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Bolzano *a priori* knowledge, and the Classical Model of Science

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Abstract This paper is aimed at understanding one central aspect of Bolzano's views on deductive knowledge: what it means for a proposition and for a term to be known *a priori*. I argue that, for Bolzano, *a priori* knowledge is knowledge by virtue of meaning and that Bolzano has substantial views about meaning and what it is to know the latter. In particular, Bolzano believes that meaning is determined by implicit definition, i.e. the fundamental propositions in a deductive system. I go into some detail in presenting and discussing Bolzano's views on grounding, *a priori* knowledge and implicit definition. I explain why other aspects of Bolzano's theory and, in particular, his peculiar understanding of analyticity and the related notion of *Ableitbarkeit* might, as it has invariably in the past, mislead one to believe that Bolzano lacks a significant account of *a priori* knowledge. Throughout the paper, I point out to the ways in which, in this respect, Bolzano's antagonistic relationship to Kant directly shaped his own views.

Keywords Bolzano · *A priori* · Grounding · Classical Model of Science · Kant · Consequence

1 Introduction

Bolzano's conception of *a priori* deductive knowledge follows the Classical Model presented by de Jong and Betti (2008). That Bolzano subscribed to the Classical Model is uncontroversial and some of the reconstructions that are already available either argue for it explicitly (de Jong 2001) or otherwise document the fact that

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Bolzano's conception of grounding in deductive sciences implies it (Tatzel 2002). With de Jong and Betti's Classical Model of deductive sciences as a background I wish, on the one hand, to argue that and explain how Bolzano's account of what it means for propositions and terms of *a priori* deductive science to be known (adequately)—i.e. what corresponds to epistemic conditions (6) and (7) of the Model—is related to his account of the proof postulate, (3b). On the other hand, while I am not interested in appraising Bolzano's interpretation of Kant *per se*, this paper is aimed at understanding the way in which Bolzano's antagonistic relationship to Kant shaped his own views on *a priori* deductive knowledge. In particular, as Bolzano sees it, Kant's recourse to so called pure intuition in order to account for knowledge in *a priori* deductive sciences makes it systematically impossible to fulfil condition (3b) by purely logical means. By contrast, Bolzano sought to provide an account of grounding and deductive knowledge founded on an axiomatic understanding of deductive systems that would, against Kant, make it possible to systematically fulfil it by purely logical means. For, as Bolzano understands it, and as we will see in more detail in what follows, grounding cannot involve non-logical steps that make use of intuition or any other proxy for logic.

According to Bolzano, besides the fundamental misunderstanding concerning the nature of demonstration in deductive disciplines just mentioned what led Kant into thinking wrongly that we need pure intuition are certain false assumptions concerning the way in which we know concepts in *a priori* deductive sciences. As Bolzano sees it, had Kant had the correct account of the latter, he would not have had to resort to the theory of pure intuition to account for *a priori* knowledge. For Bolzano, signs express concepts that correspond to their meaning (Bolzano 1837, §285) and *a priori* truths are knowable on the basis of the meaning of the terms they contain (1837, §305.3). This aspect of Bolzano's theory, as far as I know, has remained wholly unnoticed. This is in part because Bolzano's views on analyticity—what is typically taken to be the locus of an explanation of *a priori* knowledge—are (at best) odd and indeed have little to contribute to the topic. Nonetheless, his *Theory of Science* presents a subtle and largely acceptable account of *a priori* knowledge. Bolzano's conception of what it is to know the meaning of a term, at least in *a priori* deductive disciplines, depends directly on his account of what it is for propositions to belong to an axiomatic or grounding structure: meaning is determined by implicit definition, i.e. the fundamental propositions in an axiom system. This is what the paper will show.

2 Analyticity and *Ableitbarkeit*

Bolzano's theory of analyticity is a favoured topic among Bolzano scholars (cf. Bar-Hillel 1950; de Jong 2001; Künne 2006; Lapointe 2008; Morscher 1997, 2003b; Neeman 1970; Proust 1981; Textor 2001) and there is no disagreement as to the fact that Bolzano developed his own notion of analyticity after having meditated on the Kantian one.[1] Yet, it appears that the peculiarity of Bolzano's definition and the depth of his disagreement with Kant as escaped prior scrutiny. Take the following example,

[1] At least until 1812, Bolzano in fact retained the Kantian definition, see, for instance (Bolzano 1810, §5, 1812, §30).

which Bolzano uses on at least two occasions (Bolzano 1837, §148 n. 4; Bolzano and Příhonský 1850, 35*ff.*):

(B1) The father of Alexander, King of Macedonia, was King of Macedonia.

What makes (B1) an especially opportune example, and what may have been misunderstood by commentators who nowhere discuss it, is that it succeeds, in Bolzano's eyes, as a counterexample to Kantian analyticity because it shows that what Kant means when he speaks of 'decomposition', of 'inclusion' and of propositions' being true by virtue of their 'analysis', is underdetermined. Let us assume, as Bolzano thought Kant would have done, that (B1) is a categorical proposition of the form 'A is B'. It has a subject: 'The father of Alexander, King of Macedonia'; a predicate: 'King of Macedonia'; and the verb *to be* as its copula. As Bolzano sees it, since Kant nowhere explains the underlying mechanism of the decompositional method he is relying on, there seems to be no reason why the predicate 'King of Macedonia' could not be said to be 'part of' the subject 'The father of Alexander, *King of Macedonia*'.[2] According to Bolzano, in the absence of a (more) definite account of what it means for a predicate to be included in a subject—or for a concept to be decomposed into its parts—(B1) seems to satisfy the condition for being analytic in the Kantian sense. But, as Bolzano explains, and this seems right, Kant would not have intended this proposition to come out as analytic. Bolzano's point is that a definition of analyticity must altogether rest on a conception of conceptual and logical analysis that is not decompositional.[3]

One key aspect of Bolzano's theory of analyticity is a substitutional procedure that is introduced for the purpose of defining logical notions. At (1837) §147, Bolzano uses this substitutional procedure to lay the basis for his calculus of probability. It is then applied at §148 in order to pick out a set of propositions that exhibit a "particularly interesting feature", what Bolzano terms *analytic* propositions:

(Analyticity) A proposition S is analytically true/false (with respect to i, j, …) if and only if:
(i) S contains at least one arbitrarily exchangeable constituent(s) i, j, … such that
(ii) every *objectual* variant of S with respect to i, j, … is true/false

[2] Kant could have denied that 'King of Macedonia' is contained (in his sense of the term 'contained') in the subject. Depending on his treatment of the apposition, Kant may also have argued that 'King of Macedonia' is not contained in the subject at all. 'The father of Alexander, King of Macedonia, was King of Macedonia', it could be argued, expresses in fact at least three propositions: 'The father of Alexander was King of Macedonia', 'The King of Macedonia was King of Macedonia' and 'Alexander was King of Macedonia'. The question here is not whether Kant could have replied to Bolzano but what Bolzano's criticism of Kant tells us about his views. According to Bolzano, (B1) is *one* proposition of the form [A has b] whose complex subject idea [A] is itself attributive and therefore of the form [C which has d].

[3] For an exposition of Bolzano's conception of logical analysis, see Lapointe (2007). For a presentation of the way in which the conception of analysis in Kant and Bolzano, respectively, determine their views on analyticity, see Lapointe (2008).

where a proposition is *objectual* (denotative) only in case its *entire* subject-idea is denotative.[4]

Note that Bolzano defines analyticity for propositions, not sentences. For Bolzano, propositions (*Sätze*)—the short form for 'propositions in themselves' (*Sätze an sich*)—are objective (as opposed to subjective) and abstract (as opposed to causally effective) entities that correspond to the sense (*Sinn*) of sentences (1837, §§19, 28). In what follows I will use the square brackets to designate Bolzanian propositions as well as their subpropositional parts—ideas (*Vorstellungen*)—and to contrast them with their linguistic and mental counterparts.

When Bolzano says that a proposition, for instance, [The man Caius is mortal] is analytically true with respect to [Caius], he means that the set of objectual and false variants of [The man Caius is mortal] that are generated by substitution of [Caius] is empty. What must be emphasised with respect to this definition is that unlike what is the case with Kant's definition it allows Bolzano to distinguish analyticities without being constrained by a determinate syntactic structure: although condition (i) in (Analyticity) stipulates a manner of determining a set of variants of S that share some fixed constituents with S, it does not bind analyticity to any logical form in particular.[5] Bolzano's notion of *logical* analyticity, which he defines in the same section, requires that all non-logical ideas be varied (1837, §148.3) and picks out propositions whose form is such that the truth-value of their objectual variants is the same under all interpretations of the non-logical ideas they contain.[6]

When Bolzano seeks to identify inferences that systematically preserve truth from premises to conclusion, the substitutional method is again mobilised. His definition of *Ableitbarkeit* is meant to determine systematically all inferences that preserve truth from premises to conclusion:

(*Ableitbarkeit*)
T is *ableitbar* with respect to constituents i, j, … from the proposition or set of propositions S, S′, S″, … if and only if:

1. i, j, … can be exchanged so as to yield true variants of S, S′, S″, …
2. every substitution of i, j, … yielding true variants of S, S′, S″, … also yields true variants of T.

[4] Note that the objectuality constraint has for a result that only substitutions of the same formal-grammatical category are admissible. The proposition 'The man Caius is mortal' is objectual because its subject-idea 'The man Caius' is objectual and only ideas that belong to the same formal category as 'Caius' can yield objectual variants of 'The man Caius' (see Morscher 1997; Lapointe 2002).

[5] In principle, there may be different sets of variants for one and the same propositions since, on the one hand, these variants are relative to determinate exchangeable components i, j, k, … and, on the other hand, different combinations of components ({i}, {i, j}, {i, k}, {j, k}, etc.) can be taken to be arbitrarily exchangeable.

[6] Bolzano did not offer a way to demarcate logical from non-logical ideas in a non arbitrary manner since, like the young Tarski, he thinks that "the domain of the concepts that belong to logic is not so strictly delimited as to elude all disputes" (1837, §148.3). That Bolzano considers certain determinate constants to be indisputably logical is however clear. Functors such as 'has' and 'which' in, for instance 'A triangle, which has equiangularity' are logical terms. Morscher (2003b) offers a detailed and systematic discussion of Bolzano's views on logical terms and logical analyticity.

Bolzano's substitutional method, at least at first glance, seems largely workable, relying on resources that are not alien to more contemporary efforts to develop a theory of truth and consequence at large. Indeed, the literature hardly fails to mention on the one hand, that (*Ableitbarkeit*) gives us is a clear anticipation of Tarski's notion of logical consequence (see Siebel 1997, 2002) and, on the other, that extending (i) in (Analyticity) to all non logical constituents, as Bolzano does when he defines *logical analyticity*, provides us with a clear anticipation of Quine's logical truth (see Morscher 1997).[7]

3 Is logic analytic?

Interestingly, despite the apparent reach of his conception of analyticity and *Ableitbarkeit*, Bolzano explicitly denies that deductive sciences, including logic itself, follow purely analytic procedure or are analytic in his—or indeed any—sense. In fact, if we follow the *Neuer Anti-Kant*, Bolzano agreed with Kant to call deductive sciences such as arithmetic and geometry synthetic *a priori*:

> We must however agree with him when he claims that "in all the theoretical sciences of reason synthetic *a priori* judgements are involved as principles". But we find judgements of this sort not only in mathematics, in the pure natural sciences and in metaphysics, as Kant proves it incontestably, but they are also to be found in logic, namely not merely among the theorems that belong to this discipline if we understand it, with Bolzano, according to a wider concept, but in the very part of it which one calls analytic and which has been worked on since Aristotle. (Bolzano and Příhonský 1850, 42*ff.*, my transl.)

The same idea can be found in the *Wissenschaftslehre*:

> In my opinion not even one proposition in logic, or in any other science, should be a merely analytic truth. For I look upon any merely analytic proposition as much too unimportant to be laid down in any science as a proper theorems of it. Who would want to replenish geometry, for example, with propositions like: an equilateral triangle is a triangle, or is an equilateral figure, etc.? (1837, §12, my transl.)

It is important to keep in mind, when considering these passages, that Bolzano's views on what makes a proposition synthetic *a priori* are thoroughly un-Kantian and, in particular, that they have nothing to do with Kant-type pure intuitions or anything remotely similar. For a Bolzanian proposition, being synthetic *a priori* consists essentially in (i) not being analytic in Bolzano's sense and (ii) not containing any intuitions in Bolzano's sense—more on this in Sect. 6. For instance, it follows from Bolzano's terminology and his views on the nature of axioms that all axioms independently of

[7] This said, it can easily be argued that, notwithstanding some superficial similarities, there are substantial differences between the Bolzanian definition of analyticity and Quine's notion of logical truth (see Textor 2001, 101*ff.*).

the discipline to which they belong, be considered to be synthetic *a priori* propositions in his sense.[8]

Analyticity and *Ableitbarkeit* are properties defined on the basis of Bolzano's substitutional method. However, in Bolzano, the order of deductive or axiomatic systems is not defined on the basis of the substitutional method. The order of axiomatic systems is defined on the basis of the relation of 'grounding' (*Abfolge*). In what follows, I will use the term 'grounding' exclusively to refer to the Bolzanian notion and I will come back in some detail on Bolzano's theory of grounding in the next section. Despite the fact that he was not sure how they relate with one another (Bolzano 1837, §200), the distinction between *Ableitbarkeit* and grounding is crucial to Bolzano and he insisted that in order to understand his logic, one needs to be acquainted with *both* concepts (Bolzano 1851, p. 40). What is relevant here are the two following points. (i) As Bolzano sees it—and despite the fact that they, on the face of it, define structures that are at least partly isomorphic—it is grounding, not *Ableitbarkeit* which corresponds to (3b) in de Jong and Betti's Classical Model. (ii) Bolzano's definition of grounding rests on a set of resources that are not substitutional and which have strictly nothing to do with what is involved in Bolzano's definition of analyticity and *Ableitbarkeit*. One reason for this—we will see that there are other more punctual ones in Sect. 5—is the following: Bolzano thinks that deductive or axiomatic knowledge is both *a priori* and necessary. But Bolzano is aware that when it comes to explaining that a given conclusion is a necessary consequence of the premises, he cannot rely on (*Ableitbarkeit*). Similarly, when it comes to picking out propositions that are true by virtue of meaning and logic, (Analyticity) will not do the job. These are points that have often been discussed in the secondary literature. There is a broad consensus as to the nature of Bolzano's trouble and the latter can be illustrated by a couple of examples.

Let us assume that all of G. H. W. Bush's children have a take in the family fortune. Under these circumstances, the proposition:

(B2) The *first* born child of George Bush Sr. has a take in the family fortune.

is analytic in Bolzano's sense with respect to 'first', despite the fact that this is merely contingent: it, as well as all of its objectual variants, namely:

(B3) The *third* born child of George Bush Sr. (i.e. John Ellis) has a take in the family fortune.
(B4) The *fourth* born child of George Bush Sr. (i.e. Neil Mallen) has a take in the family fortune.
(B5) The *fifth* born child of George Bush Sr. (i.e. Marvin) has a take in the family fortune.
(B6) The *sixth* born child of George Bush Sr. (i.e. Dorothy) has a take in the family fortune.

are true. But this is not the case for:

[8] But the import of this is minimal. An axiom, on Bolzano's view, contains constituents that are both simple and purely conceptual and, as such, while they cannot be analytic are nonetheless *a priori*. At section §197 of the *Theory of Science* Bolzano seeks to prove the existence of synthetic propositions *in his sense* by showing that propositions whose subject and predicate are simple are necessarily of that type.

(B7) The *second* born child of George Bush Sr. (i.e. Pauline) has a take in the family fortune.

(B7) is not objectual since Pauline died in her childhood, and since it is not objectual it is not to be taken into consideration. Now, on Bolzano's account, any of (B2)–(B7) could be false and they would all be had Bush Sr. disinherited all his children or had Bush Sr. had no children at all (for in the latter case, no substitution of 'first' in (B2) would give an objectual proposition). What this shows is, for one thing, that Bolzanian analyticity has to do with actual truth under substitution: it depends on matters of facts. This holds not only for Bolzano's broader concept of analyticity but also for his narrower concept of logical analyticity.

Similarly, although *Ableitbarkeit* is an entirely reliable and unambiguously defined criterion for identifying truth-preserving inferences, it may pick out inferences that are merely materially valid (that is, such that their validity also depends on the meaning of the terms), or, what is much more peculiar, it may also pick out inferences that have mere empirical generality:[9]

(B8) *Jeb* is not taller than 3 m

is *ableitbar* with respect to 'Jeb' from

(B9) *Jeb* is Governor of the Sunshine State

although the fact that governors are no taller than 3 m is a consequence of no necessity.

The previous examples are meant to illustrate the fact that Bolzanian analyticity and *Ableitbarkeit* are properties of propositions or sets of propositions which, if they have them, they have by virtue of what is actually the case, i.e. what is actually true and what actually is not. Bolzano's notions of analyticity and *Ableitbarkeit*, via the substitutional method, may allow us to ascertain some interesting semantic regularities in propositions and inferences that share some fixed components. But this seems to miss the whole point about analyticity and valid inference, as we understand them today. Analyticity, in particular, is usually meant as an explanation not of those truths that seem to hold universally given the way the world is but of what can be known 'on the basis of meaning alone'.

Bolzanian analyticity and *Ableitbarkeit* were manifestly not meant to account for *a priori* propositions' being true by virtue of meaning alone. Bolzano nonetheless has the resources to provide such an account. His understanding of what it means to know a truth by virtue of the meaning of its terms is radically different from Kant's—who relies on decomposition to establish the content of concepts—and from many other more or less Kantian ones. For one thing, Bolzano could not agree with Kant's version of what it is for a proposition to be true by virtue of meaning since Bolzano rejects Kant's decompositional conception of what is involved in figuring out the meaning of terms, i.e. analysing the concepts in a judgement (see Lapointe 2007, 2008, pp. 103–107). Bolzano nonetheless explicitly claims on many occasions that we can know that certain kinds of *a priori* propositions are true if we know the meaning of the terms they involve. We will see this in detail in what follows. Now, what is interesting and what is retrospectively truly original about Bolzano is that he also thinks that the

[9] See Siebel (2002) for a discussion of this point.

a priori propositions in question always belong to deductive orders—e.g. arithmetics, geometry and logic—and that the terms they contain are defined implicitly by the axioms and definitions, so that everything there is to know about meaning is to be found *a priori* in the axioms of a deductive system.

4 Grounding

Much of what Bolzano has to say about truth by virtue of meaning depends on what he has to say, on the one hand, about deductive or axiomatic structures—that is, systems of propositions whose order is defined by grounding—and about the type of knowledge we acquire through grounding, on the other. Bolzano thinks that any science that is defined by a grounding order—or what amounts roughly to the same, that can be axiomatised—is synthetic *a priori* in his sense. What's more, necessary knowledge can only be achieved when we 'grasp' the objective ground of a proposition and this invariably supposes that we have epistemic access to such a system of (synthetic *a priori*) propositions that relate as grounds to their objective consequences. Bolzano took seriously the Classical idea that a deductive science is axiomatisable in the sense that it is constituted by a number of true fundamental propositions formed by primitive concepts and from which all other propositions of the given science are provable. His attempt at a definition of grounding should be considered to be an effort to secure objective scientific rationality in this sense. Now, it seems generally agreed that accounting for the explanatory power of a scientific theory involves more than the study of its logical structure. One can understand Bolzanian grounding as part of an account of scientific explanation and as corresponding, roughly, to what the truly scientific mind ought to mean when, in the conduct of a scientific inquiry, she uses the phrase '…, because ….' in response the question 'why …?'—while *Ableitbarkeit*, on its part, can be seen as a tentative account of what in natural languages corresponds to the functor 'if …, then …' (Bolzano 1837, §155). Bolzano never came to a characterization of the idea of grounding that would satisfy him (1837, §195). He nonetheless formulated a set of implicit definitions that allow us to understand the type of relation he had in mind and to contrast it with other relations (e.g. *Ableitbarkeit*, justification, causation) to which it can be compared. This said, it should be stressed that the acceptability of Bolzano's views on meaning do not ultimately depend on the acceptability of Bolzano's own idiosyncratic views about grounding, and we will not defend the latter or engage in a thorough discussion thereof (for such an exposition, see Tatzel 2002). Some details are however relevant to understanding the role grounding—as opposed to *Ableitbarkeit*—plays in Bolzano's epistemology.

As Bolzano understands it, a system whose structure is defined by grounding is a system in which all propositions either are axioms or are grounded in the axioms. In this sense, having a grounding-order is a minimal property of axiomatic systems. Bolzano however thinks that grounding holds only between *true* propositions and we will see that this is not irrelevant. In a grounding-defined system, a truth p is either immediately or mediately grounded in the axioms ϕ, ψ … In the latter case, there is a (unique) grounding-chain between p and ϕ, ψ …, each step of which is minimal in the sense that it is immediate (the following example will make clear what I mean by this term). Grounding orders can be represented linguistically by a (unique) tree structure (Bolzano 1837, §220). Let us take a simplified example (which is not from Bolzano):

L_B:

$\phi 1$

Ideas = def. the (sub-propositional) parts of propositions. *Definition*

$\phi 2$

Parts have the same ontological status as their whole. *Axiom.*

q_1

Ideas have the same ontological status as propositions. *Theorem.*

$\phi 3$

Propositions are not real. *Axiom.*

p_1

Ideas are not real. *Theorem.*

In L_B, p_1 is immediately grounded in q_1 and ϕ_3 which taken together constitute its *complete* ground (each of q_1 and ϕ_3 is a partial ground for p_1). p_1 is however only mediately grounded in the axioms ϕ_1 and ϕ_2, namely via q_1, which, on its part is immediately grounded in ϕ_1 and ϕ_2. If p_1 is immediately grounded in ϕ_1 and ϕ_2, then the inference from ϕ_1 and ϕ_2 to p_1 is minimal in the sense that there is no intermediate inferential step. Bolzano thinks that grasping these minimal inferential steps is precisely what provides the reader with an 'objective justification' for p_1.

5 *Begründung* and objective justification

The notion of grounding plays a crucial role in Bolzano's account of e.g. mathematical proof. The fact that Bolzano had interesting views on proofs is relatively well known, but few are familiar with the detail of these views. Inasmuch as mathematics (and other purely deductive sciences) are concerned, it must first be noted that Bolzano's account rests on an important distinction between (i) grounding, (ii) objective justification (*objective Erkenntnisgrund*) and (iii) and what we may call 'objective' proofs, and what Bolzano himself calls *Begründungen*. Grounding is a relation between propositions; objective justification between beliefs (i.e. certain types of mental states); and *Begründungen* are linguistic objects that are meant, according to Bolzano, to cause objectively justified knowledge of the type we find in mathematics. How are (i)–(iii) related?

According to what Bolzano says at §525, a proof in general is any device that is meant to cause someone to have a justification for certain beliefs.[10] More precisely, a proof is to increase one's confidence in the truth of the proven propositions.

[10] What Bolzano means when he uses the term 'judgement' corresponds in a substantial way to the contemporary notion of belief. For Bolzano, a judgement is a psychological entity in which a propositional content is grasped (Bolzano 1837, §34).

A proof is (typically) the linguistic representation of a set of propositions—in a given order or diagrammatic composition—we may use in order to bring our interlocutor (or ourselves) to bestow confidence on the truth of a given proposition. There are two kinds of proof according to Bolzano. On the one hand, there are those that reflect the grounding-order and therefore, according to him, also provide us with *objective* justification. Those are the proofs he calls *Begründungen*. On the other hand, there is the much greater panoply of objects that fit Bolzano's definition of proof but do not reflect the order of grounding. The latter—what Bolzano calls *Gewissmachungen*—provide us with mere *subjective* justification. What is relevant as far as *Begründungen* are concerned is the following: the reason why, according to Bolzano, a *Begründung* causes us to have both objectively justified and indeed true beliefs is that, every time it causes us to have beliefs of this sort, it also causes us to grasp (parts of) structures defined by grounding. Bolzano's point is an Aristotelian one: *Begründungen* putatively allow one to cognise not only 'that' something is true but also 'why'. *Begründungen* provide us with a special kind of justification and this is what would explain their added epistemic value.

But why, one might ask, grounding and not *Ableitbarkeit*? On Bolzano's account, our grasping an *Ableitbarkeit* relation between premises and a conclusion would not systematically warrant objective knowledge in his sense and here is why. As we've mentioned earlier, *Ableitbarkeit* and grounding define relations that are partly isomorphic. Take the two sentences:

(B10) In Dresden, it is warmer in the summer than in the winter
(B11) In Dresden, the thermometer is higher in the summer than in the winter

According to Bolzano, (B11) is both *ableitbar* from (B10) with respect to 'Dresden' and grounded in the latter. Assuredly, that (B10) is true also seems to explain why (B11) is true. Granted that both (B10), on the one hand, and the fact that if (B10) is true then (B11) is true, on the other, are also epistemically prior[11] (B10) can be said to constitute an objective justification of (B11) in Bolzano's sense. For, if I know (i) that it is warmer in Dresden and (ii) that if it is warmer then the thermometer is higher, I am also putatively bound to know *why* the thermometer is higher in Dresden, namely because it is warmer there. That we see our belief that (B10) as providing an objective justification for our belief that (B11) could not however be explained solely on the basis of the *Ableitbarkeit* relation between (B10) and (B11). In Bolzano's theory, for one thing, (B10) is also *ableitbar* (B11) with respect to 'Dresden'. But obviously, we would not want to say that (B10) is true *because* (B11) is true, at least not in the same sense, and neither does Bolzano. We might want to say that (B11) provides us with a 'sign' or 'indication' of (B10)'s truth, but this amounts to calling it a subjective justification—i.e. this amounts to calling (B11) a mere *Gewissmachung* for (B10)—and this is not what we are looking for. Put in more general terms, the problem is that some instances of *Ableitbarkeit* relations, namely all those where *one* conclusion is deduced from *one* premise, are symmetric and have therefore this consequence.

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11 Bolzano himself does not mention this epistemic restriction, but Aristotle (*Apo* 78 b 11) does. We discuss this restriction and related difficulties in (Dubucs and Lapointe 2006, Sect. 3). The article offers a discussion of the relation between objective grounding and epistemic consequence as well as a criticism of the explicativist interpretation of Bolzano's theory of *Begründung*.

According to Bolzano, minimally, a belief or a set of beliefs $q_1, q_2, \ldots, q_n$ is an objective justification for the belief p (of agent A) if and only if truths $q_1, q_2, \ldots, q_n$, taken together, are the objective ground for the truth that p. Let us call this the Bolzano Justification Condition (BJC). (BJC) corresponds to condition (6)—and assumes (4) and (3b)—of de Jong and Betti's Model and constitutes another central aspect of Bolzano's adherence to the Classical Model: one of the core constants of the Classical Model of rationality is the idea that all (non fundamental) propositions should be known to be true through their proof and, in Bolzano, (BJC) insures that this condition be systematically fulfilled. Note that, for the purpose of what follows, in order to alleviate any worries surrounding Bolzano's views on grounding, (BJC) can be generalised to accommodate any preferred account of axiomatic orders, with the proviso however that the reader bears in mind that Bolzano considers grounding to hold solely between truths.

In the light of what precedes, the trouble, from Bolzano's standpoint, with Kant's account of demonstration in deductive sciences is that none of the demonstrative procedures Kant has in mind when he discusses analytic and synthetic *a priori* knowledge fulfils (BJC). The main reason for this, on Bolzano's view, is that although the notion of ground—Kant and Bolzano use the same term '*Grund*'—plays a crucial role in Kant's theory, Kant leaves the notion undefined and confused (Bolzano 1810, appendix §3). When he discusses what counts as a ground for analytic and synthetic *a priori* judgements, Kant does not truly have a grounding-type relations in mind and what he does have in mind is unacceptable, from a Bolzanian standpoint, for the purpose of a theory of *a priori* knowledge. In particular, the reason why Kant's account of mathematical demonstration proper fails to fulfil (BJC) is, according to Bolzano, the following. Kant takes as paradigm a discipline that is undeniably axiomatic: geometry. So, in this case (BJC) could at least in principle be fulfilled. However, and this is what struck Bolzano, since Kant appeals to pure intuitions as their putative 'ground' in his geometric demonstrations, geometrical truths do not get demonstrated from the axioms via purely logical inferential steps—although they plausibly must at least be logically consistent with the axioms. Besides his appeal to the 'self-contradictory' notion of pure intuition—Bolzano's reconstruction of Kant's argument in the first paragraphs of the *Appendix* to the *Contributions…* of 1810 is, at once, one of the most scrupulous and harshest criticisms of Kant's doctrine—what Bolzano blames Kant for is the idea that the truth of a proposition could be 'grounded' in anything else than in another true proposition. Given Bolzano's definition of grounding, his reproof is somewhat trivial. But Bolzano believed that had not Kant denied this, i.e. had not Kant misunderstood this precise aspect of what it means for a truth to have a ground, Kant would not have fallen pray to the 'scabrous' doctrine of pure intuition (Bolzano 1810, appendix §8).[12]

[12] Bolzano's criticism rests on the assumption that mathematical knowledge is purely conceptual and that Kant was wrong to assume otherwise. This is not uncontroversial and some authors have recently sought to show that Kant's claims about the role of intuition in mathematics were substantial and plausible. Although I am inclined to side with Bolzano (and thus with Hintikka 1966; Friedman 1992) on this issue, I have nothing to add at this point to the debate. See Carson (2005) and Majer (2005) for a defence of the alternative 'phenomenological' interpretation of Kant's views on intuitions.

6 *A priori* knowledge

As we will see directly, Bolzano's views on *a priori* knowledge rest explicitly on the idea that the truths in deductive disciplines are known *a priori* because they are known on the sole basis of the meaning of the terms they contain. Bolzano's claim presupposes however, this must be stressed, that the truths in question invariably stand in a grounding relation and that the type of beliefs we acquire when we study the latter be objectively justified in Bolzano's sense: they must at least in principle be apt to fulfil (BJC). Only if we presuppose this can we make sense of Bolzano's idea that in order to recognise that a proposition is true, the only thing that is required is our knowing the meaning of the terms involved in that proposition.

When it comes to explaining:

> what is this unknown X on which the understanding must base itself when it believes to have found outside the concept of A a predicate B which is alien to the latter and which it considers to be nevertheless connected with it. (Bolzano 1837, §305.3)

Bolzano proposes the following explanation:

> … [i] Nothing else [is required], I say, that the understanding *has* and *knows* the concepts A and B. In my opinion, from the mere fact that we have certain concepts, we must also be in a position to judge about them. For to say that someone has certain concepts A, B, C, D, … is indeed to say that he knows and differentiates them. But to say that he knows and differentiates them is again only to say that he asserts something about the one that he does not want to assert about the other; [and he] means therefore to say that he judges about them. [ii] Since this holds universally, it holds as well in the case in which these concepts are perfectly simple. But in this case, the judgements we fall are certainly synthetic … and it seems to me therefore that we must be in a position to fall a synthetic judgement about all objects of which we have a concept. (Bolzano 1837, §305.3)

Consider again:

> [iii] If a given proposition consists of mere concepts, such as, for instance, the proposition that virtue deserves respect or that two sides of a triangle taken together are bigger than the third, etc.; [iv] then the truth or falsity of the latter depends only on the properties of these concepts; and, [v] at least in many cases, nothing else will be required in order to convince yourself of its truth that you examine attentively the concepts themselves of which it is composed. Thus, it will be possible for you to recognise the truth that virtue deserves respect from the mere fact that you have the concepts virtue, to deserve and respect. [vi] One could not say that you have a concept if you could not differentiate it from another one, that is, if you did not know that certain other concepts can be connected with it to form true propositions which cannot be connected with another. [vii] You cognise truths of this kind (purely conceptual truths) by virtue of the fact that

> you know the concepts of which they are composed. [viii] Things are different with judgements that … contain intuitions. (Bolzano 1837, §42, 180*ff.*)

Since, as we will see, the general claim in (vii) is that knowing the concepts they contain is sufficient for knowing that synthetic *a priori* propositions are true, what needs to be explained here is what it is to 'have' or know a concept, that is, the thesis Bolzano puts forward in (i) and (vi). (i) and (vi) express the thesis that to 'have' a concept x is to have some beliefs about that concept, what Bolzano, faithful to the terminology of his time, calls 'being in a position to judge about' the concept x. Of course, this sounds odd. It would appear more natural to say that to have the concept of an x is to be in a position to judge about the corresponding object(s) x—and one may plausibly speculate that this is what Bolzano intended to convey. Be it as it may, the claim can be sustained within Bolzano's theory as it stands and with the same implications: the type of beliefs about concepts that are relevant to Bolzano here are precisely those (metalogical ones) that also imply the corresponding objects systematically having certain matching properties. We will come back on this in detail in what follows.

(ii) makes it clear that whatever Bolzano means in (i) applies to the concepts we find in synthetic propositions and (iii) extends this property to synthetic *a priori* propositions which, as we have seen, include axioms and (at least some) definitions. Of course, in (iii), the expression '*a priori*' does not occur, and this requires explaining. Bolzano speaks of truths that contain only concepts as "*reine Begriffswahrheiten*" (see 1837, §133). The latter do not contain any Russellian proper name, i.e. no part or subpart designating something with which we may be 'acquainted' or, what amounts to roughly the same, any indexicals (Bolzano 1837, §72). The crucial issue here is whether Bolzano's conceptual truths are all indeed knowable *a priori*. Bolzano's claim that "the truth of [only!] *most* conceptual propositions can be decided through mere reflection" may suggest that he did not mean to imply that all conceptual propositions are knowable *a priori*. But this conclusion would be mistaken. The qualification is not aimed at implying that we may need experience, in some cases, in order to know whether a certain conceptual proposition is true. Rather, what he has in mind here is the fact that it may be simply impossible to know a conceptual proposition, and hence, neither *a priori* nor *a posteriori*. We do not know, for instance, the mathematical and therefore purely conceptual formulae from which we can infer all prime numbers; hence we do not know it *a priori* (Bolzano 1837, §133, 37 n.). One could object that, independently of what Bolzano meant or did not mean to imply, it seems unlikely that we can come to know the truth of a proposition such as, for instance, 'Something exists', which is conceptual in Bolzano's sense, by merely thinking about concepts. Consider however that existentially quantified sentences are, in Bolzano, second order sentences about the objectuality of concepts: to say that A exists or that there are As is to say that the extension of the concept of an A is not empty (Bolzano 1837, §172). Bolzano's analysis of 'Something exists' would therefore be:

(B12) The concept of something (*Etwas*) is not empty

Assuredly, Bolzano ought to have believed that his own demonstration that there is at least one truth at section §31 of the *Theory of science* implies the truth of (B12) and that it does so *a priori*. One may also point to section §99, where Bolzano proceeds to a metalogical analysis of the concept of a concept's extension (*Weite*) that allows us to conclude to the truth of (B12) without recourse to extra-conceptual considerations. He writes:

> I believe in any case that there is an idea that is the absolute widest [i.e. which has the greatest extension—SL] and the highest [i.e. under which the greatest number of objects fall—SL]; I believe namely the concept of something [*Etwas*] or of an object in general to be this very idea.

If it is a postulate of Bolzano's semantics that the concept of something (*Etwas*) is that whose extension is the widest, then it has an extension and is therefore not empty, and I need not rely on experience to come to this conclusion.

In the light of the above considerations, we may safely assume that Bolzano's theory supports the claim that if a conceptual proposition is knowable at all, then it is knowable *a priori*. It is all the safer that Bolzano takes care to mention in (viii) that we know propositions that contain intuitions—his counterpart to *a posteriori* propositions, if we follow §133—in 'another manner'. As it turns out later in the same section, their truth depends also on the nature of the objects that are represented and therefore, since these objects are intuitive in Bolzano's sense (i.e. known through acquaintance), on our empirical experience of the latter.

Assuming what precedes, (iv) sets a semantic condition for the truth of *a priori* propositions. (iv) claims that, in *a priori* deductive sciences, whether a proposition is true depends on the "nature (*Beschaffenheit*) of the concepts it contains". Now, let us piece (ii) and (iv) together. In a deductive or axiomatic system the truth of propositions depends on the concepts they contain and—following (ii)—we can cognise this truth by recognising some determinate properties of these concepts. What properties of concepts are here relevant? The significant feature of concepts Bolzano has in mind here is that we can 'infer' from them the properties of the objects to which they refer. This may sound somewhat disconcerting, but it is not a lapse. Bolzano makes this claim at many places: a property b can be 'inferred' (*werden gefolgert*; Bolzano 1837, §65.8), it may 'follow' (*folgen*) or 'ensue' (*sich ergeben*) from a concept (Bolzano 1837, §114, 531). What it means for a property to be inferred from a concept is furthermore made clear at (1837) §111 in connection with Bolzano's discussion of essential and inessential properties. There, Bolzano says that from the concept of an object it is possible to infer all the 'essential' properties of the corresponding object. Bolzano in fact defines the 'essence' of an object as the set of properties that can be inferred from its concept (Bolzano 1837, §111.3—I will come back on the Bolzanian notion of essence directly). (The property) b can be inferred from (the concept) [A] and, correlatively, b is an essential property of (the object) A iff (Bolzano 1837, §111.2.):

– the proposition [A has b] is true;
– [A] is a pure concept.

Now, since we are interested in the epistemology of grounding-defined systems, the truth [A has b] is part of a purely conceptual deductive order and [A] and [b] are then, by definition, pure concepts. Under this assumption, as we have seen above, [A has b] is knowable *a priori*. Let us also assume that [A has b] is an objective consequence of more primitive conceptual truths in the theory, i.e. that it is ultimately mediately or immediately objectively grounded on axioms and definitions. This being the case, [A has b] is a necessary consequence of the (true) axioms and it is itself therefore necessarily true (in this, Bolzano, agrees with condition (5) of the Classical Model). Following what Bolzano says, (the property) b can thus be inferred from (the concept) [A] and (the object) A has therefore the property b 'essentially'. Note that we are here dealing with a somewhat innocuous account of what counts as essential. Since in *a priori* deductive sciences concepts are ultimately and—as I will show in the next section—implicitly defined through axioms, Bolzano's claim that the essence of an object is given by its concept amounts to saying that, in *a priori* sciences, a (sound) axiom system defines the (essence, that is, the entire set of properties of the) objects over which it ranges. In defence of Bolzano, one must note that we usually say that mathematical objects have their properties essentially precisely in this sense.

7 Axioms as implicit definitions

Let us come back to our previous example L_B. If we follow what Bolzano says in (v), our having and examining the concepts, in this case, [idea] and [reality] is sufficient for us to know *a priori* whether [Ideas are not real] is true. (v) makes clear what is the crucial point: a sufficient condition for *recognising* the truth of an *a priori* proposition is to know the concepts it contains, i.e. to know the meaning of the terms occurring in the sentence that expresses this truth. This, as we've emphasised at the beginning of the previous section, should be read in the light of the fact that [idea] and [reality] belong to a grounding structure of which L_B is a segment. My knowledge of the grounding structure may, as it stands, be imperfect and partial and unsatisfactory but, according to Bolzano, full knowledge can at least in principle be attained. Now, since my belief that ideas have non-reality fulfils the Bolzano justification condition (BJC)—see Sect. 5—I also know that it is grounded in more primitive and ultimately fundamental conceptual truths, namely those I would reach once I had ultimately regressed in the order of grounds and consequences and which contain [idea] and [reality]. Indeed, whenever we ascend from consequences to their grounds in a deductive order, we ultimately arrive at the axioms and Bolzano would agree to say that, at this point, we can claim to know the meaning of the terms by virtue of implicit definitions.

Bolzano has an extensive theory of implicit definitions. One of Bolzano's motivation for putting implicit definitions forward was the need to account for the fact that we are able to understand new terms whose explicit definition we don't know or terms that cannot be explicitly defined because they are primitive. The latter is invariably the case for the terms in sentences that stand at the upper limit of *Begründungen*. Primitive terms, he thought, could alternatively and quite effectively be defined 'in use' or 'in context'.[13] Although he does not say in so many words that axioms are

[13] Sebestik (1992, 141*ff.*) discusses this aspect of Bolzano's semantics. He however leaves open the question whether Bolzano might have had views on axioms as implicit definitions. I am here suggesting that Bolzano could not have had any other view than that they are.

implicit definitions—but this will be made explicit only at the turn of the twentieth century (see Sebestik 1992, 142*ff.*)—Bolzano does use them as if they were.

Bolzano takes implicit definitions to be types of linguistic entities, not propositions in themselves. If we follow Bolzano, the job of sentences of the type of those we find at the upper limit of Bolzanian *Begründungen*, i.e. axioms or definitions is to introduce primitive, i.e. simple terms. One reading of what it is to have or cognise the meaning of the fundamental terms in this context is the following: we know the meaning of the terms involved in primitive sentences when we have determined what properties the objects to which they refer would have if the latter (and all their relevant consequences) were true. This—I take it—is what Bolzano means to explain in the following passage:

> [i] It is known that, when we come across a sign which is unknown to us in connection with other signs whose meaning we know, we are more than often in a position, if we also merely suppose that the writer does not want to express something manifestly absurd, to determine with more or less exactness what he represented himself with that sign. In such cases, we know the meaning of the term *on the basis of the use or context*. … [ii] Understandably, not every sentence in which our sign occurs in whatever connection with other known signs is equally apt to its determination and many sentences are often necessary to determine it completely. [iii] Without doubt, sentences that state a truth and a truth which is known and familiar to the reader are much more useful for this purpose than others. (Bolzano 1837, §668.9)

(i) states the general idea behind implicit definitions. (ii) states a condition for this definitional procedure to succeed: we must insure that sentence(s) used to define a term implicitly determine it fully. Part (iii) of the quotation is important because it states the (epistemic) condition under which the reader is in a position to recognise the meaning of the terms defined implicitly and thus the conditions under which implicit definitions are most likely to fulfil their epistemic role: that their truth be known, or familiar to the reader. As far as (ii) is concerned, 'to determine a sign' can be read as a semantic matter or as an epistemic one. The semantic concern is easily answered: as we have seen above, since Bolzano assumes that the essence of an object is determined by its definition and given that, for him, a definition is what establishes the meaning of a sign, then a (sound) set of axioms necessarily determines any sign it contains as much as possible (for that axiom system). Yet, from the epistemic standpoint, the full determination of, say, 'A' and 'b' in case they both are primitive and 'A has b' is an axiom rests on condition (iii): what warrants our knowledge of the terms involved in an axiomatic system is minimally our recognising the axiomatic status of the 'A has b' as well as our recognising that it is true.

The demonstration of the axiomatic status of a sentence rests on a procedure which Bolzano, in his early work, calls *Herleitung* (Bolzano 1810, II, §§11, 21, 1837, §221.5, §577).[14] The detail of Bolzano's theory is not indispensable here (see Laz 1993, pp. 53–58 for an exposition thereof). What is relevant is the following. The *Herleitung*

[14] In the *Theory of Science*, the same procedure is described, but the name is dropped.

of an axiom or of a system of axioms amounts to a metadeduction of the latter. The *Herleitung* of an axiom does not, however, entail the demonstration of its truth. What a *Herleitung* ultimately shows, when it succeeds, is that the truth of the proposition in question cannot be demonstrated via its objective ground, that is, that it fulfils BJC. According to Bolzano, as we have seen, the only means by which the truth of an *a priori* proposition can be demonstrated with necessity—and the only means by which we can therefore acquire objective knowledge thereof—is via its objective ground. Since axioms stand at the upper limit of the grounding order, they cannot be 'proven' to be true. While this is not really an issue for the contemporary logician who usually considers axioms to be postulates and not truths, it is problematic for Bolzano who believes that axioms, just like all other propositions in a grounding order, are true—and should be known as such. If we can only know for sure that a proposition is true if we know that it is grounded in another truth, then it would seem that, in the case of axioms, (iii) cannot be satisfied—at least not systematically since Bolzano denies that there is a formal procedure, deductive or metadeductive that could guarantee the truth of axioms. Bolzano ultimately appeals to a 'meta-inductive' procedure. He suggests that a proposition whose axiomatic status has been shown and whose logical consequences are also known to be true (some of the consequences may be—and indeed often are—more epistemically accessible than the axioms themselves) is very likely to be itself true (Bolzano 1837, §577.2a).

8 Conclusion

Bolzano's insistence on a meta-inductive verification procedure for the axioms reflects a feature of Bolzano's views on axiomatization that is at the heart of the Classical Model of science: it is not only that our system must be sound and complete but that it must in effect reflect the (unique) way the (mathematical, causal, etc.) world is and of which we may have only very partial and unsystematic knowledge. Bolzano's predicament, as far as the indemonstrability of the truth of axioms is concerned, was therefore not particularly original. More ought to be written on the question. It could be argued that he was an apt pupil of Aristotle and too loyal an adherent to the Classical Model of Science to be the precursor of modern axiomatic formalism. One should not however underestimate the import of his views and, in particular, of his views on *a priori* knowledge as an attempt to counter Cartesian and, as we have seen, Kantian epistemologies. If the condition of objective knowledge is that it grasp or mirror the order of grounding, then the truth of *a priori* cognitions cannot be a mere function of evidence or intuition. In general, one may understand one of the core motivations of early twentieth century analytic philosophy as consisting precisely in explaining what warrants us to hold as true *a priori* propositions without however appealing to the idea of evidence or other similar epistemic criteria such as certitude or conviction. Bolzano made it clear that one ought to look for the objective ground of a truth as long as one has not shown its axiomatic status. This pragmatic constraint ensured that the justificatory procedure never—or at least as seldom as possible—be short-circuited by our epistemic breakdowns: in the absence of a *Herleitung*, Bolzano directs us to assume that the truth in question has an objective ground and to attempt to find out what it is. At any rate, once we do away with the superficial terminological confusion which may arise from the fact that he claims that deductive knowledge is 'synthetic *a priori*' in his sense, one finds an intuition that will prove to be fruitful in a number of Bolzano's successors: *a priori* knowledge is always deductive and cannot be explained without the support of a theory of logical consequence.

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