Combing Graphs and Eulerian Diagrams in Eristic

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Abstract. In this paper, we analyze and discuss Schopenhauer's n-term diagrams for eristic dialectics from a graph-theoretical perspective. Unlike logic, eristic dialectics does not examine the validity of an isolated argument, but the progression and persuasiveness of an argument in the context of a dialogue or even controversy. To represent these dialogue situations, Schopenhauer created large maps with concepts and Euler-type diagrams, which from today's perspective are a specific form of graphs. We first present the original method with Euler-type diagrams, then give the most important graph-theoretical definitions, then discuss Schopenhauer's diagrams graph-theoretically and finally give an example of how the graphs or diagrams can be used to analyze dialogues.

Keywords: Arthur schopenhauer · Logic diagrams · Graph-theory · Eristic · Dialectics · Euler diagrams

1 Introduction

In several phases of his work, the post-Kantian philosopher Arthur Schopenhauer (1788–1860) was not only intensively concerned with logic, but also with eristic. Whereas formal logic is for him primarily the study of the correct use of concepts, judgements, and inferences, eristic examines the techniques and artifices of deliberately using them incorrectly in order to emerge victorious in a debate. Logic is thus a monological discipline, whereas eristic is a dialogical one.

Although the two disciplines pursue different goals, Schopenhauer uses similar diagrams for visualisation in both fields. In recent years, Schopenhauer's logic diagrams in particular have been intensively researched: V. Pluder and also A.-S. Heinemann have pointed out that Schopenhauer's logic was pioneering, among others because of the Euler-type diagrams used [8,18]. L. Demey has shown that Schopenhauer's logic is built compositionally from a certain number of basic diagrams. These basic diagrams use circles to depict all possible positional relations in space and also depict oppositional relations [6]. M. Dobrzański

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and K. Matsuda have illustrated how Schopenhauer's diagrams can be used to map and analyse semantic and ontological relations [7,14].

Schopenhauer's eristic diagrams are less known so far and only two research approaches can be found from the last decades: A. Moktefi and J. Lemanski have shown that Schopenhauer used some of the basic diagrams in eristic, and he was perhaps the first to introduce diagrams for n-terms [12]. M. Tarrazo has argued that these diagrams can be seen as a visualization of fuzzy logic [24].

Schopenhauer wrote several treatises on eristic, but not all of them contain diagrams (for details cf. [22, Sect. 10.2]). In the texts without diagrams, Schopenhauer describes mainly eristic fallacies, artifices or stratagemata so that one can protect oneself from those argumentation partners who deliberately use such techniques to deceive others and achieve their goal [3,9]. Although Schopenhauer's eristic diagrams are hardly known, the interest in Schopenhauer's texts on eristic, which do not contain diagrams, is all the greater in recent years: There are research approaches to these texts in the field of argumentation theory [17], proof theory [4], communication ethics [10], and pedagogy [13]. These texts on eristic are also used in the area of social sciences, especially in the field of law, economics and politics (cf. e.g. [2,23]).

This paper is a contribution to a large-scale research on diagrams in eristic, which began with the works mentioned above. Here, we discuss a graph-theoretical interpretation of eristic diagrams since it is striking that these diagrams for n-terms have a structure similar to a graph. Individual areas of these diagrams have also already been called 'routes' or 'paths' by scholars [15]. Beyond that, there is a long tradition in research of representing argumentation processes as graphs, e.g. the classic methods of argument maps by Whately, Wigmore, Toulmin or Dung [19] or current ones such as ConvGraph [16].

So it is not unlikely that Schopenhauer also had an idea in mind when he drew the diagrams, which today we would perhaps implement primarily in terms of graph theory. However, even if Schopenhauer was well versed in the mathematics of his time, his early 19th-century drawings predate the beginnings of graph theory by many years. Thus, a graph-theoretic interpretation cannot rely on Schopenhauer's descriptions of the diagram. It is our task to present and discuss the different graph-theoretic interpretation possibilities and then to select, combine and apply the best of them.

The present paper is motivated by the hope of soon having a diagrammatic tool or argument map that combines the best of both worlds – Euler-type circle diagrams and graph theory. Apart from that, however, it may simply offer a suitable means of describing eristic diagrams. Our roadmap is as follows: In Sect. 2, we introduce eristic diagrams and summarize some of the previous research on diagrams. Section 3 defines the elements of graph theory that we use in subsequent sections. Then, in Sect. 4, we present two graph-theoretic interpretations of the eristic diagrams and discuss advantages and problems. In Sect. 5, we bring together the diagrams and a particular graph-theoretical interpretation to map an exemplary controversy between two dialogue partners. However, as we also emphasize in conclusion of Sect. 6 this is only one way of combining graphs and Euler-type diagrams to apply the new technique of argument mapping in human-human or human-machine interaction.

2 Current Research Results and Problems

In this section, we introduce Schopenhauer's eristic diagrams and combine this with a presentation of results and problems that have been discussed in research in recent years.

Schopenhauer sees eristic as a discipline separate from logic. However, since eristic takes many components from logic (such as diagrams), one can say that eristic is an extension of logic by a new subject area. In the chapters on logic, Schopenhauer starts with five basic diagrams in 1819 [21, §9] and with six basic diagrams in later manuscripts of the 1820s. These six basic diagrams show the position of two circles in space to each other or to a third one (then including arc, sector, and segment). Each of these diagrams denotes the relationship of two concepts to each other or in relation to a third. Schopenhauer speaks of 'representations of possible relations' [20, p. 272] which can also be called 'relational diagrams', or RD in short. The six RD are shown in Fig. 1.

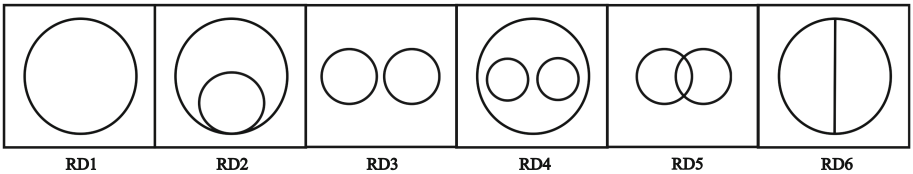


Fig. 1. Schopenhauer's Relational Diagrams (RD) taken from [20, 269–284] (*Euler diagrams* = {RD2, RD3, RD5}; *Gergonne relations* = Euler diagrams ∪ {RD1}; *Partition diagrams* = {RD4, RD6}).

A concept is symbolized by a circle (often called 'sphere' by Schopenhauer) or, as in RD6, by a semicircle. This can be concretized by some examples, but for our purposes it is sufficient to explain RD2, RD3 and RD5. A more detailed description of the RDs can be found in [11].

RD2 shows that the concept indicated by the inner circle is completely contained in the other. For example, the term 'cat' is completely contained in the concept 'animal'.

RD3 shows that two concepts are completely separate and have no commonality. For example, the concepts 'good' and 'evil' (as understood by Schopenhauer).

RD5 shows that two concepts are partially connected or have some commonality. As an example, we can take the terms 'red' and 'flower', because there are things that are only red, but are not a flower, that are both or that are only a flower, but not red.

Schopenhauer uses these three diagrams, RD2, RD3, RD5, and transfers them to eristic. Therewith he constructs diagrams to show two different perspectives: On the one hand an "in-depth view", on the other hand, a "superficial view". The in-depth view shows the actual, neutral or factual relations between two or more terms employing one RD, whereas the superficial view shows a distorted, subjective, biased or prejudiced relation by resorting to another RD. The superficial relation is the one that may seem plausible at first, i.e. when viewed superficially, but is often only used and accepted by one dialog partner, maybe only to intentionally deceive another.

> The sphere of a concept A, which lies partly in another B, but partly also in C quite different from this one, can now be represented according to its subjective intention as lying entirely in the sphere B, or in C, just as the speaker prefers [20].

Schopenhauer describes here that the thorough relation of two terms corresponds to RD5, but a dialog partner may treat the terms as if RD2 is present. One can imagine this change of the relations or views at the two diagrams of Fig. 2. In this case, the dialog partner represents $A \subset B$ (right diagram of Fig. 2) instead of $A \cap B$ (left diagram of Fig. 2). Similarly, the dialog partner represents $B \subset C$ instead of $B \cap C$, which finally leads to the superficial perspective $A \subset C$. And if the dialogue partner does this intentionally, then it is not simply a dialectical or dialogical process, but an attempt at deception, which is to be investigated by the discipline of eristic.

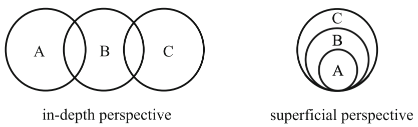
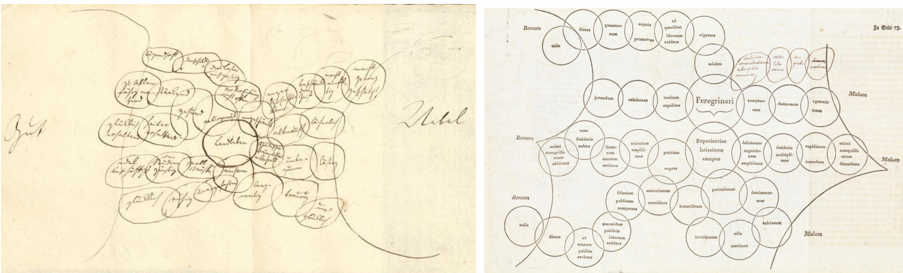


Fig. 2. Interchange of RDs

In his later works, Schopenhauer believes that this interchange of RDs is the basic principle of the entire eristic [20, p. 365]. In several treatises, Schopenhauer listed eristic artifices, which are intentionally committed fallacies, sophisms, paralogisms, etc., which are repeatedly used by dishonest discussion partners for the purpose of being right [3,17]. According to Schopenhauer's opinion, these eristic artifices can all be traced back to the interchange of RDs, which is why the diagrammatic representation of eristic was of great importance to him. This can be seen in the diagrams for n-terms, which have a strong resemblance to modern argument maps.

In Fig. 3, one finds such a diagram, which shows the in-depth view of several terms. These diagrams show numerous spheres of terms, the relationships of these terms in the form of RDs. An interchange of the RDs is not to be seen, but initially only the in-depths perspective on possible propositions of arguments, which in sum depict possible dialogues. Thus they are not only applicable to eristic, but can also be used as an argument map for any kind of dialogue. Schopenhauer, however, initially reads these diagrams in a very specific way, namely for their use in eristic. Figure 3 is supposed to show, according to Schopenhauer,

> how the conceptual spheres interlock in manifold ways and thus give room for arbitrariness to pass from each concept to this or that other. [...] I have chosen the concept of travel as an illustrative example. Its sphere reaches into the area of four others, from each of which the persuader can pass over at will: these reach again into other spheres, some of them at the same time into two and more, through which the persuader takes his way at will, always as if it were the only one, and then finally, depending on his intention, arrives at good or evil. [20]



(a) (b)

Fig. 3. Schopenhauer's Argument Maps: (a) taken from *Berlin Lectures*, StB PK, Na 50, NL Schopenhauer, 1428, Bl. 170 (urn:nbn:de:hebis:30:2-417557); (b) taken from Schopenhauer's hand copy of *The World as Will and Representation*, § 9, Fondation Martin Bodmer, S. 73 (urn:nbn:de:hebis:30:2-259336).

Schopenhauer explains that the eristic diagram describes how a possible dialogue partner **P** starts from the term in the centre and then uses several term connections, usually represented by `RD5`, to finally arrive in the periphery, i.e., on the far left or right of the diagram. Once the other dialogue partner **Q** has accepted this path, **P** can conclude that the term in the centre is a component of the periphery term. Let us take Fig. 3 again as an example: **P** wants to argue that travel is something evil. So he uses multiple `RD5` as a path from 'travelling' to 'evil'. If **Q** has accepted this, **P** can conclude that traveling is something evil. **P** thus presents the relation of travel and evil in the conclusion as `RD2`, whereas, according to Fig. 3, it is actually both terms that are connected only by `RD5`. (We will take up this example again in Sect. 5 and then see that graph theory offers us many possibilities to describe and analyse this example more precisely.)

The few interpreters of this diagram mentioned above seem to share this interpretation. However, it is problematic that numerous `RD3` appear in Fig. 3, which only in a few cases make sense from the logical perspective or often even seem irritating. In logic, RD3 (and RD4) indicate contrary relations between two terms or classes (and RD6 shows contradictory relations [6]). However, this does not make sense for all RD3 in Fig. 3, so RD3 have little crucial meaning in eristic: True, terms such as good and bad are shown to be logically correct in RD3 because they are contrary terms. However, most of the terms that stand between the middle term and the peripheral terms within a sequence of RD5 are usually not contrary [15, sect. 5]: In Fig. 4, for example, we see several RDs in a section of the diagram, but terms such as 'profitable' and 'good' are not usually taken as being contradictory.

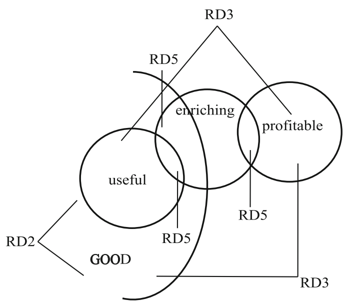


Fig. 4. RD2, RD3, and RD5 in the top left area of Fig. 3b

3 Graph Theory

We have seen in previous section that Schopenhauer established six fundamental relation diagrams, RD (Fig. 1) in logic. In eristic, we find at least three RDs again. However, it turned out that RD2 and RD3 were problematic and one would have to either clarify their meaning or ignore them altogether in the eristic diagram. If they are ignored, only a series of RD5s is relevant, which seem to make up the core idea of the diagram. Now, however, one can argue that if usually only RD5 in Fig. 3 is important, then perhaps one can get a clearer idea of Fig. 3 by ignoring the circles altogether and interpreting all RD5 as edges and vertices. That is, one turns what appear to be Euler-type eristic diagrams for n-terms into a graph. This will indeed be discussed in more detail in Sect. 4 (and we can anticipate that we will later argue for linking diagrams and graphs together). However, in order to make such a graph-theoretic interpretation of Fig. 3, we revisit certain important graph theoretic notions that we need for our interpretation in Sect. 4. In the following we define most of these notions in a much simpler way than their actual mathematical definition. Graph can be defined as

> an ordered triple G = (V(G), E(G), I_G), where V(G) is a nonempty set, E(G) is a set disjoint from V(G), and I_G is an "incidence" relation that associates with each element of E(G) an unordered pair of elements (same or distinct) of V(G) [1].

The sets, V(G) and E(G) are called 'Vertex set' and 'Edge set' respectively. We write $I_G(e) = \{u, v\}$, when the edge 'e' is connected by the two vertices 'u' and 'v'. Here, u and v are called the 'end vertices' of the edge e. A 'degree' of a vertex is basically the number of edges incident on it. Two vertices are called 'adjacent' if and only if they are end vertices on an edge. Two edges are called 'adjacent' if and only if they have a common end vertex.

A 'path' is defined as an alternating sequences of vertices and edges where neither edges nor vertices appears more than once. A graph G is said to be 'connected' if for every pair of vertices in G there is at least one path between them. Otherwise, G is said to be a 'disconnected' graph. Subgraph is defined as follows:

> A graph H is called a subgraph of G if $V(H) \subseteq V(G)$, $E(H) \subseteq E(G)$; and I_H is the restriction of I_G to E(H). If H is a subgraph of G; then G is said to be a supergraph of H: A subgraph H of a graph G is a proper subgraph of G if either $V(H) \neq V(G)$ or $E(H) \neq E(G)$ [1].

For example, in Fig. 5, $V(G) = \{v_1, v_2, v_3, v_4\}$ and $E(G) = \{e_1, e_2, e_3, e_4\}$ are the vertex set and edge set of the graph G respectively. Here, $I_G(e_1) = \{v_1, v_2\}$, $I_G(e_2) = \{v_1, v_3\}$ and so on. The degree of the vertices v_1 and v_3 is two, whereas, the degrees of the vertices v_2 and v_4 are three and one respectively. Except for v_1 and v_4, every other vertices is adjacent to one another. The two edges e_2 and e_4 are not adjacent. Rest of the edges are adjacent to one another. One example of a path in G is $v_1e_1v_2e_4v_4$. Graph G is a connected graph as for every pair of vertices $\{v_i, v_j\}$ $(1 \leq i \leq 4, 1 \leq j \leq 4$ and $i \neq j)$, there exist a path between them. Graph H is a subgraph of G [see Fig. 5].

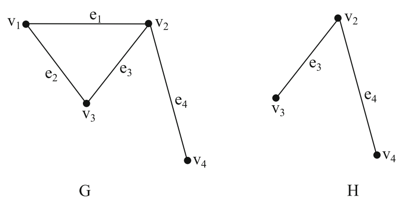


Fig. 5. Example of Graph and Subgraph

The Graph G in Fig. 5 is an undirected graph, where the incidence relation $I_G(e_k)$ associates the edge e_k to an unorderderd pair of vertices (v_i, v_j). For a 'directed graph', the incidence relation associates every edge onto some 'ordered pair' of vertices. In directed graph, every edge is represented by an line segment with an arrow to from one vertex to another vertex. In a directed graph, a 'source vertex' is the vertex where the number of incoming edges is zero and a 'sink vertex' is the vertex where the number of outgoing edges is zero. For example, Fig. 6, represents a directed graph, where v_1 is the source vertex and v_2 and v_3 are both sink vertices.

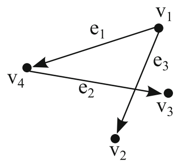


Fig. 6. Example of directed graph

4 Interpretations and Discussion

As shown in Sect. 2, Schopenhauer gave little information on how to interpret Fig. 3. Since there was no graph theory in the early 19th century either, Schopenhauer could not provide any precise statements about it. There are probably many ways of interpreting Fig. 3 in terms of graph theory. For example, three criteria such as (1) directed/ undirected graph, (2) connected/ unconnected graph, (3) display of all RDs/ display only RD5, result in 6 possible graph-theoretical interpretations. In the following, we will introduce only two interpretations (I), which we will then discuss. We cannot present these two interpretations in every detail either, but we only want to clarify certain aspects for the reader in order to awaken an understanding of how we combine the Euler-type diagrams and graph in the next chapter. We have chosen the following two interpretations as we think they are the most suitable to be applied. As envisaged in Sect. 3, only RD5s will be considered as showing dialogue transition.[1]

(I1) The first interpretation assumes that the concepts are the vertices and the edges connect the concepts with each other. Figure 3 shows almost only RD5 and in RD5s, curves represents concepts and their intersection represents the relation or connection between the concepts. Similarly, in a graph, edges acts like the intersection of the curves as it is also connects the concepts that are represented by vertices. Since ‘travelling’ is the source vertex and ‘good’ and ‘evil’ are the sink vertices, this results in a connected directed graph, as shown in Fig. 7.

(I2) In the second interpretation, we assume that the vertices are represented by the intersections of RD5 and edges connects these vertices with one another. Here we have four source vertices which we obtained by the intersection of the circle ‘travelling’ with four adjacent conceptual spheres, namely ‘healthy’, ‘expansive’, ‘ample opportunity for storing experience’ and ‘dispelling boredom’. This interpretation results in a disconnected directed graph, as shown in Fig. 8. Here, each edge and vertex are traversed only once for a single path.

Both interpretations assume a directed graph, since there is a source vertex and several possible sink vertices, but (I1) and (I2) differ in whether the graph is

[1] In the following we use the graph-theoretical labels v and e only if we directly refer to the graphs and not to the RDs.

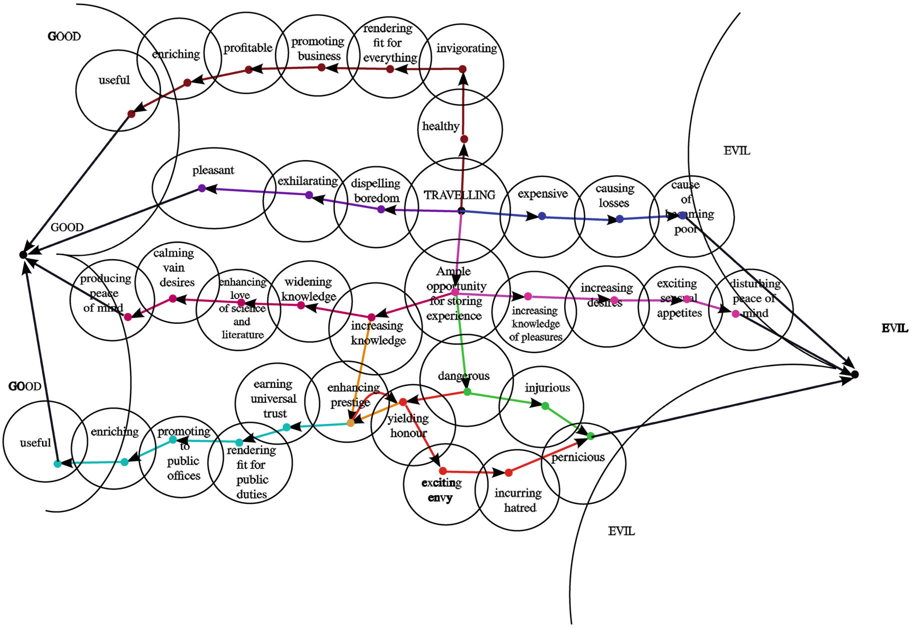


Fig. 7. (I1)

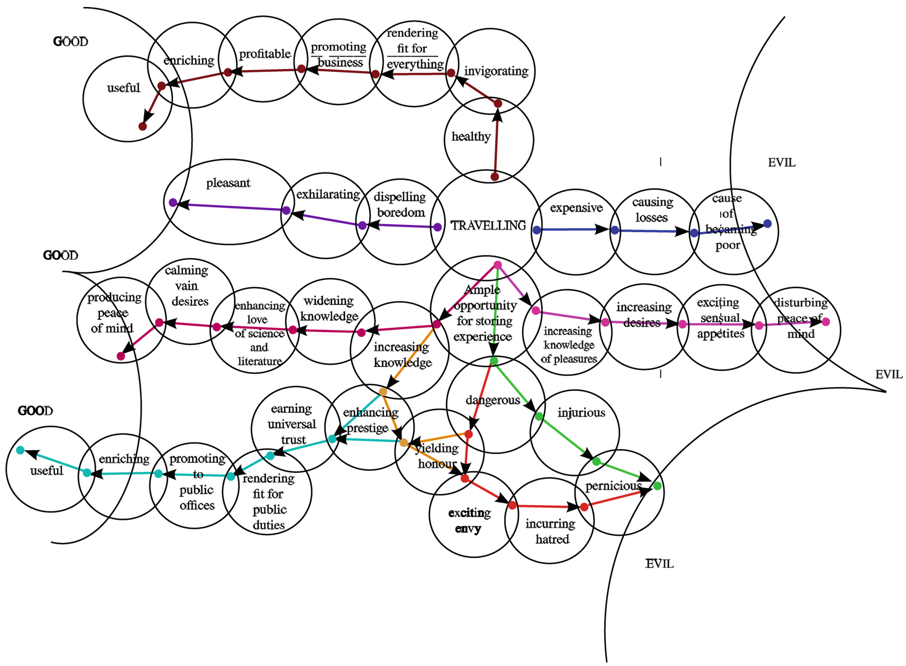


Fig. 8. (I2)

connected or not. Each of the two interpretations could have certain advantages and disadvantages, which might even vary depending on the application of Schopenhauer's eristic diagram.

In the following, we would like to present and discuss some possible advantages (A) and problems (P) of (I1) or Fig. 7 and (I2) or Fig. 8 in order to represent a dialogue.

(A1) (I2) seems to highlight the propositions of arguments under discussion (whereas (I1) put more emphasis on the concepts). The reason is that in the approach mentioned in Fig. 8 takes each intersection of the circles as the vertex of the graph. Now as we have argued in Sect. 2, a circle in this n-term diagrams represent a 'concept' not a propositions of an argument or a particular dialogue. An argument is only represented when two circle have a specific relation represented through a `RD` diagram. Thus, one can claim that Fig. 7 fails to represent the transition of dialogue which is the primary motive of this n-term diagram. In sum, Fig. 7 seems to be more suitable for concept maps [5], but Fig. 8 seems to be better for depicting a dialogue as in argument maps.

(A2) In (I2) or Fig. 8, there are four vertices (coloured as brown, blue, violate and pink) which are not connected with each other and from each of these four vertices generates four subgraphs of the main one which are not connected to one another. In a dialogue, for example, **P** have at least four different ways to convince **Q** whether 'travelling' is either good or bad. If suppose **P** beliefs that travelling is bad then **P** can consider the subgraph Fig. 9 which is the shortest path from 'travelling' to 'evil'. The disconnected graph (I2) thus shows the possibilities of having different opinions in the form of subgraphs better than the connected graph (I1) does.

(A3) Each path in (I2) could immediately indicate the direction, i.e. whether the path leads from 'traveling' to 'good' or to 'evil'. Thus, already at the first transition, the path of the dialogue would be clearly foreseeable. If Fig. 8 were used as an argument map, this would have the advantage that the course of the dialogue would be recognizable by its direction: In Fig. 8, the brown and green path are neutral at first, since they only go up or down and only approach good or evil later. But even with this advantage, as soon as a path turns to the right or to the left (e.g. as the pink one), the neutrality is removed and an 'ethical value' (good or evil) of the depicted argument arises. This is one of the problem of (I2), that will be discussed in the following.

(A4) Both (I1) and (I2) have a great advantage over the diagrams (Fig. 3) because the directed graphs can accurately represent the flow of the dialogue. They show the beginning and the end of a series of propositions or arguments. However, as mentioned in (A1), the (I1) graph does not show the transition accurately. Therefore, (I2) has advantages over (I1). Moreover, the definitions in graph theory allow a more precise description of individual elements than do the diagrams.

But there are also certain problems that accrue to one or both interpretations, which we would now like to address.

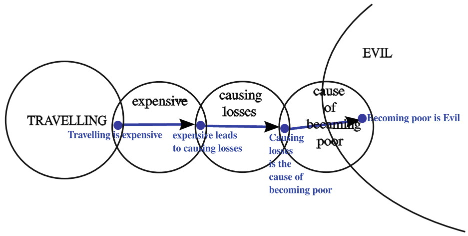


Fig. 9. Example of dialogue transition to show ‘Travelling is bad’

(P1) We have to keep the following in mind. If dialogue transition strictly depends on RD5 diagrams, we cannot connect two vertices by an edge unless the circles where these vertices lies are connected by RD5. For example, in Fig. 10, the vertex v_1 can be connected with vertex v_2 as the circle C_1 is in RD5 relation with the circle C_2. But we cannot connect v_1 with v_3 as C_1 is not in a RD5 relation with the circle C_3 but in a RD3 relation. Of course, this reduces the entire diagram to only one RD, which means that the expressivity is not very high. One can even argue that the original diagram (Fig. 3) shows more possible arguments than (I1) or (I2).

(P2) However, if one wanted to try to solve (P1) graph theoretically, one would run into a new problem. If we imagine a connected graph in which all RDs are entered, the expressivity is similar to Fig. 3, but the graph would be very confusing. We would have a network of numerous RD3s and RD5s that would be almost impossible to trace. Although one could introduce RD3s into graphs by a rule, e.g. that all vertices that are not directly connected by an edge map an RD3, this would only be implicit information. The expressivity of the original diagram thus seems to be higher than one of the graph-theoretical interpretations.

(P3) As noted above, (I2) bears most resemblance to an argument map as used today in many different variations in fields such as critical thinking, argumentation theory, argument mapping etc. [25], [19]. Overall, however, there are unfortunately numerous points that (I2) do not fulfil and which are also important for Schopenhauer’s eristic as well as for most argument maps today: the graphs show arguments, but it do not show which dialogue partner made the argument and how another reacted to it. The graph also does not show the interchange of RDs that was discussed in Sect. 2. The graph also does not show a counter-argument, e.g. an attack by another dialogue partner. In some cases, it is already sufficient to use the diagrams from Sect. 2 with the graph, but in other cases more diagrammatic elements must be used to meet all requirements.

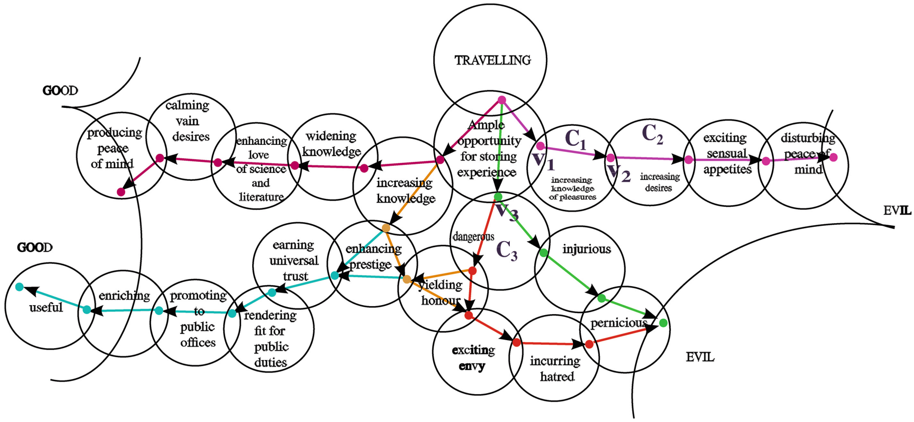


Fig. 10. Importance of RD5 in eristic

5 An Example of a Controversy

Schopenhauer's n-term diagrams can be interpreted in terms of graph theory, as we have seen in Sect. 4. This gives clearer possibilities of description by the definitions mentioned in Sect. 3 as well as some advantages mentioned in Sect. 4.

Nevertheless, the graphs discussed in Sect. 4 also have disadvantages, which concern expressivity, for example. If Schopenhauer's diagrams were to be completely replaced by graphs, as argued in Sect. 3, there would be some advantages, but also some disadvantages and problems, which would ultimately lead potential users to use graph systems that are already established in the field of argumentation, e.g. Toulmin, Scriven, Dung maps, etc. [19]. Our goal should therefore be to combine the best of both worlds and to adapt the graphs and diagrams in such a way that they are well-suited for the respective purpose.

The n-term diagrams were actually intended to be applied to Schopenhauer's own treatises on eristic. Nevertheless, the diagrams and graphs of eristic can also be applied in many other areas of human-to-human or human-to-machine interaction [19]. In this section, we will stay in the field of human agents and try to *represent* a fictitious controversy with Schopenhauer's diagrams and graphs. (However, it should be taken into account that one can also *analyse* or even *plan* possible arguments with Schopenhauer's diagrams and, on the other hand, other areas such as political debates, sales talks, negotiations, legal pleadings can be represented with the help of Schopenhauer's eristic.)

In our fictitious controversy, however, we stay in eristic with the topic of 'travelling' using Schopenhauer's example. Thus, we take up the fictional dialogue between **P** and **Q** already announced in Sect. 2, in which **P** wants to argue that travelling is something evil. The dialogue could go as follows:

Q	$\mathcal{K}1$	Dear P, what do you actually think about travelling?
P	$\mathcal{K}2$	I'd like to tell you. Travelling gives you plenty of opportunities to store experience.
Q	$\mathcal{K}3$	You could say that.
P	$\mathcal{K}4$	But experiences can also be dangerous.
Q	$\mathcal{K}5$	Well...
P	$\mathcal{K}6$	And everything that is dangerous is also injurious.
Q	$\mathcal{K}7$	No, I have to disagree. For one thing, it has nothing to do with travelling, and for another, not everything that is dangerous is also injurious. Dangerous experiences can also bring honour, and that is not injurious.
P	$\mathcal{K}8$	Yes, I agree with you. But this honour can also cause envy, so that you incur hatred.
Q	$\mathcal{K}9$	That is possible, of course.
P	$\mathcal{K}10$	If you incur hatred, that is something pernicious, and so travelling is an evil.

The entire dialogue consists of 10 actions ($\mathcal{K}$), whereby not every action represents an argument: $\mathcal{K}1$ is a question, $\mathcal{K}3$ and $\mathcal{K}5$ are agreements. On the other hand, in some cases there are several arguments in one action: whereas $\mathcal{K}2$, $\mathcal{K}4$, $\mathcal{K}6$ represent only one argument, $\mathcal{K}7$, $\mathcal{K}8$, $\mathcal{K}10$ each contain several arguments $(a, b, c, \ldots)$.

$\mathcal{K}7$ even plays a special role overall: here an attack or counter-argument is found. **Q** does not initially accept **P**'s argument in $\mathcal{K}6$. **Q** notices that **P** could have the intention to connect 'travelling' with something evil. Therefore, **Q** anticipates such an argument $\mathcal{K}7a$ (For one thing,...), excludes it, and negates $\mathcal{K}6$ in $\mathcal{K}7b$ explicitly (and for another...). As a counter-argument, **Q** falls back on $\mathcal{K}4$, which **Q** still accepted in $\mathcal{K}5$, and turns it to the positive, i.e. $\mathcal{K}7c$ (Dangerous experience can also bring honour). At the same time, **Q** uses $\mathcal{K}7c$ to refer to `RD3` between 'bringing honour' and 'is injurious', i.e. $\mathcal{K}7d$.

In $\mathcal{K}8$, **P** recognises the chance that the positive argument put forward by **Q** in $\mathcal{K}7c$ can still lead to the goal, even though **Q** has rejected $\mathcal{K}6$. In order not to give **Q** too much leeway for the new argument $\mathcal{K}7c$, **P** turns it to the negative, $\mathcal{K}8a$ (But this honour can), and immediately connects it with the next argument, $\mathcal{K}8b$ (so that you...), which is presented as a consequence. **Q** seems to have been caught off guard by this in $\mathcal{K}9$. **Q** at least admits that $\mathcal{K}8$ is possible.

This then allows **P** to present a series of arguments in $\mathcal{K}10$, i.e. $\mathcal{K}10a$, $\mathcal{K}10b$, which finally appears as a consequence of the whole controversy and also as an answer to $\mathcal{K}1$: travelling is an evil. Should **P** have the last word with $\mathcal{K}10$ in the dialogue and if **Q** not contradict, the conclusion ($\mathcal{K}10b$) should be accepted by both.

Let us look again at the transition from $\mathcal{K}6$ to $\mathcal{K}7$. What is expressed here is what we called the interchange of `RD`s in Sect. 2. This concerns the transition between 'dangerous' and 'injurious', which is evaluated differently by **P** and **Q**, which is why the controversy comes to a head here: **P** argues in $\mathcal{K}6$ that the transition between 'dangerous' and 'injurious' is justified. **P**'s argument is even so strong that it can be seen as an exaggeration: **P** makes an `RD2` out of the

RD5 between the two terms; for if everything that is dangerous is injurious, then 'dangerous' is also completely contained in 'injurious'. But **Q** does not accept this transition: **Q** points out that there are dangerous experiences that are not harmful and gives a counterexample that even constructs an RD3 argument.

This illustrates the interchange of RDs that expresses between the two speakers regarding a particular argument. Since we have chosen our example in such a way that **P** intentionally wanted to deceive **Q** with $\mathcal{K}6$, i.e. an intentional interchange from RD5 to RD2 was intended by **Q** in order to quickly support the main argument (travelling is something evil), the fictional dialogue can be taken as an example of eristic.

Our aim in this section, however, is now to apply the diagrams and their graph-theoretical interpretations to represent the dialogue just presented. To represent this dialogue, $\mathcal{K}1 - \mathcal{K}10$, we now use Schopenhauer's original diagram, Fig. 3, which represents the RDs, and an overlying subgraph of (I2), which is to represent the concrete course in the diagram. The result is Fig. 11 Here the broken line represent the path taken by **P** and the straight line represent the path taken by **Q**. We thus see in Fig. 11 two argument transitions: first the path that **P** takes, but which ends at 'dangerous' and 'injurious' without having reached the goal. The second path then continues via **Q**'s argument until **P** reaches the sink node at 'evil'[2]. The argument $\mathcal{K}7c$ remains implicit in Fig. 11, but could be supplemented by further diagrammatic elements.

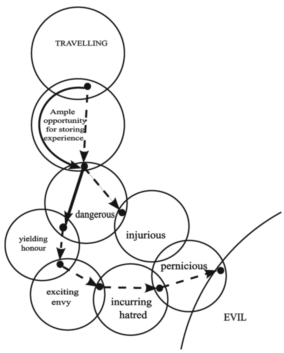


Fig. 11. Dialogue Graph

This connection of diagram and subgraph should enable a reader to read out a fictitious dialogue from Fig. 11 which, although it does not correspond to the flow of words of $\mathcal{K}1 - \mathcal{K}10$, can at least reproduce the arguments, i.e. the dialectical essence of the controversy.

[2] The first step in Fig. 11 have both dotted and straight lines as P and Q both agrees on the argument 'travelling' is 'ample opportunity for storing experience', viz. $\mathcal{K}2$ and $\mathcal{K}3$.

6 Summary and Outlook

In this paper, our aim has been to develop a graph-theoretical interpretation of the Eulerian diagrams that Schopenhauer uses in eristic and to combine the advantages of both. In doing so, we have found that there are numerous ways in which Schopenhauer's diagrams can be read and also how they can be used. While Schopenhauer primarily had application in eristic in mind, however, the diagrams can initially only show possibilities of dialogue progressions. We have understood these possible dialogues as subgraphs of a main graph, which can describe the structure of the diagrams more precisely than the diagrams do. Nevertheless, we have also seen that the Euler-type diagrams have the advantage of displaying numerous relations between terms and arguments that would no longer be intuitively understandable in complex graphs or networks.

Having explained Schopenhauer's diagrams in Sect. 2, defined the basic graph-theoretical terms in Sect. 3 and presented some possible graph-theoretical interpretations of the diagrams in Sect. 4, we have presented in Sect. 5, using an exemplary controversy, how graph and diagram can be combined to represent the course of conversation. However, numerous other applications in the field of human-human or human-machine interaction are conceivable with the help of this technique: Pointing out alternative or counterfactual arguments, strategically planning the course of arguments, analysing possible false conclusions, etc. This versatility is likely to be particularly applicable in areas where arguments play a central role in communication, such as law, politics, commerce, the sciences. In this context, Schopenhauer's eristic diagrams occupy a special position to all argument maps known so far: they combine the intuitive advantages of graphs with those of Euler-type diagrams. Moreover, their interpretation possibilities and extensions are numerous, so that one can adapt the diagram graphs depending on the field of application.

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