TYPES OF CONCEPT FUZZINESS* [1]

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Abstract: The short exposition of the triplet model of concepts and some definitions connected with it are given. In this model any concept may be depicted as having three characteristics: a base, a representing part and the linkage between them. The paper introduces the fuzzification of concepts in terms of the triplet model.

Keywords: Approximate reasoning; fuzzy sets; measure of fuzziness; psychology.

Introduction

It is well known that one of the sources of introducing the notion of a fuzzy set has been connected with modelling common concepts like **HIGH, OLD, YOUNG,** etc. [18]. On one hand,

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* The draft. The final version, see: **Types of Concept Fuzziness // *Fuzzy Sets and Systems*, 1998, 96, 2: 129-138**

[1] The authors recommend to select the definite concept that a reader possesses (or thinks to possess) and to try analysing it with ideas and definitions exposed in this paper. We would be very grateful for receiving the information on counter-examples to the triplet model.

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since that time many promising formalisations of this notion appeared [10; 12]. Moreover, a theory of named sets has been intensively developed [2]. In its view all known notions of fuzzy sets have been treated as specific partial cases of the notion of a named set [5]. On the other hand, there is the substantial progress in modelling concepts by means of fuzzy set theories [11; 13] and the theory of named sets [4]. Contemporary psychology and neurobiology also extent essentially previous models of concepts [17]. It seems that concept modelling may give some new ideas about the aspects of concept's fuzziness.

The triplet modelling of concepts

Although in a sense concepts are elementary units of knowledge, cognition and psychics, they are not formless entities. Viewing the available information on concepts and their models, one may come to the justified conclusion that concepts are very complicated and versatile entities. The results of current concept analysis do not permit one to be certain and final in his or her knowledge of concepts. There are now only reasonably detailed, formal and adequate models of a concept. These depict in various ways different properties and structures of concepts. The triplet model [13; 14; 15] unites and develops further various concept characteristics that were introduced by other models.

According to the triplet model, any concept may be described by means of three characteristics. The first characteristic is a base of a concept, the second one is a representing part of a concept, and the third one is a linkage between the base and the representing part. It should be noted that other concept models deal only partially with these characteristics.

The base of a concept

To introduce the definition of the concept base we need the following explications.

Let *U* be the set of all conceivable entities. These may have physical existence (things, events, processes, states) or mental existence (thoughts, ideas, abstractions, notions). The set *U* also contains properties of these entities, relations between entities, relations between properties, properties of relations, properties of properties, etc.

It should be noted that the discrimination between entities and their properties and relations is a relative procedure. In some contexts the selected entity may be treated as a thing with properties, and in others as a property of another thing. For example, according to classical physics and the standard interpretation of the term “property," the mass has such a property as “to be positive." At the same time, the mass is a physical magnitude, that is, a property of material bodies. With this in mind we need a rather flexible understanding of the set *U*.

There are also delicate distinctions concerning the relationships between concepts and entities falling (subsumed) under concepts, between concept names and names of entities falling under concepts, etc.

To avoid undesirable associations from here on we shall use, if necessary, capital bold symbols, letters, words, word combinations for denoting concepts. Instances of the concept denotation are **C, ATOM, ATOM OF HYDROGEN, PLANET, EARTH, ANIMAL, ELECTRON** on so forth. Entities that are falling under a concept will be denoted as **c, atoms, atoms of hydrogen, planets, earth, animals, electrons**. Correspondingly, the names of a concept might be **“C”, “ATOM”, “ATOM OF HYDROGEN”, “PLANET”, “EARTH”, “ANIMAL”, “ELECTRON.”** The names of the entities that are subsumed under a concept might be “**c”, “atom”, “atom of hydrogen”, “planet”, “earth”, “animal”, “electron”**.

The names of a concept, on occasion, may be used interchangeably with the names of entities subsumed under a concept. In these cases, for instance, **“C”** and **“c”** are to be identified. But this identification does not hold for all cases.

A concept **C** has, as a rule, many names of the kind *N*(**C**). The same is true for the entities falling under a concept. The names in question differ in their exactness, effectiveness, simplicity and so on. There are many relationships between the names of the “same” concept as well of the names of the “same” entity falling under a concept.

In this paper concepts will be considered as complicated mental instruments for thinking about elements from *U*. It is a matter of fact that by means of a given concept **C** one might think of only about a specific element or subset of *U*.

Moreover, any thought process with the help of a concept **C** takes place in some conditions **K**. Aside from describing these conditions in detail, we mention only that they have been associated with individual’s mental abilities, skills and tools, available knowledge, purposes, and even psychic state.

Bearing these distinctions and conventions in mind, we introduce

Definition 1. The ground set of the concept **C** is a set $G_{\mathbf{K}}(\mathbf{C}) \subseteq U$ that includes all elements $g \in G_{\mathbf{K}}(\mathbf{C})$ to that the name $N_{\mathbf{K}}(\mathbf{C})$ of the concept **C** refers to and about those it is possible in conditions **K** to think of by means of the concept **C**.

Elements $g \in G_{\mathbf{K}}(\mathbf{C})$ is said to fall or subsume under the concept **C**. In cognitive science and psychology these elements are also called "instances" or "exemplars" of a concept.

Let us give several illustrations associated with concepts **PLANET** and **ATOM**. For simplicity's sake we suppose some minimal level of knowledge that should have any individual graduated from a high school. Thus, we omit the reference to the conditions **K**.

The ground set *G*(**PLANET**) of the concept **PLANET** consists of the large bodies in space that move around a star. The ground set *G*(**PLANET OF THE SOLAR SYSTEM**) of the concept **PLANET OF THE SOLAR SYSTEM** consists of **planets** that move around the Sun. The members of this set are **Mercury**, **Venus**, **Earth**, etc. One of the names of the concept **PLANET** is "**PLANET"** and one of the general names of planets is "**planet**" and examples of individual names are "**Mercury**", "**Venus**", "**Earth**".

The ground set *G*(**ATOM**) of the concept **ATOM** consists of the tiny **particles** that are supposed to be constituents of macroscopic bodies. The ground set *G*(**ATOM OF HYDROGEN**) of the concept **ATOM OF HYDROGEN** is a subset of *G*(**ATOM**). Its members are described as the simplest **atoms**.

Under the traditional logical treatment one usually uses the terms "extension" or "volume" for denoting the ground set of a concept. The term "category" is in use in cognitive science and psychology. However, the association an extension with a concept is only a first step in concept modelling.

Indeed, the person's possessing of the concept **C** presupposes also that by means of **C** he or she is able to indicate and describe, at least, qualitatively some properties and relations of elements from $G_{\mathbf{K}}(\mathbf{C})$. This means that such properties and relations are important features of person's concept possession. These properties and relations are a principal part of the concept use in ordinary thinking. Besides this, the usage of scientific concepts has presupposed quantitative descriptions of some properties and relations of elements from $G_{\mathbf{K}}(\mathbf{C})$ and their values, the establishing correlation between properties under consideration, etc. As a rule, the set of some properties in question is called concept “intension” or “content”. Cognitive scientists and psychologists also separate different kinds of such properties: a prototype and a core. A prototype is a set of properties that are assumed to occur in some instances. A core is a set of properties that are singly necessary and jointly sufficient for membership of an entity in the concept’s category.

In our examples, one of the properties of **planets** is their ***size*** and two of the relations between **planets** are their ***mutual motion*** and ***distance***. One of the properties of **atoms** is their ***atomic number*** and one of the relations between **atoms** is their ***electromagnetic interaction***. It must be noted that only partially the information about these properties and relations belongs to the common knowledge.

Thus, there is a need of depicting in precise terms that person’s concept holding is associated with the information, on one hand, on the concept ground set and, on the other hand, on some properties and relations of elements and subsets from the ground set. One way to do this is to use the construction of some generalisation of a set scale $S(X)$ [3].

The set scale is built step by step through the application in definite order operations of set union, set product and constructing power-set to the basis X of the set scale $S(X)$. The basis is a collection of sets $X_1, X_2, ..., X_n$. On each step in constructing a set scale one obtains its definite level consisting of some sets. The set scale $S(X)$ is the union of all its levels.

Levels of a set scale may be associated with properties of elements from the ground set; relations between these elements; relations between elements and values of properties, etc. This is important for concept modelling.

In what follows we model properties and relations in question as abstract properties [1; 6]. An abstract property $\boldsymbol{P}$ is a triple (D, p, L) where D is a set of names of entities which may possess the property in question, L is a scale of the property, and p is a partial function assigning the element(s) $p(d) = l \in L$ to an entity with the name $d \in D$.

For example, the property of physical bodies that is usually called “***mass***” may be modelled as an abstract property in the following manner. Here D is the set of names (general, particular, symbolic, etc.) of all physical bodies existing in the world, L is the set of real numbers, and p is realised by means of some procedure of measuring the value of mass for a given body.

There is a variety of different abstract property scales and consequently a variety of property models. The choice is depended on our knowledge of a property and our goals in its modelling. In our example of the concept **PLANET**, the scale of the planet property ***size*** may be linguistic like {*little, medium, large, very large*}. Usage of such a scale is typical for common, non-scientific concepts. When a property is investigated scientifically, usually its scale is some set of mathematical constructions. For example, the usage of the property scale like the set R of real numbers is rather typical for many scientific concepts, at least, for those dealing with scalar physical magnitudes or quantities. The scale of the atom property ***atomic number*** is the set of all natural numbers. Note, that he relation between entities is modelled as an abstract property with D, consisting of pairs of names of entities in question.

In the case of triplet concept modelling, the basis X of the corresponding concept set scale necessarily includes the ground set G ($G = X_1$). It is very important, that by means of selecting the appropriate basis it is possible to depict precisely terms properties and relations of every order [6]. For this purpose along with the ground set G, the basis should include auxiliary sets that are scales of properties and relations of elements from G. Examples of auxiliary sets are real numbers, vector spaces, truth values, etc.

Definition 2. The real (in relation to the conditions **K**) base $B_{\mathbf{K}}(\mathbf{C})$ of the concept **C** includes elements of $G_{\mathbf{K}}(\mathbf{C})$ and no their properties and relations other than needed for the usage of **C** in conditions **K**. These properties and relations are modelled by means of subsets from finite number of levels of the set scale $S(G^*)$ with the basis $G^* = \{G_{\mathbf{K}}, X_2, ..., X_n\}$, where $X_2, ..., X_n$ are auxiliary sets.

One may also speak about the ideal concept base that is equal to the concept set scale. It seems plausible to assume that for the vast majority of concepts at any moment of their "life" their bases are subsets of the corresponding concept set scales, that is, they are real bases. One may use the distinction of ideal and real concept bases for describing some features of concept development.

Thus, the first triplet characteristic of a concept -- the base $B_{\mathbf{K}}(\mathbf{C})$ of a concept **C** is described with the corresponding set scale and its levels, the basis of set scale, the ground and auxiliary sets of the basis.

Elements from the bases of many common concepts are given to psychics by the help of sense perception and processing of sensory information by the brain activity. It is also necessary to consider the previous experience and available knowledge on the world. Elements from many scientific concept bases are investigated by means of special devices and equipment and conducting observations, measurements and experiments. In these cases consciousness has the immediate access only to graphic, numeric or verbal descriptions of outcomes of such procedures.

The representing part of a concept

Apparently thinking by means of a concept on its base, a person does not directly operate with elements from the concept ground set and properties and relations of these elements. He or she does operate with some their mental representations. The existence of such representations in person's mental sphere (mind, psychics, memory, consciousness, etc.) is a necessary condition for person's possession of a concept. Thus, any realistic concept model should take into account this fundamental, and usually neglected, fact. Without the loss of generality we may speak of only about the linguistic form of mental representations. Here language is understood in a very broad sense. The second triplet characteristic of a concept -- its representing part -- contains instances of this linguistic form.

Let us assume that we use some language L with the alphabet A, the vocabulary V, the set P of word combinations, the set E of expressions (sentences) and the set T of texts. The basis L^* of set scale $S(L^*)$ of language L is $\{A, V, P, E, T\}$. The language L may include sublanguages (sign, pictorial, natural, artificial, common, scientific, mathematical, etc.).

Definition 3. The representing part $R_{\mathbf{K}}(\mathbf{C}) \subseteq S(L^*)$ of the concept **C** is a set of linguistic units and structures by means of those the base $B_{\mathbf{K}}(\mathbf{C})$ of a concept **C** is depicted (mapped, represented) under conditions **K** in person's mental sphere.

For example, the representing part of the physical concept **MASS** contains the following elements: symbol m (the element of A); word "mass" (the element of V); "physical quantity" (the element of P); "mass is property of macroscopic bodies", "m is scalar positive function", "m = F/a" (the elements of E); "one of the fundamental properties of matter, i.e. all matter possesses mass; it is that property of a piece of matter which causes it to be attracted to any other piece of matter by gravitational force. ... The mass of an object also measures its inertia, i.e., the resistance to a force applied to set it in motion or change its motions” (the element of *T*) [9: 12].

The representing part of the common concept **PLANET OF THE SOLAR SYSTEM** includes the word “**planet**”, some, usually not all, names of planets in the Solar system, may be, word combinations like “bright moving star” and so on. For ancient Greeks this part also included the images and names of Greek Gods. The representing part of the scientific concept **PLANET OF THE SOLAR SYSTEM** includes various linguistic units and expressions of substantial body of knowledge from astronomy, celestial mechanics, planet physics, mathematical theories, etc. It also contains the visual images (photographs) of planets obtained by means of telescopes and interplanetary cosmic stations. The representing part of pre-scientific concept **ATOM** contains an image of small, indivisible pieces of matter. The representing part of its scientific counterpart includes quantum-mechanical wave functions, various theoretical models of **atoms**, schematic pictures of electron orbitals, etc.

Components of the concept representing part differ in their representative and expressive capacity. Some of them only denotate the base as a whole, its selected subsets, and its individual elements. Other baptise properties and relations of elements from the ground set. The third group of components gives more or less complete and/or exact descriptions of elements from the base or even their properties and relations. The fourth group models properties and relations in question.

There are closed and non-trivial links between different kinds of elements from the representing part of scientific concepts. Moreover, these elements are intimately connected to empirical and theoretical knowledge systems and classifications available in the corresponding science. In this sense the representing part of scientific concepts is knowledge dependent. For the sake of simplicity we will not here touch this point.

The linkage of a concept

Figuratively speaking, the entities (from the ground set and base) which are conceivable by means of a given concept do not bear components of its representing part. These components are human creations. As such, these are dependent on developmental levels of civilisation, culture, language, science, person’s knowledge, purposes and mental capacities. In a sense, these are conditional and ephemeral. Thus, there is a need of more careful characterisation of links between elements and structures from the concept base and components and structures from the concept representing part.

Let us point out only some aspects of links under consideration.

There are many ways of their establishing: by custom, by training, by language acquisition, by convention, by analogy, by procedure, etc. From the point of view of concept functions the usage of five letters “**earth**” for denoting **Earth** is accidental. Ukrainians and Russians use quite another set of five letters (“**çåìëÿ**”) while Germans use four letters (“**Erde**”). At the same time there are universal scientific procedures for finding values of such a property of **atoms** as ***atomic number*** for any given atom. The same is true for finding values of such a property of **planets** as ***size*** for any planet. The accuracy and exactness of these procedures may change eventually.

The almost commonly accepted approach to links between components of concept representing part and elements from the concept base treats these links as naming relations. The former components play the role of names and the latter elements play the role of entities baptised by the appropriate former components. However, naming relations that assign names to entities are a specific kind of these links. For example, if the representing part of a concept contains some abstract model of a property from the concept base, then this model not only names the property but also in principle should give the information about the values of this property and even about relationships between this and other properties.

Without going into details, one may separate various kinds of links between the components and their sets from the representing part and the elements and subsets from the base. Among these are reference links (naming, denotating, decripting, visualing, imagering), truth links, and modelling links.

From what has been said it might be assumed that the characteristic of links in question is a very important part of any reasonable concept model. To our best knowledge the triplet model is the first one that explicitly recognises and essentially uses this part.

Definition 4. The third triplet characteristic $L_{\mathbf{K}}(\mathbf{C})$ of a concept **C** is the system of links (linkage) between the base $B_{\mathbf{K}}(\mathbf{C})$ and the representing part $R_{\mathbf{K}}(\mathbf{C})$.

It is of fundamental importance, that for any concept this linkage is the outcome of very complex (sensual, perceptual, mental, scientific, etc.) activity. One usually does not realise the existence and importance of this activity while he or she operates with concepts.

For example, for the common concept **PLANET OF THE SOLAR SYSTEM** the linkage in question has been established by means of sensual perception in a course of visual observation by means of eyes and telescopes. For the synonymous scientific concept the construction of such a linkage is realised in the framework of the available scientific knowledge and connected with conducting observations and measurements with the help of various kinds of telescopes, satellite observation, spectroscopic measurements, etc.

It is supposed that **atoms** are unobservable entities. If it is true, then for different versions of scientific concept **ATOM** its linkage cannot be principally established by means of procedures of direct observation and ostensive indication. This linkage is constructed by means of measurement and application of appropriate knowledge systems (theory of measurement, atomic theory, quantum mechanics). This means that some links from the linkage of the concept **ATOM** are realised through processes of abstraction, idealisation, modelling, calculation, approximation and so forth.

For many scientific concepts there is a possibility of controlling the linkage between their bases and representing parts. In particular, the measurement and calculation procedures permit one to attribute quite specific linguistic and mathematical (numeric, vector, etc.) values to definite properties and relations of entities from the ground set of a concept. It should be noted that the concept linkage is transforming with the changes in scientific equipment, methods of its use and available scientific theories.

For instance, the physical concepts **MASS** and **TEMPERATURE** permit scientists to assign (by means of measuring devices and available knowledge systems) quite definite values of mass and average surface temperature to any planet in the Solar system. Only one hundred years ago many scientists believed that physicists never determine the surface temperature of the Sun. The same was true for experimental determination of values of many properties of **atoms**.

The triplet model of a concept

In the light of the discussion above it is apparent that the reliable concept model should take into account all three characteristics of concepts. Without any of them one may speak of only about incomplete concept modelling. Certainly, there are many successful applications of various incomplete concept models. However, the complete concept models give more profound and deep insight into concepts.

From stated above one may obtain

Definition 5. Under conditions **K** the triplet model $T_{\mathbf{K}}(\mathbf{C})$ of the concept **C** is the triple ($B_{\mathbf{K}}(\mathbf{C})$, $L_{\mathbf{K}}(\mathbf{C})$, $R_{\mathbf{K}}(\mathbf{C})$), where $B_{\mathbf{K}}(\mathbf{C})$ is the base of **C**, $R_{\mathbf{K}}(\mathbf{C})$ is the representing part of **C,** and $L_{\mathbf{K}}(\mathbf{C})$ is the linkage between $B_{\mathbf{K}}(\mathbf{C})$ and $R_{\mathbf{K}}(\mathbf{C})$.

So called non-fuzzy or sharp concepts are most studied in the logic, psychology, cognitive science, and artificial intelligence. If we consider ordinary (non-fuzzy) sets as sets for which the membership function takes only two values 0 or 1, they may be defined as follows.

Definition 6. A concept is a sharp if its triplet characteristics include only ordinary sets.

We would like to stress again the relative nature of these and other definitions connected with concepts. The treatment of a concept as sharp or fuzzy depends not only on concept itself, but also on one's approach to it.

It is possible to demonstrate without going into details that each of different concept models proposed in the literature depicts only partially and specifically the triplet characteristics of concepts. In this sense these models are partial cases of the triplet model.

For instance, the extensional classical and fuzzy models consider only the base of a concept. The representational classical and fuzzy models depict explicitly only the representing part of a concept [7].

According to the concept specialisation model concepts have been seen as having an asymmetric structure [16]. It may be shown that one component of this structure is some version of the concept representing part. Other is some description of the concept base connected, in particular, with values of properties and relations of elements that make up the base.

None of models mentioned depicts the linkage between the base and representing part of a concept, taking it as obvious and non-problematic. These models do not also isolate rather complex component and structural composition of the concept base and representing part.

Classical and fuzzy models differ in the usage of, correspondingly, ordinary and fuzzy set theories for the description of components of a concept.

Triplet fuzzification of a concept

One of the advantages of the triplet concept model is its flexibility. It may be adjusted to modelling not only "sharp", but also fuzzy concepts. Moreover, the fuzzification of triplet model gives the natural classification of fuzzy concepts. For this we will use the construction of the fuzzy dichotomous subset [6].

Let X be some set, s is a partial function and S is a scale.

Definition 7. The triple $X^* = (X, s, S)$ is called a fuzzy dichotomous subset of X, if its scale S is isomorphic to the interval [0,1] with the usual order relation. The substitution of X by X^* is called the (s,S)-fuzzification of X. The function s may be treated as a membership function of the fuzzy dichotomous set X^*.

There are many concrete types of (s, S)-fuzzification of a given set X according to the definition of the partial function s and the scale S.

Correspondingly, there are many ways of fuzzification of the triplet model of a concept. One of the simplest ways is the following. We may consider the concept base (or the concept representing part) as some fuzzy dichotomous subset of the set U (or $S(L^*)$). Here the set U (or $S(L^*)$) plays the role of X.

Let us introduce $U^* = (U, s, S)$ and $F^* = (S(L^*), f, F)$ where s and f are partial functions, and S and F are scales isomorphic to the interval [0,1].

Bearing *definitions 5* and *7* in mind, one may introduce

Definition 8. The concept $\mathbf{C^*} = (B^*_{\mathbf{K}}(\mathbf{C}), L^*_{\mathbf{K}}(\mathbf{C}), R^*_{\mathbf{K}}(\mathbf{C}))$ is called:

(1) (s, S)-fuzzy, if $B^*_{\mathbf{K}}(\mathbf{C}) = (U, s, S)$;

(2) fuzzy-(f, F), if $R^*_{\mathbf{K}}(\mathbf{C}) = (S(L^*), f, F)$;

(3) (s, S)-fuzzy-(f, F), if it is simultaneously (s, S)-fuzzy and fuzzy-(f, F).

Let **C** be a sharp concept which triplet model is ($B_{\mathbf{K}}(\mathbf{C})$, $L_{\mathbf{K}}(\mathbf{C})$, $R_{\mathbf{K}}(\mathbf{C})$), $X \subseteq B_{\mathbf{K}}(\mathbf{C})$, $Y \subseteq R_{\mathbf{K}}(\mathbf{C})$, $X^* = (X, s, S)$ and $Y^* = (Y, f, F)$.

We may take the set X (Y) and construct its appropriate fuzzy dichotomous set X^* (Y^*). Then we change the set X (Y) by the set X^* (Y^*) to the base (the representing part) of the concept **C**. As a result of this substitution we obtain the concept **C*.** Such a procedure depends on the nature of X (Y) and X^* (Y^*).

Definition 9. The concept **C*** is called:

(1) X^*-fuzzification of **C**, if $X^* = (X, s, S)$ is a fuzzy dichotomous subset of some subset X of $B_{\mathbf{K}}(\mathbf{C})$;

(2) fuzzification-Y^* of **C**, if $Y^* = (Y, f, F)$ is a fuzzy dichotomous subset of some subset Y of $R_{\mathbf{K}}(\mathbf{C})$;

(3) X^*-fuzzification-Y^* of **C**, if it is simultaneously X^*-fuzzification of **C** and fuzzification-Y^* of **C**.

The typology of concept fuzzification arising from *definition 9* is very extensive. It rests on various fuzzifications of relevant subsets of the base and the representing part of sharp concepts. One of its fragment is given in [13].

Below we will speak of about fuzzy concepts as some appropriate fuzzification of sharp concepts.

Some examples of fuzzy concepts

From what has been said follows that the consideration of any concrete concept as sharp or fuzzy is a relative procedure.

Firstly, the modelling of a concrete concept should include the reference to some conditions **K**. These conditions are connected with the usage of the concept in specific historical time and circumstances by a particular person or the group of persons. Also the aims and mental tools of a concept holder must be mentioned.

Secondly, the result of concept modelling strongly depends upon the objective information connected with the triplet characteristics of the concept in the moment of its modelling.

Thirdly, a concept model also depends upon the degree of comprehension of this information by the expert who models the concept.

At last, the model of a concept is also determined by the goals, knowledge, capabilities and possibilities of an expert who models a concept.

Let us consider several simple triplet treatments of concept fuzziness. Closer examination needs of much more space than available in this paper.

The concept **PLANET OF THE SOLAR SYSTEM** is usually considered as sharp in the sense that only nine heavenly bodies are members of its ground set *G*. Other bodies do not belong to this set. In our terminology it means that this concept is sharp in relation to *G*. However, it is only a first approximation to the modelling of this concept.

It is well known that there is a belt of asteroids, all revolving in their orbits around the Sun. The size of some of them is compatible with the size of **Mercury**. Sometimes asteroids are called "small planets". Thus, more adequate treatment should take into account this fact. It would be better to consider the ground set *G* of the concept in question as some fuzzy dichotomous subset (*U*, *s*, [0,1]) of the set *U* of all heavenly bodies revolving around the Sun. The appropriate partial function *s* assigns specific values of "planetness" to every member of *U* and takes values from the interval [0,1]. The values of *s* for the well known nine massive bodies (**Earth,** etc.) are equal to 1. For most asteroids such values are very small. However, some of them are large enough and corresponding values may be introduced, for example, as fraction between the value of their mass and the value of Mercury's mass.

The situation with modelling this concept becomes more intricate if one takes in consideration some irregularities in the Solar system. There are hypotheses that explain them by introducing the tenth planet. Depending on plausibility of such hypotheses, one should assign some non-zero value of membership of the hypothetical tenth planet to the ground set *G* of the concept **PLANET OF THE SOLAR SYSTEM**. This means again that more realistically is to treat *G* as some fuzzy dichotomous subset (*U*, *s*, [0,1]) of the set *U* of all bodies possibly revolving around the Sun. Correspondingly, the concept in question is becoming (*s*, [0,1])-fuzzy.

According to the common view, one important feature of planets is their sequence in relation to the Sun. It is usually supposed that **Mercury** is the nearest to the Sun and **Pluto** is the most remote. Let the ordered set X = {**Mercury**, ..., **Neptune**, **Pluto**} represents planet sequence for the case of the sharp concept **PLANET OF THE SOLAR SYSTEM**. The set X is a subset of its base.

However, some interval of time **Pluto** (due to the large eccentricity of its orbit) is closer to the Sun than **Neptune**. To depict this fact we may construct some fuzzy dichotomous subset X^* of X. The set X^* may have the form of (X, s, [0,1]) where the function s assigns to any planet some number from the interval [0,1]. Such a number may be interpreted as the measure of possibility for a planet to take quite definite place in planet sequence. The values of this function are equal to 1 for all planets except **Neptune** and **Pluto**. However, for these planets such values are smaller than 1. One might assigns definite values in dependence on his or her understanding of planet sequence. Possible variants are to take as the appropriate measure the fraction of the mean distance between the planet and the Sun to the "length" of the Solar system or to take the fraction of the interval of time while **Pluto** is closer to the Sun than **Neptune** to the period of **Pluto**.

Even the concept **ATOM** reveals some types of X-fuzziness under closer examination. One may consider such a property of **atoms** as the ***stability***. In a first approximation this property has two linguistic values {*stable, unstable*}. But it would be more realistically to treat the interval [0,1] as the scale of this property and to model it as an abstract property P of the form (A, t, [0,1]). Here A is the set of all **atoms**, and the function t assigns to each atom the fraction of the value of its half-decay time to the life time of the Universe. This means that the concept in question is P-fuzzy.

The concepts mentioned above also reveal their fuzziness-Y. It may be associated with the existence of different knowledge systems about **planets** and **atoms**. Let us show this for the concept **PLANET OF THE SOLAR SYSTEM**.

Such linguistic and theoretical units named as "celestial body", "material point", "system of material points", "**Venus**", etc. are commonly used in its representing part. In view of the current scientific knowledge some of such units are more suitable than others. For example, in the framework of classical mechanics the available data about planet orbits is better described by using "system of material points" but not "material point" or "celestial body". There are also physical descriptions of the Solar system in terms of quantum mechanics. In this case the representing part of the concept

PLANET OF THE SOLAR SYSTEM includes the highly abstract units like “wave function”, “Hilbert space”, etc. Depending on situations there are different values of appropriateness of the usage of these components. This means that a closer approximation to this concept should treat it as fuzzy-*Y*.

It is also possible to introduce other types of concept fuzziness. In particular, the linkage of a concept may be realised by means of some fuzzy operation, that is, operation that associates, for example, definite element and some fuzzy set. Any measurement process may be analysed as some fuzzy operation. If one connects with the concept **PLANET OF THE SOLAR SYSTEM** the knowledge of values of mass of planets, he or she inevitably deals with fuzziness associated with its linkage.

Perspectives

So called object concepts were discussed above. Their ground sets consist of entities supposed to be objects, things, elements, etc. Such a consideration may be extended to any other kind of concepts like property concepts (**CHARGE, BEAUTIFUL**), process concepts (**MOTION, TRANSITION**), relation concepts (**FORCE, EXCHANGE**), mental state concepts (**THOUGHT, INTENSION**) and so on.

The triplet model of a concept also may be analysed in general and precise terms from the theory of named sets [2]. Moreover, concepts with the fuzzy base and representing part may be described with named sets of second order. All this opens the way of the exact description of such notions as **CONCEPT FORMATION, CONCEPT TRANSFORMATION, COMPLEXITY OF CONCEPT, LINK OF CONCEPTS, SYSTEM OF CONCEPTS**, of various relations among concepts as “**TO BE MORE GENERAL”, “TO BE COMPOSITION OF**”, etc. and the formulation of exact propositions about the various properties of concepts and relations among them (including relations among fuzzy concepts and non-fuzzy concepts).

The modelling of concepts proposed may be applied in the study of approximate reasoning, acquisition and learning of knowledge in artificial intelligence, cognitive science, and cognitive psychology, where it is important to have realistic and manageable concept models.

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