



## Real quantitiveness: what formal investigations can(not) show

J. E. Wolff: *The metaphysics of quantity*. New York: Oxford University Press, 2020. 240 pp, \$72.00 HB

Derek Lam<sup>1</sup>

Accepted: 22 December 2021

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The mathematicalization of nature is the hallmark of modern science. Given analytic philosophers' deferential attitude toward the sciences, one would have thought that discussions about quantities would be at the forefront of the field. Nothing can be further from the truth. Wolff's book is not the first monograph on the subject. But it is the first that is devoted to offering a comprehensive treatment of this subject. It is a magnificent book that both presents a unified picture of the state of play and defends a version of structuralism about quantities.

What makes an attribute a quantity? Answering this question requires not just any inquiry into the metaphysics of quantity, but a *metaphysics of quantitiveness*. Wolff uses this question to tie the literature together. The book can be divided into four parts.

**Chapters 1 to 4** survey two contrasting approaches—realism versus empiricism—to the quantitiveness question and defend the prospects of realism against Hasok Chang's and Bas van Fraassen's epistemic objection. **Chapters 5 and 6** introduce the basics of representational measurement theory (RMT) and use a formal result from it to justify drawing the line of quantitiveness between the ordinal scale and the interval scale. **Chapters 7 and 8** develop a metaphysics of quantity, which Wolff calls "sophisticated substantivalism," that says the (fundamental) quantities exist as a multi-dimensional quality-space. **Chapters 9 and 10** hone in on the fact that the formal feature that distinguishes quantitiveness in Chapters 5 and 6 is a *higher-order* structural feature and discuss the metaphysical significance of this fact.

This review focuses on two general aspects of the book: (1) its attempt at a unified narrative in the metaphysics of quantity; and (2) its formalism-driven methodology.

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✉ Derek Lam  
lams@uww.edu

<sup>1</sup> University of Wisconsin-Whitewater, Whitewater, USA

Those who write on quantities have different agendas, interests, and methodologies, and their writings do not belong to a single literature. Forging a unified narrative out of this literature requires some tradeoffs. I will offer two examples.

First, Wolff presents the metaphysics of quantitateness as the unifying theme of the book and what the metaphysics of quantity per se is truly about. But not all who work on the metaphysics of quantity care about quantitateness. And there is no reason why they all should. For example, according to a popular metaphysic, the relation between the quantities and their respective magnitudes is the determinable–determinate relation. No one thinks that this relation is unique to quantities. It cannot be what makes quantities. This should not be a problem. But in Wolff's narrative, this becomes objectionable.

Second, when Wolff presents the debate between realism and empiricism about quantitateness, she uses Joel Michell's view as the face of realism. His view says that the magnitudes of a quantity stand in ratio relations to each other; and these ratio relations are *literally* numbers, not just something we *represent* with numbers. This metaphysical relation with numbers is the basis of quantitateness. Making this the representative of realism, Wolff portrays the empiricism versus realism discussion as a debate about whether numbers are merely representational devices for quantities or constitutive of their quantitateness. This is misleading.

A highly influential realist view is Brent Mundy's, which Wolff discusses at length in Chapter 7. For Mundy, magnitudes are Platonic universals. There are axioms that quantify over them and allow us to justify *representing* them with numbers in certain ways. Such formal work can provide a basis for quantitateness. This is a realist position about the quantitateness of quantities because the axioms are interpreted to be independent of our measurement practices. Yet, numbers are merely representational devices on this view.

Why was Michell's view paraded as the face of realism then? My *hypothesis* is that at that early stage of the book, before she has introduced RMT, she is not in the position to introduce views like Mundy's. Michell's view is the one realist position that can stand on its own without RMT. This is a by-product of Wolff's attempt to weave a unified narrative that introduces the formal work of RMT *organically* as an answer to an *existing debate* about quantitateness.

I draw attention to these examples not necessarily as an objection but as a remark on an inescapable tradeoff. It is valuable to have a comprehensive treatment of the field. But it comes with some distortions to the dialectic of an inherently disunified literature. The benefit outweighs the cost. But this is something that readers who are new to the terrain should be aware of.

Let us turn now to Wolff's formalism-driven methodology. Wolff's substantive contribution is in Chapters 6 through 10, where first she motivates a specific criterion for quantitateness and then develops a substantive metaphysics of quantity. What she defends per se is not entirely new. It is common in science to draw the line of *genuine* quantitative research where she draws it. And the idea that quantities form an abstract quality-space is not new either. What is unique, though, is how she gets there.

A methodologically noteworthy feature of the book is that, on the one hand, Wolff rightly reminds us that formalisms are in certain ways metaphysically neutral.

A set of measurement theoretic axioms can be interpreted in many ways. On the other hand, Wolff is keen to read her metaphysics off RMT. First, her preferred criterion for quantitateness is supposed to fall out from the Alper–Narens theorem. Second, her metaphysics of quantities as a quality-space is supposed to be superior to its major competitor because the latter is “at odds with the formal representation of such quantities” (193).

To achieve the first task, Wolff appeals to the Alper–Narens theorem to make precise a feature of the kind of scale structures that is *supposed to be what we want* for measurement, namely *homogeneous* scale structures. Instead of unpacking in detail what homogeneity is and why only homogeneous structures matter, which would have helped her make some hidden premises in her reasoning more explicit. Wolff glosses over the issue and talks about more technical details, for example, degrees of homogeneity, which seem to do no real work in her arguments other than serving as signals of rigor. Such expository imbalance is my one minor complaint about this otherwise brilliant book. But here is the general idea. The more permissible transformations there are in a scale, the less information the representations in that scale carry. The Alper–Narens theorem entails that for homogeneous scale structures, there is a *steep drop in informativeness* for anything more permissive to transformation than the interval scale. *If* we care about drawing a line between quantitative and non-quantitative research *somewhere*, there is a pragmatic reason to draw it somewhere scientifically significant. The radical drop in informativeness provides one option. There are many things worth exploring in Wolff’s argument. The bottom-line is that she is convincing in demonstrating that RMT, *assuming* that we focus on homogeneous structures, encourages drawing the line for quantitateness where she does.

By contrast, it is less clear that we can read her sophisticated substantialism off RMT. Hence, whereas there is a dialectical continuity from Chapters 5 and 6 to Chapters 9 and 10, Chapters 7 and 8 appear out of place in the book’s RMT-driven arch. So, it is unclear how to pin down the view’s place in the overall project of metaphysics of quantitateness. More importantly, since RMT already provides a criterion for quantitateness, it is unclear how sophisticated substantialism further contributes to the issue, a point Wolff raised as an *objection* to other metaphysics of quantity. But I will set this aside.

Let us focus on Wolff’s treatment of the absolutism versus comparativism debate to substantiate my remark. Sorting out the interwoven threads in Wolff’s discussion requires work beyond this review. I will further restrict my attention to her treatment of absolutism. Wolff argues that sophisticated substantialism is the only option left standing in the face of RMT. Whereas I agree that RMT rules out comparativism because the world of individuals is not rich enough, absolutism *is not* at odds with RMT.

Dave Baker’s and Niels Martens’s classic escape velocity case shows that absolutism has a major advantage. Unlike comparativism, absolutism is consistent with the fact that we *cannot* arbitrarily scale up the mass of things, but we *can* arbitrarily scale up our mathematical representations of mass. Wolff argues that this supposedly positive feature of absolutism is at odds with RMT. Technical work on formalism is useful. But, to put a formalism to use, we need to *translate* the relevant issue,

ideas, arguments, etc., into the formal language of our choosing. Wolff translates the claim about scaling up mass and mathematical representations in the following way in terms of morphisms:

The anti-comparativist argument put forward by Martens relies on granting comparativism about passive transformations, where denying that there are active transformations corresponding to these passive transformations; that is to say, he grants that there are many homomorphisms from the physical structure to a given numerical structure, while **denying that there are corresponding automorphisms of the physical structure**. (149; my emphasis)

Wolff shows that the emphasized part is provably false in RMT. Mathematically, homogeneous structures *must* have non-trivial automorphisms (149).

The problem is that the emphasized part is not a proper translation of what absolutism implies. When absolutists say we *cannot* scale up the mass of things, they do not mean there are no corresponding automorphisms per se. They mean there are no corresponding automorphisms *without empirical significance*. Whether scaling up a quantity makes a difference is always an empirical matter. Quantities indeed have comparative structures isomorphic to mathematical representations that can be arbitrarily scaled up. But according to absolutism, such structures do not exhaust all that is in quantities. A structure-preserving transformation of something, i.e., an automorphism *may* fail to preserve features that are not constitutive of the structure preserved. Mistranslating the absolutist claim creates a formal strawman. Hence, RMT does not disprove absolutism and deliver sophisticated substantivalism. Whereas RMT gives us a dividing line for quantitiveness as Wolff argues, I am not convinced that RMT delivers anything more substantive about the metaphysics of quantity.

One take-away from this is that we should be cautious about our fascination with formal methods in metaphysics of quantity. Formalism-driven methodology tends to hide potentially debatable philosophical moves in the translation, obscuring what does the work in an argument and producing an illusion of mathematical rigor.

None of my methodological remarks diminishes what Wolff has accomplished in this excellent contribution. These methodological reflections would not have been possible without her ambitious attempt at a unified and formalism-driven treatment of the metaphysics of quantity. This book is a must-read for anyone who is interested in scientific measurement or scientific representation in general and will no doubt be the reference point for the discussions on this topic for years to come.

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