(d_1) \rightarrow d_2 \in \mathrm{Der} \wedge {}^*\Omega(d_1) = {}^*\Omega(d_2) \wedge \Omega(d_2) = l_k \wedge \Omega^*(d_1) = \Omega^*(d_2)) \wedge$
(3) $(l_i = \Omega(d_1) \wedge l_i \in \Omega^*(d_1) \rightarrow d_2 \in \mathrm{Der} \wedge {}^*\Omega(d_1) = {}^*\Omega(d_2) \wedge \Omega(d_2) = l_k \wedge \Omega^*(d_2) = (\Omega^*(d_1) - \{l_i\}) \cup \{l_k\}) \wedge$
(4) $(l_i \neq \Omega(d_1) \wedge l_i \in \Omega^*(d_1) \rightarrow d_2 \in \mathrm{Der} \wedge {}^*\Omega(d_1) = {}^*\Omega(d_2) \wedge \Omega(d_2) = \Omega(d_1) \wedge \Omega^*(d_2) = (\Omega^*(d_1) - \{l_i\}) \cup \{l_k\})$

If a result of substitution is the empty set, then the substitution will be called ineffective.

2.2.2. *Substructural derivations.* There are four types of substructural derivations. (1) derivations of the type $\mathrm{Str_t}$ that posses the form $\mathrm{t} \Rightarrow \{\mathrm{d}_1, \ldots, \mathrm{d_i}\}$, where $\mathrm{i} \geq 1$ and $\mathrm{t} \in \mathrm{T}$, $\mathrm{d}_1, \ldots, \mathrm{d_i} \in \mathrm{D}$; (2) derivations of the type $\mathrm{Str_d}$ that fall under fomrs: $\mathrm{d_i} \Rightarrow \{\mathrm{w}_1, \ldots, \mathrm{w_k}\}$; $\mathrm{d_i} \Rightarrow \{0\}$, where $\mathrm{i} \geq 1$ and $\mathrm{k} \geq 1$, $\mathrm{d_i} \in \mathrm{D}$, $\mathrm{w}_1, \ldots, \mathrm{w_k} \in \mathrm{W}$; (3) derivations of the type $\mathrm{Str_w}$ which may appear in forms: $\mathrm{w_i} \Rightarrow \{\mathrm{r}_1, \ldots, \mathrm{r_k}\}$; $\mathrm{w_i} \Rightarrow \{\mathrm{e}_1, \ldots, \mathrm{e_k}\}$; $\mathrm{w_i} \Rightarrow \{0\}$, where $\mathrm{i} \geq 1$ and $\mathrm{k} \geq 1$, $\mathrm{w_i} \in \mathrm{D}$, $\mathrm{r}_1, \ldots, \mathrm{r_k} \in \mathrm{R}$, $\mathrm{e}_1, \ldots, \mathrm{e_k} \in \mathrm{E}$; (4) derivations of the type $\mathrm{Str_r}$ falling under forms: $\mathrm{r_i} \Rightarrow \{\mathrm{e}_1, \ldots, \mathrm{e_k}\}$; $\mathrm{r_i} \Rightarrow \{0\}$, where $\mathrm{i} \geq 1$ and $\mathrm{k} \geq 1$, $\mathrm{r_i} \in \mathrm{R}$ and $\mathrm{e}_1, \ldots, \mathrm{e_k} \in \mathrm{E}$. Formal definitions of these types of derivations are as follows:

(Df 12) $d \in \mathrm{Str_t} \equiv d \in \mathrm{Der_{for}} \wedge \Omega(d) \in \mathrm{T} \wedge \Omega^*(d) \subset \mathrm{D}$
(Df 13) $d \in \mathrm{Str_d} \equiv d \in \mathrm{Der_{for}} \wedge \Omega(d) \in \mathrm{D} \wedge (\Omega^*(d) \subset \mathrm{W} \vee \Omega^*(d) = \{0\})$
(Df 14) $d \in \mathrm{Str_w} \equiv d \in \mathrm{Der_{for}} \wedge \Omega(d) \in \mathrm{W} \wedge (\Omega^*(d) \subset \mathrm{R} \vee \Omega^*(d) = \{0\} \vee \Omega^*(d) \subset \mathrm{E})$
(Df 15) $d \in \mathrm{Str_r} \equiv d \in \mathrm{Der_{for}} \wedge \Omega(d) \in \mathrm{R} \wedge (\Omega^*(d) \subset \mathrm{E} \vee \Omega^*(d) = \{0\})$

It is easy to prove many theorems that describe formal properties of substructural derivations especially in reference to operations of sum, intersection and substitution.

The class Str of all derivations is the sum of sets: $\mathrm{Str_t}$, $\mathrm{Str_d}$, $\mathrm{Str_w}$, $\mathrm{Str_r}$. All distinguished classes of substructural derivations are mutually exclusive and they are also non-empty classes.

2.2.3. *Interpretive derivations.* The class of interpretive derivations Int is defined in the following way:

(Df 16) $d \in \mathrm{Int} \equiv d \in \mathrm{Der} \wedge \Omega(d) \in \mathrm{L} - \{0\} \wedge \Omega^*(d) \subset \mathrm{P} \wedge {}^*\Omega(d) = \rightarrow$

Interpretive derivations regulate processes of filling architectonic forms with philosophical contents. There are five types of interpretive derivations with respect to their shapes: (1) $\mathrm{t} \rightarrow \{\mathrm{p}_1, \ldots, \mathrm{p}_\mathrm{i}\}$, (2) $\mathrm{d} \rightarrow \{\mathrm{p}_1, \ldots, \mathrm{p}_\mathrm{i}\}$, (3) $\mathrm{w} \rightarrow \{\mathrm{p}_1, \ldots, \mathrm{p}_\mathrm{i}\}$, (4) $\mathrm{r} \rightarrow \{\mathrm{p}_1, \ldots, \mathrm{p}_\mathrm{i}\}$, (2) $\mathrm{e} \rightarrow \{\mathrm{p}_1, \ldots, \mathrm{p}_\mathrm{i}\}$, where $\mathrm{t} \in \mathrm{T}$, $\mathrm{d} \in \mathrm{D}$, $\mathrm{w} \in \mathrm{W}$, $\mathrm{r} \in \mathrm{R}$, $\mathrm{e} \in \mathrm{E}$, $\mathrm{p}_1, \ldots, \mathrm{p}_\mathrm{i} \in \mathrm{P}$. Formal definitions of these types of derivations are omitted.

In reference to classes Der, $\mathrm{Der_{for}}$, $\mathrm{Der_{sem}}$, Str, the category of interpretive derivations Int stands in relationships described in theorems:

(TD 9) $\mathrm{Int} \cap \mathrm{Str} = \varnothing$
(TD 10) $\mathrm{Int} \cap \mathrm{Der_{for}} = \varnothing$
(TD 11) $(\forall d)[d \in \mathrm{Str} \rightarrow (\exists d_1)(d_1 \in \mathrm{Int} \wedge \Omega(d) = \Omega(d_1))]$
(TD 12) $(\forall d)(\forall l)[d \in \mathrm{Str} \wedge l \in \Omega^*(d) \wedge \Omega^*(d) \neq \{0\} \rightarrow (\exists d_1)(d_1 \in \mathrm{Int} \wedge l = \Omega(d_1))]$

If a type of derivation is represented as an ordered triple of the form ⟨a category of argument, a category of value, a way of derivation⟩ (formally: $\langle \Omega(d), \Omega^*(d), {}^*\Omega(d)\rangle$), then it is possible to distinguish following types of derivations: $\langle \mathrm{L}, \mathrm{L}, \Rightarrow\rangle$, $\langle \mathrm{L}, \mathrm{P}, \rightarrow\rangle$, $\langle \mathrm{P}, \mathrm{P}, \rightarrow\rangle$, $\langle \mathrm{L}, \mathrm{P}, \Rightarrow\rangle$, $\langle \mathrm{P}, \mathrm{L}, \Rightarrow\rangle$, $\langle \mathrm{P}, \mathrm{P}, \Rightarrow\rangle$, $\langle \mathrm{L}, \mathrm{L}, \rightarrow\rangle$. It is easy to notice that two first representations stand for $\mathrm{Der_{for}}$ and Int, respectively. The third one and the sixth one refer, in turn, to $\mathrm{Der_{sem}}$. The others designate types of derivations not participating in mechanisms of generating architectonic forms.

2.3. *The level of architectonic representations and its model*

Architectonics are cognitive representations that are transformed into philosophical systems by philosophical minds. Mechanisms of these transformations are regulated by various semantic derivations. Philosophical systems are sets of propositions in a locutionary sense, ordered by variety of semantic relations. They may be expressed in different ways, in various languages. A starting-point of the process of doing philosophy is the synthesis of an architectonic in a philosopher's mind. These structures are sets of both substructural and interpretive derivations, constructed in the appropriate way. The process of generating an architectonic courses through two phases. In the first phase, a philosophical subject selects some finite set of substructural derivations from the whole universum of them, encoded in mind. This process is regulated by algorithms determined by some peculiar logic. In the second phase, an architectonic form is interpreted with philosophical concepts. This phase consists in the selection of the appropriate sets of interpretive derivations.

2.3.1. *The axiomatics of the model of architectonics.* Architectonics are sets of substructural and interpretive derivations.

(AA 1) $(\forall a)\,[a \in \mathrm{ARCH} \rightarrow (\forall d)(d \in a \rightarrow d \in \mathrm{Str} \cup \mathrm{Int})]$

To a given architectonic there do not belong any two different substructural derivations and any two interpretive derivations, arguments of which are one and the same formal lexical element. In other words, in architectonics derivations behave like functions.

(AA 2) $(\forall a)\{a \in \mathrm{ARCH} \rightarrow (\forall d_1, d_2)[d_1 \in a \wedge d_2 \in a \wedge \Omega(d_1) = \Omega(d_2) \wedge [(d_1 \in \mathrm{Str} \wedge d_2 \in \mathrm{Str}) \vee (d_1 \in \mathrm{Int} \wedge d_2 \in \mathrm{Int})] \rightarrow d_1 = d_2]\}$

All lexical elements that are values or belong to arguments of substructural derivations constituting a given architectonic but are not identical with the empty element, are arguments of some interpretive derivations.

(AA 3) $(\forall a)\{a \in \mathrm{ARCH} \rightarrow (\forall l)[(\exists d)(d \in a \wedge d \in \mathrm{Str} \wedge (l \in \Omega^*(d) \vee l = \Omega(d)) \wedge \Omega^*(d) \neq \{0\}) \rightarrow (\exists d)(d \in a \wedge d \in \mathrm{Int} \wedge l = \Omega(d))]\}$

To any architectonic there belongs the exactly one derivation of the class $\mathrm{Str_t}$ called an initial derivation.

(AA 4) $(\forall a)[a \in \mathrm{ARCH} \rightarrow (\exists d)(d \in \mathrm{Str_t} \wedge d \in a) \wedge (\forall d_1, d_2)(d_1 \in \mathrm{Str_t} \wedge d_1 \in a \wedge d_2 \in \mathrm{Str_t} \wedge d_2 \in a \rightarrow d_1 = d_2)]$

Every architectonic consists of at least one derivation of the class $\mathrm{Str_d}$ and if a derivation of the class $\mathrm{Str_d}$ belongs to a given architectonic, then its argument is derived from the initial derivation as an element of its value.

(AA 5) $(\forall a)\{a \in \mathrm{ARCH} \rightarrow (\exists d)(d \in \mathrm{Str_d} \wedge d \in a) \wedge (\forall d)[d \in a \wedge d \in \mathrm{Str_d} \rightarrow (\exists d_i)(d_i \in a \wedge d_i \in \mathrm{Str_t} \wedge \Omega(d) \in \Omega^*(d_i))]\}$

Each dimension derived from the initial derivation is an argument of the derivation of the type $\mathrm{Str_d}$ that belongs to a given architectonic.

(AA 6) $(\forall a)\{a \in \mathrm{ARCH} \rightarrow (\forall d_1, l)[d_1 \in a \wedge d_1 \in \mathrm{Str_t} \wedge l \in \Omega^*(d_1) \wedge l \in \mathrm{D} \rightarrow (\exists d_2)(d_2 \in \mathrm{Str_d} \wedge d_2 \in a \wedge \Omega(d_2) = l)]\}$

For any pair of derivations of the type $\mathrm{Str_d}$ belonging to one and the same architectonic, it is a case that derived sets of worlds as their values do not possess a common non-empty element.

(AA 7) $(\forall a)\{a \in \mathrm{ARCH} \rightarrow (\forall d_1, d_2)[d_1 \in \mathrm{Str_d} \wedge d_1 \in a \wedge d_2 \in \mathrm{Str_d} \wedge d_2 \in a \wedge d_1 \neq d_2 \rightarrow \Omega^*(d_1) \cap \Omega^*(d_2) = \varnothing \vee \Omega^*(d_1) \cap \Omega^*(d_2) = \{0\}]\}$

Any world derived from a derivation of the class $\mathrm{Str_d}$ is an argument of the exactly one derivation of the type $\mathrm{Str_w}$.

(AA 8) $(\forall a)\{a \in \mathrm{ARCH} \rightarrow (\forall d_1, l)[d_1 \in a \wedge d_1 \in \mathrm{Str_d} \wedge l \in \Omega^*(d_1) \wedge l \in \mathrm{W} \rightarrow (\exists d_2)(d_2 \in \mathrm{Str_w} \wedge d_2 \in a \wedge \Omega(d_2) = l)]\}$

For any derivation of the class $\mathrm{Str_w}$ belonging to a given architectonic, its argument is derived from some dimension that is, in turn, an argument of a derivation of the type $\mathrm{Str_d}$ belonging to a given architectonic.

(AA 9) $(\forall a)\{a \in \mathrm{ARCH} \rightarrow (\forall d_1, l)[d_1 \in a \wedge d_1 \in \mathrm{Str_w} \wedge \Omega(d_1) = l \rightarrow (\exists d_2)(d_2 \in \mathrm{Str_d} \wedge d_2 \in a \wedge l \in \Omega^*(d_2)]$

It is not a case for any explanator to be derived from two different worlds that are, in turn, derived from different ontological dimensions.

(AA 10) $(\forall a)\{a \in \mathrm{ARCH} \rightarrow (\forall d_1, d_2, l)[d_1 \in \mathrm{Str_w} \wedge d_1 \in a \wedge d_2 \in \mathrm{Str_w} \wedge d_2 \in a \wedge d_1 \neq d_2 \wedge l \in \mathrm{E} \wedge l \in \Omega^*(d_1) \wedge l \in \Omega^*(d_2) \rightarrow (\forall d_3, d_4)(d_3 \in \mathrm{Str_d} \wedge d_3 \in a \wedge d_4 \in \mathrm{Str_d} \wedge d_4 \in a \wedge \Omega(d_1) \in \Omega^*(d_3) \wedge \Omega(d_2) \in \Omega^*(d_4) \rightarrow d_3 = d_4)]\}$

It is not excluded that some ontic region is derived within a given architectonic more than one time.

(AA 11) $(\forall a)\{a \in \mathrm{ARCH} \rightarrow (\forall d_1, l)[d_1 \in a \wedge d_1 \in \mathrm{Str_w} \wedge l \in \Omega^*(d_1) \wedge l \in \mathrm{R} \rightarrow (\exists d_2)(d_2 \in \mathrm{Str_r} \wedge d_2 \in a \wedge \Omega(d_2) = l)]\}$

If from two different regions, there are derived some sets of explanators and these two regions are derived from two different worlds entangled as elements of values in two different derivations of the type $\mathrm{Str_d}$, then these sets of explanators are exclusive.

(AA 12) $(\forall a)\{a \in \mathrm{ARCH} \rightarrow (\forall d_1, d_2)[d_1 \in a \wedge d_1 \in \mathrm{Str_r} \wedge d_2 \in a \wedge d_2 \in \mathrm{Str_r} \wedge (\exists d_3, d_4, d_5, d_6)(d_3 \in a \wedge d_3 \in \mathrm{Str_w} \wedge d_4 \in a \wedge d_4 \in \mathrm{Str_w} \wedge \Omega(d_1) \in \Omega^*(d_3) \wedge \Omega(d_2) \in \Omega^*(d_4) \wedge d_5 \in a \wedge d_5 \in \mathrm{Str_d} \wedge d_6 \in a \wedge d_6 \in \mathrm{Str_d} \wedge \Omega(d_3) \in \Omega^*(d_5) \wedge \Omega(d_4) \in \Omega^*(d_6) \wedge \Omega(d_5) \neq \Omega(d_5)) \rightarrow \Omega^*(d_1) \cap \Omega^*(d_2) = \varnothing]\}$

2.3.2. *Ways of syntactic modeling of architectonics.* Architectonic forms may be represented with help of derivational trees. The universal, full derivational tree of an architectonic form may be illustrated as in Figure 1.

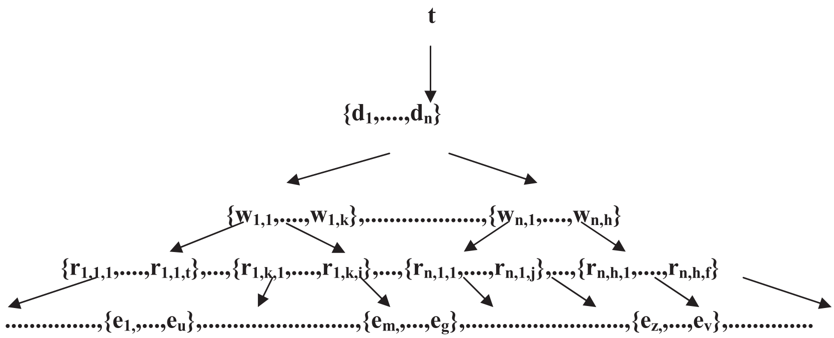

Figure 1. *The derivational tree should be read as follows: From the element **t** belonging to the category of totality there is derived a set of ontological dimensions* $\{d_1, \ldots, d_n\}$. *Subsequently, from each dimension there are derived sets of worlds. Each of these worlds is an argument of some derivations leading to sets of regions. And finally, from regions there are derived sets of explanators.*

Another way of representing architectonic forms employs the linear form of symbolism. Let a term of the form "$\alpha[\beta_1, \ldots, \beta_n]$" designate a substructural derivation $\alpha \Rightarrow \{\beta_1, \ldots, \beta_n\}$. The universal matrix that represents an architectonic form may be illustrated in such a way:

$t[d_1[w_{1,1}[r_{1,1,1}[e_1, \ldots, e_u], \ldots, r_{1,1,t}[e_m, \ldots, e_g]], \ldots, w_{1,k}[r_{1,k,1}[e_z, \ldots, e_v], \ldots, r_{1,k,i}[e_x, \ldots, e_y]]], \ldots, d_n[w_{n,1}[r_{n,1,1}[e_j, \ldots, e_p], \ldots, r_{n,1,j}[e_d, \ldots, e_q]], \ldots, w_{n,h}[r_{n,h,1}[e_c, \ldots, e_b], \ldots, r_{n,h,f}[e_o, \ldots, e_m]]]].$

If terms are subsequently introduced of the form "$\alpha(\gamma_1, \ldots, \gamma_n)$," which designates interpretive derivations of the shape $\alpha \rightarrow \{\gamma_1, \ldots, \gamma_n\}$, then the universal matrix representing an architectonic will be designated by such a term:

$t(\ldots)[d_1(\ldots)[w_{1,1}(\ldots)[r_{1,1,1}(\ldots)[e_1(\ldots), \ldots, e_u(\ldots)], \ldots, r_{1,1,t}(\ldots)[e_m(\ldots), \ldots, e_g(\ldots)]], \ldots, w_{1,k}(\ldots)[r_{1,k,1}(\ldots)[e_z(\ldots), \ldots, e_v(\ldots)], \ldots, r_{1,k,i}(\ldots)[e_x(\ldots), \ldots, e_y(\ldots)]]], \ldots, d_n(\ldots)[w_{n,1}(\ldots)[r_{n,1,1}(\ldots)[e_j(\ldots), \ldots, e_p(\ldots)], \ldots, r_{n,1,j}(\ldots)[e_d(\ldots), \ldots, e_q(\ldots)]], \ldots, w_{n,h}(\ldots)[r_{n,h,1}(\ldots)[e_c(\ldots), \ldots, e_b(\ldots)], \ldots, r_{n,h,f}(\ldots)[e_o(\ldots), \ldots, e_m(\ldots)]]]]$

Let us give the following matrix of an architectonic form as an example: $t(\ldots)[d_1(\ldots)[w_1(\ldots)[r_1(\ldots)[0],r_2(\ldots)[0]],w_2(\ldots)[e_1(\ldots)]],d_2(\ldots)[w_3(\ldots)[0],w_4(\ldots)[r_3(\ldots)[e_2(\ldots),e_3(\ldots)],r_4(\ldots)[0]]]$. By interpretation, one may receive the architectonic matrix: t(*being, truth*)[d_1(*cosmos*)[w_1(*space, time, transience, materiality*)[r_1(*quanta, electromagnetism*)[0], r_2 (*gravitation, macroscopic*)[0]], w_2(*space, time, eternity, spirituality*) [e_1(*souls*)]], d_2(*logos*)[w_3(*ideas*)[0], s_4(*God*)[r_3(*intellect*)[e_2(*logic, consistency*), e_3(*creative force*)], r_4(*volition*)[0]]]. This example of an architectonic matrix may be a transformational basis for the philosophical system consisting, for instance, of the theses:

What is being is also a truth; Cosmos exists; In cosmos there are two worlds: the material, transient world in space and time and the spiritual, eternal world also in space and time; The material world divides into two regions: a stratum of electromagnetic quanta and a stratum of macroscopic bodies influenced by gravitational effects; The spiritual and eternal world is composed of individualized souls; There exists logos consisted of two worlds: the world of ideas and the world inhabited by God; God possesses the intellect and the will; The intellect of God acts in the logical and consistent way; The will of God is creative.

2.3.3. *Relations between architectonics.* In metaphilosophical considerations it is often said that some conceptions are modified or extended versions of others. The theory of philosophical grammar enables one to define various relations holding between architectonics. They may serve as tools of explanation of different metaphilosophical facts asserted in the metaphilosophical comparative research. To define these relations it is necessary to introduce additional notions. In any architectonic, one may distinguish its form and its content. An architectonic form is a part of a given architectonic that consists of the only substructural derivations that are, in turn, composed of lexical elements constituting a lexical formal basis of a given architectonic. An architectonic content comprises only interpretive derivations.

(Df 17) $(\forall a)\{a \in \mathrm{ARCH} \rightarrow [\mathrm{FrARCH}(a) = \mathrm{x} \equiv [(\forall d)(d \in \mathrm{x} \equiv d \in a \wedge d \in \mathrm{Str}) \wedge \mathrm{x} \neq \varnothing]]\}$

(Df 18) $(\forall a)\{a \in \mathrm{ARCH} \rightarrow [\mathrm{CnARCH}(a) = \mathrm{x} \equiv [(\forall d)(d \in \mathrm{x} \equiv d \in a \wedge d \in \mathrm{Int}) \wedge \mathrm{x} \neq \varnothing]]\}$

(Df 19) $(\forall a)\{a \in \mathrm{ARCH} \rightarrow [l \in \mathrm{BasisARCH}(a) \equiv (\exists d)[d \in \mathrm{FrARCH}(a) \wedge (l = \Omega(d) \vee l \in \Omega^*(d))]\}$

It is easy to prove some theorems concerned with lexical bases and forms.

(TA 1) $(\forall a)[a \in \mathrm{ARCH} \rightarrow \mathrm{BasisARCH}(a) \neq \varnothing]$

(TA 2) $(\forall a)\{a \in \mathrm{ARCH} \rightarrow [(\forall d)(d \in \mathrm{FrARCH}(a) \rightarrow \Omega(d) \in \mathrm{BasisARCH}(a) \wedge \Omega^*(d) \subset \mathrm{BasisARCH}(a))]\}$

(TA 3) $(\forall a)(\forall b)[a \in \mathrm{ARCH} \wedge b \in \mathrm{ARCH} \rightarrow (\mathrm{FrARCH}(a) = \mathrm{FrARCH}(b) \rightarrow \mathrm{BasisARCH}(a) = \mathrm{BasisARCH}(b))]$

(TA 4) $(\exists a)(\exists b)[a \in \mathrm{ARCH} \wedge b \in \mathrm{ARCH} \wedge \mathrm{BasisARCH}(a) \neq \mathrm{BasisARCH}(b)]$

It is not difficult to notice that a given sufficiently complex lexical basis may be correlated with many architectonic forms. There are also bases from which there may be generated only one architectonic form. For instance, the basis {t, d, w, r, e} determines only one architectonic form, namely: t(. . .)[d(. . .)[w(. . .)[r(. . .)[e(. . .)]]]].

Let us define the following relations holding between architectonics: relation of being version, relation of similarity, relation of incommensurability, relation of inclusion.

(Df 20) $(\forall a,b)\{a, b \in \mathrm{ARCH} \rightarrow [a\ vers\ b \equiv \mathrm{FrARCH}(a) = \mathrm{FrARCH}(b)]\}$
(Df 21) $(\forall a,b)\{a, b \in \mathrm{ARCH} \rightarrow [a\ sim\ b \equiv \mathrm{BasisARCH}(a) = \mathrm{BasisARCH}(b)]\}$
(Df 22) $(\forall a,b)\{a, b \in \mathrm{ARCH} \rightarrow [a\ incom\ b \equiv (\mathrm{BazaARCH}(a) \cap \mathrm{BazaARCH}(b) = \varnothing \vee \mathrm{BazaARCH}(a) \cap \mathrm{BazaARCH}(b) = \{0\})]\}$
(Df 23) $a, b \in \mathrm{ARCH} \rightarrow [a \angle b \equiv (\forall d)[d \in \mathrm{FrARCH}(a) \rightarrow (\exists d_1)(d \bullet_{\mathrm{Der}} d_1 = d \wedge d_1 \in \mathrm{FrARCH}(b))]$

If the relation *vers* holds between architectonics, they must have different contents. The relation *vers* is reflexive, symmetric, and transitive in the set ARCH. Architectonics that stand in a similarity relation must be generated from one and the same lexical formal basis. The relation of similarity is also reflexive, symmetric, and transitive. Two architectonics are incommensurable if and only if the intersection of their lexical bases is the empty set or is a singleton composed of the empty element. The notion of incommensurability may be used in the explanation of facts consisting of the incomparability of philosophical systems. An architectonic *a* is grammatically included in an architectonic *b* if and only if for every substructural derivation belonging to FrARCH(*a*) there exists such a substructural derivation belonging to FrARCH(*b*) that the intersection of both derivations is identical with the first of them. The relation of grammatical inclusion is reflexive and transitive. However, it is not symmetric. It is easy to define numerous theorems concerned with defined concepts.

2.4. *The level of architectonic spaces*

In the philosophical mind, there is encoded some set of lexical formal elements that are identified by ways of their interpretation. Some of subsets of this set are lexical formal bases that determine sets of architectonic forms. And families of sets of architectonic forms that satisfy special conditions, are architectonic spaces. These sets of architectonics forms that constitute an architectonic space are called semantic locations. Each semantic location is a set of architectonic forms that are correlated with a given lexical basis. The definition of architectonic spaces requires the construction of two additional notions designating some special operations. The first of them is the operation of generating substructural derivations. It is represented in a mind by the substructural derivator. The second of them is the operation of generating semantic locations. It is represented in a mind by the derivator of semantic locations.

The model of doing philosophy may be schematically described in the following way: In philosophical mind there are encoded lexical representations by virtue of various educational processes. The activation of the process of doing philosophy stems from activation of two cognitive operators encoded in a philosophical mind, namely, the substructural derivator and the derivator of semantic locations. With help of the first one, a mind synthesizes substructural derivations and, by the use of the second one, a mind constitutes the set of semantic locations. This set is just the architectonic space that functions in a philosophical mind as the cognitive representation of an individualized philosophical tradition. Acts of philosophical practice may be, for instance, modeled as acts of selecting some special semantic location within a given architectonic space, and subsequently as acts of transformations of architectonics into philosophical systems, or as acts of passing from a starting-location to other semantic locations within a given space, or as acts of extending or narrowing an architectonic space given at input.

2.4.1. *The substructural derivator and its formal properties.* The substructural derivator Φ is a function assigning a set of substructural derivations to any set of formal lexical elements in such a way that arguments of these derivations belong to a given set of lexical elements and values of these derivations are contained in the set of lexical elements.

(Df 24) $d \in \Phi(a) \equiv d \in \mathrm{Str} \wedge \Omega(d) \in a \wedge \Omega^*(d) \subset a \wedge a \subset \mathrm{L}$

Sets of lexical elements upon which the substructural derivator Φ operates are said to induce appropriate sets of substructural derivations. The list of theorems describing basic relationships between lexical bases and the derivator Φ is as follows.

(TA 5) $(\forall a)(\forall b)[a \in \mathrm{ARCH} \wedge b \subset \mathrm{L} \rightarrow (\mathrm{BasisARCH}(a) \subset b \rightarrow \mathrm{FrARCH}(a) \subset \Phi(b))]$

(TA 6) $(\forall a)(\forall b)[a \in \mathrm{ARCH} \wedge \mathrm{FrARCH}(a) \subset \Phi(b) \rightarrow \mathrm{BasisARCH}(a) \subset b]$

(TA 7) $(\forall a)(\forall d)[a \in \mathrm{ARCH} \wedge d \in \Phi(\mathrm{BasisARCH}(a)) \rightarrow (\exists b)(b \in \mathrm{ARCH} \wedge d \in \mathrm{FrARCH}(b) \wedge \mathrm{BasisARCH}(b) \subset \mathrm{BasisARCH}(a))]$

The subsequent theorem expresses that the derivator Φ is a monotonic function.

(TA 8) $(\forall a)(\forall b)[a \subset b \wedge b \subset \mathrm{L} \rightarrow \Phi(a) \subset \Phi(b)]$

2.4.2. *The derivator of semantic locations and its formal properties.* The derivator of semantic locations λ operates upon non-empty sets of lexical, formal elements in such a way that it produces families of sets satisfying two conditions: (i) every element of this family is a set of architectonic forms that (ii) are correlated with identical lexical bases.

(Df 25) $X \in \lambda(L_i) \equiv \{L_i \subset L \wedge L_i \neq \varnothing \wedge (\forall a)[a \in X \rightarrow (\exists b)(b \in \text{ARCH} \wedge a = \text{FrARCH}(b) \wedge \text{BasisARCH}(b) \subset L_i)] \wedge (\forall a)(\forall b)(\forall c)(\forall d)[c \in \text{ARCH} \wedge d \in \text{ARCH} \wedge a = \text{FrARCH}(c) \wedge b = \text{FrARCH}(d) \wedge \text{BasisARCH}(c) = \text{BasisARCH}(d) \rightarrow (a \in X \equiv b \in X)] \wedge (\forall a)(\forall b)(\forall c)(\forall d)[c \in \text{ARCH} \wedge d \in \text{ARCH} \wedge a = \text{FrARCH}(c) \wedge b = \text{FrARCH}(d) \wedge \text{BasisARCH}(c) \neq \text{BasisARCH}(d) \rightarrow \sim(a \in X \wedge b \in X)]\}$

It is easy to notice that every architectonic form belongs to exactly one semantic location among all semantic locations induced by a given non-empty lexicon.

(TA 9) $(\forall L_i)(\forall X)(\forall Y)(\forall a)[X \in \lambda(L_i) \wedge Y \in \lambda(L_i) \wedge a \in \text{ARCH} \wedge \text{FrARCH}(a) \in X \wedge \text{FrARCH}(a) \in Y \rightarrow X = Y]$

The following two theorems describe relationships between Φ and λ.

(TA 10) $(\forall X)[X \in \lambda(L_i) \rightarrow (\forall a)(a \in X \rightarrow (\exists b)(b \in \text{ARCH} \wedge a = \text{FrARCH}(b) \wedge a \subset \Phi(L_i)))]$

(TA 11) $(\forall d)[(\exists X)(\exists a)(X \in \lambda(L_i) \wedge a \in X \wedge d \in a) \rightarrow d \in \Phi(L_i)]$

2.4.3. *Architectonic spaces.* (Df 26) An architectonic space determined by a set of lexical elements L_i, is an ordered pair $\langle L_i, \lambda(L_i)\rangle$ that satisfies the following conditions ($\langle L_i, \lambda(L_i)\rangle \in \text{SPACE}$ iff):

(1) $L_i \cap T \neq \varnothing$,
(2) $L_i \cap D \neq \varnothing$,
(3) $0 \in L_i$
(4) $(\forall a)(\forall b)[a \in \text{ARCH} \wedge b = \text{FrARCH}(a) \wedge b \subset \Phi(L_i) \rightarrow (\exists X)(b \in X \wedge X \in \lambda(L_i))]$
(5) $(\forall d)[d \in \Phi(L_i) \rightarrow (\exists X)(\exists a)(X \in \lambda(L_i) \wedge a \in X \wedge d \in a)]$
(6) $(\forall l)\{l \in L_i \rightarrow (\exists d)[(l = \Omega(d) \vee l \in \Omega^*(d)) \wedge d \in \Phi(L_i)]\}$

Conditions (1), (2), and (3) claim that at least one element belonging to the category T and at least one element belonging to the category D and also the empty element 0 are members of a lexicon constituting an architectonic space. According to condition (4), each architectonic form composed of derivations induced by the derivator Φ in application to L_i, belongs to some semantic location of the family $\lambda(L_i)$. In light of condition (5), every substructural derivation induced by derivator Φ in application to L_i, belongs to some architectonic form comprised by some semantic location of a given architectonic space. And finally on condition (6), any lexical element belonging to the lexicon of a given architectonic space must be entangled in some substructural derivation that is, in turn, an element of some architectonic form belonging to some semantic location.

2.4.4. *The derivator of architectonic forms.* The derivator of architectonic forms ∏ operates upon any set of lexical formal elements and produces some set of architectonic forms whose substructural derivations are constituted by elements belonging to a given set of lexical elements.

(Df 27) $a \in \prod(L_i) \equiv [L_i \subset L \wedge (\exists b)(b \in \mathrm{ARCH} \wedge a = \mathrm{FrARCH}(b) \wedge \mathrm{BasisARCH}(b) \subset L_i]$

The derivator ∏ is encoded in a philosophical mind and it is responsible for cognitive processes of the synthesis of architectonics. But it does not act in isolation. It cooperates with other derivators: λ and Φ. Therefore, formal interconnections, especially between ∏ and λ, may represent some explanatory factors for metaphilosophical facts concerned with the influence of the philosophical tradition on individual processes of philosophizing.

3. The logic of philosophical transformations and transitions

The LTTF-logic is a system of algorithms regulating cognitive processes of generating and activating architectonic forms in various semantic locations of a given space. It may be said that the LTTF-logic is the grammatical mechanism of functioning the derivator ∏ of architectonic forms. On the ground of the LTTF-logic there may be formulated LTTF-theories of architectonic spaces encoded in philosophical minds.

3.1. *The language and the inferential mechanism of the LTTF-logic*

The language L_{LTTF} consists of three syntactic levels. The first level is composed of expressions designating lexical formal elements. The second one comprises expressions designating substructural derivations, and the last level is consisted of expressions designating grammatical transitions whose counterparts in formal standard logic are inferences. The set of transitions is divided into valid and invalid ones. The LTTF-logic is the set of valid transitions that may be proved on its ground. In many cases, arguments of these transitions are architectonic forms.

3.1.1. *The LTTF-syntax.* The primitive symbols of the language L_{LTTF} are as follows:

(1) individual constants and variables of the category T: Lowercase Roman letters of the form t, $t_1, \ldots, t_n$ represent constants whereas italicized letters of the form $l_t, l_{t,1}, \ldots, l_{t,n}, l_{t,i}$ represent variables; (2) individual constants and variables of the category D: Lowercase Roman letters of the form d, $d_1, \ldots, d_n$ represent constants whereas italicized letters of the form $l_d, l_{d,1}, \ldots, l_{d,n}, l_{d,i}$ represent variables; (3) individual constants and variables of the category W. Lowercase Roman letters of the shapes: w, $w_1, \ldots, w_n$ represent constants whereas italicized letters of shapes: $l_w, l_{w,1}, \ldots, l_{w,n}, l_{w,i}$ represent variables; (4) individual constants and variables of the category R. Lowercase Roman letters of the shapes: r, $r_1, \ldots, r_n$ represent constants whereas italicized letters of shapes: $l_r, l_{r,1}, \ldots, l_{r,n}, l_{r,i}$ represent variables; (5) individual constants and variables of the category E. Lowercase Roman letters of the shapes: e, $e_1, \ldots, e_n$ represent constants whereas italicized letters of shapes: $l_e, l_{e,1}, \ldots, l_{e,n}, l_{e,i}$ represent variables; (6) the constant designating the empty lexical element 0; (7) variables ranging over the set of substructural derivations: $d_1, \ldots, d_n, d_i, \ldots, d_j$ $h_1, \ldots, h_n, \ldots, h_{t+i}, \ldots, h_{t+j}$. These symbols belong to the syntactical category Str. (8) the constant designating transition-operation //, which forms expressions of the category Tr and whose arguments belong to the category Str; (9) the constant designating derivation-operation [. . .], which forms expressions of the category Str and whose arguments belong to appropriate lexical categories.

Rules of syntax for the language L_{LTTF} are expressed in the following formulas:

(1) $\alpha_1[\beta_1, \ldots, \beta_n]$ is a well-formed expression belonging to the category Str if and only if at least one of the following conditions is satisfied: (a) α_1 is a constant or a variable of the category T and $\beta_1, \ldots, \beta_n$ are constants or variables of the category D; (b) α_1 is a constant or a variable of the category D and $\beta_1, \ldots, \beta_n$ are constants or variables of the category W; (c) n = 1 and α_1 is a constant or a variable of the category D and β_n is the constant designating the empty element 0; (d) α_1 is a constant or a variable of the category W and $\beta_1, \ldots, \beta_n$ are constants or variables of the category R; (e) α_1 is a constant or a variable of the category W and $\beta_1, \ldots, \beta_n$ are constants or variables of the category E; (f) n = 1 and α_1 is a constant or a variable of the category W and β_n is the constant designating the empty element 0; (g) $\alpha_{1\ 1}$ is a constant or a variable of the category R and $\beta_1, \ldots, \beta_n$ are constants or variables of the category E; (h) n = 1 and α_1 is a constant or a variable of the category R and β_n is the constant designating the empty element 0; (2) The expression of the shape α // β is a well-formed formula belonging to the category Tr if and only if α and β belong to the category Str or α belongs to the category T and β belong to the category Str; (3) The expression of the shape $\alpha_1, \ldots, \alpha_j$ // $\beta_1, \ldots, \beta_n$ is

a well-formed formula of L_{LTTF} belonging to the category Tr if and only if $\alpha_1, \ldots, \alpha_j$ and $\beta_1, \ldots, \beta_n$ belong to the category Str.

3.1.2. *Rules of inference.* The list of primitive rules of inference may be grouped in the following way:

(1) The rule of starting: (Start) $l_t \,//\, l_t[l_d]$. This rule allows for inferring a substructural derivation of the category Str_t from any element belonging to T.

(2) The rule of multiplication: (Mult) $\alpha[\beta_1, \ldots, \beta_n] \,//\, \alpha[\beta_1, \ldots, \beta_n, \beta_{n+1}]$. This rule is responsible for enriching architectonic forms with elements of various ontological categories. Its particular versions are:

(Mult$_{\mathbf{t}}$): $l_t[l_{d,1}, \ldots, l_{d,n}] \,//\, l_t[l_{d,1}, \ldots, l_{d,n}, l_{d,n+1}]$
(Mult$_{\mathbf{d}}$): $l_d[l_{w,1}, \ldots, l_{w,n}] \,//\, l_d[l_{w,1}, \ldots, l_{w,n}, l_{w,n+1}]$
(Mult$_{\mathbf{w,r}}$) $l_w[l_{r,1}, \ldots, l_{r,n}] \,//\, l_w[l_{r,1}, \ldots, l_{r,n}, l_{r,n+1}]$
(Mult$_{\mathbf{w,e}}$) $l_w[l_{e,1}, \ldots, l_{e,n}] \,//\, l_w[l_{e,1}, \ldots, l_{e,n}, l_{e,n+1}]$
(Mult$_{\mathbf{r}}$) $l_r[l_{e,1}, \ldots, l_{e,n}] \,//\, l_r[l_{e,1}, \ldots, l_{e,n}, l_{e,n+1}]$

(3) The rule of elimination: (Elim) $\alpha[\beta_1, \ldots, \beta_n] \,//\, \alpha[\beta_1, \ldots, \beta_{n-1}]$. This rule reduces number of formal lexical elements in architectonics. Its particular versions are:

(Elim$_{\mathbf{t}}$) $l_t[l_{d,1}, \ldots, l_{d,n}] \,//\, l_t[l_{d,1}, \ldots, l_{d,n-1}]$
(Elim$_{\mathbf{d}}$) $l_d[l_{w,1}, \ldots, l_{w,n}] \,//\, l_d[l_{w,1}, \ldots, l_{w,n-1}]$
(Elim$_{\mathbf{r}}$) $l_r[l_{e,1}, \ldots, l_{e,n}] \,//\, l_r[l_{e,1}, \ldots, l_{e,n-1}]$
(Elim$_{\mathbf{w,r}}$) $l_w[l_{r,1}, \ldots, l_{r,n}] \,//\, l_w[l_{r,1}, \ldots, l_{r,n-1}]$
(Elim$_{\mathbf{w,e}}$) $l_w[l_{e,1}, \ldots, l_{e,n}] \,//\, l_w[l_{e,1}, \ldots, l_{e,n-1}]$

(4) The rule of introduction: (Intr) $\alpha[0] \,//\, \alpha[\beta]$. This rule introduces elements of new categories into an architectonic form. Here are its particular versions:

(Intr$_{\mathrm{d}}$) $l_d[0] \,//\, l_d[l_{w,1}]$
(Intr$_{\mathrm{w,r}}$) $l_w[0] \,//\, l_w[l_{r,1}]$
(Intr$_{\mathrm{w,e}}$) $l_w[0] \,//\, l_s[l_{w,1}]$
(Intr$_{\mathrm{r}}$) $l_r[0] \,//\, l_r[l_{e,1}]$

(5) The rule of destruction: (Dest) $\alpha[\beta] \,//\, \alpha[0]$. This rule enables us to eliminate all elements of a given category from an architectonic. Its particular versions are:

(Dest$_{\mathrm{d}}$) $l_d[l_{w,1}] \,//\, l[0]$
(Dest$_{\mathrm{w,r}}$) $l_w[l_{r,1}] \,//\, l[0]$
(Dest$_{\mathrm{w,e}}$) $l_w[l_{e,1}] \,//\, l[0]$
(Dest$_{\mathrm{r}}$) $l_r[l_{e,1}] \,//\, l[0]$

(6) Rules of transition, which enable the inference of the substructural derivations of an appropriate type from derivations of another type.

(Trans$_d$) $l_t[l_d]$ // $l_d[0]$ (Trans$_w$) $l_d[l_w]$ // $l_w[0]$
(Trans$_r$) $l_w[l_r]$ // $l_r[0]$

(7) The rule of an architectonic synthesis: (Synt) $\xi\{\ldots\alpha[\beta_1, \ldots, \beta_i, \ldots, \beta_n]\ldots\}, \beta_i[\gamma_1, \ldots, \gamma_k]$ // $\xi\{\ldots\alpha[\beta_1, \ldots, \beta_i[\gamma_1, \ldots, \gamma_k], \ldots, \beta_n]\ldots\}$, where the sign '$\xi\{\ldots\ldots\}$' represents any derivational context. This rule allows one to build architectonic matrices from substructural derivations.

3.2. *LTTF-provability*

The construction of the notion of LTTF-provability requires defining some auxiliary concepts: Mult, Elim, Intr, Dest, Start, Trans. These concepts refer to special types of transitions.

(Df 28) $t \in \text{Mult}_t \equiv (\exists l_t, l_{d,1}, \ldots, l_{d,n}, l_{d,n+1})[t = l_t[l_{d,1}, \ldots, l_{d,n}]$ // $l_t[l_{d,1}, \ldots, l_{d,n}, l_{d,n+1}]]$
(Df 29) $t \in \text{Mult}_d \equiv (\exists l_d, l_{w,1}, \ldots, l_{w,n}, l_{w,n+1})[t = l_d[l_{w,1}, \ldots, l_{w,n}]$ // $l_d[l_{w,1}, \ldots, l_{w,n}, l_{w,n+1}]]$
(Df 30) $t \in \text{Mult}_{w,r} \equiv (\exists l_w, l_{r,1}, \ldots, l_{r,n}, l_{r,n+1})[t = l_w[l_{r,1}, \ldots, l_{r,n}]$ // $l_w[l_{r,1}, \ldots, l_{r,n}, l_{r,n+1}]]$
(Df 31) $t \in \text{Mult}_{w,e} \equiv (\exists l_w, l_{e,1}, \ldots, l_{e,n}, l_{e,n+1})[t = l_w[l_{e,1}, \ldots, l_{e,n}]$ // $l_w[l_{e,1}, \ldots, l_{e,n}, l_{e,n+1}]]$
(Df 32) $t \in \text{Mult}_r \equiv (\exists l_r, l_{e,1}, \ldots, l_{e,n}, l_{e,n+1})[t = l_r[l_{e,1}, \ldots, l_{e,n}]$ // $l_r[l_{e,1}, \ldots, l_{e,n}, l_{e,n+1}]]$
(Df 33) $\text{Mult} = \text{Mult}_t \cup \text{Mult}_d \cup \text{Mult}_{w,r} \cup \text{Mult}_{w,e} \cup \text{Mult}_r$
(Df 34) $t \in \text{Elim}_t \equiv (\exists l_t, l_{d,1}, \ldots, l_{d,n}, l_{d,n+1})[t = l_t[l_{d,1}, \ldots, l_{d,n}, l_{d,n+1}]$ // $l_t[l_{d,1}, \ldots, l_{d,n}]]$
(Df 35) $t \in \text{Elim}_d \equiv (\exists l_d, l_{w,1}, \ldots, l_{w,n}, l_{w,n+1})[t = l_d[l_{w,1}, \ldots, l_{w,n}, l_{w,n+1}]$ // $l_d[l_{w,1}, \ldots, l_{w,n}]]$
(Df 36) $t \in \text{Elim}_{w,r} \equiv (\exists l_w, l_{r,1}, \ldots, l_{r,n}, l_{r,n+1})[t = l_w[l_{r,1}, \ldots, l_{r,n}, l_{r,n+1}]$ // $l_w[l_{r,1}, \ldots, l_{r,n}]]$
(Df 37) $t \in \text{Elim}_{w,e} \equiv (\exists l_w, l_{e,1}, \ldots, l_{e,n}, l_{e,n+1})[t = l_w[l_{e,1}, \ldots, l_{e,n}, l_{e,n+1}]$ // $l_w[l_{e,1}, \ldots, l_{e,n}]]$
(Df 38) $t \in \text{Elim}_r \equiv (\exists l_r, l_{e,1}, \ldots, l_{e,n}, l_{e,n+1})[t = l_r[l_{e,1}, \ldots, l_{e,n}, l_{e,n+1}]$ // $l_r[l_{e,1}, \ldots, l_{e,n}]]$
(Df 39) $\text{Elim} = \text{Elim}_t \cup \text{Elim}_d \cup \text{Elim}_{w,r} \cup \text{Elim}_{w,e} \cup \text{Elim}_r$
(Df 40) $t \in \text{Intr}_d \equiv (\exists l_w, l_{s,1})\ t = l_d[0]$ // $l_d[l_{w,1}]$
(Df 41) $t \in \text{Intr}_{w,r} \equiv (\exists l_w, l_{r,1})\ t = l_w[0]$ // $l_w[l_{r,1}]$
(Df 42) $t \in \text{Intr}_{w,e} \equiv (\exists l_w, l_{e,1})\ t = l_w[0]$ // $l_w[l_{e,1}]$

(Df 43) $t \in \mathrm{Intr}_\mathrm{r} \equiv (\exists l_r, l_{e,1})\ t = l_r[0]\ //\ l_r[l_{e,1}]$
(Df 44) $\mathrm{Intr} = \mathrm{Intr}_\mathrm{d} \cup \mathrm{Intr}_{\mathrm{w,r}} \cup \mathrm{Intr}_{\mathrm{w,e}} \cup \mathrm{Intr}_\mathrm{r}$
(Df 45) $t \in \mathrm{Dest}_\mathrm{d} \equiv (\exists l_d, l_{w,1})\ t = l_d[l_{w,1}]\ //\ l[0]$
(Df 46) $t \in \mathrm{Dest}_{\mathrm{w,r}} \equiv (\exists l_w, l_{r,1})\ t = l_w[l_{r,1}]\ //\ l[0]$
(Df 47) $t \in \mathrm{Dest}_{\mathrm{w,e}} \equiv (\exists l_w, l_{e,1})\ t = l_w[l_{e,1}]\ //\ l[0]$
(Df 48) $t \in \mathrm{Dest}_\mathrm{r} \equiv (\exists l_r, l_{e,1})\ t = l_r[l_{e,1}]\ //\ l[0]$
(Df 49) $\mathrm{Dest} = \mathrm{Dest}_\mathrm{d} \cup \mathrm{Dest}_{\mathrm{w,r}} \cup \mathrm{Dest}_{\mathrm{w,e}} \cup \mathrm{Dest}_\mathrm{r}$
(Df 50) $t \in \mathrm{Start} \equiv (\exists l_t, l_d)\ \{t = l_t\ //\ l_t[l_d]\}$
(Df 51) $t \in \mathrm{Trans}_\mathrm{d} \equiv (\exists l_t, l_d)\ \{t = l_t[l_d]\ //\ l_d[0]\}$
(Df 52) $t \in \mathrm{Trans}_\mathrm{w} \equiv (\exists l_d, l_w)\ \{t = l_d[l_w]\ //\ l_w[0]\}$
(Df 53) $t \in \mathrm{Trans}_\mathrm{r} \equiv (\exists l_w, l_r)\ \{t = l_w[l_r]\ //\ l_r[0]\}$
(Df 54) $t \in \mathrm{Trans} \equiv t \in \mathrm{Trans}_\mathrm{d} \cup \mathrm{Trans}_\mathrm{w} \cup \mathrm{Trans}_\mathrm{r}$

The definition of LTTF-provability is formulated in two steps. In the first one, there is expressed the condition for transitions whose arguments are lists of derivations and in the second one there is formulated the condition for transitions whose arguments are lists with a lexical element belonging to the category of totality T.

(Df 55)

(1) LTTF $\vdash$ ‘$d_1, \ldots, d_\mathrm{n}\ //\ d_\mathrm{i}, \ldots, d_\mathrm{j}$’ $\equiv (\exists\langle h_1, \ldots, h_\mathrm{n}, \ldots, h_{\mathrm{t+i}}, \ldots, h_{\mathrm{t+j}}\rangle)[h_1 = d_1 \wedge \cdots \wedge h_\mathrm{n} = d_\mathrm{n} \wedge (\forall \mathrm{k}, \mathrm{t+j} \geq \mathrm{k} > \mathrm{n})(\exists \mathrm{m}, \mathrm{m} < \mathrm{k})(h_\mathrm{m}\ //\ h_\mathrm{k} \in \mathrm{Mult} \cup \mathrm{Elim} \cup \mathrm{Intr} \cup \mathrm{Dest} \cup \mathrm{Trans}) \wedge h_{\mathrm{t+i}} = d_\mathrm{i} \wedge \cdots \wedge h_{\mathrm{t+j}} = d_\mathrm{j}]$

(2) LTTF $\vdash$ ‘$l_t, d_1, \ldots, d_\mathrm{n}\ //\ d_\mathrm{i}, \ldots, d_\mathrm{j}$’ $\equiv (\exists\langle h_0, h_1, \ldots, h_\mathrm{n}, \ldots, h_{\mathrm{t+i}}, \ldots, h_{\mathrm{t+j}}\rangle)[h_0 = l_t \wedge h_1 = d_1 \wedge \cdots \wedge h_\mathrm{n} = d_\mathrm{n} \wedge (\forall \mathrm{k}, \mathrm{t+j} \geq \mathrm{k} > \mathrm{n})(\exists \mathrm{m}, \mathrm{m} < \mathrm{k})(h_\mathrm{m}\ //\ h_\mathrm{k} \in \mathrm{Mult} \cup \mathrm{Elim} \cup \mathrm{Intr} \cup \mathrm{Dest} \cup \mathrm{Start} \cup \mathrm{Trans}) \wedge h_{\mathrm{t+i}} = d_\mathrm{i} \wedge \cdots \wedge h_{\mathrm{t+j}} = d_\mathrm{j}]$

In light of (Df 55), transitions are provable expressions of the LTTF-language. However, it should be noted that proofs are sequences of derivations and proved expressions belong to different syntactical category. They are transitions. If some transition is proved, then it will be said that a consequent (a value) of a given transition is provable on ground of an antecedent (an argument) of it.

3.2.1. *Instances of proofs.*

(1) The rule of the form $\{l_{t,1}[l_{d,2}, l_{d,3}], l_{t,4}[l_{d,5}, l_{d,6}]\}\ //\ \{l_{t,1}[l_{d,2}, l_{d,5}], l_{t,4}[l_{d,3}, l_{d,6}]\}$

1. $l_{t,1}[l_{d,2}, l_{d,3}]$ assumption
2. $l_{t,4}[l_{d,5}, l_{d,6}]$ assumption

3. $l_{t,1}[l_{d,2}]$ Elim$_{\mathbf{t}}$: 1
4. $l_{t,4}[l_{d,6}]$ Elim$_{\mathbf{t}}$: 2
5. $l_{t,1}[l_{d,2}, l_{d,5}]$ Mult$_{\mathbf{t}}$: 3
6. $l_{t,4}[l_{d,3}, l_{d,6}]$ Mult$_{\mathbf{t}}$: 4

Let us prove the architectonic form $t[d_1[w_1[0]], d_2[w_2[0], w_3[0]], d_3[0]]$ on ground of the lexical element t:

(1)	t	
(2)	$t[d_1]$	(Start: 1)
(3)	$t[d_1, d_2]$	(Mult$_t$: 2)
(4)	$t[d_1, d_2, d_3]$	(Mult$_t$: 3)
(5)	$t[d_2]$	(Start: 1)
(6)	$t[d_3]$	(Start: 1)
(7)	$d_1[0]$	(Trans$_d$: 2)
(8)	$d_1[w_1]$	(Intr$_d$: 7)
(9)	$d_2[0]$	(Trans$_d$: 5)
(10)	$d_2[w_2]$	(Intr$_d$: 9)
(11)	$d_2[w_2, w_3]$	(Mult$_d$: 10)
(12)	$d_2[w_3]$	(Elim$_d$: 11)
(13)	$w_2[0]$	(Trans$_w$: 10)
(14)	$w_3[0]$	(Trans$_w$: 12)
(15)	$w_1[0]$	(Trans$_w$: 8)
(16)	$d_3[0]$	(Trans$_t$: 6)
(17)	$t[d_1[w_1], d_2, d_3]$	(Syn: 4, 8)
(18)	$t[d_1[w_1], d_2[w_2], d_3]$	(Syn: 17, 10)
(19)	$t[d_1[w_1], d_2[w_2, w_3], d_3[0]]$	(Syn: 18, 16)
(20)	$t[d_1[w_1[0]], d_2[w_2, w_3], d_3[0]]$	(Syn: 19, 15)
(21)	$t[d_1[w_1[0]], d_2[w_2[0], w_3], d_3[0]]$	(Syn: 20, 13)
(22)	$t[d_1[w_1[0]], d_2[w_2[0], w_3[0]], d_3[0]]$	(Syn: 21, 14)

Proof-lines from (17) to (22) constitute the algorithm generating the architectonic matrix.

3.2.2. *Formal properties of LTTF-provability.* The following theorems express some important formal properties of LTTF-provability in the set of substructural derivations. According to (LTTF 1), LTTF-provability is monotonic. (LTTF 2) and (LTTF 3) express that LTTF-provability is closed under the sum of arguments and values of the functor //. In light of (LTTF 4) and (LTTF 5), LTTF-provability is reflexive. (LTTF 6) expresses in turn that LTTF-provability is transitive.

(LTTF 1) $(\forall d_i, \ldots, d_n, d_{n+1}, d_j)$[LTTF $\vdash$ '$d_i, \ldots, d_n \,//\, d_j$' $\rightarrow$ LTTF $\vdash$ '$d_i, \ldots, d_n, d_{n+1} \,//\, d_j$']

(LTTF 2) $(\forall d_i, \ldots, d_n, d_i, d_j)$[LTTF $\vdash$ '$d_i, \ldots, d_n \,//\, d_i$' $\wedge$ LTTF $\vdash$ '$d_i, \ldots, d_n \,//\, d_j$' $\rightarrow$ LTTF $\vdash$ '$d_i, \ldots, d_n \,//\, d_i, d_j$']

(LTTF 3) $(\forall d_1, \ldots, d_n, d_i, \ldots, d_j, d_k, \ldots, d_h, d_f, \ldots, d_z)$[LTTF $\vdash$ '$d_i, \ldots, d_n \,//\, d_i, \ldots, d_j$' $\wedge$ LTTF $\vdash$ '$d_k, \ldots, d_h \,//\, d_f, \ldots, d_z$' $\rightarrow$ LTTF $\vdash$ '$d_i, \ldots, d_n, d_k, \ldots, d_h \,//\, d_i, \ldots, d_j, d_f, \ldots, d_z$']

(LTTF 4) $(\forall d)$ LTTF $\vdash$ '$d \,//\, d$'

(LTTF 5) $(\forall d_1, \ldots, d_n)$ LTTF $\vdash$ '$d_1, \ldots, d_n \,//\, d_1, \ldots, d_n$'

(LTTF 6) $(\forall d_1, \ldots, d_n, d_i, \ldots, d_j, d_f, \ldots, d_z)$ [LTTF $\vdash$ '$d_i, \ldots, d_n \,//\, d_i, \ldots, d_j$' $\wedge$ LTTF $\vdash$ '$d_i, \ldots, d_j \,//\, d_f, \ldots, d_z$' $\rightarrow$ LTTF $\vdash$ '$d_i, \ldots, d_n \,//\, d_f, \ldots, d_z$']

It is easy to notice that every substructural derivation may be inferred from some element belonging to the category T. Let us define the function **.

(Df 56) (i) $(\forall X)[\sim X \subset T \rightarrow (X)^{**} = \varnothing]$
(ii) $(\forall X)\{X \subset T \rightarrow [d \in (X)^{**} \equiv (\exists l_t)(l_t \in X \wedge$ LTTF $\vdash$ '$l_t \,//\, d$')]\}

It is obvious that the category Str is identical with the set of values of the function **.

(LTTF 7) Str = (T)**

(LTTF 8) Φ(L) = (T)**

According to (LTTF 8), the category of totality T may be treated as the generator of the universum of all substructural derivations.

3.3. *The semantic model of LTTF-logic*

The universum of structures, called philosophical transitions, is the model of LTTF-logic. It is possible to prove that all transitions are provable in LTTF-logic and everything that is provable in this logic is a transition. In this way, one may formulate theories of such objects on ground of the LTTF-logic. Such theories may be used in researching historical grammatical processes of the evolution of philosophical systems.

3.3.1. *The construction of the universum of philosophical transitions.* First, let us define the category of elementary transformations e-TR founded on substructural derivations exclusively:

(Df 57) $(d_1 \,//\, d_2) \in \text{e-TR} \equiv [d_1 \in \text{Str} \wedge d_2 \in \text{Str} \wedge \Omega(d_1) = \Omega(d_2) \wedge (\Omega^*(d_1) = \{0\} \vee \Omega^*(d_2) = \{0\} \vee \Omega^*(d_1) \subset \Omega^*(d_2) \vee \Omega^*(d_2) \subset \Omega^*(d_1))]$

Subsequently, let us define the category of secondary transformations s-TR founded in some special way on e-TR:

(Df 58) $(d_1, \ldots, d_n \,//\, d_i, \ldots, d_j) \in \text{s-TR} \equiv [d_1, \ldots, d_n, d_i, \ldots, d_j \in \text{Str} \wedge (\exists\langle h_1, \ldots, h_n, \ldots, h_{t+i}, \ldots, h_{t+j}\rangle)\, [h_1 = d_1 \wedge \cdots \wedge h_n = d_n \wedge (\forall k, t + j \geq k > n)(\exists m, m < k)(h_m \,//\, h_k \in \text{e-TR}) \wedge h_{t+i} = d_i \wedge \cdots \wedge h_{t+j} = d_j]]$

The sum of e-TR and s-TR is called the category of transformations TR.
Now, let us define two categories of transitions: s-Trans and t-Trans.

(Df 59) $(d_1, \ldots, d_n \,//\, d_i, \ldots, d_j) \in \text{s-Trans} \equiv [d_1, \ldots, d_n, d_i, \ldots, d_j \in \text{Str} \wedge (\exists\langle h_1, \ldots, h_n, \ldots, h_{t+i}, \ldots, h_{t+j}\rangle)[h_1 = d_1 \wedge \cdots \wedge h_n = d_n \wedge (\forall k, t + j \geq k > n)(\exists m, m < k)(h_m \,//\, h_k \in \text{TR} \cup \text{Trans}) \wedge h_{t+i} = d_i \wedge \cdots \wedge h_{t+j} = d_j]]$

(Df 60) $(l_t, d_1, \ldots, d_n \,//\, d_i, \ldots, d_j) \in \text{t-Trans} \equiv [d_1, \ldots, d_n, d_i, \ldots, d_j \in \text{Str} \wedge l_t \in \text{C} \wedge (\exists\langle h_0, h_1, \ldots, h_n, \ldots, h_{t+i}, \ldots, h_{t+j}\rangle)[h_0 = l_c \wedge h_1 = d_1 \wedge \cdots \wedge h_n = d_n \wedge (\forall k, t + j \geq k > n)(\exists m, m < k)\, (h_m \,//\, h_k \in \text{TR} \cup \text{Trans} \cup \text{Start}) \wedge h_{t+i} = d_i \wedge \cdots \wedge h_{t+j} = d_j]]$

The set of all transitions Trans is the sum s-Trans ∪ t-Trans.
The following theorems are obvious:

(LTTF 9) $\text{TR} \subset \text{Trans}$
(LTTF 10) $\text{TR} \cap \text{t-Trans} = \varnothing$
(LTTF 11) $\text{TR} \subset \text{s-Trans}$
(LTTF 12) $\text{e-Trans} \subset \text{s-Trans}$
(LTTF 13) $\text{Start} \subset \text{t-Trans}$

It is possible to prove the semantic completeness of the LTTF-logic in the universum of transitions Trans

(LTTF 14) $(\forall d_1, \ldots, d_n, d_i, \ldots, d_j)[(d_1, \ldots, d_n \,//\, d_i, \ldots, d_j) \in \text{s-Trans} \rightarrow \text{LTTF} \vdash \text{‘}d_1, \ldots, d_n \,//\, d_i, \ldots, d_j\text{’}]$
(LTTF 15) $(\forall l_t, d_1, \ldots, d_n, d_i, \ldots, d_j)[(l_t, d_1, \ldots, d_n \,//\, d_i, \ldots, d_j) \in \text{t-Trans} \rightarrow \text{LTTF} \vdash \text{‘}l_t, d_1, \ldots, d_n \,//\, d_i, \ldots, d_j\text{’}]$
(LTTF 16) $(\forall d_1, \ldots, d_n, d_i, \ldots, d_j)[\text{LTTF} \vdash \text{‘}d_1, \ldots, d_n \,//\, d_i, \ldots, d_j\text{’} \rightarrow (d_1, \ldots, d_n \,//\, d_i, \ldots, d_j) \in \text{s-Trans}]$

(LTTF 17) $(\forall l_t, d_1, \ldots, d_n, d_i, \ldots, d_j)[\text{LTTF} \vdash `l_t, d_1, \ldots, d_n \,//\, d_i, \ldots, d_j\text{'} \rightarrow (l_t, d_1, \ldots, d_n \,//\, d_i, \ldots, d_j) \in \text{t-Trans}]$

(LTTF 18) $(\forall t)[t \in \text{Trans} \equiv \text{LTTF} \vdash `t\text{'}]$

(LTTF 18) expresses the semantic completeness of the LTTF-logic.

3.4. *Formal properties of transitions*

In opposition to transitions t-Trans, transitions belonging to the category s-Trans are reflexive and transitive.

(LTTF 19) $(\forall d_1, \ldots, d_n)[d_1, \ldots, d_n \in \text{Str} \rightarrow (d_1, \ldots, d_n \,//\, d_1, \ldots, d_n) \in \text{s-Trans}]$

(LTTF 20) $(\forall d_1, \ldots, d_n, d_i, \ldots, d_j, d_k, \ldots, d_h)[(d_1, \ldots, d_n \,//\, d_i, \ldots, d_j) \in \text{s-Trans} \wedge (d_i, \ldots, d_j \,//\, d_k, \ldots, d_h) \in \text{s-Trans} \rightarrow (d_1, \ldots, d_n \,//\, d_k, \ldots, d_h) \in \text{s-Trans}]]$

(LTTF 21) $(\forall l_t, d_1, \ldots, d_n, d_i, \ldots, d_j, d_k, \ldots, d_h)[(t_c, d_1, \ldots, d_n \,//\, d_i, \ldots, d_j) \in \text{t-Trans} \wedge (d_i, \ldots, d_j \,//\, d_k, \ldots, d_h) \in \text{s-Trans} \rightarrow (l_t, d_1, \ldots, d_n \,//\, d_k, \ldots, d_h) \in \text{t-Trans}]]$

The list of important theorems for the whole class of transitions is as follows:

(LTTF 22) $(\forall D_1, D_2)\{(\exists d)[D_1 \,//\, d \notin \text{Trans} \wedge d \in D_2] \rightarrow D_1 \,//\, D_2 \notin \text{Trans}\}$

(LTTF 23) $(\forall d_1, D)\{D \,//\, d_1 \in \text{Trans} \rightarrow (\exists d)[d \in D \wedge d \,//\, d_1 \in \text{Trans}]\}$

According to (LTTF 22), if some derivation is not inferable from some set of derivations, then any set to which this inferable derivation belongs is not inferable from the set at input. The theorem (LTTF 23) expresses that if some derivation is inferable from some set of derivations, it is also inferable from some single derivation belonging to the set at input.

Another group of theorems determines sufficient conditions of being a transition.

(LTTF 24) $(\forall d_1, d_2)[d_1 \in \text{Str}_t \wedge d_2 \in \text{Str}_d \rightarrow (d_1 \,//\, d_2) \in \text{Trans}]$

(LTTF 25) $(\forall d_1, d_2)[d_1 \in \text{Str}_t \wedge d_2 \in \text{Str}_w \rightarrow (d_1 \,//\, d_2) \in \text{Trans}]$

(LTTF 26) $(\forall d_1, d_2)[d_1 \in \text{Str}_t \wedge d_2 \in \text{Str}_r \rightarrow (d_1 \,//\, d_2) \in \text{Trans}]$

(LTTF 27) $(\forall d_1, d_2)[d_1 \in \text{Str}_d \wedge d_2 \in \text{Str}_w \rightarrow (d_1 \,//\, d_2) \in \text{Trans}]$

(LTTF 28) $(\forall d_1, d_2)[d_1 \in \text{Str}_d \wedge d_2 \in \text{Str}_r \rightarrow (d_1 \,//\, d_2) \in \text{Trans}]$

(LTTF 29) $(\forall d_1, d_2)[d_1 \in \text{Str}_w \wedge d_2 \in \text{Str}_r \rightarrow (d_1 \,//\, d_2) \in \text{Trans}]$

(LTTF 30) $(\forall d_1, d_2)[d_1 \in \text{Str}_t \wedge d_2 \in \text{Str}_t \wedge \Omega(d_1) = \Omega(d_2) \rightarrow (d_1 \,//\, d_2) \in \text{Trans}]$

(LTTF 31) $(\forall l_c, d)[l_c \in \text{T} \wedge d \in \text{Str}_t \wedge \Omega(d) = l_t \rightarrow (l_t \,//\, d) \in \text{Trans}]$

Since in architectonic forms derivations are attributed to some derivational level in accordance with the hierarchy determined by the order $\langle \text{Str}_t, \text{Str}_d, \text{Str}_w, \text{Str}_r \rangle$, so it may be said that the above-mentioned theorems express that each derivation from the lower level in a given architectonic is inferable from some derivation of the higher level. That is why it is possible to prove the theorem according to which two architectonic forms possessing an identical top are inferable from each other.

(LTTF 32) $(\forall a, b)[a \in \text{ARCH} \wedge b \in \text{ARCH} \wedge \text{BasisARCH}(a) \cap \text{BasisARCH}(b) \cap \text{T} \neq \varnothing \rightarrow (\text{FrARCH}(a) \,//\, \text{FrARCH}(b)) \in \text{Trans} \wedge (\text{FrARCH}(b) \,//\, \text{FrARCH}(a)) \in \text{Trans}]$

It is also evident that incommensurable architectonics cannot be inferred each other.

(LTTF 33) $(\forall a, b)[a \in \text{ARCH} \wedge b \in \text{ARCH} \wedge a \textit{ incom } b \rightarrow (\text{FrARCH}(a) \,//\, \text{FrARCH}(b)) \notin \text{Trans} \wedge (\text{FrARCH}(b) \,//\, \text{FrARCH}(a)) \notin \text{Trans}]$

Other theorems express that it is impossible to infer any derivation in a given architectonic form from a derivation of some lower level in this architectonic form. This theoretic fact may be used in explaining the intuition that the philosophical enterprise is not an inductive cognitive activity. The process of the creation of a philosophical system starts from some universal or general visions and sometimes reaches detailed conceptions.

4. Conclusion

The formal theory outlined above may be applied not only to philosophical systems but also to various ideological or religious systems or even to literary worlds of fiction. Its explanatory range of application comprises all texts of human culture. If the theory will be verified in all mentioned areas, it will mean that the whole of culture created by mankind possesses a generative character. At the end it should be also indicated that the proposed formal construction may be rebuilt in various ways. Architectonic forms may be constructed as structures with fractalized substructures. Moreover, on the interpretive level of any architectonic, it is possible to construct semantic rules of coherence of an architectonic. Furthermore, if semantic locations in architectonic spaces are comprehended as possible worlds, then it is easy to define the relation of accessibility between semantic locations. The presented theory enables one to build the theory of world-lines as developmental paths of the history of philosophy. These final notes show the explanatory force and power of inventing new theoretic tools of the proposed theory.

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